Predicting Output Using the Entire Yield Curve*

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Abstract

Many studies find that yields for government bonds predict real economic activity. Most of these studies use the yield spread, defined as the difference between two yields of specific maturities, to predict output. In this paper, I propose a different approach that makes use of information contained in the entire term structure of U.S. Treasury yields to predict U.S. real GDP growth. My proposed dynamic yield curve model produces better out-of-sample forecasts of real GDP than those produced by the traditional yield spread model. The main source of this improvement is in the dynamic approach to constructing forecasts versus the direct forecasting approach used in the traditional yield spread model. Although the predictive power of the yield curve for output is concentrated in the yield spread, there is also a gain from using information in the curvature factor for the real GDP growth prediction.
(JEL C22, E43, E44, E47)

Keywords: Yield curve, Yield spread, Nelson-Seigel model, Forecasting

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1 Introduction

There are numerous papers which explore the question: "What information does the yield spread contain about future real economic activity?" These studies are based on the intuition that, when agents price assets, they take into account expectations about future states of the economy, and therefore interest rates potentially contain useful information about future economic growth. Estrella and Hardouvelis (1991) find evidence that the U.S. government bond yield spread contains information about future U.S. real economic activity at horizons of up to four years. Estrella and Mishkin (1997) confirm that the yield spread has the predictive power for real economic activity in the United States and in a number of European countries. Wheelock and Wohar (2009) provide a comprehensive survey of the literature on the predictive power of the yield spread for output growth.

In most of the previous literature on the predictive power of the yield curve for real economic activity, researchers have considered simple OLS regressions of future output on a yield spread defined as the difference between a specific long-term government bond rate and a short-term T-bill rate. Although this approach has the advantage of its simplicity, it does not have enough flexibility to use the information contained in the entire term structure of interest rates.

In this paper, I propose an approach to predicting output based on information contained in the entire yield curve. In particular, I examine the predictive power of the yield curve for real output by jointly modeling real GDP growth and the yield curve using the dynamic yield curve model proposed by Diebold and Li (2006) (hereafter DL(2006)). This model, which I refer to as the "NS dynamic yield curve model" for the purpose of this study, is based on the Nelson and Siegel (1987) three-latent-factor framework. The choice of the NS dynamic model for this study is driven by its relative parsimony compared to other yield curve models and its good out-of-sample forecasting performance for future yields. The model describes the entire term structure of interest rates using only three factors. DL(2006) introduce dynamics to the evolution of these factors and show that the NS dynamic model has a more accurate in-sample fit and produces better

forecasts of future yields at long horizons relative to other simple models. In terms of predicting output, the NS dynamic model has two advantages over the yield spread framework: (i) the model contains information about the entire term structure of interest rates and (ii) real GDP growth can be modeled jointly with yields in a parsimonious way using the three endogenously-defined factors. Another potential choice of term structure modeling would be the affine arbitrage-free class of models, which is popular in finance literature. However, as reported by Duffee (2002), arbitrage-free models produce poor out-of-sample forecasts of future yields.

Ang, Piazzesi, and Wei (2006) (hereafter APW(2006)) study the predictive power of the short-term yield and yield spread for real GDP growth using an affine arbitrage-free dynamic yield curve model. Their approach is based on modeling real GDP growth jointly with an exogenously-defined short-term yield and yield spread and imposing no-arbitrage constraints on the pricing of bonds. They find, in contrast to the previous findings in the literature on the predictive power of the yield curve for output, that the short-term interest rate has more predictive power for the GDP growth than the yield spread. The authors also report that imposing the no-arbitrage restriction only marginally improves forecasts of real GDP. Huang, Lee, and Li (2006) also analyze the gains from using information in the entire yield curve for aggregate personal income and inflation in their forecast combination study. They find that combining forecasts, where each individual forecast uses information in the yield curve, can improve forecasts of aggregate personal income growth and inflation. Chauvet and Senyuz (2009) construct a common factor from information in the yield curve to improve forecasts of recessions and industrial production. In particular, they estimate a common factor from the exogenously-defined yield curve level, slope, and curvature, using the data for three yields.

The focus of my analysis is to find out whether forecasting real GDP growth using the entire yield curve is better than using a yield spread forecasting model. For this analysis, I perform pseudo out-of-sample forecast comparisons for real GDP growth based on root mean square errors (RMSEs) for the NS dynamic yield curve model and the yield spread model based on OLS regressions of the GDP growth rate on a yield spread. I consider

various versions of the dynamic yield curve model in which real GDP growth is explained by different yield factors in order to analyze the marginal impact of each of the factors on the forecasting performance.

I find that the dynamic yield curve model significantly improves out-of-sample forecasts of real GDP growth at all horizons relative to the yield spread model. The main source of this improvement can be attributed to the dynamic way yield factors and real GDP growth are modeled. Although the predictive power of the yield curve for output is concentrated in the yield spread, there is also a gain from extracting more information from the entire yield curve relative to a specific exogenously-defined yield spread. In particular, there is a gain from using information in the curvature factor.

The rest of this paper is organized as follows. Section 2 describes the data. Section 3 motivates and presents the traditional yield spread model and reports the predictive power of this model for output. Section 4 describes the dynamic yield curve model. Section 5 reports estimation results for the dynamic yield curve model. Section 6 reports out-of-sample forecasting results and compares various versions of the dynamic yield curve and yield spread models. Section 7 concludes.

