

# Aggregate Implications of Micro Asset Market Segmentation\*

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## Abstract

This paper develops a consumption-based asset pricing model to explain and quantify the aggregate implications of a frictional financial system, a system comprised of many financial markets partially integrated with one-another. Each of our micro financial markets is inhabited by traders who are specialized in that market's type of asset. We specify exogenously the level of segmentation that ultimately determines how much idiosyncratic risk traders bear in their micro market and derive aggregate asset pricing implications. We pick segmentation parameters to match facts about systematic and idiosyncratic return volatility. We find that if the same level of segmentation prevails in every market, traders bear 20% of their idiosyncratic risk. With otherwise standard parameters, this benchmark model delivers an unconditional equity premium of 3.7% annual. We further disaggregate the model by allowing the level of segmentation to differ across markets. This version of the model delivers the same aggregate asset pricing implications but with only half the amount of segmentation: on average traders bear 10% of their idiosyncratic risk.

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# 1 Introduction

Asset trade occurs in a wide range of security markets and is inhibited by a diverse array of frictions. Upfront transaction costs, asymmetric information between final asset holders and financial intermediaries, and trade in over-the-counter or other decentralized markets that make locating counterparties difficult, all create “limits to arbitrage” (Shleifer and Vishny, 1997). A considerable empirical and theoretical literature on market microstructure has studied these frictions and conclusively finds that “local” factors, specific to the market under consideration, matter for asset prices in that market.<sup>1</sup> But these market-specific analyses do not give a clear sense of whether micro frictions and local factors matter in the aggregate. Indeed, by focusing exclusively on market-specific determinants of asset prices, these analyses are somewhat disconnected from traditional frictionless consumption-based asset pricing models of Lucas (1978), Breeden (1979) and Mehra and Prescott (1985). Research in that tradition, of course, takes the opposite view that micro asset market frictions and local factors do not matter in the aggregate and that asset prices are determined by broad macroeconomic factors. The truth presumably lies somewhere between these two extremes: asset prices reflect both macro and micro-market specific factors (Cochrane, 2005).

This paper constructs a simple consumption-based asset pricing model in order to explain and quantify the macro impacts of micro market-specific factors. At the heart of our paper is a stylized model of a financial system comprised of a collection of many small micro financial markets that are partially integrated with one another. We strip this model of a fragmented financial system down to a few essential features and borrow some modeling tricks from Lucas (1990) and others to build a tractable aggregate model. In short, we take a deliberately macro approach: we do not address any particular features of any specific asset class but we are able to spell out precisely the aggregate implications of fragmentation and limits-to-arbitrage frictions.

In our benchmark model, there are many micro asset markets. Each market is inhabited by traders specialized in trading a single type of durable risky asset with supply normalized to one. Of course, if the risky assets could be frictionlessly traded across markets all idiosyncratic market-specific risk would be diversified away and each asset trader would be exposed only to aggregate risk. We prevent this full risk sharing by imposing, exogenously, the following pattern of market-specific frictions: we assume that for each market  $m$  an exogenous fraction  $\lambda_m$  of the cost of trading in that market must be borne by traders specialized in that market. In return, these traders receive  $\lambda_m$

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<sup>1</sup>See Collin-Dufresne, Goldstein, and Martin (2001) and Gabaix, Krishnamurthy, and Vigneron (2007), for example.

of the benefit of trading in that market. We show that, in equilibrium, the parameter  $\lambda_m$  measures the amount of non-tradeable idiosyncratic risk: when  $\lambda_m = 0$  all idiosyncratic risk can be traded and traders are fully diversified. When  $\lambda_m = 1$  traders cannot trade away their idiosyncratic risk and simply consume the dividends thrown off by the asset in their specific market.

Our theoretical market setup is made tractable by following [Lucas \(1990\)](#) in assuming that investors can pool the *tradeable* idiosyncratic risk within a large family. In equilibrium, the “state price” of a unit of consumption in each market  $m$  is a weighted average of the marginal utility of consumption in that market (with weight  $\lambda_m$ ) and a term that reflects the cross-sectional average marginal utility of consumption (with weight  $1 - \lambda_m$ ). Generally, both the average level of  $\lambda_m$  and its cross-sectional variation across markets play crucial roles in determining the equilibrium mapping from the state of the economy, as represented by the realized exogenous distribution of dividends across markets, to the endogenous distribution of asset prices across markets. In the special case where  $\lambda_m = 0$  for all markets  $m$ , then the state price of consumption is equal across markets and equal to the marginal utility of the aggregate endowment so that this economy collapses to the standard [Lucas \(1978\)](#) consumption-based asset pricing model. The specification of  $\lambda_m$ , representing the array of micro frictions which impede trade in claims to assets across markets, constitutes our one new degree of freedom relative to a standard consumption-based asset pricing model.

We then calibrate a special case of the general model where  $\lambda_m = \lambda$  for all markets. We choose standard parameters for aggregates and preferences: independently and identically distributed (IID) lognormal aggregate endowment growth, time- and state-separable expected utility preferences with constant relative risk aversion  $\gamma = 4$ . We then use the parameters governing the distribution of individual endowments and the single  $\lambda$  to simultaneously match the *systematic* return volatility of a well-diversified market portfolio and key time-series properties of an individual stock *total* return volatility (see [Goyal and Santa-Clara, 2003](#); [Bali, Cakici, Yan, and Zhang, 2005](#)). This procedure yields segmentation of approximately  $\lambda = 0.20$ . We find that this model generates a sizable unconditional equity premium, some 3.7% annual. However, as is familiar from many asset pricing models with expected utility preferences and trend growth, the model has a risk free rate that is too high and too volatile. Next, we extend this benchmark model by allowing for multiple types of market segmentation  $\lambda_m$ , which generates cross-sectional differences in stock return volatilities. This motivates us to pick values for  $\lambda_m$  in order to match the volatilities of portfolios sorted on measures of idiosyncratic volatility, as documented by [Ang, Hodrick, Xing, and Zhang](#)

(2006). Our main finding is that aggregation matters: with cross-sectional variation in  $\lambda_m$ , we need an *average* amount of segmentation of approximately  $\bar{\lambda} = 0.10$  to hit our targets, only half that of the single  $\lambda$  model. Moreover, this version of the model delivers essentially the same aggregate asset pricing implications as the single  $\lambda$  benchmark despite having only about half the average amount of segmentation. The characteristics of the micro markets in this disaggregate economy are quite distinct: some 50% of the aggregate market by value has a  $\lambda_m$  of zero, with the amount of segmentation rising to a maximum of about  $\lambda_m = 0.33$  for about 2% of the aggregate market by value.

**Market frictions in the asset pricing literature.** Traditionally, macroeconomists have taken the view that frictions in financial intermediation or other asset trades are small enough to be neglected in the analysis: asset prices are set “as if” there were no intermediaries but instead a grand Walrasian auction directly between consumers. In particular, early contributions to the literature, such as Rubenstein (1976), Lucas (1978) and Breeden (1979), characterize equilibrium asset prices using frictionless models. The quantitative limitations of plausibly calibrated traditional asset pricing models were highlighted by the “equity premium” and “risk-free rate” puzzles of Mehra and Prescott (1985), Weil (1989) and others.

Since then an extensive literature has attempted to explicitly incorporate one or other market frictions into an asset pricing model in an attempt to rationalize these and related asset pricing puzzles.<sup>2</sup> Models introducing market frictions have tended to follow one of two approaches. On the one hand, the financial economics literature followed deliberately micro-market approaches, focusing on the impact of specific frictions in specific financial markets. This microfoundations approach is transparent and leads to precise implications does not lead to any clear sense of whether or why micro asset market frictions matter in the aggregate. Moreover, with the exception of He and Krishnamurthy (2008), these models are typically not well integrated with the standard Lucas (1978) consumption-based asset pricing framework. On the other hand, the macroeconomics literature focused on frictions faced by households (perhaps a transaction cost or borrowing constraint), with unabashedly aggregate approaches. Intermediaries generally play no explicit role and the friction “stands in” for a diverse array of real-world micro frictions on the households’ side. Since there are large discrepancies between the predictions of frictionless asset pricing models and the data, in the calibration of such a macro model the friction also tends to have to be large. This

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<sup>2</sup>Other attempts to rationalize asset pricing puzzles retain frictionless markets but depart from traditional models by using novel preference specifications (e.g., Epstein and Zin (1989), Weil (1989, 1990), Campbell and Cochrane (1999)) and/or novel shock processes (e.g., Bansal and Yaron (2004)).

approach has the advantage that the friction has macro implications, by construction, but has the disadvantage that the friction has no transparent interpretation. In particular, it is difficult to evaluate the plausibility of the calibrated friction in terms of the constraints facing real-world households and firms. Our approach takes a middle course — we embed a stylized model of a collection of micro-markets that together form a financial system into an otherwise standard asset pricing model.

The remainder of the paper is organized as follows. In [Section 2](#) we present our model and show how to compute equilibrium asset prices. In [Section 3](#) we calibrate a special case of the model with a single type of market segmentation and in [Section 4](#) we show that this model can generate a sizable equity premium. [Section 5](#) then extends this benchmark model by allowing for multiply types of market segmentation and a non-degenerate cross-section of volatilities. Technical details and several extensions are given in the Appendix.

## 2 Model

The model is a variant on the pure endowment asset pricing models of [Lucas \(1978\)](#), [Breedon \(1979\)](#) and [Mehra and Prescott \(1985\)](#).

### 2.1 Setup

**Market structure and endowments.** Time is discrete and denoted  $t \in \{0, 1, 2, \dots\}$ . There are many distinct micro asset markets indexed by  $m \in [0, 1]$ . Each market  $m$  is specialized in trading a single type of durable asset with supply normalized to  $S_m = 1$ . Each period the asset produces a stochastic realization of a non-storable dividend  $y_m > 0$ . The aggregate endowment available to the entire economy is:

$$y := \int_0^1 y_m S_m dm = \int_0^1 y_m dm.$$

The aggregate endowment  $y > 0$  follows an exogenous stochastic process. Conditional on the aggregate state, the endowments  $y_m$  are independently and identically distributed (IID) across markets.

