Risk Matters:

The Real Effects of Volatility Shocks*

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Abstract

This paper shows how changes in the volatility of the real interest rate at which small open emerging economies borrow have a quantitatively important effect on real variables like output, consumption, investment, and hours worked. To motivate our investigation, we document the strong evidence of time-varying volatility in the real interest rates faced by a sample of four emerging small open economies: Argentina, Ecuador, Venezuela, and Brazil. We postulate a stochastic volatility process for real interest rates using T-bill rates and country spreads and estimate it with the help of the Particle filter and Bayesian methods. Then, we feed the estimated stochastic volatility process for real interest rates in an otherwise standard small open economy business cycle model. We calibrate eight versions of our model to match basic aggregate observations, two versions for each of the four countries in our sample. We find that an increase in real interest rate volatility triggers a fall in output, consumption, investment, and hours worked, and a notable change in the current account of the economy.

Keywords: Small Open Economy, DSGE Models, Stochastic Volatility.

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1. Introduction

This paper shows how changes in the volatility of the real interest rate at which emerging economies borrow have a substantial effect on real variables like output, consumption, investment, and hours worked. These effects appear even when the level of the real interest rate itself remains constant. We argue that, consequently, the time-varying volatility of real interest rates is an important force behind the distinctive size and pattern of business cycle fluctuations of emerging economies.

To prove our case this paper makes two points. First, we document the strong evidence of time-varying volatility in the real interest rates faced by a sample of four emerging small open economies: Argentina, Ecuador, Venezuela, and Brazil. We postulate a stochastic volatility process for real interest rates and estimate it using T-bill rates and country spreads with the help of the Particle filter and Bayesian methods. We uncover large movements in the volatility of real interest rates and a systematic relation of those movements with output, consumption, and investment. Second, we feed the estimated stochastic volatility process for real interest rates in an otherwise standard small open economy business cycle model as in Mendoza (1991) calibrated to match data from our set of countries. We find that an increase in real interest rate volatility triggers a fall in output, consumption, investment, and hours worked, and a notable change in the current account. The effects are more salient for Argentina and Ecuador and milder for Venezuela and Brazil.

We think of our exercise as capturing the following sequence of events. Prior to period t, households live in an environment characterized by the average standard deviation of real interest rates. At time t, the standard deviation of the innovation associated with the country's spread increases by one standard deviation, while the level of the real interest rate itself remains constant. Then, agents optimally adjust their consumption, labor, investment, and savings decisions to face the new level of risk of real interest rates.

The intuition for our result is as follows. Small open economies rely on foreign debt to smooth consumption and to hedge against idiosyncratic productivity shocks. When the volatility of real interest rates rises, debt becomes riskier as the economy becomes exposed to potentially fast fluctuations in the real interest rate and their associated and unpleasant movements in marginal utility. To reduce this exposure, the economy lowers its outstanding debt by cutting consumption. Moreover, since debt is suddenly a worse hedge for the productivity shocks that drive returns to physical capital, investment falls. A lower investment reduces output and, through a fall in the marginal productivity of labor, hours worked.

To strengthen our argument, we perform a battery of robustness checks. First, we highlight that movements in the volatility of real interest rates are highly correlated with variations

in levels. We reestimate our stochastic volatility model while allowing for this correlation and recompute the model with the new processes. Our main conclusion that changes in risk affect real variables remains unchallenged. If anything, our results are reinforced by the correlation of shocks to levels and volatility. Second, we present two extensions of the model: working capital and Uzawa preferences. We find that both extensions amplify the effects of stochastic volatility. Third, we assess the importance of several parameter values for our quantitative conclusions. This check clarifies many of the lessons learned in the main part of the paper. Finally, we explore the consequences of imposing different priors in our estimation exercise. Again, for a wide class of reasonable priors, our results are basically unaltered.

Our investigation begets a number of riveting additional points. First, due to the non-linear nature of stochastic volatility, we apply the Particle filter to evaluate the likelihood function of the process driving the real interest rates (see the description of the Particle filter in Doucet *et al.*, 2001, and, applied to economics, in Fernández-Villaverde and Rubio-Ramírez, 2007 and 2008). By doing so, we introduce a new technique that can have many applications in international finance where non-linearities abound (sudden stops, exchange rate regime switches, large devaluations, etc.)

Second, capturing time-varying volatility creates a computational challenge. Since we are interested in the implications of a volatility increase while keeping the level of the real interest rate constant, we have to consider a third-order Taylor expansion of the solution of the model. In a first-order approximation, stochastic volatility would not even play a role since the policy rules of the representative agent follow a certainty equivalence principle. In the second-order approximation, only the product of the innovations to the level and to the volatility of real interest rates appears in the policy function. Only in the third-order approximation, the innovations to the volatility play a role by themselves.

Third, we document that time-varying volatility moves the ergodic distribution of the endogenous variables of the model away from their deterministic steady state. This is crucial for business cycles analysis and for the empirical implementation of the model. Thus, we calibrate the model according to that ergodic distribution and not, as commonly done, to match steady-state values.

Our paper does not offer a theory of why real interest rate volatility evolves over time. Instead, we model it as an exogenously given process. By doing so, we join an old tradition in macroeconomics, starting with Kydland and Prescott (1982), who took their productivity shocks as exogenous, then to Mendoza (1995), who did the same with his terms of trade shocks, or Neumeyer and Perri (2005), who consider country spread shocks as given. Part of the reason is that an exogenous process for volatility sharply concentrates our attention on the mechanism through which real interest rate risk shapes the trade-offs of agents in small

open economies. More pointedly, the literature has not developed, even at the prototype level, an equilibrium model to endogenize volatility shocks. If we had tried to build such a model in this paper simultaneously with our empirical documentation of volatility and the measurement of its effects, we would lose focus and insight in exchange for a most uncertain reward. In comparison, a thorough understanding of the effects of volatility changes *per se* will be a solid foundation for more elaborated theories of time-dependent variances.[1]

Fortunately, our strategy is justified empirically by the findings of Uribe and Yue (2006) and Longstaff *et al.* (2007). Uribe and Yue estimate a VAR with panel data from emerging economies to investigate how much of the country spreads are driven by domestic factors and how much by international conditions. They conclude that at least two thirds of the movements in country spreads are explained by innovations that are exogenous to domestic conditions. Longstaff *et al.* look at CDs spreads for 26 open economies sovereign debt and document that country-spreads are driven much more by global financial market variables and global risk premia than by local forces. The evidence in these two papers is strongly supportive of the view that a substantial component of changes in volatility is exogenous to the country.

Uribe and Yue's result should not be a surprise because the aim of the literature on financial contagion is to understand phenomena that distinctively look like exogenous shocks to small open economies (Kaminsky *et al.*, 2003). For instance, after Russia defaulted on its sovereign debt in the summer of 1998, Argentina, Brazil, or Hong Kong (countries that have little if anything in common with Russia or Russian fundamentals besides appearing in the same table in the back pages of *The Economist* as an emerging market) suffered a significant increase in the volatility of the real interest rates at which they borrowed. At a first pass, thinking about those volatility spikes as exogenous events and tracing their consequences within the framework of a standard business cycle model seems empirically plausible and worthwhile.

Our paper is linked with three literatures. First, our worked is related with the literature on time-varying volatility in finance and macroeconomics. While the effects of time-varying volatility have been widely studied in finance (Shephard, 2008), the issue has been nearly neglected in macroeconomics. Justiniano and Primiceri (2007) and Fernández-Villaverde and Rubio-Ramírez (2007) estimate dynamic equilibrium models where heteroskedastic shocks drive the dynamics of the economy to account for the "Great Moderation" that has characterized the last twenty years in the U.S. economy (Stock and Watson, 2002). Both papers

[1] We have the additional obstacle of data limitations on real aggregate variables. For the countries in our data set, it is even difficult to compute the evolution of TFP. Since we have to use high-frequency data for volatility, the problem becomes more acute.

conclude that time-varying volatility helps to explain the reduction observed in the standard deviation of output growth and other macroeconomics variables. However, these papers also show that for the U.S. economy, stochastic volatility mainly affects the second moments of the variables with little effect on their first moments. In comparison, Bloom (forth.) exploits firm-level data to estimate a model where a spike in uncertainty affects real variables by freezing hiring and investment. Also, for the U.S., Bloom *et al.* (2008) find that uncertainty (as measured by several proxies) is countercyclical. Then, they show how, in an augmented version of neoclassical growth model, a rise in uncertainty decreases investment and employment. Our paper complements this line of work by offering an alternative mechanism through which time-varying volatility has a first-order impact.[2]

Second, we have many points of contact with the literature that studies the relation between growth and volatility. The empirical evidence suggests that countries with higher volatility have lower growth rates, as documented by Ramey and Ramey (1995) and Fatás (2002). To link our findings with Ramey and Ramey, we could modify our model by introducing mechanisms through which the short-run fluctuations may have long-run impacts. Investment in research and development or irreversible investment are natural candidates for such extensions.

Third, we engage in the discussion of why the business cycles of emerging economies present characteristics that diverge from the pattern of business cycle fluctuations in developed small open economies (Aguiar and Gopinath, 2007, Neumeyer and Perri, 2005, and Uribe and Yue, 2006, among others). Our paper suggests that the higher time-varying volatility of the real interest rate faced by Argentina in comparison, let's say, with Canada is an important source of differences. Stochastic volatility may help explain, for example, why consumption is more volatile than output in emerging economies.

However, we do not postulate time-varying volatility of the real interest rate as a substitute for any of the theories proposed by previous authors. Instead, we see it as a complement, as many of the channels explored by the literature may become stronger in its presence. We document that this is precisely the case for the real interest rate shocks that are the focus of Neumeyer and Perri (2005): real interest rate shock and volatility shocks interact in a non-linear way that exacerbates the effects of both.

The rest of the paper is organized as follows. Section 2 presents our data, the stochastic

[2] From a more reduced-form perspective, several papers have documented the effects of volatity on real variables. Guerrón-Quintana (2009) finds that volatility shocks à la Bloom induce depreciations in the real exchange rate in US particularly vis-a-vis the Canadian dollar. Lee *et al.* (1995) showed that the conditional volatility of oil prices matter for the effect of oil shocks on the economy. Grier and Perry (2000) and Fountas and Karanasos (2007) relate inflation and output volatility with average output growth while Elder (2004) links nominal and real volatility. We thank Jim Hamilton for the last references.

volatility process for real interest rates that we estimate, and the relation of this process to other aggregate variables. Section 3 lays down our benchmark small open economy model and explains how to calibrate and compute it. Section 4 discusses our results and sections 5 to 7 offer extensions and sensitivity analysis. Section 8 concludes.

2. Estimating the Law of Motion for Real Interest Rates

In this section, we estimate the law of motion for the evolution of real interest rates in four emerging economies: Argentina, Brazil, Ecuador, and Venezuela. We select our countries based on data availability and because they represent a relatively coherent set of South American economies. We build the real interest rate faced by each country as the sum of the international risk-free real rate and a country-specific spread. Next, we estimate the law of motion of the international risk free real rate, which is common across countries, and the law of motion of the country spread, one for each economy. Therefore, this section plays a dual role. First, it documents that changes in the volatility of real interest rates are quantitatively significant. Second, it provides us with the processes that we feed, later in the paper, into the calibrated versions of our model.

2.1. Data on Interest Rates

For any given country, we decompose the real interest rate, r_t, it faces on loans denominated in U.S. dollars as the international risk-free real rate plus a country-specific spread. We use the T-bill rate as a measure of the international risk-free nominal interest rate. This is a standard convention in the literature. We build the international risk-free real rate by subtracting expected inflation from the T-bill rate. Following Neumeyer and Perri (2005), we compute expected inflation as the average U.S. CPI inflation in the current month and in the eleven preceding months. This assumption is motivated by the observation that inflation in the U.S. is well approximated by a random walk (Atkeson and Ohanian, 2001).[3] Both the T-bill rate and the inflation series are obtained from the St. Louis Fed's FRED database. We use monthly rather than the more popular quarterly data because monthly data are more appropriate for capturing the volatility of interest rates as required by our investigation. Otherwise, quarterly means would smooth out much of the variation in interest rates.

For data on country spreads, we use the Emerging Markets Bond Index (EMBI) Global

[3] We checked that more sophisticated methods to back up expected inflation, such as the IMA(1,1) process proposed by Stock and Watson (2007), deliver results that are nearly identical. The consequences of using these alternative processes for expected inflation, given the size of the changes in country-spreads, are irrelevant from a quantitative perspective.

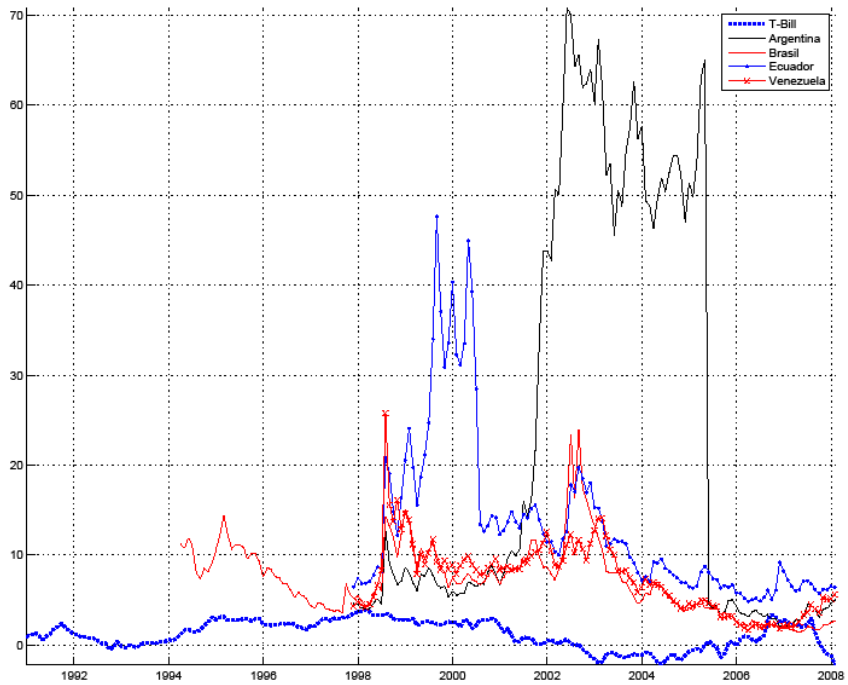

Figure 1: Country Spreads and T-Bill Real Rate

Spread reported by J.P. Morgan at a monthly frequency. This index tracks secondary market prices of actively traded emerging market bonds denominated in U.S. dollars. Neumeyer and Perri (2005) explain in detail the advantages of EMBI data in comparison with the existing alternatives. Unfortunately, except for Brazil, EMBI is available only from 1998. Thus, our sample misses the Tequila crisis and the early stages of the Asian crisis. Yet the sample is large enough to cover the 2000-2001 equity price correction in the U.S. and the Argentinean crisis of 2001-2002. The EMBI data coverage is as follows: Argentina 1997.12-2008.02; Ecuador 1997.12-2008.02; Brazil 1994.04-2008.02; and Venezuela 1997.12-2008.02.