2 Data

The raw interest rate data are monthly-average yields on U.S. government bonds for maturities 3, 6, 12, 24, 36, 60, 84, and 120 months obtained from the FRED database.[1] The yields are constant maturity rates, except for the 3 and 6 month maturities that are secondary market rates.[2] Yield data for the maturities 3, 12, 36, 60, and 120 months cover the period of 1953:04 to 2009:12, for 6 months from 1959:01 to 2009:12, for 24

[1]Gurkaynak, Sack, and Wright (2007) is another source of publicly available data on the term structure of interest rates, which has yields for long-term bonds. These data are constructed using the Svensson (1994) model, which is an extension of the Nelson and Siegel (1987) model. Since the model used for my study is also based on the Nelson and Siegel (1987) model, I opt not to use these data in order to avoid fitting the data with the approach used to generate data in the first place.

[2]I use secondary market rate data for the 3 and 6 month maturities because the constant maturity rate data for these maturities are available for a substantially shorter sample period than the sample period that I consider for this study. I compared the secondary market 3 and 6 month maturity yield series with the constant maturity rate series for the common sample period and found that the dynamics of the series are close to each other. Therefore, this heterogeneity in data should not significantly affect my results.

months from 1976:07 to 2009:12, for 84 months from 1969:07 to 2009:12.[3] Monthly data on yields are transformed to the quarterly frequency by using observations from the last month of each quarter. Quarterly data on real GDP from 1952:Q1 to 2009:Q4 are also from the FRED database. Real GDP data are seasonally adjusted and chained in 2005 prices. Annualized real GDP growth is calculated as the difference of natural log output multiplied by 400. Similar to the previous studies on the predictive power of the yield curve for output growth (e.g., Stock and Watson (2003); Ang et al. (2006); and Diebold and Li (2006)), I use the revised data on real GDP rather than real-time data. As discussed in Ang et al. (2006), the focus of the analysis is on predictions of what actually happens to the economy, not preliminary announcements of economic growth. Table 1 reports descriptive statistics for the yields and real GDP growth.

Table 1: Descriptive statistics for yields and RGDP growth

Maturities (months)	Period	Mean	Std. Dev.	Min.	Max.	ADF
3	1953-M04 : 2009-M12	4.96	2.86	0.03	16.30	-2.51
6	1959-M01 : 2009-M12	5.44	2.78	0.15	15.52	-1.87
12	1953-M04 : 2009-M12	5.51	3.03	0.31	16.72	-2.03
24[1]	1976-M07 : 2009-M12	6.56	3.31	0.80	16.46	-0.88
36	1953-M04 : 2009-M12	5.92	2.88	1.07	16.22	-1.72
60	1953-M04 : 2009-M12	6.13	2.79	1.52	15.93	-1.65
84[1]	1959-M07 : 2009-M12	7.19	2.68	1.89	15.65	-1.16
120	1953-M04 : 2009-M12	6.36	2.69	2.29	15.32	-1.55
RGDP growth	1953-Q2 : 2009-Q4	3.02	3.76	-10.97	15.44	-10.31*

RGDP growth is calculated as the difference of natural log output multiplied by 400. The Augmented Dickey-Fuller (ADF) unit root test is based on SIC lag selection. The critical values for rejection of hypothesis of a unit root are: -3.44 at 1 percent level and -2.87 at 5 percent level. The hypothesis that yields have unit roots cannot be rejected at 5 percent level. The hypothesis that real GDP growth has a unit root is rejected at the 1 percent level, denoted by an asterisk.

/1 Average yields of 24 and 84 month bonds are higher than those of 36 and 120 month, respectively, because of the difference in sample periods.

[3]The data on yields have different staring dates; however, I do not extrapolate yields with shorter sample periods to the same beginning date, as the focus of this study is predictive power of yields on output using all available information.

3 Motivation

The standard explanations for why a yield spread might predict economic growth are focused on monetary policy and the expectation hypothesis. Under the expectation hypothesis, the term structure of interest rates is determined by agents' expectations of future short-term interest rates. Therefore, current long-term interest rates are averages of expected future short-term rates. If a monetary contraction sends the current short-term rate higher than the expected future short-term interest rate, then today's investment will decline causing a decline in future economic growth. Conversely, if a monetary expansion produces low current short-term interest rates leading to higher economic growth in future, then future short-term interest rates are expected to increase.

Harvey (1988) proposes another explanation for why the slope of yield curve and future economic activities can be related, which is based on the theory of smoothing intertemporal consumption and the real term structure of interest rates. In this setting, if agents expect that future economic activity will decline, then they have an incentive to save in the current period by selling short-term assets and buying bonds which will pay off in the low-income period. This will lower the yields for the bonds that will mature in the future and increase the short rate. Thus, in theory the yield curve contains information about future economic growth.

The term premium for holding long-term bonds is also a component that contributes to the determination of the term structure of interest rates in addition to the expectation factor. APW(2006) suggest that the expectation hypothesis component of the term structure of interest rates is the main driving force for output predictability. Hamilton and Kim (2002) suggest that the term premium, in addition to the expectation component, is also important for output prediction.

Most previous studies of the predictive power of the yield curve for real economic activity have employed OLS regressions of the future real GDP growth rate on the yield spread, defined as the difference between interest rates on the long-term (10 year) Treasury bond and the short-term (3 month) Treasury bill:

$$g_{t,t+k} = \alpha_{0,k} + \alpha_{1,k}\left(y_t\left(120\right) - y_t\left(3\right)\right) + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right), \tag{3.1}$$

where $g_{t,t+k}$ is the annualized real GDP (RGDP) growth rate defined as

$$g_{t,t+k} = 400/k\left(\ln RGDP_{t+k} - \ln RGDP_t\right), \tag{3.2}$$

where $y_t\left(120\right)$ and $y_t\left(3\right)$ are interest rates on the 10-year treasury bond and the 3-month treasury bill, respectively.

Figure 1 plots the yield spread as defined above, along with the annualized real GDP growth rate over subsequent four quarters. It is evident that real GDP growth and the yield spread are positively correlated. The correlation coefficient is 0.33.