**Preferences.** We follow [Lucas \(1990\)](#) and use a representative family construct to provide consumption insurance beyond our market-segmentation frictions. The single representative family consists of many, identical, traders who are specialized in particular asset markets. The period utility for the family views the *utility* of each type of

trader as perfectly substitutable:

$$U(c) := \int_0^1 u(c_m) dm,$$

where  $u : \mathbb{R}^+ \rightarrow \mathbb{R}$  is a standard increasing concave utility function. Only in the special case of risk neutrality does the family view the *consumption* of each type of trader as being perfect substitutes. In general risk aversion will lead the family to smooth consumption across traders in different markets. Intertemporal utility for the family has the standard time- and state-separable form,  $\mathbb{E}_0 [\sum_{t=0}^{\infty} \beta^t U(c_t)]$ , with constant time discount factor  $0 < \beta < 1$ . The crucial role of the representative family is to eliminate the wealth distribution across markets as an additional endogenous state variable (see, e.g., [Alvarez, Atkeson, and Kehoe, 2002](#)).

**Segmentation frictions.** We interpret the representative family as a partially integrated financial system. Each trader in market  $m$  works at a specialized trading desk that deals in the asset specific to that market ([Figure 1](#) illustrates). Traders in market  $m$  are assumed to bear an exogenous fraction  $0 \leq \lambda_m \leq 1$  of the cost of trading in that market and in return receive  $\lambda_m$  of the benefit. The remaining  $1 - \lambda_m$  of the cost and return of trading in that market is shared between family members.

More precisely, given segmentation parameter  $\lambda_m$ , the period budget constraint facing a representative trader in market  $m$  is:

$$c_m + \lambda_m p_m s'_m + (1 - \lambda_m) a' \leq \lambda_m (p_m + y_m) s_m + (1 - \lambda_m) a, \quad (1)$$

where  $p_m$  is the ex-dividend price of a share in the asset in market  $m$ , and  $s_m, s'_m$  represent share holdings in that asset. The two terms  $a$  and  $a'$  are the cum-dividend value of the *family portfolio* brought into the period and the ex-dividend value of the family portfolio acquired this period, respectively.

As can be seen from the budget constraint, a trader in market  $m$  holds directly a number  $\lambda_m s'_m$  of shares of asset  $m$ . The cost of acquiring all the remaining shares in the economy,

$$\int_0^1 (1 - \lambda_m) p_m s'_m dm,$$

is divided among all the family members. More specifically, the term  $(1 - \lambda_m) a'$  on the left-hand side of the budget constraint (1) means that the trader in market  $m$  is asked to contribute  $1 - \lambda_m$  of the cost of acquiring all the remaining shares. Symmetrically, the term  $(1 - \lambda_m) a$  on the right-hand side of the budget constraint means that the

trader receives  $1 - \lambda_m$  of the benefit. An explicit expression for  $a'$  can be found using the simple accounting identity:

$$\int_0^1 (1 - \lambda_m) a' dm = \int_0^1 (1 - \lambda_m) p_m s'_m dm.$$

In words, the total value of all family members' contributions to the family portfolio (the left-hand-side) has to equal the total asset value of the family portfolio (the right-hand-side). Defining  $\bar{\lambda} := \int_0^1 \lambda_m dm$  we can rewrite this identity as:

$$a' := \int_0^1 \frac{1 - \lambda_n}{1 - \bar{\lambda}} p_n s'_n dn. \quad (2)$$

Similarly, the cum-dividend value of the family portfolio brought into the period is:

$$a := \int_0^1 \frac{1 - \lambda_n}{1 - \bar{\lambda}} (p_n + y_n) s_n dn. \quad (3)$$

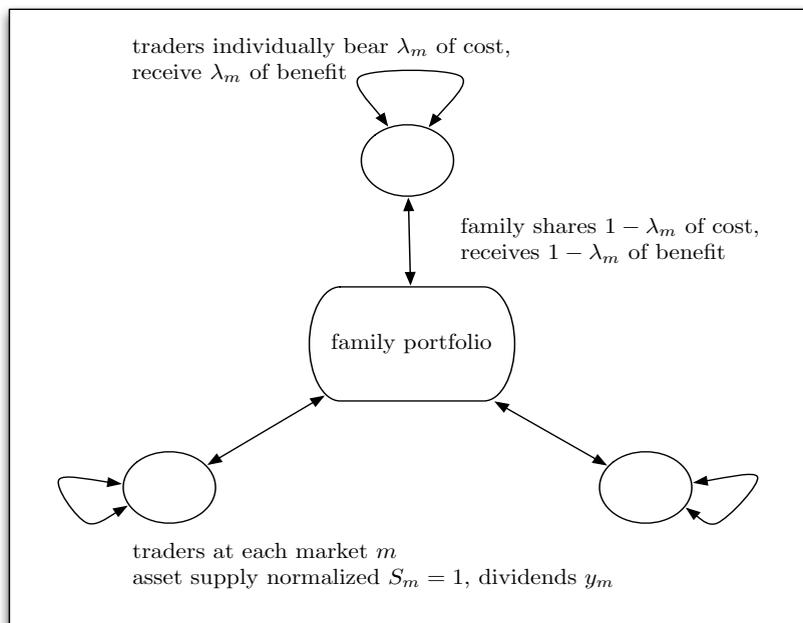


Figure 1: Market structure and segmentation frictions.

## 2.2 Equilibrium asset pricing

The Bellman equation for the family's value function is

$$v(s, y) = \max_{c, s'} \left\{ \int_0^1 u(c_m) dm + \beta \mathbb{E}[v(s', y') | y] \right\}. \quad (4)$$

The maximization is over choices of consumption allocations  $c : [0, 1] \rightarrow \mathbb{R}^+$  and asset holdings  $s' : [0, 1] \rightarrow \mathbb{R}$ , specifying  $c_m$  and  $s'_m$  for each  $m \in [0, 1]$ . The maximization is taken subject to the collection of budget constraints (1), one for each  $m$ , and the accounting identities for the family portfolio in (2) and (3). The expectation on the right-hand-side of the Bellman equation is formed using the conditional distribution for  $y'$  given the current aggregate endowment  $y$ .

An *equilibrium* of this economy is a collection of functions  $\{c, p, s'\}$  such that (i) taking  $p : [0, 1] \rightarrow \mathbb{R}^+$  as given,  $c : [0, 1] \rightarrow \mathbb{R}^+$  and  $s' : [0, 1] \rightarrow \mathbb{R}$  solve the family's optimization problem (4), and (ii) asset markets clear:  $s'_m = 1$  for all  $m$ .

**Equilibrium allocation.** Before solving for asset prices, we provide the equilibrium allocation of consumption across market. Substituting the accounting identities (2) and (3) into the budget constraint (1) and imposing the equilibrium condition  $s'_m = 1$ , we obtain:

$$c_m = \lambda_m y_m + (1 - \lambda_m) \int_0^1 \frac{1 - \lambda_n}{1 - \lambda} y_n dn.$$

And since the realized idiosyncratic  $y_n$  are independent of  $\lambda_n$ , an application of the Law of Large Numbers gives:

$$c_m = \lambda_m y_m + (1 - \lambda_m) y. \quad (5)$$

This formula is intuitive: equilibrium consumption in market  $m$  is a weighted average of the idiosyncratic and aggregate endowments with weights reflecting the degree of market segmentation. The  $\lambda_m$  represent the extent to which traders are not fully diversified and hence the segmentation parameters determine the degree of risk sharing in the economy. If  $\lambda_m = 0$ , traders are fully diversified and will have consumption equal to the aggregate endowment  $c_m = y$  (i.e., full consumption insurance). But if  $\lambda_m = 1$ , traders are not at all diversified and will simply consume the dividends realized in their specific market  $c_m = y_m$  (i.e., autarky).

**Asset prices.** To obtain asset prices, we use the first-order condition of the family's optimization problem. Let  $\mu_m \geq 0$  denote the Lagrange multiplier on the family's budget constraint for market  $m$ . We show in [Appendix A](#) that the family's Lagrangian

can be written:

$$\mathcal{L} = \int_0^1 \left[ u(c_m) + q_m(p_m + y_m)s_m - q_m p_m s'_m - \mu_m c_m \right] dm + \beta \mathbb{E}[v(s', y')|y],$$

where

$$q_m := \lambda_m \mu_m + (1 - \lambda_m) \int_0^1 \frac{1 - \lambda_n}{1 - \bar{\lambda}} \mu_n dn, \quad (6)$$

is a weighted average of the Lagrange multipliers in market  $m$  and the multipliers for other markets with weights reflecting the various degrees of market segmentation. More specifically,  $q_m$  is the marginal value to the family of earning one (real) dollar in market  $m$ . The first term in (6) arises because a fraction  $\lambda_m$  goes to the local trader, with marginal utility  $\mu_m$ . The second term arises because the remaining fraction is shared among other family members, with marginal utility  $\mu_n$ , according to their relative contributions  $(1 - \lambda_n)/(1 - \bar{\lambda})$  to the family portfolio. We will refer to  $q_m$  as the *state price* of earning one real dollar in market  $m$ .

Just as equilibrium consumption in market  $m$  is a weighted average of the idiosyncratic or “local” endowment and aggregate endowment with weights  $\lambda_m$  and  $1 - \lambda_m$ , so too the state price for market  $m$  is a weighted average of the idiosyncratic multiplier and an aggregate multiplier with the same weights. To highlight this, define

$$q := \int_0^1 \frac{1 - \lambda_n}{1 - \bar{\lambda}} \mu_n dn \quad (7)$$

so that we can write  $q_m = \lambda_m \mu_m + (1 - \lambda_m)q$ . If any particular market  $m$  has  $\lambda_m = 0$  then the state price in that market is equal to the aggregate state price  $q_m = q$  and is independent of the local endowment realization. If the segmentation parameter is common across markets,  $\lambda_m = \lambda$  all  $m$ , then  $q$  is the cross-sectional *average marginal utility* and  $q = \int_0^1 q_m dm$ . More generally,  $q$  is not a simple average over  $\mu_m$  since different markets have different relative contribution  $(1 - \lambda_m)/(1 - \bar{\lambda})$  to the family portfolio.

