The Effects of Monetary Policy Regime Shifts on the Term Structure of Interest Rates☆

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April 2011

Abstract

We investigate how the entire term structure of interest rates is influenced by changes in monetary policy regimes. To do so, we develop and estimate an arbitrage-free dynamic term-structure model which accounts for regime shifts in monetary policy, volatility, and the price of risk. Our results for U.S. data from 1985-2008 indicate that (*i*) the Fed's reaction to inflation has changed over time, switching between "more active" and "less active" monetary policy regimes, (*ii*) on average, the slope of the yield curve in the "more active" regime was steeper than in the "less active" regime, and (*iii*) the yield curve in the "more active" regime was considerably more volatile than in the "less active" regime. The steeper yield curve in the "more active" regime reflects higher term premia that result from the risk associated with a more volatile future short-term interest rate given a more sensitive response to inflation.
(JEL G12, C11, E43)

Keywords: Term structure of interest rates, Affine no-arbitrage model, Markov switching process, Bayesian MCMC estimation

☆The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of Richmond or the Federal Reserve System. We are especially grateful to James Morley for his valuable discussions and suggestions. We would also like to thank Steve Fazzari, William Gavin, Werner Ploberger, Guofu Zhou, and participants at the 2011 ASSA meetings for their helpful comments. All remaining errors are our own.

1. Introduction

As stated in the Federal Reserve Act, the Federal Reserve System shall maintain moderate long-term interest rates, as one of the monetary policy objectives. A number of research papers show importance of long-term interest rates for the real economy and study the relationship between the short-term interest rate - the Fed's monetary policy instrument and long-term interest rates. Goodfriend (1991) argues that variations in long-term interest rates are more important determinants for the aggregate output and prices than the short-term interest rate fluctuations. Cook and Hahn (1989), Rudebusch (1985), and Goodhart (1996) provide evidences that changes in monetary policy targets for the short-term interest rate affect long-term interest rates. Also, it is believed that the monetary policy stance has changed in terms of the response to the output and inflation gap in the last four decades (e.g. Clarida et al. (2000) and Boivin and Giannoni (2006)). Therefore, the effects of the monetary policy changes on the yield curve is an interesting and important area for examination.

While a number of empirical studies (e.g. Clarida et al. (2000); Cogley and Sargent (2005)) focus mainly on the response of the macroeconomic fundamentals to the changes in monetary policy stance, however only a few studies (e.g. Bikbov and Chernov (2008) and Ang et al. (2010) hereafter ABDL(2010)) look at the implications of monetary policy changes for the term structure of interest rates.

As discussed in ABDL(2010), the entire term structure of interest rates may respond to the changes in short-term interest rate in two main ways. First, the inflation and output fluctuations caused by the short-term interest rate changes may influence term premia. This effect is supported by many recent studies which provide evidence of the impact of macroeconomic factors on the term structure of interest rates (e.g. Ang and Piazzesi (2003); Ang et al. (2008); and Bikbov and Chernov (2010)). Second, according to the no-arbitrage condition, long-term interest rates should be affected by changes in the short-term interest rate.

Similarly, the way monetary policy is conducted can have two potential implications for long-term interest rates. First, the monetary authority may reduce inflation risk premia for long-term interest rates through aggressively changing the short rate in re-

sponse to macroeconomic fluctuations. Second, for a given expected inflation, a more sensitive short rate in response to macroeconomic fluctuations may cause expectations of a more volatile future short rate, which could result in higher risk premia for long-term interest rates. Thus, the monetary authority may face a trade-off between these two opposite effects on long-term interest rates in their choice of how aggressively to respond to macroeconomic fluctuations. In addition, as discussed in Bikbov and Chernov (2008), if long-term interest rates respond to the changes in monetary policy stance, then the term structure of interest rates may contain more useful information for identifying the monetary policy regimes as compared to only considering the short rate.

The main objective of this paper is to analyze effects of monetary policy regime changes on the entire term structure of interest rates. Specifically, we aim to identify which of the two above-described effects on long-term rates dominates when the monetary authority responds aggressively to macroeconomic fluctuations. For this analysis, we propose an affine no-arbitrage term structure model with regime shifts in monetary policy, volatility of yield factors, and the market price of risk governed by three separate Markov-switching processes. This framework enables us to identify the effects of monetary policy regime shifts on long-term interest rates. In our model, the short-term interest rate, which is considered as the monetary policy instrument, is set by a Taylor (1993) rule with coefficients switching between two monetary policy regimes. These regimes are labeled as "more active" and "less active" regimes, depending on how aggressively the monetary authority changes the short rate in response to inflation and output gap fluctuations. Similar to previous studies, the regime-switching processes are assumed to be exogenous in our model, and therefore we do not aim to study why the monetary policy stance change over time.

Our results can be summarized as follows. First, our results indicate that even during "the Great Moderation" period of the past quarter century, the Fed's reaction to inflation has varied over time, switching between "more active" and "less active" regimes. This result concurs with Sims and Zha (2006), Bianchi (2009), and ABDL(2010), who conclude that regime shifts of monetary policy should be considered probabilistically rather than by only a single break in the early 1980s.

Second, monetary policy regime shifts have quantitatively important effects on the term spread and the volatility of the yield curve. For the sample of U.S. data from 1985:Q4 to 2008:Q4, the short-term interest rate was considerably more volatile in the "more active" regime than in the "less active" regime, while the average short-term interest rates in the two monetary policy regimes were close to each other. The long-term interest rate, defined as the 10-year maturity yield, was on average 129 basis points higher in the "more active" regime than in the "less active" regime, resulting in a steeper slope of the yield curve, on average, in the "more active" regime. In general, the yield curve was more volatile in the "more active" regime than in the "less active" regime. These results can be explained by a more sensitive response of the short rate to inflation fluctuations in the "more active" regime creating higher risk for the future short rate fluctuations. This risk drives up long-term yields. Thus, the Fed appears to face a policy trade-off between a "more active" reaction to the macroeconomic fluctuations and higher long-term interest rates as well as a more volatile yield curve caused by this reaction. This argument is consistent with Woodford (1999), who claims that it may be more optimal for the monetary authority to conduct policies that do not require the short rate to be too volatile.

Our study is distinguished in several dimensions from Bikbov and Chernov (2008) and ABDL(2010), who also investigate the interaction between the term structure of interest rates and monetary policy. In particular, our model employs discrete-time regime-switching processes in contrast to ABDL(2010), who describe monetary policy shifts as continuously changing Taylor rule coefficients. Also, our model is differentiated from ABDL(2010) by incorporating volatility regime shifts, which, as indicated by Sims and Zha (2006), is important for evaluating the impact of monetary policy changes on macroeconomic behavior. Unlike Bikbov and Chernov (2008), who also apply discrete regimes, our model accounts for the regime shifts in the price of risk that are independent of volatility changes. Duffee (2002) reports that it is essential to allow for variation in the price of risk independent of factor volatility for fitting the yield curve and modeling plausible term premium. Through model comparisons, we confirm that accounting for the regime shifts in volatility and the price of risk considerably improves fitting of

the model. Also, as we show in our analysis, these two regime-switching processes substantially help the model to produce a plausible variation in the term premium, and therefore provide necessary flexibility for the monetary policy regimes to be identified by the policy response to macroeconomic fluctuations in greater extend than by the term premium.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the estimation method. Section 4 presents the empirical results. Section 5 concludes. The Appendices provide details for the model derivation and the estimation method.

2. Model

In this section, we present our model used to quantify effects of monetary policy regime shifts on the term structure of interest rates. In particular, we develop a three-factor affine no-arbitrage term structure model with regime shifts in monetary policy response to macroeconomic fluctuations. The model also accounts for changes in volatility of yield factors and the market price of risk, governed by two other regime-switching processes. This modeling choice allows us to separate the identification of monetary policy changes from changes in volatility of yield factors and the market price of risk. To derive bond prices that account for the effects of monetary policy regime shifts and satisfy no-arbitrage condition, we make assumptions about a monetary policy response function, evolutions of regime processes, dynamics of factor process, and a stochastic discount factor, described in the following subsections.

2.1. Short-Term Interest Rate and the Monetary Policy Rule

We assume that the monetary authority use the short-term interest rate as their policy instrument and set it according to the Taylor rule (1993) with coefficients subject to regime shifts:

$$r_t^{m_t} = \overline{r}^{m_t} + \alpha^{m_t}\left(\pi_t - \overline{\pi}^{m_t}\right) + \beta^{m_t} g_t + u_t \ , \tag{2.1}$$

where $r_t^{m_t}$ is the short rate, π_t is inflation, $\overline{\pi}^{m_t}$ is the inflation target, g_t is the output gap, $\overline{r}^{m_t}$ is the target level of the nominal short rate for the case when inflation and output

gaps are zero, α^{m_t} and β^{m_t} are policy response coefficients to inflation and output gaps, respectively, and u_t captures all other determinants of the short rate, including monetary policy inertia and the monetary policy shock, that are not related to the current output and inflation gaps. Superscript m_t denotes the monetary policy regime.

In this specification of the policy rule, similarly to ABDL(2010), the monetary authority is assumed to respond to contemporaneous inflation and output gap, in contrast to expected inflation and output gap used in some studies on the Taylor rule (e.g. Clarida et al. (2000)). Sims and Zha (2006) argue that using expected inflation in the policy rule may result in distorted conclusions because expected inflation will be measured as a set of all influences on monetary policy and also it has less variation than current nominal variables, potentially causing spuriously scaled up response coefficients.

In our specification of the policy rule, the response coefficients to inflation and output gaps switch between two monetary policy regimes. These monetary policy regimes m_t are governed by a two-state Markov chain with transition matrix

$$\Pi_m \equiv \begin{bmatrix} 1 - p_m^{12} & p_m^{12} \\ p_m^{21} & 1 - p_m^{21} \end{bmatrix}, \tag{2.2}$$

where $p_m^{jk} = \mathsf{Pr}[m_t = k | m_{t-1} = j] \in [0, 1]$.

As pointed out by ABDL(2010), if u_t is correlated with inflation and output, then estimation of the standard Taylor rule equation (i.e. equation (2.1) with single regime) does not produce consistent estimates of the response coefficients. This correlation may be caused by contemporaneous effect of the monetary shocks on macroeconomic variables. However, Ang et al. (2007b), Bikbov and Chernov (2008), and ABDL(2010) show that u_t can be identified by utilizing the information in the entire term structure of interest rates through a no-arbitrage restriction. Also, Bikbov and Chernov (2008) empirically show that with the information in the short rate only the monetary policy regimes are not identified well.

2.2. Factor Dynamics

Similarly to many studies on the term structure of interest rates in the macro-finance literature (e.g., Ang and Piazzesi (2003); Ang et al. (2007b); and Bikbov and Chernov (2008)), we describe the dynamics of bond prices by three factors $\mathbf{f}_t = (u_t, \pi_t, g_t)'$, two

of which are observable macro variables (π_t, g_t) and one is a latent variable u_t . The factor dynamics are assumed to follow a regime-dependent Gaussian vector autoregressive process and can be described by

$$\mathbf{f}_{t+1} - d^{m_{t+1}} = G\left(\mathbf{f}_t - d^{m_t}\right) + L^{v_{t+1}}\varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}_{3\times 1}(0, I) \ , \tag{2.3}$$

where G is 3×3 matrix; $L^{v_{t+1}}$ is the lower-triangular Cholesky decomposition of $\Omega^{v_{t+1}}$ matrix that denotes the variance-covariance matrix of the factor shocks, d^{m_t} is the mean of factors within each monetary policy regime.[1] We assume that the factors volatilities can change their values between "low" and"high" volatility regimes denoted by v_t and governed by a two-state Markov-switching process with transition probability matrix

$$\Pi_v \equiv \begin{bmatrix} 1 - p_v^{12} & p_v^{12} \\ p_v^{21} & 1 - p_v^{21} \end{bmatrix}. \tag{2.4}$$

In this setting, similar to Bikbov and Chernov (2008) and ABDL(2010), the factors are assumed to be exogenous, and therefore we analyze effects of monetary policy regime shifts on the yield curve for a given expected inflation and output gap.[2] Also, by setting the persistence parameter matrix G to be regime-independent we avoid having potential changes in persistence influence the identification of the monetary policy regimes.[3]

2.3. Market Price of Risk

To model risk premia for long rates, we specify the market price of risk to have a time-varying form. Similarly to Ang et al. (2008), the market price of risk is assumed

[1]We describe the model more generally with regime-switching d^{m_t}, while for the estimation we assume this parameter to be regime-independent. As we described in subsection (3.2), this assumption helps identify the regimes.