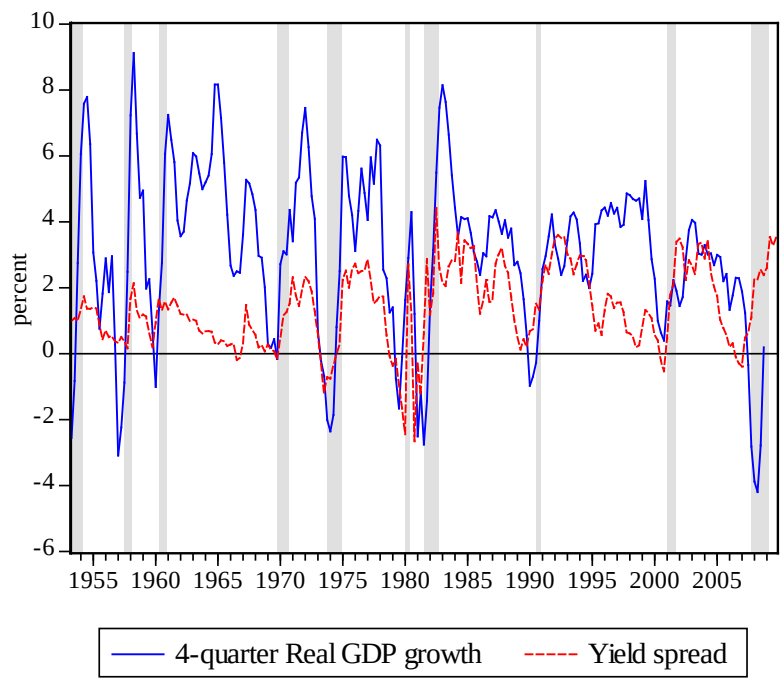

Figure 1: Real GDP growth and Yield Spread

This figure displays the subsequent four-quarter real GDP growth rate and the yield spread, defined as the difference between interest rates on the 10-year Treasury bond and the 3-month Treasury bill. Shaded areas correspond to NBER recession dates.

Table 2 reports the estimation results for the OLS regressions of future real GDP growth on the yield spread according to equation (3.1), the spread and one lag of the real GDP growth rate, the short rate only (defined as the 3-month T-bill interest rate), and

Table 2: Parameter estimates for OLS regressions of k-quarter-ahead annualized RGDP growth on the yield spread

k	OLS(Spread)		OLS(Spread,g_{t-1})			OLS($y_t(3)$)		OLS(Spread,$y_t(3)$)		
	$\alpha_{1,k}$	$\overline{R}^2$	$\alpha_{1,k}$	$\alpha_{2,k}$	$\overline{R}^2$	$\alpha_{3,k}$	$\overline{R}^2$	$\alpha_{1,k}$	$\alpha_{3,k}$	$\overline{R}^2$
1	0.472		0.517	0.218		-0.178		0.551	-0.104	
	(0.251)	0.02	(0.227)	(0.073)	0.06	(0.138)	0.01	(0.279)	(0.147)	0.04
4	0.686		0.688	0.020		-0.216		0.687	-0.129	
	(0.189)	0.10	(0.187)	(0.057)	0.10	(0.111)	0.05	(0.219)	(0.108)	0.14
8	0.621		0.620	-0.021		-0.165		0.571	-0.096	
	(0.136)	0.18	(0.137)	(0.039)	0.18	(0.081)	0.07	(0.175)	(0.076)	0.19
12	0.363		0.361	-0.028		-0.078		0.320	-0.038	
	(0.121)	0.10	(0.122)	(0.030)	0.10	(0.071)	0.02	(0.142)	(0.069)	0.08

Sample period: 1953:Q2-2009:Q4. Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors are in parentheses. The spread is defined as the difference between yields on the 10-year bond and the 3-month Treasury bill. The short rate is defined as the yield on the 3-month Treasury bill, denoted as $y_t(3)$; $\alpha_{1,k}$, $\alpha_{2,k}$, and $\alpha_{3,k}$ denote the coefficients from respective OLS regressions, with explanatory variables listed in parentheses; g_{t-1} is one lag of the annualized continuously-compounded real GDP growth rate; $\overline{R}^2$ denotes adjusted-R^2.

the spread and the short rate for the period from 1953:Q2 to 2009:Q4. The estimates for the yield spread coefficient from the yield spread regression are statistically significant for all horizons up to 12 quarters ahead and the adjusted-R^2s are considerably higher for 4, 8, and 12 quarter horizons than for the one quarter horizon. The estimates for the yield spread coefficient remain robust to controlling for one lag of the real GDP growth rate, with an increase in the adjusted-R^2 only at the one quarter horizon. This increase can be explained by short-term persistence of real GDP growth. I also consider the explanatory power of the short-term interest rate for future real GDP growth. The short-term interest rate is statistically insignificant in the regression with the short-term rate only for the most of the considered horizons. Also, the adjusted-R^2 of this regression is lower than the one for the regression model with the yield spread only. The yield spread remains strongly statistically significant after controlling for the short-term rate up to 8 quarters ahead. These results, which are in line with previous findings on

the predictive power of yield spread for output, confirm that the yield spread may be used to predict real output.