We plot our data in figure 1. We use annualized rates in percentage points to facilitate comparison with the most commonly quoted rates. The international risk free real rate is low (with negative interest rates in 2002-2006) and relatively stable over the sample. In comparison, all country spreads are large and volatile. The spreads are nearly always larger than the real T-bill rate itself and fluctuate, at least, an order of magnitude more. The most prominent case is Argentina, where the 2001-2002 crisis raised the country spreads to 70 percentage points. In the figure, we also see the problems of Ecuador in 1998-1999 and the turbulence in all four countries during the virulent international turmoil of 1998.

2.2. The Law of Motion for Real Interest Rates

We write the real interest rate faced by domestic residents in international markets at time t as $r_t = r + \varepsilon_{tb,t} + \varepsilon_{r,t}$. In this equation, r is the mean of the international risk-free real rate plus the mean of the country-spread. The term $\varepsilon_{tb,t}$ equals the international risk-free real rate subtracted of its mean and $\varepsilon_{r,t}$ equals the country-spread subtracted from its mean. To ease notation, we omit a subindex for the country-specific variables and parameters.

We specify that both $\varepsilon_{tb,t}$ and $\varepsilon_{r,t}$ follow $AR(1)$ processes described by:

$$\varepsilon_{tb,t} = \rho_{tb}\varepsilon_{tb,t-1} + e^{\sigma_{tb,t}} u_{tb,t} \tag{1}$$

and:

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t} \tag{2}$$

where both $u_{r,t}$ and $u_{tb,t}$ are normally distributed shocks with mean zero and unit variance. The main feature of our process is that the standard deviations $\sigma_{tb,t}$ and $\sigma_{r,t}$ are not constant, as commonly assumed, but follow an $AR(1)$ processes:

$$\sigma_{tb,t} = \left(1 - \rho_{\sigma_{tb}}\right)\sigma_{tb} + \rho_{\sigma_{tb}}\sigma_{tb,t-1} + \eta_{tb} u_{\sigma_{tb},t} \tag{3}$$

and

$$\sigma_{r,t} = \left(1 - \rho_{\sigma_r}\right)\sigma_r + \rho_{\sigma_r}\sigma_{r,t-1} + \eta_r u_{\sigma_r,t} \tag{4}$$

where both $u_{\sigma_r,t}$ and $u_{\sigma_{tb},t}$ are normally distributed shocks with mean zero and unit variance. Thus, our process for interest rates displays stochastic volatility. The parameters σ_{tb} and η_{tb} control the degree of mean volatility and stochastic volatility in the international risk free real rate: a high σ_{tb} implies a high mean volatility of the international risk free real rate and a high η_{tb}, a high degree of stochastic volatility. The same can be said about σ_r and η_r and the mean volatility and stochastic volatility in the country spread.

Our specification is parsimonious yet powerful enough to capture some salient peculiarities of the data (Shepard, 2008). Alternative specifications, like estimating realized volatility, are difficult to implement because we do not have intraday data. Also, realized volatility is less useful for us since we need a parametric law of motion for volatility to feed into the equilibrium model of section 3.

Two shocks affect each of the components of the real interest rate: one influencing its level and another its volatility. For instance, the deviation due to the international risk-free real rate, $\varepsilon_{tb,t}$, is hit by $u_{tb,t}$ and $u_{\sigma_{tb},t}$. The first innovation, $u_{tb,t}$, changes the level of the deviation, while the second innovation, $u_{\sigma_{tb},t}$, affects the standard deviation of $u_{tb,t}$. The

shocks $u_{r,t}$ and $u_{\sigma_r,t}$ have a similar reading. We call $u_{tb,t}$ and $u_{r,t}$ shocks to the level of the international risk-free real rate and the country-spread, respectively.[4] We call $u_{\sigma_{tb},t}$ and $u_{\sigma_r,t}$ shocks to the volatility of international risk free real rate and the country spread, respectively. Sometimes, for simplicity, we call this second type of innovation stochastic volatility shocks.

Following the literature, we can interpret a shock to the volatility of real interest rates from at least two different perspectives. First, higher volatility may reflect more risk surrounding the world financial markets. Times generally understood as uncertain, such as the Asian and the Long Term Capital Management (LTCM) crises, are associated with high volatility. A second interpretation builds on the idea that volatility is related to information (Ross, 1989, and Andersen, 1996). During turbulent times, news arrives frequently (or perhaps more attention is devoted to it), inducing high volumes of trade in foreign debt and rising volatility in interest rates.

As our benchmark exercise, we assume that $u_{tb,t}$, $u_{r,t}$, $u_{\sigma_{tb},t}$, and $u_{\sigma_r,t}$, are independent of each other. How strong is this assumption? We checked that $u_{tb,t}$ and $u_{r,t}$ are uncorrelated in our data. This result confirms the findings of Neumeyer and Perri (2005). At the same time, we will report below that 1) the pair $u_{tb,t}$ and $u_{\sigma_{tb},t}$ is strongly correlated and 2) the pair $u_{r,t}$ and $u_{\sigma_r,t}$ is strongly correlated as well. Motivated by this evidence, we will reestimate our stochastic volatility process allowing for correlation. However, we keep the case without correlation as our benchmark because it more neatly separates the effects of the changes to levels from the effects of changes to volatility.

2.3. Estimation

We estimate the parameters of the process in equations (1) to (4) with a likelihood-based approach. The likelihood of these processes is challenging to evaluate because of the presence of two innovations, the innovation to levels and to volatility, that interact in a non-linear way. We address this problem using the Particle filter. This filter is a Sequential Monte Carlo algorithm that allows for the evaluation of the likelihood given some parameter values through resampling simulation methods. The appendix offers further details and references. We follow a Bayesian approach to inference by combining the likelihood function with a prior. In our context, Bayesian inference is convenient because we have short samples that can be complemented with pre-sample information.

[4] Strictly speaking, they are shocks to the deviation of the real interest rate with respect to its mean due to the international risk-free rate and the country-spreads. Hereafter, to facilatate exposition, we omit the word "deviation" where we do not risk ambiguity.

2.3.1. Priors

We now elicit our priors. We start by concentrating on the priors for the parameters driving the law of motion of the country spread deviation. Then, we analyze the priors for the parameters of the process for international risk-free real rate deviations.

Table 1: Priors

	ρ_r	σ_r	ρ_{σ_r}	η_r
Argentina	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-5.30, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$
Brazil	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-6.60, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$
Ecuador	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-5.80, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$
Venezuela	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-6.50, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$
Note: 1) $\mathcal{B}$, $\mathcal{N}$, and $\mathcal{N}^+$ stand for Beta, Normal, and truncated Normal distributions.				
2) Mean and standard deviation in parentheses.				

Table 1 reports our priors for the parameters of the processes corresponding to each of the four countries' spreads. Except for σ_r, we adopt the same prior for all four countries. This facilitates the comparison of the posteriors. For ρ_r and ρ_{σ_r} we choose a Beta prior with mean 0.9 and a moderate standard deviation, 0.02, for ρ_r, and a fairly large one, 0.1 , for ρ_{σ_r}. These priors reflect our view that there is a mild persistence in interest rates (since we have a monthly model, a monthly value of 0.9 is equivalent to a quarterly value of 0.73). The small standard deviation for ρ_r pushes the posterior toward lower values of the parameter. Otherwise, the median of the posterior would become virtually identical to 1, exacerbating the effects of stochastic volatility. Hence, our choice is conservative in the sense that it biases the results *against* our hypothesis that stochastic volatility is quantitatively relevant. The value of 0.1 for the standard deviation for ρ_{σ_r} embodies our relative ignorance regarding the persistence of the shock to volatility.

For η_r, we pick a truncated normal (to ensure that the parameter is positive). The mean of the prior for η_r implies that, on average, the standard deviation of the innovation to the level of the country spread increases by a factor of roughly 1.7 after a positive stochastic volatility shock of one standard deviation ($\exp(0.5)$ =1.6487). This rise is modest compared to the large swings in interest rate volatility displayed in figure 1. For the case of Argentina, the standard deviation of the country spread is 7 times larger in the period $2002-2005$ compared to that $1998-2002$. The standard deviation of 0.3 allows the posterior to move away from the mean of the prior. Last, σ_r is chosen to be a country-specific normal distribution. At the prior mean, the unconditional variance of $\varepsilon_{r,t}$ matches that of the data if we assume no stochastic volatility shocks. The standard deviation of the mean is fixed to be sufficiently high

to give flexibility to the posterior. Thus, our priors capture the observation that Argentina and Ecuador have larger country spread variances than Brazil and Venezuela.

Overall, we view our priors are sufficiently loose to accommodate all countries in our sample. We found that increasing the standard deviation of the priors for σ_r, ρ_{σ_r}, and η_r had no significant impact on our results, while increasing the the standard deviation of the prior for ρ_r favors our case. We further elaborate on the effects of the priors on our quantitative results in section 7.

The priors for the parameters of the law of motion of the international risk free real rate are chosen following an identical approach than for the country specific spreads. Thus, the justifications we provided before for these priors also hold here. We choose Beta priors for ρ_{tb} and $\rho_{\sigma_{tb}}$ with mean 0.9 and standard deviations of 0.02 and 0.1 respectively. For η_{tb}, we picked a truncated normal with mean 0.5 and standard deviation 0.3. Finally, σ_{tb} is such that, at the prior mean, -8, the unconditional variance of $\varepsilon_{tb,t}$ matches the one observed in the data without stochastic volatility shocks. The standard deviation of the prior of σ_{tb} is 0.4, a 5 percent of the mean.

2.3.2. Posterior Estimates

We draw 20,000 times from the posterior of each of the five processes that we estimate (one for the international risk-free real rate and one for each country spread) using a random walk Metropolis-Hastings. The draw was run after an exhaustive search for appropriate initial conditions and an additional 5,000 burn-in draws. We select the scaling matrix of the proposal density to induce the appropriate acceptance ratio of proposals (Roberts *et al.*, 1997). Each evaluation of the likelihood is performed with 2,000 particles. We implemented standard tests of convergence of the simulations, both of the Metropolis-Hastings and of the Particle filter. Given the low dimensionality of the problem, even a relatively short draw like ours converges without further problems.

The sample mean for the real return of the T-bill, our measure of the international risk-free real interest rate, is 0.001, a number that coincides, for example, with Campbell (2003). Table 2 presents the mean of the monthly real interest rate for each country, r. Each of them pays a considerable risk premium, from the 0.007 of Brazil and Venezuela to the 0.02 of Argentina. In annualized terms, the mean differential varies from 840 to 2400 basis points.

Table 2: Mean of Real Interest Rate

	Argentina	Ecuador	Venezuela	Brazil
r	0.020	0.011	0.007	0.007

Table 3 reports the posterior medians of the parameters for the law of motion of the country spread. First, for the case of Argentina and Ecuador (and for Brazil and Venezuela to a lesser degree), the average standard deviation of a shock to the level of country spread, σ_r, is large. This finding reveals a large degree of volatility in the country spread data. Moreover, the posterior is tightly concentrated. Second, for all four countries, there is a substantial presence of stochastic volatility in the country spread series (a large η_r). The shocks to the level and standard deviation of the country spread are highly persistent (large ρ_r and ρ_{σ_r}). The standard deviation of the posteriors of ρ_r is small (the 95 percent probability sets are entirely above 0.9). The standard deviation of the posteriors of ρ_{σ_r} is larger, but even at the 2.5 percentile, the persistence of the process in the range of 0.77 to 0.99.

Table 3: Posterior Medians
(95 percent set in parenthesis)

	Argentina	Ecuador	Venezuela	Brazil		T-Bill
ρ_r	0.97 [0.96,0.98]	0.95 [0.93,0.97]	0.94 [0.91,0.96]	0.95 [0.93,0.96]	ρ_{tb}	0.95 [0.93,0.97]
σ_r	-5.71 [-6.39,-4.89]	-6.06 [-6.73,-5.27]	-6.88 [-7.40,-6.22]	-6.97 [-7.49,-6.19]	σ_{tb}	-8.05 [-8.44,-7.55]
ρ_{σ_r}	0.94 [0.83,0.99]	0.96 [0.87,0.99]	0.91 [0.77,0.98]	0.95 [0.84,0.99]	$\rho_{\sigma_{tb}}$	0.94 [0.76,0.97]
η_r	0.46 [0.33,0.63]	0.35 [0.23,0.52]	0.32 [0.19,0.47]	0.28 [0.18,0.40]	η_{tb}	0.13 [0.04,0.29]

We now examine each country in particular. We start with Argentina, the most volatile country in our sample. The estimated value of σ_r implies that the innovation to the level of the spread has an average annualized standard deviation of 398 basis points ($= 120,000 \exp(\sigma_r)$), where the loading factor of $120,000$ transforms the estimate into annualized basis points. A positive stochastic volatility shock of one standard deviation magnifies the standard deviation of the innovation to the level of the spread by a factor of 1.58 ($= \exp(\eta_r)$). Consequently, a combined positive shock to both the level and volatility would raise Argentina's spread by 629 basis points ($= 120,000 \exp(\sigma_r + \eta_r)$).

Table 4: Argentina before the Corralito
(95 percent set in parenthesis)

	Prior	Median Posterior
ρ_r	$\mathcal{B}(0.9, 0.02)$	0.91 [0.86,0.94]
σ_r	$\mathcal{N}(-5.3, 0.4)$	-5.51 [-6.31,-4.69]
ρ_σ	$\mathcal{B}(0.9, 0.1)$	0.95 [0.84,0.99]
η	$\mathcal{N}(0.5, 0.3)$	0.47 [0.27,0.75]

Our findings for Argentina are not dependent on the effects of the Corralito and the partial default on sovereign debt. In table 4, we re-estimate the process for the spread of Argentina without the data after the onset of the Corralito (December 1st, 2001). The medians of the posteriors for the stochastic volatility parameters, ρ_σ and η_r, are 0.95 and 0.47, nearly the same as 0.94 and 0.46 in the case with Corralito data. Not surprisingly, the variances of the posterior are bigger, since we use many less observations for the estimation. The medians of ρ_r and σ_r change a bit more (the persistence of interest rate shocks falls to 0.91), but they are still quite close to the original ones.

Let us come back to Table 3 and turn to Brazil, the country with less volatility. Its innovation to the level of the spread has an mean standard deviation of 113 annual basis points. Furthermore, a positive volatility shock amplifies the effects of a level shock by a factor of 1.32, indicating that a combined positive shock to both the level and volatility would raise Brazilian's spread by 149 basis points. Ecuador and Venezuela lay in the middle of our sample. Ecuador has an average standard deviation of 280 basis points and a combination of positive shocks increases the spread by 398 basis points. These results put Ecuador in line with Argentina. Venezuela's numbers are closer to Brazil's. It has an average standard deviation of 123 basis points and a combined positive shock increases the interest rate spread by 170 basis points.