The first order conditions of the family’s problem are straightforward. For each  $c_m$  we have:

$$u'(c_m) = \mu_m. \quad (8)$$

And for each  $s'_m$  we have:

$$q_m p_m = \beta \mathbb{E} \left[ \frac{\partial v}{\partial s'_m}(s', y') \middle| y \right]. \quad (9)$$

The Envelope Theorem then gives:

$$\frac{\partial v}{\partial s_m}(s, y) = q_m(p_m + y_m). \quad (10)$$

Combining equations (9) and (10) gives the Euler equation characterizing asset prices in each market:

$$p_m = \mathbb{E} \left[ \beta \frac{q'_m}{q_m} (p'_m + y'_m) \middle| y \right]. \quad (11)$$

The Euler equation for asset prices takes a standard form, familiar from [Lucas \(1978\)](#), with the crucial distinction that the stochastic discount factor (SDF),  $\beta q'_m/q_m$ , is market specific.

Note that, combining the formulas for equilibrium consumption, (5), market-specific state prices, (6), and the pricing equation (11), we obtain a mapping from the primitives of the economy (the  $\lambda_m$ ,  $y_m$  etc) into equilibrium asset prices. In particular, one easily verify that the [Lucas \(1978\)](#) asset prices are obtained in the further special case  $\lambda_m = 0$  all  $m$ , so that  $c_m = y$  all  $m$  and  $\mu_m = u'(y)$  all  $m$  and  $q = \int_0^1 u'(y) dn = u'(y)$ .

**Shadow prices of risk-free bonds.** To simplify the presentation of the model, we have not explicitly introduced risk-free assets. But we can still compute “shadow” bond prices. Let  $\pi_k$  denote the price of a zero-coupon bond pays one unit of the consumption good for sure in  $k \geq 1$  period’s time and that is held in the family portfolio. As shown in [Appendix A](#) these bonds would have price:

$$\pi_k = \mathbb{E} \left[ \beta \frac{q'}{q} \pi'_{k+1} \middle| y \right], \quad (12)$$

with  $\pi_0 := 1$ . Bonds are priced using the aggregate state price  $q$ . In particular, the one-period shadow gross risk-free rate is given by  $1/\pi_1 = 1/\mathbb{E}[\beta q'/q]$ . Although the one-period SDF for bonds  $\beta q'/q$  does not depend on any particular idiosyncratic endowment realization, it does depend on the *distribution* of idiosyncratic endowments and in general is *not* simply the [Lucas \(1978\)](#)-[Breedon \(1979\)](#) SDF.

### 3 Calibration

To evaluate the significance of these segmentation frictions, we calibrate the model.

### 3.1 Parameterization of the model

**Preferences and endowments.** Let period utility  $u(c)$  be CRRA with coefficient  $\gamma > 0$  so that  $u'(c) = c^{-\gamma}$ . The log aggregate endowment is a random walk with drift and IID normal innovations:

$$\log g_{t+1} := \log(y_{t+1}/y_t) = \log \bar{g} + \epsilon_{g,t+1}, \quad \epsilon_{g,t+1} \sim \text{IID and } N(0, \sigma_{\epsilon g}^2), \quad \bar{g} > 0. \quad (13)$$

Log market-specific endowments are the log aggregate endowment plus an idiosyncratic term:

$$\log y_{m,t} := \log y_t + \log \hat{y}_{m,t}, \quad (14)$$

so that market-specific endowments inherits the trend in the aggregate endowment. The log idiosyncratic endowment  $\log \hat{y}_{m,t}$  is conditionally IID normal in the cross-section:

$$\log \hat{y}_{m,t} \sim \text{IID and } N(-\sigma_t^2/2, \sigma_t^2), \quad (15)$$

where the mean of  $-\sigma_t^2/2$  is chosen to normalize  $\mathbb{E}_t[\hat{y}_{m,t}] = 1$  and  $\mathbb{E}_t[y_{m,t}] = y_t$ .

**Idiosyncratic endowment volatility.** Volatility of the idiosyncratic endowment in a given market is time-varying and given by an AR(1) process in logs:

$$\log \sigma_{t+1} = (1 - \phi) \log \bar{\sigma} + \phi \log \sigma_t + \epsilon_{v,t+1}, \quad \epsilon_{v,t+1} \sim \text{IID and } N(0, \sigma_{\epsilon v}^2), \quad \bar{\sigma} > 0. \quad (16)$$

In a frictionless model ( $\lambda_m = 0$  all  $m$ ), all idiosyncratic risk would be diversified away so that asset prices would be independent of  $\sigma_t$ . With segmentation frictions ( $\lambda_m > 0$ ), by contrast, both the level and dynamics of  $\sigma_t$  will affect asset prices.

**Solving the quantitative model.** Since  $\hat{y}_{m,t} = y_{m,t}/y_t$  we can use equation (5) to write equilibrium consumption in market  $m$  as the product of the aggregate endowment  $y_t$  and an idiosyncratic component that depends only on the local idiosyncratic endowment realization  $\hat{y}_{m,t}$  and the amount of segmentation:

$$c_{m,t} = [1 + \lambda_m(\hat{y}_{m,t} - 1)]y_t. \quad (17)$$

Similarly, we can then use this expression for consumption and the fact that utility is CRRA to write the local state price as:

$$q_{m,t} = \theta_{m,t} y_t^{-\gamma}, \quad (18)$$

where:

$$\theta_{m,t} := \lambda_m [1 + \lambda_m (\hat{y}_{m,t} - 1)]^{-\gamma} + (1 - \lambda_m) \int_0^1 \frac{1 - \lambda_n}{1 - \bar{\lambda}} [1 + \lambda_n (\hat{y}_{n,t} - 1)]^{-\gamma} dn. \quad (19)$$

The SDF for market  $m$  is then:

$$M_{m,t+1} := \beta g_{t+1}^{-\gamma} \frac{\theta_{m,t+1}}{\theta_{m,t}}. \quad (20)$$

This is the usual [Lucas \(1978\)](#)-[Breedon \(1979\)](#) aggregate SDF  $M_{t+1} := \beta g_{t+1}^{-\gamma}$  with a market-specific multiplicative “twisting” factor  $\hat{M}_{m,t+1} := \theta_{m,t+1}/\theta_{m,t}$  that adjusts the SDF to account for idiosyncratic endowment risk.

To solve the model in stationary variables, let  $\hat{p}_{m,t} := p_{m,t}/y_t$  denote the price-to-aggregate-dividend ratio for market  $m$ . Dividing both sides of equation (11) by  $y_t > 0$  and using  $g_{t+1} := y_{t+1}/y_t$  this ratio solves the Euler equation:

$$\hat{p}_{m,t} = \mathbb{E}_t \left[ \beta g_{t+1}^{1-\gamma} \frac{\theta_{m,t+1}}{\theta_{m,t}} (\hat{p}_{m,t+1} + \hat{y}_{m,t+1}) \right], \quad (21)$$

which is the standard CRRA-formula except for the multiplicative adjustment  $\theta_{m,t+1}/\theta_{m,t}$ . This is a linear integral equation to be solved for the unknown function mapping the state into the price/dividend ratio. In general this integral equation cannot be solved in closed form, but numerical solutions can be obtained in a straightforward manner along the lines of [Tauchen and Hussey \(1991\)](#). We discuss these methods in greater detail in [Appendix B](#) below.

## 3.2 Calibration strategy and results

We calibrate the model using monthly postwar data (1959:1-2007:12, unless otherwise noted). Following a long tradition in the consumption-based asset pricing literature, we interpret the aggregate endowment as per capita real personal consumption expenditure on nondurables and services. We set  $\log \bar{g} = (1.02)^{1/12}$  to match an annual 2% growth rate and  $\sigma_{eg} = 0.01/\sqrt{12}$  to match an annual 1% standard deviation over the postwar sample. We set  $\beta = (0.99)^{1/12}$  to reflect an annual pure rate of time preference of 1% and we set the coefficient of relative risk aversion to  $\gamma = 4$ .

For our benchmark calibration we assume that all markets in the economy share the same segmentation parameter,  $\lambda$ . Given the values for preference parameters  $\beta, \gamma$  and the aggregate endowment growth process  $\bar{g}, \sigma_{eg}$  above, we still need to assign values to this single  $\lambda$  and the three parameters of the cross-sectional dispersion process  $\bar{\sigma}, \phi, \sigma_{ev}$ .

**Calibrating the idiosyncratic volatility process.** The crucial consequence of market segmentation is that local traders are forced to bear some idiosyncratic risk. Thus, to explain the impact of market segmentation on risk premia, it is important that our model generates realistic levels of idiosyncratic risk. This leads us to choose the parameters of the stochastic process for idiosyncratic endowment volatility in order to match key features of the volatility of a typical stock return. To see why there is a natural mapping between the two volatilities, write the gross returns on stock  $m$  as:

$$R_{m,t} = g_t \frac{\hat{y}_{m,t} + \hat{p}_{m,t}}{\hat{p}_{m,t-1}}. \quad (22)$$

Thus, the volatility of  $\hat{y}_{m,t}$  directly affects stock returns through the dividend term of the numerator. It also indirectly affects stock returns through the asset price,  $\hat{p}_{m,t}$ .

We obtain key statistics about stock return volatility from [Goyal and Santa-Clara \(2003\)](#). Their measure of monthly stock volatility is obtained by adding up the obtained by adding up the cross-sectional stock return dispersion over each day of the previous month. In [Figure 2](#) we show the monthly time series (1963:1-2001:12) of their measure of the cross-sectional standard deviation of stock returns, as updated by [Bali, Cakici, Yan, and Zhang \(2005\)](#).

