A Quantitative Theory of Hard and Soft Sovereign Defaults

Grey Gordon and Pablo Guerron-Quintana
Federal Reserve Bank of Richmond and Boston College*

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Abstract

Empirical research on sovereign default shows "hard defaults"—defined as defaults with above-average haircuts—have worse outcomes for GDP growth than "soft defaults" and that sovereigns continue to borrow post-default. We propose a model capable of capturing these and other empirical regularities. In it, the sovereign makes period-by-period decisions of whether to make the prescribed debt payments or not. Hard defaults arise when the sovereign repeatedly *chooses* to not pay over the course of many years. Unlike in the standard model, default does not exogenously result in autarky. Rather, autarky-like conditions arise endogenously as the shocks leading to default result in higher spreads than the sovereign is willing to pay. The calibrated model predicts that growth shocks are the main determinant of whether default is hard or soft. We use the model and the particle filter to decompose how much of the empirical correlation between default intensity and output growth is selection and how much is causal. Decomposition of model forces shows that one-third (one-tenth) of hard (soft) defaults are explained by actual default costs with the rest explained by selection. A historical decomposition of shocks reveals that transitory shocks and trend shocks were the primary drivers of the Argentinean defaults in the 1980s and the 2000s, respectively. Estimated haircuts were 20 percentage points higher in the 2001 default than in the one in the 1980s, consistent with the data. Our estimated productivity shocks coincide with major events such as the convertibility plan and the Asian crisis.

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*Grey Gordon, greygordon@gmail.com; Pablo Guerron, pguerron@gmail.com. We thank Andres Blanco, Pablo D'Erasmo, Gernot Müller, Kris Nimark, Bruce Preston, and seminar participants at Monash University for helpful comments, and especially Gabriel Mihalache for particularly detailed and helpful ones.

1 Introduction

Recent research (Trebesch and Zabel, 2017) has revealed a striking pattern in the data that can be seen in Figure 1. In particular, the path for output following hard defaults—i.e., defaults characterized by large haircuts—and soft defaults—defaults characterized by small haircuts—are completely different. Whereas hard defaults are associated with a sharp and extremely persistent decline in output relative to a year before default, soft defaults are associated with a small decline on impact and growing output post default. The benchmark sovereign default models (Arellano, 2008; Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012) have nothing to say about this pattern as all defaults result in 100% haircuts. In this paper, we construct a default model with an intensive margin of default that rationalizes these patterns while simultaneously shedding light on how much of these patterns are causal—i.e., hard (soft) defaults literally reduce output—versus how much of these patterns are driven by selection—i.e., persistently low output growth leads to hard defaults.

In the standard model, the sovereign's debt repudiation decision is a once-and-for-all choice to never repay any existing debt. We replace this assumption with an alternate one, having the sovereign decide on a period-by-period basis whether they wish to make the prescribed debt payments or not. Large (small) haircuts occur when the sovereign finds it optimal to default on the prescribed payments for many (few) periods. Including growth and transitory shocks then naturally produces a pattern where (1) bad growth shocks reduce output and incentivize default for long periods of time—leading to large haircuts—whereas (2) bad transitory shocks reduce output for a short periods of time and result in small haircuts. This selection into default, however, is only part of the story as default costs also play a direct role in reducing output. Disciplining the model by making it match a host of empirical regularities, including the pattern in Figure 1, lets us determine how much of the pattern is driven by selection versus causal effects.

Our proposed model has new and richer predictions than the benchmark models while also being significantly *simpler*. Unlike in the benchmark models, there is no need to keep track of whether the sovereign defaulted in the past, and the sovereign faces the same choices every period. Despite this, the model produces near-autarky endogenously—with debt issuance falling sharply in default—, endogenous autarky durations, and endogenous haircuts. It also rationalizes the large post-default spreads seen in the data. In our formulation, 100% haircuts never occur—as in the data (Arellano, Mateo-Planas, and Ríos-Rull, 2013)—because that requires the sovereign defaulting in every future period. Moreover, our model retains the benchmark model's successes such as matching the correlation

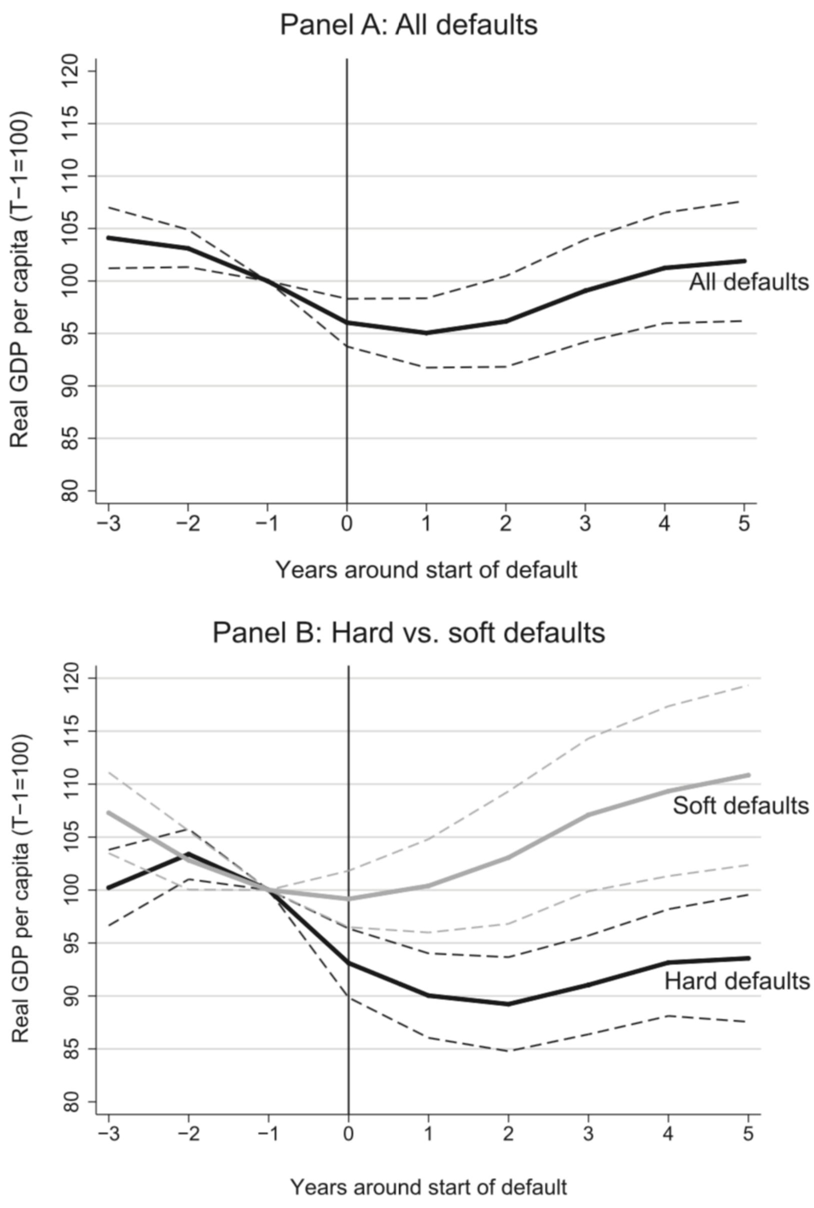

Figure 1: Hard and Soft Default Episodes (From Trebesch and Zabel)

between spreads and output.

We take our model to the data using a tractable approach, which we call SMM-pf. In the first step, we use the simulated method of moments to match the model's predictions to moments in the data like debt-to-output ration, the mean and standard deviation of spreads, and the probability of default. In the second stage, we combine the calibrated model, the bootstrap particle filter, and Argentina's data on output and spreads to recover the path of structural shocks and other unobserved variables like consumption, haircuts, and the cost of default. This approach strikes a good balance between the computational complexities behind solving/calibrating/filtering and the desire to recover the dynamic paths of relevant variables in a reasonable amount of time.

From this empirical exercise, we learn several patterns regarding Argentina. The filtering step correctly identifies the 2001-2005 epoch as a default episode. This default can be classified as a hard default, characterized by haircuts in excess of 60% and a sudden collapse of fundamentals, in particular, those related to trend growth. The default results from a sudden reversal of fortune with adverse shocks affecting trend growth in 2001. In all, the drop in fundamentals explains about 95% of the decline in GDP, with the remaining 5% explained by the cost of defaulting.

In contrast, the default in the 1980s looks more like a soft default with smaller haircuts (around 40%). The cost of default accounted for 4% of the contraction in GDP during the last part of the decade. This default episode had no trend growth and persistently adverse transitory shocks, resulting in depressed output throughout the decade. The recovery in the early 1990s is the product of an initial strong recovery in transitory innovations followed by moderately favorable trend shocks.

In the last part of the paper, we relate our estimated productivity shocks to economic events in the recent history of Argentina. For example, the speculative attacks against the peso in 1995, resulting from the Tequila crisis, led to a sequence of adverse trend and transitory shocks. The economic debacle in the late 1990s started with the Asian crisis and accelerated with the Russian crisis. These events triggered a long-lasting decline in transitory productivity, followed by adverse trend shocks in the eve of the 2001 default.

Existing literature

Our work is closely related to Arellano et al. (2013). They show that default in the data is always partial in that defaulted debt relative to payments due is always less than one. Further, they propose a model that explicitly keeps track of debt in arrears, which rolls over at an exogenous rate. Relative to them, we have three main contributions. First, we show that the composition of growth and transitory shocks is the main driver for how "partial" default is (in the sense of how large the haircuts are). Second, our model captures many of the features of partial default without having to *explicitly* keep track of debt in arrears (it is still included, but in the overall stock of debt). Third, our model is computationally more facile, and the long-term debt specification can be computed without trouble (in their online draft, only the short-term debt version has results as of this writing). This tractability makes the model amenable to estimation and extensions along other dimensions. We also decompose the observed negative correlation between haircut size and output growth into causal and selection effects.

Our work is also related to a substantial literature on debt renegotiation including Yue (2010), Asonuma and Trebesch (2016), and many other papers . Most of this literature focuses on an explicit bargaining problem formulated between a sovereign and large creditors. Our model's assumption that sovereign's decide how much to pay on existing obligations period by period essentially assumes that either (1) there is perfect competition among creditors, or (2) the bargaining power of creditors is zero, or (3) that the sovereign makes a take-it-or-leave it offer, extracting all possible surplus. Creditors in our model act passively, simply computing the present value of existing debt obligations and offering that as payment on any obligations.

To our knowledge, our is the first paper filtering data using a fully-fledge sovereign default model. Although there have been previous attempts, they impose stringent restrictions on the default decision for tractability reasons. A common abstraction is to assume an exogenous default rule, according to which the small open economy defaults if the state variables cross some threshold (Bi and Traum (2014) for Greece and Bocola (2016) for Italy). This tractability comes at the expense of ignoring the dependence of the rule on fundamentals of the economy like the discount factor and risk aversion. As a result, the rule is subject to the Lucas critique.

The rest of the paper proceeds as follows. We show the model and theoretical results in Section 2. The calibration exercise is in Section 3. In Section 4, we discuss the model's implications for hard and soft defaults. Section 5 presents estimates for the paths of several variables during the defaults in the 1980s and 2001 in Argentina. We conclude that section with a narrative account of economic events in Argentina and relate them to the filtered shocks and decomposition from our model.

2 Theoretical model and results

Following Aguiar and Gopinath (2006), we assume output is given by

$$y_t = z_t \Gamma_t$$

The transitory shock z_t is an AR1 process with

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_{z,t}$$

The "permanent component" is

$$\Gamma_t = g_t \Gamma_{t-1}$$

with g_t an AR1 process

$$\log g_t = (1 - \rho_g) \log(\mu_g) + \rho_g \log(g_{t-1}) + \sigma_g \varepsilon_{g,t}$$

The appendix provides evidence that growth and transitory shocks affect bond prices, as will come out of the model.

The utility function is

$$u(c) = c^{1-\sigma}/(1 - \sigma).$$

2.1 The model with trend

The model with trend is given in the appendix, as well as a proof that it corresponds to the detrended model in the next subsection.

2.2 The detrended model

The asset/debt structure is the same as in Chatterjee and Eyigungor (2012). In particular, debt is $-b$ in some finite set $\mathcal{B}$. Debt matures at a geometric rate $\lambda \in (0, 1]$. The $(1 - \lambda)$ fraction of debt that does not mature has a prescribed coupon payment $\kappa \in \mathbb{R}^+$. Define $\tilde{\lambda} := \lambda + (1 - \lambda)\kappa$ so that $\tilde{\lambda}$ is the payment due in a period from a unit obligation. Unlike in Chatterjee and Eyigungor (2012), a "default" does not eliminate future obligations to pay debt. Rather, a default—and we will return to the definition in the calibration section to determine default durations—is any payment less than the prescribed payment.