4 Model

4.1 The Dynamic Yield Curve Model

I consider the three-latent-factor dynamic yield curve model developed by DL(2006) based on the Nelson and Siegel (1987) framework. In this NS dynamic yield curve model, yields are represented by the following functional form:

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t} \underbrace{\left(\frac{1-\exp(-\lambda_t \tau)}{\lambda_t \tau}\right)}_{L_2(\tau,\lambda)} + \beta_{3,t} \underbrace{\left(\frac{1-\exp(-\lambda_t \tau)}{\lambda_t \tau} - \exp(-\lambda_t \tau)\right)}_{L_3(\tau,\lambda)}, \tag{4.1}$$

where $y_t(\tau)$ is an interest rate of zero-coupon bond with maturity τ at period t; $\beta_{1,t}$, $\beta_{2,t}$, $\beta_{3,t}$ are three latent dynamic factors interpreted as the level, slope, and curvature of the yield curve; and λ_t is a parameter responsible for fitting the yield curve at different maturities. Small values of λ_t fit the yield curve better at long maturities, while large values produce a better fit at short maturities. In this paper, I follow DL(2006) and, for simplicity, estimate λ_t as a time-invariant parameter. Therefore, its time subscript is dropped in further discussions. $L_2(\tau,\lambda)$ and $L_3(\tau,\lambda)$ denote the loadings for factors $\beta_{2,t}$ and $\beta_{3,t}$, respectively. The loading for factor $\beta_{1,t}$ is 1.

The choice of the NS dynamic model is motivated by its parsimony and good out-of-sample forecasting performance for the future yields. The alternative yield curve model to consider for this study would be the affine arbitrage-free class of yield curve models. However, as reported by Duffee (2002), arbitrage-free yield curve models perform poorly out-of-sample. Also, APW(2006), who study predictive power of the yield curve for output, find that imposing no-arbitrage restriction improves GDP forecasting only marginally over a VAR model. As will be shown in the empirical section, the NS dynamic model does better relative to a VAR model.

In the NS framework, the entire panel of yields is modeled by three latent factors

with imposed structure of loadings as follows:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & L_2(\tau_1,\lambda) & L_3(\tau_1,\lambda) \\ 1 & L_2(\tau_2,\lambda) & L_3(\tau_2,\lambda) \\ \vdots & \vdots & \vdots \\ 1 & L_2(\tau_n,\lambda) & L_3(\tau_n,\lambda) \end{pmatrix} \begin{pmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_n) \end{pmatrix}, \quad (4.2)$$
$$\varepsilon_t \backsim N_n(0,\Sigma).$$

Similarly to DL(2006) and Diebold, Rudebusch, and Aruoba (2006) (hereafter DRA(2006)), the measurement errors of yields of different maturities are assumed to be independent from each other. Therefore, the variance-covariance matrix of measurement errors in this equation, denoted as Σ, is a diagonal.

The latent factors are modeled as Gaussian first-order autoregressive processes:

$$\beta_{i,t} = \mu_i + \phi_i \beta_{1,t-1} + u_t, \qquad u_t \backsim N\left(0, \sigma_i^2\right) \quad \text{for } i \in \{1,2,3\}, \quad (4.3)$$

where σ_i^2 denotes the variance of the error-term for the factor process $\beta_{i,t}$.

In their study of the relationship between macro variables and the yield curve, DRA(2006) assume that the factors are governed by a VAR(1) process, allowing for interaction between all three factors and macro variables, and between their shocks. However, DL(2006) report that a model with a VAR(1) factor process forecasts yields poorly compared to a simple AR(1). My result suggests that a model based on independent factor processes also forecasts output better than a model with a VAR(1) factor process.

DL(2006) show that this general model can generate all possible yield curve shapes, has good in-sample fit, and forecasts future yields better out of sample than other models at 6 months or longer horizons. They also show that the $\beta_{1,t}$ factor is highly correlated with yields of different maturities. Therefore, it is interpreted as level factor; $-\beta_{2,t}$ is highly correlated with the yield spread; and $\beta_{3,t}$ is correlated with the curvature. In this model, all three latent factors are assumed to be stationary. As will be shown next, this model is also flexible in terms of incorporating macro variables.

4.2 The Dynamic Yield Curve Model with Real GDP growth

This subsection describes how to incorporate real GDP growth into the NS dynamic yield curve model. Since output growth is correlated with yields and yields are described by three factors, output growth should be correlated with the yield factors of the model.[4] Therefore, I modify the yield curve model to jointly model yields with the real GDP growth rate using the three yield factors. Previous analysis suggested that adding lagged real GDP growth improves forecasts of output, and therefore the modified model also controls for one lag of the real GDP growth rate.[5] After this modification, equation (4.2) has the following form:

$$\begin{aligned}
\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_n) \\ g_t \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \mu_g \end{pmatrix} + \begin{pmatrix} 1 & L_2(\tau_1,\lambda) & L_3(\tau_1,\lambda) \\ 1 & L_2(\tau_2,\lambda) & L_3(\tau_2,\lambda) \\ \vdots & \vdots & \vdots \\ 1 & L_2(\tau_n,\lambda) & L_3(\tau_n,\lambda) \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} \begin{pmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \end{pmatrix} \\
&\quad + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \gamma_4 g_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_n) \\ \varepsilon_t(g) \end{pmatrix}, \qquad (4.4) \\
\varepsilon_t &\backsim N_{n+1}\left(0, \widetilde{\Sigma}\right),
\end{aligned}$$

where g_t denotes the real GDP growth rate defined as

$$g_t = 400\left(\ln RGDP_t - \ln RGDP_{t-1}\right).$$

In this specification, output growth only enters into equation (4.4) while the factor dynamics equations remain the same as before. Thus, in this setting, real GDP growth

[4] DRA(2006) find evidence of interactions between the yield curve and macro variables based on analysis of impulse response functions and variance decompositions. They do not study forecasting performance of the macro-yield-curve model. They model macro variables as additional factors in the state dynamics of the yield curve model.