[2]The interaction between the real economy and the bond markets can be modeled endogenously in the general equilibrium framework (e.g., Wu (2006), Wachter (2006), Rudebusch and Swanson (2008), and Bekaert et al. (2010)). Despite this advantage, the general equilibrium modeling approach does not have flexibility for incorporating all regime-switching processes described in our work because of difficulties of model solutions.

[3]The persistence of latent factor and inflation could be assumed to be policy dependent. Watson (1999) finds that persistence of the short rate increased over the two sample periods: 1965-1978 and 1985-1998. For the sample period considered in our study, preliminary estimates of the model with regime-switches in persistence parameters indicates that the estimates of these parameters are close to each other in the two identified monetary policy regimes.

to have the regime-switching and essentially affine in the factors form:

$$\Lambda_t^{l_{t+1}} = \lambda_0^{l_{t+1}} + \lambda_f \mathbf{f}_t \ , \tag{2.5}$$

where λ_f is a 3×3 matrix and $\lambda_0^{l_{t+1}}$ is 3×1 vector, which switches between "high" and "low" price of risk regimes denoted by l_t and governed by a two-state Markov-switching process with transition matrix

$$\Pi_l \equiv \begin{bmatrix} 1 - p_l^{12} & p_l^{12} \\ p_l^{21} & 1 - p_l^{21} \end{bmatrix}. \tag{2.6}$$

In the context of the general equilibrium framework, the market price of risk is mainly determined by the consumer's preference as well as the monetary policy reaction coefficients. Thus, in our model, allowing for regime shifts in the market price of risk helps accommodate potential changes in the agents' preference. On the empirical side, as we show in Section 4, accounting for the regime-switching in $\lambda_0^{l_{t+1}}$ considerably improves the model fitting. It provides grater flexibility for the model to generate plausible time-variation in risk premium in contrast to the time-variation in the price of risk that is originated only from the factors. This feature distinguishes our work from Bikbov and Chernov (2008). For tractability we assume that the matrix λ_f is regime independent.

2.4. Bond Prices

We follow Davig and Doh (2009) and assume that the monetary policy (m_t), volatility (v_t), and price of risk (l_t) regime processes are independent from each other.[4] Because each regime process has two regimes, the aggregate regime process, denoted as s_t, becomes a first-order eight-state Markov process and its transition probability matrix is given by $\Pi = \Pi_l \otimes \Pi_v \otimes \Pi_m$.

Bond pricing with a no-arbitrage restriction is derived by assuming the existence of a stochastic discount factor $\kappa_{t,t+1} = \kappa(\mathbf{f}_t, s_t; \mathbf{f}_{t+1}, s_{t+1})$ that establishes a recursion for

[4]The monetary policy regime changes can presumably depend on other two regime-processes, therefore it would be more desirable that the regimes are assumed to be dependent. However, computationally this specification is difficult to implement in our high dimensional case with eight regimes.

pricing bonds of different maturities:

$$P_{\tau,t}^{s_t} = \mathbb{E}\left[\kappa_{t,t+1} P_{\tau-1,t+1}^{s_{t+1}} | \mathbf{f}_t, s_t\right], \tag{2.7}$$

where $P_{t,\tau}^{s_t}$ denotes the price of bond at time t in regime s_t that matures at period $(t+\tau)$ and $\mathbb{E}$ is an expectation operator. Note that this expectation is conditional on the current factors and regimes since they are assumed to be known to agents. Meanwhile, the future values of the factors and regimes are unknown and follow the stochastic processes described in the previous subsections, and thus the expectation is over the future uncertainties. However, the whole time path of the factors and regimes (even the past values of the latent factor and regimes) are not observable to econometricians and to be estimated.

In order to impose the no-arbitrage condition, we follow Ang et al. (2008) and assume that the stochastic discount factor has the form[5]:

$$\kappa_{t,t+1} = \exp\left(-r_t^{s_t} - \frac{1}{2}\Lambda_t^{s_{t+1}\prime}\Lambda_t^{s_{t+1}} - \Lambda_t^{s_{t+1}\prime}\varepsilon_{t+1}\right), \tag{2.8}$$

where $\Lambda_t^{s_{t+1}}$ is given by equation (2.5).

The logarithms of bond prices are assumed to be affine in the factors and they depend on three regime processes:

$$\log P_{\tau,t}^{s_t} = -A_\tau^{s_t} - B_\tau^{s_t\prime}\mathbf{f}_t\ , \tag{2.9}$$

where $A_\tau^{s_t}$ and $B_\tau^{s_t}$ are regime specific coefficients a the bond of maturity τ .

In order to represent the continuously-compounded short rate as an affine function of the factors, the Taylor rule equation (2.1) is transformed to the form:

$$r_t^{s_t} = \delta_0^{s_t} + \delta_f^{s_t\prime}\mathbf{f}_t\ , \tag{2.10}$$

[5]In contrast to Dai et al. (2007) and Ang et al. (2010), our model specification does not allow us to price the risk of regime shifts explicitly. Explicit pricing the regime-shift risk in our setting would require assuming a factor process in which the next-period-regime uncertainty does not affect the conditional distribution of factors $\mathbf{f}_{t+1}$. As discussed in Bansal and Zhou (2002), the implication of this assumption is not consistent with the evidence reported by Hamilton (1988) and Gray (1996). These two studies empirically show that the short-rate dynamics are successfully described as a mixture of conditional Normal distributions.

where it can easily be seen that $\delta_0^{s_t} = \overline{r}^{s_t} - \alpha^{s_t}\overline{\pi}^{s_t}$ and $\delta_f^{s_t} = \begin{pmatrix} 1 & \alpha^{s_t} & \beta^{s_t} \end{pmatrix}'$.

To solve for A_τ^j and B_τ^j, we substitute for $P_{t,\tau}^{s_t}$ and $P_{t,\tau-1}^{s_{t+1}}$ in equation (2.7) and, following Bansal and Zhou (2002), we use the law of iterated expectations, the method of undetermined coefficients, and log-linearization as discussed in Appendix A. The solution has a form of recursive system:

$$\begin{aligned} A_\tau^j &= \delta_0^j + \sum_{k=1}^{S} p^{jk} \Big(A_{\tau-1}^k + \left(d^k - Gd^j - L^k\lambda_0^k\right)' B_{\tau-1}^k \\ &\quad - \frac{1}{2} B_{\tau-1}^{k\prime} L^k L^{k\prime} B_{\tau-1}^k \Big) \qquad (2.11) \\ B_\tau^j &= \delta_f^j + \sum_{k=1}^{S} p^{jk} \left(G - L^k\lambda_f\right)' B_{\tau-1}^k \qquad (2.12) \end{aligned}$$

with the initial conditions given by $A_1^j = \delta_0^j$ and $B_1^j = \delta_f^j$. Given this recursion, the continuously-compounded yield for a τ-maturity zero-coupon bond is determined by

$$R_{\tau,t}^{s_t} = -\frac{1}{\tau} \log\left(P_{\tau,t}^{s_t}\right) = a_\tau^{s_t} + b_\tau^{s_t\prime}\mathbf{f}_t \ , \qquad (2.13)$$

where $a_\tau^{s_t} = \frac{A_\tau^{s_t}}{\tau}$, $b_\tau^{s_t} = \frac{B_\tau^{s_t}}{\tau}$, and $R_{1,t}^{s_t} = r_t^{s_t}$. This equation and the solution for $a_\tau^{s_t}$ and $b_\tau^{s_t}$ provide a basis for estimating the model and analyzing the effects of monetary policy regime shifts on the term structure of interest rates.

In each time period, the sequence of bond pricing by agents can be described as follows:

Stage 1 At the beginning of time t, agents learn regime s_t, where the realization of s_t depends on s_{t-1} and the transition probabilities;

Stage 2 The regime s_t determines the corresponding model parameters θ^{s_t};

Stage 3 Given θ^{s_t}, the factors $\mathbf{f}_t$ are generated by regime-specific autoregressive process $\mathbf{f}_t = f_{\mathbf{f}}\left(\theta^{s_t}, \mathbf{f}_{t-1}\right)$ in equation (2.3);

Stage 4 Next, given parameters θ^{s_t}, one can calculate the values of $A_\tau^{s_t}$ and $B_\tau^{s_t}$ recursively for all maturities τ based on the recursions in equations (2.11) and (2.12);

Stage 5 Finally using $\mathbf{f}_t$, $A_\tau^{s_t}$, and $B_\tau^{s_t}$ the agents price bonds $P_{t,\tau}^{s_t} = f_P\left(\mathbf{f}_t, A_\tau^{s_t}, B_\tau^{s_t}\right)$ as in equation (2.9).

2.5. Expected Excess Return and Term Premium

This subsection presents the solution for expected excess return and term premium implied by our model. As is well-known, the term spread, which is a difference between long-term and short-term yields, can be decomposed into expectation hypothesis and term premium components:

$$R^{s_t}_{\tau,t} - r^{s_t}_t = \underbrace{\left[\frac{1}{\tau}\sum_{i=0}^{\tau-1}\mathbb{E}_t\left[r_{t+i}\right] - r^{s_t}_t\right]}_{\text{Expectation Hypothesis Component}} + \underbrace{\frac{1}{\tau}\sum_{i=1}^{\tau-1}\mathrm{ER}^{s_t}_{\tau+1-i,t}}_{\text{Term Premium}}, \tag{2.14}$$

where $\mathbb{E}_t$ denotes an expectation operator conditional on s_t and $\mathbf{f}_t$; $\mathrm{ER}^{s_t}_{\tau+1-i,t}$ denotes one-period expected excess return for the $(\tau+1-i)$-period bond in regime s_t.

The expected excess returns is derived following the approach of Dai et al. (2007). A risk-neutral agent should be indifferent between two strategies: *i)* holding a bond at time t, which matures at time period $(t+1+\tau-1)$ and *ii)* holding one-period bond at time t and purchasing a bond at time $(t+1)$ that matures at time period $(t+1+\tau-1)$. After accounting for the risk, the difference between these two strategies represents the expected excess return; and therefore the one-period expected excess return on the τ-period bond in regime $s_t = j$ is given by

$$\mathrm{ER}^j_{\tau,t} = \mathbb{E}[\overline{p}_{\tau-1,t+1}|s_t = j, \mathbf{f}_t] + \overline{p}^j_{1,t} - \overline{p}^j_{\tau,t}\ , \tag{2.15}$$

where $\overline{p}^j_{1,t} \equiv \log P^j_{\tau,t}$. Appendix B provides details of the solution for the expected excess return which has the form:

$$\mathrm{ER}^j_{\tau,t} = -\sum_{k=1}^{S} p^{jk}\left(B^{k\prime}_{\tau-1}L^k\Lambda^k_t + \frac{1}{2}B^{k\prime}_{\tau-1}L^kL^{k\prime}B^k_{\tau-1}\right). \tag{2.16}$$

The term premium for τ-period holding is simply the average of the expected excess returns over all maturities from 2 to $\tau-$periods.

