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Interest Rate Smoothing and Price Level Trend-Stationarity*

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Abstract

This paper discusses the definition and mechanics of central bank interest rate smoothing under rational expectations. A tension arising between interest rate smoothing and macroeconomic stabilization objectives induces non-trend-stationary price level and money stock behavior. The paper thereby helps explain why such nominal non-stationarities are widely observed. Further implications are drawn for base drift, distribution of real returns on long-term fixed-rate nominal debt, and operating characteristics of interest rate pegs and policy instruments.

1. Introduction

For industrial countries in the post-war period, the price level and the money stock have displayed little tendency to revert to given growth paths.[1] Indeed, this stylized fact is frequently referred to by monetarist critics of central banks, who point out that periods of temporarily high or low money growth, rather than being subsequently reversed, typically alter the level of the money stock and prices permanently.

Why should such a money supply rule be optimal from the standpoint of central banks and consequently be widely observed? This paper sees the answer in the interaction of price level and nominal interest rate smoothing policies commonly practiced by the world's central banks. Given their responsibility for macroeconomic stabilization, central banks regard price level instability as costly. As custodians of the financial system, central banks cushion nominal interest rates against economic shocks. The paper analyzes a central bank seeking to smooth price level and nominal interest rate movements occasioned by transitory disturbances to money

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[1] Using formal statistical procedures, in a sample of industrialized countries in the post-war period Wasserfallen (1986) cannot, in general, reject the hypothesis that monthly consumer price level and M1 money stock data have no tendency to revert to given growth paths.

demand, aggregate supply, and the real interest rate. The tension that arises between these objectives induces non-trend-stationary stochastic processes for the price level and the money stock, which is the modern time-series characterization of the previously mentioned stylized fact. Basically, it is desirable for the central bank to regard past money growth, in part, as 'bygones' so that the money stock and the price level wander over time without any tendency to return to given growth paths.

The organization of the remainder of the paper is as follows. In section 2, a simple rational expectations macromodel is laid out and a class of policy rules is discussed which contains both trend-stationary and non-trend-stationary processes. Central bank preferences for price level and interest rate smoothing are motivated in section 3. In section 4, the model is solved assuming that the central bank pursues price level smoothing objectives alone. Section 5 discusses monetary policy with interest rate smoothing. Issues of definition and mechanics are explored in section 5.1. Section 5.2 demonstrates the ‘optimality’ of a non-trend-stationary money supply rule with interest rate smoothing. Other implications of interest rate smoothing are discussed in section 6. A brief summary concludes the paper.

2. The macromodel

The analysis is conducted in a simple macromodel. The model includes a goods market equilibrium-interest rate arbitrage condition:

$$r_t = a_0 + \underset{t}{\mathrm{E}}\, p_{t+1} - p_t + q_t, \tag{1}$$

where

$\mathrm{E}_t p_{t+1}$ ≡ period t mathematical expectation of the log period $t+1$ price level,

r_t ≡ period t nominal interest rate,

p_t ≡ log period t price level,

q_t ≡ serially uncorrelated, zero mean, real interest rate disturbance,

a_0 > 0.

Eq. (1) is a zero profit arbitrage condition requiring that the nominal interest rate minus expected inflation equal the goods market clearing real interest rate. The sum $a_0 + q_t$ represents the period t ex ante real rate of interest that clears the period t goods market. The q_t term is intended to capture random real interest rate disturbances associated with goods market clearing.

The model includes a money demand function:

$$m_t^{\mathrm{D}} - p_t = a_1 + a_2 r_t + a_3 y_t + v_t, \tag{2}$$

where

m_t^{D} ≡ log nominal period t money stock demanded,

$y_t \quad \equiv$ log period t real income,

$v_t \quad \equiv$ serially uncorrelated, zero mean, money demand disturbance, $\sigma_{qv} = 0$,

$a_1 \quad > 0, a_2 < 0$, and $a_3 > 0$.

Real income is generated by a typical 'surprise' aggregate supply function:

$$y_t = \bar{y} + h\left(p_t - \underset{t-1}{\mathrm{E}} p_t\right) + w_t, \tag{3}$$

where

$p_t - E_{t-1}p_t \equiv$ period t price level forecast error,

$\bar{y} \quad \equiv$ mean log of real income,

$w_t \quad \equiv$ serially uncorrelated, zero mean, aggregate supply disturbance, $\sigma_{qw} = \sigma_{uw} = 0$,

$h \quad \geq 0$.

