Interest Rate Fluctuations and Equilibrium in the Housing Market*

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Abstract

I study the general equilibrium of the housing market in an economy populated by overlapping generations of households. A contribution of the present paper is to solve for the housing market equilibrium in the presence of aggregate (interest rate) uncertainty with a realistic mortgage contract. In addition, households also face idiosyncratic uncertainty resulting from stochastic changes over the lifecycle in tastes (or need) for housing. In this environment, profit maximizing banks offer fixed-rate mortgage (FRM) contracts to home buyers. As seems plausible, each housing market transaction is subject to a fixed cost, which gives rise to S-s policy rules for housing transactions: existing home owners change the size of their houses only if there is a sufficiently large change in the state of the economy (i.e., in interest rates, in their taste for housing, etc.) A plausibly calibrated version of the model is consistent with three empirically documented features of the housing market: (i) highly volatile housing prices and transaction volume, (ii) a strong positive correlation between transaction volume and housing prices, and (iii) a significant negative relationship between interest rates and housing prices, which can rationalize a large part of the recent boom in housing prices in the U.S. and around the world.

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1 Introduction

In this paper I study the effects of interest rate fluctuations on the equilibrium of the housing market in an economy populated by overlapping generations of households. The particular model studied here combines three realistic features of the housing market: first, I allow for stochastic fluctuations in mortgage interest rates, second, I model long-term mortgage contracts explicitly, and third, I model housing as an illiquid asset (which is also consumed). I find that while the illiquidity of the housing increases the responsiveness of the housing prices to the interest rates, the mortgage contract enables to match the housing price-transaction volume (number of houses sold) comovement observed in the data.

I use this model to study several important trends observed in the housing market in the last decades. The first one of these trends is the rise in housing prices observed in the United States since the late 1970's, which turned into a substantial "boom" between 1995 and 2005. This trend was especially pronounced in large cities: for example, in places like Boston, New York, San Francisco, and Los Angeles real median housing prices have tripled, and in some cases quadrupled, between 1975 and 2004 (see figures 1 and 3.) The upward spiraling in prices led to suggestions of a bubble in the housing market in the popular press.[1] Although it seems hard to rule out the existence of a bubble definitively, it is important to note that similar trends in housing prices have been observed in many other countries: from 1997 to 2006, housing prices rose by 116 percent in Sweden, by 184 percent in Britain, by 173 percent in Spain, by 244 percent in Ireland, and by 315 percent in South Africa, exceeding the 98 percent rise observed in the United States. Overall, thirteen out of the eighteen countries in the survey saw price increases of 88 percent or more during this period (see the survey of the global housing market in the Economist magazine (2006b).)[2]

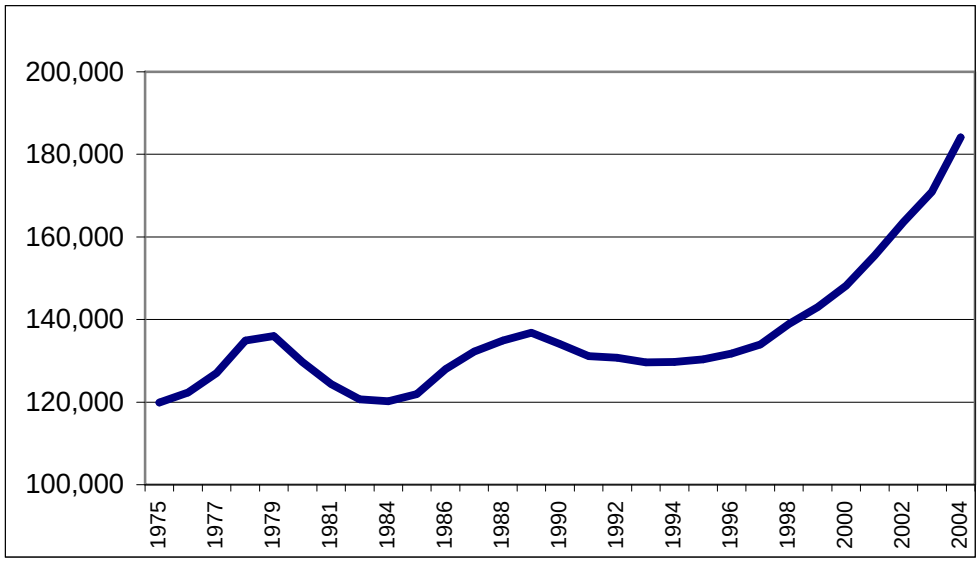
This global nature of the housing price boom suggests that changes in some fundamentals common to these countries could be possible driving forces behind these trends. Clearly, the (mortgage) interest rate is one such variable with a global impact, which is also especially natural to think of in the context of housing market transactions. Figure 2 plots the real 30-year fixed rate mortgage rates in the U.S. from 1975 to 2005. Comparing the mortgage rate movements in this graph to the changes in real median housing prices shown earlier in figure 1 reveals a surprisingly strong (negative) relationship: housing prices were falling from 1979 to 1985 when real mortgage rates were high; and housing prices were "booming" in late 1990's when mortgage rates were low (and falling.) Therefore, figures 1 and 2 are at least qualitatively consistent with the hypothesis that the housing price boom of the last decade could be due to falling interest rates. Of course, the real question is how big the effect is quantitatively, which is the question I undertake in this paper.

A parallel set of trends during this period have been observed in the transaction volume—which I define as the number of existing homes sold in a given time period. Transaction volume has increased

[1] See for example, the article entitled "America's house-price bubble" in the Economist magazine (2006a).

[2] The data that we report from the Economist (2006b) is in nominal terms.

Figure 1: Real, Median Housing Prices, US, in 2004 Dollars (Yearly). Source: National Association of Realtors. Deflated by CPI (BLS).



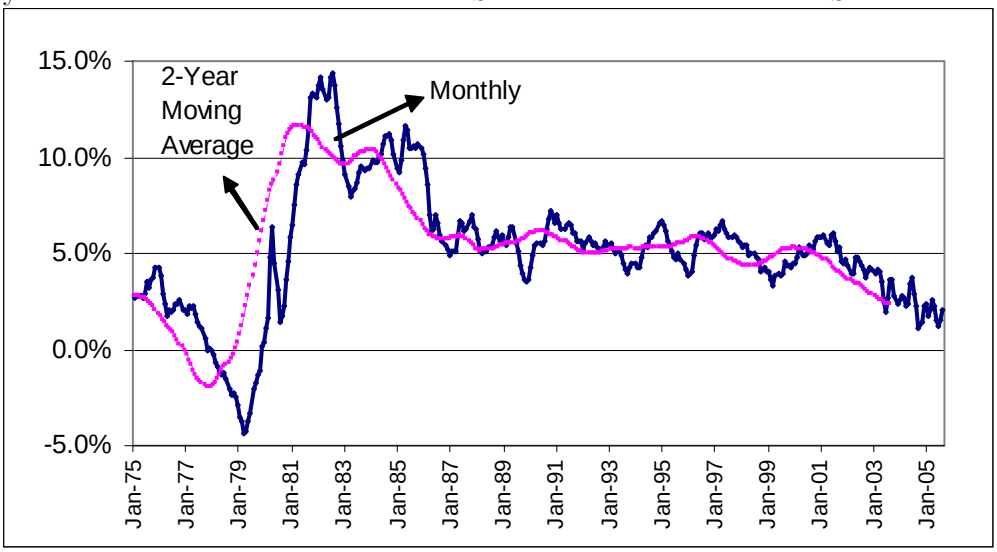
steadily during this period (except for a brief but sharp fall from 1979 to 1982 after the sharp rise in mortgage rates) and more than *doubled* from 1982 to 2004 (figure 4.) Furthermore, this trend in transaction volume appears to be closely associated with the trend in prices: the correlation between the two series is 0.90 in the national U.S. data (figure 5), and as shown by Genesove and Mayer (2001) the comovement is even more pronounced in local housing markets. The same comovement is also documented for Britain and France (Ortalo-Magne and Rady(2004); Stein (1995).) Finally, the comovement of the two series is not entirely driven by the trend components, but they also strongly appear at higher frequency (see for example Rios-Rull and Sanchez-Marcos (2005).

Clearly, this paper is not the first to observe that (mortgage) interest rates are likely to have an important impact on housing prices and transaction volume. To give just one example, in his testimony to the Congress in 2005, the then-Chairman of the Federal Reserve Board Alan Greenspan said:

> Historically, it has been rising real long-term interest rates that have restrained the pace of residential building and have suppressed existing home sales, high levels of which have been the major contributor to the home equity extraction that arguably has financed a noticeable share of personal consumption expenditures and home modernization outlays.

Nevertheless, the difficulty of solving a model with interest rate uncertainty and realistic mortgage contracts (as well as some other key features of housing market transactions, such as fixed costs,

Figure 2: Real 30 Year Fixed-Rate Mortgage Rates. Real rates are obtained by subtracting inflation in a given year from the nominal interest rate. Source: Fannie Mae and BLS.



which I consider here) has previously prevented a quantitative assessment of the channels explored here.[3]

The basic framework studied in this paper is a two-period overlapping-generations model.[4] At the beginning of each period, the economy is hit by an exogenous interest rate shock. Households derive utility from housing and non-housing consumption and they are allowed to borrow and lend in the bond market. Households can also borrow in the mortgage market only for the purpose of financing their house purchase. Risk-neutral profit maximizing banks offer fixed-rate mortgages that are paid in two periods (end of periods 1 and 2.) Because young households enter the economy with limited resources, they borrow in the mortgage market to finance the housing purchase. At the beginning of period 2 (old age) households receive an idiosyncratic shock to their taste for housing, which could be thought of as resulting from changes in family size, etc. Consequently, households would like to adjust the size of their houses. As noted earlier, housing market transactions are subject to a fixed cost, which implies that only households with a sufficiently large shock (in absolute value) will choose to adjust. These "movers," will sell their homes, receive the proceeds, pay the balance of their first mortgage debt, buy a new house and get a new mortgage (which in this case is a one-period mortgage paid at the end of period 2.) Non-movers simply pay their existing mortgage and enjoy their existing home and non-housing consumption. Before death, individuals sell their homes and derive utility from the sales price which can be thought of as the utility from leaving a bequest. For

[3] To my knowledge, there has been no work on the role of the interest rate fluctuations on the transaction volume.

[4] I calibrate a richer version of the framework described here in quantitative analysis. In particular, I allow for individuals to live 30 years, but let them trade at 2 points during their lifecycles.

simplicity I assume that aggregate house supply is fixed and there is no rental market.