3. Estimation

3.1. Data

We use quarterly data on yields of zero-coupon bonds and macroeconomic variables for the sample period of 1985:Q4 to 2008:Q4. The term structure data on eight yields

of 1, 4, 8, 12, 16, 24, 36, and 40 quarter maturities are obtained from Gurkaynak et al. (2007). The yield for one-quarter Treasury bills is our measure of the short rate. The measure of inflation is the year on year log difference in the CPI. We follow Rudebusch and Swanson (2002) and ABDL(2010) and express the output gap as a percentage of the potential output as

$$g_t = \frac{1}{4}\frac{RGDP_t - RGDP_t^p}{RGDP_t^p} \ , \tag{3.1}$$

where $RGDP_t$ is real GDP in 2005 constant prices obtained from the St. Louis FED database and $RGDP_t^p$ is potential GDP computed similarly to Ang et al. (2007b) by applying the Hodrick and Prescott (1997) filter.[6] The gap is factored by 1/4 to make estimated coefficients interpretable as coefficients for annualized interest rates.

3.2. Identification Restrictions

Our study focuses on the interaction between monetary policy and term structure dynamics in the post-1985 period. The estimation of the model over the post-1985 period avoids identifying the monetary policy regimes with the major oil shocks in the 1970s, the monetary policy "experiment" in 1979, and the structural break in the monetary policy found by many studies (e.g. Fuhrer (1996) and Clarida et al. (2000)), which is associated with the beginning of the "Volcker" disinflation policy.

The factor dynamics and Taylor rule equation (2.1) are linked through identification restrictions $\overline{\pi}^{m_t} = d_2^{m_t}$ and $d_3^{m_t} = 0$. The latter of the two restrictions is imposed because the last factor is the output gap and one can reasonably assume that it has to be targeted at zero independently of the monetary policy regimes. For identification of the latent factor, $d_1^{m_t}$ is restricted to zero in both regimes. The inflation target $\overline{\pi}^{m_t}$ and the short rate target $\overline{r}^{m_t}$ are assumed to be regime-independent, which is a more reasonable assumption for the sample period under consideration than if we had included the 1970s. Setting these parameters to be regime-independent avoids identifying monetary policy regimes by potential switching in the mean of inflation and/or the short rate rather

[6] We are not claiming that the HP filter actually captures potential output or the output gap. However, we assume that it proxies for the Fed's and the market's perceptions of the output gap. This approach is taken in other papers on Taylor rules, such as Cecchetti et al. (2008), which applies the HP filter for real-time data.

than switching in the policy reaction coefficients. It also avoids the monetary policy regimes indirectly affecting the identification of the regime shifts in the market price of risk through the factor dynamics. Indeed, by setting parameters $\overline{\pi}$ and $\overline{r}$ to be regime-independent the factor dynamics is not effected by changes in the monetary policy, and therefore the policy changes do not affect the identification of the market price of risk through equation (2.5). We also set $\overline{\pi}$ and $\overline{r}$ to their sample average values, as in Dai et al. (2007), Bikbov and Chernov (2008), and Ang et al. (2008). Clarida et al. (2000) also restrict the real rate to its sample average to identify the inflation target.

To reduce the dimension of the parameter space, the variance-covariance matrix Ω^{v_t} is constrained to be a diagonal. In this setting, interactions between factors are determined by the G matrix. This constraint is not too restrictive given estimation results of many studies that report statistically insignificant and, in most cases, relatively small off-diagonal elements of the variance-covariance matrix (e.g. Ang et al. (2007b), Chib and Kang (2009)).

It is well known that it is hard to estimate the risk parameters in small samples, and therefore, similarly to Ang et al. (2007a), for tractability we also constrain λ_f to be a diagonal matrix. This restriction is also in line with the empirical approach of Dai et al. (2007), who constrained most of the off-diagonal elements of the λ_f matrix to zero based on their preliminary estimation results.

In order to label monetary policy regime m_t=1 to be "more active" with respect to response to inflation than regime m_t=2, we restrict $\alpha^1 > \alpha^2$. To label volatility regime v_t=1 to have higher volatility than in regime v_t=2, we restrict $\Omega^1_{i,i} > \Omega^2_{i,i}$ for each diagonal element i. We also label market price of risk regime l_t=1 to have higher price of risk of inflation than in regime l_t=2 by restricting $\lambda^1_{0,2} < \lambda^2_{0,2}$ because more negative value of $\lambda_0^{s_t}$ is associated with higher price of risk.

The factor dynamics are assumed to be a stationary process by constraining all eigenvalues of the G matrix to be less than unity in absolute value. The recursion for $B_\tau^{s_t}$ is also restricted to be stationary to ensure that the implied yields for long-term bonds are non-explosive.

3.3. Estimation Method

No-arbitrage term-structure models are known to have a likelihood surface with many local maxima. The problem becomes more severe in our high dimensional parameter space. Our statistical inference is Bayesian, and to fit such models we use the tailored randomized block Metropolis-Hasting (TaRB-MH) algorithm recently developed by Chib and Ramamurthy (2010). The idea behind this implementation is to update parameters in blocks where both the number of blocks and the members of the blocks are randomly drawn within each MCMC cycle. The use of this MCMC method is essential to improve the mixing of the draws in the context of term structure models in which there is no natural way of grouping the parameters. For more details about the TaRB-MH algorithm, see Chib and Ramamurthy (2010).

One important feature of our estimation method is that proposal densities are constructed from the output of simulated annealing, described in detail in Goffe (1996). For our problem this stochastic optimization method is more reliable than the standard Newton-Raphson class of deterministic optimizers due to high irregularity of the likelihood surface.

3.4. State Space Form

This subsection provides details for the state space form, which comprises the transition and measurement equations and is the basis for model estimation. The transition equation of the state space form is given by equation (2.3). To derive the measurement equation, we follow Dai et al. (2007) and assume that one yield, in particular the 12 quarter maturity yield ($R_{12,t}$), is priced without error. This yield is entitled basis yield. We choose the 12 quarter maturity yield to be priced without error based on the finding in Chib and Kang (2009) that the yields in the middle of the yield curve have the lowest variance of the measurement errors. As a result, the pricing equation for this yield has the form:

$$R_{12,t} = a_{12}^{s_t} + b_{12}^{s_t\prime}\mathbf{f}_t = a_{12}^{s_t} + b_{u,12}^{s_t} u_t + b_{\overline{m},12}^{s_t\prime}\overline{m}_t \ , \tag{3.2}$$

where

$$b_{12}^{s_t} = \begin{pmatrix} b_{u,12}^{s_t} \\ b_{\overline{m},12}^{s_t} \end{pmatrix}$$

and $\overline{m}_t$ denotes the vector of macro factors $(\pi_t, g_t)'$. This assumption allows the latent factor to be expressed in terms of observable yields and macro variables:

$$u_t = \left(b_{u,12}^{s_t}\right)^{-1} \left(R_{12t} - a_{12}^{s_t} - b_{\overline{m},12}^{s_t\prime}\overline{m}_t\right). \tag{3.3}$$

Thus,

$$\mathbf{f}_t = \begin{pmatrix} u_t \\ \overline{m}_t \end{pmatrix} = \begin{pmatrix} \left(b_{u,12}^{s_t}\right)^{-1} \left(R_{12t} - a_{12}^{s_t} - b_{\overline{m},12}^{s_t\prime}\overline{m}_t\right) \\ \overline{m}_t \end{pmatrix}. \tag{3.4}$$

By denoting the vector of all yields other than R_{12t} by R_t and $y_t \equiv (R_t, \mathbf{f}_t)'$, the measurement equation can be expressed as

$$y_t = \underbrace{\begin{pmatrix} \overline{a}^{s_t} \\ \mathbf{0} \end{pmatrix}}_{\overline{A}^{s_t}} + \underbrace{\begin{pmatrix} \overline{b}^{s_t} \\ I_3 \end{pmatrix}}_{\overline{B}^{s_t}} \mathbf{f}_t + \begin{pmatrix} I_7 \\ 0_{3\times 7} \end{pmatrix} \widetilde{\varepsilon}_t \ , \qquad \widetilde{\varepsilon}_t \sim iidN\left(0, \Sigma\right) \ , \tag{3.5}$$

where Σ is the variance-covariance matrix for the measurement errors, which is assumed to be a diagonal and regime independent, and $\overline{a}^{s_t}$ and $\overline{b}^{s_t}$ denote the vector and matrix of all stacked $a_\tau^{s_t}$ and $b_\tau^{s_t\prime}$ excluding $a_{12}^{s_t}$ and $b_{u,12}^{s_t}$.

3.5. Prior Distribution

We set the prior distributions of the model parameters based on the general observation that, on average, the yield curve is upward sloping. Following Chib and Ergashev (2009) we simulate parameters and model-implied yield curves from the prior distributions to ensure that our prior produces, on average, a reasonably shaped yield curve. At the same time we set the variances of key parameter distributions to be relatively large so that the distributions cover economically reasonable values of parameters. The prior for the diagonal elements of G is based on the fact that interest rates, inflation, and the output gap are all persistent time series. Since $\lambda_0^{s_t}$ and Ω^{s_t} are key parameters determining the term premium, their means are set based on the simulation outcomes of the model-implied yield curve. Full details of the prior distributions are provided in Appendix C. To see the prior implied outcomes, we sample the parameters 25,000 times from the prior distributions and simulate factor dynamics and yield curves. This simulation exercise produces, on average, a slightly upward-sloping yield curves with substantial variation between -3% and 15%.

3.6. Posterior Distribution

The posterior distributions of parameters are simulated by Markov Chain Monte Carlo (MCMC) methods. The joint posterior distribution to be simulated is described by

$$\pi\left(\theta, \mathbf{S}_T|\mathbf{y}\right) \propto f\left(\mathbf{y}|\theta, \mathbf{S}_T\right) f\left(\mathbf{S}_T|\theta\right) \pi\left(\theta\right) \ , \tag{3.6}$$

where $f\left(\mathbf{y}|\theta, \mathbf{S}_T\right)$ is the likelihood function for data, denoted by $\mathbf{y}$ comprising time series of all yields and macro factors, given all parameters of interest θ and time series of regimes $\mathbf{S}_T = \{s_t\}_{t=0,1,..,T}$; $f\left(\mathbf{S}_T|\theta\right)$ is the density function for regime-indicators given the parameters; $\pi\left(\theta\right)$ is the prior density of the parameters.

The MCMC procedure is discussed in detail in Appendix D and summarized as follows:

Step 1: Initialize $(\theta, \mathbf{u}_T, \mathbf{S}_T)$; where $\mathbf{u}_T = \{u_t\}_{t=0...T}$ is the time series of the latent factor and $\mathbf{S}_T = \{s_t\}_{t=0...T}$ is the time series of regimes;

Step 2: Sample θ conditional on $(\mathbf{S}_T, \mathbf{F}_T, \mathbf{R}_T)$, where $\mathbf{F}_T = \{\mathbf{f}_t\}_{t=0...T}$ is the time series of factors and $\mathbf{R}_T = \{R_t\}_{t=0...T}$ is the time series of yields;

Step 3: Sample $\mathbf{S}_T$ conditional on $(\theta, \mathbf{F}_T, \mathbf{R}_T)$;

Step 4: Compute $\mathbf{u}_T$ conditional on $(\theta, \mathbf{S}_T, \overline{\mathbf{m}}_T, \mathbf{R}_{12,T})$ using equation (3.3), where $\overline{\mathbf{m}}_T = \{\overline{m}_t\}_{t=0...T}$ is the time series of macro factors and $\mathbf{R}_{12,T} = \{R_{12,t}\}_{t=0...T}$ is the time series of basis yield;

Step 5: Repeat Steps 2-4 $(n_0 + n)$ times, then disregard the first n_0 iterations, which are burn-in iterations, and save n draws of the parameters.