Eq. (3) captures the effect of nominally denominated labor contracts which may convert price level forecast errors into aggregate real income fluctuations.

Finally, the model includes a simple money supply rule which allows the central bank to choose whether to make the money stock trend-stationary or not:

$$m_t^{\mathrm{S}} = m_{t-1} + \theta_1\left(r_t - \underset{t-1}{\mathrm{E}} r_t\right) - \theta_2\left(m_{t-1} - \underset{t-2}{\mathrm{E}} m_{t-1}\right), \tag{4}$$

where

$m_t^{\mathrm{S}} \quad \equiv$ log nominal period t money stock supplied,

$m_{t-1} \equiv$ realized log nominal period $t-1$ money stock.

The rule includes two policy parameters that the central bank can choose independently. The first, θ_1, describes the contemporaneous money stock response to an interest rate innovation. The second, θ_2, describes the extent to which the contemporaneous money stock response to an interest rate innovation is offset in the following period. The money supply rule is trend-stationary, i.e., the offset is exact, if and only if $\theta_2 = 1$. Formally, when $\theta_2 = 1$ the money supply rule becomes $m_t = \mathrm{E}_{t-2}m_{t-1} + \varepsilon_t$, which has the trend-stationary solution $m_t = \bar{m} + \varepsilon_t$, where ε_t, is white noise and $\bar{m}$ is a constant equal to an initial condition on $\mathrm{E}_{t-2}m_{t-1}$. The rule is clearly not trend-stationary if $\theta_2 \neq 1$. Note that the unit coefficient on the m_{t-1} term in (4) allows the central bank to make the money stock non-trend-stationary in the simplest sensible way. Coefficients on m_{t-1} that are inside the unit circle automatically imply convergence to a fixed trend, in this case with zero growth, while those outside the unit circle imply explosive money growth.

The money market equilibrium condition closes the model:

$$m_t^S = m_t^D. \tag{5}$$

Expectations are assumed to be formed rationally. For the monetary authority, expectations are conditioned on information set $I_t = \{r_t, r_{t-1}, ...; m_t, m_{t-1}, ...; p_{t-1}, p_{t-2}, ...;$ and $y_{t-1}, y_{t-2}, ...\}$. Individual information sets include observations on current local prices and incomes as well. However, given the model specification, this individual information advantage plays no role in the model's solution.

The system of eq. (1) through (5) determines p_t, r_t, m_t, and y_t each period as functions of $q_t, u_t \equiv v_t + a_3 w_t, m_{t-1} - E_{t-2} m_{t-1}, m_{t-1}$, and the parameters $a_0, a_1, a_2, a_3, \bar{y}, h, \theta_1$, and θ_2. Quasi solution functions or generating processes for p_t, r_t, m_t, and y_t are

$$\begin{aligned} p_t = \quad & -(a_1 + a_0 a_2 + a_3 \bar{y}) + \left[\frac{a_2 - \theta_1}{a_2 - \theta_1 - (1 + a_3 h)\big(1 - \theta_1(1 - \theta_2)\big)}\right] q_t \\ & + \left[\frac{1 - \theta_1(1 - \theta_2)}{a_2 - \theta_1 - (1 + a_3 h)\big(1 - \theta_1(1 - \theta_2)\big)}\right] u_t \\ & - \theta_2 \left(m_{t-1} - \underset{t-2}{\mathrm{E}} m_{t-1}\right) + m_{t-1}, \end{aligned} \tag{6}$$

$$\begin{aligned} r_t = a_0 & + \left[\frac{-(1 + a_3 h)}{a_2 - \theta_1 - (1 + a_3 h)\big(1 - \theta_1(1 - \theta_2)\big)}\right] q_t \\ & + \left[\frac{-1}{a_2 - \theta_1 - (1 + a_3 h)\big(1 - \theta_1(1 - \theta_2)\big)}\right] u_t, \end{aligned} \tag{7}$$

$$m_t = m_{t-1} + \theta_1 \left(r_t - \underset{t-1}{\mathrm{E}} r_t\right) - \theta_2 \left(m_{t-1} - \underset{t-2}{\mathrm{E}} m_{t-1}\right), \tag{8}$$

$$y_t = \bar{y} + h \left(p_t - \underset{t-1}{\mathrm{E}} p_t\right) + w_t.^{2} \tag{9}$$

Note that since the money demand level-specified disturbance is transitory, aggregate real money balances are trend-stationary. This implies that the price level is trend-stationary if and only if the nominal money stock is.