[5] The model is flexible to incorporating information in other macroeconomic variables. However, I limit the list of explanatory variables for the real GDP growth to the yield factors and one lag of real GDP growth, because the focus of this study is to analyze a gain from using information in the entire yield curve compared to the yield spread, rather than finding the best forecasting model for the real GDP growth.

is modeled only by the latent factors, which are mainly identified by the term structure of interest rates due to the rich panel of yields. This approach focuses on the one-way interaction from yields to macro variables. An alternative way of incorporating output growth into the yield curve model would be to follow DRA(2006) and add output growth to the factor process as an additional factor. This specification would allow for two-way interaction between output growth and other yield factors. However, preliminary results suggested that the forecasts produced by such a model were inferior to those produced by the model in equation (4.4).

5 In-sample Results

Estimation of the dynamic yield curve model is based on quarterly yield data for the sample period from 1953:Q2 to 2009:Q4. I estimate the model using a one-step Kalman filter maximum-likelihood procedure, which produces more efficient inferences than those from the two-step estimation procedure applied by DL(2006) and APW(2006).

The estimates of the factor process parameters, reported in Table 3, suggest that $\beta_{1,t}$ is a very persistent series with an autoregressive coefficient of 0.980 and a standard deviation for its shocks of 0.267. $\beta_{2,t}$ and $\beta_{3,t}$ are less persistent and more volatile than the level factor. The Augmented Dickey-Fuller (ADF) tests for unit roots in $\beta_{1,t}$, $\beta_{2,t}$, $\beta_{3,t}$ suggest that $\beta_{1,t}$ may have unit root with p-value 0.439 while $\beta_{2,t}$ and $\beta_{3,t}$ appear to be stationary with p-values <0.001. The ADF tests for unit roots in all yields, reported in the last column of Table 1, indicate that all yields may have unit roots.

Cointegration tests using the Johansen (1998) method suggest that the yields are cointegrated with each other.[6] Based on these results, I also considered a version of the model where yields are assumed to be cointegrated unit root processes.[7] Forecast results for real GDP growth in the stationary and unit root specifications are close to each other and there is no dominant model; therefore, I focus only on the model with the stationary specification in the remaining analysis.[8]

[6]The cointegration test suggests that elements of the vector of 3, 12, 36, 120 month yields are cointegrated with each other at a 5 percent level.

[7]In the unit root specification, it is assumed that $\beta_{1,t}$ is unit root process by restricting ϕ_1 to unity.

[8]The unit root dynamic yield curve model produces lower RMSEs of yields than the stationary model

Table 3: Parameter estimates for the factor processes

	ϕ_i	μ_i	σ_i
for $\beta_{1,t}$	0.980	0.118	0.267
	(0.012)	(0.082)	(0.027)
for $\beta_{2,t}$	0.836	- 0.291	0.764
	(0.037)	(0.085)	(0.076)
for $\beta_{3,t}$	0.812	0.000	1.564
	(0.041)	(0.040)	(0.160)

Sample period: 1953:Q2-2009:Q4. The parameters are denoted according to equation (4.3). Standard errors of estimates are reported in parentheses.

Table 4 reports estimates of the factor loadings for real GDP growth in the dynamic yield curve model. The estimates of the slope and curvature factor loadings for real GDP growth are statistically significant at a 10 percent level and they are economically significant given their point estimates are considerably different from zero. The negative sign of the slope coefficient γ_2 for real GDP growth is consistent with the interpretation of $\beta_{2,t}$ as minus the slope of the yield curve. The estimate of the level factor loading for real GDP growth is statistically insignificant.

Table 4: Parameter estimates for RGDP growth

γ_1	γ_2	γ_3	γ_4	μ_g
-0.120	-0.305	0.297	0.341	0.341
(0.138)	(0.165)	(0.150)	(0.063)	(0.063)

Sample period: 1953:Q2-2009:Q4. The parameters correspond to equation (4.4). Standard errors of estimates are reported in parentheses.

The estimate of the coefficient for the lagged real GDP growth rate, denoted as γ_4, is statistically significant and its value is comparable with the estimate in AR(1) model, suggesting that the autocorrelation component remains important after controlling for the yield factors.

at long horizons. Forecasting performances of the models for real GDP growth relative to each other are mixed.

Table 5: Statistics for measurement errors of yields and RGDP growth

maturity	Dynamic Model		OLS		AR(1)	
and RGDP	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
3	- 0.01	1.02				
12	0.16	1.02				
36	0.01	0.84				
60	- 0.01	0.75				
120	0.02	0.63				
RGDP growth						
1 quarter ahead	0.01	3.46	0.00	3.63	0.00	3.48
4 quarter ahead	0.05	2.33	0.00	2.35	0.05	2.31
8 quarter ahead	0.11	1.68	0.00	1.58	0.08	1.64
12 quarter ahead	0.14	1.35	0.00	1.29	0.08	1.29

Sample period: 1953:Q2-2009:Q4. The dynamic yield curve and the yield spread models include one lag of real GDP growth. OLS denotes a regression of RGDP growth on the yield spread and one lag of RGDP growth.

Table 5 reports statistics for the measurement errors of yields and real GDP growth based on the in-sample fit of the dynamic yield curve model, OLS yield spread model, and an AR(1) model. All these models control for one lag of real GDP growth.

The dynamic yield curve model has a better fit for real GDP growth at the one-quarter horizon, while the OLS yield spread model has a better fit at most of the other horizons. The fit of real GDP growth by the OLS yield spread model at long horizons is explained by the forecasting specification of the model and the nature of OLS regression, which is to minimize squared residuals. Specifically, the OLS yield spread model has an advantage in terms of in-sample fit over the dynamic yield curve model, because the former is a forecasting model at targeted horizons, while the dynamic yield curve model fits the current data.[9] Meanwhile, both the dynamic yield curve model and the OLS yield spread model have a better fit than the univariate autoregressive model because

[9]To check this point, I estimated a dynamic yield curve model with the specification changed to be similar to a direct forecasting model. Even with a forecasting specification at one period ahead and iterating for longer horizon forecasts, the in-sample fit for the forecasting dynamic yield curve model improved over the results of the OLS yield spread models for most horizons. Despite the obvious advantage of the forecasting specification of the dynamic yield curve model, I use the contemporaneous version of the model for this study as it uses all available current information for out-of-sample forecasting. Also, out-of-sample forecasting results suggest that the contemporaneous model outperforms the model with a forecasting specification.

they nest the AR(1) model.