In every period, the sovereign solves

$$V(b,z,g) = \max_{d\in\{0,1\},b'\in\mathcal{B}} u(c) + \beta\mathbb{E}_{[z',g'|z,g]} g'^{(1-\sigma)} V(b',z',g')$$

$$\text{s.t. } c + q(b',z,g)(b' - (1-\lambda)\frac{b}{g}) = z(1-\chi(d;z,g)) + \tilde{\lambda}\frac{b}{g}(1-d)$$

Note that a default does not reduce future "obligations" to pay: The prescribed debt payments next period are b' irrespective of d. Rather, the sovereign decides period by period, given the nominal obligations to pay, whether the sovereign wants to or not. Note that default is associated with a cost $\chi(d;z,g)$.[1] The cost is purely within period. However, the calibrated model will generate financial autarky endogenously by q falling close to zero when default occurs. We interpret the output loss χ as a literal reduction in output that is reflected in the national accounting identities. Because our focus is on long-term debt, allowing for only a partial default on the prescribed payments has a quantitatively negligible effect. So we have made the d within the period be a discrete choice. Let the policy functions associated with this problem be denoted $a'(b,z,g)$ and $d(b,z,g)$.

The bond prices correspond to the net present value of payment streams.

$$q(b',z,g) = \frac{1}{1+r^*}\mathbb{E}_{[z',g'|z,g]}(\tilde{\lambda}(1-d') + (1-\lambda)q(b'',z',g')))$$

where $b'' = a'(b',z',g')$ and $d' = d(b',z',g')$.

The optimal intratemporal default decision—because there is no exclusion post default so that d is purely intratemporal—is to default whenever $z\chi(d;z,g) > \tilde{\lambda}\frac{b}{g}$. Consequently, the problem can be rewritten as

$$V(b,z,g) = \max_{b'\in\mathcal{B}} u(c) + \beta\mathbb{E}_{[z',g'|z,g]} g'^{(1-\sigma)} V(b',z',g')$$

$$\text{s.t. } c + q(b',z,g)(b' - (1-\lambda)\frac{b}{g}) = \max\{z(1-\chi(1;z,g)), \tilde{\lambda}\frac{b}{g}\}.$$

Note that having to treat all creditors equally —the sovereign cannot issue new debt that is treated better than existing debt holders—means the declining sequence of geometric payments holds and only b is needed as a state variable.

Proposition 1. *V is increasing in b.*

Sketch. A larger b gives higher utility for any b', d choice and must therefore increase V holding continuation utility as given. □

[1] We assume χ is positive and bounded above by some number less than 1. While we could have χ separable, thinking of the Mendoza and Yue QJE mechanism it makes more sense to have it proportional.

Proposition 2. *If q is increasing in b', then a' is increasing in b and m.*

Sketch. I think the sufficient conditions in Gordon and Qiu (2018) hold here. □

Proposition 3. *If a' is increasing in b and m, then q is increasing in b'.*

Sketch. Because the default decision is decreasing in b, a higher b' is going to translate into larger $q(b', z, g)$ provided that the continuation price $(1 - \lambda)q(b'', z', g')$ increases. This happens if $b'' = a(b', z', g')$ increases in b' (for each z', g'). □

For haircuts, we use a "market haircut" definition.[2] Specifically, we define it as one less the market value of debt relative to the risk-free value of debt. In the appendix, we show starting from the not-detrended problem that it is given by

$$H_{sz}(b, z, g) := 1 - \frac{\text{market value of debt}}{\text{risk-free value of debt}} = 1 - \frac{\tilde{\lambda}(1 - d) + (1 - \lambda)q(b'(b, z, g), z, g)}{\tilde{\lambda} + (1 - \lambda)\bar{q}}$$

where $\bar{q}$ is the risk-free price of debt, which solves $\bar{q} = (1 + r^*)^{-1}(\tilde{\lambda} + (1 - \lambda)\bar{q})$ (i.e., $\bar{q} = \tilde{\lambda}/(r^* + \lambda)$). (From this definition it is clear that in every period there is a "haircut," and so that when a default occurs the stated haircut was partially anticipated. This is why Tomz and Wright, 2013 note that this formulation overstates creditor losses.) Note that the price schedule and haircuts are related via

$$q(b', z, g) = \frac{\tilde{\lambda} + (1 - \lambda)\bar{q}}{1 + r^*} \mathbb{E}_{[z', g' | z, g]} [1 - H_{sz}(b', z', g')]$$

2.3 Formulation with taste shocks

As discussed in Chatterjee and Eyigungor (2012), the dependence of q on future policy functions creates serious convergence problems that are not well mitigated by increasing grid sizes or increasing the number of AR(1) shocks. However, they show that by incorporating an i.i.d. shock to the endowment, and explicitly treating it as continuous, convergence can be obtained. For our purposes, we produce the same qualitative properties at much lower computational cost by using taste shocks. The reader not interested in the details of how we do this can go to the next section without much loss.

With taste shocks $\vec{\varepsilon}$ over bond choices, we have the following specification. For each

[2] While there are alternative definitions, Tomz and Wright (2013) say in their data they deliver surprisingly similar results.

$b' \in \mathcal{B}$ let $\varepsilon_{b'} \overset{i.i.d.}{\sim} Gumbel(-\gamma - \log(\#\mathcal{B}), 1)$ where γ is the Euler-Mascheroni constant.[3] Let

$$w(b', b, z, g, d) = u(c) + \beta \mathbb{E}_{[z', g'|z, g]} g'^{(1-\sigma)} V(b', z', g')$$
$$\text{where } c = -q(b', z, g)(b' - (1-\lambda)\frac{b}{g}) + z(1 - \chi(d; z, g)) + \tilde{\lambda}\frac{b}{g}(1-d)$$

and

$$W(b, z, g, d, \vec{\varepsilon}) = \max_{b' \in \mathcal{B}} w(b', b, z, g, d) + \sigma_\varepsilon \varepsilon_{b'}$$

While there is a policy function $a'(b, z, g, d, \vec{\varepsilon})$, we will use the associated choice probabilities, call them $p(b'|b, z, g, d) \in [0, 1]$ with $\sum_{b'} p(b'|b, z, g, d) = 1$.

Let $v(b, z, g, d) = \int W(b, z, g, \vec{\varepsilon}) dF(\vec{\varepsilon})$. We assume there are also taste shocks $\vec{\eta}$ that influence the default decision with $\varepsilon_\eta \overset{i.i.d.}{\sim} Gumbel(-\gamma - \log(2), 1)$:

$$\hat{V}(b, z, g, \vec{\eta}) = \max_{d \in \{0,1\}} v(b, z, g, d) + \sigma_\eta \eta_d.$$

Consequently, "the default decision" is given by choice probabilities $p(d|b, z, g) \in [0, 1]$ with $p(0|b, z, g) + p(1|b, z, g) = 1$. The V in the continuation utility of the first problem is then $V(b, z, g) := \int \hat{V}(b, z, g, \vec{\eta}) dF(\eta)$. Of course, if $\sigma_\eta = \sigma_\varepsilon = 0$, the problem is the same as if there were no taste shocks and the choice probabilities are degenerate (unless there is exact indifference).[4]

The price schedule then becomes

$$q(b', z, g) = \frac{1}{1+r^*} \mathbb{E}_{[z', g'|z, g]} \left[\sum_{d', b''} p(d'|b', z', g') p(b''|b', z', g', d')(\tilde{\lambda}(1-d') + (1-\lambda) q(b'', z', g')) \right].$$

The well-known formulas due to McFadden (1974) and Rust (1987) give the choice

[3] The usual distribution is $Gumbel(-\gamma, 1)$ (Rust, 1987). However, note that the preference shocks have unbounded support so, as the number of choices grows, a law of large numbers gives that W would go to infinity else equal even if w were bounded. This modest adjustment ensures that if w is uniformly equal to some $\bar{w}$, that $\int W dF(\vec{\varepsilon}) = \bar{w}$ as well.

[4] While w is not well-defined for every b', d choice, we note that if non-feasible choices are made feasible with severe penalties, the same numerical approximation will be obtained as if treating $w = -\infty$ when not-feasible.

probabilities and value functions as

$$p(b'|b,z,g,d) = \frac{\exp(w(b',b,z,g,d)/\sigma_\varepsilon)}{\sum_{\tilde{b}'}\exp(w(\tilde{b}',b,z,g,d)/\sigma_\varepsilon)}$$
$$v(b,z,g,d) = \sigma_\varepsilon \log\left(\frac{1}{\#\mathcal{B}}\sum_{b'}\exp(w(b',b,z,g)/\sigma_\varepsilon)\right)$$
$$p(d|b,z,g) = \frac{\exp(v(b,z,g,d)/\sigma_\eta)}{\sum_{\tilde{d}}\exp(v(b,z,g,\tilde{d})/\sigma_\eta)}$$
$$V(b,z,g) = \sigma_\eta \log\left(\frac{1}{2}\sum_{d}\exp(v(b,z,g,d)/\sigma_\eta)\right).$$

Note that as long as the value functions move continuously in q, the choice probabilities and hence the implied q do as well. This makes convergence extremely easy to obtain computationally provided that σ_η and σ_ε are sufficiently large. The complication behind this new formulation is that we need to adapt the particle filter to track the entire distribution of states/choices, rather than the current state as in start filtering problems.

3 Calibration and validation

This section describes how we calibrate the model and gives the model's fit of both targeted and untargeted moments.

3.1 Functional forms

Let $\chi(d;z,g)$ be given by $d\overline{\chi}\delta(z,g)$. The parameter $\overline{\chi}$ determines the overall magnitude the default costs. The "slope" term $\delta(z,g)$ depends on the growth and transitory shocks via[5]

$$\delta(z,g) = \max\{0, 1 + \delta_1(z-1) + \delta_2(g-\mu_g)\}. \tag{1}$$

[5]A justification for this assumption is the default excusability discussed in Tomz and Wright (2013). Specifically, if default came as a surprise or was "unavoidable" (thereby making it excusable), empirical evidence suggests default costs are lower. Additionally, this flexible parameterization of δ may reflect a negotiation process where bad output shocks reduce the sovereign's surplus from reaching an agreement thereby lowering effective default costs. Specifically, a bad transitory shocks produces an incentive to borrow (since output mean-reverts in expectation)—thereby increasing the surplus from reaching a settlement with creditors—while a bad growth shock does not. We don't take a stance apriori on whether low z or low g lowers default costs more.

3.2 GDP process estimation

Our procedure to separate out trend from transitory shocks in the data is to first log and HP-filter the observed quarterly output series in Argentina from 1980:1 to 2005:2. We then fit an AR1 to the trend and an AR1 to the first differences of the deviations from trend. Absent default costs that depress output and assuming the HP-filter correctly distinguishes between the trend and deviations, this exactly identifies the shocks. This is misspecified in default periods because we have assumed default costs directly lower output.

The values are reported in the first rows of Table 1. The transitory component of output has values similar to the existing literature (in large part because this is the only part of output the literature usually looks at—the deviation in HP-filtered output). The growth shocks are more interesting and reveal an extremely persistent growth process with a small conditional standard deviation (but a .005 unconditional standard deviation). The appendix plots the actual output series for Argentina along with the HP-filtered components, which clearly shows why the regressions pick up a very persistent growth shock.

Parameter	Value(s)	Reason / Data
(ρ_z, σ_z)	(.853,.0243)	AR1 fitted to HP-filtered log output deviations
$(\mu_g, \rho_g, \sigma_g)$	(.0133,.989,.0008)	AR1 fitted to HP-filtered log output trend differences
(σ_b, σ_d)	(.0003,.0003)	Large enough to ensure convergence
λ	0.0357	Tomz and Wright (2013)
κ	0.03	Chatterjee and Eyigungor (2012)
σ	2	Commonly used CRRA value
Note: .		

Table 1: Parameters fixed a priori

3.3 Exogenously determined parameters

We set the constant relative risk aversion (CRRA) parameter to 2 in line with most of the literature. The coupon payment κ is set to 0.03, which follows Chatterjee and Eyigungor (2012) (who calibrate to Argentina). To have a chance of matching (1) a mean duration of staying in default for 28 quarters and (2) haircut sizes of (38%), we need a debt maturity that is not too large—28 quarters of continual non-payment result in $(1-\lambda)^{28}$ of the face value being left to pay.[6] Hence, we use a λ smaller than Chatterjee and Eyigungor (2012) (who use .05) by setting $\lambda = .0357$. This number is the average from Tomz and Wright

[6]We will not require continual non-repayment to stay in default. Rather default will end when $d_t = 0$ for 8 quarters in a row.

(2013), who discuss how debt maturity calculations are sensitive with no clear winning strategy. The taste shock magnitudes were chosen to enable reliable convergence while also being small.

3.4 Calibrated parameters and calibration targets

The remaining parameters are three default cost parameters—$\delta_1, \delta_2, \overline{\chi}$—and the discount factor β. We use these to match five statistics. The first three, the debt-output ratio (1), the mean spread (8.15), and the spread standard deviation (4.43), are from CE and are based on Argentinean data.[7] We also match two other statistics.