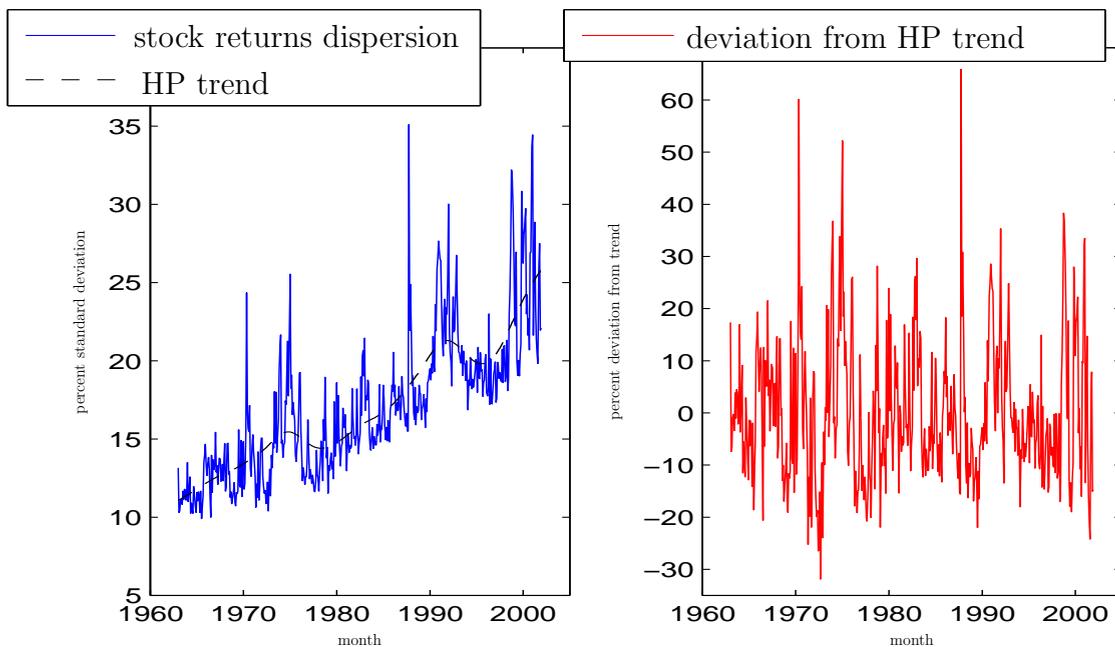


Figure 2: Cross-sectional standard deviation of stock returns (1963:1-2001:12) from [Goyal and Santa-Clara \(2003\)](#), as updated by [Bali et al. \(2005\)](#).

We choose the idiosyncratic endowment volatility process so that, when we calculate the same stock return volatility measure in our model, we replicate key features of this data. We replicate three features of the data: the unconditional average return volatility of 16.4%, the unconditional standard deviation of return volatility 4.2% monthly, and AR(1) coefficient of return volatility 0.84 monthly. We replicate these three features by simultaneously choosing the three parameters governing the stochastic process for endowment volatility: the unconditional average  $\bar{\sigma}$ , the innovation standard deviation  $\sigma_{ev}$ , and the AR(1) coefficient  $\phi$ .

**Calibrating the segmentation parameter: basics.** The segmentation parameter  $\lambda$  governs the extent to which local traders can diversify away the return volatility of their local asset. Thus,  $\lambda$  determines the extent to which the idiosyncratic volatility factor,  $\sigma_t$ , has an impact on asset prices and creates systematic variation in asset returns. This leads us to identify  $\lambda$  using a measure of systematic volatility, the return volatility of a well diversified portfolio.

To understand precisely how the identification works, recall first what would happen in the absence of market segmentation,  $\lambda = 0$ . Then, we would be back in the [Lucas \(1978\)](#)-[Mehra and Prescott \(1985\)](#) model with IID lognormal aggregate endowment growth. As is well known (see, e.g., [LeRoy, 2006](#)), this model can't generate realistic amounts of systematic volatility. Specifically, with  $\lambda = 0$  the return from a diversified market portfolio is  $g_t(1+\bar{p})/\bar{p}$  where  $\bar{p} = \beta\mathbb{E}[g^{1-\gamma}]/(1-\beta\mathbb{E}[g^{1-\gamma}])$  is the constant price/dividend ratio for the aggregate market. With our standard parameterization of the preference parameters and aggregate endowment growth,  $(1+\bar{p})/\bar{p} \approx 1.0058$  so that the monthly standard deviation of the diversified market portfolio return is approximately the same as the monthly standard deviation of aggregate endowment growth, about 0.29% monthly as opposed to about 4.28% monthly in the data.

In contrast with the  $\lambda = 0$  case, when  $\lambda > 0$  idiosyncratic endowment volatility creates systematic volatility. Indeed, because of persistence, a high idiosyncratic endowment volatility this month predicts a high idiosyncratic endowment volatility next month. Thus in every market  $m$  local traders expect to bear more idiosyncratic risk, and because of risk aversion the price dividend ratio  $\hat{p}_{m,t}$  has to go down everywhere. Because this effect impacts all stocks at the same time, it endogenously *creates* systematic return volatility. Clearly, the effect is larger if markets are more segmented and traders are forced to bear more idiosyncratic risk: thus, a larger  $\lambda$  will result in a larger increase in systematic volatility.

**Calibrating the segmentation parameter: specifics.** We consider two calibration schemes. In our benchmark calibration, we identify  $\lambda$  by matching the return volatility of a well-diversified portfolio, specifically the 4.3% monthly standard deviation of the real value-weighted return of NYSE stocks from CRSP. In a second, more conservative, calibration, we identify  $\lambda$  by matching *only* the fraction of the aggregate volatility of stock returns that is systematically explained by idiosyncratic stock return volatility (i.e., the [Goyal and Santa-Clara, 2003](#), factor). [Goyal and Santa-Clara](#) find that the correlation between the systematic volatility of value-weighted stock returns and idiosyncratic stock return volatility is about 0.4 at a monthly frequency. Therefore, if idiosyncratic stock return volatility was the only source of systematic return volatility, systematic volatility should only be about  $0.4 \times 4.3\% = 1.72\%$  monthly. Implicitly, this procedure leaves other factors outside the model to explain the remaining  $0.6 \times 4.3\% = 2.58\%$  monthly of the systematic volatility observed in the CRSP data. Since our second calibration selects  $\lambda$  to reproduce this counterfactually lower level of systematic volatility, it naturally leads to a lower calibrated value for  $\lambda$ .

**Calibration results.** The calibrated parameters from these two schemes are listed in [Table 1](#). In our benchmark calibration, the level of  $\lambda$  is 0.21. That is, 21% of idiosyncratic endowment risk is non-tradeable. In terms of portfolio weights, we also find that  $\lambda = 0.21$  implies that, in a typical market  $m$ , a trader invests approximately 21% of his wealth in the local asset, and the rest in the family portfolio.<sup>3</sup>

Our more conservative calibration necessarily selects a lower level of  $\lambda$ , but in practice the difference is small: the conservative calibration of  $\lambda$  is 0.19. Thus there is a relatively small difference in  $\lambda$  despite the fact that the conservative calibration targets systematic stock return volatility that is only 40% of actual systematic stock volatility. The differences between the calibrations are more significant for the cross-sectional endowment volatility process. In the benchmark, the volatility process is more persistent and has bigger innovations but fluctuates around a lower unconditional mean.

[Table 2](#) shows that with these parameters, the benchmark model matches the target moments closely but not exactly. Notice that in the benchmark calibration the persistence of the cross-sectional endowment distribution  $\phi$  is close to one.<sup>4</sup> That is, to match the persistence in the data, the calibration procedure tries to select a very high  $\phi$ . As explained above, persistence is necessary for idiosyncratic volatility to matter

<sup>3</sup>The derivation of portfolio weights for the family is given in [Appendix A](#) below.

<sup>4</sup>The discrete-state approximation of the process for  $\sigma_t$  (equation 16) used to solve the model is stationary by construction, even when  $\phi = 1$  exactly. See [Appendix B](#) for more details on our solution method.

for systematic stock return volatility. Thus, the value of  $\phi$  is high largely because the model is matching a high level of systematic volatility. This intuition is confirmed by the conservative calibration: when we target a lower level of systematic volatility, the calibration procedure gives a lower value,  $\phi = 0.89$ , and the model remains able to hit the persistence of the cross-sectional standard deviation of returns exactly.

## 4 Quantitative examples

**Aggregate statistics.** In comparing our model to data, we adopt the perspective of an econometrician who observes a collection of asset returns but who ignores the possibility of market segmentation. In particular, to make our results comparable to those typically reported in the empirical asset pricing literature, we focus on the properties of the aggregate market portfolio. We define the gross market return  $R_M := (p' + y')/p$  where  $p := \int_0^1 p_m dm$  is the *ex-dividend* value of the market portfolio and  $y$  is the aggregate endowment. We define the gross (shadow) risk free rate for market  $m$  by  $R_{f,m} := \mathbb{E}[\beta q'_m/q_m]^{-1}$  and the average risk free rate by  $R_f := \int_0^1 R_{f,m} dm$ . We then calculate the *unconditional* equity risk premium  $\mathbb{E}[R_M - R_f]$  by and similarly for other statistics. We call  $p/y$  the price dividend ratio of the market.

### 4.1 Results

**Equity premium.** With these definitions in mind, [Table 3](#) shows our model's implications for aggregate returns and price/dividend ratios. We report annualized monthly statistics from the model and compare these to annualized monthly returns and to annual price/dividend ratios (we use annual data for price/dividends because of the pronounced seasonality in dividends at the monthly frequency). The benchmark model produces an annual equity risk premium of 3.69% annual as opposed to about 5.27% annual in the postwar NYSE CRSP data. Clearly this is a much larger equity premium than is produced by a standard [Lucas \(1978\)](#)-[Mehra and Prescott \(1985\)](#) model. For comparison, that model with risk aversion  $\gamma = 4$  and IID consumption growth with annual standard deviation of 1% produces an annual equity premium of about 0.04%. The segmented markets model with  $\lambda = 0.21$  is able to generate an equity premium some two orders of magnitude larger.

**Why is there a large equity premium?** Relative to standard consumption-based asset pricing models with time-separable expected utility preferences, our model delivers a large equity premium. We use our parameter  $\lambda$  to generate realistically high levels

of systematic market volatility. Is matching systematic return volatility what helps us generate a large equity premium? No. Recall from the GMM estimates of risk aversion in Hansen and Singleton (1982) (and others) that the standard consumption-based model fails to match the equity premium in the data even when — by construction — it matches the systematic return volatility in the data. But of course model risk premia are generated by *covariances* so no matter how much systematic return volatility you feed into a model, the equity risk premia will be zero if the model’s SDF is not correlated with that systematic return volatility (see Cochrane and Hansen, 1992, for a forceful argument).