Using a plausibly calibrated version of the model I find that housing prices are quite sensitive to the interest rates: a 10 percent decline in interest rates (close to the decline observed in the US in the last 2 decades) causes a 50 percent increase in the real median housing prices. This quantity is large enough to explain almost entire real housing price appreciation in the US and around half of the appreciation in big cities observed during the last two decades (see figures 1 and 3.)[5] An interesting implication of the model is that current interest rates not only affect the current housing prices but also affect the next period's housing prices. Higher interest rates in the current period induce young agents to save more, which increases the total wealth in the next period. As wealthier agents will demand more housing, prices will go up in the next period. Quantitatively, if current interest rates increase 1 percent, housing prices will increase around 4 percent in the next period.

In the model, transaction volume also varies significantly with interest rates. With low interest rates in the current period, homeowners are more willing to sell their houses since their mortgage contract they got in the previous period is relatively more costly to hold. With a similar intuition, if home buyers purchase their home with low interest rates in the current period, they will be less likely to sell their houses in the next period, because they will already be holding a good contract. The model predicts that, on average, a 1 percent decline in the current interest rates causes a 3 percent increase in transaction volume in the current period, and a 5 percent decrease in transaction volume in the next period. The combination of the two effects further implies that transaction volume will decline when interest rates rise, which is the observation of Alan Greenspan that I quoted earlier.

In the model, consistent with the US data, transaction volume and housing prices comove with a large positive correlation (around 0.8) as interest rates affect both variables similarly. Moreover, although we abstracted from the sources of dispersion other than interest rates, the dispersions of the housing prices and transaction volume—measured by the coefficient of variation—are very close to the dispersions observed in the US data.

The model predicts that the welfare effects of housing price movements differ across agents. In case of a housing price increase, an old agent who downsizes his house will be better off since his return from the housing transaction will be higher (due to the difference of the sizes of houses.) Old agents who move to bigger houses and young agents will be worse off. Finally, I look at the welfare implications of a possible tax reform which eliminates the tax deductibility of the mortgage interest payments. A comparison of the two steady states (before and after the tax reform) shows that the young agents are willing to give up around 1.3 percent of their consumption to *implement* the tax reform. However, old agents are willing to give up around 1.2 percent of their consumption to *avoid* the tax reform, because after the tax reform, they will not be getting the tax returns and the returns

[5] Since we assumed that aggregate housing supply is fixed the results mentioned in the paper fits better to large cities where zoning restrictions and land regulations limit the response of housing supply (see for example Glaeser and Gyourko (2003a)).

they expect to get by selling their houses in the next period will be lower.[6]

Himmelberg, Mayer and Sinai (2005) and Martin (2005) emphasize the importance of interest rates in explaining the recent increase in housing prices. Himmelberg, Mayer and Sinai (2005) develop a formula to account for the changes in the annual cost of owning a house. In their formula, interest rates affect the homeowner's forgone earnings. Martin (2005) studies the baby boom's impact on the US housing prices by using a simple Lucas asset pricing model. He argues that the baby boom affected the demand for housing as well as the interest rates. Due to lower interest rates, his model can predict the housing price increase in the last decade as opposed to Mankiw and Weil (1988.) In the mentioned studies housing is a liquid asset whereas in this study (and in real life) it is not. The illiquidity of housing good combined with an FRM contract amplifies the effect of interest rates since the consumer will be locked in with an interest rate for a longer time.

There is also an extensive literature which tries to explain the comovement of housing prices and transaction volume. Stein (1995) and Ortalo-Magne and Rady (2004) explain the comovement by using credit constraints and large down payment requirements. Wheaton (1990) and Berkovec and Goodman (1996) use search models to explain the comovement. Using a data set for the Boston condominium market, Genesove and Mayer (2001) argue that loss aversion can explain price-volume correlation. Each of the mentioned studies relies on some frictions. In my model, it is the mortgage contracts and the interest rate movements that cause the price-volume comovement. Thus, in addition to the mentioned mechanisms, this paper introduces a new quantitatively important mechanism which causes housing price-transaction volume comovement.

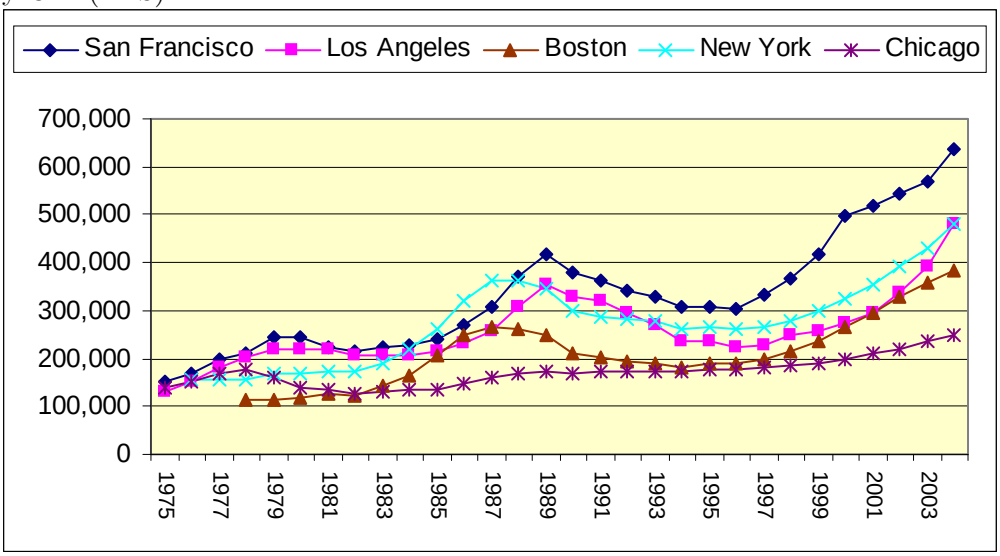
The paper is organized as follows. The next section documents some observations in the housing market. Section 3 describes the model. In Section 4, I give an outline of the algorithm that I use to solve the model. I do the calibration in Section 5. The main results of the paper are reported in Section 6. In Section 8 I look at the effect of housing prices movements on the welfare of the consumers. I do a policy analysis where I eliminate the tax deductibility of the mortgage interest payments in Section 8. In Section 9, I conclude.

2 Some Features of the Housing Market

Office of Federal Housing Enterprise Oversight (OFHEO), National Association of Realtors (NAR) and Census Bureau are the major sources of housing price data in the US. McCarthy and Peach (2005) show that the Census constant quality index consistently predicts less housing price appreciation than the other two. The NAR index and the OFHEO index have very close housing price predictions. But the bottom line is that all of them predict that housing prices have been rising exceptionally, especially in the last 10 years. Figure 1 shows that the real median price of a house in the US increased more than 50 percent in the last 20 years. Housing prices have been increasing

[6] Housing prices decrease around 14 percent after the tax reform.

Figure 3: Real, Median, City Level Housing Prices, in 2004 Dollars (Yearly). Source: OFHEO. Deflated by CPI (BLS)



more than the national average for most of big cities. The median price in New York, San Francisco, Boston and Los Angeles almost doubled in last 10 years (figure 3).

Transaction volume (number of existing units sold) has been increasing steadily during the last three decades other than the sharp decline around 1980's. After that decline, transaction volume more than doubled in the last two decades (figure 4).[7] The data comes from The 2005 State of the Nation report prepared by the Joint Center of Housing of Harvard University. It includes existing detached single-family homes and townhomes, but does not include condos and co-ops.

An interesting and also puzzling feature of the housing market is the high positive correlation between transaction volume and housing prices (the correlation coefficient is around 0.9 for the national data). In figure 5 I plot the percentage deviations of housing prices and transaction volume from their trends. The figure shows the strong comovement of the two series. Another way of showing the comovement in the high frequency is using the Hodrick-Prescot filter to remove the trends. By using this method, Rios-Rull and Sanchez-Marcos (2006) show that the comovement of both series holds also for higher frequencies.

Housing prices and transaction volume have been highly volatile. Genesove and Mayer (2001) report that in a real estate cycle, from peak to through, it is not unusual to see a 50 percent decline in transaction volume, especially in local markets. Rios-Rull and Sanchez-Marcos (2006) show that housing prices and transaction volume are much more volatile than the GDP both in Canada and the US. In table 1.1 I report the dispersion of some housing market variables measured by coefficient

[7] I control for the increase in number of the households in the US. The data of the number of the households can be obtained at http://www.census.gov/population/www/socdemo/hh-fam.html#ht (Table HH-4.)

Figure 4: Transaction Volume (Number of Existing Houses Sold), US (Yearly). Source: The Joint Center of Housing of Harvard University (2005). Controlled for the increase in number of the households in the US.

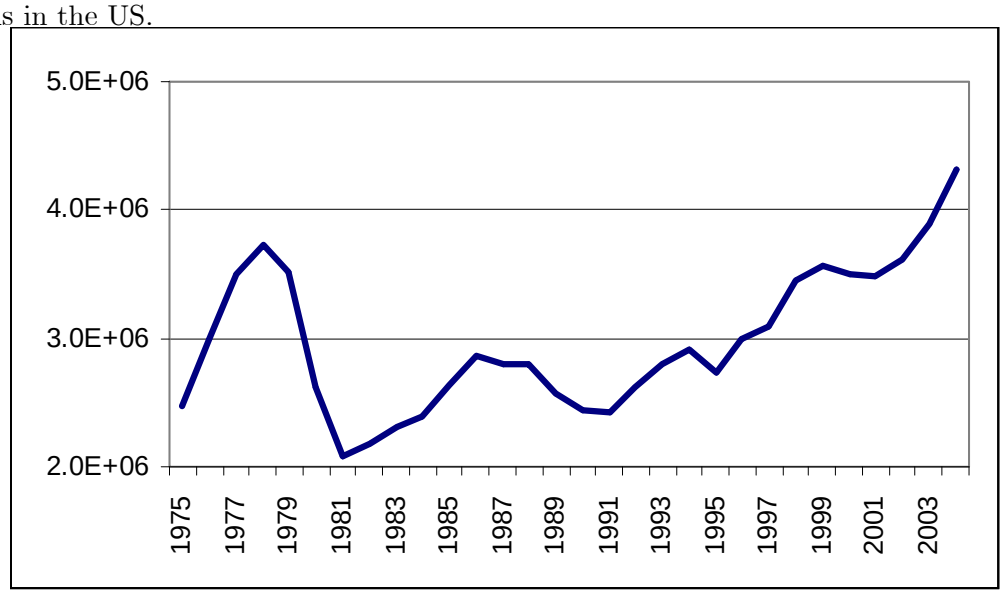


Figure 5: Comovement of Housing Prices and Transaction Volume (percentage deviations from their trends.)