6 Out-of-sample Forecasting Results

6.1 Forecasting Procedure and Notation

Pseudo out-of-sample forecasts of real GDP growth are performed for the period from 1990:Q1 through 2009:Q4. This period includes two mild recessions in 1990 and 2001 and the severe economic downturn in 2007-2009. The forecast performance of models is compared using root mean square errors (RMSEs) relative to a benchmark model. Following Stock and Watson (2003) and Ang et al. (2006), I use the RMSEs for the AR(1) model at different horizons as benchmarks.

The RMSE statistic for the dynamic yield curve model is generated using the following procedure. First, the parameters of the state-space model are estimated using Kalman filter method and then yields and real GDP growth are forecasted for 1 to 12 quarters ahead. Next, one more observation is added to the in-sample data and the estimation and forecasting are repeated.[10] This procedure produces 73-k observations of k-quarter-ahead out-of-sample forecasts for k from 1 to 12 quarter horizons.

For the forecast performance comparisons at a given horizon, I use a cumulative real GDP growth averaged for the whole horizon rather than marginal one-period forecasts at that horizon. This choice of the forecast comparison is explained by the iterative approach to constructing k-quarter-ahead forecasts using the NS dynamic yield curve model. For this approach, the quality of one-quarter forecasts at a given horizon depends directly on the quality of the forecasts at all previous periods. In contrast, constructing one cumulative k-quarter-ahead real GDP growth forecast using the OLS yield spread model does not require these iterations because the parameter estimates for equation

[10] In this procedure, the beginning of the in-sample period is fixed for all forecasts, in contrast to the rolling-window approach with a shifting in-sample period. As discussed in subsection 6.4, yield curve literature reports evidence for structural changes in the predictive power of the yield curve for output. Indeed, the correlation between the yield spread and 4-quarter-ahead real GDP growth has values of 0.36, 0.69, and 0.14 for the sub-samples 1953Q2-1968:Q1, 1968:Q2-1984:Q4, and 1985:Q1-2009:Q4, respectively, indicating a strong predictive power of the yield spread in the middle of the sample. Keeping the beginning of the in-sample period fixed should reduce any bias in the model parameter estimates by using more observations with less predictive power of the yield curve relative to the middle of the sample.

(3.1) quantify the relationship between the yield spread and cumulative k-quarter-ahead real GDP growth.

I compare the out-of-sample forecast performance of the two classes of models: the NS dynamic yield curve models and the OLS yield spread models. For each class of models, I consider several specifications of the models with different explanatory variables for real GDP growth. I denote the class of dynamic yield curve models as NS and the class of yield spread models as PR, which stands for "predictive regression". To denote the specification of a model in each class of models, the explanatory variables used to model real GDP growth are listed in parentheses. For example, the notation $NS(g(\beta_2, \beta_3, g_{t-1}))$ means that this is the NS dynamic yield curve model with real GDP growth modeled by $\beta_{2,t}$, $\beta_{3,t}$ factors and one lag of real GDP growth g_{t-1}.

6.2 Forecasts of Real GDP Growth

In this subsection, I analyze the effects of different explanatory factors for the real GDP growth forecasts. Table 6 reports RMSE results for different versions of the NS dynamic yield curve and the OLS yield spread models.

The dynamic yield curve model with lagged real GDP growth has lower RMSEs than models without lagged real GDP growth. Most of the improvement is observed at short horizons. Similarly, adding lagged real GDP growth in the OLS yield spread model improves forecasts at shorter horizons. The positive effect of the autoregressive component in the short-term horizon forecasts reflects the short-term persistence of real GDP growth.

The RMSEs for models with a curvature factor $\beta_{3,t}$ are smaller than for models without this factor at all considered horizons. Thus, adding the curvature factor to the slope factor for real GDP growth forecasting extracts additional information contained in yield curve for real GDP modeling, while the OLS yield spread model does not contain this information. This result concurs with Huang et al. (2006) who find some usefulness of the curvature factor for forecasting output in their study of forecast combination using information in the yield curve.

Adding the level factor $\beta_{1,t}$ as an explanatory variable for real GDP growth in the

Table 6: Out-of-sample forecasts of RGDP growth rates: Root Mean Square Error Ratios. Out-of-sample period 1990:Q1-2009:Q4

	Forecast horizon k-quarters ahead			
	1	4	8	12
Dynamic Yield Curve Models				
NS($g(\beta_2)$)	1.157	1.068	1.000	0.994
NS($g(\beta_2, g_{t-1})$)	1.010	1.008	0.973	0.974
NS($g(\beta_2, \beta_3, g_{t-1})$)	**0.996**	**0.970**	**0.947**	**0.955**
NS($g(\beta_1, \beta_2, \beta_3, g_{t-1})$)	1.002	0.987	0.985	1.032
VAR($Spread, g_{t-1}$)	1.022	1.022	0.991	0.993
Yield Spread Models				
PR($Spread$)	1.239	1.254	**1.103**	**1.045**
PR($Spread, g_t$)	**1.076**	**1.197**	1.112	1.051
PR($Shrt.Rate, Spread, g_t$)	1.113	1.293	1.211	1.126

NS and PR denote the NS dynamic yield curve and PR yield spread models, respectively. The denominators are the RMSEs for an AR(1) model. The lowest RMSE ratios within each class of models are in bold.

dynamic yield curve model increases RMSEs, indicating its negative effect on the forecasting performance of the model. Similarly, adding the short rate to the yield spread model increases the RMSEs.