What, then, is the equity premium from the point of view of aggregate consumption? One can show that, in our model, if one computes the unconditional average equity premium using the model generated market return and the Lucas (1978)-Breedon (1979) SDF  $\beta g_{t+1}^{-\gamma}$  instead of the true model SDF, then the equity premium is on the order of 0.04% (4 basis points) annual rather than the 3.69% annual in the benchmark model. While aggregate consumption does not command a big risk premium, the volatility factor does. To see this, consider the premium implied by the SDF  $\beta q_{t+1}/q_t$  where  $q$  is aggregate state price that determines the (shadow) price of risk free bonds. In general this is given by equation (7) but with a single common  $\lambda$  it reduces to  $q = \int_0^1 \mu_m dm = \int_0^1 c_m^{-\gamma} dm$ , the cross-section average marginal utility. In our benchmark model, this SDF implies an equity premium of 2.17% annual. This comes from the convexity of the marginal utility function: a high realization of  $\sigma_t$  makes equilibrium consumption highly dispersed across markets so that average marginal utilities are high. At the same time, a high  $\sigma_t$  depresses asset prices in every market, so that the return on the market portfolio is high.

**Level of the risk-free rate.** Although the benchmark model delivers reasonable implications for the level of the equity premium, it is not so successful on other dimensions of the data. The level of the risk free rate is very high as compared to the data. In the model the risk free rate is about 8 or 9% annual, while in the data it is more like 2%. As emphasized by Weil (1989), this is a common problem for models with expected-utility preferences. In short, attempts to address the equity premium puzzle by increasing risk aversion also tend to raise the risk free rate so that even if it’s possible to match the equity *premium*, the model may well do so at absolute *levels* of returns that are too high. This effect comes from the relationship between real interest rates and growth in a deterministic setting with expected utility, high risk aversion means low intertemporal elasticity of substitution so that it takes high real interest rates to compensate for high

aggregate growth. With risk, there is an offsetting precautionary savings effect that could, in principle, pull the risk free rate back down to more realistic levels. But in our calibration this precautionary savings effect is quantitatively small: raising  $\lambda$  from zero (the Mehra and Prescott case) to  $\lambda = 0.21$  (our benchmark) lowers the risk free rate by about 1% annual.

**Volatility of the risk-free rate.** In the data, the risk free rate is smooth and the volatility of the equity premium reflects the volatility of equity returns. In the benchmark model, the risk free rate is too volatile, about 11% annual as opposed to 1% annual in the data.

**Price/dividend ratio.** The benchmark model produces an annual price/dividend ratio of about 12 as opposed to an unconditional average of more like 31 in the NYSE CRSP data. Given the large, persistent, swings in the price/dividend ratio in the data, what constitutes success on this dimension is not entirely clear. The model generates if anything slightly too much unconditional volatility in the log price/dividend ratio, some 43% annual as opposed to 39% in the data. This is another reflection of the fact that the model by construction matches the aggregate volatility of the market. While the unconditional volatility of the price/dividend ratio is similar in the model and in the data, the temporal composition of this volatility is somewhat different. In particular, the persistence of the log price/dividend ratio is about 0.50 annual in the benchmark model, lower than the 0.88 in the data. That is, the unconditional volatility of the price/dividend ratio in the data comes from large, low-frequency movements whereas the unconditional volatility in the model comes from higher-frequency movements.

## 4.2 Discussion

**Constant endowment volatility.** Our benchmark model has two departures from a standard consumption-based asset pricing model: segmentation and a time-varying endowment volatility. To show that both these departures are essential for our results, we solved our model with constant endowment volatility, i.e.,  $\sigma_t = \sigma$  all  $t$ . For this exercise, we fix the volatility at the same level as the unconditional average from the benchmark model  $\bar{\sigma} = 1.71$  and keep the level of segmentation at the benchmark  $\lambda = 0.21$ . In Table 2 we show that this “constant  $\sigma$ ” version of the model produces essentially the same amount of unconditional cross-sectional stock return volatility as in the data (suggesting that this moment is principally determined by  $\bar{\sigma}$  alone) but

produces only as much systematic stock volatility as would a benchmark [Lucas \(1978\)](#)-[Mehra and Prescott \(1985\)](#) model, about 0.29% monthly as opposed to 4.28% in the data. Thus  $\lambda > 0$  is necessary but not sufficient for our model to create systematic stock volatility from idiosyncratic endowment volatility. In [Table 3](#) we see that despite producing negligible systematic stock volatility, the model with constant  $\sigma$  is capable of generating a modest equity premium, some 1.29% annual. The benchmark model with time variation in endowment volatility generates another 2.4% on top of this for a total of 3.69% annual.

**Countercyclical endowment volatility.** Many measures of cross-sectional idiosyncratic risk increase in recessions.<sup>5</sup> This counter-cyclicality is also a feature of the cross-sectional standard deviation of returns data from [Goyal and Santa-Clara \(2003\)](#). However, the stochastic process we use for the cross-sectional volatility evolves independently of aggregate growth. To see if our results are sensitive to this, we modify the stochastic process in [\(16\)](#) to:

$$\log \sigma_{t+1} = (1 - \phi) \log \bar{\sigma} + \phi \log \sigma_t - \eta(\log g_t - \log \bar{g}) + \epsilon_{v,t+1}. \quad (23)$$

with  $\epsilon_{v,t+1}$  IID normal, as before. If  $\eta > 0$ , then aggregate growth below trend in period  $t$  increases the likelihood that volatility is above trend in period  $t + 1$  so that volatility is counter-cyclical. We identify the new parameter  $\eta$  by requiring that, in a regression of the cross-section standard deviation of stock returns on *lagged* aggregate growth, the regression coefficient is  $-0.50$  as it is in the data. The calibrated parameters for this “feedback” version of the model are also shown in [Table 1](#). The calibrated  $\eta$  elasticity is 1.88 so aggregate growth 1% below trend tends to increase endowment volatility by nearly 2%. The other calibrated parameters are indistinguishable from the benchmark parameters. Moreover, the model’s implications for asset prices as shown in [Table 3](#) are also very close to the results for the benchmark model. This suggests that while the model can be reconciled with the countercyclical behavior of cross-sectional stock volatility, this feature is not necessary for our main results.

**Relationship to incomplete markets models.** There is a large literature in macroeconomics that analyzes the asset pricing implications of market incompleteness when households receive uninsurable idiosyncratic income shocks.<sup>6</sup> One might have the impression that all our model does is shift the focus of incomplete markets models from

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<sup>5</sup>See for example [Campbell, Lettau, Malkiel, and Xu \(2001\)](#) and [Storesletten et al. \(2004\)](#).

<sup>6</sup>See for example [Telmer \(1993\)](#) and [Heaton and Lucas \(1996\)](#).

the idiosyncratic labor income risk facing households to the idiosyncratic income risk faced by the financial sector. We argue that, while our segmented markets model indeed results in uninsurable shocks, it is conceptually different from standard incomplete markets models.

To see why, note that in standard incomplete markets models the intertemporal marginal rate of substitution (IMRS) of *every* household  $i$  prices the excess return of the market portfolio:

$$\mathbb{E}[M_i R^e] = 0. \quad (24)$$

As forcefully emphasized by [Mankiw \(1986\)](#), [Constantinides and Duffie \(1996\)](#) and [Kruger and Lustig \(2008\)](#), with CRRA utility when idiosyncratic consumption growth is statistically independent from aggregate consumption growth, idiosyncratic risk has no impact on the equity premium. Indeed, in that case the IMRS can be factored into  $\hat{M}_i M$ , where  $M = \beta g^{-\gamma}$  is the standard [Lucas \(1978\)](#)-[Breedon \(1979\)](#) stochastic discount factor, and  $\hat{M}_i$  is an idiosyncratic component that is independent from  $M$ . Expanding the expectation in (24) we have:

$$\mathbb{E}[\hat{M}_i M R^e] = \mathbb{E}[\hat{M}_i] \mathbb{E}[M R^e] + \text{Cov}[\hat{M}_i, M R^e] = 0.$$

From independence  $\text{Cov}[\hat{M}_i, M R^e] = 0$ . Using this and dividing both sides by  $\mathbb{E}[\hat{M}_i] > 0$  we obtain:

$$\mathbb{E}[M R^e] = 0. \quad (25)$$

As shown by [Kocherlakota \(1996\)](#), this asset pricing equation cannot rationalize the observed equity premium.

In our benchmark model, we maintain the assumption that the idiosyncratic component of dividends,  $\hat{y}_m$ , is statistically independent from aggregate consumption growth. Yet, we obtain a much larger equity premium than [Mehra and Prescott](#). The reason for this is that in our asset pricing model the local stochastic discount factor does not have to price the excess return on the *aggregate* market portfolio, as in equation (24), but instead price the excess return on the *local* asset market. The local discount factor is correlated with the local excess return (through the local endowment realization) and this makes it impossible to strip-out the influence of market-specific factor.

Specifically, instead of equation (24) we have a pricing equation of the form:

$$\mathbb{E}[M_m R_m^e] = 0, \quad (26)$$

where  $M_m$  is the local stochastic discount factor and  $R_m^e$  is the local excess return. From

equation (20) we can factor the local discount factor into  $\hat{M}_m M$  where  $M$  is again the Lucas (1978)-Breedon (1979) discount factor and  $\hat{M}_m$  is a market-specific factor. Now proceeding as above and expanding the expectation in (26) we have:

$$\mathbb{E}[\hat{M}_m MR_m^e] = \mathbb{E}[\hat{M}_m] \mathbb{E}[MR_m^e] + \text{Cov}[\hat{M}_m, MR_m^e] = 0.$$

But  $\hat{M}_m$  and  $R_m^e$  depend on the same local risk factor so  $\text{Cov}[\hat{M}_m, MR_m^e] \neq 0$  and we cannot factor out  $\mathbb{E}[\hat{M}_m]$ . Because this makes it impossible to aggregate the collection of equations (26) into (24), in our model the standard incomplete markets logic does not apply.

## 5 Cross-sectional volatilities

In our first set of quantitative examples we used a common amount of segmentation,  $\lambda$ , for all asset markets. This implies that conditional on the aggregate state of the economy, each market  $m$  is characterized by a common amount of volatility (essentially determined by the economy-wide  $\sigma_t$  and  $\lambda$ ) so that there is no *cross-sectional variation in volatility*. We now pursue the implications of the general model with market-specific  $\lambda_m$  and hence a non-degenerate cross-section of volatility.