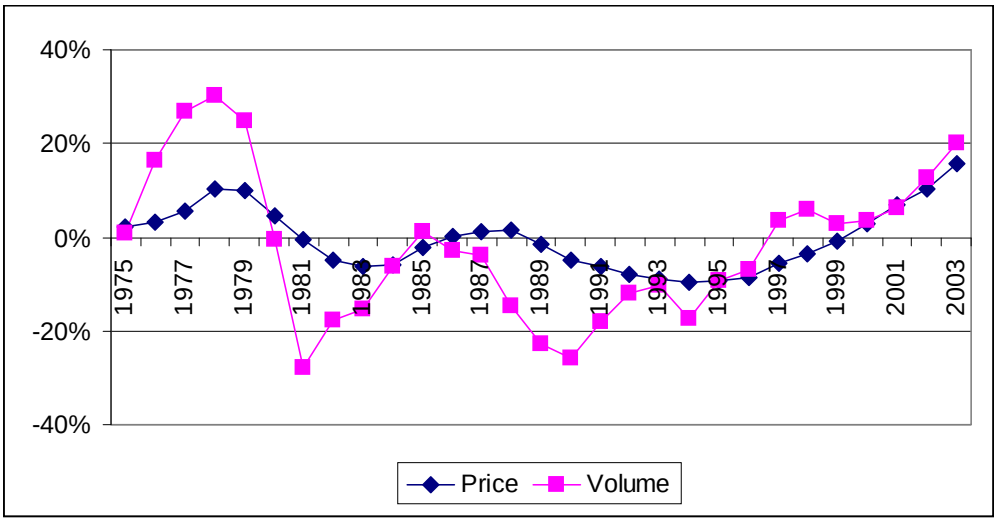


Table 1: Housing Market Statistics

	Data
CV(real 15 year mortgage rate)	0.50
CV(housing price, national)	0.08
CV(housing price, big cities)	0.18
CV(transaction volume, national)	0.15

Note.– CV is coefficient of variation and calculated as standard deviation divided by mean All series are detrended.

of variation. Housing prices in big cities are more dispersed than the national prices and transaction volume is more dispersed than housing prices.[8]

3 The Model

The model is a two period overlapping generations model.[9] At the beginning of each period, the economy is hit by an exogenous interest rate shock. A young agent enters into the economy with a limited amount of resources which makes him borrow in the mortgage market to finance his housing purchase. After he buys his house he receives his remaining income, saves in the bond market and consumes the non-housing consumption good. At the end of the period he makes the first mortgage payment. Old agents receive idiosyncratic housing taste shocks, which can be interpreted as changes in family sizes or changes in need of housing, causing optimal housing to be different from the one obtained when young. Since there are fixed transaction costs, some of the agents will choose to sell their existing houses and some will choose to stay. If an old agent sells his house, he gets the return from selling the house and pays the remaining mortgage debt coming from the previous house. Then he borrows in the mortgage market to finance his new house. If he does not sell his house, he makes the second mortgage payment and uses the remaining income to purchase non-housing consumption. At the end of their lives, all agents sell their houses and leave the economy. There are risk-neutral profit-maximizing banks which offer mortgage contracts. At the equilibrium, banks are indifferent between lending in the bond market and the mortgage market. For simplicity, I assume that aggregate house supply is fixed and there is no rental market.

Mortgage Market

Risk-neutral profit-maximizing banks offer fixed-rate 30 year mortgage contracts. I have two conditions to find the mortgage rates and the corresponding payments. The first condition is the

[8] Coefficient of variation of interest rates are calculated by using the net interest rates. If we use gross rates the coefficient of variation is around 0.03.

[9] In this section I present the two period version of the model for clarity. In section 5, I extend it to 30 periods to have a more realistic calibration.

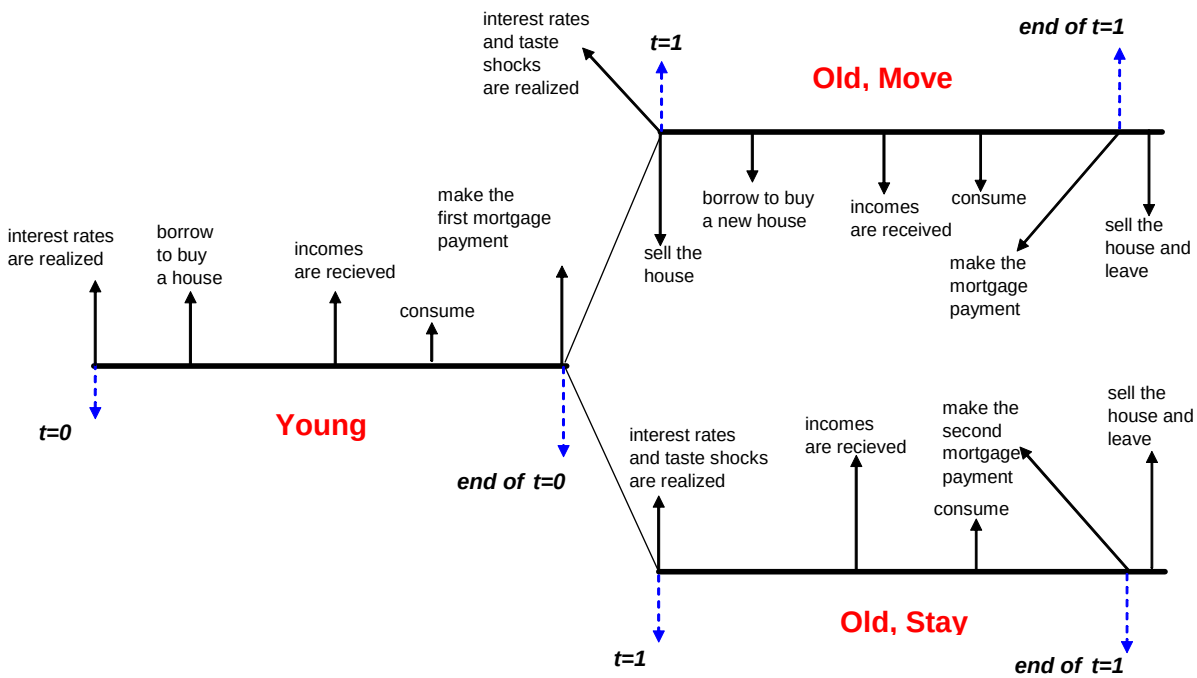


Figure 6: Timing of the Events

present value condition, which means that the present value of the payments should be equal to the loan amount.

$$1 = \frac{D_t}{1+d_t} + \frac{D_t}{(1+d_t)^2} \tag{1}$$

An agent should pay D_t if he borrows 1 dollar in the mortgage market when the mortgage rate is d_t. The next condition is the *no-arbitrage condition*. At equilibrium, banks should be indifferent between lending in the bond market and lending in the mortgage market.

$$1 = \frac{D_t}{1+r_t} + (1-\pi_t)\frac{D_t}{(1+r_t)E[(1+r_{t+1})]} + \pi_t \frac{1-D_t+d_t}{1+r_t} \tag{2}$$

The lender will receive the first mortgage payment D_t at the end of the first period, and he will discount this with current interest rate, $1+r_t$.[10] With probability $1-\pi_t$ the agent will not move and he will do his second mortgage payment, which is $\frac{D_t}{(1+r_t)E[(1+r_{t+1})]}$. With π_t probability the agent will move and prepay his loan by the beginning of the second period. The discounted value of this payment is $\frac{1-D_t+d_t}{1+r_t}$. In the solution of the model, I should also find D_t, d_t and π_t as functions of

[10] As the agents in the model can transact only after 15 years, the corresponding discount factor will be "15 to the power of *yearly interest rate*".

state variables. The old agents who buy a house get a 15 year mortgage contract (since they are going to leave the economy), and the interest on the mortgage is the same as the interest rate on the bond.

Old Agent's Problem

An old agent decides whether to move or not, and if he moves how big a house to purchase depending on the taste shock θ_h he received and the state of the market. The maximization problem is

$$V_{old}(\theta_h, h_{t-1}, s_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = Max\left\{V_{move}, V_{stay}\right\}. \tag{3}$$

The aggregate state of the economy is summarized by $P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t$. The price of housing, P_{t-1}, the quantity of housing, H_{t-1}, and the interest rate, r_{t-1} are all part of the aggregate state. I used $t-1$, because it is the time when they first bought their houses. The price, the quantity, and the interest rate at $t-1$ are all effective for the old agents' decisions today. The aggregate amount of housing held by the oldest generation which is leaving the economy is H_{t-2}. The current period interest rate is denoted as r_t. The quantities of housing and saving of an old agent are h_{t-1}, s_{t-1}.[11] The value of moving can be formulated as,

$$V_{move}(\theta_h, h_{t-1}, s_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = \underset{h_{t,o,m}}{Max}\left\{U(c_{t,o,m}, h_{t,o,m}) + \beta E[U(c_{t,o,l}, 0)]\right\}. \tag{4}$$

In equation 4, $c_{t,o,m}$ is the consumption, $h_{t,o,m}$ is the housing choice of an old moving agent. I assume that when the agents leave the economy, they sell their houses and use the returns from the transaction to purchase non-housing consumption good (no housing for the leavers since they sell at the beginning of the period).[12] The value of the houses to the agents when they leave the economy is then $\beta E[U(c_{t,o,l}, 0)]$. The utility function is quadratic in both consumption and housing. By quadratic utility function I am able to find most of the decision rules and all of the value functions analytically which simplifies the computation significantly.

$$U(c_{t,o,m}, h_{t,o,m}) = -\alpha_c/2(\theta_c - c_{t,o,m})^2 - \alpha_h/2(\theta_h - h_{t,o,m})^2 \tag{5}$$

The parameters of the utility function of an old agent are $\alpha_c, \theta_c, \alpha_h, \theta_h$. The taste shock that an old agent receives at the beginning of the period is θ_h. We assumed a taste shock in this form because of its convenience.

[11] In the equilibrium, the housing held by an old agent at the beginning of the period, h_{t-1}, and the aggregate amount of housing held by old the agents at the beginning of the period, H_{t-1}, will be equal to each other.