3. Central bank preferences

The choice of policy parameters θ_1 and θ_2 is determined by central bank preferences for price level and interest rate smoothing. Central banks prefer smooth price level movements in two senses. As described in eq. (3), price level forecast errors may have destabilizing employment effects. In addition, price level forecast errors can have potentially destabilizing wealth redistribution effects associated with nominally denominated credit market contracts.

Central banks appear to prefer minimal price level forecast error to protect against both employment and credit market instabilities. To minimize distortions arising from imperfect

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[2] The generating processes are derived by the method of undetermined coefficients.

indexation of nominally denominated contracts, and expenditure on indexation itself, central banks also appear to prefer minimal variability of expected inflation.

Central banks smooth nominal interest rates to maintain ‘orderly money markets.’ Interest rate smoothing minimizes financial market stress due to interest rate prediction errors and associated surprise wealth redistributions. As custodians of the financial system, central banks appear to prefer smooth interest rates to minimize unexpected asset price movements that raise the risk of bankruptcies and banking crises.

In this paper, absence of q and u serial correlation and preference for minimal expected inflation variability make it optimal for the central bank to generate serially uncorrelated expected inflation and nominal interest rates. That is why the IMA(1,1) restriction on the money supply rule is optimal. It follows for the nominal interest rate that forecast error variance is equivalent to unconditional variance, so a nominal interest rate smoothing objective is equivalent to an objective for $\mathrm{var}[r_t - a_0]$.

4. Monetary policy with price level smoothing objectives alone

In order to provide a benchmark against which to judge the effects of interest rate smoothing, this section characterizes monetary policy with price level smoothing objectives alone. In this case, θ_1 and θ_2 are chosen to minimize price level forecast error variance, $\mathrm{var}[p_t - \mathrm{E}_{t-1}p_t]$, and the variances of expected inflation, $\mathrm{var}[\mathrm{E}_t p_{t+1} - p_t]$. Using eq. (6), the price level forecast error can be written as

$$p_t - \underset{t-1}{\mathrm{E}} p_t = [1 - (1 + a_3 h)A]q_t - Au_t, \tag{10}$$

where

$$A \equiv \frac{\theta_1(1-\theta_2) - 1}{a_2 - \theta_1 - (1 + a_3 h)\big(1 - \theta_1(1-\theta_2)\big)}.$$

The value of A that minimizes the price level forecast error variance is

$$A^* \equiv \frac{(1 + a_3 h)\sigma_q^2}{(1 + a_3 h)^2\sigma_q^2 + \sigma_u^2}, \tag{11}$$

where σ_q^2 and σ_u^2 are the real interest rate disturbance variance and the composite money demand-aggregate supply disturbance variance, respectively.[3]

Using eqs. (6) and (8), expected inflation may be written as

[3] Second-order conditions are satisfied throughout the paper.

$$\underset{t}{\mathrm{E}}p_{t+1} - p_t = -[1 + (1 + a_3 h)B]q_t - Bu_t, \tag{12}$$

where

$$B \equiv \frac{1}{a_2 - \theta_1 - (1 + a_3 h)\big(1 - \theta_1(1 - \theta_2)\big)}.$$

The value of B that minimizes the variance of expected inflation is

$$B^* \equiv \frac{-(1 + a_3 h)\sigma_q^2}{(1 + a_3 h)^2 \sigma_q^2 + \sigma_u^2} \tag{13}$$

The values of θ_1 and θ_2 that satisfy conditions (11) and (13) are

$$\theta_1^* = a_2 + \frac{\sigma_u^2}{(1 + a_3 h)\sigma_q^2} \quad \text{and} \quad \theta_2^* = 1. \tag{14}$$

The value θ_1^* represents the optimal contemporaneous money stock response to a nominal interest rate innovation. In Poole's (1970) terminology, a zero θ_1^* is a pure money stock policy. Poole's pure interest rate policy, i.e., a ‘peg’, could be optimal here if the variance of the composite money demand aggregate supply disturbance vastly exceeded the real interest rate disturbance variance. In general, however, θ_1^* is neither zero nor infinite, so that a partial money stock response to a contemporaneous interest rate innovation is optimal. In Poole's words, some combination policy is generally called for.