First, we want default to persistent as in the data, and so we match the probability of staying in default. Annually, using the data of Trebesch and Zabel (2017), we find this number is 0.882 for Argentina and 0.823 for all countries in their sample on average. Note that the definition in terms of when default ends is not clear—Tomz and Wright (2013) say the S&P definition is when no further paymentsare likely to occur—, which is why we we focus on transition rates to and from default. We say default is exited at time t when $d_t = 0$ for the next 8 quarters in the simulation. Table 2 reports the transition rates.

Transition rates	Argentina	All
$\mathbb{P}(d=1\|d_{-1}=0)$	.167 (.048)	.027 (.001)
$\mathbb{P}(d=1\|d_{-1}=1)$	.882 (.214)	.823 (.057)
Implied $\mathbb{P}(d=1)$	.586	.132

Note: standard error of the mean is in parentheses; observations are annual; all countries is the sample from Trebesch and Zabel (2017).

Table 2: Default transition rates

Second, we want to match the pattern observed in Figure 1. To this end, we target the correlation between output 4 years ahead (relative to the previous years output) and haircut sizes conditional on default. In the data, we find this measure is -0.547.Because

[7] Chatterjee and Eyigungor (2012) only target 70% of the debt-output ratio. In particular, they say

> We seek to match only a portion of debt because we do not model repayment. In reality, sovereign debt that goes into default eventually pays off something. In Argentina's case, the repayment on debt defaulted on in 2001 has been around 30 cents to the dollar. Thus, we treat only 70 cents out of each dollar of debt as the truly unsecured portion of the debt. But, as part of our sensitivity analysis, we also examine the case in which we fully match average external debt-to-output ratio.

the Trebesch and Zabel (2017) data is annual and our model is quarterly, we must use some form of aggregation. For GDP is to obtain an annual series y_t^A compute the mean of $y_{t:1}^Q, \ldots, y_{t:4}^Q$. For d_t^A, we take the maximum over $d_{t:1}^Q, \ldots, d_{t:4}^Q$. From now on, we use a tilde ("˜") to distinguish a variable as annual. For haircuts, we also take the maximum.

3.5 Model fit of targeted and untargeted moments

The targeted and untargeted moments are displayed in Table 3. Although small relative to standard RBC values, the discount factor is large relative to existing default models despite matching the largest debt stock to date. The "mean" default cost $\overline{\chi}$ is 0.131 (i.e., the default costs when $z = 1$ and $g = \mu_g$), which does not seem unreasonably high to us. Of course, the actual default costs realized on average (0.043) are smaller as the sovereign tends to default when costs are low. The default cost slopes δ_1 and δ_2 (recall the specification $\delta(z, g)$ in (1)) must be interpreted in light of the unconditional shock deviations. For the transitory (growth) shock this is 0.0466 (0.0054). Hence a 1 standard deviation increase in the transitory (growth) shock increases default costs by 0.26 (0.21). This makes default in states with good shocks almost infeasible while making default in states with bad shocks be at zero cost. These costs should be reflected in bond spreads in the data, and they are—the appendix gives regressions (and raw data) showing the significant impact model-consistent measures of shocks have on spreads.

The model fits the data well along virtually every reported dimension. The model's 56.3% time spent in default is very close to "the data's" 58.6%. The latter measure comes from computing the invariant distribution associated with the flows in Table 2, and is subject to substantial uncertainty. Using a broad sample of countries, the average time in default in Tomz and Wright (2013) is 18%, but we suspect Argentina is an unusually common defaulter.[8]

A similar issue arises with haircuts, where the data measure we report (38.0%) is from Trebesch and Zabel (2017). The model's average haircut size is 57.6%, which seems high relative to the data. However, we match the mean and standard deviation of spreads, and given the tight relationship between spreads and haircuts, it would seem we must be doing better along that dimension than it seems.[9] However, we cannot say for sure because the

[8]Tomz and Wright (2013) Figure 2 plots the historical proportion of borrowers that are in default. In the modern era 1980 and on it varies from just a few percentage points to around 42%. They also say "That this moment [the frequency of default] is sensitive to reasonable changes in the definition of default suggests that an alternative moment—one more robust to changes in definition—should be used to calibrate models of default. One possibility is the fraction of time debtors spend in default, which is 18% across the entire sample" (p. 258). We use the average implied by transition rates.

[9]Tomz and Wright (2013) report findings on average haircut sizes using three different methodologies.

Targeted moments	Target	Model	Parameter	Value
Debt-output ratio (normal times)	1.000	0.985	β	0.970
Spread mean (normal times)	8.150	9.601	$\overline{\chi}$	0.131
Spread std. dev. (normal times)	4.430	4.857	δ_1	5.583
Corr$(100\tilde{y}_{t+4}/\tilde{y}_{t-1}, H_{SZ,t} \vert \tilde{d}_t = 1)$	-0.547	-0.448	δ_2	38.906
Prob. stay in default $\mathbb{P}(\tilde{d}_t = 1 \vert \tilde{d}_{t-1} = 1)$	0.882	0.789		

Untargeted moments	Data	Model
Corr$(\log y_t, r_t)$	-0.700	-0.538
Prob. go to default $\mathbb{P}(\tilde{d}_t = 1 \vert \tilde{d}_{t-1} = 0)$	0.167	0.272
Haircut size $\mathbb{E}H_{SZ,t}$	0.380	0.576
Time in default $\mathbb{P}(\tilde{d}_t = 1)$	0.586	0.563
Time to decisively exit default (see note)	28.000	23.247
Spread at default	28.600	19.997
Realized default costs $\mathbb{E}[\chi_t \vert def_t = 1]$	-	0.043

Note: A tilde represents an annual rather than quarterly value. The debt-output ratio is the debt stock divided by quarterly GDP in the data and model (the data measure is from Chatterjee and Eyigungor, 2012 who do the same—the World Bank's debt series DT.DOD.DPPG.CD series over the GNI series NY.GNP.ATLS.CD all annual over 1993-2001 gives an average of 0.257). GDP is aggregated via computing an average, default by taking a max. When a default occurs at time t, we calculate time to decisively exit default as the first j such that $d_\tau = 0$ for all $\tau \in \{t+j, \ldots, t+j+8\}$; if such a j cannot be found (because we truncate histories at 200 periods), we assign $j = 200$.

Table 3: Targeted and untargeted moments

sample size for Argentina would only have a few data points. In Argentina, according to Trebesch and Zabel's data, the haircut size was 32% in the 1982 default and 76.8% in the 2002 default. If one is willing to average these two data points, one arrives at 54.4%, almost exactly matching the model's 57.6%.

In sum, the model matches important targeted moments and many of the untargeted ones. We now look more in depth at the model's predictions.

4 Equilibrium hard and soft defaults

This section considers the equilibrium policies and prices and shows how the model produces hard and soft defaults.

4.1 Equilibrium policies and prices: Transitory vs. growth shocks

Figure 2 plots the price schedule, debt issuance, and debt distributions for the four combinations of highest and lowest growth and transitory shocks. The worst growth shocks greatly depress bond prices and trigger default for any nonnegligible debt positions. Interestingly, the sovereign still borrows a bit (and this at very large spreads), but the market value is comparatively small. For the lowest growth shock and a high productivity shock, the sovereign does not default at conventional debt levels and saves a fair amount. (The unconditional level of output is normalized to around 1.) These observations back up our claim that our model endogenous generates periods of autarky where, while in default, there is little to no borrowing. Here, rather than by assumption, it comes from current default being positively correlated with future default, depressing bond prices and disincentivizing borrowing. Periods of fast growth are characterized by large amounts of borrowing driven by (1) attractive interest rates and (2) debt being easy to afford as reflected in b/g in the detrended problem. The maximum "sustainable" level of debt, i.e., the debt where the sovereign is indifferent between $d_t = 0$ and $d_t = 1$ differ tremendously depending on what the shocks are, ranging from essentially zero to more than twice an average level of output.

4.2 Hard and soft default episodes

Figure 3 shows the model reproduces almost exactly the hard and soft default episodes observed in the data (which can be seen in Figure 1). Since we matched the correlation be-

" Benjamin & Wright (2008) estimate an average market haircut of 38%, whereas Cruces & Trebesch (2013) estimate a 40% market haircut and a 37% SZ-haircut."

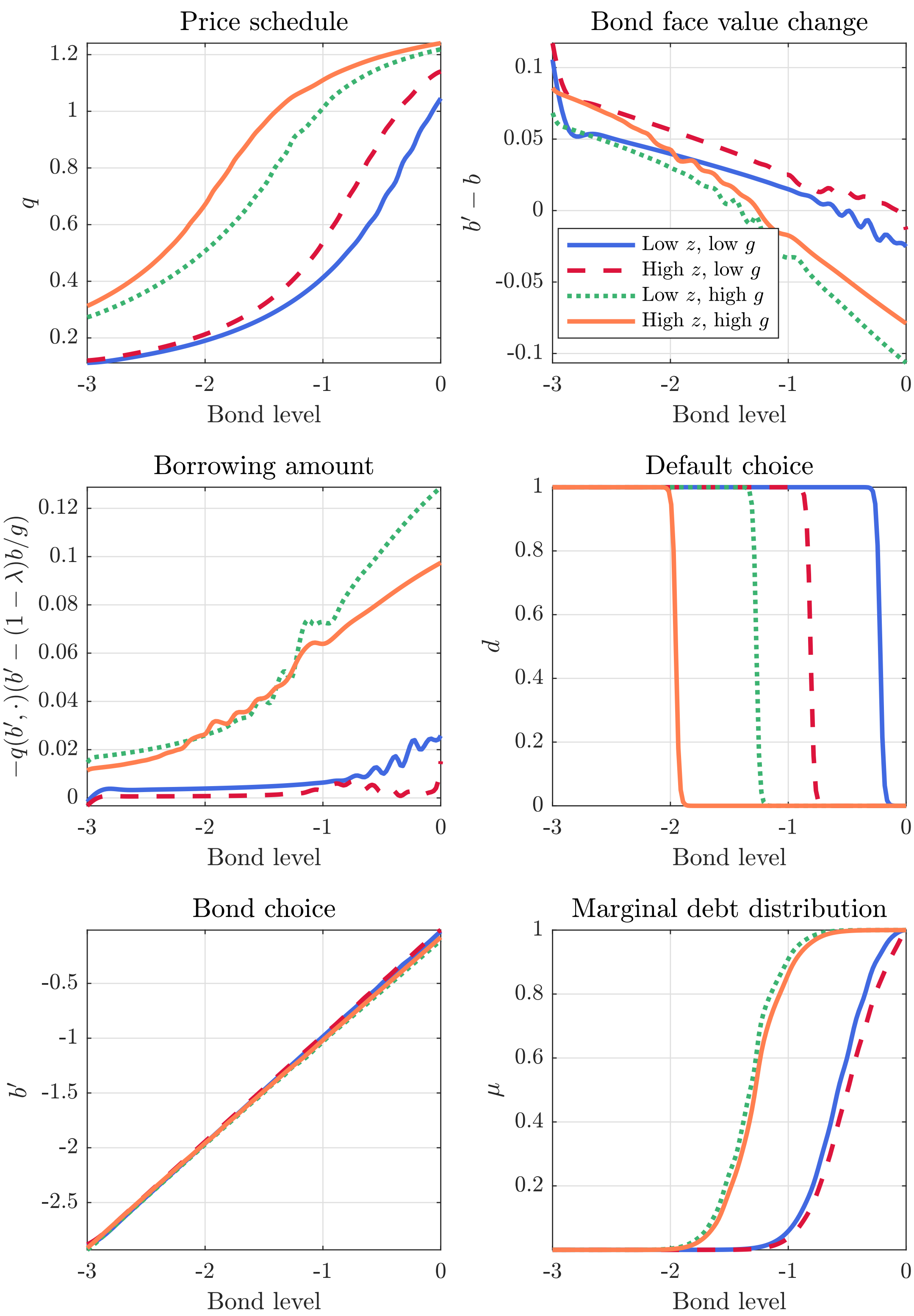

Figure 2: Equilibrium objects, policies, and distribution

tween haircut sizes at default and output growth 4 years ahead, this is not terribly surprising in itself. Rather, the model's ability to rationalize that statistic is the accomplishment.