[12] Allowing agents to sell their houses at the end of their lives gives additional wealth motive to the home buyers.

$$c_{t,o,m} = w_{old} + s_{t-1}(1+r_{t-1}) - \lambda P_t h_{t,o,m} - \frac{(1-\lambda)P_t h_{t,o,m}(1+r_t)}{1+r_t} + P_t h_{t-1} - (1-\lambda)P_{t-1}h_{t-1}(1-D_{t-1}+d_{t-1}) - \Delta \tag{6}$$

If an old agent chooses to move, first he will sell his old house, then pay off the remaining mortgage debt. The seller will receive $P_t h_{t-1}$ as the price. The remaining debt from the old mortgage is $(1-\lambda)P_{t-1}h_{t-1}(1-D_{t-1}+d_{t-1})$.[13] Once he is done with the old house, he purchases a new house by borrowing in the mortgage market. His mortgage payment at the end of the period is $(1-\lambda)P_t h_{t,o,m}(1+r_t)$. We discount this payment with $1+r_t$ to measure the real cost of mortgage payment to the agent. The buying process has a fixed transaction cost of Δ and agents should make a down payment equivalent to $\lambda P_t h_{t,o,m}$, where λ is the down payment ratio. In equation 6, w_{old} is old agent's income and $s_{t-1}(1+r_{t-1})$ is the return from the savings made when young. The consumption of an agent who leaves the economy is equal to his return from selling his house.

$$c_{t,o,l} = P_{t+1}h_{t,o,m} \tag{7}$$

The optimal housing decision of an old agent is,

$$h_{t,o,m} = \frac{\alpha_c P_t(-\Delta + h_{t-1}[P_t + (-1+\lambda)P_{t-1}(1-D_{t-1}+d_{t-1})])}{\alpha_h + \alpha_c(P_t^2 + \beta E[P_{t+1}^2])} + \frac{\alpha_h\theta_h + \beta\alpha_c\theta_c E[P_{t+1}]}{\alpha_h + \alpha_c(P_t^2 + \beta E[P_{t+1}^2])} + \frac{\alpha_c P_t(s_{t-1}r_{t-1} - \theta_c)}{\alpha_h + \alpha_c(P_t^2 + \beta E[P_{t+1}^2])}. \tag{8}$$

Although equation 8 seems complicated there are intuitive comparative static results. For example, if the transaction cost Δ increases, $h_{t,o,m}$ will decrease. This is intuitive since higher transaction costs will leave the moving agents with less assets, and with less assets agents cannot afford larger houses. With a similar intuition, higher mortgage debt from the previous period decreases the optimal quantity of housing of the mover. If expected prices $E[P_{t+1}]$ increase, movers' optimal housing will increase, since the return of holding a larger house becomes larger. Higher taste shock θ_h implies higher optimal quantity of housing. Keeping the expected price constant, if $E[P_{t+1}^2]$ increases, demand for housing decreases. Here, $E[P_{t+1}^2]$ can be thought of as a measure of variance of the price. As variance increases (if mean stays same) risk averse agents will buy less housing.

Because of the existence of the transaction costs, not all agents will transact. The value of staying in their old houses is

[13] The total payment he made is $(1-\lambda)P_{t-1}h_{t-1}D_{t-1}$, but $(1-\lambda)P_{t-1}h_{t-1}d_{t-1}$ of this payment is gone for the interest

$$V_{stay}(\theta_h, h_{t-1}, s_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) = U(c_{t,o,s}, h_{t,o,s}) + \beta E[U(c_{t,o,l}, 0)], \tag{9}$$

where

$$c_{t,o,s} = w_{old} + s_{t-1}(1 + r_{t-1}) - \frac{(1-\lambda)P_{t-1}h_{t-1}D_{t-1}}{1 + r_t}. \tag{10}$$

If the agents stay they will pay the remaining mortgage of their house, which is $(1-\lambda)P_{t-1}h_{t-1}D_{t-1}$ and consume the rest.

$$h_{t,o,s} = h_{t-1} \tag{11}$$

Their housing will be the same housing they lived in during the previous 15 periods, h_{t-1}.

Given the state of the economy there will be θ_{low} and θ_{high} such that the old agents who receive a taste shock θ in between, will choose to stay, while the others will move (an S-s rule).[14] As the old agents with taste parameters θ_{low} and θ_{high} should be indifferent between moving and staying, the value of θ_{low} and θ_{high} can be found by equating the value of staying to value of moving[15].

$$\theta_{low}(s_{t-1}, h_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t), \tag{12}$$

$$\theta_{high}(s_{t-1}, h_{t-1}; P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t). \tag{13}$$

Once the boundaries of inactivity region are found we can also find the transaction volume implied by the model.

Young agent's problem

Young agents (no heterogeneity) enter into the economy with limited amount of resources. They borrow in the mortgage market to finance their housing purchases. Then they receive their income, save for the next period, consume and make their first mortgage payments. Young agents solve the following optimization problem,

[14] The existence and uniqueness of the boundaries hold under very general conditions.

[15] We have the boundaries in closed form, but because of their complexity we do not present them here.

$$V_{young}(P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) =$$
$$\max_{s_{t,y},h_{t,y}} \left\{ \begin{array}{l} U_y(c_{t,y}, h_{t,y}) + \\ +\beta E \left[\int_{\theta_{\min}}^{\theta_{low}} V_{move} f(\theta) d\theta + \int_{\theta_{high}}^{\theta_{\max}} V_{move} f(\theta) d\theta \right] \\ +\beta E \left[\int_{\theta_{low}}^{\theta_{high}} V_{stay} f(\theta) d\theta \right] \end{array} \right\} \tag{14}$$

where

$$c_{t,y} = w_{young} - s_{t,y} - \lambda P_t h_{t,y} - \frac{(1-\lambda) P_t h_{t,y} D_t}{1 + r_t} - \Delta. \tag{15}$$

In the utility function of the young agents $c_{t,y}$ is the consumption, $h_{t,y}$ is the housing, and $s_{t,y}$ is the saving at time t. Young agent's income is w_{young}. They have a quadratic utility function, similar to the old's utility function, the difference being the value of θ_h. We assume,

$$U_y(c_{t,y}, h_{t,y}) = -\alpha_c/2(\theta_c - c_{t,y})^2 - \alpha_h/2(\theta_y - h_{t,y})^2. \tag{16}$$

While making their choices, they take into consideration that it will affect their moving decisions when they are old. We assume that the taste shock is uniformly distributed. Here we know the boundaries in closed form, so we may find the integrals algebraically which allows us to write the first order condition of the young agent in closed form.

Market Clearing Condition

There are two market clearing conditions that should be satisfied. The sum of the quantity of housing held by each generation should be equal to aggregate house supply ($\overline{H_{tot}}$). The first market clearing condition can be written as,

$$\overline{H_{tot}} = \sum_{i=0}^{1} \overline{H_{t-i}}. \tag{17}$$

The total amount of housing owned by age i generation is $\overline{H_{t-i}}$. The market for available houses should also be cleared, as in each period only limited amount housing will be available for trade. This is equivalent to,

$$\overline{H_{t-2}} + \overline{H_{t-1}} = H_{t,o,s} + H_{t,o,m} + H_{t,y}. \tag{18}$$

The left hand side of the equation is the total available housing for trade. H_{t-2} is the total housing owned by the leavers and H_{t-1} is the housing owned by period t old agents. In the right had side of equation 18, $H_{t,o,s}$ is the amount of housing held by the old generation who did not move, $H_{t,o,m}$

is the demand of the movers. The young agents' housing consumption which we compute by using their optimization problem is $H_{t,y}$.

$$H_{t,o,s} = \int_{\theta_{low}}^{\theta_{high}} h_{t-1} f(\theta) d\theta \tag{19}$$

$$H_{t,o,m} = \int_{\theta_{\min}}^{\theta_{low}} h_{t,o,m} f(\theta) d\theta + \beta \int_{\theta_{high}}^{\theta_{\max}} h_{t,o,m} f(\theta) d\theta \tag{20}$$

4 Calibration

Although I have most of the decision rules in closed form, I cannot solve housing prices and the young agent's housing decision analytically. For those, I need to calibrate the model to have a numerical solution comparable to the US data. If available, I take the parameter values directly from the U.S. data and from the literature. For the other ones, I pick some target statistics from the US data, and try to match them with the model's predictions by choosing the free parameter values.

To make the calibration more realistic I assume that the model period is a year (each year a new interest rate shock is realized). I allow agents to transact houses only twice in their life time to make the computation easier. First transaction is done when they are young (30 years old) and the second transaction can be done (they may choose not to transact) when they are old (45 years old.) I assume that agents leave the economy at the age 60 after selling their houses. This set-up implies that, given that the model period is a year, there will be 30 generations living together in a period, but only three of them (young, movers and leavers) will be transacting.

Specifically, this extension helps us to calibrate the interest rates more realistically. Otherwise, the model would be a 2-period model with a period length of 15 years and it would be impossible to calibrate the persistence of interest rates properly. Moreover, extending the model in this particular way helps us to solve the model with the computational algorithm that is described in Appendix C.

Normalizations

Housing supply in the model is set to 30 to allow each generation to hold 1 unit of housing on average. I set $\alpha_h = 2$.

Parameters taken from the literature

The yearly discount factor, β is 0.96. The down payment ratio, λ is 0.2 (Campbell and Cocco (2003).) The fixed transaction cost, Δ is set to 0.015. This value corresponds to approximately 8 percent of the value of the house purchased, which is in the range of reported values (Smith, Rosen, and Fallis (1988).) I assume that each generation has 1 unit of income ($\frac{w_{young}+w_{old}}{2} = 1$.) An increase

Table 2: Normalizations

Description	Parameter	Value
A parameter of the utility function	α_h	2.0
Aggregate housing supply	$\overline{H}$	30

Table 3: Parameters taken from the Data and Literature

Description	Parameter	Value
Discount Factor	β	0.96
Downpayment ratio	λ	0.2
Fixed transaction cost	Δ	0.015
Young's income	w_{young}	0.85
Old's income	w_{old}	1.15
Average of the real interest rates (1975-2006)	μ	5%
Annual autocorrelation of the interest rates (1975-2006)	ρ	0.88
Coefficient of variation of interest rates (1975-2006)	CV	0.5

in income– from the age 30 to the age 45– in the range of 50 percent seems plausible (US Census Bureau median household income by age of the householder.)

Statistics used for calibration

I use the following 5 target statistics to calibrate the remaining 5 parameters; θ_y, θ_c, α_c, $\theta_{\min}$, $\theta_{\max}$: (i) the coefficient of risk aversion in consumption is on average 1, (ii) the national average of housing price to income is around 2.6 (Joint Center for Housing Studies of Harvard University, table A-1, since I have 15 years of income, the adjusted target price to income ratio is 0.22,) (iii) approximately, the aggregate housing wealth of people between ages 45-54 is 25 percent higher than aggregate housing wealth of people between ages 35-44 (Hurst, Luoh and Stafford (1998), table 1.2,) (iv) PSID data shows that 5.44 percent of the respondents moved during the last year (Cocco (2000), The corresponding number for 15 years would be 56 percent), (v) Hanushek and Quigley (1980) shows that price elasticity of housing is around -0.7.