The interesting feature of the optimal value θ_2^* is that it is unity, regardless of the relative size of the real interest rate and money demand disturbance variances. If it is optimal to generate contemporaneous money stock responses to interest rate innovations, i.e., if $\theta_1^* \neq 0$, then targeted future money growth should respond so that money stock innovations are expected to be offset exactly in the following period. In other words, for monetary policy with price level smoothing objectives alone, the optimal price level and money stock generating processes are trend-stationary.

5. Monetary policy with interest rate smoothing

5.1. Definition and mechanics

As mentioned in section 3, a nominal interest rate smoothing objective is equivalent to an objective for the nominal interest rate variance. The central bank can attain any degree of interest rate smoothing by using (7) to calculate the set of (θ_1, θ_2) pairs that achieves the desired nominal interest rate variance, and then choosing the (θ_1, θ_2) pair from that set. The degree of interest rate smoothing should be understood to be zero if the desired interest rate variance matches the variance attained when $\theta_1 = \theta_1^*$ and $\theta_2 = \theta_2^*$. This definition seems sensible because policy characterized by the (θ_1^*, θ_2^*) pair makes nominal interest rate movements correspond as closely as possible to real interest rate movements.[4] The central bank is said to be engaged in interest rate smoothing if $|B^*| > |B|$, where B^* is the value of B associated with (θ_1^*, θ_2^*). The degree of interest rate smoothing is inversely related to $|B|$ because the nominal interest rate variance falls with $|B|$.

One may note from (7) that price level smoothing objectives alone, yielding $\theta_1^* > 0$ and $\theta_2^* = 1$, reduce the interest rate variance relative to its value when $\theta_1 = 0$, i.e., when the money stock is held constant. From that perspective, it is possible to say that there is already some interest rate smoothing when $|B| = |B^*|$. However, in this context defining interest rate smoothing as a further reduction in variance seems appropriate for two reasons. First, it is only additional smoothing that leads to a non-trend-stationary price level. Second, as shown in section 5.2, the additional smoothing corresponds to the case where interest rate variance itself is a cost in the central bank objective function. It seems natural to reserve the definition of interest rate smoothing for that case.

As is evident from (12), the variance of expected inflation is completely determined once the degree of nominal interest rate smoothing, i.e., B, is chosen. Eq. (12) also makes clear that the central bank faces a tradeoff between the expected inflation variance and the nominal interest rate variance. The inverse relation between the two variances is due to the fact that nominal interest rate smoothing is achieved by creating expected inflation or deflation to offset the effect of real interest rate disturbances on the nominal interest rate.

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4 The monetary authority's conditional expectation of inflation can be expressed as

$$\underset{t}{\mathrm{E}} p_{t-1} - \underset{t}{\mathrm{E}} p_t = -[1 + (1 + a_3 h)B]\underset{t}{\mathrm{E}} q_t - B\underset{t}{\mathrm{E}} u_t$$

where

$$\underset{t}{\mathrm{E}} q_t = -\frac{1}{B}\left[\frac{(1 + a_3 h)\sigma_q^2}{(1 + a_3 h)^2\sigma_q^2 + \sigma_\mu^2}\right]\left(r_t - \underset{t-1}{\mathrm{E}} r_t\right),$$
$$\underset{t}{\mathrm{E}} u_t = -\frac{1}{B}\left[\frac{\sigma_u^2}{(1 + a_3 h)^2\sigma_q^2 + \sigma_u^2}\right]\left(r_t - \underset{t-1}{\mathrm{E}} r_t\right).$$

Substituting for $\mathrm{E}_t q_t$ and $\mathrm{E}_t u_t$ yields

$$\underset{t}{\mathrm{E}} p_{t-1} - \underset{t}{\mathrm{E}} p_t = \left[\frac{[B^{-1} + 1 + a_3 h](1 + a_3 h)\sigma_q^2 + \sigma_u^{2-}}{(1 + a_3 h)^2\sigma_q^2 + \sigma_u^2}\right](r_t - a_0).$$

Finally, substituting the optimal B^* from (13) into the bracket term above makes it zero. In other words, the objective of minimizing the variance of the public's expected inflation, $\mathrm{E}_t p_{t+1} - p_t$, is equivalent to the monetary authority targeting its conditional expectation of inflation at zero. So nominal interest rate movements correspond as closely as possible to real interest rate movements.