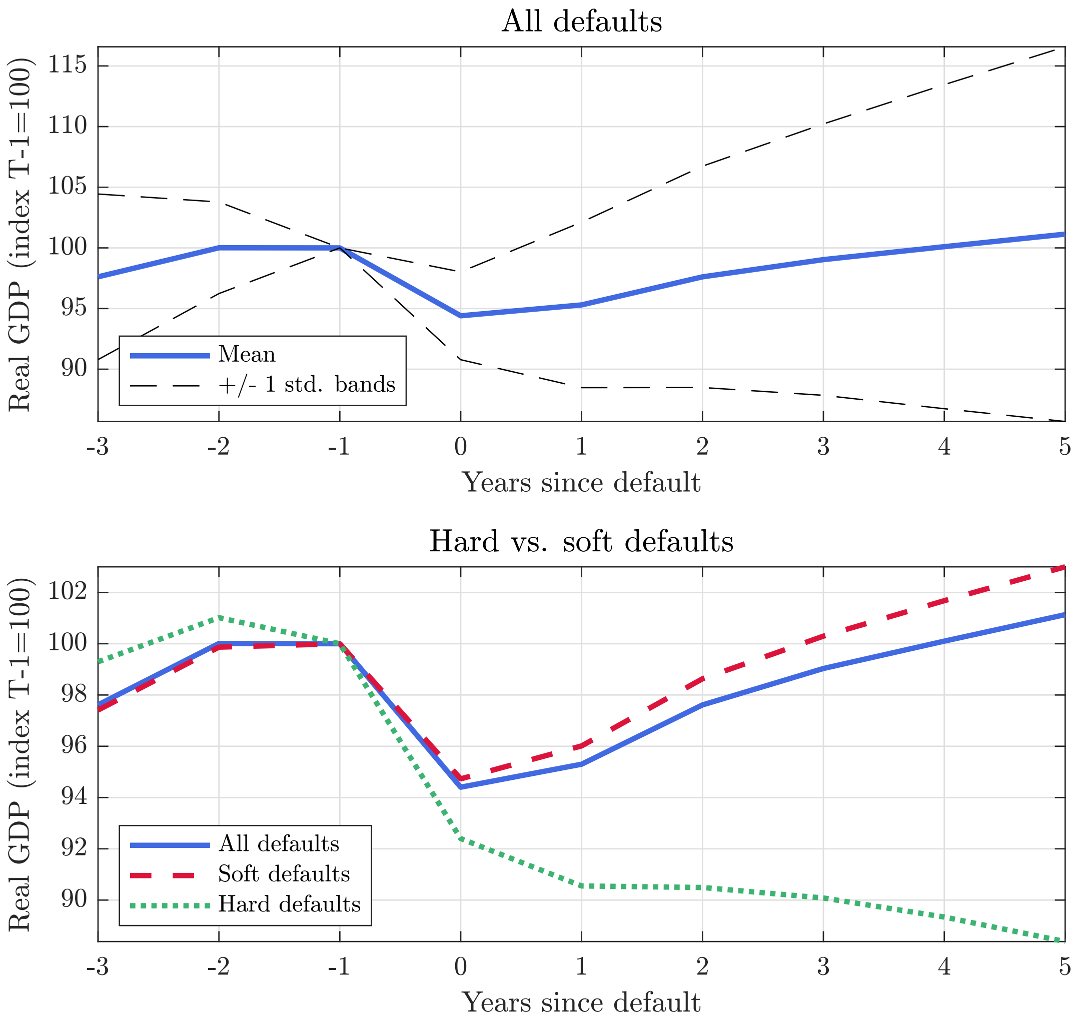

Figure 3: Hard and soft default episodes

As reflected in Figure 4, the logic behind hard and soft defaults is more complicated than it might first appear. In particular, growth shocks are the main determinant of whether default is hard or soft (with hard defaults driven by negative growth shocks). But defaults of all types are triggered with negative growth and transitory shocks.

Post-default, the face value of debt to output tends to be the same or higher a few years after default (and significantly higher following a soft default. This may seem counterfactual. However, Benjamin and Wright (2008), as quoted in Tomz and Wright (2013), show for a sample of 90 defaults that the face value of debt to GDP "does not fall and may even rise after a default. The median country ends the year of the settlement with a debt-to-GDP ratio 5 percentage points higher than when it entered default." For the average default

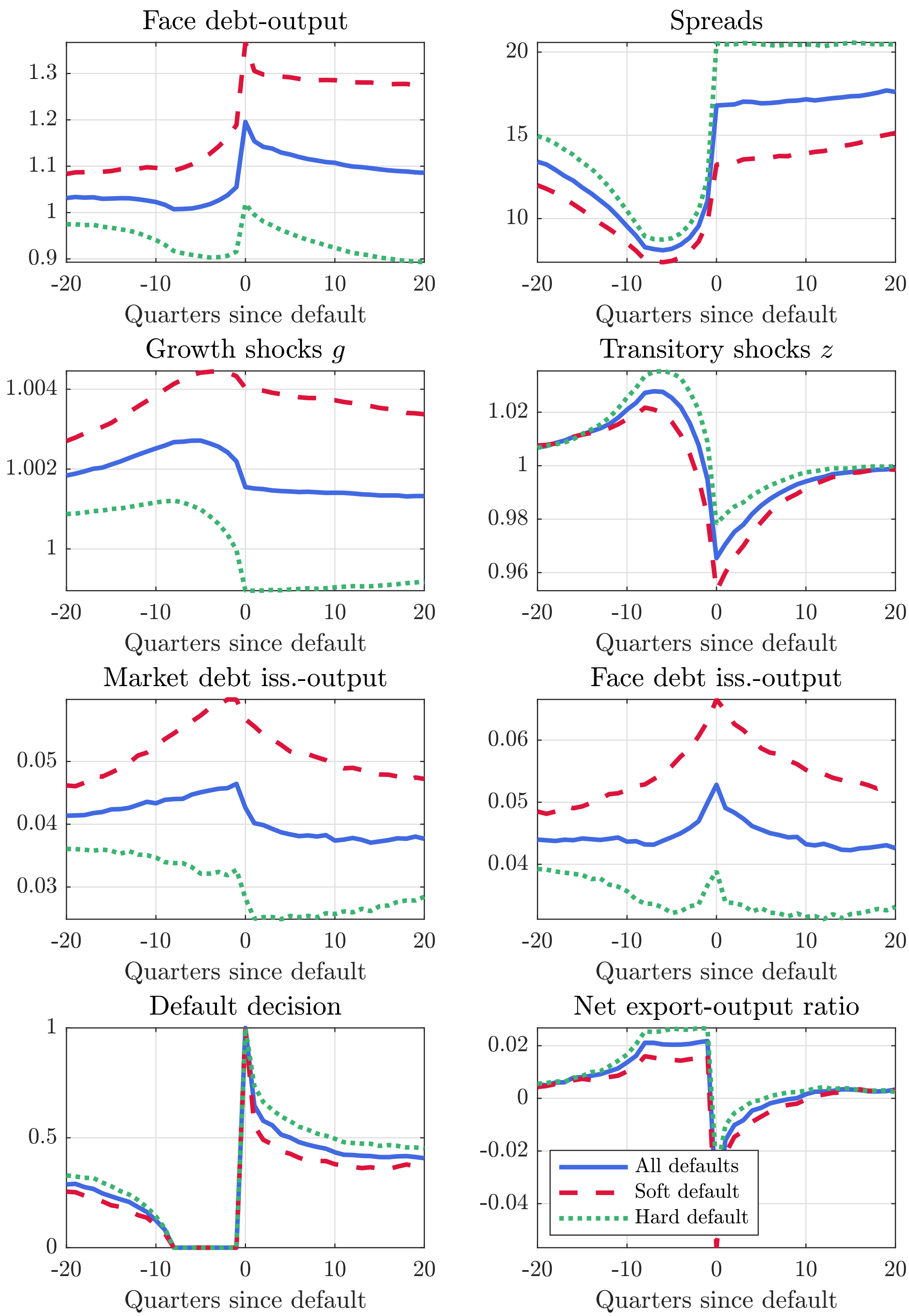

Figure 4: Default event comparison

episode, the debt-output ratio goes from around 1.03 predefault to 1.10 five years after, an increase of around 7%. Consequently, this is not a failure of the model but a success.

The default episodes also reveal that hard all defaults have a marked reduction in debt issuance, giving endogenous "autarky" in the model. Clearly, this autarky is not total as in the standard model, but as discussed in the previous paragraph, it is not total in the data, either. Note that spreads increase and remain high post default for an extended period of time, especially in hard defaults; and also note that the standard model does not have well-defined spreads while the sovereign is in default. The behavior of net exports is counterfactual in this class of models because, with autarky-like conditions post default, net exports must be negative upon reentry as credit markets expand and so does borrowing.[10] However, this presumably could be fixed by endogenizing default costs along the lines of Mendoza and Yue (2012).

In sum, the model reproduces many features of the data, including the hard and soft default trajectories for growth, while being richer and simpler than the benchmark model. We now turn to Argentina's actual path to analyze its hard and soft defaults.

5 Argentina's hard and soft defaults

In this section, we consider the actual path that Argentina's economy has undertaken over the past few decades. This is an interesting test case because at face value Argentina presents a counter-narrative to the hard and soft default story of Trebesch and Zabel (2017): Argentina's soft default in the 1980s was characterized by a sluggish recovery while its hard default in the 2000s had a rapid return to growth. To analyze it, we use the calibrated parameters and run the particle filter to obtain a historical shock decomposition. The observables for this exercise are quarterly output and spreads corresponding to the period 1984:Q1-2005.Q1 (blue crossed line in Figure 5).[11] We add i.i.d. Gaussian measurement errors with mean 0 and standard deviation equal to 5% of the volatility of each observable. Details on the filtering step are provided in appendix C.

The red lines in Figure 5 display the filtered paths for output and spreads. Clearly, the filtered series track closely the data, but there are some differences around the 2001 default. The gray areas correspond to default episodes as predicted by our model (see notes in Table 3). Although we don't have spread series post 2002, our filtering exercise estimates that as the economy started to recover in 2003, spreads declined. The low spreads in 2005 coincide with the beginning of Argentina's debt restructuring process. That our

[10] For more on this, see the discussion on pp. 78-79 of Gordon and Guerron-Quintana, 2018.

[11] We have output data going back to 1980.Q1, which we use to initialize the filter.

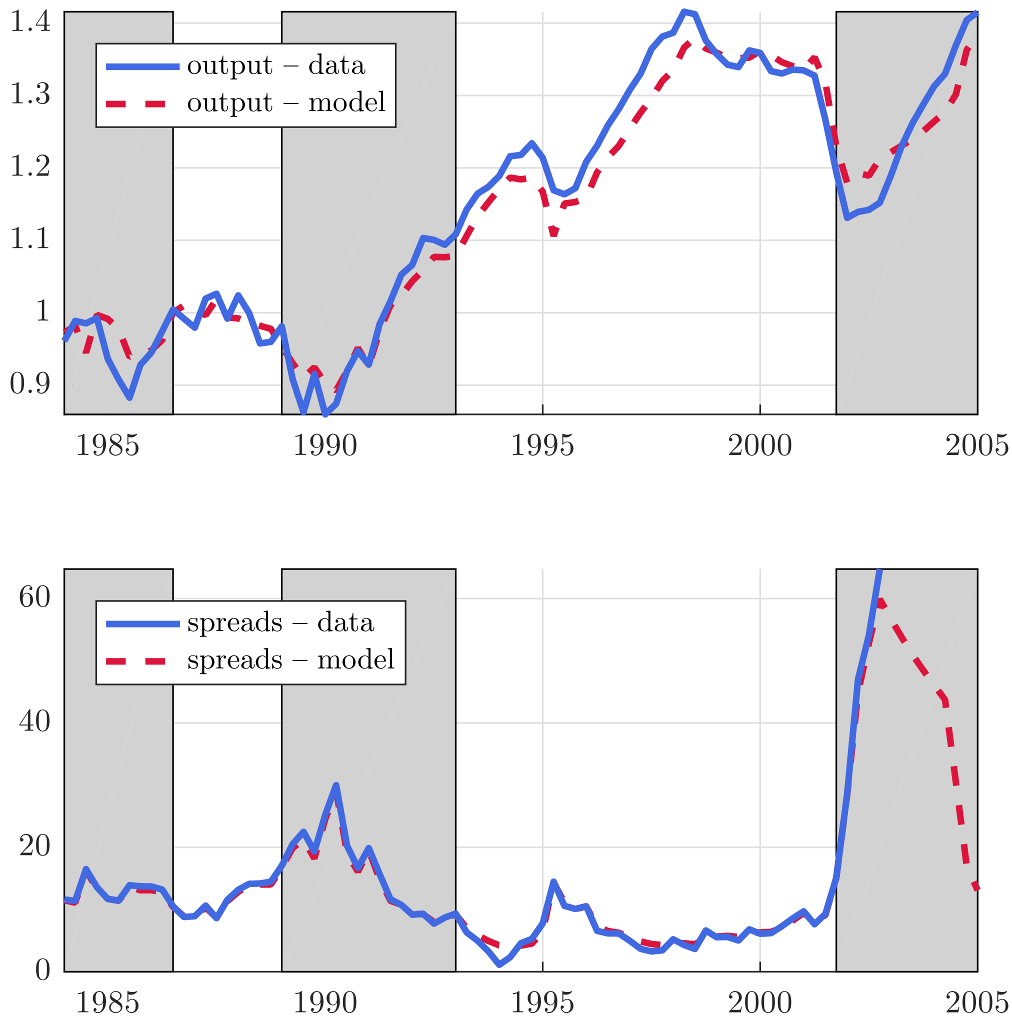

Figure 5: Observable variables

model matches the data is expected given that its calibration targeted several features of the output and spread (see Table 3). Importantly, the good fit of the data shows the power of our SMM-filtering approach.