4. Empirical Results

4.1. Model Comparisons

To confirm an importance of accounting for regime shifts in the monetary policy, volatility of yield factors, and market price of risk for fitting the data, we estimate models with different combinations of regime-processes and conduct model comparisons. We

compare the model with the three regime-switching processes and models with all combination of two regime-switching processes out of the three processes using the deviance information criterion (DIC) proposed by Spiegelhalter et al. (2002).[7] Table 1 confirms that the model with the three regime-switching processes is the most supported by the data.[8] The pairwise comparison of the models with two regime-processes and the three regime-processes support the importance of each regime-process for the model fitting. Indeed, adding the volatility or the price of risk regime-switching processes to the model considerably increases the likelihood of the model and improves the DIC value.

The following subsections discuss estimation results for the model with the three regime-switching processes and analyze the effects of monetary policy regime shifts on the term structure of interest rates.

Table 1: The deviance information criterion (DIC) and Log likelihood

Model	DIC	LnL
Regimes: m_t, v_t, l_t	-11618.7	5830.4
Regimes: m_t, v_t	-11513.6	5675.8
Regimes: m_t, l_t	-11115.7	5608.7
Regimes: v_t, l_t	-11446.8	5696.6

m_t, v_t, and l_t denote regimes of monetary policy, volatility, and the market price of risk, respectively. The model with the smallest value of the DIC is the most supported by the data. LnL denotes log likelihood evaluated at the mode of the posterior distribution.

[7]The deviance information criterion (DIC) is defined as: $DIC = 2\frac{1}{n}\sum_{i=1}^{n} D(\mathbf{y}, \theta^{(i)}) - D(y, \overline{\theta})$, where $D(\mathbf{y}, \theta) = -2 \log f(\mathbf{y}|\theta)$, $\theta^{(i)}$ is the vector of parameters from the posterior distribution, and $\overline{\theta}$ is the mean of the posterior distribution of parameters. The model with the smallest value of DIC is the most supported by the data. Alternative criterion for a model comparison, used widely in the Bayesian literature, is the Bayes factor, which is based on the marginal likelihood. However, it was found that the majority of methods to compute marginal likelihoods based on values of likelihoods cannot be used for this study. For example, the harmonic mean estimator with different modifications is not reliable enough due to its instability. The method for estimating the marginal log likelihood proposed by Chib and Jeliazkov (2001), which seems more desirable, is computationally costly for our model due to the high dimensionality with eight regimes.

[8]The results for the non-switching model, which is much less supported by the data, is not reported for conciseness of the analysis.

4.2. Parameter Estimates and Regimes

Table 2 reports the parameter estimates of the model. Specifically, the table reports the posterior means of parameters and their standard deviations in parentheses based on 15,000 iterations of the MCMC algorithm beyond a burin-in of 5,000 iterations. To evaluate the efficiency of the MCMC-produced results, we use the acceptance rates in the MH step of the sampler and the inefficiency factor as discussed in Chib (2001).[9] These parameters have, on average, values of 53.7 percent and 180.0 respectively indicating good mixing.

We start the interpretation of the estimation results with analysis of the parameter estimates in the two monetary policy regimes. The inflation coefficients α^1 and α^2, which have values of 0.18 and 0.88, respectively, are considerably different in the two monetary policy regimes. The output gap coefficients β^1 =0.63 and β^2 =0.75 are also different in the two monetary policy regimes; however, this difference is not as strong as for the inflation coefficients. Thus, the monetary policy regimes are mainly identified by switching in the Fed's reaction to inflation.

These coefficients are not directly comparable to those from a single-equation Taylor rule that accounts for interest rate smoothing. The single-equation Taylor rule with interest rate smoothing is specified as a linear combination of the target rate and past value of the short rate as

$$r_t^{m_t} = (1-\rho)\left[\widetilde{r}^{m_t} + \widetilde{\alpha}^{m_t}\left(\pi_t - \overline{\pi}^{m_t}\right) + \widetilde{\beta}^{m_t} g_t\right] + \rho r_{t-1}^{m_{t-1}} + \xi_t \ , \tag{4.1}$$

where ξ_t denotes monetary policy shocks for this specification of the policy rule. It is easy to see that $\widetilde{r}^{m_t} = \frac{\overline{r}^{m_t}}{(1-\rho)}$, $\widetilde{\alpha}^{m_t} = \frac{\alpha^{m_t}}{(1-\rho)}$, $\widetilde{\beta}^{m_t} = \frac{\beta^{m_t}}{(1-\rho)}$, $u_t = \rho r_{t-1}^{m_{t-1}} + \xi_t$, and it is easy to show that $\rho = G_{1,1}$.[10] After this transformation the coefficients $\widetilde{\alpha}^1$ =3.30 and $\widetilde{\alpha}^2$ =16.33

[9]The inefficiency factor is defined as $1+2\sum_{k=1}^{M}\rho(k)$, where $\rho(k)$ is the k-order autocorrelation computed from the sampled distribution and M is a large number, which we set to be 500. Thus, if the sampler did not mix at all then the inefficiency factor would have a value of 500. Given this choice for M, empirically, a value of the inefficiency factor of 250 is usually considered as an upper-bound for a reasonable level of mixing.

[10]We do not use the specification of the Taylor with smoothing because, in our structure, the short rate has an affine form in the factors and also the latent factor is identified from the VAR(1) dynamics

Table 2: Parameter estimates

(a) Monetary Policy

α^1	α^2	β^1	β^2
0.178	0.882	0.628	0.750
(0.098)	(0.164)	(0.167)	(0.226)

(b) G matrix

G		
0.946	0.006	0.016
(0.030)	(0.010)	(0.026)
-0.039	0.958	0.041
(0.036)	(0.023)	(0.046)
0.133	0.014	0.838
(0.036)	(0.030)	(0.039)

(c) Factors' Volatilities $\times 400$

L^1			L^2		
0.692	0.688	0.411	0.739	1.154	0.668
(0.065)	(0.060)	(0.053)	(0.068)	(0.164)	(0.097)

(d) Measurement Errors' Volatilities $\times 400$

σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
0.438	0.174	0.052	0.026	0.064	0.115	0.132
(0.038)	(0.015)	(0.004)	(0.002)	(0.006)	(0.009)	(0.011)

(e) Market Price of Risks

λ_0^1			λ_0^2			λ_f		
0.237	-0.342	-0.374	0.193	-0.442	-0.498	0.314	0.733	0.251
(0.060)	(0.086)	(0.155)	(0.076)	(0.103)	(0.176)	(1.997)	(1.974)	(1.837)

(f) Transition Probabilities

p_m^{11}	p_m^{22}	p_v^{11}	p_v^{22}	p_l^{11}	p_l^{22}
0.988	0.986	0.943	0.959	0.978	0.975
(0.004)	(0.004)	(0.016)	(0.016)	(0.007)	(0.009)

The Table reports posterior means and their standard deviations in parentheses based on 15,000 posterior draws beyond 5,000 draws as a burn-in.

both have values grater than unity, and therefore they do not potentially create a risk of

rather than from the single short-rate equation.

indeterminacy of the equilibrium based on the generalized Taylor principle, introduced by Davig and Leeper (2007).[11,12] Given this result, the regime with the smaller inflation coefficient is entitled a "less active" monetary policy regime and the one with the bigger coefficient, a "more active" regime. The transformed coefficients for the output gap $\widetilde{\beta}^1$ and $\widetilde{\beta}^2$ have values of 11.63 and 13.89, respectively. In our model structure, the policy response coefficients are responsible for fitting the short-term interest rate as well as the long-term interest rate through a no-arbitrage restriction rather than only the short rate in the single-equation Taylor rule. Therefore, this model structure can lead to different estimates of the coefficients than those from the single-equation model.[13]

Figure 1 displays the probabilities of regimes for all three regime processes. In general, the monetary policy regimes are well-identified and very persistent throughout the sample period with 99 percent probabilities of staying in the same regime from quarter to quarter, as reported in Table 2. The period from 1986 through 1994 is characterized by the "more active" monetary policy regime. In this period, inflation was, on average, relatively high and the Fed was adjusting the short rate relatively close to inflation and output gap dynamics. The period from 1995 through 2000, where the "less active" monetary policy regime prevails, is characterized by the relatively stable short rate and inflation, while the output gap was steadily increasing in magnitude. At the beginning of 2001, when the recession hit the U.S. economy, the Fed responded to the decline in output and inflation by reducing the short rate and switching to the "more active" policy regime, which lasted until 2004. In the period from 2002 through 2004, inflation remained, on average, relatively low and the Fed kept the short rate at a low

[11] As discussed in Clarida et al. (2000), if the inflation coefficients are below unity, then increase in expected inflation causes a decline in the real interest rate. The decline in the real interest rate leads to growth in aggregate consumption, which consequently leads to further increase in inflation.

[12] Davig and Leeper (2007) introduced the concept of the generalized Taylor principle to rule out indeterminate equilibria in a version of the new-Keynesian dynamic general equilibrium model where the parameters of the policy rule follow a Markov-switching process. According to the generalized Taylor principle monetary policy can satisfy the Taylor principle in the long run, while deviating from it substantially for relatively short periods or modestly for prolonged periods.

[13] Although the estimates of the policy response coefficients for inflation and output gap after transformation are higher than those often reported from a single-equation Taylor rule model, they are of the same magnitude as those reported by ABDL(2010) for their specification of a no-arbitrage model.

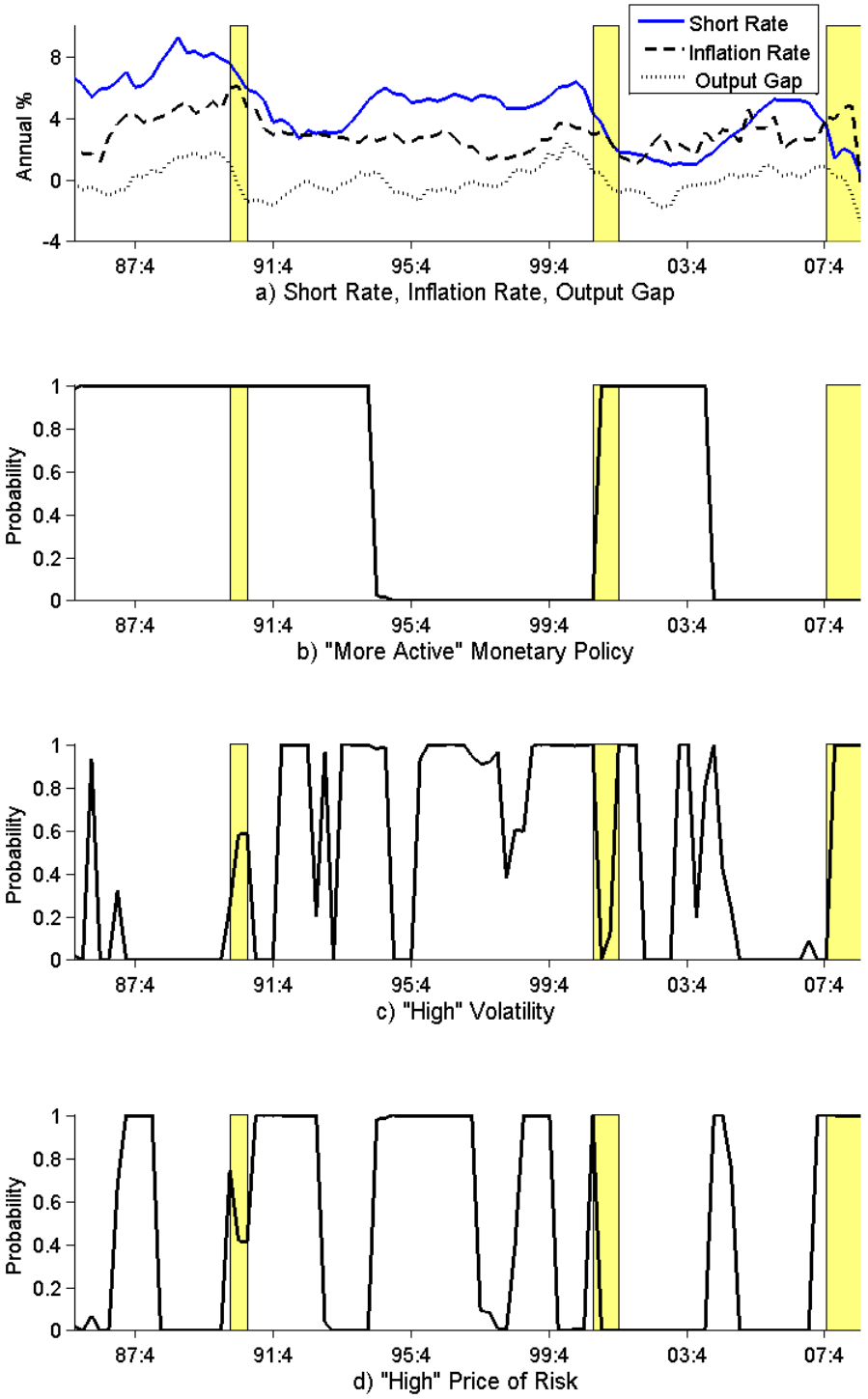
level to accommodate the still low output gap. The identification of the monetary policy regime in this period as "more active" is also affected by the increased term spread. As we noted above, in the no-arbitrage framework, the Taylor rule coefficients are identified by the short rate as well as the slope of the yield curve. Also, it is important to note that this paper employs a different structure of regimes and time-varying volatility specification from the previous literatures, which explains why our results for the timing of the regime switches might differ from those in ABDL(2010), Bikbov and Chernov (2008), and Davig and Doh (2009).