To understand the mechanics of interest rate smoothing, consider the special but revealing case where $\theta_2 = 1$, so that the money supply rule is trend-stationary. Any degree of interest rate smoothing can be achieved by setting θ_1 appropriately. So interest rate smoothing per se does not require money stock and price level non-trend-stationarity. However, because A cannot remain equal to A^* in (11) when $\theta_2 = 1$ and $\theta_1 > \theta_1^*$, interest rate smoothing increases the price level forecast error variance when the money supply rule is restricted to be trend-stationary. The key to this result is that money supply rule trend-stationarity anchors the expected future price level at a fixed target. Therefore, the expected inflation or deflation necessary to smooth the nominal interest rate against real interest rate disturbances must be generated by lowering or raising the current price level. Hence, the one-period-ahead price level forecast error variance cannot be minimized. This is the tension, mentioned in the introduction, that induces a central bank to choose a non-trend-stationary money supply rule as described below.

5.2. The 'optimality' of a non-trend-stationary money supply rule

In this section I derive the 'optimal' money supply rule for a central bank that considers price level forecast error and expected inflation variability to be costly, but also views nominal interest rate variability as costly. We may assume that the central bank wishes to minimize the cost function

$$C = \alpha \operatorname{var}[r_t - a_0] + \beta \operatorname{var}\left[p_t - \underset{t-1}{\mathrm{E}} p_t\right] + \gamma \operatorname{var}\left[\underset{t}{\mathrm{E}} p_{t+1} - p_t\right], \tag{15}$$

where $\alpha, \beta, \gamma > 0$. Using (7), (10), and (12) to express the respective variances as functions of the parameters of the model, (15) can be rewritten as

$$\begin{aligned} C = \;& \alpha\left[B^2\left((1 + a_3 h)^2\sigma_q^2 + \sigma_u^2\right)\right] + \beta\left[(1 - (1 + a_3 h)A)^2\sigma_q^2 + A^2\sigma_u^2\right] \\ & + \gamma\left[(1 + (1 + a_3 h)B)^2\sigma_q^2 + B^2\sigma_u^2\right]. \end{aligned} \tag{16}$$

The central bank's problem is to choose θ_1 and θ_2, and thereby A and B, to minimize (16). The values of θ_1 and θ_2 that satisfy first-order conditions for a minimum are

$$\tilde{\theta}_1 = a_2 + \left(1 + \frac{\alpha}{\gamma}\right)\left(\frac{1}{1 + a_3 h}\right)\frac{\sigma_u^2}{\sigma_q^2},$$

and (17)

$$\tilde{\theta}_2 = 1 + \frac{\alpha}{\gamma}\left(\frac{1}{\tilde{\theta}_1}\right)$$

for

$$\tilde{\theta}_1 \neq 0.$$

Inspection of $\tilde{\theta}_1$ and $\tilde{\theta}_2$ in (17) yields two important features of the 'optimal' money supply rule. First,

$$|\tilde{B}| = \left(\frac{\gamma}{\alpha + \gamma}\right)|B^*| < |B^*|,$$

so the optimal rule involves interest rate smoothing. Second, if it is optimal to generate contemporaneous money stock responses to interest rate innovations, i.e., if $\tilde{\theta}_1 \neq 0$, then the optimal rule is not trend-stationary. These features characterize the optimal rule regardless of the relative sizes of α, β, and γ.

Here is an answer to the question posed in the introduction. The tension between price level and interest rate smoothing does, in fact, induce nontrend-stationary processes for the money stock and the price level. Why? Notice that $\tilde{A} = A^*$, so price level forecast error variance continues to be minimized with interest rate smoothing. Basically, it is optimal for central banks to make the expected future money stock and price level respond to current interest rate innovations in order to generate expected inflation and deflation necessary to smooth interest rates without creating current price level surprises.

6. Other implications of interest rate smoothing

6.1. Base drift

Since the Federal Reserve began targeting money growth in 1975, it has accepted 'base drift' in the level of money stock when moving from one targeting period to the next. That is, the Federal Reserve has not adjusted its money growth targets to offset money stock innovations as required to hold the money stock to a predetermined trend target path.[5] Of course, as Walsh (1986) points out, in the presence of permanent shocks to aggregate supply or money demand, base drift would be necessary to achieve price level stationarity. In fact, Walsh defends base drift by showing how a monetary authority could maintain price level stationarity by generating base drift optimally to insulate the price level from permanent shocks. His defense would appear to have little relevance for Federal Reserve monetary targeting since 1975, however, since the Federal Reserve did not generate price level stationarity over the period.