In comparison with the country spread, the international risk-free real rate has both lower average standard deviation of the innovation to its level (σ_{tb} is smaller than σ_r for all four countries) and less stochastic volatility (η_{tb} is smaller than η_r for all four countries). The posterior median for σ_{tb} equals -8.05 and for η_{tb} equals 0.13. Thus, the innovation to the level of the international risk-free real rate has an average annualized standard deviation of only 38 basis points, and when combined with a positive shock to volatility, the international risk free real rate increases to 44 basis points. The persistence ρ_{tb}, 0.95, is in line with other estimates in the literature (Neumeyer and Perri, 2005, find a persistence of 0.81 at a quarterly rate). The persistence of the volatility shocks, $\rho_{\sigma_{tb}}$, is also high.

If we compare the volatility of the international risk-free real rate and the volatility of the country spreads, the latter is between 3 to 10 times more volatile than the former and has a time-varying component that is between 2 to 4 times bigger. These relative sizes justify why, in our theoretical model, we concentrate on the study of shocks to the level and volatility of country spreads and forget about shocks to the international risk free real rate.

2.4. Empirical Regularities

We exploit the output from our econometric exercise to document several empirical regularities about business cycles and country spread volatility in our four economies. The objective

is to analyze the correlations between country spreads, output, investment, and consumption with country spread volatility. The challenge is that the country spread volatility, $\sigma_{r,t}$, is not an observable variable but a latent one. However, we can take advantage of our model for country spreads, specified by equations (2) and (4), and the Particle filter to smooth the distribution of country spread volatilities conditional on our whole sample. We report the value of the average smoothed volatility conditional on the median of the posterior of the parameters. Since we use monthly data for interest rates and quarterly data for aggregate variables, we linearly interpolate output, investment, and consumption.

A first exercise is to plot, in figure 2, the time series of output and the smoothed country spread volatility in annualized basis points. The figure indicates a negative correlation between output and country spread volatility. For all four countries, times of high volatility are times of low output. A similar picture would emerge if we printed volatility against consumption or investment.

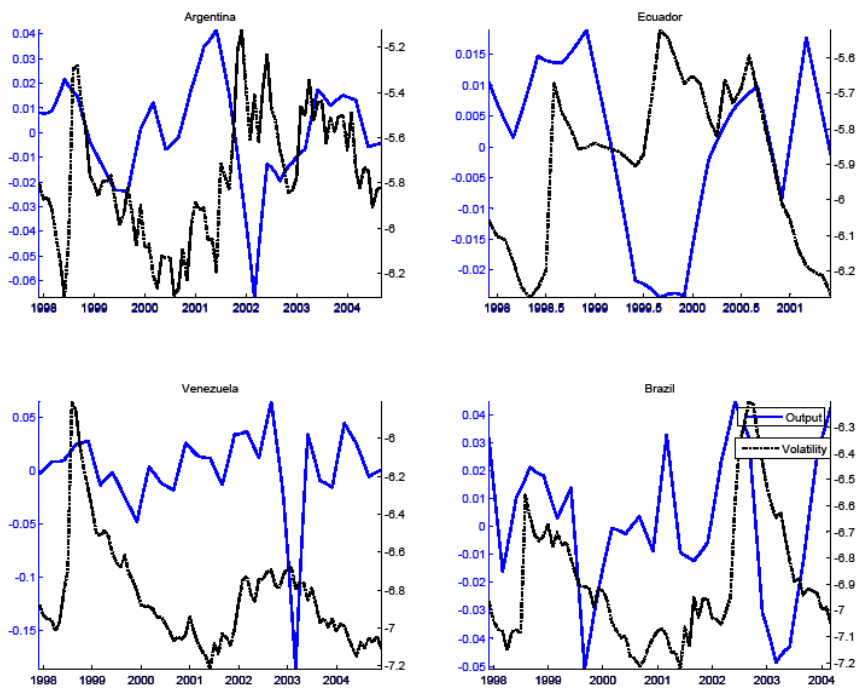

Figure 2: Output and Volatility

An alternative view of this negative correlation is to plot, in figure 3, the cross-correlation between output and country spread volatility at different lags for the countries in our sample. Country spread volatility is countercyclical and leads the cycle by about five months. The contemporaneous correlation coefficients between output and volatility range from around

zero in Brazil to -0.3/-0.4 in Argentina or Ecuador. The average contemporaneous correlation is -0.17. Figure 3 also plots the cross-correlation between investment and country spread volatility and consumption and country spread volatility. As before, country spread volatility leads the cycle with respect to investment and consumption. For the case of consumption, the contemporaneous correlation varies from slightly below zero for Brazil to -0.43 in Ecuador. The average is around -0.2. For the case of investment, the contemporaneous correlation moves from roughly 0 for Brazil to -0.23 in Ecuador.

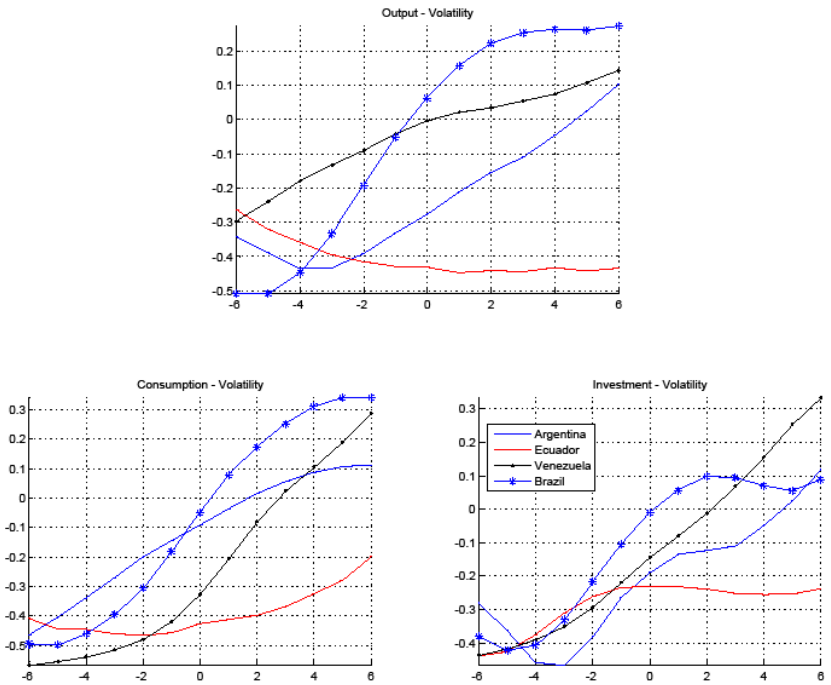

Figure 3: Cross-correlations: Output-Volatility, Consumption-Volatility, Investment-Volatility

Figure 4 plots the time series of country spread and the computed average country spread volatility. Figure 4 reveals a positive comovement between country spread and country spread volatility. Hence, periods of high country spreads are associated with periods of high country spread volatility. This suggests that we need to relax our assumption that the innovation to the level and volatility of the country spread are uncorrelated. We undertake this task in the next subsection. Fortunately, this generalization only strengthens our argument.

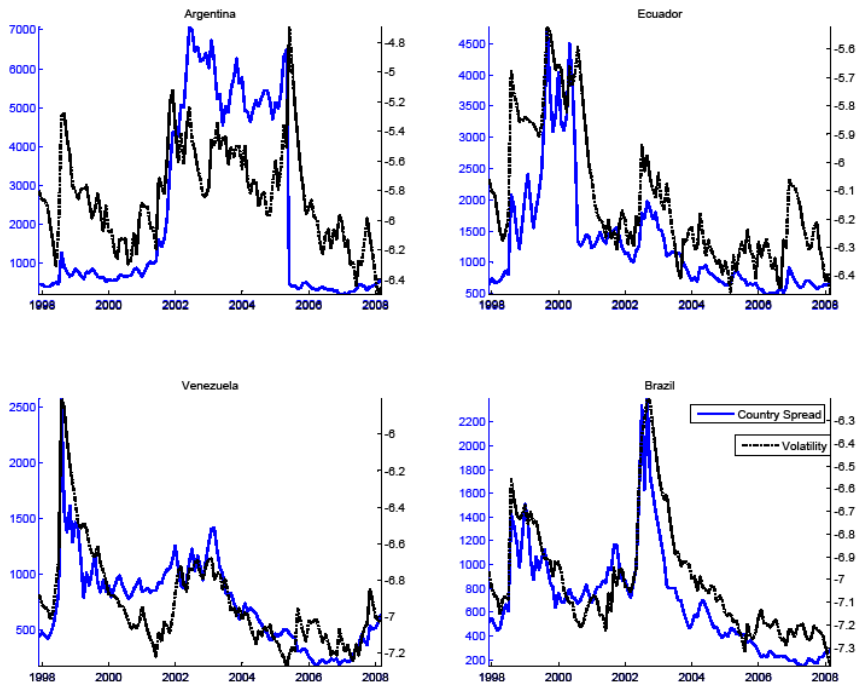

Figure 4: Country Spread and Volatility

2.5. Re-estimating the Processes with Correlation of Shocks

Motivated by the evidence in figure 4, we repeat our estimation assuming that the shocks come from a multivariate normal:

$$\begin{pmatrix} u_{r,t} \\ u_{\sigma_r,t} \end{pmatrix} \sim \mathcal{N} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{matrix} 1 & \kappa \\ \kappa & 1 \end{matrix}$$

In our formulation, κ controls the strength of the correlation and, therefore, the size of the "leverage effect" of level shocks on volatility shocks.[5] We do not correlate the shocks to levels and volatility of the international risk-free real rate, since their empirical size is small and they would not play a quantitatively significant role in the simulation of the model. We impose a uniform prior for κ in $(-1, 1)$ to reflect a roughly neutral stand on the size of the correlation.

[5] In this case, since we are thinking about risk premia and not returns, the "leverage effect" intuition of balance-sheet problems implies that we should expect a positive κ: bad shocks about a country increase both its spread and the volatility risk.

Table 5: Posterior Medians with Correlation
(95 percent set in parenthesis)

	Argentina	Ecuador	Venezuela	Brazil		T-bill
ρ_r	0.97 [0.96,0.98]	0.95 [0.92,0.96]	0.95 [0.93,0.97]	0.96 [0.94,0.97]	ρ_{tb}	0.95 [0.93,0.97]
σ_r	-5.80 [-6.28,-5.28]	-5.93 [-6.32,-5.50]	-6.61 [-7.00,-6.02]	-6.57 [-6.88,-6.26]	σ_{tb}	-8.05 [-8.44,-7.55]
ρ_{σ_r}	0.90 [0.79,0.97]	0.89 [0.83,0.95]	0.92 [0.81,0.96]	0.91 [0.85,0.94]	$\rho_{\sigma_{tb}}$	0.94 [0.76,0.97]
η_r	0.45 [0.28,0.65]	0.34 [0.23,0.48]	0.32 [0.21,0.47]	0.28 [0.22,0.38]	η_{tb}	0.13 [0.04,0.29]
κ	0.69 [0.39,0.89]	0.89 [0.75,96]	0.75 [0.53,0.89]	0.89 [0.76,0.95]		

Table 5 reports our posterior. The median values of the posterior of the parameters ρ_r, σ_r, ρ_{σ_r}, and η_r for each of the four countries are close to our benchmark estimates. Thus, the quantitative patterns of figures 2 to 4 redone with the new process remain virtually identical and we do not include them to save space. The new parameter, κ, is estimated to be highly positive, between 0.69 and 0.89. When we simulate the model, we will see how the clustering of level and volatility shocks reinforces our case because both affect the economy in the same direction and have a significant interaction effect that reinforces each other. By keeping as a benchmark scenario the situation without correlation, we isolate more clearly the direct effects of stochastic volatility. At the same time, for completeness, we will also report the case when the shocks are correlated.

2.6. Summary of Empirical Results

In this section, we have estimated the law of motion for country spreads and international risk-free rates for the four countries in our sample. We have reached four conclusions. First, the average standard deviation of a shock to the level of country spread is large. Second, there is substantial stochastic volatility in the country spread data. Third, international risk-free rates have both less mean volatility and less stochastic volatility than the country spread for any of the four countries. Fourth, country spread volatility is countercyclical and leads the cycle with respect to output, investment, and consumption. Given these findings, we move to use a canonical small open economy model to measure the business cycle implications of the large degree of volatility and stochastic volatility that we find in country spreads.

3. The Model

We formulate a prototypical small open economy with incomplete asset markets in the spirit of Mendoza (1991), Correia *et al.* (1995), Neumeyer and Perri (2005), and Uribe and Yue (2006). The small open economy is populated by a representative household whose preferences are captured by the utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t - \omega^{-1} H_t^{\omega}\right]^{1-v} - 1}{1-v}. \tag{5}$$

Here, $\mathbb{E}_0$ is the conditional expectations operator, C_t denotes consumption, H_t stands for hours worked, and $\beta \in (0,1)$ corresponds to the discount factor.

Our choice of the Greenwood-Hercowitz-Huffman (GHH) preferences follows the finding by Correia *et al.* (1995) that such utility function is better suited to match the second moments of small open economies. The main appealing feature of the GHH preferences is the absence of wealth effects on the labor supply decision. In this way, labor supply depends only on the real wage, and the model, as suggested by the data, is capable of generating a contraction in consumption, labor, and output after a positive shock to the interest rate level.

The real interest rate r_t faced by domestic residents in financial markets follows equations (1) to (4) specified in section 2. This assumption, motivated by our empirical evidence, is the main difference of our model with respect to the standard small open economy business cycle model.

The household can invest in two types of assets: the stock of physical capital, K_t, and an internationally traded bond, D_t. We maintain the convention that positive values of D_t denote debt. Then, the household's budget constraint is given by:

$$\frac{D_{t+1}}{1+r_t} = D_t - W_t H_t - R_t K_t + C_t + I_t + \frac{\Phi_D}{2}\left(D_{t+1} - D\right)^2 \tag{6}$$

where W_t represents the real wage, R_t denotes the real rental rate of capital, I_t is our notation for gross domestic investment, $\Phi_D > 0$ is a parameter that controls the costs of holding a net foreign asset position, and D is a parameter that determines average debt. The cost is paid to some foreign international institution (for example, an investment bank that handles the issuing of bonds for the representative household).

We highlight two points about (6). First, the household has access to a one-period, uncontingent bond. This reflects the extremely limited ability of the countries in our sample to issue debt at long horizons; when they do so, it is only accepted by the market at steep discounts. For a theoretical investigation of why this is so, see Alfaro and Kanczuk (forthcom-

ing) and Broner *et al.* (2007). Consequently, the household will not have the possibility of structuring its debt maturity to minimize the effects of volatility (or, equivalently, the market for volatility contracts on the debt does not exist or it is too small.) Second, we assume that the household faces this cost of holding a net foreign asset position with the purpose of eliminating the unit root otherwise built into the dynamics of the small open economy model. This unit root is inconvenient because it makes it difficult to analyze transient dynamics. Section 7 will quantitatively compare our specification with other ways to close the model.