6.3 Does the Dynamic Yield Curve Model Forecast Output better than the Yield Spread Model?

To answer the question of whether the dynamic yield curve model improves forecasts of real GDP growth over the OLS yield spread model, I compare RMSEs for the following pairs of models with comparable explanatory variables for real GDP growth: $NS(g(\beta_2))$ and $PR(Spread)$; $NS(g(\beta_2, g_{t-1}))$ and $PR(Spread, g_t)$; $NS(g(\beta_2, \beta_3, g_{t-1}))$ and $PR(Spread, g_t)$. Table 6 reports noticeably lower RMSEs for the dynamic yield curve models than for OLS yield spread models at all horizons. The Diebold and Mariano (1995) (hereafter DM(1995)) test of forecast accuracy comparison, reported in Table 7, suggests that these differences in RMSEs are statistically significant in most of the

Table 7: Diebold-Mariano tests for comparative forecast accuracy

Models	Forecast horizon k-quarters ahead			
	1	4	8	12
NS($g(\beta_2)$)	-1.092	-1.699	-0.505	-0.156
against PR($Spread$)	(0.005)	(0.006)	(0.087)	(0.171)
NS(g(β_2,g_{t-1}))	-0.761	-1.642	-0.669	-0.235
against PR($Spread, g_t$)	(0.004)	(0.004)	(0.021)	(0.046)
NS(g(β_2,β_3,g_{t-1}))	-0.924	-1.934	-0.786	-0.287
against PR($Spread, g_t$)	(0.015)	(0.005)	(0.006)	(0.022)

NS and PR denote the NS dynamic yield curve and PR yield spread models, respectively. The null hypothesis of the Diebold-Mariano(1995) test is that the mean of square loss-differential of two models is zero, against alternative that it is not zero. Negative (positive) value of the estimate indicates that the first model produces more (less) accurate forecasts than compared model. The p-values for the test are reported in parentheses. The test is based on the Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors.

cases.[11] Thus, the dynamic yield curve model outperforms the OLS yield spread model in forecasting real GDP growth.

To check the robustness of the result to the sample choice, I also consider a shorter out-of-sample period 1998:Q1-2009:Q4, allocating more observations for the in-sample period. The RMSE ratios, reported in the first panel in Table 8 , confirm that the NS dynamic yield curve model performs better than the yield spread model for this out-of-sample period as well.

Haubrich and Dombrosky (1996) and Dotsey (1998) find a decline in the predictive ability of the yield curve for output in the period after 1985. Estrella, Rodrigues, and Schich (2003), using the test for an unknown break date, also find some evidence of structural instability in the yield spread and industrial production relationship in 1983. To analyze the effect of this structural instability on the forecast performance of the OLS yield spread model, I perform out-of-sample forecasts of real GDP using the in-sample period from 1985:Q1 to 1997:Q4. The beginning of this period is chosen based on the

[11] While there are several tests of forecast accuracy (e.g. West (1996) and Giacomini and White (2006)), the choice of the Diebold and Mariano (1995) test for this study is explained by the focus on out-of-sample performance and simplicity of the test application.

Table 8: Out-of-sample forecasts of RGDP growth rate: Root Mean Square Error Ratios. Out-of-sample period 1998:Q1-2009:Q4. Different in-sample periods

	Forecast horizon k-quarters ahead			
	1	4	8	12
In-sample period: 1953:Q2-1997:Q4				
NS($g(\beta_2, \beta_3, g_{t-1})$)	0.986	0.953	0.933	0.942
PR($Spread, g_t$)	1.062	1.143	1.009	1.003
In-sample period: 1985:Q1-1997:Q4				
PR($Spread, g_t$)	1.015	0.993	0.964	0.938

NS and PR denote the NS dynamic yield curve and PR yield spread models, respectively. Denominators are RMSEs for the AR(1) model.

previous literature on structural break. The extension of the end of the in-sample period from 1989:Q4, used in my previous analysis, to 1997:Q4 allocates a sufficient number of observations for the in-sample estimation of the yield spread model, given a relatively small number of the model parameters. The second panel in Table 8 reports RMSE ratios from the PR yield spread model based on the in-sample period 1985:Q1-1997:Q4. The RMSE ratios for the PR yield spread model based on the post 1985 in-sample period is noticeably smaller than those based on the longer in-sample period, suggesting a possible structural break in the relationship between the yield curve and output. However, the NS dynamic yield curve model still performs better than the yield spread model at most of the horizons. Although the in-sample period 1953:Q1-1997:Q4 does not fully address structural change in parameters for the NS dynamic yield curve model, it still reduces any bias of parameter estimates given that the sample period contains more post regime-shift observations. Forecasting using the NS dynamic yield curve model based on the in-sample period 1985:Q1-1997:Q4 is not performed due to the high number of model parameters relative to the small number of in-sample observations. The large standard errors of parameter estimates based on this short in-sample period overweigh the potential benefit from just using the post-structural-break data.[12]

Given that the dynamic NS model with real GDP growth modeled by the slope

[12]Presumably, both the NS dynamic yield curve model and the PR yield spread model should improve forecasting performance if structural changes are addressed using non-linear regime-switching models. For conciseness of this study I leave more complicated non-linear models for future research.