Identification of monetary policy as "less active" for the period from the middle of 2004 through 2005 is also affected by the slope of the yield curve. In this period, entitled a "conundrum" by then-Fed Chairman Alan Greenspan, the long-term yields slightly declined while the short rate was steadily increasing from 1 percent to around 4 percent. These dynamics of the yield curve, as discussed by Rudebusch et al. (2006) in detail, are perceived to be unusual given economic expansion, the falling unemployment rate, and the increasing fiscal gap, which all normally correspond a higher long rate. Similar to Kim and Wright (2005), our results suggest that the term premium, displayed in Figure 2, was low in this period. While this result suggests that part of the "conundrum" can be related to a decline in the term premium, full assessment of its contribution to the pricing anomaly is beyond the scope of this study.[14]

The volatility estimates of exogenous shocks to all factors, reported in Table 2 suggest that identification of the volatility regimes is presumably driven by the volatility of inflation shocks. The volatility estimates for the inflation shocks factored by 400 have values of 0.69 and 1.15 - the values with the largest difference in the two volatility regimes among all factors. The transition probabilities of staying in the same volatility regime are estimated at 94 and 98 percent for the "low" and "high" volatility regimes, respectively.

[14]Kim and Wright (2005) finds that the decline in term premium is a key factor explaining the "conundrum". In contrast, Rudebusch et al. (2006) find that no arbitrage macro-finance models are not able to explain it. They consider macroeconomic factors other than those included in the macro-finance models and find that declines in long-term bond volatility may explain a part of the "conundrum".

Figure 1: The Probabilities of monetary policy, volatility, and risk regimes



Graph (a) displays the time series of the short rate, inflation and the output gap; graphs (b), (c), and (d) display probabilities of regimes in "more active" monetary policy, "high" volatility, and "high" price of risk, respectively. Shaded areas correspond to NBER recession dates.

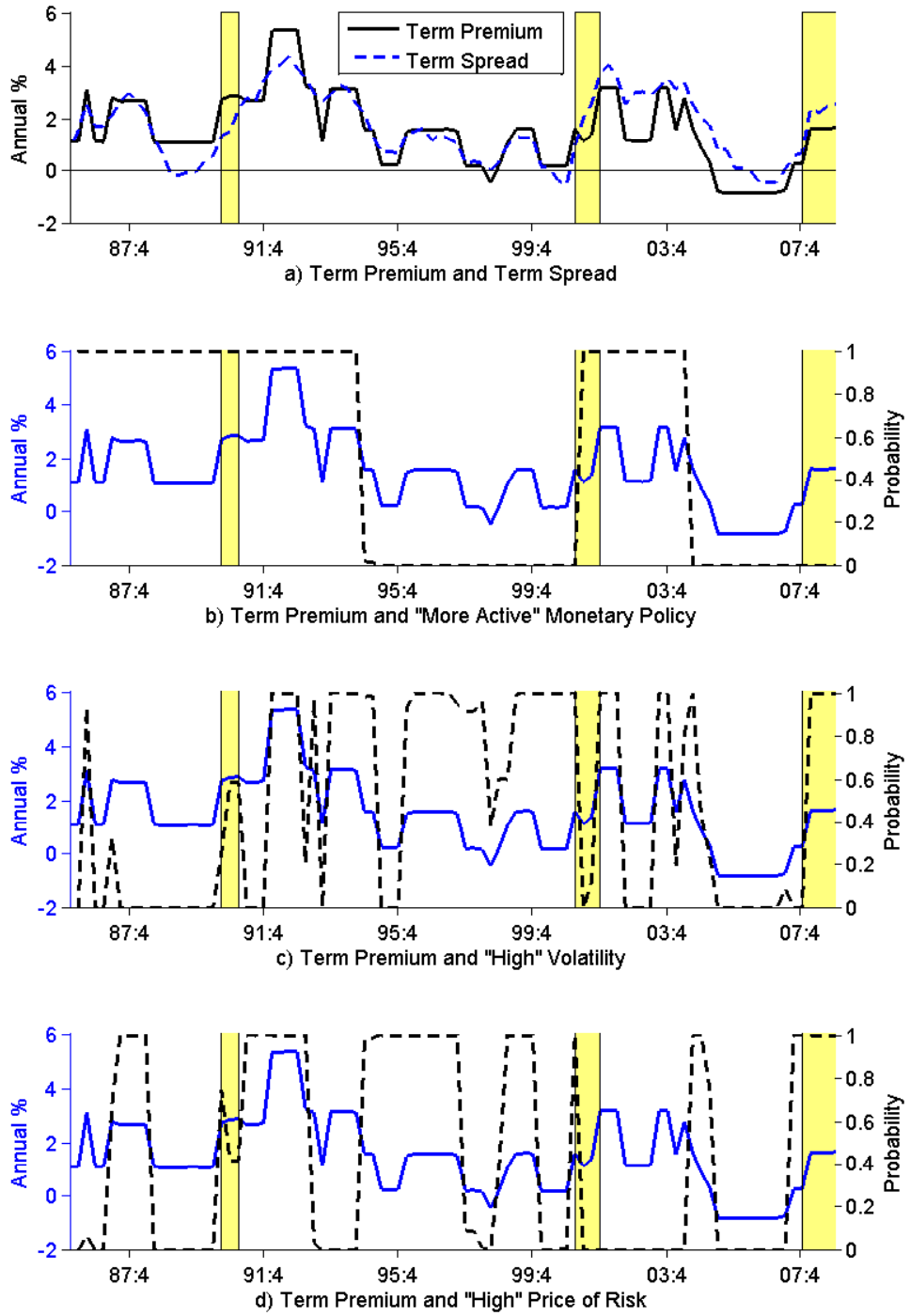
The bottom graph of Figure 1 displays probabilities of the "high" price of risk regime based on switching of risk parameters $\lambda_0^{s_{t+1}}$. The probabilities of the "high" price of risk regime clearly and persistently change between 0 and 1 in most of the sample period, indicating that the regimes are well identified. In the next section we analyze the role of the price of risk regime process for modeling the term premium.

4.3. Term Premium and Regimes

Graph (a) of Figure 2 displays the model-implied term premium and the term spread between the long-term and short-term interest rates. The highly correlated co-movement between the term premium and the term spread indicates that the model implies a plausible term premium. From equation (2.16) one can see that the time-variation in the term premium theoretically depends on the regime-switching of all processes as well as the continuous variation in the yield factors. The continuous variation is originated from the functional form of risk parameter $\Lambda_{t,t+1}$ which comprises of the regime-switching component as well as the continuously time-varying component as a function of the factors. While the model theoretically allows both discrete and continuous variations in the term premium, the discrete shape of the estimated term premium suggests that the regime-switching processes play the considerably more important role in large variations of the term premium than the factors.

Graphs (b) through (d) of Figure 2 display the probabilities of regimes and the term premium, illustrating the relative role of each regime-process in the time-variation of the term premium. One can see that the volatility and the price of risk regime-processes are most closely related to the variation of the term premium, complementing each other. In contrast, the monetary policy regimes highly persistent relative to the term premium, indicating that the monetary policy regimes have less responsible for the time-variation of the term premium, given the other two regime-processes. This result suggests that the monetary policy regimes are identified by the correlation between the short rate and the macroeconomic variables in larger extend than by the term premium. In contrast to this result, the monetary regimes estimated in the models with only two-regime processes, the regime-switching monetary policy and the regime-switching volatility or market price of risk, are more affected by the term premium than in the

Figure 2: The Term Premium, Term Spread, and Regimes



The figure displays the model-implied term premium, the term spread for 10-year bonds, and probabilities of regimes. On the graphs (b) through (d) the solid line displays the term premium and the dashed-lines display probabilities of regimes. Shaded areas corresponds to NBER recession dates.

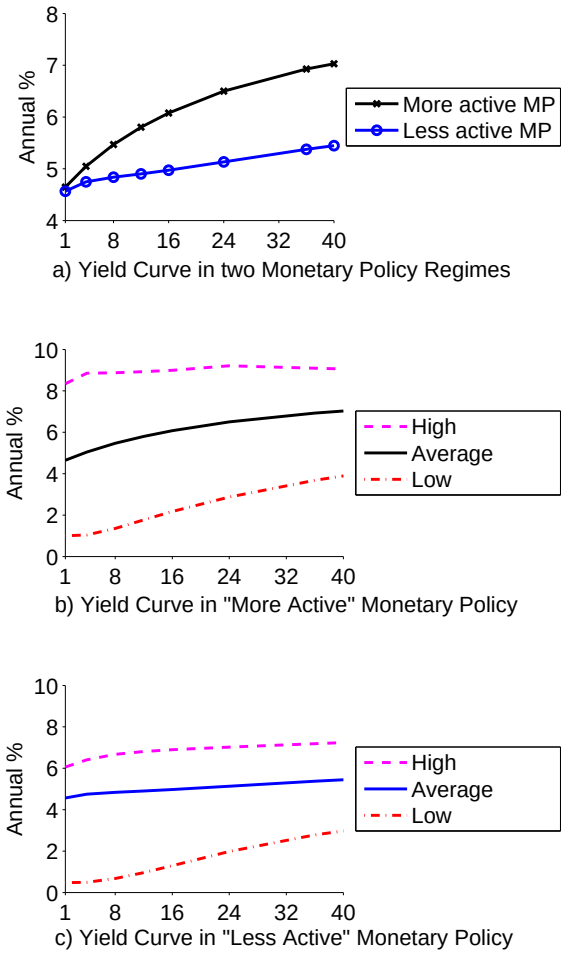
model with the three-regime processes. For example, in the two-regime models, similar to Bikbov and Chernov (2008), the monetary policy regime in the late 1980s is identified as the less active regime when the term spread was low.[15] Also, as we pointed earlier, the model comparisons suggest that accounting for the regime-switching of all three regime-switching processes considerably improves the data fitting by the model. Thus, we conclude that the volatility and the price of risk regime-switching processes are important components of the model for the monetary policy regimes identification.