The stock of capital evolves according to the law of motion:

$$K_{t+1} = (1-\delta)K_t + \left(1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t$$

where δ is the depreciation rate and the process of capital accumulation is subject to adjustment costs. The parameter $\phi > 0$ controls the size of these adjustment costs. The introduction of capital adjustment costs is commonplace in business cycle models of small open economies. They are a convenient and plausible way to avoid excessive investment volatility in response to changes in the real interest rate. The representative household is also subject to the typical no-Ponzi-game condition.

Firms rent capital and labor from households to produce output in a competitive environment according to the technology $Y_t = K_t^{\alpha}\left(e^{X_t}H_t\right)^{1-\alpha}$ where X_t corresponds to a labor-augmenting productivity shock that follows an $AR(1)$ process:

$$X_t = \rho_x X_{t-1} + e^{\sigma_x} u_{x,t} \tag{7}$$

where $u_{x,t}$ is a normally distributed shock with zero mean and variance equal to one.

Firms maximize profits by equating wages and the rental rate of capital to marginal productivities. Thus, we can rewrite equation (6) as:

$$NX_t = Y_t - C_t - I_t = D_t - \frac{D_{t+1}}{1+r_t} + \frac{\Phi_D}{2}\left(D_{t+1} - D\right)^2$$

where NX_t are net exports. Also, we can define the current account as $CA_t = D_t - D_{t+1}$ where the order of the terms is switched from conventional notation because positive values of D_t denote debt. Combining the definitions of net exports and current account:

$$CA_t = (1+r_t)\,NX_t - r_t D_t - (1+r_t)\,\frac{\Phi_D}{2}\left(D_{t+1} - D\right)^2$$

3.1. Equilibrium

A competitive equilibrium can be defined in a standard way as a sequence of allocations and prices such that both the representative household and the firm maximize and markets clear. The set of equilibrium conditions that characterize the time paths for C_t, D_{t+1}, K_{t+1}, H_t, and I_t are given by the first-order conditions for the household and the firm:

$$\left[C_t - \frac{H_t^\omega}{\omega}\right]^{-v} = \lambda_t, \tag{8}$$

$$\frac{\lambda_t}{1+r_t} = \lambda_t \Phi_D \left(D_{t+1} - D\right) + \beta \mathbb{E}_t \lambda_{t+1}, \tag{9}$$

$$-\varphi_t + \beta \mathbb{E}_t \left[\left(1-\delta\right)\varphi_{t+1} + \alpha \frac{Y_{t+1}}{K_{t+1}} \lambda_{t+1}\right] = 0, \tag{10}$$

$$H_t^\omega = \left(1-\alpha\right) Y_t, \tag{11}$$

$$\varphi_t \left[1 - \frac{\phi}{2}\left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)^2 - \frac{\phi I_t}{I_{t-1}}\left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)\right] + \beta \mathbb{E}_t \left[\varphi_{t+1}\phi \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)\right] = \lambda_t \tag{12}$$

together with the resource constraint, the law of motion for capital, the production function, and the stochastic processes for the interest rate. The Lagrangian λ_t is associated with the debt level and the Lagrangian φ_t with physical capital.

The deterministic steady state is given by the solution to the following set of equations:

$$\left[C - \frac{H^\omega}{\omega}\right]^{-v} = \lambda,$$

$$\beta \left[\left(1-\delta\right)\varphi + \alpha \frac{Y}{K}\lambda\right] = \varphi,$$

$$H^{\omega-1}\left[C - \frac{H^\omega}{\omega}\right]^{-v} = \left(1-\alpha\right)\lambda \frac{Y}{H},$$

$$\lambda = \varphi,$$

$$\frac{D}{1+r} = D - Y + C + I,$$

$$Y = K^\alpha H^{1-\alpha},$$

$$I = \delta K.$$

We will calibrate the value of D to ensure that the model generates an ergodic distribution of debt with an average that matches the mean value of net exports to output observed in the data. In addition, r is set at the mean of the country's real interest rate (T-bill plus EMBI). Hence, we have a system of 7 equations for 7 unknowns: C, H, λ, φ, K, I, and Y.

3.2. Solving the Model

We solve the model by relying on perturbation methods to approximate the policy functions of the agents and the laws of motion of exogenous variables around the deterministic steady state defined above. Aruoba *et al.* (2006) report that perturbation methods are highly accurate and deliver a fast solution in a closed economy version of the model considered here.[6]

One of the exercises we are keenly interested in is to measure the effects of a volatility increase (a positive shock to either $u_{\sigma_r,t}$ or $u_{\sigma_{tb},t}$), while keeping the level of the real interest rate unchanged (fixing $u_{r,t} = 0$ and $u_{tb,t} = 0$). Consequently, we need to obtain a *third* approximation of the policy functions. A first-order approximation to the model would miss all of the dynamics induced by volatility because this approximation is certainty equivalent. Thus, the policy functions would exclusively depend on the normally distributed shocks $u_{tb,t}$, $u_{r,t}$, and $u_{X,t}$. Shocks to volatility, $u_{\sigma_r,t}$ and $u_{\sigma_{tb},t}$, do not appear in this approximation (more precisely, the coefficients in front of these variables are equal to zero). A second order approximation would only capture the volatility effect indirectly via cross product terms of the form $u_{r,t}u_{\sigma_r,t}$ and $u_{tb,t}u_{\sigma_{tb},t}$, that is, through the joint interaction of both shocks. Thus, up to second order, volatility does not have an effects as long as the real interest rate does not change. It is only in a third-order approximation that the stochastic volatility shocks, $u_{\sigma,t}$ and $u_{\sigma_{tb},t}$, enter as independent arguments in the policy functions with a coefficient different from zero. Hence, if we want to explore the direct role of volatility, we need to consider cubic terms. Furthermore, given the estimated stochastic volatility processes, the cubic terms in the policy functions are quantitatively significant. This is one of the most relevant findings of our paper. In the appendix, we show how the simulation paths of the model are affected by these higher order terms.

Also, the third-order approximation and our estimated stochastic processes move the mean of the ergodic distributions of the endogenous variables of the model away from their deterministic steady-state values. Thus, our calibration must target the moments of interest generated by the ergodic distributions and not the moments of the deterministic steady state, since those last ones are not representative of the stochastic dynamics.

There are two possible objections to our perturbation solution: first, whether approximating the policy function around the steady state is the best choice; second, whether a third-order solution is accurate enough. The first objection can be dealt with by observing that 1) the approximation around the steady state is the asymptotically valid one (something

[6] Value function iteration or projection methods are too slow to run with the required level of accuracy (we have 8 state variables). Moreover, as we will see momentarily, the calibration of the model requires a fair amount of simulations. A slow solution method would make this task too onerous.

that cannot be said for sure about other approximation points) and that 2) the second order terms include a constant that corrects for precautionary behavior. To answer the second objection, we computed a sixth order approximation to the model. We found that the fourth, fifth, and sixth order terms contributed next to nothing to the dynamics of interest.[7] Once you have the terms on volatility that the third order delivers, fourth and higher order terms have extremely small coefficients. Since the additional terms considerably slowed down the solution and limited our ability to simulate and explore the model (in the sixth order we have 1,899,240 terms to compute), we stopped at the third order.

The states of the model are $States_t = \left(\widehat{K}_t, \widehat{I}_{t-1}, \widehat{D}_t, X_{t-1}, \varepsilon_{r,t-1}, \varepsilon_{tb,t-1}, \sigma_{r,t-1}, \sigma_{tb,t-1}, \Lambda\right)'$ and the exogenous shocks are $\xi_t = (u_{X,t}, u_{r,t}, u_{tb,t}, u_{\sigma_r,t} u_{\sigma_{tb},t})'$, where $\widehat{K}_t$, $\widehat{I}_{t-1}$, and $\widehat{D}_t$ are deviations of the logs of K_t and I_{t-1}, and the level of D_t with respect to the log of K and I and the level of D (we do not take logs of D because it may be negative). Also, Λ is the perturbation parameter.

We take a perturbation solution around $\Lambda = 0$, that is, around the steady state implied when all the variances of the shocks are equal to zero. Since the optimal decision rules depend on the states and the exogenous shocks, we define $s_t = (States_t', \xi_t')'$ as the vector of arguments of the policy function. Also, we call s_t^i to the $i-th$ entry of s_t and ns to the cardinality of s_t. Thus, we can write the third-order approximation to the laws of motion of the endogenous states. First, we have a law of motion for capital:

$$\widehat{K}_{t+1} = \psi_i^K s_t^i + \frac{1}{2}\psi_{i,j}^K s_t^i s_t^j + \frac{1}{6}\psi_{i,j,l}^K s_t^i s_t^j s_t^l,$$

where each term $\psi_{i,\ldots}^K$ is a scalar and where we have followed the tensor notation:

$$\begin{aligned} \psi_i^K s_t^i &= \sum_{i=1}^{ns} \psi_i^K s_t^i \\ \psi_{i,j}^K s_t^i s_t^j &= \sum_{i=1}^{ns}\sum_{i=1}^{ns} \psi_{i,j}^K s_t^i s_t^j \end{aligned}$$

and

$$\psi_{i,j,l}^K s_t^i s_t^j s_t^l = \sum_{i=1}^{ns}\sum_{j=1}^{ns}\sum_{l=1}^{ns} \psi_{i,j,l}^K s_t^i s_t^j s_t^l$$

that eliminates the symbol $\sum_{i=1}^{ns}$ when no confusion arises. Similarly, we have a law of motion

[7] We want to be careful here. We found that for our calibration and estimated processes, these higher orders were not important. There might exist parameter values for which these orders are relevant.

of investment and foreign debt:

$$\begin{aligned}\widehat{I}_t &= \psi_i^I s_t^i + \frac{1}{2}\psi_{i,j}^I s_t^i s_t^j + \frac{1}{6}\psi_{i,j,l}^I s_t^i s_t^j s_t^l, \\ \widehat{D}_t &= \psi_i^D s_t^i + \frac{1}{2}\psi_{i,j}^D s_t^i s_t^j + \frac{1}{6}\psi_{i,j,l}^D s_t^i s_t^j s_t^l\end{aligned}$$

Finally, we have the law of motion for the technology shock, (7), the deviation of the real interest rate due to the country spread, (2), the deviation of the real interest rate due to the international risk-free real rate, (1), and the volatilities, (3) and (4). For the case of the law of motion for the deviation of the real interest rate due to the country spread, (2), and the deviation of the real interest rate due to the international risk-free real rate, (1), we also consider third-order approximations instead of their exact form to keep the order of the approximation consistent across equations. Our solution, including calculating all the analytic derivatives, is implemented in `Mathematica`.

3.3. Calibration

We calibrate eight versions of the model, two for each country, one using our benchmark estimates of the law of motion for interest rates (without correlation of the shocks to level and volatility), and one for the alternative estimates (with correlation). Thereafter, we will call the first version of the model, the process without correlation of shocks, M1, and the second version, where we feed in the processes with correlation, M2.[8] Since the estimated processes for the interest rate are monthly, we set one period in our model to be one month and calibrate the parameters accordingly. Below, when we compare the moments of the model with the moments of the data, we aggregate three periods of the model to create a quarter.

We fix the value of the following five parameters in all eight calibrations: 1) the parameter that determines the elasticity of labor to wages, $\omega = 1.6$; 2) the depreciation factor; $\delta = 0.014$; 3) the capital income share, $\alpha = 0.32$; 4) the inverse of the elasticity of intertemporal substitution, $v = 2$; 5) and $\rho_x = 0.95$, the autoregressive of the productivity process. The values for ω, α, and v are those used in Mendoza (1991), Schmitt-Grohé and Uribe (2003), and Aguiar and Gopinath (2007). The depreciation rate is taken from Neumeyer and Perri (2005), who find this high value appropriate for Argentina. The absence of equivalent measures for the other countries forces us to use Argentina's depreciation rate across the eight different versions of our model. The autoregressive process is more difficult to pin down because of the absence of good data on the Solow residual. Following the suggestion of Mendoza (1991), we

[8] Ideally, we would like to estimate the structural parameters of the model. However, the lack of reliable high-frequency data and the non-linear nature of our solution method make such an enterprise infeasible.

select a value slightly lower than the one commonly chosen for rich economies. We checked that our results are robust to this choice by recalibrating and recomputing the model for values of ρ_x as low as 0 without finding much difference in the effects of volatility shocks.

The rest of the parameters differ across each version of the model. First, we set the parameters for the law of motion of the real interest rate equal to the median of the posterior distributions reported in section 2. Second, we set the discount factor equal to the inverse of the gross mean real interest rate of each country $\beta = (1 + r)^{-1}$. Conditional on the previous choices, we pick the last four parameters to match moments of the ergodic distribution of the model with moments of the data. We select four moments in the data: 1) output volatility; 2) the volatility of consumption relative to the volatility of output; 3) the volatility of investment with respect to output; and 4) the ratio of net exports over output. The parameters are 1) σ_x, the standard deviation of productivity shocks; 2) ϕ, the adjustment cost of capital; 3) D, the parameter that controls average value debt; and 4) the holding cost of debt, Φ_D.

If we were using the steady state to calibrate the model, we could pick each parameter to match almost independently each of the four moments of interest in the data (for example, σ_x would nail down output volatility and D would determine the ratio of net exports over output). In the ergodic distribution, in contrast, the moments are all affected by a non-linear combination of the parameters. Hence, moving one parameter to improve, say, the fit of volatility of consumption relative to the volatility of output may worsen the fit of the volatility of investment with respect to output. We fix this problem by minimizing a quadratic form of the distance of the moments of the model with those of the moments of the data. In addition, to discipline the exercise further, we pick only two levels of Φ_D, one for the two most volatile countries, Argentina and Ecuador, and another for Venezuela and Brazil that is 50 percent of the first value. Our choices for Φ_D are consistent with the values reported in Uribe and Yue (2006). Their small value helps to close the model without significantly affecting its dynamic properties.

The four empirical moments to be matched are reported in table 6 and they are based on H-P filtered quarterly data from the sources described in section 2. The row nx/y displays the average of net exports as a percentage point of output. A positive value means that the country is running a trade surplus.

Table 6: Empirical Second Moments

	Argentina	Ecuador	Venezuela	Brazil
σ_y	4.80	2.50	4.72	4.79
σ_c/σ_y	1.30	2.50	0.87	1.10
σ_i/σ_y	3.80	9.32	3.42	1.65
nx/y	1.80	3.90	4.00	0.10

To compute the moments of the ergodic distribution generated by our model, we proceed as follows. First, we simulate the model, starting from the steady state, for 2096 periods. We disregard the first 2000 periods as a burn-in and use the last 96 periods, which correspond to 8 years in our data, to compute the moments of the ergodic distribution.[9] Since our data come in quarterly frequency, we transform the model-simulated variables from a monthly to a quarterly basis and we H-P filter them. We repeat this exercise 200 times to obtain the mean of the moments over the 200 simulations. We checked the stability of our simulations. The country-specific results of our calibration are summarized in table 7.