5.1 Predictions for nonobservables

Figure 6 shows the dynamics paths for some nonobservables in our model. According to the model, Argentina's debt remained roughly stable during the 1980s and early 1990s (solid blue line in right upper panel). However, debt skyrocketed in the second part of Carlos Menem's tenure. Indeed, debt almost doubled between 1994 and 2001. Following default in 2001, our model/filtering approach has Argentina entering a deleveraging epoch, with its debt dropping 30% by the end of our sample. For comparison purposes, the red circles correspond to debt-to-GDP ratio in the data. Although our model initially over states the amount of debt in Argentina, we correctly capture the dynamics of debt around the 2001 crisis. Moreover, the deleveraging in the data is consistent with that predicted by the model.

Looking at the default flag (left lower panel), we observe that our approach correctly detect the default episodes of the 1980s picking up one in 1984 and in 1988. The latter coincides well with Argentina's 1989 default. Moreover, the model correctly identifies the 2001 default episode.

The panel in the lower right corner displays the default cost χ from (1) in percent. When Argentina defaulted in 2001, default costs accounted for roughly 5% of the drop in GDP in the early part of the default period. In other words, fundamentals account for 95% of the contraction of the Argentinean economy. In the following years, the output loss from default hovered around 3%. We estimate that the default loss between 2001 and 2005 in net present value was equivalent to roughly 60% of GDP (3% for 20 quarters).

The implied haircut is plotted in Figure 7. During the 1980s default, the haircut averaged 40% and reached 60% in 1990. The 2001 default was characterized by an average haircut of 64%, but it was as high as 73% in 2002. By comparison, Edwards (2014) reports that the 2003 renegotiation (known as the Dubai guidelines, black circle) implied a haircut of 75% (red dashed line in figure 7). Over the long run, haircuts are on average 37%, which is in line with the cross-country average reported by Edwards.[12] Based on these estimated haircuts, it seems reasonable to label the first and second defaults as a soft default and a hard default, respectively.

[12] Edwards (2014) compiles a sample of 180 debt restructurings between 1978 and 2010, the average haircut in the sample is 37%

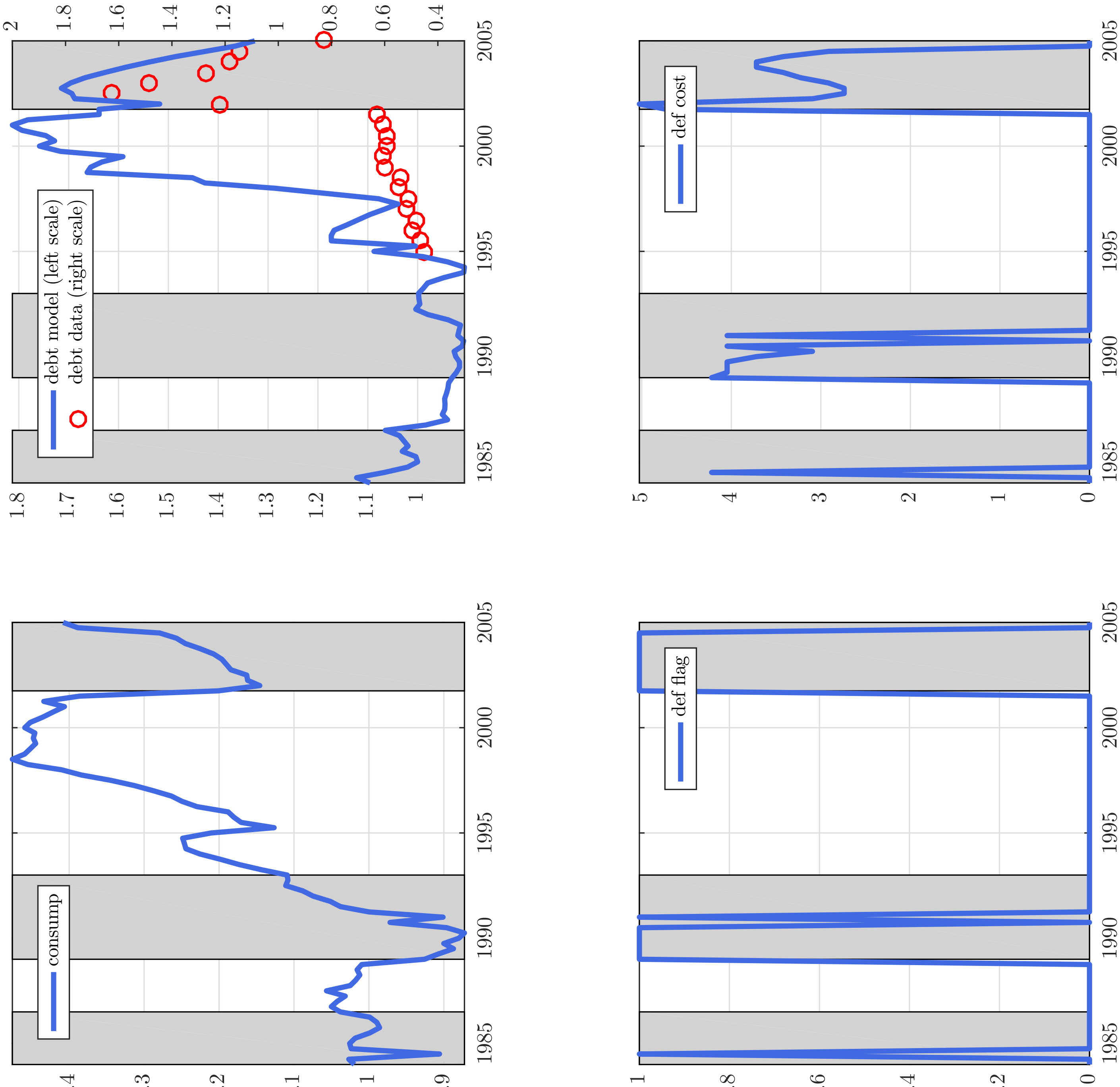

Figure 6: Nonobservable variables

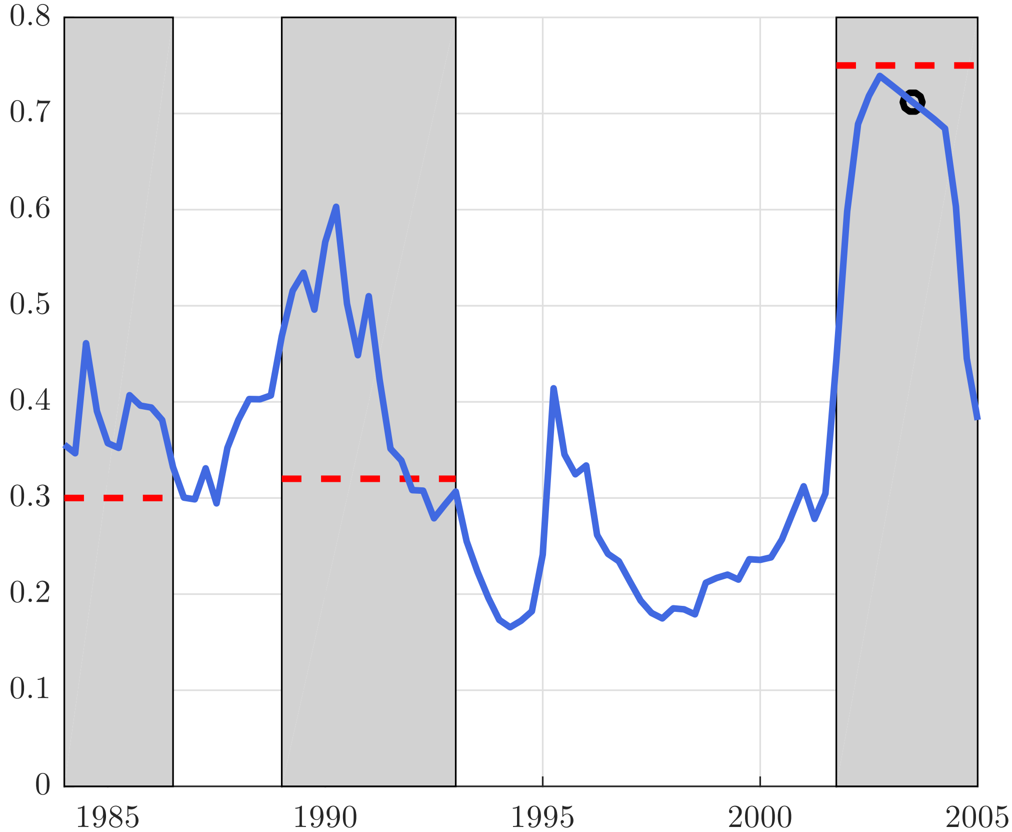

Figure 7: Haircuts

5.2 Historical shock decomposition

We now examine what shocks drove the soft default in the 1980s and the hard default in the 2000s. Figure 8 shows the filtered paths for the structural shocks: permanent g_t in the upper panel and transitory z_t in the bottom.

The structural reforms introduced by Carlos Menem in the early 1990s are reflected in strong tailwinds in persistent and temporary productivity. The recovery in the estimated productivity shocks coincides with strong readings in output per worker, which averaged 7% per year between 1990 and 1994 (Llach and Gerchunoff (2018)). All of this happening while the level of debt relative to GDP remained relatively flat (figure 6). The sudden drop in productivity in 1995 coincides with capital flights and speculative attacks to the peso, resulting from the Tequila crisis.[13]

As we move through the second half of the 1990s, Argentina recovered from 1995 recession benefited from favorable persistent shocks to the trend. Temporary innovations reinforced Argentina's expansion during the last part of that decade. The advent of the new century brings a change of fortune to Argentina. First, transitory innovations weakened and eventually turned into headwinds. Second, and more important, there is a sudden reversal in the trend shocks around 2001. Together, these forces brought the economy to a hard default with a large haircut and severe contraction. By 2005, transitory shocks had mean reverted but permanent shocks continued to be a drag on the economy.

One way to understand these transitory and permanent disturbances is to relate them to Argentina's terms of trade. This is a reasonable approach given Argentina's dependence on exports, primarily agriculture. The red dashed lines in Figure 8 corresponds to the terms of trade between 1985 - 2005.[14] A quick look at the bottom panel reveals that our transitory shocks are correlated with the terms of trade in Argentina (with a correlation coefficient of 0.60). For example, one can see that Argentina's transitory disturbances in the 1990s mostly reflects developments in the external sector (in particular, those related to the soy and wheat markets). Both the recovery in the early 1990s and the demise in the late 1990s coincide with periods of terms of trade appreciations and depreciations, respectively. When we turn to shocks to the growth rate (upper panel), the connection between the terms of trade and the shocks in our model become less clear (the correlation drops to 0.3). However, it is not difficult to see that the terms of trade seem to lead the growth shocks (the one-year ahead correlation is 0.65). These results suggest that disturbances in the international commodity markets initially fed to the Argentinean economy via what

[13] The Argentinean Central Bank lost one forth of its foreign reserves and the EMBI shut up from 8% to 55% (Llach and Gerchunoff (2018)).

[14] The data is taken Fernandez, Schmitt-Grohe, and Uribe (2017).

our model interprets as transitory innovations. But as the adverse terms of trade lingered, they affected the long run growth of the economy. This narrative is consistent with the recovery post-2001 default.