Interest Rates

I assume the stochastic process in the model is the real interest rate on a 15 year fixed-rate mortgage. This fits into my model, because when old agents borrow in the market I assumed they could get loans with the period interest rate. In reality it will correspond to a 15 year mortgage rate. To find the real rate on a 15 year fixed-rate mortgage, I simply subtract the inflation in that year from the nominal rate.[16] The average of real 15 year fixed-rate mortgage during 1975-2005 period

[16] This is equivalent to assuming current period's inflation is the best predictor of the future inflation.

Table 4: Targets

Statistics	Data	Model
The coefficient of risk aversion in consumption	1 to 5	1.5
Income to housing price ratio	0.22	0.29
(Old agents' housing wealth)-(Young agents' housing wealth)	25%	24%
Average transaction volume	56%	80%
Price elasticity of housing	~ -0.7	-0.7

Table 5: Parameters Calibrated Using the Targets

Description	Parameter	Value
A parameter of the young agent's utility function	θ_y	1.0
Lower bound of the taste shock	$\theta_{\min}$	0.1
Upper bound of the taste shock	$\theta_{\max}$	2.1
A parameter of the utility function	θ_c	1.4
A parameter of the utility function	α_c	3.0

is 5 percent. Then I estimate an AR(1) process of these real rates which gives the autocorrelation coefficient as 0.88. Finally, I use Hussey and Tauchen (1991) method to approximate the AR(1) process with a 5 state Markov chain. The coefficient of variation of real 15 year fixed-rate mortgage is around 0.5. Because of my computational algorithm[17], I cannot exactly match both the mean and the coefficient of variation at the same time. In the next sections I report the results for 6 percent average interest rate with 0.43 coefficient of variation. I also solved for different combinations of mean and coefficient of variation and none of the results depend on the specific choice that is made here.

5 Results

Before going into more through analysis of the model's findings I will demonstrate the model's response to an interest rate history similar to the history observed in the US. Because of computational reasons I assume the maximum value of the interest rate is 10 percent and the minimum value is 2 percent, whereas in the US, interest rates were as high as 15 percent at early 80's and as low as 1.5 percent at 2005. With these values I can approximate the 1982-2005 period. Figure 7 plots housing prices and transaction volume implied by the model. To make it comparable to the U.S. data, I rescale the value of housing price and the transaction volume to match their values in 1982. The

[17] If I increase the coefficient of variation to 0.5 then I have boundary solutions. Even though the algorithm can handle boundary solutions for most of the states, for a small number of states it cannot, which decreases the accuracy of the solution.

average median housing price in big cities was approximately $200,000 and the transaction volume (number of units sold) was 2,000,000.

The median housing price in the model increases around 50 percent. I should remind that the decline in the interest rates that I feed into the model is smaller than the one observed in the US. If I had the original history (at least for the way I calculated it) the model would predict around 70 percent increase in the housing prices. I must caution that the way I calculated the real interest rates is the simplest way and may give different results than more sophisticated techniques. On the other hand, one important parameter that I am more interested is the elasticity of housing prices with respect to interest rates and it does not change much with the specific interest rates. If the true real interest rate history is different from the one that I calculated then the model will not be predicting a 70 percent increase but will be predicting another amount.

Interestingly, the model predicts a price decline in the 90's. This prediction is also consistent with the U.S. data. In the 90's, most the of big cities as well as the US median housing prices declined.[18] Transaction volume increases by around 25 percent in the model which is lower than the observed increase in the housing transactions which is presented in figure 4. The increase in the transaction volume in figure 4, includes the new home buyers who switch from renting because of lower down payment and attractiveness of the housing market which I do not have in the model. Similar to the housing prices, the model predicts the decline in the transaction volume in the 90's. Finally, even in this short time period, it can be seen that housing prices and transaction volume move together.

5.1 Housing Prices

In this section, I show the functional relation between housing prices and interest rates. Figure 8 depicts housing price as a function of current interest rates where I assume that the last period interest rate is 6% (it looks very similar for the others histories.) On the y axis I have *Log* of the price and on the x axis I have *Log* of the current interest rate. Unit prices are higher than the median prices because the size of the median house is almost always smaller than 1.

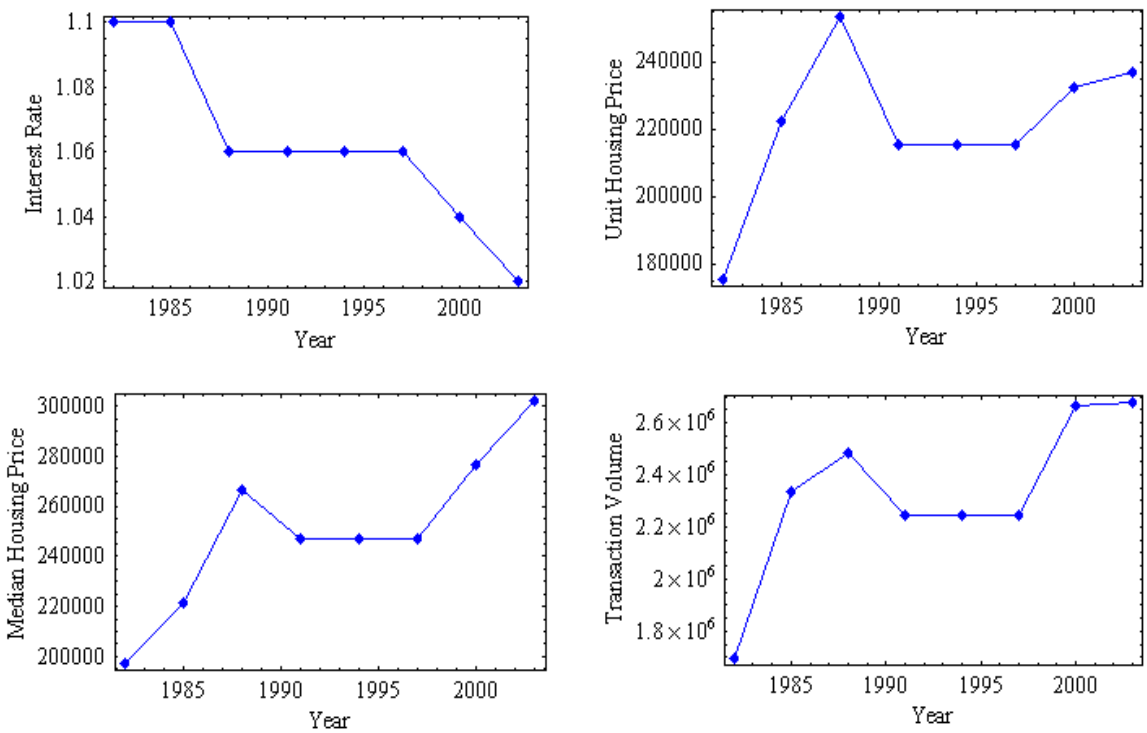
The slope of the graph gives the interest elasticity of housing prices. The elasticity is around -5, which means a 1 percentage point decrease in interest rates will cause prices to increase by 5 percent. The real interest rates decreased more than 10 percentage points during last two decades.[19] In this case, the model predicts housing prices would appreciate approximately 50 percent. This amount of housing price appreciation explains a large part of the housing price appreciation in big cities (it explains almost all of the increase in the national median housing prices.)

There are other factors that affect the current period housing prices. The biggest effect comes from the last period's interest rate, as it affects the savings of the agents in the last period. If the

[18] I explain the reason of this prediction in the next section.

[19] The real interest rates in the model corresponds to real interest rates on a 15 year FRM contract.

Figure 7: Time Series of the Interest Rates, Housing Prices, and Transaction Volume. The history of the interest rates is aproximated to the data. Median price is calculted by multiplying unit price with the median housing size. Transaction volume is calculated by $\frac{\theta_{high}-\theta_{low}}{\theta_{\max}-\theta_{\min}}$. Transaction volume and housing price series are rescaled to approximately match their values at the starting date, 1982.



interest rate during the last period was high it means that the old agents will have more assets in the current period, which increases their willingness to pay higher housing prices. Because of this reasoning, high interest rates in the past will increase the housing prices today. In this case, the interest (last period) elasticity of *unit* housing prices is 4.8. For the median housing prices the elasticity is smaller, 2.5. This is due to the change in the size of the median house as a response to an increase in prices. If housing prices increase because of the last period's interest rate, the young agent's mortgage payments increase linearly with the prices, which decreases the housing demand of the young agent. As in the model the median house is always bought by a young agent, the size of the median house will decrease, pushing down the median housing price.

It is this *last period's interest rate effect* that caused the model to match the decrease in housing prices in 1990 in figure 7. In the approximate interest rate history, during the 1982-1985 period, interest rates are 10 percent. During this period, because of the high interest rates, agents in the model save more, bringing extra assets to the next period (1988-1992.) In 1988, interest rate

Figure 8: Housing Prices and Interest Rates. Median housing price is found by multiplying the median size of housing with the unit price. Unit price is the price of 1 unit house. For the clarity of the figure, last period interest rate is assumed to be 6%.