Specifically, we allow for a finite number of market *types*. In a slight abuse of notation we continue to index these market types by  $m$ . We assume that each market contains the same number of assets, but that there is a total measure  $\omega_m$  of traders in market  $m$ , with a supply *per trader* normalized to 1. With this notation, then, the aggregate endowment is  $y = \sum_m y_m \omega_m$ , etc.

**Calibration of market-specific  $\lambda_m$ : strategy.** In the case of a single common  $\lambda$  above, the value of  $\lambda$  was identified by matching a measure of systematic volatility, the return volatility of a well-diversified portfolio of stocks. We now need to identify a *vector* of segmentation parameters and we do this using a closely-related strategy. In particular, we identify market types with quintile portfolios of stocks sorted on measures of idiosyncratic volatility from Ang, Hodrick, Xing, and Zhang (2006). They compute value-weighted quintile portfolios by sorting stocks based on idiosyncratic volatility relative to the Fama and French (1993) three-factor pricing model in postwar CRSP data. To give a sense of this data, Ang, Hodrick, Xing, and Zhang report an average standard deviation of (diversified) portfolio returns for the *first* quintile of stocks of about 3.83% monthly (as opposed to about 4.28% for the market as a whole). By

construction this portfolio is 20% of a simple count of stocks but it constitutes about 53.5% of the market by value. At the other end of the volatility spectrum, the average monthly standard deviation of a well-diversified portfolio of the *fifth* quintile of stocks is about 8.16% and these constitute only about 1.9% of the market by value.

We choose the value of  $\lambda_m$  for  $m = 1, \dots, 5$  to match the unconditional average idiosyncratic volatility of the  $m$ 'th quintile portfolio in [Ang, Hodrick, Xing, and Zhang](#). Since our model has no counterpart of [Fama and French](#)'s size and book/market factors, we construct a measure of idiosyncratic volatility relative to a single-factor CAPM model. For each market  $m$  in the model we obtain the residuals from a CAPM regression of individual market returns on the aggregate market return and use these residuals to compute idiosyncratic volatility for each market. Similarly, we choose values of  $\omega_m$  so that the unconditional average portfolio weight<sup>7</sup> of the family in assets of market  $m$  matches the average market share for the  $m$ 'th quintile portfolio from [Ang, Hodrick, Xing, and Zhang \(2006\)](#). Our calibration procedure chooses these parameters simultaneously with the parameters  $\bar{\sigma}, \phi, \sigma_{ev}$  of the stochastic process for cross-sectional endowment volatility. We keep the values of the preference parameters  $\beta, \gamma$  and the aggregate growth parameters  $\bar{g}, \sigma_{eg}$  at their benchmark values.

**Calibration of market-specific  $\lambda_m$ : results.** The calibrated parameters from this procedure are listed in [Table 4](#). We find that the  $m = 1$  market, with the lowest idiosyncratic volatility, has a segmentation parameter  $\lambda_1 = 0.00$  (to two decimal places). These assets are essentially frictionless. This  $m = 1$  market consists of 20% of assets by number, by construction, but it accounts for 50% of total market by value. By contrast, the  $m = 5$  market has segmentation parameter  $\lambda_5 = 0.33$  but accounts for only 2% of total market value. Across markets the segmentation parameters  $\lambda_m$  are monotonically increasing in  $m$  while the weights  $\omega_m$  are monotonically decreasing in  $m$ . Averaging over the five markets  $\bar{\lambda} = \sum_m \lambda_m \omega_m = 0.11$ . Thus this economy, which matches the same aggregate moments as the benchmark model, hits its targets with an average amount of segmentation  $\bar{\lambda} = 0.11$  roughly half that of the single parameter benchmark  $\lambda = 0.21$ . This suggests that there may be a significant bias when aggregating a collection of heterogeneously segmented markets into a “representative” segmented market.

Relative to the benchmark, the model's endowment volatility process now has a lower unconditional average, more time-series variation, and slightly less persistence. [Table 5](#) shows that with these parameters the model matches the target moments closely but not exactly. In particular, while the average idiosyncratic volatility across the  $m$

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<sup>7</sup>The derivation of portfolio weights for the family is given in [Appendix A](#) below.

markets is about 4.6% monthly as opposed to 4.7% in the data, this is achieved with slight discrepancies at the level of each market, e.g., the 1st market has volatility of 3.8% against 4.2% in the data.

**Market-specific asset pricing implications.** In Panel A of [Table 6](#) we show the risk premia and CAPM parameters for each market type in the model and their empirical counterparts. In the data, the equity premia for the low volatility  $m = 1$  market is 0.53% monthly (roughly 6.5% annual) whereas in the model it is 0.18% monthly. This market accounts for half of total market value and has an alpha of about 0.14 in the data and 0.11 in the model. As we go to markets with higher volatility, the model predicts that risk premia *monotonically* increase, reaching 0.85% monthly (or 10.6% annual) for the  $m = 5$  market. However, the data predicts a *hump-shaped* pattern for the cross-section of equity premia, with the premia reaching a maximum at about 0.69% monthly for the  $m = 3$  market before falling to  $-0.53\%$  for the 5th and most volatile market. Thus the model fails to account even qualitatively for the equity premia of the highest idiosyncratic volatility markets.

**Aggregate asset pricing implications.** In Panel B of [Table 6](#) we show the aggregate asset pricing implications of the model with market-specific  $\lambda_m$ . The aggregate equity premium is essentially the same as in the benchmark single  $\lambda$  model, 3.6% annual, despite the fact that the average segmentation here is only  $\bar{\lambda} = 0.11$ , half the single  $\lambda$  benchmark. For comparison, the table shows the asset pricing implications for an otherwise identical single  $\lambda$  economy with  $\lambda = \bar{\lambda} = 0.11$ . The aggregation of the micro segmentation frictions across the different markets adds some 1.6% annual to the equity premium, taking it from 2% to 3.6%. This model generates about twice as much time-series volatility in the equity premium as the benchmark, some 1.8% as opposed to 0.9% annual, but still an order of magnitude lower than the 14% annual in postwar NYSE CRSP data. Compared to the benchmark model, the risk free rate has about the same level and if anything is even more volatile.

## 6 Conclusion

We propose a tractable consumption-based model in order to explain and quantify the macro impact of financial market frictions. We envision an economy comprised of many micro financial markets that are partially segmented from one another. Because of segmentation, traders in each micro market have to bear some local idiosyncratic risk.

Assets in every markets are priced by a convex combination of the local trader marginal utility (who has to bear some of the idiosyncratic risk), and of the average marginal utility in the rest of the economy (who can diversify the remaining idiosyncratic risk in a large portfolio). We calibrate the model when all markets share the same level of segmentation and show that it can generate a sizeable equity premium. We also allow segmentation to differ across markets and show that aggregation matters: we can obtain essentially the same aggregate asset pricing implication with a much smaller *average* level of segmentation.

## Tables

Panel A: *Preferences and aggregate endowment growth.*

Parameter	Monthly value	Notes
$\beta$	0.9992	annual discount rate 1%
$\gamma$	4	coefficient relative risk aversion
$\bar{g}$	1.0017	annual aggregate growth 2%
$\sigma_{eg}$	0.0029	annual std dev aggregate growth 1%

Panel B: *Segmentation and idiosyncratic endowment volatility.*

Parameter	Model			Data moment	
	Benchmark	Conservative	Feedback		
$\lambda$	0.21	0.19	0.21	std dev diversified market portfolio return	4.28
$\bar{\sigma}$	1.71	2.30	1.71	average cross-section std dev returns	16.40
$\sigma_{ev}$	0.10	0.04	0.10	time-series std dev cross-section std dev returns	4.17
$\phi$	0.99	0.89	0.99	AR(1) cross-section std dev returns	0.84
$\eta$			1.88	regression cross-section std dev returns on lagged growth	-0.50

Table 1: **Parameter choices.**

Moment	Data	Model			
		Benchmark	Conservative	Constant	Feedback
std dev diversified market portfolio return	4.28	4.21	1.81	0.29	4.21
average cross-section std dev returns	16.40	16.37	16.40	14.46	16.36
time-series std dev cross-section std dev returns	4.17	4.18	4.17	0.00	4.18
AR(1) cross-section std dev returns	0.84	0.78	0.84		0.78
regression cross-section std dev returns on lagged growth	-0.50				-0.50

Table 2: **Fit of calibrated models.**

Moment		Data	Model			
			Benchmark	Conservative	Constant	Feedback
equity premium	$\mathbb{E}[R_M - R_f]$	5.27	3.69	1.87	1.29	3.68
market return	$\mathbb{E}[R_M]$	7.04	12.29	11.08	10.54	12.33
risk free rate	$\mathbb{E}[R_f]$	1.78	8.59	9.21	9.25	8.64
	$\text{Std}[R_M - R_f]$	14.25	0.88	0.28	0.00	0.89
	$\text{Std}[R_M]$	14.44	14.60	6.27	1.01	14.60
	$\text{Std}[R_f]$	1.06	11.29	5.92	0.00	11.71
sharpe ratio	$\mathbb{E}[R_M - R_f]/\text{Std}[R_M - R_f]$	0.37	4.20	6.80		4.12
price/dividend ratio	$\mathbb{E}[p/y]$	30.81	12.58	12.18	12.40	12.59
	$\text{Std}[\log(p/y)]$	38.63	43.21	12.29	0.00	43.15
	$\text{Auto}[\log(p/y)]$	0.88	0.50	0.20		0.50

Table 3: **Aggregate asset pricing implications of single  $\lambda$  model.**

Panel A: *Segmentation parameters.*

Market $m$	Model		Data moment	
	$\lambda_m$	$\omega_m$	Portfolio std dev	Market share
1	0.00	0.50	3.8	0.54
2	0.17	0.28	4.7	0.27
3	0.25	0.13	5.9	0.12
4	0.30	0.06	7.1	0.05
5	0.33	0.02	8.2	0.02
average	0.11		4.6	