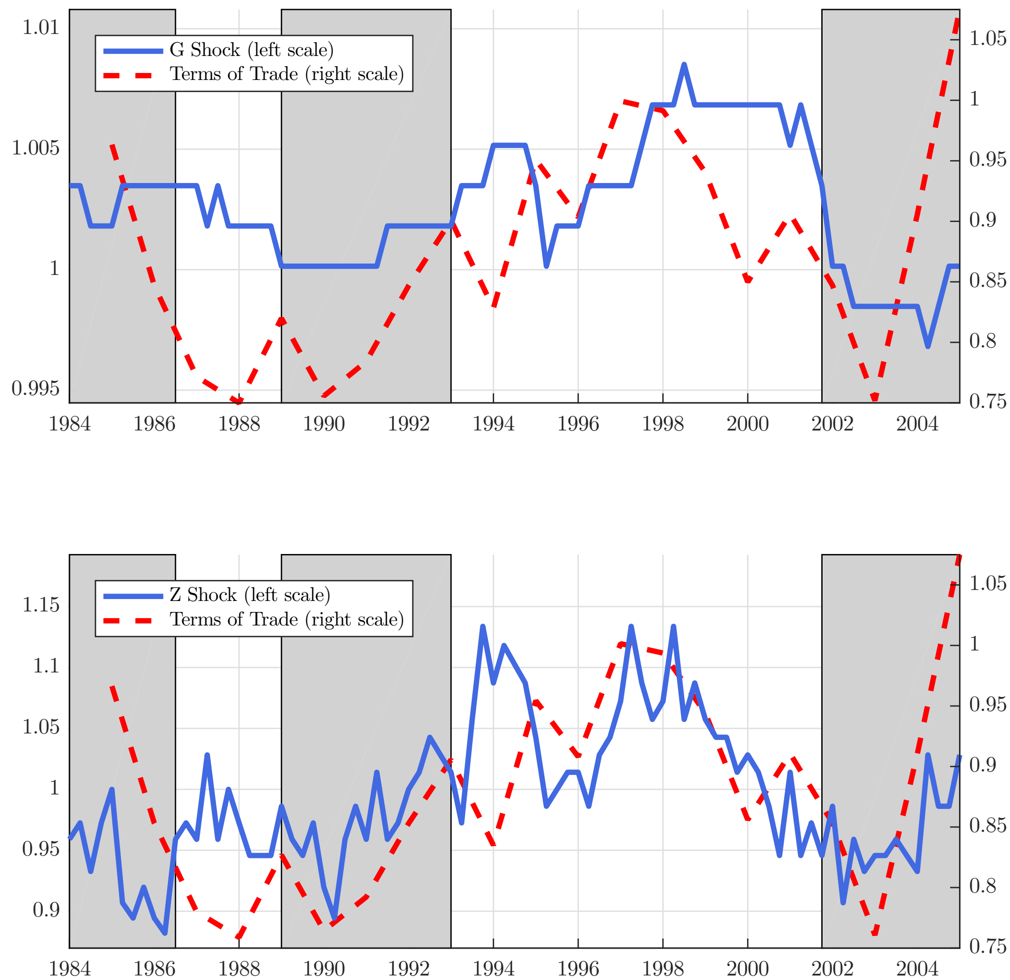

Figure 8: Filtered Shocks

Figure 9 shows the counterfactual paths of output when only growth shocks (red dashed line) or only transitory shocks (green dotted line) are in place. As one can see, our exercise reveals that the Argentinean economy was driven by transitory shocks during the 1980s and the early part of the 1990s. Soon after, trend shocks favored growth in the economy. The path to the 2001 default was paved by a slowdown in transitory shocks in the late 1990s and the abrupt decline in growth in 2001. Between 2001 and 2005, growth was muted with the mild recovery in 2004 driven by transitory innovations.

To further interpret our results, Figure 9 also displays some major events in Argentina during the 1980s and 1990s. We begin our narrative in 1985 with the introduction of Plan Austral, which aimed to contain inflation and restart growth. Based on our decomposi-

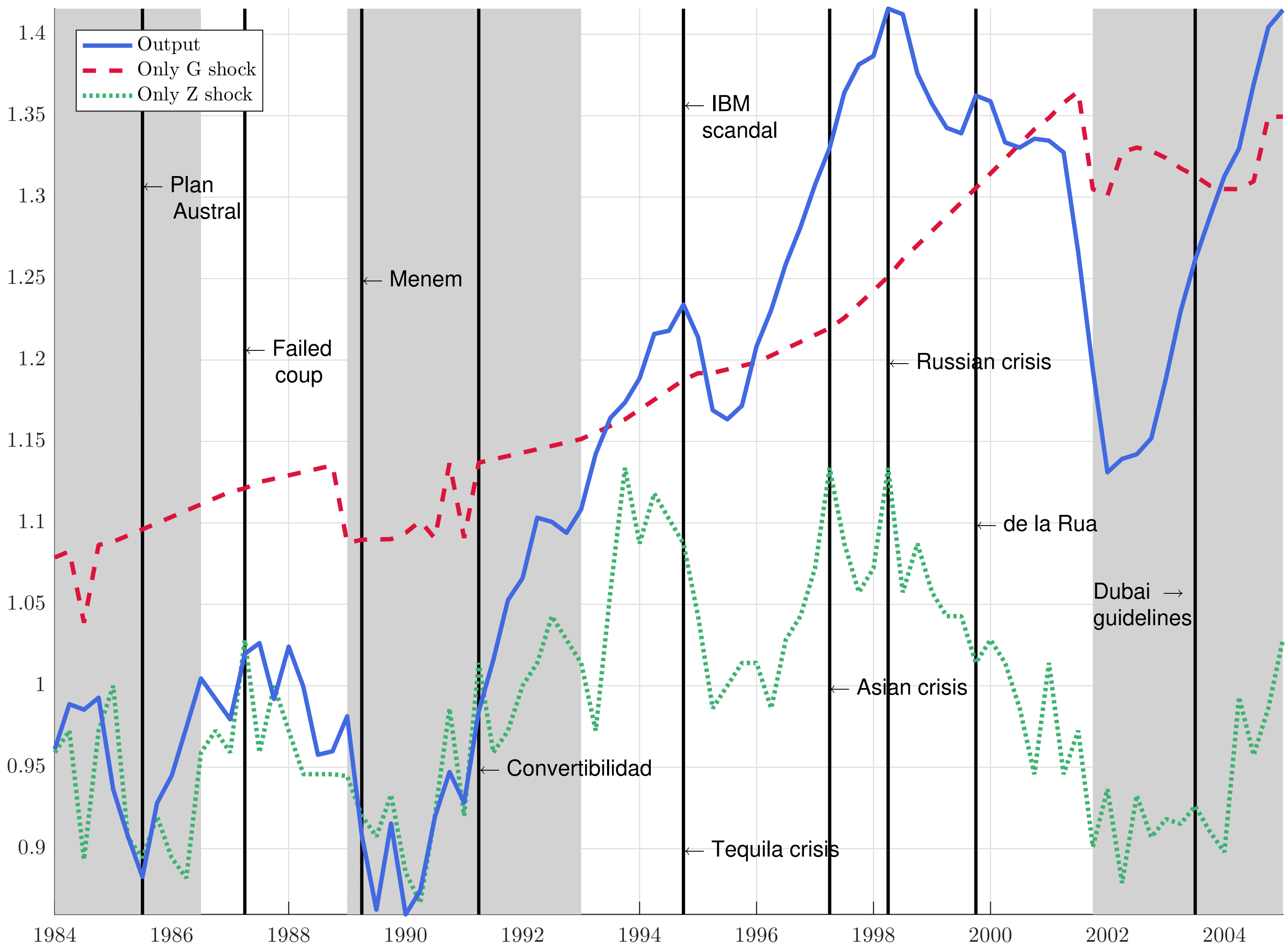

Figure 9: Output and Economic Events

tion, the plan worked initially through trend growth and later via temporary productivity advances. However, a change of fortunes in 1987, which coincides with a failed military coup against Alfonsin's presidency (Llach and Gerchunoff, 2018), reverses the recovery the economy enjoyed in 1986. Weak growth and rising inflation (prices claimed 10% in August 1987) were the first signs that the Plan Austral was failing.

By the end of a dismal economic decade, Carlos Menem was elected president, with the mission of reining on inflation and modernizing the economy.[15] After two tumultuous years in office, Menem appointed Domingo Cavallo as minister of the treasury, introducing the Convertibility plan as well as a series of privatizations in areas such as commercial aviation and telecommunication. After years of stagnation, the economy finally began to grow, which our model attributes to a combination of trend growth and positive temporary productivity innovations. The good times are also reflected in low spreads (figure 5), strong consumption growth (figure 6), and historically low haircuts (figure 7).

The Tequila crisis entered full swing in 1995, hitting the Argentinean economy at the same time that corruption scandals (like the IBM case; Sims, 1996) were rampant. These forces brought higher spreads (figure 5) and adverse (temporary and permanent) productivity shocks, resulting in a decline in GDP of 6%.[16] Interestingly, the plan convertibilidad was effective in keeping inflation on track, reaching 1.6% per year in 1995 (Llach and Gerchunoff, 2018). Prior to the Asian crisis, Argentina was back in trend thanks to favorable shocks and spreads. But Argentina ran out of luck as the Asian crisis brought adverse temporary shocks. After a quick rebound, the Russian crisis (1998) brought a second round of adverse temporary shocks, which marked the beginning of the collapse of Argentina's economy. By the time de la Rua takes power, GDP was 4 % below its peak. The combination of some trend growth and mild adverse temporary disturbances led to two years of muted output growth. However, the cumulative effect of the temporary negative productivity and a sudden reversal en trend productivity brings a deep recession, eventually forcing the economy to default.

5.3 Causation vs. selection in output/default intensity relationship

We now answer the question of *why* output growth is significantly lower following hard defaults compared to output growth in soft defaults. Part of the answer lies in selection, which is what we have stressed the most thus far. However, output is also causally lower

[15] Llach and Gerchunoff (2018) refers to this era as the new macroeconomy

[16] Mauro (1995) presents some evidence showing the negative relation between corruption and economic growth. De Rosa, Gooroochurn, and Gorg (2010) find that bribe taxes, defined as informal payments to government officials, have a negative effects on firm-level productivity.

because of the default costs $\chi(d_t; z_t, g_t)$. Because we have the shocks, we can say what $\chi_t(d_t; z_t, g_t)$ is at each point in time and, consequently, disentangle the direct effect on output versus selection. To this end, figure 10 shows the filtered path of output (blue line) and the filtered path excluding default costs (red dashed line). During the 2001 default, output in the model declined by 16 % from peak (1998) to trough (2002). Of this drop, about 5 percentage points corresponds to the cost of default. In other words, the decision to default contributed to about 1/3 of the contraction in GDP with the rest coming from fundamentals in the economy. This means that the default decision contributed to the recession but it was not its main driver. As the economy recovered, the gap between the two output measures closed as well. If we consider the default episode in 1990, the decline in GDP is close to 11 % of which 1 percentage point comes from default costs. Based on this decomposition, we can see that the 2001 episode was more costly in term of output loss from the decision to default. However, the sovereign still finds default optimal because at that time leverage was very high following the protracted growth of the 1990s and associated low spreads.

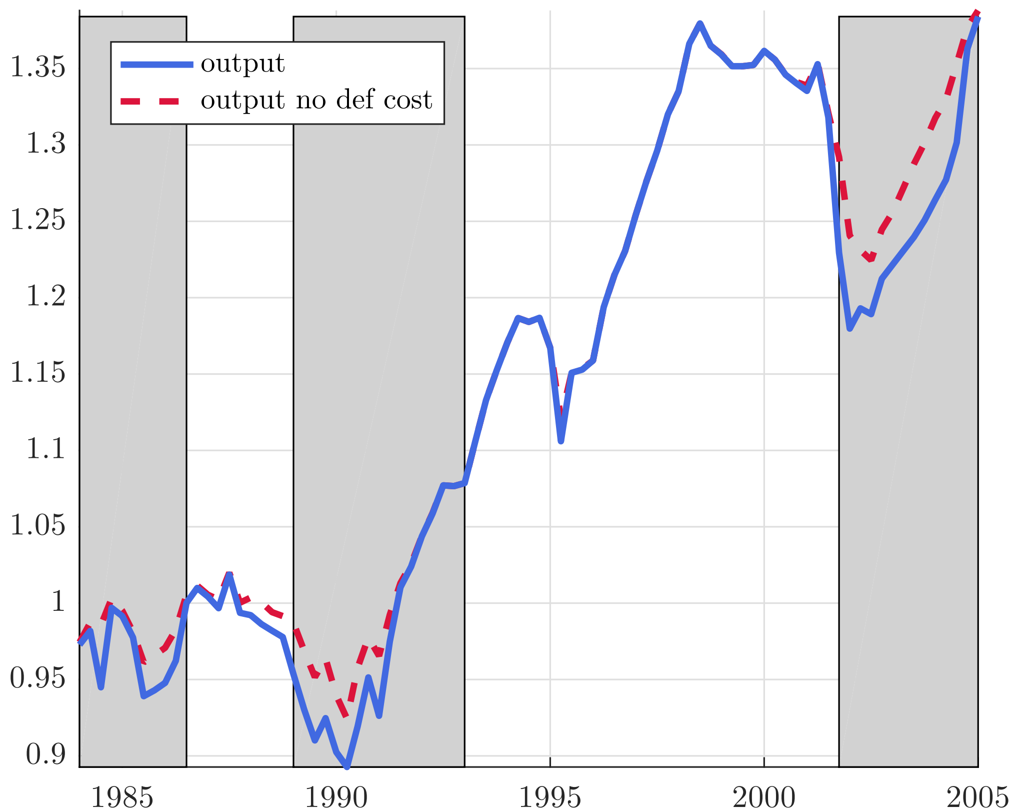

Figure 10: Output loss

6 Conclusion

We proposed a novel theory of hard and soft sovereign defaults that, while quite simple, captures many features of the data. First and foremost of these is the low output growth following hard defaults and comparatively high output growth following soft ones. The model rationalizes this feature by having negative growth shocks lead to protracted periods of non-payment and hence hard defaults with less negative growth shocks resulting in shorter period periods of non-payment. Using a historical shock decomposition to recover the shocks in, before, and after Argentina's defaults, the model successfully reproduced the behavior of the observable spreads and output as well as non-observables such as the the paths of debt, the magnitude of haircuts, and the timing of defaults episodes. The historical shock decomposition led to a natural narrative of Argentinean history that notably captured the election of Menem and the Tequila crisis. Finally, we used the model to quantify how much of the output declines observed in default are caused by default versus how much is selection into default. We found selection played the larger role in the soft default in the late 1980s while actual default costs played the larger role in the early stages of the 2001 hard default.