Table 7: Summary Calibration

	Argentina		Ecuador		Venezuela		Brazil	
	M1	M2	M1	M2	M1	M2	M1	M2
β	0.980	0.980	0.989	0.989	0.993	0.993	0.993	0.993
Φ_D	$1.4e-4$	$1.4e-4$	$1.4e-4$	$1.4e-4$	$7e-5$	$7e-5$	$7e-5$	$7e-5$
D	27	24	53	50	75	75	10	10
ϕ	280	240	60	55	20	35	65	150
σ_x	0.0075	0.0072	0.0014	0.001	0.007	0.0075	0.0075	0.0075

Our values for D roughly align with the ratio of net exports to output (a higher ratio signaling a higher foreign debt). Higher values for Φ_D mainly reflect higher volatility of consumption. Higher volatility of output appears in higher values of σ_x. The values of ϕ are more difficult to interpret.

[9] We follow Kim *et al.*'s (2003) pruning approach to get rid of spurious higher order terms in our simulations. Furthermore, we rule out volatility shocks larger than two standard deviations because of convergence issues (technically, the convergence results of perturbation depend on the shocks to the model being bounded).

4. Results

In this section, we analyze the quantitative implications of our model. First, we report the moments generated by the model and compare them with the data. Second, we look at the impulse response functions (IRFs) of shocks to the level and volatility of country spreads. Third, we decompose the variance of aggregate variables among different shocks.

4.1. Moments

Our first exercise is to compute the model-based moments with those of the data. For each country, table 8 reports the results for both versions of the model (M1 and M2) and the data moments. For both calibrations, the model does a fair job at matching the moments of the data. Even if we have used four of the moments for calibration, the relative success of the model is no small accomplishment, as small open economy models often have a tough time reproducing the moments in the data for any combination of parameter values. We found it challenging to match simultaneously the volatility of consumption over the volatility of output and the ratio of net exports-to-output, in particular for the Argentinean calibration.

Table 8: Second Moments

	Argentina			Ecuador			Venezuela			Brazil		
	Data	M1	M2	Data	M1	M2	Data	M1	M2	Data	M1	M2
σ_y	4.80	4.81	4.67	2.50	2.20	2.51	4.72	4.52	4.86	4.79	4.52	4.44
σ_c/σ_y	1.30	1.94	1.65	2.50	2.34	2.04	0.87	0.84	0.95	1.10	0.77	0.88
σ_i/σ_y	3.80	3.30	3.31	9.32	9.46	9.27	3.42	3.58	3.86	1.65	1.63	1.62
σ_{nx}/σ_y	0.39	0.27	0.16	0.65	0.22	0.19	0.19	0.12	0.11	0.22	0.21	0.13
nx/y	1.80	1.56	2.00	3.90	3.84	3.48	4.00	3.16	3.04	0.1	0.39	0.31

We highlight two results from table 8. First, the model roughly accounts for the relative volatility of net exports over output, although it tends to underestimate it. This finding is relevant because this is a moment that we did not use in the calibration and that small open economy models have difficulties matching. Second, it is interesting that the moments with and without correlation of shocks are quite similar.

4.2. Impulse Responses

Our second exercise looks at the IRFs of the model to shocks in the level and volatility of country spreads. Computing these IRFs in a non-linear environment is somewhat involved since the IRFs are not invariant to re-scaling and to the previous history of shocks. We refer the reader to the appendix for details on how we construct them.

4.2.1. Argentina

We start by analyzing the effects of shocks in Argentina. The graphs for the other three countries will follow the same format in the order of presentation. In figure 5 we plot the IRFs to three shocks (rows) of consumption (first column of panels), investment (second column), output (third column), labor (fourth column), the interest rate (fifth column), and debt (the sixth column). Interest rates are expressed in basis points while all other variables are expressed as percentage deviations from the mean of their ergodic distributions.

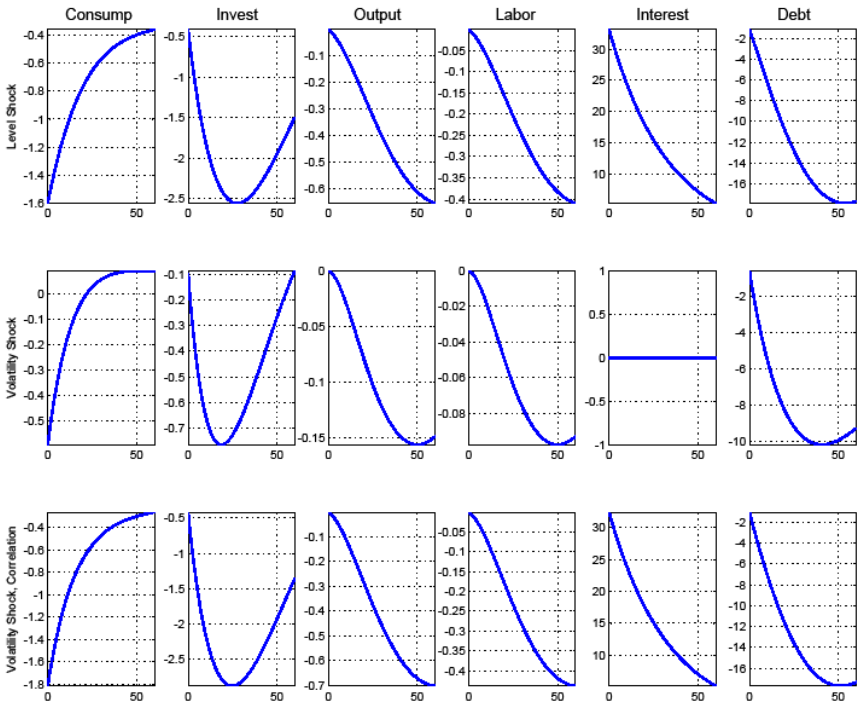

Figure 5: IRFs Argentina

The first row of panels plot the IRFs to a one standard deviation shock to the level of the Argentinean country spread, $u_{r,t}$ in the M1 version of the model. Following a 33 basis point rise in the level of Argentina's monthly spread, the country experiences a persistent contraction, with consumption dropping 1.60 percent upon impact and investment falling for two and a half years. To match the second moments found in the Argentinean data, our model requires a significant degree of adjustment costs in investment. Consequently, we find that the decline in output is highly persistent. Only after 66 months does output reach its lowest level (-0.67 percent). Labor mimics the dynamics of output, which results from our reliance on GHH preferences. Debt falls for five years, with a total reduction of 18 percent of the original value of the liability.

The intuition for the drop in output, consumption, and investment is well understood (see Neumeyer and Perri, 2005). A higher r_t raises the service payment of the debt, reduces consumption, forces a decrease in the level of debt (since now it is more costly to finance it), and lowers investment through a non-arbitrage condition between the returns to physical capital and to foreign assets. We include this exercise to show that our model delivers the same answers as the standard model when hit by equivalent level shocks and to place in context the size of the IRFs to volatility shocks.

The contraction in economic activity may seem large relative to those found in the literature. Uribe and Yue (2006), for instance, estimate that a 1 percentage point rise in the country spread reduces output by 0.15 percent and investment by 0.5 percent. However, we must keep in mind that our time frame is a month, which implies that the interest rate in fact rises by 4.1 percentage points on an annual basis. When we normalize the spread shock so that the interest rate increases by 8.3 basis points upon impact (or 1 percentage point in a yearly basis), consumption falls by 0.41 percent while output and investment contracts by 0.16 and 0.64 percent, respectively. These findings are more in line with the empirical estimates reported by Uribe and Yue (2006). Furthermore, Uribe and Yue find that it takes about two years for output to reach its lowest level. Their result raises the question of whether our model may overpredict the persistence of output because of a large investment adjustment cost. We will discuss the effects of smaller adjustment costs in section 7.

The second row of panels plots the IRFs to a one standard deviation shock to the volatility of the Argentinean country spread, $u_{\sigma,t}$. To put a shock of this size in perspective, our econometric estimates of section 2 indicate the collapse of LTCM in 1998 meant a positive volatility shock of 1.5 standard deviation and that the 2001 financial troubles amounted to two repeated shocks of roughly 1 standard deviation.

This second row is one of the main points of our paper. First, note that there is no movement on the level of the domestic interest rate faced by Argentina or its expected value. Second, there is a) a contraction in monthly consumption (0.60 percent at impact), b) a slow decrease of investment (after six quarters it falls 0.76 percent), c) a slow fall in output (after four years, it falls 0.16 percent) and labor, and d) debt shrinks upon impact and keeps declining until it reaches its lowest level (-10.21 percent), roughly four years after the shock. These IRFs show how increments in risk have real effects on the economy even when the level of the real interest rate remains constant.

To understand the economic logic behind this mechanism, we go back to the equilibrium conditions of the model. Our starting point is equation (9), which we can rewrite as:

$$\frac{1}{1+r_t} - \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \Phi_D \left(D_{t+1} - D \right) \tag{13}$$

A volatility shock leaves r_t unchanged but it raises $\mathbb{E}_t\lambda_{t+1}/\lambda_t$, as illustrated in figure 6. Why? The Lagrangian λ_t is the marginal utility of consumption. A higher real interest rate risk causes more volatile consumption in the future. Our estimate for η_r implies that a typical stochastic volatility shock in Argentina raises the standard deviation of a shock to the level of interest rates by a factor of 1.58 ($= \exp(\eta_r)$). Thus, households may face a 52 (1.58*33) basis point surge in the monthly interest rates on their debt obligations if a one standard deviation level shock to interest rate materializes tomorrow. Since marginal utility is convex, Jensen's inequality tells us that $\mathbb{E}_t\lambda_{t+1}$ rises. The total increment of the ratio $\mathbb{E}_t\lambda_{t+1}/\lambda_t$ is smaller because, as we saw in the IRFs, consumption drops at impact and recovers in the following periods, which increases marginal utility today and λ_t. In our calibration, this second effect is dominated by the dispersion of marginal utilities. Hence, the left-hand side of (13) falls and we can make only the equation hold with equality if D_{t+1} falls as well. The intuition is that holding foreign debt is now riskier than before. Hence, the representative household wants to reduce its exposure to this risk.[10]

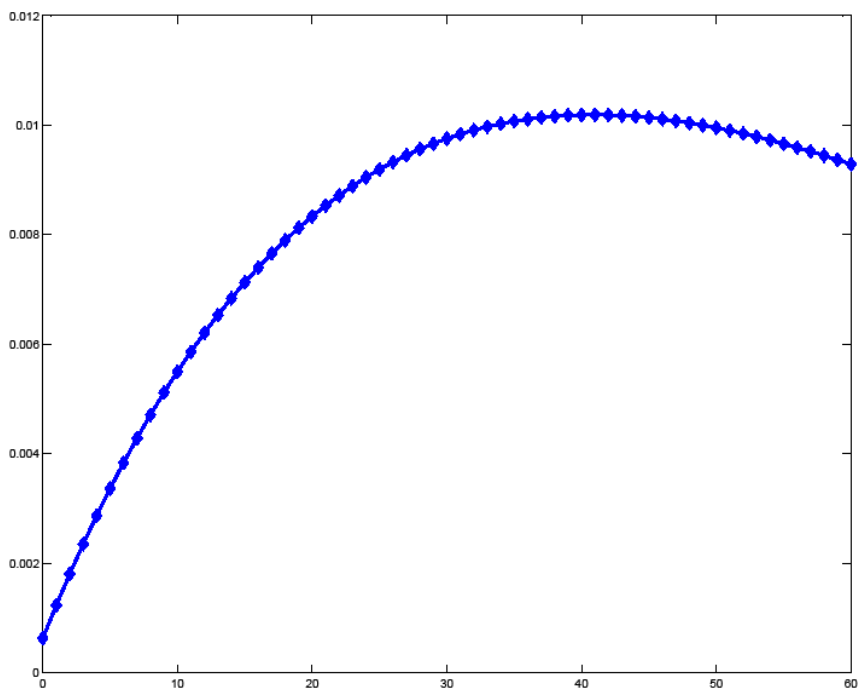

Figure 6: Evolution of $\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}$

[10] This argument is independent of technology shocks. Even with $\sigma_x = 0$, a volatility shock increases the dispersion of future marginal utilities through more dispersed real interest rate levels.

How can the representative household reduce its foreign debt? Since the country is not more productive than before, the only way to do so is to increase net exports by either working more or by reducing national absorption (the sum of consumption and investment). The first alternative, working more, is precluded by our GHH utility function, since these preferences do not have a wealth effect. Hence, the household must reduce national absorption. This can be done in three different ways; 1) consuming and investing less, 2) investing more and consuming sufficiently less that national absorption falls, or 3) consuming more and investing sufficiently less that national absorption falls. Option 3) does not smooth utility over time for standard parameter values (although there are unrealistic combinations of parameter values where they may be the optimal response).[11] Option 2) is eliminated because, as we will show below, investment must fall. Option 1) is, therefore, the only alternative.

To further understand why investment falls, we rewrite the Euler equation as:

$$\beta \mathbb{E}_t \left[\frac{(1-\delta)\, q_{t+1} + R_{t+1}}{q_t} \frac{\lambda_{t+1}}{\lambda_t} \right] = 1.$$

where we have defined the marginal cost of a unit of installed capital K_{t+1} in terms of consumption units as $q_t = \frac{\varphi_t}{\lambda_t}$ and R_t is the rental rate of capital. Then:

$$\beta \mathbb{E}_t \frac{(1-\delta)\, q_{t+1} + R_{t+1}}{q_t} \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} + cov \left(\frac{(1-\delta)\, q_{t+1} + R_{t+1}}{q_t}, \frac{\lambda_{t+1}}{\lambda_t} \right) = 1$$

In this expression, the conditional covariance of the return to capital and the ratio of Lagrangians decreases when volatility rises. Households use debt to smooth productivity shocks. Imagine that we are in a situation with low volatility. Then, after a negative shock to X_t and the subsequent fall in the return to capital, consumption drops by a small amount (and hence the ratio of Lagrangians rises by a small amount) because debt increases to smooth consumption. However, when volatility is high, the household accepts a bigger reduction in consumption after a productivity shock, since increasing the debt level carries a large interest rate risk. At the same time, we just saw that $\mathbb{E}_t \lambda_{t+1} / \lambda_t$ increases only by a small amount because of the interaction of mean-reverting consumption with the increased dispersion of marginal utilities. Therefore, the only term that can change in our previous equation to accommodate the lower covariance is to raise the term $\mathbb{E}_t \left((1-\delta)\, q_{t+1} + R_{t+1} \right) / q_t$. This goal is accomplished with a lower investment today.[12]

[11] In the absence of adjustment costs, investment still falls but consumption increases at impact. However, without adjustment costs, the model does very poorly accounting for the moments of the data.