Panel B: *Idiosyncratic endowment volatility.*

Parameter		Data moment	
$\bar{\sigma}$	1.12	average cross-section std dev returns	16.40
$\sigma_{\epsilon v}$	0.18	time-series std dev cross-section std dev returns	4.17
$\phi$	0.97	AR(1) cross-section std dev returns	0.84

Table 4: **Market-specific segmentation.**

Panel A: *Segmentation parameters.*

Market $m$	Data moment		Model moment	
	Portfolio std dev	Market share	Portfolio std dev	Market share
1	3.8	0.54	4.2	0.54
2	4.7	0.27	4.6	0.27
3	5.9	0.12	5.7	0.12
4	7.1	0.05	7.0	0.05
5	8.2	0.02	8.1	0.02
average	4.6		4.7	

Panel B: *Idiosyncratic endowment volatility.*

Moment	Data	Model
average cross-section std dev returns	16.40	16.50
time-series std dev cross-section std dev returns	4.17	4.20
AR(1) cross-section std dev returns	0.84	0.85

Table 5: **Fit of market-specific segmentation model.**

Panel A: *Market-specific asset pricing implications.*

Market $m$	Risk premia		CAPM alpha		CAPM beta
	Data	Model	Data	Model	Model
1	0.53	0.18	0.14	0.11	0.89
2	0.65	0.27	0.13	0.02	0.98
3	0.69	0.44	0.07	-0.24	1.27
4	0.36	0.65	-0.28	-0.56	1.62
5	-0.53	0.85	-1.21	-0.86	1.95

Panel B: *Aggregate asset pricing implications.*

Moment		Data	Model	
			$\lambda_m$	$\bar{\lambda}$
equity premium	$\mathbb{E}[R_M - R_f]$	5.27	3.63	2.04
market return	$\mathbb{E}[R_M]$	7.04	12.14	10.97
risk free rate	$\mathbb{E}[R_f]$	1.78	8.51	8.94
	$\text{Std}[R_M - R_f]$	14.25	1.78	1.30
	$\text{Std}[R_M]$	14.44	16.13	12.08
	$\text{Std}[R_f]$	1.06	14.82	11.38
sharpe ratio	$\mathbb{E}[R_M - R_f]/\text{Std}[R_M - R_f]$	0.37	2.02	1.56
price/dividend ratio	$\mathbb{E}[p/y]$	30.81	13.70	13.90
	$\text{Std}[\log(p/y)]$	38.63	42.81	31.39
	$\text{Auto}[\log(p/y)]$	0.88	0.42	0.40

Table 6: **Asset pricing implications of market-specific segmentation.**

# Technical Appendix

## A General model with detailed derivations

We add three features relative to the model presented in the main text: (i) for each market  $m$  there is a density  $\omega_m \geq 0$  of traders, (ii) the asset supply is  $S_m \geq 0$ , not normalized to 1, and (iii) there are bonds in positive net supply held in the family portfolio. The  $\omega_m$  and  $S_m$  must satisfy the following two restrictions. First, the total measure of traders is one, i.e.,

$$\int_0^1 \omega_m dm = 1. \quad (27)$$

Each period one share of the asset produces a stochastic realization of a non-storable dividend  $y_m > 0$ . The aggregate endowment available to the entire economy is:

$$y = \int_0^1 y_m S_m \omega_m dm. \quad (28)$$

As in the text, traders in market  $m$  are assumed to bear an exogenous fraction  $0 \leq \lambda_m \leq 1$  of the cost of trading in that market and in return receive  $\lambda_m$  of the benefit. The remaining  $1 - \lambda_m$  of the cost of trading in that market is borne by the family. As show in the text, this results in a sequential budget constraint of the form:

$$c_m + \lambda_m p_m s'_m + (1 - \lambda_m) a' \leq \lambda_m (p_m + y_m) s_m + (1 - \lambda_m) a - t_m, \quad (29)$$

where the new term  $t_m$  are lump-sum taxes levied on market  $m$  by the government. As in the main text  $a$  and  $a'$  represent the cum-dividend value of the family portfolio brought into the period and the ex-dividend value of the family portfolio acquired this period, respectively. Proceeding as in the text, we find that  $a$  and  $a'$  satisfy:

$$\begin{aligned} (1 - \bar{\lambda})a &= \int_0^1 (1 - \lambda_n)(p_n + y_n)s_n \omega_n dn + b_1 + \sum_{k \geq 1} \pi_k b'_{k+1} \\ (1 - \bar{\lambda})a' &= \int_0^1 (1 - \lambda_n)p_n s'_n \omega_n dn + \sum_{k \geq 1} \pi_k b'_k, \end{aligned}$$

where  $\pi_k$  and  $b_k$  denote the price and quantity of purchases of zero-coupon bonds that pay the family one (real) dollar for sure in  $k$  period's time.

**Government.** The government collects lump-sum taxes from each market and issues zero-coupon bonds of various maturities subject to the period budget constraint:

$$B_1 + \sum_{k \geq 1} \pi_k B_{k+1} \leq \sum_{k \geq 1} \pi_k B'_k + \int_0^1 t_m \omega_m dm, \quad (30)$$

where  $B_k$  denotes the government's issue of  $k$ -period bonds. The lump-sum taxes are designed to not redistribute resources across markets. This is achieved by setting

$$t_m = \frac{1 - \lambda_m}{1 - \bar{\lambda}} \left( B_1 + \sum_{k \geq 1} \pi_k B_{k+1} - \sum_{k \geq 1} \pi_k B'_k \right). \quad (31)$$

**Optimization.** The Bellman equation for the family is now

$$v(s, b, y) = \max_{c, s', b'} \left\{ \int_0^1 u(c_m) \omega_m dm + \beta \mathbb{E}[v(s', b', y') | y] \right\}. \quad (32)$$

where the maximization is taken subject to the collection of budget constraints and the accounting identities for the family portfolio.

**Equilibrium allocations.** Market clearing requires  $s'_m = S_m$  for each  $m$  and  $b'_k = B'_k$  for each  $k$ . We plug these conditions in the market-specific budget constraints and then use the government budget constraint combined with the expressions for lump-sum taxes that do not redistribute resources across markets, as in (31). After canceling common terms we get:

$$c_m = \lambda_m y_m S_m + (1 - \lambda_m) \int_0^1 \frac{1 - \lambda_n}{1 - \bar{\lambda}} y_n S_n \omega_n dn.$$

**First-order condition and asset pricing.** Let  $\mu_m \geq 0$  denote the multiplier on the budget constraint for market  $m$  and use the market-specific budget constraints and accounting identities for the family portfolio to write the Lagrangian:

$$\begin{aligned} \mathcal{L} &= \int_0^1 u(c_m) \omega_m dm + \beta \mathbb{E}[v(s', b', y') | y] \\ &+ \int_0^1 \mu_m \left[ \lambda_m (p_m + y_m) s_m + \frac{1 - \lambda_m}{1 - \bar{\lambda}} \left( \int_0^1 (1 - \lambda_n) (p_n + y_n) s_n \omega_n dn + b_1 + \sum_{k \geq 1} \pi_k b_{k+1} \right) \right] \omega_m dm \\ &- \int_0^1 \mu_m \left[ c_m + \lambda_m p_m s'_m + \frac{1 - \lambda_m}{1 - \bar{\lambda}} \left( \int_0^1 (1 - \lambda_n) p_n s'_n \omega_n dn + \sum_{k \geq 1} \pi_k b'_k \right) + t_m \right] \omega_m dm. \end{aligned}$$

Now collecting terms and rearranging:

$$\begin{aligned}
\mathcal{L} &= \int_0^1 u(c_m)\omega_m dm + \beta\mathbb{E}[v(s', b', y')|y] \\
&+ \int_0^1 \mu_m \left[ \lambda_m(p_m + y_m)s_m + \frac{1 - \lambda_m}{1 - \bar{\lambda}} \left( b_1 + \sum_{k \geq 1} \pi_k(b_{k+1} - b'_k) \right) - t_m - c_m - \lambda_m p_m s'_m \right] \omega_m dm \\
&+ \int_0^1 \mu_m \frac{1 - \lambda_m}{1 - \bar{\lambda}} \int_0^1 (1 - \lambda_n)[(p_n + y_n)s_n - p_n s'_n] \omega_n \omega_m dn dm.
\end{aligned}$$

Now, in the last term, we permute the roles of the symbols  $m$  and  $n$  and then interchange the order of integration:

$$\begin{aligned}
&\int_0^1 \mu_m \frac{1 - \lambda_m}{1 - \bar{\lambda}} \int_0^1 (1 - \lambda_n)[(p_n + y_n)s_n - p_n s'_n] \omega_n \omega_m dn dm \\
&= \left[ \int_0^1 \mu_n \frac{1 - \lambda_n}{1 - \bar{\lambda}} \omega_n dn \right] \int_0^1 (1 - \lambda_m)[(p_m + y_m)s_m - p_m s'_m] \omega_m dm.
\end{aligned}$$

We now define the weighted average of Lagrange multipliers:

$$q_m = \lambda_m \mu_m + (1 - \lambda_m)q, \quad q := \int_0^1 \frac{1 - \lambda_n}{1 - \bar{\lambda}} \mu_n \omega_n dn,$$

as used in the main text. Substituting for  $q_m$  and  $q$  we get:

$$\begin{aligned}
\mathcal{L} &= \int_0^1 \left[ u(c_m) + q_m(p_m + y_m)s_m - q_m p_m s'_m - \mu_m(c_m + t_m) \right] \omega_m dm \\
&+ q \left( b_1 + \sum_{k \geq 1} \pi_k(b_{k+1} - b'_k) \right) + \beta\mathbb{E}[v(s', b', y')|y],
\end{aligned}$$

Apart from the term reflecting the presence of bonds, this is the same Lagrangian as in the main text. We take derivatives (point-wise) to obtain the first order necessary conditions reported in the main text.