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A Data

The top panel of Figure 11 plots the log real GDP series for Argentina and the trend implied by the HP filter. Visually it appears Argentina goes through different growth regimes, and the filter picks this up. The bottom panel gives a scatter plot of the deviations from trend against their lag. The slope of the best fit line is the estimate for the transitory shock persistence (and the mean is zero). The bottom right panel does similarly, but for the first differences in the *trend* against their lag—the first difference is the growth shock. The slope of the best fit line is the persistence estimate for the growth shock, which reveals why the process persistence (.989) is so large.

Figure 12 plots Argentina's spread data in the raw time series (top panel) and against the growth shocks (bottom left panel) and transitory shocks (bottom right panel). For transitory shocks, there is a clear negative relationship in the raw data. For growth shocks there is also some evidence, but it is not as striking. This may be because of transitory shocks that create significant dispersion in the raw data.

Table 4 builds on these figures by regressing spreads on the growth shocks and transitory shocks, which here are linearly transformed to zero mean and unit variance for easier interpretation of the coefficients. To limit dependence on "outliers," we do a full sample regression in column (1) and through 2001 in column (2). These two shock values, independent of any debt measure or lagged spreads explains 50% of the data's spreads variation. Both shocks have a significant effect on spreads, but the transitory shock's impact is several times larger (but the shocks are less persistent, so the cumulative effect is unclear).

	(1) Spreads	(2) Spreads
Log real GDP trend 1st difference, normalized	-2.151	-1.081
	(0.001)	(0.006)
Log real GDP deviations, normalized	-7.802	-4.273
	(0.000)	(0.000)
Constant	12.79	10.89
	(0.000)	(0.000)
Observations	80	75
R^2	0.511	0.492

p-values in parentheses

Robust standard errors were used. (1) is full sample; (2) is up to 2001:4.

Table 4: Regressions of spreads on shocks

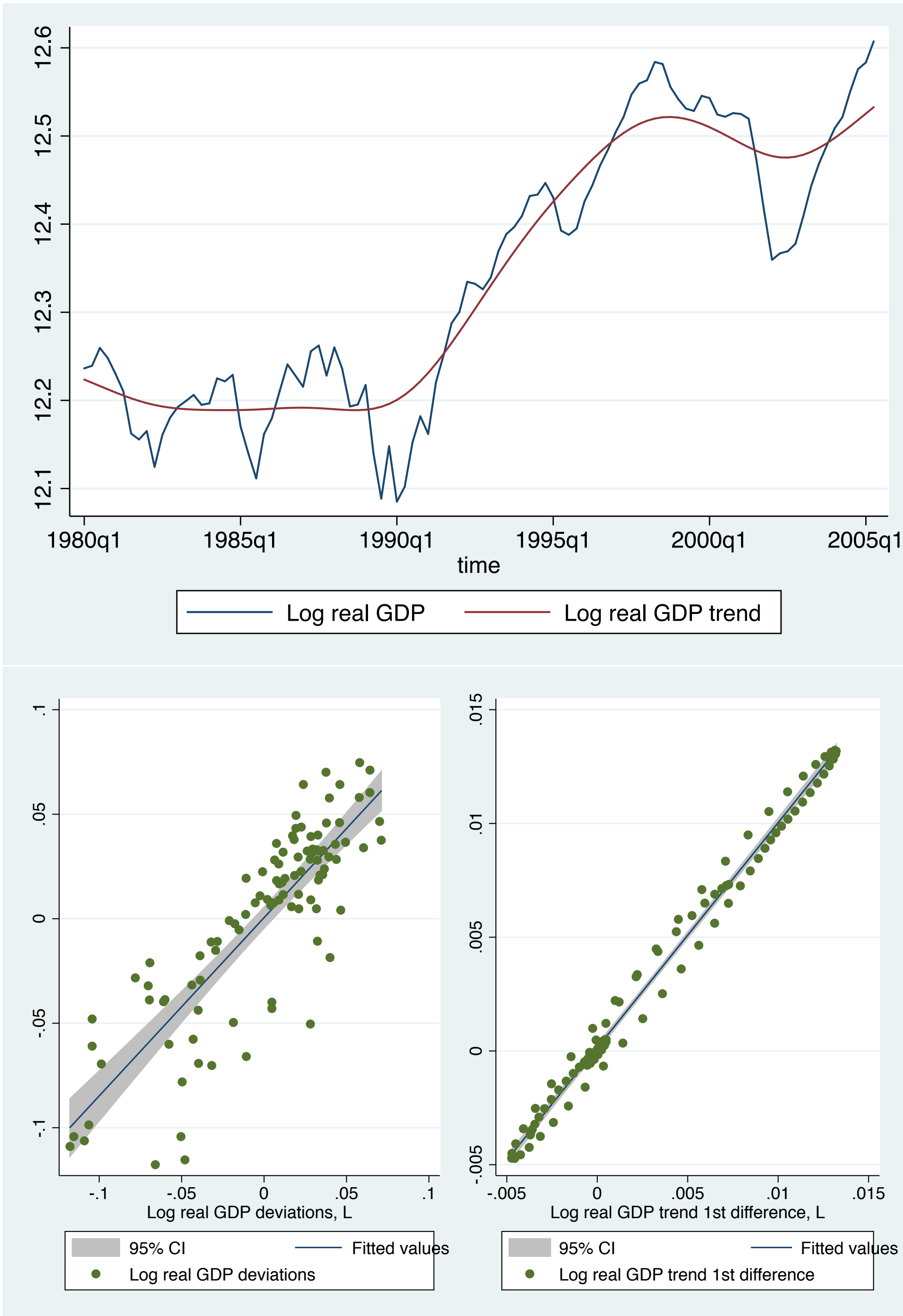

Figure 11: Argentina's output and shock identification

Figure 12: Argentina's spreads

B Detrending the equilibrium

B.1 Not detrended

Say $u(c) = c^{1-\sigma}/(1-\sigma)$. For any Γ, define $\tilde{\mathcal{B}}(\Gamma) = \{b'|b'/\Gamma \in \mathcal{B}\}$ for some $\mathcal{B}$. Let the state space be $\tilde{\mathcal{S}} = \{(b, z, g, \Gamma)|b \in \mathcal{B}(\frac{\Gamma}{g}), z \in \mathcal{Z}, g \in \mathcal{G}, \Gamma \in \mathbb{R}^{++}\}$.

$$\tilde{V}(b, z, g, \Gamma) = \max_{b' \in \tilde{\mathcal{B}}(\Gamma), d \in D} u(c) + \beta \mathbb{E}_{[z', g'|z, g]} \tilde{V}(b', z', g', \Gamma')$$
$$\text{s.t. } c + q(b', z, g, \Gamma)(b' - (1-\lambda)b) = z\Gamma(1 - \chi(d; z, g)) + \tilde{\lambda} b(1-d)$$
$$\Gamma' = g'\Gamma$$

Let the associated policies be denoted $\tilde{a}(b, z, g, \Gamma)$ (for savings), $\tilde{d}(b, z, g, \Gamma)$, and $\tilde{c}(b, z, g, \Gamma)$. Note that by the construction of $\tilde{\mathcal{B}}$, $b' \in \tilde{\mathcal{B}}(\Gamma)$ if and only if $b' \in \tilde{\mathcal{B}}(\Gamma'/g')$ for all g'. Because of this, the continuation utility is only evaluated at points in the state space, i.e., for any $b' \in \tilde{\mathcal{B}}(\Gamma)$, $(b', z', g', \Gamma') \in \tilde{\mathcal{S}}$ wherever the expectation operator has positive support.

The price schedule q, assuming a risk-neutral intermediary who discounts at rate $1+r^*$ is

$$\tilde{q}(b', z, g, \Gamma) = \frac{1}{1+r^*} \mathbb{E}_{[z', g'|z, g]} \left[(1 - \tilde{d}(b', z', g', \Gamma'))\tilde{\lambda} + (1-\lambda)\tilde{q}(b'', z', g', \Gamma') \right]$$

where $b'' = \tilde{b}'(b', z', g', \Gamma')$. A haircut in the model is

$$\tilde{H}_{sz}(b, z, g, \Gamma) := 1 - \frac{\text{market value of debt}}{\text{risk-free value of debt}} = 1 - \frac{\tilde{\lambda}(1 - \tilde{d}(b, z, g, \Gamma)) + (1-\lambda)\tilde{q}(\tilde{a}(b, z, g, \Gamma), z, g, \Gamma)}{\tilde{\lambda} + (1-\lambda)\bar{q}}$$

Note that $\tilde{q}$ is just a transformation of the conditional expectation of $\tilde{q}$.

B.2 Detrended

The above problem is equivalent (in a certain sense to be made precise shortly) to a detrended problem. Let the state space of the detrended problem be $\mathcal{S} = \mathcal{B} \times \mathcal{Z} \times \mathcal{G}$. The sovereign solves

$$V(b, z, g) = \max_{b' \in \mathcal{B}, d \in D} u(c) + \beta \mathbb{E}_{[z', g'|z, g]} g'^{(1-\sigma)} V(b', z', g')$$
$$\text{s.t. } c + q(b', z, g)(b' - (1-\lambda)\frac{b}{g}) = z(1 - \chi(d; z, g)) + \tilde{\lambda}\frac{b}{g}(1-d)$$

Let the policy functions associated with this problem be denoted $a(b,z,g), d(b,z,g)$, and $c(b,z,g)$. The price schedule satisfies

$$q(b',z,g) = \frac{1}{1+r^*}\mathbb{E}_{[z',g',m'|z,g]}(1 - d(\tilde{b'},z',g',m'))\tilde{\lambda} + (1-\lambda)q(b'',z',g')))$$

where $b'' = a(b',z',g')$. The haircut is

$$H_{sz}(b,z,g) = 1 - \frac{\tilde{\lambda}(1-d(b,z,g)) + (1-\lambda)q(a(b,z,g),z,g)}{\tilde{\lambda} + (1-\lambda)\bar{q}}$$

B.3 Equivalence

Proposition 4. *Any equilibrium (V,d,c,a,q) in the detrended problem corresponds to an equilibrium $(\tilde{V},\tilde{d},\tilde{c},\tilde{a},\tilde{q})$ in the not-detrended problem with*

$$\begin{aligned}
\tilde{V}(b,z,g,\Gamma) &= \Gamma^{1-\sigma}V(\frac{bg}{\Gamma},z,g)\\
\tilde{a}(b,z,g,\Gamma) &= \Gamma a(\frac{bg}{\Gamma},z,g)\\
\tilde{c}(b,z,g,\Gamma) &= \Gamma c(\frac{bg}{\Gamma},z,g)\\
\tilde{d}(b,z,g,\Gamma) &= d(\frac{bg}{\Gamma},z,g)\\
\tilde{q}(b',z,g,\Gamma) &= q(\frac{b'}{\Gamma},z,g)\\
\tilde{H}_{sz}(b,z,g,\Gamma) &= H_{sz}(\frac{bg}{\Gamma},z,g)
\end{aligned}$$

for all $(b,z,g,\Gamma) \in \mathcal{S}$ and $b' \in \mathcal{B}(\Gamma)$.

Proof. First, consider that the definition is well-defined in that the detrended functions are only being evaluted where defined. To see this, take $(b,z,g,\Gamma) \in \mathcal{S}$. Then $b \in \mathcal{B}(\Gamma/g)$, implying $b/(\Gamma/g) = bg/\Gamma \in \tilde{\mathcal{B}}$. So, $(bg/\Gamma,z,g) \in \tilde{\mathcal{S}}$. Similarly, $q(b',z,g,\Gamma)$ is defined for all $b' \in \mathcal{B}(\Gamma)$ while $\tilde{q}(\tilde{b'},z,g)$ is defined for all $b' \in \tilde{\mathcal{B}}$. But then $b' \in \mathcal{B}(\Gamma) = \{b'|b'/\Gamma \in \tilde{\mathcal{B}}\}$ if and only if $b'/\Gamma \in \tilde{\mathcal{B}}$. So, all the not-detrended policies, value, and price functions are well-defined.