[12] In comparison with the behavior of $\mathbb{E}_t \lambda_{t+1} / \lambda_t$, the fall of investment requires either a positive standard deviation of the productivity shock and/or adjustment costs. If none of these mechanism is present, the return to capital is risk-free and the covariance is zero.

A slightly different way to understand the fall in investment after a volatility shock is to note that foreign debt allows the household to hedge against the risk of holding physical capital. This hedging property raises the desired level of physical capital. The total effect is, however, small because debt also allows the representative household to rely less on physical capital as a self-insurance device. In calibration M1 for Argentina, the presence of debt increases the average holdings of capital by 1.25 percent in comparison with a closed economy version of our model. A higher volatility of the real interest rate makes the hedge provided by foreign debt less attractive, it induces the household to reduce its level of debt, and, hence, it also lowers its holdings of physical capital with a fall in investment.

To quantify the debt reduction mechanism, we show in figure 7 the evolution of debt, current account, and net exports (all linked with debt through the budget constraint). Debt is expressed as a percentage of monthly output and the bottom two panels are in percentage points of their ergodic means. After a volatility shock, debt falls for a value equal to 7.5 points of monthly output, the current account improves 0.63 percent at impact, and net exports raise to 0.69 percent. This figure suggests that volatility is a potentially substantial factor behind movements in current accounts and net exports in countries like Argentina.

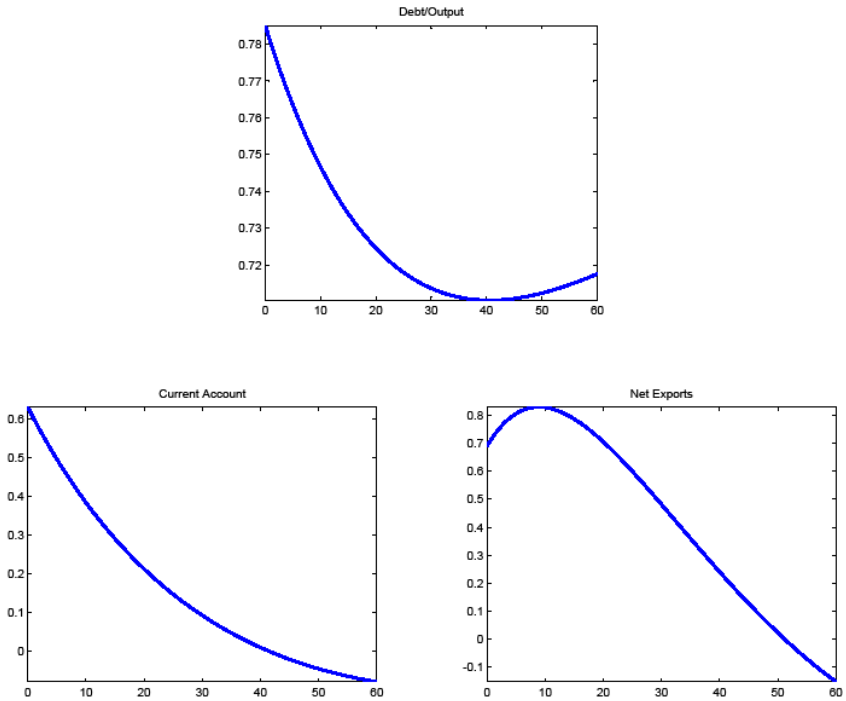

Figure 7: IRFs Debt/Output, Current Account, Net Exports

The last row in figure 5 plots the IRFs in the M2 version of the model where there is correlation in the shocks to the level and volatility of r_t. In this row, we plot the IRFs after a one standard deviation level shock that is accompanied by a $\kappa-$standard deviation shock to volatility. The pattern of the IRFs is qualitatively the same as in the first row. The quantitative size is now bigger as we combine two shocks. The lesson from this third row is that our results are robust to the correlation between shocks to the level and volatility of r_t. If anything, they become larger because of the interaction effects of the two shocks.

4.2.2. Ecuador

Next, we turn to Ecuador, whose IRFs are plotted in figure 8. The IRFs are similar to those in the Argentinian case. There is a decline in economic activity with responses qualitatively similar although somehow smaller than those for Argentina. After a shock to volatility, consumption drops 0.21 percent upon impact, investment 0.11 percent, and debt 0.05 percent. Investment falls for 15 months and output and labor for around three and a half years, when debt also reaches its lowest level, 1.34 percent below its original level. It is perhaps surprising given Ecuador's large debt-to-output ratio (net exports are 3.9 percent of output), that the results, even if still large, are smaller than for Argentina. The key for this finding is that Ecuador enjoys a smaller standard deviation in the innovation to volatility shocks, η_r.

It is interesting, however, to look at the third row of IRFs, when the shocks to the level and to volatility are correlated. While a shock to the level raises the interest rate only by 23.6 basis points, a correlated shock raises it by 33 basis points. This is due to the high estimated correlation of 0.89. After a one standard deviation shock to levels and a 0.89 standard deviation shock to volatility, output takes a dive, falling 0.8 percent after four years. When we evaluate this last row in conjunction with the results of our econometric exercise, we can venture the hypothesis that Ecuador's debacle in the late 1990s started with a sharp volatility shock in 1998 2.5 standard deviations in size.

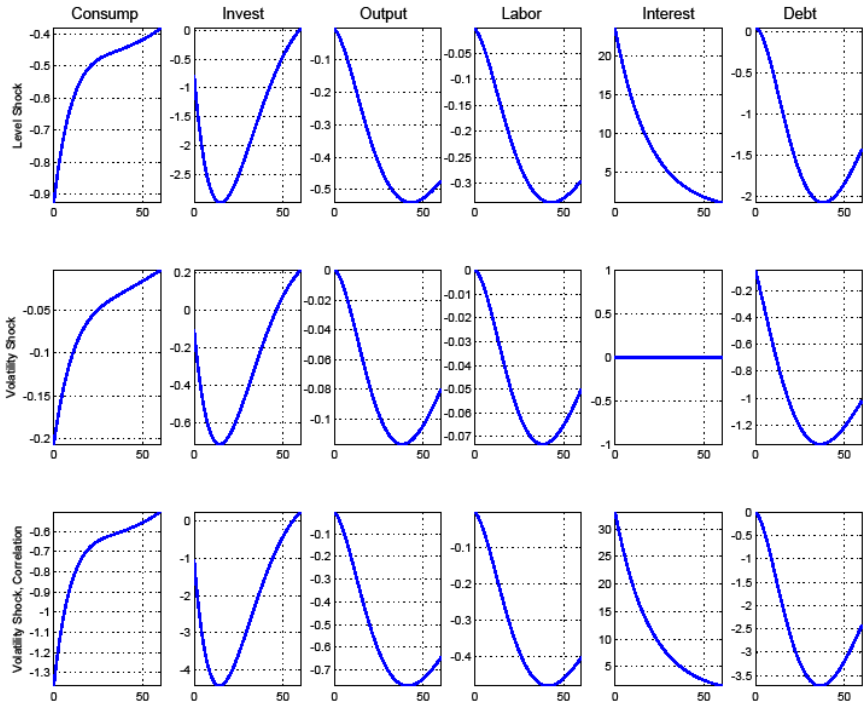

Figure 8: IRFs Ecuador

4.2.3. Venezuela

Our next IRFs are those of Venezuela in figure 9. Although the qualitative shape of the IRFs is similar to the two previous cases, now the response in to a volatility shock is milder. The similar net export-to-output ratios in Ecuador and Venezuela could have made us suspect that these countries should experience equivalent contractions following a volatility shock. Yet a look at figures 8 and 9 reveals that consumption drops 12 times as much in Ecuador as in Venezuela; large indebtedness alone does not generate large recessions. Furthermore, the size of the volatility shock, η_r, is essentially the same for the two countries. What matters in this case are the differences in the average standard deviation of the level shock, σ_r (the posterior median of σ_r for Venezuela is -6.88 while for Ecuador it is -6.06). A higher σ_r increases the mean volatility of the economy and, with it, the size of the IRFs.

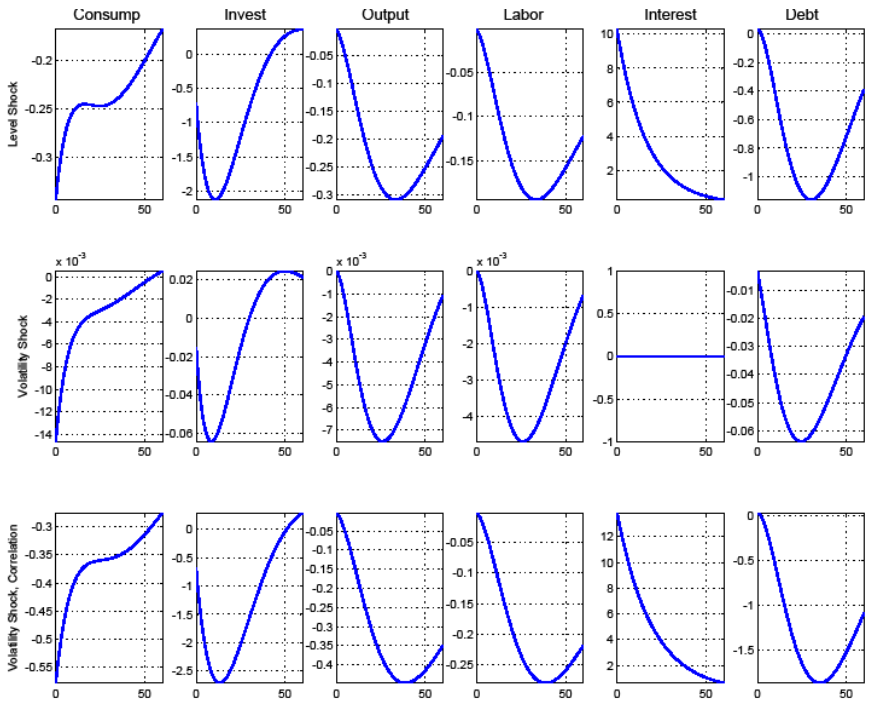

Figure 9: IRFs Venezuela

To better compare the IRFs across countries, we propose the following experiment. At time t, the economy is hit by a one standard deviation volatility shock, which is followed by a shock to the interest rate level, u_r, at time $t+1$. An Ecuadorian household facing this scenario understands that annualized interest rates will increase tomorrow by as much as 4 percentage points. The same sequence of events means that Venezuelans will see an increase in annualized interest rates of 1.7 percentage points.[13] Clearly, Ecuador faces a rather stringent situation, which explains the larger recession in this country.

4.2.4. Brazil

Figure 10 presents Brazil's responses to level and volatility shocks. The main result for Brazil's case is, once more, the similarity of the IRFs to previous findings, although now the response of output is quite muted, even more so than in the case of Venezuela. The stronger response to volatility shocks in Venezuela than in Brazil is accounted for by Venezuela's larger shocks and debt-to-output ratio. This remark further illustrates how the mechanism through which volatility affects real variables is the increased exposure to consumption risk implied by D_t when volatility rises.

[13] As before, to transform into annualized percentage points, we use the loading term $1200\exp(\sigma_r + \rho_{\sigma_r}\eta_r)$.

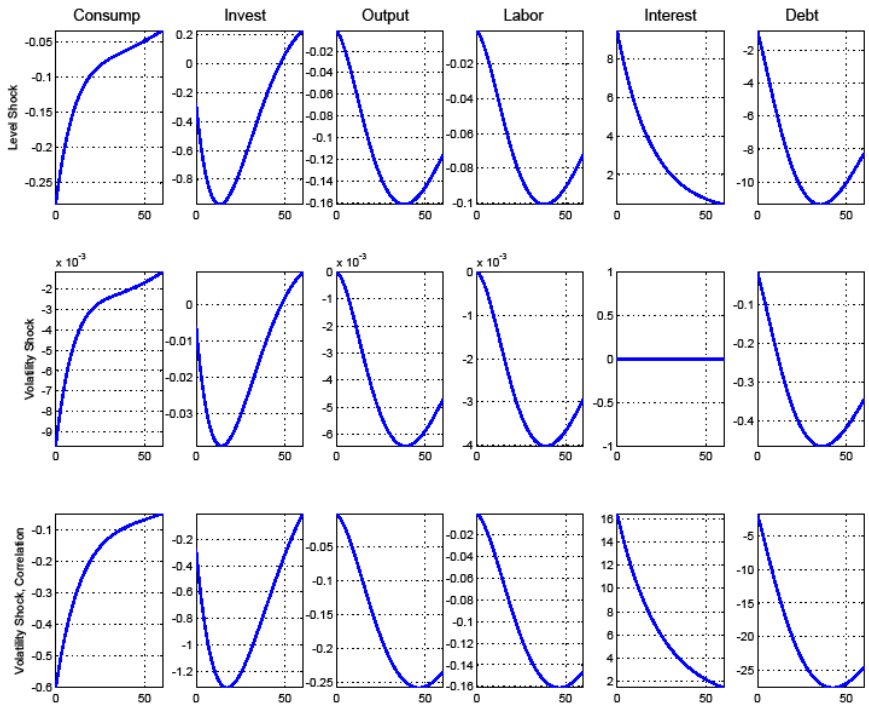

Figure 10: IRFs Brazil

4.3. Variance Decomposition

An additional exercise is to measure the contribution of each of the three shocks in our model to aggregate fluctuations. The task is complicated because, with a third-order approximation to the policy function and its associated non-linear terms, we cannot neatly divide total variance among the three shocks as we would do in the linear case.

A possibility is to set the realizations of one or two of the shocks to zero and measure the volatility of the economy with the remaining shocks. The agents in the model still think that the shocks are distributed by the law of motion that we specified: it just happens that their realizations are zero in the simulation. We explore six possible combinations: 1) the benchmark case with all three shocks, 2) when we have only a shock to productivity, 3) when we have a shock to productivity and to the level of the interest rate (with volatility fixed at its unconditional value), 4) when we have a shock only to the level of the interest rate, 5) when we have shocks to levels and to volatility, 6) when we have shocks only to volatility.

Table 9: Variance Decomposition: Argentina

	All three shocks	Prod.	Prod. and Level	Level	Level and Volatility	Volatility
σ_y	4.80	4.48	4.65	0.97	1.77	0.17
σ_c	8.80	3.89	7.25	6.07	8.13	3.06
σ_i	14.9	1.44	9.18	9.00	15.1	2.57
σ_{nx}	1.12	0.14	2.35	1.10	0.84	0.21

Table 9 reports the results for Argentina. When we allow only productivity to change over time, output has fluctuations that are around 93 percent of the observed ones. Remember that, in the absence of good data on the Solow residual, we are calibrating productivity shocks to match output volatility, and hence this 93 percent is not sensu stricto a measurement of the impact of productivity innovations. A more informative finding is that, counterfactually, the standard deviation of consumption falls below the standard deviation of output. This result is of interest because one of the most salient characteristics of the business cycle of emerging economies is that consumption is more volatile than output. In a model with such a strong desire for consumption smoothing as this one, it is difficult to get around this result when only productivity shocks are considered.