**Portfolio weights and returns.** To streamline the exposition we return to the model used in the main text. The total value of the family portfolio is:

$$\int_0^1 \frac{1 - \lambda_m}{1 - \bar{\lambda}} p_m s'_m dm.$$

Thus, in the family portfolio, asset  $m$  is represented with a weight:

$$\psi_m := \frac{\frac{1-\lambda_m}{1-\lambda} p_m s'_m}{\int_0^1 \frac{1-\lambda_n}{1-\lambda} p_n s'_n dn}.$$

Letting  $r'_m = (p'_m + y'_m)/p_m$  be the return on asset  $m$ , the return on the family portfolio can be written:

$$R' = \int_0^1 r'_m \psi_m dm.$$

Now recall that trader  $m$  holds  $\lambda_m p_m s'_m$  real dollars of asset  $m$ , and the rest of his investment:

$$(1 - \lambda_m) \int_0^1 \frac{1 - \lambda_n}{1 - \lambda} p_n s'_n dn,$$

is in the family portfolio. Thus, the return of trader's  $m$  portfolio can be written:

$$R'_m = \Psi_m r'_m + (1 - \Psi_m) R',$$

where:

$$\Psi_m := \frac{\lambda_m p_m s'_m}{\lambda_m p_m s'_m + (1 - \lambda_m) \int_0^1 \frac{1-\lambda_n}{1-\lambda} p_n s'_n dn},$$

is the portfolio weight in the local asset.

## B Computational details

**Setup.** Let utility be CRRA with coefficient  $\gamma > 0$  so  $u'(c) = c^{-\gamma}$ . Assume markets come in  $M$  different types  $m \in \{1, \dots, M\}$ . Note that this is an abuse of notation given that we previously used  $m$  to index a single market within the  $[0, 1]$  continuum. There is an equal measure of assets,  $1/M$ , in each market type: think of the unit interval being divided in  $M$  equally sized intervals. The total measure of traders in a market of type  $m$  is denoted by  $\omega_m$ . Thus, we have the restriction:

$$\sum_{m=1}^M \omega_m = 1.$$

The supply of asset per trader in a market of type  $m$  is  $S_m$ , so the total supply in that market is  $S_m \omega_m$ . The dividend is  $y_m = y \hat{y}_m$  where  $\mathbb{E}[\hat{y}_m | g, \sigma] = 1$ . Since the aggregate endowment is  $y$ , we have the restriction:

$$\sum_{m=1}^M S_m \omega_m = 1.$$

The segmentation parameter in a market of type  $m$  is  $\lambda_m$  and the supply per trader is  $S_m$ . In equilibrium, consumption in a market of type  $m$  is given by:

$$c_m = y (A_m + B_m \hat{y}_m),$$

where

$$A_m := (1 - \lambda_m) \sum_{n=1}^M \frac{1 - \lambda_n}{1 - \bar{\lambda}} S_n \omega_n,$$

and

$$B_m := \lambda_m S_m.$$

We then have  $q_m = \theta_m y^{-\gamma}$  where:

$$\theta_m = \lambda_m (A_m + B_m \hat{y}_m)^{-\gamma} + (1 - \lambda_m) \sum_{n=1}^M \frac{1 - \lambda_n}{1 - \bar{\lambda}} \mathbb{E}_n [(A_n + B_n \hat{y}_n)^{-\gamma}] \omega_n,$$

where  $\mathbb{E}_n [x]$  is the expectation of  $x$ , conditional on past and current realizations of the aggregate state. By the LLN this expectation calculates the cross-sectional average of  $x$  within type  $n$  markets. We explain below how to compute this expectation. Now let  $\hat{p}_m := p_m/y$  be the price/dividend ratio in a type  $m$  market. This solves:

$$\hat{p}_m = \mathbb{E} \left[ \beta g^{1-\gamma} \frac{\theta'_m}{\theta_m} (\hat{p}'_m + \hat{y}'_m) \right]. \quad (33)$$

**Specification.** The aggregate state is a VAR for log consumption growth and log idiosyncratic volatility:

$$\begin{aligned} \log g_{t+1} &= (1 - \rho) \log \bar{g} + \rho \log g_t + \varepsilon_{g,t+1} \\ \log \sigma_{t+1} &= (1 - \phi) \log \bar{\sigma} + \phi \log \sigma_t - \eta (g_t - \log \bar{g}) + \varepsilon_{v,t+1}, \end{aligned}$$

where  $0 \leq \rho, \phi < 1$  and where the two component of innovation  $\varepsilon_{g,t+1}$  and  $\varepsilon_{v,t+1}$ , are assumed to be contemporaneously uncorrelated. The dividend in market  $m$  is:

$$\log y_{m,t} = \log y_t + \log \hat{y}_{m,t}, \quad (34)$$

where the log idiosyncratic component is conditionally IID normal in the cross section:

$$\begin{aligned} \log \hat{y}_{m,t} &\sim \text{IID across } m \text{ and } N(-\sigma_{mt}^2/2, \sigma_{mt}^2) \\ \sigma_{mt} &= \sigma_t \hat{\sigma}_m, \end{aligned}$$

for some time-invariant market specific volatility level  $\hat{\sigma}_m$ .

**Approximation.** Each market is characterized by 3 states: two aggregate states  $(g, \sigma)$  and one idiosyncratic state  $\hat{y}_m$  (to simplify notation, we omit the ‘log’). Given the specification above, the transition density is of the form:

$$f(g', \sigma', \hat{y}' | g, \sigma, \hat{y}) = f(g', \sigma' | g, \sigma) f(\hat{y}' | \sigma')$$

Our approximation follows [Tauchen and Hussey \(1991\)](#). First, we pick quadrature nodes and weights for the aggregate state: consumption growth,  $Q_g$  and  $W_g$  (column vectors of size  $N_g$ ) and volatility,  $Q_\sigma$  and  $W_\sigma$  (column vectors of size  $N_\sigma$ ). Following the recommendation of [Tauchen and Hussey](#), these nodes and weights are generated according to the transition density evaluated at the mean, i.e., a bivariate Gaussian density  $f(g', \sigma' | \bar{g}, \bar{\sigma})$  which is the product of two independent normal densities with means  $\log \bar{g}$ ,  $\log \bar{\sigma}$ , respectively, and variances  $\sigma_g^2$  and  $\sigma_v^2$ .

Then, for every value of  $\sigma$ , we generate quadrature nodes and weights in each market,  $Q_{\hat{y}|\sigma}^m$  and  $W_{\hat{y}|\sigma}^m$  for the log idiosyncratic state  $\log \hat{y}$ , according to a Gaussian density with mean  $-\sigma_{mt}^2/2$  and variance  $\sigma_{mt}^2$ . The resulting nodes and weights column vectors have length  $N_\sigma \times N_{\hat{y}}$ . We adopt the convention that “idiosyncratic endowment comes first:” that is, in the quadrature node vector, idiosyncratic endowment  $i$  under volatility  $j$  is found in entry  $i + N_{\hat{y}}(j - 1)$ .

Combining these together, we have for each market a state space of size  $N_g \times N_\sigma \times N_{\hat{y}}$ . We Kroneckerize the weight and nodes vectors into vectors of length  $N \equiv N_g \times N_\sigma \times N_{\hat{y}}$ , the size of the state space:

$$\begin{aligned} V_g &= Q_g \otimes e_{N_\sigma} \otimes e_{N_{\hat{y}}} \\ V_\sigma &= e_{N_g} \otimes Q_\sigma \otimes e_{N_{\hat{y}}} \\ V_{\hat{y}}^m &= e_{N_g} \otimes Q_{\hat{y}|\sigma}^m, \end{aligned}$$

where  $e_N$  denotes a  $N \times 1$  vector of ones. The order of the Kronecker products follows our convention that the state of idiosyncratic endowment  $i \in \{1 \dots, N_{\hat{y}}\}$ , volatility  $j \in \{1, \dots, N_\sigma\}$ , and aggregate consumption growth  $k \in \{1, \dots, N_g\}$  is found in entry

$$n = i + N_{\hat{y}}(j - 1) + N_{\hat{y}}N_\sigma(k - 1).$$

For instance, entry  $n$  of vector  $V_\sigma$  contains consumption growth if the state of market  $m$  is  $n$ . In other words, idiosyncratic endowment comes first, volatility second, and consumption growth third. To get the quadrature weights, we use the following calculation:

$$\begin{aligned} A &= W_g \otimes e_{N_\sigma} \otimes e_{N_{\hat{y}}} \\ B &= e_{N_g} \otimes W_\sigma \otimes e_{N_{\hat{y}}} \\ C^m &= e_{N_g} \otimes W_{\hat{y}|\sigma}^m, \end{aligned}$$

so that the quadrature weight for the state are:

$$W^m = A .* B .* C^m$$

where  $.*$  denotes MATLAB coordinate-per-coordinate product.

**Transition Probability Matrix.** To implement the method of [Tauchen and Hussey \(1991\)](#), we define a MATLAB function:

$$f^m(s' | s) = f^m(\hat{y}' | \sigma') \times f(\sigma' | \sigma, g) \times f(g' | g),$$

as well as the quadrature weighting function:

$$\omega^m(s) = \omega^m(\hat{y} | \sigma) \times \omega(\sigma) \times \omega(g).$$

Letting  $N \equiv N_{\hat{y}} \times N_{\sigma} \times N_g$ , the matrix formula for the transition matrix is:

$$G = f^m(e_N V'_{\hat{y}} | e_N V'_{\sigma}) .* f(e_N V'_{\sigma} | V_{\sigma} e'_{N_{\sigma}}, V_g e'_{N_g}) .* f(e_N V'_g | V_g e'_{N_g}) \\ .* (e_N * W'). ./ [e_N .* \omega(V'_{\hat{y}} | V'_{\sigma}) .* \omega(V'_{\sigma}) .* \omega(V'_g)],$$

which we then normalize so that the rows sum to 1.

**Calculating cross-sectional moments.** In many instance in the program we need to calculate

$$\mathbb{E}[x_m | g, \sigma],$$

for some random variable  $x_m$ . To do this, we consider:

$$K_{\sigma} = (I_{N_g \times N_{\sigma}} \otimes e'_{N_{\hat{y}}}) [x_m .* W^m],$$

where

$$W^m = e_{N_g} \otimes W^m_{\hat{y} | \sigma}.$$

The coordinate-wise product multiplies each realization of  $x_m$  by its probability conditional on  $(g, \sigma)$ , and the pre-multiplication adds up. We then re-Kroneckerize this in order to obtain a  $N \times 1$  vector:

$$K_{\sigma} \otimes e_{N_{\hat{y}}}.$$

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