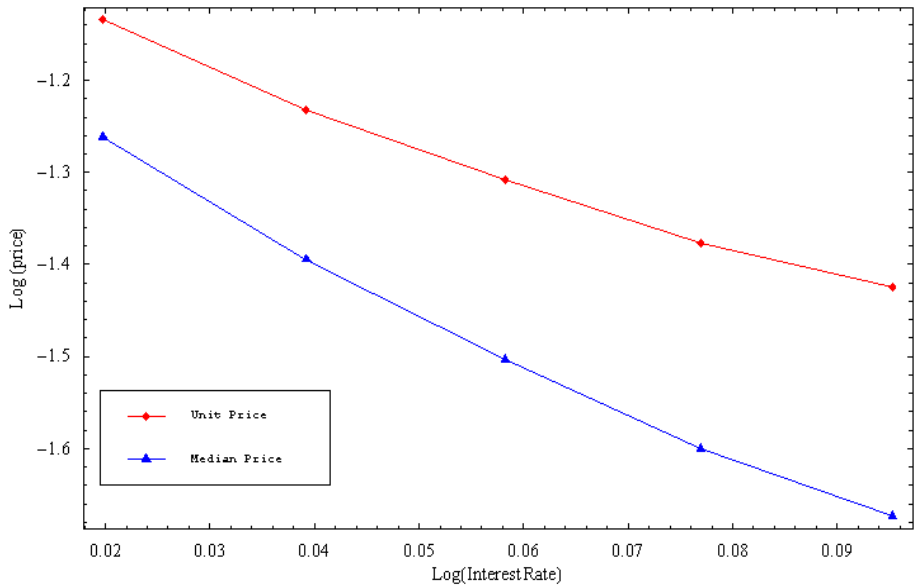
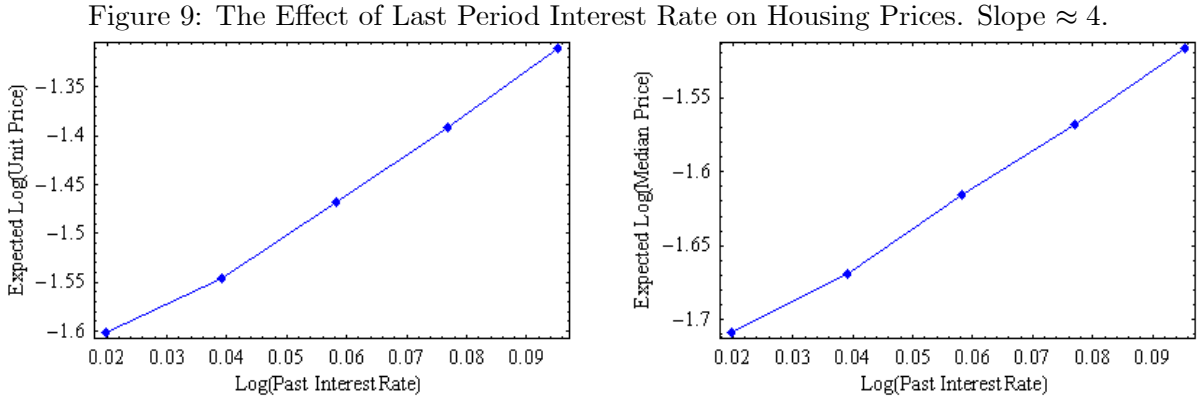


Figure 9: The Effect of Last Period Interest Rate on Housing Prices. Slope ≈ 4.



decreases to 6 percent. The low interest rates together with extra assets of old agents cause housing prices to increase. As this period is a low interest rate period, young agents in this period save less compared to the young agents of 1985. When the next period (1991-1994) comes although interest rates are still 6 percent, in this period there will be less asset available, which will cause a price decline.

In the literature, while questioning the existence of a housing price bubble, one of the measures that researchers used is the housing price-income ratio. Especially for the high housing price appreciation cities, such as New York and San Francisco, looking at the housing price-income ratio they find evidence of a housing price bubble, since that ratio became unusually high compared to its historical values. In the model, I did not have any income change. The average housing price to income ratio is 2.6. As housing prices fluctuate because of interest rates, the ratio becomes more than 3.5 when housing price reaches its maximum, and it becomes less than 1.5 when housing prices reaches its minimum. My conclusion is, housing price-income ratio cannot be a real measure of the real cost of housing since that measure does not take into account the effect of interest rates on the housing prices.[20]

One interesting implication of the model is that the last period interest rate has predictive power on the current period's price. If the last period's interest rate was high, people expect (on average) higher prices. This is because young agents in the last period saved more, making them more wealthy in the current period which then increases their willingness to pay higher prices. Figure 9 showed the relation between housing prices and the last period's interest rate. The elasticity of prices with respect to last period interest rate is approximately 4. To show the importance of savings, I solve the model with the same parameters, but I assume agents cannot save. Figure 10 shows that there is almost no predictability anymore and the new elasticity is approximately -0.5. One may think that the predictability can be coming from fixed transaction costs. To check that claim I solve the model with no transaction cost ($\Delta = 0$.) The new interest rate (last period) elasticity of housing is around 5. So the transaction cost does not cause the predictability in the model.

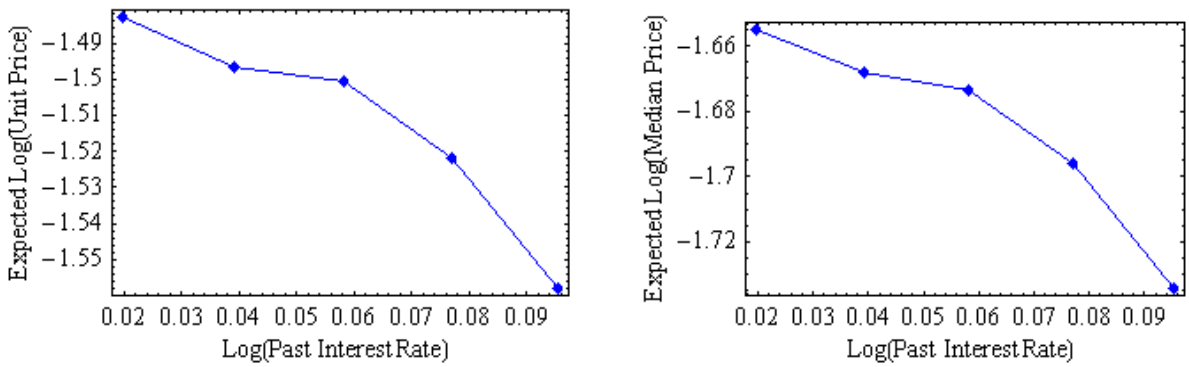
5.2 Transaction Volume

Stein (95), Ortalo-Magne and Rady (2004) use the credit constraints and large down payments to explain the positive relation. Wheaton (90) and Goodman and Berkovec (96) use information imperfections to explain the comovement. Genesove and Mayer (2001) claims that transaction volume-price correlation is consistent with the loss aversion of the sellers.

The scatter plot in figure 11 shows that housing prices and transaction volume are positively related in the model as in the U.S. data. But the mechanism which generates the positive relation

[20] Although there is no rental market in the model, a similar conclusion will hold for housing price to rent ratio. The key thing is, interest rates affect housing prices, as it determines the cost of borrowing, while it would not affect the price of rent that much.

Figure 10: Housing Price Predictability without Saving. Slope ≈ -0.5



is different from the mechanisms used in Stein (95), Ortalo-Magne, Rady (2004).[21] To check the effect of down payment on transaction volume I solve the model with no down payment requirement. The results look very similar, meaning the down payment requirement is not an important part of the mechanism of the model that generates the comovement of prices and transaction volume (see appendix figure 17.)

Figure 12 shows the effect of the current and the last period's interest rate on the transaction volume. The graph in the right panel of figure 12 shows that the current period interest rate and transaction volume are negatively related where I assumed that the last period interest rate is 6% (it looks very similar for the others histories.) The interest (current) elasticity of transaction volume is around -3, which implies a 10 percent decrease in interest rates, similar to the U.S. data, will cause a 30 percent increase in the transaction volume, explaining a significant part of the increase in the transaction volume. In the left panel of figure 12, I plot the expected transaction volume as a function of last period's interest rate. The last period interest elasticity of transaction volume is around 3. Both effects combined helps the model to predict the large increase in the transaction volume observed in the U.S. data.

To see the intuition clearly I write the costs and returns of a transaction. To make it simpler I assume that the down payment requirement, λ, is 0 (I already showed that it is not crucial in the model.) If an agent sells his house, the return from the transaction is

$$P_t h_{t-1} - P_t h_{t,o,m} - P_{t-1} h_{t-1}(1 - D_{t-1} + d_{t-1}).$$

The return the agent gets when he sells his old house is $P_t h_{t-1}$. The agent will pay $P_t h_{t,o,m}$ for his new house. The last term, $P_{t-1} h_{t-1}(1 - D_{t-1} + d_{t-1})$, is the payment that the agent has to make to the lender for the debt coming from the old house he sold. When the agents stays in his house his

[21] The mechanism is obviuosly different from Wheaton (90) and Goodman and Berkovec (96), as the model does not have any information frictions.

Figure 11: Comovement of Housing Prices and Transaction Volume.

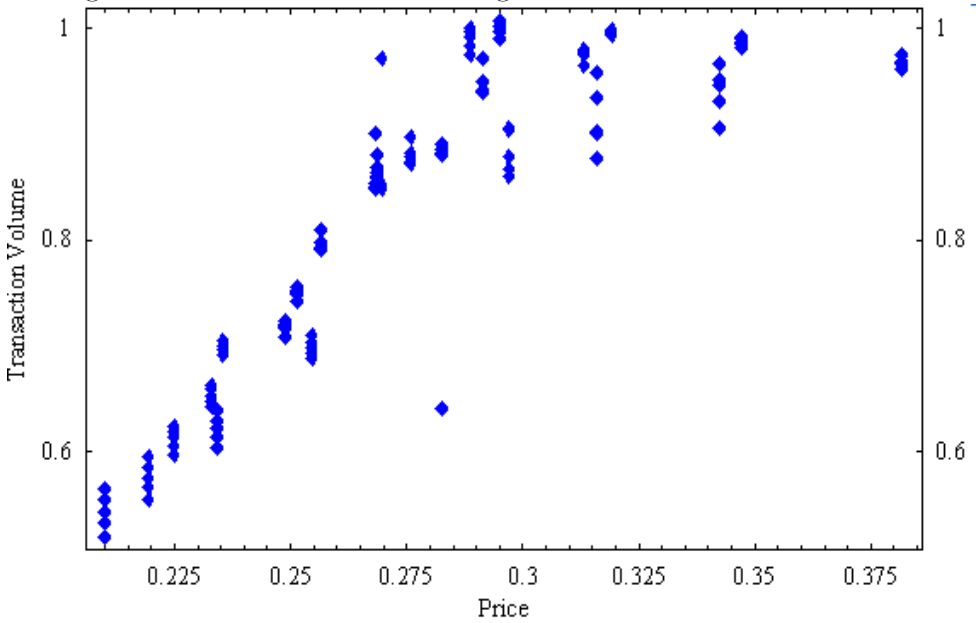
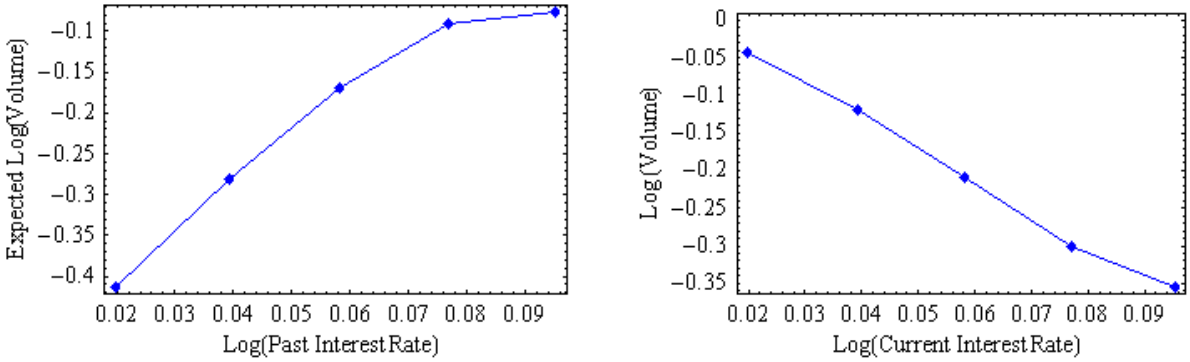


Figure 12: Transaction Volume and Interest Rates. For clarity, last period interest rate is assumed to be 6% for the graph in the left panel.



cost of housing will be,

$$\frac{P_{t-1}h_{t-1}D_{t-1}}{1+r_t}.$$

I discount the second mortgage payment $P_{t-1}h_{t-1}D_{t-1}$ with $1+r_t$ to find present value of the payment.