4.4. Monetary Policy Regimes and the Yield Curve

Figure 3 displays the average realized yield curves in the two monetary policy regimes. The left-hand-side graph demonstrates that the average yield curves in the two regimes mainly differ in terms of their long rates and slopes. In particular, while the average short rates in two regimes are close to each other, the long rate in the "more active" regime is, on average, 129 basis points higher than in the "less active" regime, resulting in a considerably steeper sloped yield curve, on average, in the "more active" regime. This result suggests that long-term yields are more sensitive to monetary policy shifts than the short-rate, which is in line with findings of ABDL(2010) and can be explained as follows. Because the policy coefficients switch to higher values in response to greater macroeconomic factor risk in the "more active" regime, they also magnify this risk for the long-term yields through a no-arbitrage restriction. This effect can be seen from equation (2.16), where term premia for the long-term yields increase with increase in the coefficients for the macroeconomic factors in short rate equation (2.1). The middle- and right-hand-side graphs of Figure 3 demonstrate that the short rate in the "more active" regime was considerably more volatile than in the "less active" regime. The sample standard deviation of the short rate in the "more active" regime is 2.48 percent compared to 1.39 percent in the "less active" regime. In general, the yield curve in the "more active" regime is more volatile than in the "less active" regime with the standard deviations of the long-term yields of 1.65 and 1.10 percent in the "more active" and

[15] To conserve space we do not report results of the two-regime models, which are less supported by the data.

Figure 3: Average realized yield curves



The graphs are constructed using the term structure of interest rates computed at each iteration of the posterior distribution and then separately averaging them over the two monetary policy regimes. Graphs (b) and (c) display the average and 2.5%, and 97.5% quantile yield curves in the two monetary policy regimes. The X-axes display yield maturities in quarters.

"less active" regimes, respectively. In summary, these results can be explained by a more sensitive response of the short rate to inflation in the "more active" regime that creates higher risk for the future short rate fluctuations. This risk drives the higher long-term rate relative to the short rate. Thus, the Fed faces a policy trade-off between a "more active" reaction to macroeconomic fluctuations and higher long-term interest rates as well as a more volatile yield curve caused by this reaction. This argument

is consistent with Woodford (1999), who claims that it may be more optimal for the monetary authority to conduct policies that do not require the short rate to be too volatile.

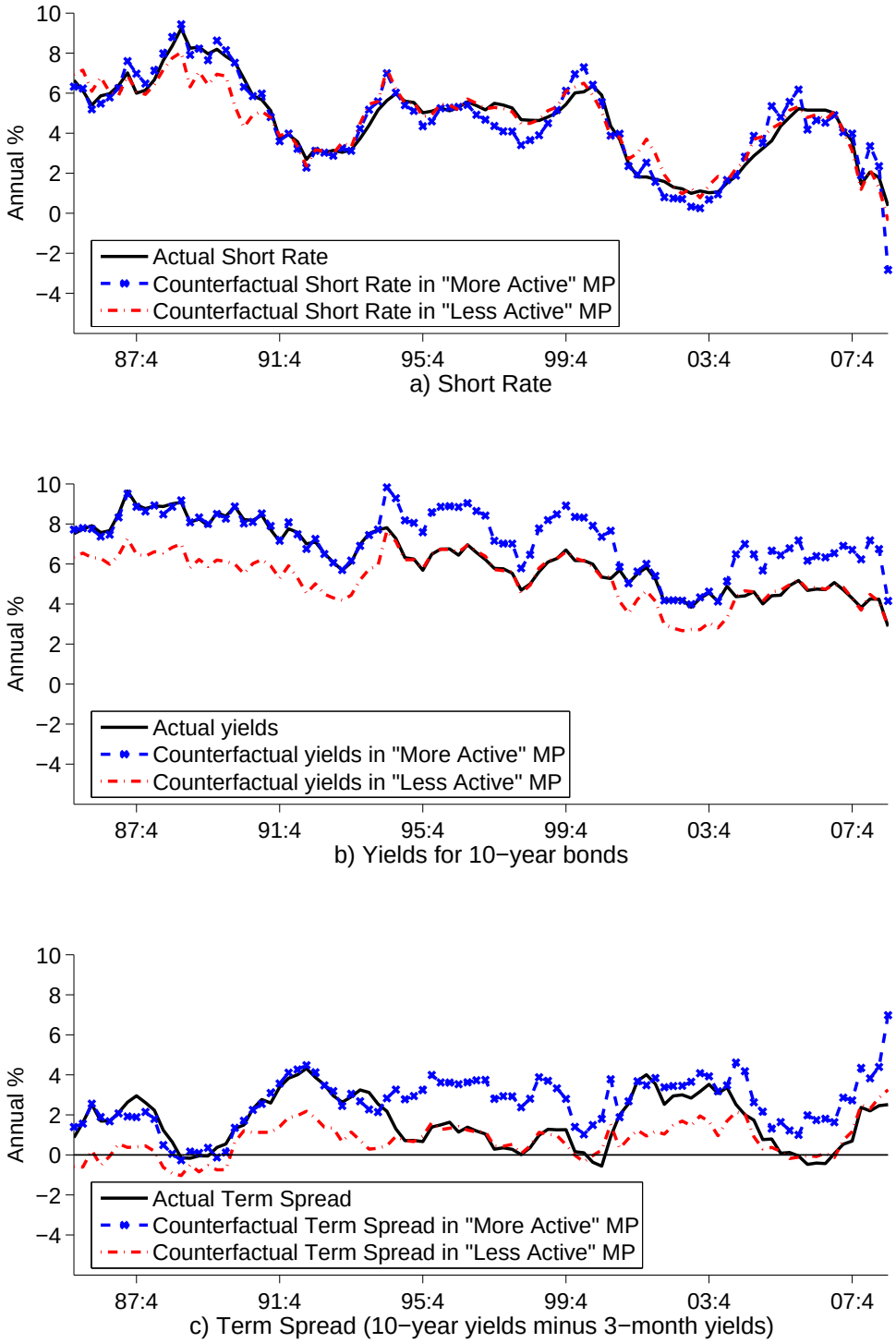
To see what effect monetary policy would have had on the term structure of interest rates if a single regime were maintained throughout the sample, we conduct a counterfactual analysis. Figure 4 displays the short and long rates and the term spreads generated by fixing parameters to one of the two monetary policy regimes. Throughout most of the sample, the short rate in the "more active" regime would have been more volatile than in the "less active" regime. The long rate and consequently the term spread would have been higher than the actual ones in those periods when the regime was "less active".

5. Conclusion

In this paper, we proposed a no-arbitrage affine term structure model with regime shifts in monetary policy, factor volatilities, and the price of risk. This model allowed us to quantitatively assess the influence of monetary policy regime shifts on the entire term structure of interest rates.

We found that, in the "more active" monetary policy regime, the slope of the yield curve was steeper than in the "less active" regime. Also, the short rate and the entire yield curve in general were more volatile in the "more active" regime than in the "less active" regime. The explanation for these results is that a higher sensitivity of the short rate in response to inflation fluctuations in the "more active" regime leads to a higher term premium in anticipation of a more volatile future short rate. These results also suggest that the Fed faces a policy trade-off between a "more active" reaction to macroeconomic fluctuations and a more volatile yield curve caused by this reaction.

Figure 4: Counterfactual short rates, long rates, and term spreads



The time series of counterfactual interest rates are simulated by fixing parameters to one of the two monetary policy regimes.

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Appendices

Appendix A. Bond Pricing

We solve for A_τ^j and B_τ^j using the law of iterated expectatios, method of undetermined coefficients, and log-linearization:

$$P_{t,\tau}^{s_t} = \mathbb{E}\left[\exp\left(-r_t^{s_t} - \frac{1}{2}\Lambda_t^{s_{t+1}\prime}\Lambda_t^{s_{t+1}} - \Lambda_t^{s_{t+1}\prime}\varepsilon_{t+1}\right) P_{\tau-1,t+1}^{s_{t+1}} | \mathbf{f}_t, s_t\right]$$

$$\begin{aligned}
1 &= \mathbb{E}\left[\exp\left(-r_t^j - \frac{1}{2}\Lambda_t^{s_{t+1}\prime}\Lambda_t^{s_{t+1}} - \Lambda_t^{s_{t+1}\prime}\varepsilon_{t+1}\right) \frac{P_{\tau-1,t+1}^{s_{t+1}}}{P_{\tau,t}^j} | \mathbf{f}_t, s_t = j\right] \\
&= \sum_{k=1}^{S} p^{jk}\mathbb{E}\left[\exp\left(-r_t^j - \frac{1}{2}\Lambda_t^{k\prime}\Lambda_t^k - \Lambda_t^{k\prime}\varepsilon_{t+1}\right) \frac{P_{\tau-1,t+1}^k}{P_{\tau,t}^j} | \mathbf{f}_t, s_t = j, s_{t+1} = k\right]
\end{aligned}$$

$$\begin{aligned}
&= \sum_{k=1}^{S} p^{jk}\mathbb{E}\left[\exp\left(\begin{array}{c} -r_t^j - \frac{1}{2}\Lambda_t^{k\prime}\Lambda_t^k - \Lambda_t^{k\prime}\varepsilon_{t+1} \\ +A_\tau^j + B_\tau^{j\prime}\mathbf{f}_t - A_{\tau-1}^k - B_{\tau-1}^{k\prime}\mathbf{f}_{t+1} \end{array}\right) | \mathbf{f}_t, s_t = j, s_{t+1} = k\right] \\
&= \sum_{k=1}^{S} p^{jk}\left\{\begin{array}{c} \exp\left(-r_t^j - \frac{1}{2}\Lambda_t^{k\prime}\Lambda_t^k + A_\tau^j - A_{\tau-1}^k + B_\tau^{j\prime}\mathbf{f}_t - B_{\tau-1}^{k\prime}\mu_t^{j,k}\right) \\ \times\mathbb{E}\left[\exp\left(-\left(\Lambda_t^{k\prime} + B_{\tau-1}^{k\prime}L^k\right)\varepsilon_{t+1}\right) | \mathbf{f}_t, s_t = j, s_{t+1} = k\right] \end{array}\right\} \qquad (A.1) \\
&= \sum_{k=1}^{S} p^{jk}\left\{\exp\left(\begin{array}{c} -r_t^j - \frac{1}{2}\Lambda_t^{k\prime}\Lambda_t^k + A_\tau^j - A_{\tau-1}^k + B_\tau^{j\prime}\mathbf{f}_t - B_{\tau-1}^{k\prime}\mu_t^{j,k} \\ +\frac{1}{2}\left(\Lambda_t^{k\prime} + B_{\tau-1}^{k\prime}L^k\right)\left(\Lambda_t^{k\prime} + B_{\tau-1}^{k\prime}L^k\right)' \end{array}\right)\right\} \qquad (A.2) \\
&= \sum_{k=1}^{S} p^{jk}\exp\left(\begin{array}{c} -r_t^j + A_\tau^j - A_{\tau-1}^k + B_\tau^{j\prime}\mathbf{f}_t \\ -B_{\tau-1}^{k\prime}\mu_t^{j,k} + B_{\tau-1}^{k\prime}L^k\Lambda_t^k + \frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k \end{array}\right) \qquad (A.3) \\
&\approx \sum_{k=1}^{S} p^{jk}\left\{\begin{array}{c} -\delta_0^j - \delta_f^{j\prime}\mathbf{f}_t + A_\tau^j - A_{\tau-1}^k + B_\tau^{j\prime}\mathbf{f}_t \\ -B_{\tau-1}^{k\prime}d^k - B_{\tau-1}^{k\prime}G\left(\mathbf{f}_t - d^j\right) \\ +B_{\tau-1}^{k\prime}L^k\left(\lambda_0^k + \lambda_f\mathbf{f}_t\right) + \frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k + 1 \end{array}\right\}. \qquad (A.4)
\end{aligned}$$

$(A.1)$ is transformed into $(A.2)$ using the property of moment generating function for Normally distributed ε_{t+1}:

$$\varphi_t^{jk}(x) \equiv \mathbb{E}\left[\exp\left(x'\varepsilon_{t+1}\right) | \mathbf{f}_t, s_t = j, s_{t+1} = k\right] = \exp(\frac{x'x}{2}) \ , \ x \in \mathbb{R}^3$$

evaluated at $x = -\left(\Lambda_t^{k\prime} + B_{\tau-1}^{k\prime}L^k\right)'$. Following Bansal and Zhou (2002), $(A.3)$ is transformed into $(A.4)$ using log-approximation $\exp(y) \approx y + 1$ for a sufficiently small y and substituting for r_t^j using equation (2.10).