In contrast to Walsh's model, which does not explain price level non-trend-stationarity, the view presented here explains base drift and price level non-trend-stationarity as the joint product of a tension between macroeconomic stabilization and interest rate smoothing objectives.[6] Allowing for permanent shocks to aggregate supply or money demand in my model

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5 Broaddus and Goodfriend (1984) document and analyze base drift in detail. Base drift is also discussed in Council of Economic Advisers (1985 pp. 53-54) and in Walsh (1986).

6 The Federal Reserve employed an adjustable Federal funds rate peg, i.e., a funds rate instrument, to smooth interest rates during the 1970 s. As Goodfriend et al. (1986) document in detail, the Federal Reserve continued to smooth interest rates during the non-borrowed reserve targeting period running from October 1979 to the Fall of 1982. Since then, policy has been implemented through borrowed reserve targets and discount rate adjustments, procedures which use a kind of noisy Federal funds rate instrument.

Federal Reserve policy procedures have generally involved interest rate smoothing. In the 1920s, 50s, and 60s, the Federal Reserve used free reserve targeting together with discount rate adjustments as the instruments of credit control. As has been emphasized by Brunner and Meltzer (1964), such procedures amount to a kind of interest rate instrument. In the 1930s, nominal interest rates were extremely low, excess reserves were extremely high, and the Federal Reserve essentially did no day-to-day intervention would yield optimal base drift in the absence of interest rate smoothing, i.e., if $\alpha = 0$. However, allowing for such permanent shocks would not yield a non-trendstationary price level generating process in the absence of interest rate smoothing. In short, permanent shocks to aggregate supply or money demand are neither necessary nor sufficient to explain price level non-trend-stationarity. However, the tension between macroeconomic and interest rate smoothing objectives appears to be.

6.2. Price level forecast error variance as the horizon recedes

Price level forecast error variance cannot be minimized at all horizons in the presence of nominal interest rate smoothing. Consider eq. (18) below, which is derived from (6) and (8),

$$\begin{aligned}
&\operatorname{var}\left[p_{t+k} - \underset{t}{E} p_{t+k}\right] \\
&= [1 - (1 + a_3 h)A]^2 \sigma_q^2 + A^2 \sigma_u^2 \qquad \text{for } k = 1, \\
&= [1 - (1 + a_3 h)A]^2 \sigma_q^2 + A^2 \sigma_u^2 + [(\theta_2 - 1)\theta_1 B]^2 \\
&\times \left[(1 + a_3 h)^2 \sigma_q^2 + \sigma_u^2\right](k - 1) \qquad \text{for } k > 1.
\end{aligned} \tag{18}$$

With $\theta_2 \neq 1$, price level forecast error variance rises as the forecast horizon lengthens. With $\theta_2 = 1$, the forecast error variances are equal for all horizons. However, they are minimized only if $\theta_1 = \theta_1^*$ as given in (14), which is inconsistent with $|B^*| > |B|$ as required for interest rate smoothing. One period-ahead price level forecast error variance can be minimized with interest rate smoothing, but only if the price level is made non-trend-stationary. In that case, the greater the perceived relative costliness of interest rate variability, i.e., α/γ in (15), the more steeply price level forecast error variance rises as the horizon recedes.

Consider what this implies for the distribution of prospective real returns on a long-term fixed-rate nominally denominated loan. The variance of the real return associated with such a commitment is minimized in the policy environment of section 4 where monetary policy involves price level smoothing alone. Since price level smoothing alone minimizes price level forecast error at all horizons, it minimizes price level risk associated with the loan. The fixed-rate feature of the loan further insulates its real return from unexpected real interest rate disturbances. In contrast, the real return variance minimizing feature of a long-term fixed-rate nominally denominated loan diminishes in the interest rate smoothing environment of section 5. With interest rate smoothing, real interest rate disturbances are converted into unexpected price level movements that cumulate over the term of the commitment as non-trend-stationary price level movements.