Suppose that the current interest rate is low. In this case the second mortgage payment will be more expensive (compared to a higher interest rate state) for an agent who borrowed in the last period since the denominator in $\frac{P_{t-1}h_{t-1}D_{t-1}}{1+r_t}$ will be smaller. As the cost of the second mortgage payment increases more people find it optimal to move, which increases the transaction volume. I also showed in the previous section that low interest rates cause high prices. If I combine both of the findings, it is the interest rates which cause the comovement of the housing prices and transaction volume. But this is only half of the story. I showed in figure 12 and figure 9 that last period's interest rate positively affect the transaction volume and housing prices. If the last period's interest rate was high, it means that the agents who borrowed in the mortgage market during the last period have higher interest payments in the current period. If the cost of staying is high, then more people choose to move. This is the other channel that causes the comovement of the housing prices and transaction volume.

The key to the model's success in generating the comovement of the housing prices and the transaction volume is the mortgage contract. To quantify the effect of the mortgage contract in the model, I eliminate the mortgage market from the model by assuming that the down payment requirement, λ, is 1. In this case the agents will have to pay all the price of their purchase in cash, so they will not borrow in the mortgage market. I calibrate the model using the same targets that I used to calibrate the baseline model. The results show that: (i) a model without the mortgage contract has a similar interest elasticity of housing prices (about -5,) (ii) without the mortgage contract the model is incapable of explaining the negative correlation between transaction volume and interest rates, (iii) transaction volume has a much smaller coefficient of variation (around 0.03.) These findings show that mortgage market is a key component of the baseline model. (See figures 15 and 16 in the appendix)

5.3 The Dispersion of Interest Rates, Housing Prices and Transaction Volume

The coefficient of variation of housing prices is 0.08 for the national data and 0.18 for the big cities. The model predicts a coefficient of variation of 0.14 for housing prices. This is very close to the observed value for the big cities (since we assumed fixed housing supply, the results of the model are more comparable to the results of the big cities.) The model predicts a coefficient of variation of 0.16 for the transaction volume, which is almost the same as the observed value. It appears that interest rates can be a major factor in explaining the volatilities in the housing market.

Table 6: Housing Market Statistics

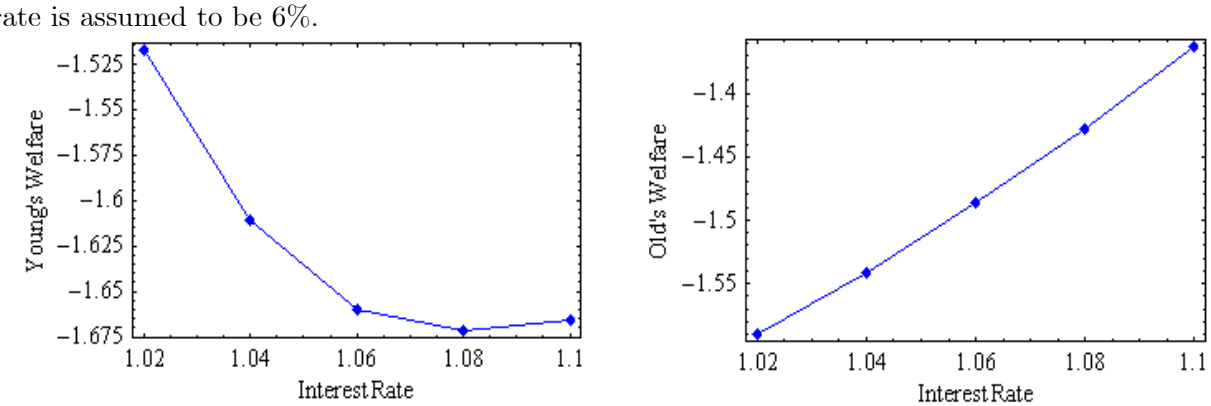
	Data	Model
CV(real 15 year mortgage rate)	0.5	0.43
CV(housing price, big cities)	0.18	0.14
CV(transaction volume)	0.15	0.16

Note. –CV is coefficient of variation and calculated as standard deviation divided by mean

6 A Welfare Analysis

In this section, I analyze the effect of interest rate movements on the welfare of the consumers (I use the value functions as a measure of welfare.) I first look at the welfare of the each generation (figure 13 .) Rising interest rates decrease the welfare of the young agents. The first reason is the higher cost of borrowing in the bond market when interest rates increase. The second reason is the higher cost of borrowing in the mortgage market. Only when the interest rates are around 10, percent they start to save and higher interest rates start to be welfare improving for them. For the old agents, rising interest rates are welfare improving as most of the agents upgrade their houses. With higher interest rates, they pay less for their larger houses that they purchase.

Figure 13: Old's and Young's Welfare. Welfare is measured with the value function. Uniform weights are used to calculate the welfare of a generation. For the clarity of the figure, last period interest rate is assumed to be 6%.



I go further and ask whether all moving old agents are affected in the same way from the interest rates movements. In this part, I separate the old agents into two groups. First, I plot the welfare of the old agents who receive a low taste shock (low θ_h) and then I plot the welfare of the old agents who receive a high taste shock (high θ_h.) I find that, while the old agents with low taste shocks are better off if the economy is hit by low interest rate shock causing high housing prices, the old agents with high taste shocks are worse off.

Figure 14: Differential Welfare Effects of Interest Rate Movements. $\theta_h = 7$ for High Taste Shock. $\theta_h = 0.1$ for Low Taste Shock. For the clarity of the figure, last period interest rate is assumed to be 6%.

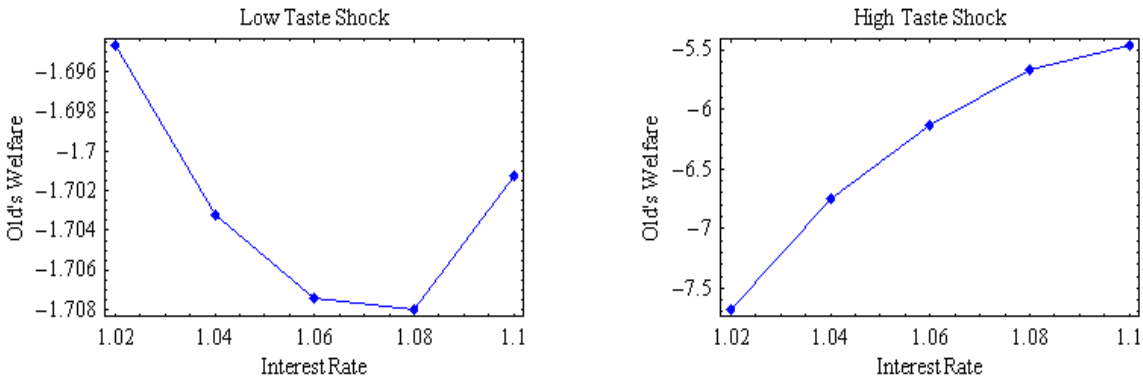


Table 7: The Effect of a Tax Reform

	Before Tax Reform	After Tax Reform
Median Price	0.300	0.255
Young's Consumption	0.696	0.743
Old's Consumption	0.982	1.015
Young's Housing	0.851	0.849
Old's Housing	1.149	1.151

7 Tax Reform

The tax deductibility of the mortgage interest payments gives additional incentives to the consumers to be homeowners, which then affects both prices and allocations. Here I ask what would happen to the economy, if there is a tax reform which eliminates this incentive. This question is also motivated by the current tax reform proposal in the US.

In the baseline economy I did not have any taxes. To study the effect of the tax reform I introduce taxes into the model. I assume each homeowner will receive 20 percent of his mortgage interest payment as a tax return.[22] I solve the model with the same parameters that I used for the baseline. Then I implement the tax reform by assuming that the agents cannot get any tax returns anymore. Instead, they receive an income transfer equaling to the amount of the tax return. I solve the model again with the same parameters. I report the results in table 7.

As a result of the tax reform, median housing prices decline around 14 percent. Young agents' non-housing consumption increases around 7 percent. Old agents' non-housing consumption in-

[22] We make this assumption because of computational reasons. Higher values gives less accurate results. We also assume that government finances the tax return from other resources, not from the homeowners.

creases around 2 percent. The tax reform affect young and old agents differently because they pay different amount of interests on their mortgages, they have different house sizes, and they have different horizons. As old agents get a 15 year mortgage contract and as they buy larger houses the amount of interest payment is larger for them which makes the tax return more important. This is consistent with the intuition of Nicolas Retsinas, director of the Joint Center for Housing Studies at Harvard University. He says, "The people who have the biggest homes, who make the most money are the greatest beneficiaries of this tax subsidy". The housing consumption of both agents remains almost same after the tax reform.

To further compare both policies I ask what percent of their consumption the consumers are willing to give (or receive) to stay with the current policy. This is an exercise similar to the one performed by Lucas (1987.) Formally, the welfare cost of switching from policy A to the policy B can be formulated as

$$U((1+\mu)c_A) = U(c_B), \tag{21}$$

where μ is the measure of welfare gain (a comparison of the steady states.) It appears that while young agents are better of with the tax reform, old agents are worse off. Young agents would be willing to give up their 1.3 percent of their consumption to *implement* the tax reform. Old agents are willing to give up around 1.2 percent of their consumption to *avoid* the tax reform. Despite the increase in the consumption of the old agents because of a tax reform, they do not like it, because it will decrease the value of their assets (the resale value.)