When we add a real interest rate level shock, volatility of output does not increase much and its standard deviation goes up a mere 4 percent, to 4.64. The reason is that, since both shocks are independent, their effects often cancel each other (for instance, a positive technological shock happens at the same time as a rise in the real interest rate). In comparison, the simultaneous presence of both shocks substantially raises the volatility of consumption, which now becomes bigger than output. While the household wants to smooth out productivity shocks, it prefers to pay back the debt and adjust consumption as a response to a positive level shock to the real interest rate. For a similar reason, investment becomes more volatile. These two mechanisms are seen more clearly in the case with only level shocks. While output variability drops to only 0.97, the standard deviation of consumption is still 6.07 and the standard deviation of investment 9.00.

The fourth case is when we have level and volatility shocks. The standard deviation of output rises to 1.77, 37 percent of the observed volatility, consumption goes to 8.13, and investment to 15.1. The final case is when we have only volatility shocks. In this situation, the standard deviation of output is low, 0.17 (after all, volatility per se only appears in the third-order term of the policy function). For output, the interaction effect of the level and volatility shocks is noticeable: jointly they generate a standard deviation of 1.77 while separately they induce standard deviations of 0.97 and 0.17. The difference is accounted for by the cross-terms of level and volatility shocks that appear in the policy function of

the agents. Volatility alone, however, makes a substantial contribution to the fluctuations of consumption (the standard deviation is 3.06 with volatility shocks alone) and investment (standard deviation of 2.57).

For completeness, we include the results of the variance decomposition in the other three countries of our sample. Table 10 reports the results for Ecuador. The main difference with respect to Argentina is that productivity shocks are less important than the level and volatility shocks at accounting for output, consumption, and investment volatility (remember that Ecuador has a low productivity shock variance).

Table 10: Variance Decomposition: Ecuador

	All three shocks	Prod.	Prod. and Level	Level	Level and Volatility	Volatility
σ_y	2.20	0.86	1.50	1.22	2.06	0.22
σ_c	5.07	0.70	4.49	4.44	5.02	0.96
σ_i	20.4	0.72	16.1	16.1	20.4	2.79
σ_{nx}	0.47	0.13	0.55	0.55	0.47	0.43

Table 11 has our findings for Venezuela and table 12 for Brazil, both of which follow patterns very similar to our previous results.

Table 11: Variance Decomposition: Venezuela

	All three shocks	Prod.	Prod. and Level	Level	Level and Volatility	Volatility
σ_y	4.52	4.19	4.28	0.94	1.46	0.04
σ_c	3.77	3.06	3.65	1.95	2.12	0.1
σ_i	15.5	5.08	14.8	13.8	14.9	0.43
σ_{nx}	0.47	0.18	0.60	0.54	0.45	0.31

Table 12: Variance Decomposition: Brazil

	All three shocks	Prod.	Prod. and Level	Level	Level and Volatility	Volatility
σ_y	4.52	4.49	4.43	0.42	0.60	0.10
σ_c	3.46	3.21	3.40	1.33	1.40	0.40
σ_i	7.11	2.95	6.60	6.06	6.61	1.50
σ_{nx}	1.33	1.26	1.60	0.58	0.49	0.70

5. A First Extension: Working Capital

Our benchmark model does not have working capital: firms do not need to borrow to pay their wage bill in advance. We did not add this channel, which Neumeyer and Perri (2005)

have emphasized as a source of fluctuations in emerging economies, to keep the model as simple as possible. However, it is relatively easy to extend the model to capture this feature.

Let Θ be the fraction of the wage bill that must be paid in advance. This means that if the firm borrows funds at the international rate $1 + r_t$, its problem is:

$$\max Y_t - R_t K_t - \Theta (1 + r_t) W_t H_t - (1 - \Theta) W_t H_t$$
$$\text{s.t. } Y_t = K_t^{\alpha} \left(e^{X_t} H_t\right)^{1-\alpha}$$

The optimality conditions of the firm are then:

$$R_t = \alpha \frac{Y_t}{K_t},$$
$$W_t = \frac{(1-\alpha)}{1 + \Theta r_t} \frac{Y_t}{H_t}$$

All the other equilibrium conditions of the model are the same except the static first order condition of the household determining labor supply:

$$\theta H_t^{\omega} = \frac{(1-\alpha)}{1 + \Theta r_t} Y_t,$$

We re-compute the model with working capital for the Argentinean case. We set $\Theta = 1$, that is, the extreme case where all of the wage bill needs to be paid in advance (the opposite case, $\Theta = 0$, gives us back the model of section 3). We picked this value because even with $\Theta = 1$, our main results regarding the importance of volatility shocks are virtually unchanged. We run the simulations under two alternative calibrations. In the first calibration, we keep the same parameter values as in the calibration of section 3, except for $\Theta = 1$. However, this calibration generates excess volatility of output since, as observed by Neumeyer and Perri (2005), working capital increases the impact of real interest rate shocks. To compensate for this effect, we re-calibrate the model to match the same moments as in the benchmark case. In particular, we change three parameters. The cost of holding debt, Φ_D, goes from $1.4e-4$ to $1.3e-4$; the parameter that controls average debt, D, goes from 27 to 30; and the standard deviation of the productivity shock, σ_x, goes from 0.0075 to 0.0072. In terms of moments, the re-calibrated model performs roughly the same as the benchmark case except that consumption becomes a bit too volatile (σ_c/σ_y is 2.5 instead of 2.30).

We plot our results for the case of a volatility shock in figure 11. The differences, as seen in this figure, are quite small. In the first calibration, we hardly observe any change whatsoever. A volatility shock has an effect because it changes the uncertainty tomorrow, while working capital affects the costs of the firm today. Since the problem of the firm is static, the IRFs

have only minor differences because of the differences in the ergodic distribution induced by working capital. Therefore, the IRFs are nearly on top of each other. In the new calibration, differences are a bit bigger because of the three different parameter values. However, the IRFs still provide us with the same fundamental result that our findings do not depend on the presence or absence of working capital; if anything, our results suggest slightly higher effects of volatility shocks, particularly in the case of debt, which goes down 18 percent instead of the 10 percent of the benchmark case.

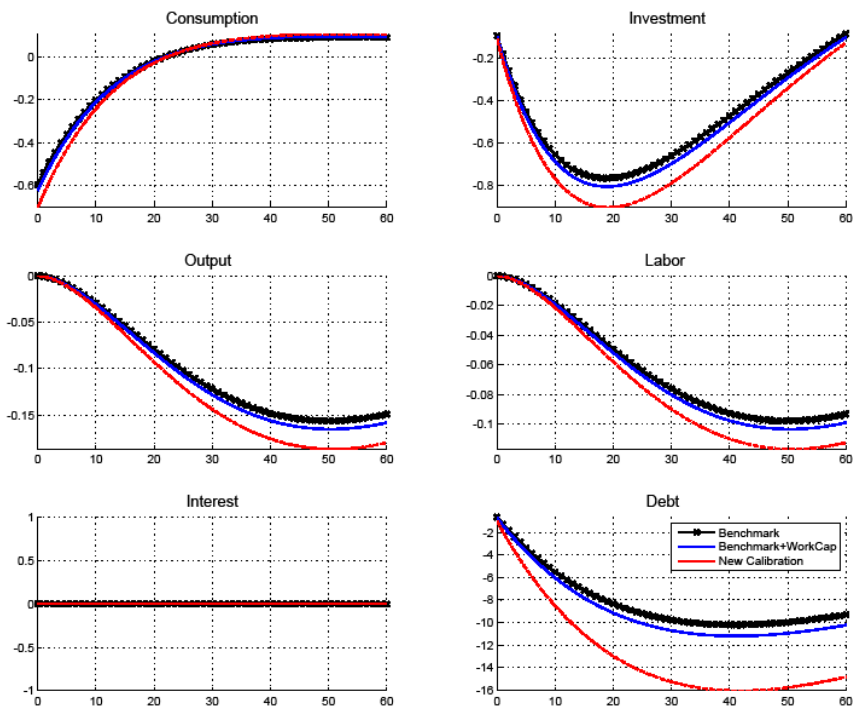

Figure 11: IRFs for Argentina, Three Different Calibrations.

6. A Second Extension: Uzawa Preferences

We closed the open economy component of our model by assuming that there is a quadratic cost to holding debt. However, other alternatives deserve evaluation. Schmitt-Grohé and Uribe (2003) present three of them. One, complete markets, is of less interest to us in this paper because emerging economies do not issue state-contingent debt. The second one, a debt-elastic interest rate such as:

$$r_t = r + \Phi_d \left(e^{D_{t+1}-D} - 1\right) + \varepsilon_{r,t} + \varepsilon_{tb,t} \tag{14}$$

has the problem that the economy's response to a volatility shock would contain an indirect channel through its effect on the level of the interest rate. After a variation in volatility, as we saw in the previous sections, the level of debt changes. Since D_{t+1} appears in (14), any change to volatility would trigger a change in the country spread itself. This effect complicates the interpretation of the experiment. In any case, for empirically plausible values of Φ_d (Schmitt-Grohé and Uribe, 2003, calibrate Φ_d to be 0.000742), the effect of changes of volatility in the level of D_{t+1} trigger a quite small response in the interest rate and this specification ends up giving us nearly the same answers as our benchmark case.

The third, and in our opinion most relevant, alternative is to close the model with Uzawa preferences as proposed by Mendoza (1991). The household preferences are represented by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left[\beta\left(C_t, H_t\right)\right]^t \frac{\left[C_t - \omega^{-1} H_t^{\omega}\right]^{1-v} - 1}{1-v}$$

where the discount factor, $\beta\left(C_t, H_t\right) = \left(1 + C_t - \omega^{-1} H_t^{\omega}\right)^{-\mu}$, is a function of consumption and labor. We assume an external discount factor: households take $\beta\left(C_t, H_t\right)$ as exogenous (Schmitt-Grohé and Uribe, 2003, show that internalization by households of the effects on the discount factor of their consumption and labor supply decisions has absolutely minimum quantitative effects.) The first-order conditions are exactly the same as before except that the β is replaced by $\beta\left(C_t, h_t\right)$.

We re-computed the model with Uzawa preferences. The new version of the model has one new parameter, μ, and it drops two: D and Φ_D. With one less parameter, it is more difficult to hit our original calibration targets, but a value $\mu = 0.01346$ delivers the same steady state as in the benchmark model and only small differences in other moments. The results are plotted in figure 12, where, as in the previous section, we graph both the IRFs of the benchmark model and the IRFs of the model with Uzawa preferences. Uzawa preferences make our results stronger. Consumption falls a bit more at impact but investment falls more steeply (except in the first quarter, when the fall is smaller), around two times more at the bottom, and for more quarters. Lower investment leads to lower marginal productivity of labor, lower labor supply, and lower output. Lower output means that debt is reduced less than in the benchmark case, even if investment also falls more. The reason why investment falls less at impact and more later is that, after a fall in consumption, the discount factor rises. But as consumption recovers and labor goes down, the discount factor falls and the household reduces its holdings of physical capital. Therefore, our choice for closing down the model with debt holding cost is a conservative one, since it makes the effects of volatility smaller.

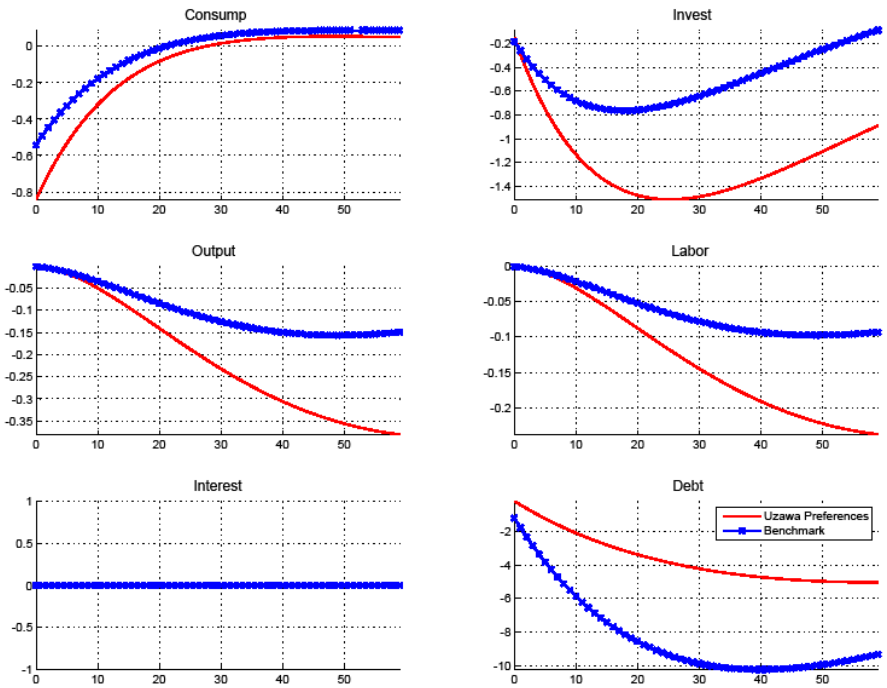

Figure 12: IRFs for Argentina, Uzawa Preferences

7. Robustness Checks

In the interest of space, we consider only robustness analysis for Argentina. However, the lessons that we learn from the Argentinian case about the performance of the model under alternative scenarios are general for all four countries in our sample.

The first, and perhaps the most natural, experiment is to gauge the effects of risk aversion; this parameter controls how strongly the variance of the shocks matters for the policy functions of the agents. In the first row of panels of figure 13, we plot the IRFs of Argentina after a one standard deviation volatility shock when we lower risk aversion, v, from 2 to 1, while keeping the rest of the parameters at their original levels. As the representative household becomes less risk adverse, the ratio $\mathbb{E}_t\lambda_{t+1}/\lambda_t$ increases less than in the benchmark case, while debt, consumption, investment, and output drop less. However, we can still see that output falls up to 0.6 percent and debt close to 4 percent.