6.3. Pegging, buffering money demand shocks, and interest rate instruments

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in the money market. The Federal Reserve pegged the Treasury bill rate at $3/8$ of 1 percent from 1942 until 1947. See Goodfriend (1987) for more on Federal Reserve interest rate smoothing.

Mankiw and Miron (1986) present empirical evidence supporting the institutional and theoretical views that Federal Reserve policy procedures have generally involved interest rate smoothing. They find that the short rate is approximately a random walk after the founding of the Federal Reserve but not before. They suggest that the random walk character of the short rate may be attributable to the Federal Reserve's commitment to stabilizing interest rates.

The p, r, and m generating processes with perfect interest rate smoothing, i.e., a 'peg', can be derived by substituting $\tilde{\theta}_1$ and $\tilde{\theta}_2$ from (17) into (6), (7), and (8) and then letting α/γ go to infinity to yield

$$\begin{aligned} p_t = \quad & -(a_1 + a_0 a_2 + a_3 \bar{y}) + \left[\frac{1}{1-(1+a_3 h)(1-\hat{\theta}_2)}\right] q_t \\ & + \left[\frac{1-\hat{\theta}_2}{1-(1+a_3 h)(1-\hat{\theta}_2)}\right] u_t - \hat{\theta}_2 \left(m_{t-1} - \underset{t-2}{E} m_{t-1}\right) + m_{t-1} \qquad (19) \\ r_t = \quad & a_0, \qquad (20) \\ m_t = \quad & m_{t-1} + \left[\frac{1+a_3 h}{1-(1+a_3 h)(1-\hat{\theta}_2)}\right] q_t \\ & + \left[\frac{1}{1-(1+a_3 h)(1-\hat{\theta}_2)}\right] u_t - \hat{\theta}_2 \left(m_{t-1} - \underset{t-2}{E} m_{t-1}\right), \qquad (21) \end{aligned}$$

where

$$\hat{\theta}_2 = \lim_{\alpha/\gamma \to \infty} \tilde{\theta}_2 = 1 + (1 + a_3 h)\frac{\sigma_q^2}{\sigma_u^2}.$$

Contrary to conventional thinking in a static context, a peg alone is not sufficient to perfectly buffer the price level and output against money demand shocks (which are contained in u_t). It is necessary that the money supply rule also be trend-stationary, i.e., $\theta_2 = 1$. However, a peg plus trend-stationarity does not minimize price level forecast error or the variance of output, since $\hat{\theta}_2 > 1$. In other words, it is inefficient stabilization policy to use a peg to buffer output against money demand shocks. In further contrast to conventional thinking in a static context, the money supply is not entirely demand determined under a peg. It depends on the θ_2 money supply rule parameter and on other restrictions of the money supply rule.[7]

Before concluding, we may interpret the interest rate peg described above as a simple example of an interest rate instrument, i.e., implementing policy by setting the interest rate. As should be clear, it creates expected inflation variability and current or future price level forecast error, and so should not be used unless interest rate variability itself were viewed as costly by a central bank. Therefore, even though the use of an interest rate instrument is conceptually distinct from interest rate smoothing, evidently the former must be widely observed because central banks are highly concerned about the latter.

Of course, in practice central banks adjust the level of the interest rate over time in response to the persistent changes in economic conditions. It remains for future research to introduce persistence into a model like this one to explain the interest rate adjustment process that characterizes actual central bank operations.

[7] See Dotsey and King (1983) and McCallum (1986) for more on interest rate pegs.

7. Summary

Historically, central banks have utilized monetary policy to stabilize both the financial markets and the macroeconomy. To these ends, they have pursued nominal interest rate and price level smoothing policies respectively. This paper has highlighted tension inherent in pursuing these objectives that induces non-trend-stationary processes for the money stock and the price level. The analysis, thereby, contributes to our understanding of the money stock and price level drift that has characterized the post-war era. It also points out that interest rate smoothing must increase both the price level forecast error variance at some horizon and the variability of expected inflation. So interest rate smoothing tends to create macroeconomic instability. In addition, interest rate smoothing and associated non-trend-stationary price level policies have implications, outlined in the paper, for money stock base drift, the distribution of prospective real returns on long-term fixed-rate nominally denominated loans, and the operating characteristics of interest rate pegs and instruments. The paper has, however, merely identified a constraint across central bank price level and interest rate smoothing objectives. It remains for future research to investigate such issues as the mix of smoothing behavior that is socially optimal.

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