Using above result for the bond pricing equation and collecting terms for $\mathbf{f}_t$:

$$
\begin{aligned}
0 &= \sum_{k=1}^{S}\left\{p^{jk}\mathbb{E}\left[\exp\left(-r_t^j-\frac{1}{2}\Lambda_t^{k\prime}\Lambda_t^k-\Lambda_t^{k\prime}\varepsilon_{t+1}\right)\frac{P_{t+1,k}^{\tau-1}}{P_{t,j}^{\tau}}|\mathbf{f}_t,s_t=j,s_{t+1}=k\right]\right\}-1\\
&\approx \sum_{k=1}^{S}p^{jk}\left(\begin{array}{c}-\delta_0^j-\delta_f^{j\prime}\mathbf{f}_t+A_\tau^j-A_{\tau-1}^k+B_\tau^{j\prime}\mathbf{f}_t-B_{\tau-1}^{k\prime}d^k-B_{\tau-1}^{k\prime}G\left(\mathbf{f}_t-d^j\right)\\ +B_{\tau-1}^{k\prime}L^k\left(\lambda_0^k+\lambda_f\mathbf{f}_t\right)+\frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k\end{array}\right)\\
&= \sum_{k=1}^{S}p^{jk}\left(\begin{array}{c}-\delta_0^j+A_\tau^j-A_{\tau-1}^k-B_{\tau-1}^{k\prime}d^k+B_{\tau-1}^{k\prime}Gd^j\\ +B_{\tau-1}^{k\prime}L^k\lambda_0^k+\frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k\end{array}\right)\\
&\quad+\sum_{k=1}^{S}p^{jk}\left(-\delta_f^{j\prime}+B_\tau^{j\prime}-B_{\tau-1}^{k\prime}G+B_{\tau-1}^{k\prime}L^k\lambda_f\right)\mathbf{f}_t\ .
\end{aligned}
$$

The above identity has to be true for every value of $\mathbf{f}_t$, which will be the case only if the first and second terms are 0:

$$
0 = \sum_{k=1}^{S}p^{jk}\left(\begin{array}{c}-\delta_0^j+A_\tau^j-A_{\tau-1}^k-B_{\tau-1}^{k\prime}d^k+B_{\tau-1}^{k\prime}Gd^j\\ +B_{\tau-1}^{k\prime}L^k\lambda_0^k+\frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k\end{array}\right)
$$

and

$$
0 = \sum_{k=1}^{S}p^{jk}\left(-\delta_f^{j\prime}+B_\tau^{j\prime}-B_{\tau-1}^{k\prime}\left(G-L^k\lambda_f\right)\right).
$$

This leads to the solution for A_τ^j and B_τ^j in the form of recursive system:

$$
\begin{aligned}
A_\tau^j &= \delta_0^j+\sum_{k=1}^{S}p^{jk}\left(A_{\tau-1}^k+\left(d^k-Gd^j-L^k\lambda_0^k\right)'B_{\tau-1}^k-\frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k\right)\\
B_\tau^j &= \delta_f^j+\sum_{k=1}^{S}p^{jk}\left(G-L^k\lambda_f\right)'B_{\tau-1}^k\ .
\end{aligned}
$$

To derive the initial conditions for A_0^j and B_0^j, we let $\tau=0$. Given $P_{\tau,t}^j=\exp(-\tau r_t^j)$, we have $P_{0,t}^j=\exp(-0\times r_t^j)=1$. From $P_{j,t}^\tau=\exp(-A_\tau^j-B_\tau^{j\prime}\mathbf{f}_t)$ for $\tau=0:1=P_{0,t}^j=\exp(-A_0^j-B_0^{j\prime}\mathbf{f}_t)$ has to be true for every $\mathbf{f}_t$, therefore $A_0^j=0$ and $B_0^j=\mathbf{0}$, consequently $A_1^j=\delta_0^j$ and $B_1^j=\delta_f^j$.

Appendix B. Expected Excess Return

The one-period expected excess return on the n-period bond:

$$\mathrm{ER}^j_{\tau,t} = \mathbb{E}[\overline{p}_{\tau-1,t+1}|\mathbf{f}_t, s_t = j] + \overline{p}^j_{1,t} - \overline{p}^j_{\tau,t} \ ,$$

where $\overline{p}^j_{\tau,t}$ and $\overline{p}^j_{1,t}$ are log prices of bonds derived in the following ways:

$$\begin{aligned}
\overline{p}^j_{\tau,t} &= \mathsf{log}\, P^j_{\tau,t} = \mathsf{log}\, \mathbb{E}\left[\mathsf{exp}\left(-r^j_t - \frac{1}{2}\Lambda^{k\prime}_t\Lambda^k_t - \Lambda^{k\prime}_t\varepsilon_{t+1}\right) P_{\tau-1,t+1}|\mathbf{f}_t, s_t = j\right] \\
&= -r^j_t + \mathsf{log}\left(\sum_{k=1}^{S} p^{jk}\mathbb{E}\left[\mathsf{exp}\left(-\frac{1}{2}\Lambda^{k\prime}_t\Lambda^k_t - \Lambda^{k\prime}_t\varepsilon_{t+1}\right) P^k_{t+1,\tau-1}|\mathbf{f}_t, s_t = j, s_{t+1} = k\right]\right) \\
&= -r^j_t + \mathsf{log}\left(\sum_{k=1}^{S} p^{jk}\mathbb{E}\left[\begin{array}{c}\mathsf{exp}(-\frac{1}{2}\Lambda^{k\prime}_t\Lambda^k_t - \Lambda^{k\prime}_t\varepsilon_{t+1} \\ -A^k_{\tau-1} - B^{k'}_{\tau-1}\mathbf{f}_{t+1})|\mathbf{f}_t, s_t = j, s_{t+1} = k\end{array}\right]\right) \\
&= -r^j_t + \mathsf{log}\left(\begin{array}{c}\sum_{k=1}^{S} p^{jk}\,\mathsf{exp}\left(-\frac{1}{2}\Lambda^{k\prime}_t\Lambda^k_t - A^k_{\tau-1} - B^{k\prime}_{\tau-1}\mu^{j,k}_t\right) \\ \times\mathbb{E}\left[\mathsf{exp}(-\left(\Lambda^{k\prime}_t + B^{k\prime}_{\tau-1}L^k\right)\varepsilon_{t+1})|\mathbf{f}_t, s_t = j, s_{t+1} = k\right]\end{array}\right) \\
&= -r^j_t + \mathsf{log}\left(\sum_{k=1}^{S} p^{jk}\,\mathsf{exp}\left(\begin{array}{c}-A^k_{\tau-1} - B^{k'}_{\tau-1}\mu^{j,k}_t \\ +B^{k\prime}_{\tau-1}L^k\Lambda^k_t + \frac{1}{2}B^{k'}_{\tau-1}L^kL^{k\prime}B^k_{\tau-1}\end{array}\right)\right)
\end{aligned}$$

and

$$\overline{p}^j_{t,1} = \mathsf{log}\left(\mathsf{exp}\left(-r^j_t\right)\right) = -r^j_t \ .$$

Then the expected value of the log price is given by

$$\begin{aligned}
\mathbb{E}[\overline{p}_{\tau-1,t+1}|\mathbf{f}_t, s_t &= j] = \sum_{k=1}^{S} p^{jk}\mathbb{E}[\overline{p}^k_{\tau-1,t+1}|\mathbf{f}_t, s_t = j, s_{t+1} = k] \\
&= \sum_{k=1}^{S} p^{jk}\left(-A^k_{\tau-1} - B^{k'}_{\tau-1}\mathbb{E}\left[\mathbf{f}_{t+1}|\mathbf{f}_t, s_t = j, s_{t+1} = k\right]\right) \\
&= \sum_{k=1}^{S} p^{jk}\left(-A^k_{\tau-1} - B^{k'}_{\tau-1}\mu^{j,k}_t\right) \ .
\end{aligned}$$

Next, the expected excess return is derived in the following way:

$$\begin{aligned}
\mathbb{E}[\overline{p}_{\tau-1,t+1}|\mathbf{f}_t, s_t &= j] + \overline{p}^j_{1,t} - \overline{p}^j_{\tau,t} \\
&= \sum_{k=1}^{S} p^{jk}_t\left(-A^k_{\tau-1} - B^{k'}_{\tau-1}\mu^{j,k}_t\right) - r^j_t
\end{aligned}$$

$$
\begin{aligned}
& -\left\{-r_t^j + \mathsf{log}\left(\sum_{k=1}^{S} p^{jk}\,\mathsf{exp}\left(\begin{array}{c} -A_{\tau-1}^k - B_{\tau-1}^{k\prime}\mu_t^{j,k} + B_{\tau-1}^{k\prime}L^k\Lambda_t^k \\ +\frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k \end{array}\right)\right)\right\} \\
= & \sum_{k=1}^{S} p_t^{jk}\left(-A_{\tau-1}^k - B_{\tau-1}^{k\prime}\mu_t^{j,k}\right) \\
& -\mathsf{log}\left(\sum_{k=1}^{S} p^{jk}\,\mathsf{exp}\left(\begin{array}{c} -A_{\tau-1}^k - B_{\tau-1}^{k\prime}\mu_t^{j,k} + B_{\tau-1}^{k\prime}L^k\Lambda_t^k \\ +\frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k \end{array}\right)\right) \\
\approx & \sum_{k=1}^{S} p_t^{jk}\left(-A_{\tau-1}^k - B_{\tau-1}^{k\prime}\mu_t^{j,k}\right) \\
& -\mathsf{log}\sum_{k=1}^{S} p^{jk}\left(\begin{array}{c} -A_{\tau-1}^k - B_{\tau-1}^{k\prime}\mu_t^{j,k} + B_{\tau-1}^{k\prime}L^k\Lambda_t^k \\ +\frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k + 1 \end{array}\right) \\
\approx & \sum_{k=1}^{S} p_t^{jk}\left(-A_{\tau-1}^k - B_{\tau-1}^{k\prime}\mu_t^{j,k}\right) \\
& -\sum_{k=1}^{S} p^{jk}\left(\begin{array}{c} -A_{\tau-1}^k - B_{\tau-1}^{k\prime}\mu_t^{j,k} + B_{\tau-1}^{k\prime}L^k\Lambda_t^k \\ +\frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k \end{array}\right) \\
= & -\sum_{k=1}^{S} p^{jk}\left(B_{\tau-1}^{k\prime}L^k\Lambda_t^k + \frac{1}{2}B_{\tau-1}^{k\prime}L^kL^{k\prime}B_{\tau-1}^k\right) .
\end{aligned}
$$

To derive the above result, we applied log-linearization for $\mathsf{exp}\,(y)$ and $\mathsf{log}\,(x)$. The argument of the exponent is a return, which is a sufficiently small number, therefore it can be approximated as $\mathsf{exp}\,(y) \approx y + 1$. $\sum_{k=1}^{S} p^{jk}\,(y+1) \equiv x$ is a number sufficiently close to 1, therefore it can be approximated as $\mathsf{log}\,(x) \approx x - 1$.