Now consider the price schedule mapping. Use $\mathbb{E}$ in place of $\mathbb{E}_{[z',g'|z,g]}$. Let $q(b',z,g)$ be an equilibrium price schedule. Then

$$q(b',z,g) = \frac{1}{1+r^*}\mathbb{E}\left[(1-d(b',z',g'))\tilde{\lambda} + (1-\lambda)q(b'',z',g')))\right]$$

where $b'' = a(\tilde{b}', z', g')$. Fixing some $\Gamma \in \mathbb{R}^{++}$ and $b' \in \mathcal{B}(\Gamma)$, evaluating the above at $\tilde{b}' = b'/\Gamma$ gives

$$q(\frac{b'}{\Gamma}, z, g) = \frac{1}{1+r^*}\mathbb{E}\left[(1 - d(\frac{b'}{\Gamma}\frac{\Gamma'}{\Gamma'}, z', g'))\tilde{\lambda} + (1-\lambda)q(a(\frac{b'}{\Gamma}\frac{\Gamma'}{\Gamma'}, z', g'), z', g')\right]$$

Then, since $\Gamma'/\Gamma = g'$,

$$q(\frac{b'}{\Gamma}, z, g) = \frac{1}{1+r^*}\mathbb{E}\left[(1 - d(\frac{b'g'}{\Gamma'}, z', g'))\tilde{\lambda} + (1-\lambda)q(a(\frac{b'g'}{\Gamma'}, z', g'), z', g')\right].$$

Since $a(\frac{b'g'}{\Gamma'}, z', g') = \tilde{a}(b', z', g', \Gamma')/\Gamma'$ and $d(\frac{b'g'}{\Gamma'}, z', g') = \tilde{d}(b', z', g', \Gamma')$, one has

$$q(\frac{b'}{\Gamma}, z, g) = \frac{1}{1+r^*}\mathbb{E}\left[(1 - d(b', z', g', \Gamma'))\tilde{\lambda} + (1-\lambda)q(\frac{\tilde{a}(b', z', g', \Gamma')}{\Gamma'}, z', g')\right].$$

Then, using $\tilde{q}(b', z, g, \Gamma) = q(\frac{b'}{\Gamma}, z, g)$ and $\tilde{q}(b'', z', g', \Gamma') = q(\frac{b''}{\Gamma'}, z', g')$, one has

$$\tilde{q}(b', z, g, \Gamma) = \frac{1}{1+r^*}\mathbb{E}\left[(1 - \tilde{d}(b', z', g', \Gamma'))\tilde{\lambda} + (1-\lambda)\tilde{q}(\tilde{a}(b', z', g', \Gamma'), z', g', \Gamma')\right],$$

which is what the equilibrium price schedule must satisfy.

Now, by the optimality of the detrended value functions, the value functions solve a fixed point problem $V = T \circ V$ taking q as given. Now, we want to show that the definitions $\tilde{V}$ satisfy the fixed point problem associated with the trend problem $\tilde{V} = \tilde{T} \circ \tilde{V}$ taking $\tilde{q}$ as given.

The V problem

$$V(b, z, g) = \max_{b' \in \mathcal{B}, d \in D} u(c) + \beta\mathbb{E}g'^{(1-\sigma)}V(b', z', g')$$
$$\text{s.t. } c + q(b', z, g)(b' - (1-\lambda)\frac{b}{g}) = z(1 - \chi(d; z, g)) + \tilde{\lambda}\frac{b}{g}(1-d).$$

For any $\Gamma \in \mathbb{R}^{++}$, multiplying through by $\Gamma^{1-\sigma}$ and using $\Gamma^{1-\sigma}V(b, z, g) = \tilde{V}(b\Gamma/g, z, g, \Gamma)$, this can equivalently be written

$$\tilde{V}(\tilde{b}, z, g, \Gamma) = \max_{c, b', \tilde{c}, \tilde{b}' \in \tilde{\mathcal{B}}, d \in D} u(c\Gamma) + \beta\mathbb{E}\Gamma^{1-\sigma}g'^{(1-\sigma)}V(\frac{b'}{\Gamma}, z', g')$$
$$\text{s.t. } c + q(b', z, g)(b' - (1-\lambda)\frac{b}{g}) = z(1 - \chi(d; z, g)) + \tilde{\lambda}\frac{b}{g}(1-d)$$
$$\tilde{b} = b\Gamma/g$$

Noting that $\Gamma^{1-\sigma} g'^{(1-\sigma)} V(b', z', g') = \Gamma'^{(1-\sigma)} V(b', z', g') = \tilde{V}(b'\Gamma'/g', z', g', \Gamma') = \tilde{V}(b'\Gamma, z', g', \Gamma')$, one has

$$\begin{aligned}
\tilde{V}(\tilde{b}, z, g, \Gamma) &= \max_{c, b', \tilde{c}, \tilde{b}' \in \tilde{\mathcal{B}}, d \in D} u(\tilde{c}) + \beta \mathbb{E} \tilde{V}(\tilde{b}', z', g', \Gamma') \\
&\tilde{b} = b\Gamma/g, \ \tilde{b}' = b'\Gamma, \ \tilde{c} = c\Gamma \\
\text{s.t. } \frac{\tilde{c}}{\Gamma} + q(\frac{\tilde{b}'}{\Gamma}, z, g)(\frac{\tilde{b}'}{\Gamma} - (1-\lambda)\frac{\tilde{b}}{\Gamma}) &= z(1 - \chi(d; z, g)) + \lambda \frac{\tilde{b}}{\Gamma}(1-d)
\end{aligned}$$

where superfluous choice variables $\tilde{c}$ and $\tilde{b}'$ have been added. Since $a(\tilde{b}, z, g)$ and $c(\tilde{b}, z, g)$ are optimal policies for b' and c, it is clear that $\tilde{b}' = \Gamma a'(\tilde{b}, z, g)$ and $\tilde{c} = \Gamma c(\tilde{b}, z, g)$ are optimal.

Using $q(\tilde{b}'/\Gamma, z, g) = \tilde{q}(\tilde{b}', z, g)$ and multiplying the budget constraint through by Γ, one then has

$$\begin{aligned}
\tilde{V}(\tilde{b}, z, g, \Gamma) &= \max_{c, b', \tilde{c}, \tilde{b}' \in \tilde{\mathcal{B}}, d \in D} u(\tilde{c}) + \beta \mathbb{E} \tilde{V}(\tilde{b}', z', g', \Gamma') \\
&\tilde{b} = b\Gamma/g, \ \tilde{b}' = b'\Gamma, \ \tilde{c} = c\Gamma \\
\text{s.t. } \tilde{c} + \tilde{q}(\tilde{b}', z, g)(\tilde{b}' - (1-\lambda)\tilde{b}) &= z\Gamma(1 - \chi(d; z, g)) + \lambda \tilde{b}(1-d)
\end{aligned}$$

Noting that, we have

$$\begin{aligned}
\tilde{V}(\tilde{b}, z, g, \Gamma) &= \max_{\tilde{c}, \tilde{b}' \in \tilde{\mathcal{B}}, d \in D} u(\tilde{c}) + \beta \mathbb{E} \tilde{V}(\tilde{b}', z', g', \Gamma') \\
\text{s.t. } \tilde{c} + \tilde{q}(\tilde{b}', z, g)(\tilde{b}' - (1-\lambda)\tilde{b}) &= z\Gamma(1 - \chi(d; z, g)) + \lambda \tilde{b}(1-d)
\end{aligned}$$

This is exactly the form of the not-detrended problem, and optimal policies for it (from the discussion above) $\tilde{a}(\tilde{b}, z, g, \Gamma) = \Gamma a(bg/\Gamma, z, g)$, $\tilde{c}(\tilde{b}, z, g, \Gamma) = \Gamma c(bg/\Gamma, z, g)$, and $d(\tilde{b}, z, g, \Gamma) = d(bg/\Gamma, z, g)$.

The equivalence for haircuts is given by

$$\begin{aligned}
\tilde{H}_{sz}(b, z, g, \Gamma) &= 1 - \frac{\tilde{\lambda}(1 - \tilde{d}(b, z, g, \Gamma)) + (1-\lambda)\tilde{q}(\tilde{a}(b, z, g, \Gamma), z, g, \Gamma)}{\tilde{\lambda} + (1-\lambda)\bar{q}} \\
&= 1 - \frac{\tilde{\lambda}(1 - d(bg/\Gamma, z, g)) + (1-\lambda)\tilde{q}(\Gamma a(bg/\Gamma, z, g), z, g, \Gamma)}{\tilde{\lambda} + (1-\lambda)\bar{q}} \\
&= 1 - \frac{\tilde{\lambda}(1 - d(bg/\Gamma, z, g)) + (1-\lambda)q(a(bg/\Gamma, z, g), z, g)}{\tilde{\lambda} + (1-\lambda)\bar{q}} \\
&=: H_{sz}(bg/\Gamma, z, g)
\end{aligned}$$

$\square$

Note that the proposition does not establish that a "trend" equilibrium can be mapped into an "untrended" one. This is because, in the case of multiplicity, Γ might act as a sunspot variable on which creditors coordinate.

Tabel 5 gives the mapping from detrended variables to observables.

Observable	Trend variable	Mapping to trend
Debt state	$-\tilde{b}_t$	$-\Gamma_t b_t/g_t$
Debt choice	$-\tilde{b}_{t+1}$	$-\Gamma_t b_{t+1} = \tilde{b}_{t+1} g_{t+1}$
Observed bond price	$\tilde{q}(\tilde{b}_{t+1}, z_t, g_t)$	$q(b_{t+1}, z_t, g_t)$
Consumption	$\tilde{c}_t$	$\Gamma_t c_t$
Output	$\tilde{y}_t$	$\Gamma_t z_t(1-\chi_t)$
Net exports	$\tilde{y}_t - \tilde{c}_t$	$\Gamma_t(z_t(1-\chi_t) - c_t)$
Debt issuance at face value	$-(\tilde{b}_{t+1} - (1-\lambda)\tilde{b}_t)$	$-\Gamma_t(b_{t+1} - (1-\lambda)\frac{b_t}{g_t})$
Debt issuance market at value	$-\tilde{q}(\tilde{b}_{t+1}, z_t, g_t)\left(\tilde{b}_{t+1} - (1-\lambda)\tilde{b}_t\right)$	$-\Gamma_t q(b_{t+1}, z_t, g_t)(b_{t+1} - (1-\lambda)\frac{b_t}{g_t})$

Table 5: Mapping detrended variables to the data

C Filtering

In this section, we describe the details involved in filtering Argentina's data.

The model solution consists on the grids of permanent shocks, g, and transitory shocks, z; the grid of bond choices; the transition matrix between the shock states, $p(z', g'|z, g)$; the probability distribution of default choice given past debt and shocks, $p(d|b, z, g)$; the probability distribution of debt given past debt, shocks, and default choice, $p(b'|b, z, g, d)$; and the stationary distribution of debt and shocks, $p(b, g, z)$. The joint grid of g and z has 441 elements, the grid of debt has 250.

Because we are dealing with discrete variables, the standard particle filter does not apply directly. First, the discreteness of the state variables limits the number of particles. We choose the maximum, which is given by the size of the (g, z) grid times the size of the debt grid, i.e. $N_p = 441 \times 250 = 110,250$. To draw from the discrete distributions, we use the inversion method (Cappe, Moulines, and Ryden (2005)). We proceed according to the following algorithm:

Result: Filtered states and likelihood
initialization: draw initial particles $\{(b^i, g^i, z^i)\}$ for $i = 1, \ldots, M$ from $p(b, z, g)$;
set weights $\omega_0^i = 1$;
for $t = 1$ **to** T **do**
  forecasting;
  for $i = 1, \ldots, M$ **do**
    draw default choices d_t^i from $p(d|b_t^i, g_{t-1}^i, z_{t-1}^i)$;
    draw debt choices b_{t+1}^i from $p(b'|b_t^i, g_{t-1}^i, z_{t-1}^i, d_t^i)$;
    compute output, consumption, spread, haircut: $y_t^i, c_t^i, sp_t^i, H_t^i$;
    use y_t^i, sp_t^i and data to compute likelihood ℓ_t^i;
    set weights $\omega_t^i = \ell_t^i$;
  end
  updating for $i = 1, \cdots, M$ **do**
    compute normalized weights: $\widetilde{\omega}_t^{(i)} = \frac{\omega_t^{(i)} \omega_{t-1}^{(i)}}{\frac{1}{M} \sum \omega_t^{(i)} \omega_{t-1}^{(i)}}$;
    resample from multinomial distribution $\left\{((b')^i, g^i, z^i), \widetilde{\omega}^{(i)}\right\}$;
    set $\omega_t^{(i)} = 1$;
  end
  approximate state distribution and likelihood are:

$$p(x_t|Y_{1:t}) \approx \sum_{i=1}^{M} \omega_t^{(i)} \delta(x_t - x_t^{(i)}), \quad p(y_t|Y_{1:t-1}) \approx \frac{1}{N} \sum_{i=1}^{M} \ell_t^i. \tag{2}$$

  here, $x_t = \{b_t, g_t, z_t\}$
end

Algorithm 1: Filtering Argentina's default

We implement a computationally efficient bootstrap particle filter in Cuda 9.1 and do the filtering using a Nvdia GPU.