8 Conclusion

In this paper, I have examined the role of interest rate movements in determining the fluctuations in housing prices and in transaction volume. First, I found that the substantial decrease in interest rates in the last two decades can account for a large part of the increases in real housing prices. In the next step, I analyzed the model's implication about transaction volume. The model can account for both the comovement of housing prices and transaction volume and the increase in the level of the transaction volume. Moreover, the model implies that housing prices and transaction volume have high volatilities.

The present model abstracts from some important aspects of the housing market to isolate the effect of interest rates. First of all, I assumed that income does not fluctuate. It is straightforward to write and solve a very similar model with income uncertainty instead of interest rate uncertainty. It is also possible to solve a model with both income and interest rate uncertainty.[23] On the other hand, there are some ingredients in the model that mimic the behavior of income such as savings. I

[23] In this case we should use 2 or 3 state Markov processes for both income and interest rates.

showed that the effect of interest rates on the housing prices are similar to the effect of savings on the housing prices. I also assumed that housing supply is fixed. This assumption makes the model work better for places where land is scarce or zoning restrictions are more effective. It would be interesting to include home production into the model and see the effect of zoning restrictions on the housing market.

I assumed that there are only FRM contracts available. I solved (but did not report) a very similar model for adjustable-rate mortgage (ARM) contracts. Housing price and transaction volume movements implied by ARM contracts are close to the movements implied by the FRM contracts. But it is my interest to see in a future research the implications of type of mortgages on the macro economy.

For computational reasons, I assumed that the agents can transact only twice during their life time. Adding extra transaction choices to the model would bring large computational burden. Another computationally costly extension is adding mortgage choice to the model. I solved a case where I had mortgage choice, but in that case I had to assume that interest rates were following a two state Markov process. I am still looking for ways by which I can incorporate these extensions without much computational burden.

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Appendix A The Effect of Mortgage Contract

As could be seen from figure 15, interest rates have a very small effect on the transaction volume if the model does not have a mortgage contract. The housing price implications of a model without mortgage contracts are similar to the one with the mortgage contract (figure 16.)

Figure 15: Transaction Volume without the Mortgage Contract.

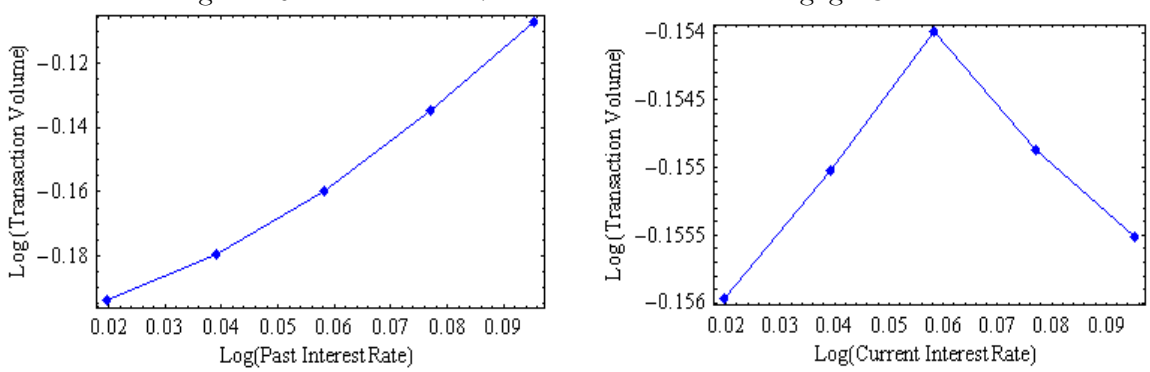
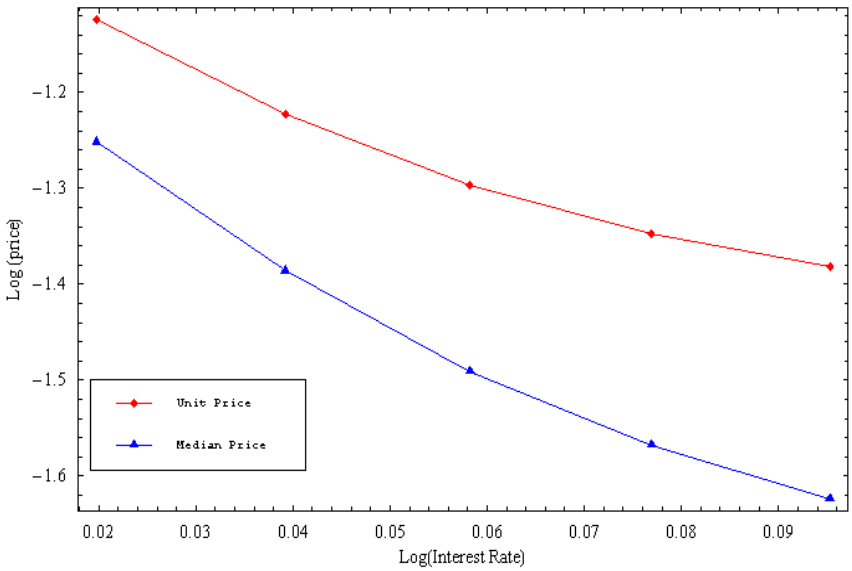


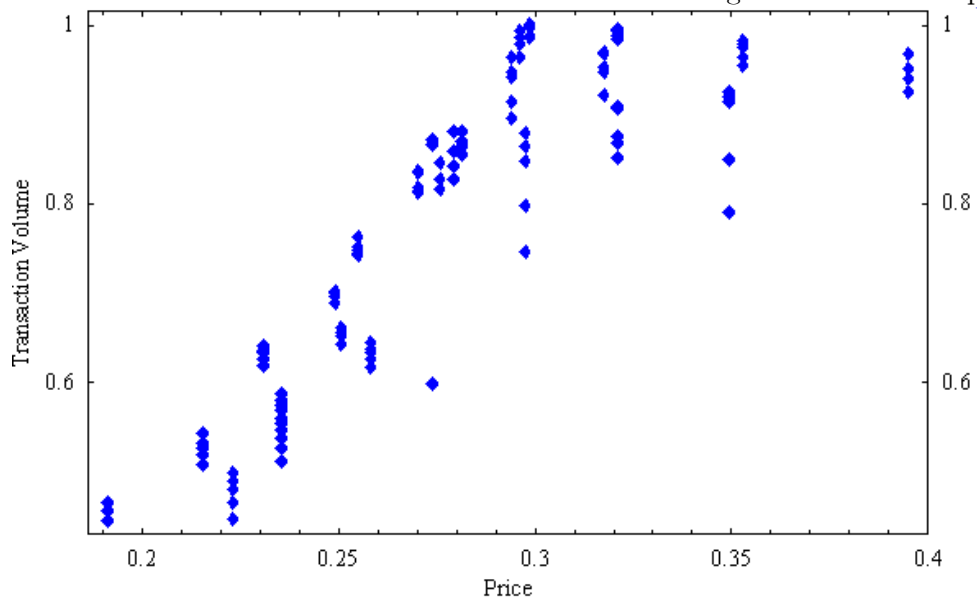
Figure 16: Housing Prices without the Mortgage Contract



Appendix B The Effect of Downpayment Requirement

Several papers used large downpayment requirements to explain the comovement of the transaction volume and housing prices. In figure 17, I show that the existence of the downpayments is not the driving force in the model.

Figure 17: Comovement of the Transaction Volume and Housing Prices. No Downpayment



Appendix C Computational Algorithm

The exogenous process in the model is the interest rates. In a given period I expect that the endogenous variables are mostly affected by current interest rates. Normally, if I knew all the history of interest rates then I could (theoretically) know the current prices and choices, and I would not need any other state variables. However, it would be computationally impossible to solve. Here I assume that the most important part of the interest rate history is the part which is close to the current date.

In the model's formulation, current period price should be formulated as,

$$P_t = F(P_{t-1}, H_{t-2}, H_{t-1}, r_{t-1}, r_t) \tag{22}$$

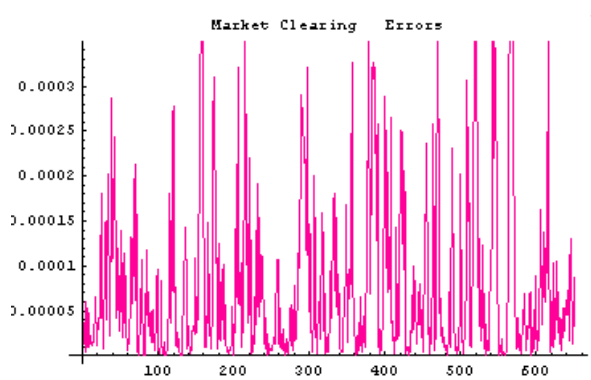
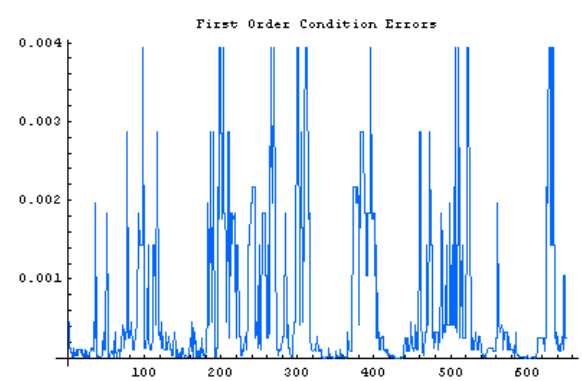
As P_{t-1}, H_{t-1} are also endogenous variables they would mostly be affected from interest rates. This reasoning will go until time 0. At that point current price would be a function of the history of interest rates.

$$P_t = F(r_0, ..., r_{t-1}, r_t) \tag{23}$$

Now suppose that agents are boundedly rational and they consider only the recent history of the interest rates while making their decisions. I find that r_{t-1}, r_t gives a very accurate solution. I do not make any functional form guesses for any function. I find the values of the functions at those states only.

$$P_t = F(r_{t-1}, r_t) \tag{24}$$

$$H_{t,y} = G(r_{t-1}, r_t) \tag{25}$$

To form the objective function I simulate an interest rate history. At all points in the interest rate history I *algebraically find the first order conditions and market clearing condition.* The objective function is the sum of squared errors of first order conditions and market clearing conditions across time. Then I use a minimization technique (I used both local and global techniques) to solve for prices and allocations jointly that minimize the errors. This algorithm is much faster than the *weighted residuals* method, which seemed to be the best method (compared to the other available algorithms) for the particular model, and gives more accurate results. I demonstrate the errors in the next section. I made some robustness analysis by adding more history to the agent's decision. Additional histories gave similar results with higher accuracy. To check the possibility of multiple equilibria I solved the model also with global optimization techniques. I could not find any other equilibria.

Appendix D First Order Condition and Market Clearing Errors