Appendix C. Details for the Prior Distributions

First, we describe the approach for estimating the transition probabilities.We estimate the transition probabilities separately for each regime process as functions of Normally distributed parameters

$$
p_{rg}^{jk} = \frac{1}{1 + \mathsf{exp}\left(\eta_{rg}^{jk}\right)}, j \neq k \ , \tag{C.1}
$$

which truncates the transition probability values to be within 0 and 1 bounds.

We assume that all parameters, denoted as θ, are distributed independently from each other. Table C.3 provides detail for the prior distributions of the parameters. We set the prior for all variances to be defuse to ensure that the prior implied yield

curve and the factor processes have considerable variations. Parameters Ω^1, Ω^2, Σ are reparameterized using coefficients

$$d_\Omega = \begin{pmatrix} 5\times 10^5 & 5\times 10^5 & 7\times 10^4 \end{pmatrix} \tag{C.2}$$

and

$$d_\Sigma = \begin{pmatrix} 7\times 10^5 & 4\times 10^6 & 3\times 10^7 & 6\times 10^7 & 10^7 & 10^7 & 10^7 \end{pmatrix} . \tag{C.3}$$

Table C.3: Prior distributions

Parameter	density	mean				Std.	
α^1, α^2	normal	0.40	0.40		1.00	1.00	
β^1, β^2	normal	0.30	0.30		1.00	1.00	
G	normal	0.80	0.00	0.00	0.20	0.10	0.10
		0.00	0.80	0.00	0.10	0.20	0.10
		0.00	0.00	0.80	0.10	0.10	0.20
λ_0^1	normal	-0.10	-0.10	-0.10	0.30	0.30	0.30
λ_0^2	normal	-0.10	-0.10	-0.10	0.30	0.30	0.30
λ_f	normal	1.00	1.00	1.00	2.00	2.00	2.00
η_m^{12}, η_m^{21}	normal	3.48	3.48		0.50	0.50	
η_v^{12}, η_v^{21}	normal	3.48	3.48		0.50	0.50	
$\eta_\lambda^{12}, \eta_\lambda^{21}$	normal	3.48	3.48		0.50	0.50	
$d_\Omega \times \Omega^1, d_\Omega \times \Omega^2$	defuse prior	1.10	1.10		0.23	0.23	
$d_\Sigma \times \Sigma$	defuse prior	1.00			0.17		

All elements of the reparameterized $d_\Omega \times \Omega^1, d_\Omega \times \Omega^2$, and $d_\Sigma \times \Sigma$ matrices have the same prior means and standard deviations within each matrix stated in the Table, where d_Ω and d_Σ are defined by (C.2) and (C.3). The prior for η_{rg}^{12} and η_{rg}^{21} implies the prior for p_{rg}^{12} and p_{rg}^{21} with means and standard deviations equal to 0.03 and 0.02, respectively.

Appendix D. MCMC Sampling

This Section provides details of the MCMC algorithm summarized in Section 3.6 and the construction of the likelihood function.

Step 2: Sampling θ

Parameters θ conditional on $(\mathbf{S}_T, \mathbf{F}_T, \mathbf{R}_T)$ are sampled using the Metropolis-Hastings (MH) algorithm. Because it is difficult to find an optimal parameter blocking scheme due to the high dimension of parameter space of the model, we use the tailored randomized block M-H (TaRB-MH) method developed by Chib and Ramamurthy (2010). The

general idea of this method is in setting a number and composition of blocks randomly in each sampling iteration. We let the proposal density $q\left(\theta_i|\theta_{-i},\mathbf{y}\right)$ for parameters θ_i in the ith block, conditional on the value of parameters in the remaining blocks θ_{-i} to take the form of a multivariate student t distribution with 15 degrees of freedom

$$q\left(\theta_i|\theta_{-i},\mathbf{y}\right) = St\left(\theta_i|\widehat{\theta}_i, V_{\widehat{\theta}_i},15\right) \ ,$$

where

$$\widehat{\theta}_i = \arg\max_{\theta_\mathbf{i}} \ln\{f(\mathbf{y}|\theta_i,\theta_{-i},\mathbf{S_T})\pi(\theta_i)\}$$
$$\text{and } V_{\widehat{\theta}_i} = \left(-\frac{\partial^2 \ln\{f(\mathbf{y}|\theta_i,\theta_{-i},\mathbf{S_T})\pi(\theta_i)\}}{\partial\theta_i\partial\theta_i'}\right)^{-1}_{|\theta_i=\widehat{\theta}_i} \ .$$

Following Chib and Kang (2009) and Chib and Ergashev (2009), we solve numerical optimization problem using the simulated annealing algorithm, which has better performance in this problem than deterministic optimization routines due to high irregularity of the likelihood surface.

Next, we draw a proposal value $\theta_i^\dagger$ from the multivariate student t distribution with 15 degrees of freedom, mean $\widehat{\theta}_i$ and variance $V_{\widehat{\theta}_i}$. If the proposed value does not satisfy the model imposed constrains, then it is immediately rejected. The proposed value, satisfying the constraints, is accepted as the next value in the Markov chain with probability

$$\alpha\left(\theta_i^{(g-1)},\theta_i^\dagger|\theta_{-i},\mathbf{y}\right)$$
$$= \min\left\{\frac{f\left(\mathbf{y}|\theta_i^\dagger,\theta_{-i},\mathbf{S}_T\right)\pi\left(\theta_i^\dagger\right)}{f\left(\mathbf{y}|\theta_i^{(g-1)},\theta_{-i},\mathbf{S}_T\right)\pi\left(\theta_i^{(g-1)}\right)}\frac{St\left(\theta_i^{(g-1)}|\widehat{\theta}_i,V_{\widehat{\theta}_i},15\right)}{St\left(\theta_i^\dagger|\widehat{\theta}_i,V_{\widehat{\theta}_i},15\right)},1\right\} \ ,$$

where g is an index for the current iteration. The completed simulation of θ in the gth iteration with h_g blocks produces sequentially updated parameters in all blocks:

$$\pi\left(\theta_1|\theta_{-1},y,\mathbf{S}_T\right),\pi\left(\theta_2|\theta_{-2},y,\mathbf{S}_T\right),...,\pi\left(\theta_{h_g}|\theta_{-h_g},y,\mathbf{S}_T\right).$$

Now we derive the log-likelihood function conditional on θ and $\mathbf{S}_T$, which has the form:

$$\log f\left(\mathbf{y}|\theta,\mathbf{S}_T\right) = \sum_{t=1}^{T}\log f(y_t|I_{t-1},\theta,\mathbf{S}_T) \ ,$$

where $I_{t-1} = \{y_n\}_{n=0}^{t-1}$ denotes the information set available for the econometricians at time t-1. Given the model specification, y_t conditional on $s_{t-1} = j, s_t = k, I_{t-1}$, and θ is distributed Normally with the mean and variance defined as

$$\begin{aligned} y_{t|t-1}^{jk} &\equiv E\left[y_t|s_{t-1} = j, s_t = k, I_{t-1}, \theta\right] = \overline{A}^k + \overline{B}^k \mu_{t-1}^{j,k} \\ V_{t|t-1}^{jk} &\equiv Var\left[y_t|s_{t-1} = j, s_t = k, I_{t-1}, \theta\right] = \overline{B}^k L^k L^{k\prime} \overline{B}^{k\prime} + \underbrace{\begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix}}_{\overline{W}} . \end{aligned}$$

Thus, the conditional density of y_t becomes

$$\begin{aligned} f\left(y_t|s_{t-1} = j, s_t = k, I_{t-1}, \theta\right) &= \frac{1}{(2\pi)^{10/2} \left|V_{t|t-1}^{jk}\right|^{1/2}} \left(-\frac{1}{2}\left(y_t - y_{t|t-1}^{jk}\right)'\right. \\ &\quad \left.\left[V_{t|t-1}^{jk}\right]^{-1}\left(y_t - y_{t|t-1}^{jk}\right)\right) . \end{aligned} \tag{D.1}$$

Step 3: Sampling regimes $\mathbf{S}_T$

Regimes $\mathbf{S}_T$ are sampled from $f\left(\mathbf{S}_T|I_T, \theta\right)$ in a single block in backward order. First, the regime probabilities conditional on I_t and θ are obtained by applying the filtering procedure developed by Hamilton (1989) as follows:

Step 1: Probabilities of regime s_0 conditional on available information at time $t = 0$ and parameters are initialized at unconditional probabilities of regimes denoted by $p_{steady-state}$:

$$\Pr\left(s_0|I_0, \theta\right) = p_{steady-state} .$$

Step 2: The joint density of s_{t-1} and s_t conditional on information at time $t-1$ and parameters is given by

$$\Pr\left(s_{t-1} = j, s_t = k|I_{t-1}, \theta\right) = p^{jk} \Pr\left(s_{t-1} = j|I_{t-1}, \theta\right) . \tag{D.2}$$

Step 3: Then, the density of y_t conditional on information at time $t-1$ and parameters is given by

$$f\left(y_t|I_{t-1}, \theta\right) = \sum_{j,k} f\left(y_t|s_{t-1} = j, s_t = k, I_{t-1}, \theta\right) \Pr\left(s_{t-1} = j, s_t = k|I_{t-1}, \theta\right), \tag{D.3}$$

where the first and second terms are given by equations (D.1) and (D.2), respec-

tively.

Step 4: The joint density of s_{t-1} and s_t conditional on information at time t and parameters is obtained by using the Bayes rule:

$$\mathsf{Pr}\left(s_{t-1}=j, s_t=k|I_t,\theta\right) = \frac{f\left(y_t, s_{t-1}=j, s_t=k|I_{t-1},\theta\right)}{f\left(y_t|I_{t-1},\theta\right)}$$
$$= \frac{f\left(y_t|s_{t-1}=j, s_t=k, I_{t-1},\theta\right)\mathsf{Pr}\left(s_{t-1}=j, s_t=k|I_{t-1},\theta\right)}{f\left(y_t|I_{t-1},\theta\right)} \ ,$$

where the first and second terms of the nominator are given by equations (D.1) and (D.2) and the denominator is given by equation (D.3).

Step 5: By integrating out regime s_{t-1} we obtain the probabilities of regime s_t conditional of information at time t and parameters:

$$\mathsf{Pr}\left(s_t=k|I_t,\theta\right) = \sum_j \mathsf{Pr}\left(s_{t-1}=j, s_t=k|I_t,\theta\right) \ .$$

Next, the regimes are drawn backward based on regime probabilities. In particular, regime s_T is sampled from $\mathsf{Pr}\left(s_T|I_T,\theta\right)$ and then for t from T-1 to 1 regimes are sampled from probabilities computed sequentially backward as

$$\mathsf{Pr}\left(s_t=j|I_t, s_{t+1}=k,\theta\right) = \frac{\mathsf{Pr}\left(s_{t+1}=k|s_t=j\right)\mathsf{Pr}\left(s_t=j|I_t,\theta\right)}{\sum\limits_{j=1}^{n}\mathsf{Pr}\left(s_{t+1}=k|s_t=j\right)\mathsf{Pr}\left(s_t=j|I_t,\theta\right)} \ ,$$

where n is the total number of regimes.