factor outperforms the yield spread model with a direct forecasting approach at all horizons, there are two potential sources for the forecast improvement. In particular, the improvement could originate from i) an iterative forecasting scheme versus the direct forecasting approach used in the yield spread model, and/or ii) from using an endogenously estimated slope factor versus a particular observable yield spread. To check these two possibilities, I evaluate out-of-sample forecasts using a simple VAR(1) model with an observable term spread and real GDP growth as variables:

$$\begin{pmatrix} Spread_t \\ g_{t-1} \end{pmatrix} = \begin{pmatrix} \mu_s \\ \mu_g \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} Spread_{t-1} \\ g_{t-2} \end{pmatrix}. \tag{6.1}$$

Given the dynamic structure of the VAR model, the RMSEs for this model are produced using the same forecasting procedure applied for the dynamic yield curve model. The RMSEs for the VAR model, denoted as $VAR\,(Spread, g_{t-1})$ and reported in Table 6, are considerably smaller than those from the yield spread model $PR(Spread, g_t)$. Thus, the dynamic approach for forecasting real GDP appears to be the main source of the forecast improvement over the direct forecasting approach. The OLS regression for a targeted forecasting horizon may cause overfitting of in-sample data due to the "least squares" nature of the inferences. This point is supported by the fact that the yield spread model performs considerably worse than the dynamic yield curve model in out-of-sample forecasts, while it has the best in-sample fit. Thus, poor out-of-sample performance of the yield spread model indicates that the yield curve is less useful for GDP forecasting than suggested by the in-sample OLS regression.

In addition, although the RMSEs for the $VAR\,(Spread, g_{t-1})$ and $NS(g(\beta_2, g_{t-1}))$ models are close to each other, the NS dynamic yield curve model still outperforms the VAR at all horizons. Thus, there is also a gain from using the endogenously-estimated slope factor versus the observable yield spread. Also, modeling real GDP growth by endogenously determined factors avoids the problem of dependence of results on the choice of the maturities for the yield spread.

6.4 Output forecasts for the period of the "Great Recession" in 2007-2009

Previous research findings show that the yield spread is a relatively good predictor of recessions (e.g.Estrella and Mishkin (1996) and Estrella and Mishkin (1998)), suggesting that the predictive power of the yield spread for output is mainly concentrated in periods of large changes in economic conditions. Meanwhile, the AR(1) model, our benchmark model, has a good predictive performance in periods of low volatility. To analyze forecasting performance of the yield curve models during the recent severe economic downturn, I consider predictions of real GDP implied by the NS dynamic yield curve model, the yield spread model, and the AR(1) model for the period of the "Great Recession" in 2007-2009 which involved large movements in real GDP. Because of the short out-of-sample period, I analyze output predictions keeping the number of out-of-sample forecasts the same for all horizons. This requires in-sample periods to be different for different forecast horizons. For example, the in-sample period ends 2006:Q4 for the first 1-quarter-ahead forecast, which is for the period 2007:Q1, and 2006:Q1 for the first 4-quarter-ahead forecast, which is for the period 2006:Q2-2007:Q1. This procedure contrasts the one used in the previous analysis with the same in-sample period and different out-of-sample periods for different forecast horizons. The latest recession allows the beginning of the in-sample period to be changed to 1985, in contrast to 1953 used in my previous analysis. Despite this change in the beginning date of the in-sample period, the extension of the end of the in-sample period allocates a reasonable number of observations for the in-sample fit of the dynamic yield curve model relative to the number of model parameters. Using the in-sample period starting after 1985 allows the models to address structural instability in the yield curve and output relationship discussed in my previous analysis.

Table 9 reports RMSE results for real GDP growth for the period of 2007-2009 using the NS dynamic yield curve model NS($g(\beta_2, \beta_3, g_{t-1})$) and the yield spread model PR($Spread, g_t$). The RMSE ratios for the NS dynamic yield curve model relative to those for the AR(1) model are lower than those for the yield spread model for all considered horizons. These results suggest that the NS dynamic yield curve model predicted real

GDP in this period better than the yield spread model, confirming robustness of the previous result to the sample choice. In addition, both the NS dynamic yield curve model and the yield spread model performed better than the AR(1) model for most of forecast horizons for the "Great Recession" period. In the previous analysis with the longer out-of-sample period, only the NS dynamic yield curve model out-performed AR(1) model. The RMSE ratios for the "Great Recession" out-of-sample period are noticeably smaller than those reported for the longer out-of-sample period, confirming that the yield curve is more useful for forecasting output when there are large changes in output than when it is relatively stable.

Table 9: Out-of-sample forecasts of RGDP growth rates: Root Mean Square Error Ratios. Out-of-sample period 2007-2009

	Forecast horizon k-quarters ahead			
	1	4	8	12
NS($g(\beta_2, \beta_3, g_{t-1})$)	0.957	0.900	0.845	0.901
PR($Spread, g_t$)	1.012	0.958	0.890	0.919

NS and PR denote the NS dynamic yield curve and PR yield spread models, respectively. The denominators are the RMSEs for an AR(1) model. The in-sample period starts from 1985:Q1 and ends 1, 4, 8, and 12 quarters prior to a forecasted period for respective horizons, allocating the same number of out-of-sample forecasts for all horizons.

7 Conclusion

Most studies that investigate the predictive power of the yield curve for real GDP growth consider a simple direct forecasting structure with the yield spread as the predictive variable. In this paper, I have considered a different approach. In particular, I have jointly modeled real GDP growth and yields using the dynamic three-factor yield curve model.

My empirical findings suggest that the dynamic yield curve model produces better out-of-sample forecasts of real GDP growth than the traditional yield spread model. This result is mainly attributed to the dynamic structure of the yield curve model. Although the predictive power of the yield curve is concentrated in the yield spread, there is also a gain from extracting more information from the term structure of interest rates versus

an exogenously-defined yield spread used in the yield spread model. In particular, there is a gain from using information in the curvature factor. In general, through, the yield curve is less useful for out-of-sample prediction of real GDP than the predictive power suggested by in-sample OLS regression analysis.

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