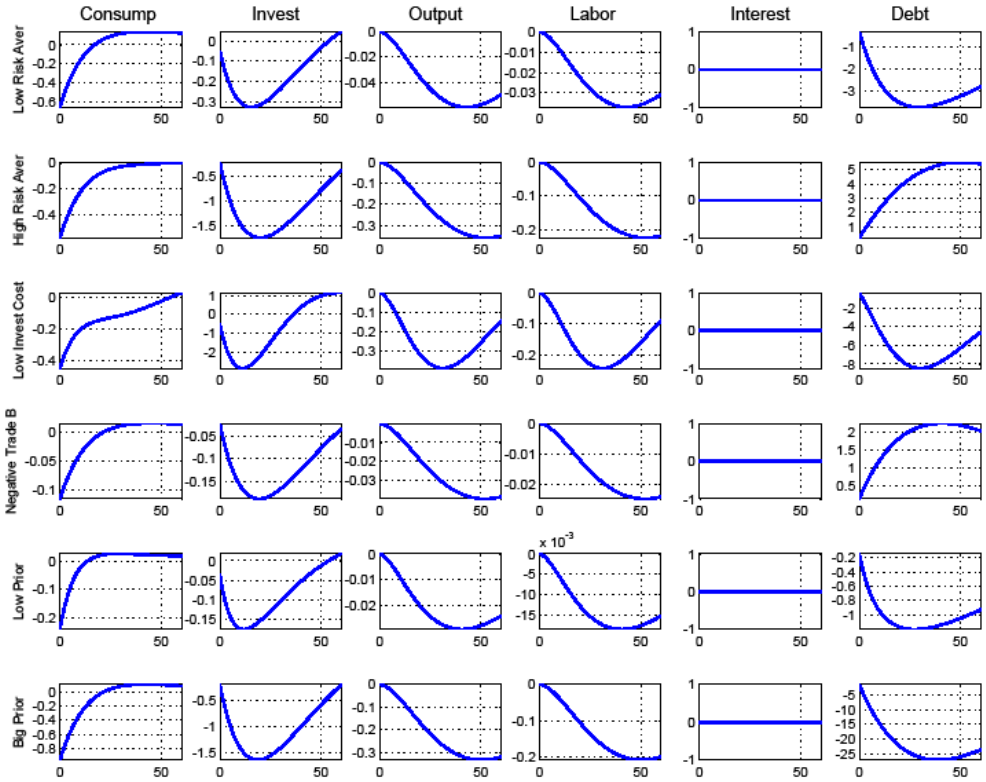

Figure 13: Robustness Checks, Argentina

We can undertake the opposite exercise by raising risk aversion to 5, also keeping the rest of the parameters constant. We report the new IRFs in the second row of panels of figure 13. Inspection of this second row shows that again the qualitative patterns of the IRFs are unchanged. The only point to highlight is that we see the evolution of debt as having the opposite sign than in the benchmark case. This is a product of having defined debt as a positive number. When we set risk aversion to 5, the mean of debt in the ergodic distribution becomes negative (the country holds positive foreign assets on average). Then, as the household wants to reduce its exposure to the increased real interest rate risk induced by a higher volatility, it will unload part of these assets.

Our third robustness experiment is motivated by the observation that, relative to the empirical evidence (Uribe and Yue, 2006), our model predicts a more persistent response of investment following a shock to the interest rate spread. This persistence arises from the large adjustment cost in investment required to match the second moment properties found in the Argentinean data. To understand the consequences of such a cost, we repeat our simulations with an adjustment cost that makes investment's response to a spread shock consistent with

the evidence in Uribe and Yue. The results are reported in the third row of panels in figure 13. We observe that 1) all variables but consumption become more responsive to a stochastic volatility shock and 2) investment reaches its minimum in only one year after the shock. The faster response of investment is a direct implication of the smaller adjustment costs.

The large contraction in economic activity when the adjustment cost is lower is explained as follows. A smaller adjustment cost allows investment to easily drop after a volatility shock. Such a drop has two effects on households. First, it ameliorates the need to reduce consumption in the aftermath of the shock and the household can use the additional proceeds from lower investment to buy back debt. Second, capital will shrink tomorrow, thanks to a smaller investment. Low capital in turn triggers low labor productivity, which reduces the demand for labor and hence households' wealth. This decline in income ultimately exacerbates the contraction in future consumption. In the middle run, this second effect dominates the smaller reduction in consumption at impact, inducing a longer duration in its later drop.

We previously argued that a volatility shock to the interest rate is contractionary because households consume less and save more in anticipation of possibly larger future interest rate shocks. Then, an interesting question is what happens if the country has positive net assets, $D_t < 0$. To answer this question, we repeat the experiment for the Argentinean calibration M1, but we now assume that the economy starts with a net export-to-output ratio of -1.56 (the negative value of what we previously used). We report the results in the fourth row of panels in figure 13. Two features are worth mentioning: 1) a very mild recession follows the volatility shock. Indeed, output and labor barely change. 2) Households de-accumulate assets. The decline in assets results from risk-adverse households who fear that a negative spread shock tomorrow may drive down the return of their foreign asset positions. The very small declines in consumption and investment are caused by the household adjusting to a possibly rather negative shock to their country spread in the next period that will reduce its income from D.[14]

As a final robustness check, we discuss the implications of the priors on our model's predictions. To that end, we re-estimate the processes (2) and (4) for Argentina with two alternative priors and report the results in table 13. For the first option (Case I), we select relatively uninformative priors for ρ_r and ρ_{σ_r} centered in 0.5 while the other parameters' priors remain the same as in the original exercise. For the same reason as in the original prior (to minimize the impact of stochastic volatility), we endow ρ_r with a tighter prior. Under

[14] This experiment begets the question of why the country is getting such a high rate of return on its foreign position. A simple answer is that for every debtor paying a high interest rate, there is a creditor receiving a high interest rate. If we reduced the spread a country gets when $D_t < 0$, the results are qualitatively similar but the IRFs are even smaller than those in the fourth row.

this prior, the posterior ρ_r still concentrates around 1. For the second alternative (Case II), we center ρ_r around its OLS estimates, and the other priors are left as in the baseline setup. Overall, the estimates are again similar to those in table 3.

Table 13: Alternative Priors

	Case I		Case II	
	Prior	Median Posterior	Prior	Median Posterior
ρ_r	$\mathcal{B}(0.5, 0.1)$	0.98 [0.97,0.99]	$\mathcal{B}(0.96, 0.02)$	0.99 [0.98,0.99]
σ_r	$\mathcal{N}(-5.3, 0.4)$	-6.00 [-6.55,-5.25]	$\mathcal{N}(-5.3, 0.4)$	-5.73 [-6.48,-4.86]
ρ_{σ_r}	$\mathcal{B}(0.5, 0.2)$	0.87 [0.72,0.96]	$\mathcal{B}(0.9, 0.1)$	0.94 [0.83,0.99]
η_r	$\mathcal{N}(0.6, 0.3)$	0.52 [0.38,0.72]	$\mathcal{N}(0.6, 0.3)$	0.48 [0.35,0.65]

We present the results from using the new priors in the fifth (case I) and sixth (case II) row of panels in figure 13. For the first alternative, note that the impulse responses are qualitatively similar to those we found under our benchmark formulation. For example, consumption experiences a decline of 0.24 percent while investment contracts by up to 0.18 percent after three years.

More interesting are the results from the second set of priors. Note the strong response of all variables following the volatility shock. The decline in consumption is 0.96 percentage points, quite larger than in our baseline scenario. Similarly, investment's response is more than two times larger than the one we observe in figure 5. The substantially high posterior medians for ρ_r and ρ_{σ_r} explain these results. If a level shock, $u_{r,t}$, follows the volatility shock, interest rates will remain above their pre-shock level for quite a few periods. Thus, households will endure substantially larger payments on their debt obligations. Even if the shock level does not materialize tomorrow, households know that the large persistence of the volatility process means that future level shocks will be almost equally painful. In anticipation of these scenarios, households choose to make large debt repayments today and substantially contract consumption and investment.

8. Summary and Directions for Future Research

Our empirical evidence shows that time-varying volatility is a key feature of the real interest rate faced by emerging economies. This changing volatility has a quantitatively important effect on the dynamics of the economy as measured by an otherwise standard small open economy business cycle model, even when the level of the real interest rate remains constant.

The mechanism behind the real effects of volatility is that households with precautionary behavior will change their holding of foreign debt as a response to changes in volatility to reduce future fluctuations of marginal utility.

Our investigation opens the door to a set of interesting questions. First, and most obviously, why does volatility change over time? Is it related to some states of the economy? How does it interact with other phenomena, such as debt default, debt renegotiation, or financial market integration? Second, we would like to evaluate the possibilities of having time-varying volatilities in other aspects of the economy. For example, in a recent and influential paper, Aguiar and Gopinath (2007) have argued that one factor behind business cycle fluctuations in emerging economies is recurrent changes in the productivity growth trend, possibly caused by policy. It would be profitable to explore the consequences of introducing stochastic volatility in these changes.

9. Appendix

For completeness, this appendix includes the description of NIPA data used in section 2, a brief introduction to the Particle filter, a more detailed discussion of the consequences of using a third-order approximation for the dynamics of the model, and the explanation of how we compute the IRFs of the model.

9.1. NIPA Data

We obtain aggregate data from the International Financial Statistics (IFS) service of the International Monetary Fund, except for Venezuela, for which data come from the Central Bank of Venezuela (the IFS data set has some gaps for Venezuela). The data coverage is: Argentina: 1993.Q1-2004.Q3; Brazil: 1995.Q1-2004.Q1; Ecuador: 1992.Q1-2001.Q2; and Venezuela: 1991.Q1-2004.Q4. Consumption corresponds to household expenditure on goods and services; investment is the sum of gross fixed capital formation and changes in inventories; net exports equals exports of goods and services minus imports of goods and services; finally, output equals the addition of consumption, investment, and net exports. Real variables were obtained by dividing nominal ones by the GDP deflator. All variables were seasonally adjusted using the U.S. Census Bureau's X-12 program. Unless otherwise mentioned, output, consumption, and investment are H-P filtered.

9.2. Particle Filter

We present a brief introduction to the particle filter. We will concentrate on the main idea of the algorithm and skip most of the technical details. Doucet *et al.* (2001) is an excellent reference of the interested reader. Fernández-Villaverde and Rubio-Ramírez (2007 and 2008) are examples of applications in economics.

We want to evaluate the likelihood of the international risk-free real interest rate $\varepsilon_{tb,t}$ and the country spread deviations $\varepsilon_{r,t}$. Since the explanation of the filter for the likelihood of one process or the other is equivalent, we just take the first case.

The likelihood is costly to evaluate because of the non-linear interaction of volatility and levels. Let us start by stacking all T observations of $\varepsilon_{tb,t}$ in ε_{tb}^{T} and the parameters of the process in Ψ. Given the Markov structure of our state space representation, we can factorize the likelihood function as:

$$p\left(\varepsilon_{tb}^{T};\Psi\right)=\prod_{t=1}^{T}p\left(\varepsilon_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)$$

Now, we can derive the factorization:

$$p\left(\varepsilon_{tb}^{T};\Psi\right)=\int p\left(\varepsilon_{tb,1}|\varepsilon_{tb,0},\sigma_{tb,0};\Psi\right)d\sigma_{tb,0}\prod_{t=2}^{T}\int p\left(\varepsilon_{tb,t}|\varepsilon_{tb,t-1},\sigma_{tb,t};\Psi\right)p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)d\sigma_{tb,t}$$

and using equation (2):

$$\begin{aligned} p\left(\varepsilon_{tb}^{T};\Psi\right) &= \int\frac{1}{(2\pi)^{0.5}}\exp\left[-\frac{1}{2}\left(\frac{\varepsilon_{tb,1}-\rho_{tb}\varepsilon_{tb,0}}{e^{\sigma_{tb,0}}}\right)^{2}\right]d\sigma_{tb,0}\,* \\ &\quad *\prod_{t=2}^{T}\int\frac{1}{(2\pi)^{0.5}}\exp\left[-\frac{1}{2}\left(\frac{\varepsilon_{tb,t}-\rho_{tb}\varepsilon_{tb,t-1}}{e^{\sigma_{tb,t}}}\right)^{2}\right]p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)d\sigma_{tb,t} \quad (15) \end{aligned}$$

Consequently, if we had access to the sequence $\left\{p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)\right\}_{t=1}^{T}$, we could compute (15). Unfortunately, this sequence of conditional densities cannot be characterized analytically.

The Particle filter is a sequential Monte Carlo procedure that substitutes the density $p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)$ by an empirical draw from it. In other words, the filter relies on the observation that if we have available a draw of N simulations $\left\{\sigma_{tb,t|t-1}^{i}\right\}_{i=1}^{N}$ from $p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)$, then a Law of Large numbers ensures that:

$$\int p\left(\varepsilon_{tb,t}|\varepsilon_{tb,t-1},\sigma_{tb,t};\Psi\right)p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)d\sigma_{tb,t}\simeq\frac{1}{N}\sum_{i=1}^{N}p\left(\varepsilon_{tb,t}|\varepsilon_{tb,t-1},\sigma_{tb,t|t-1}^{i};\Psi\right)$$

where our notation for each draw i indicates in the subindex the conditioning set (i.e., $t|t-1$ is a draw at moment t conditional on information until $t-1$).

To draw from $p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)$, the Particle filter uses the idea of sequential important sampling proposed by Rubin (1988):

Proposition 1. *Let $\left\{\sigma_{tb,t|t-1}^{i}\right\}_{i=1}^{N}$ be a draw from $p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t-1};\Psi\right)$. Let the sequence $\left\{\widetilde{\sigma}_{tb,t}^{i}\right\}_{i=1}^{N}$ be a draw with replacement from $\left\{\sigma_{tb,t|t-1}^{i}\right\}_{i=1}^{N}$ where the resampling probability is given by*

$$\omega_{t}^{i}=\frac{p\left(\varepsilon_{tb,t}|\varepsilon_{tb,t-1},\sigma_{tb,t|t-1}^{i};\Psi\right)}{\sum_{i=1}^{N}p\left(\varepsilon_{tb,t}|\varepsilon_{tb,t-1},\sigma_{tb,t|t-1}^{i};\Psi\right)},$$

Then $\left\{\sigma_{tb,t|t}^{i}\right\}_{i=1}^{N}=\left\{\widetilde{\sigma}_{tb,t}^{i}\right\}_{i=1}^{N}$ is a draw from $p\left(\sigma_{tb,t}|\varepsilon_{tb}^{t};\Psi\right)$.

Proposition 1, which is just a simple application of Bayes' theorem, builds the draws $\left\{\sigma_{tb,t|t}^{i}\right\}_{i=1}^{N}$ recursively from $\left\{\sigma_{tb,t|t-1}^{i}\right\}_{i=1}^{N}$ by incorporating the information on $\varepsilon_{tb,t}$. The

resampling step is crucial. If we just draw a whole sequence of $\left\{\left\{\sigma^i_{tb,t|t-1}\right\}_{i=1}^N\right\}_{t=1}^T$ without resampling period by period, all the sequences would become arbitrarily far away from the true sequence of volatilities, since it is a zero measure set. Then, the sequence that happened to be closer to the true states would dominate all of the remaining ones in weight and the evaluation of the likelihood would be most inaccurate. Evidence from simulation shows that this degeneracy problem already appears after a small number of observations.

Now that we have $\left\{\sigma^i_{tb,t|t}\right\}_{i=1}^N$, we can draw N exogenous shocks $u^i_{\sigma_{tb},t+1}$ from a standard normal distribution and find:

$$\sigma^i_{tb,t+1|t} = \left(1-\rho_{\sigma_{tb}}\right)\sigma_{tb} + \rho_{\sigma_{tb}}\sigma^i_{tb,t|t} + \eta_{tb}u^i_{\sigma_{tb},t+1} \tag{16}$$

to generate $\left\{\sigma^i_{tb,t+1|t}\right\}_{i=1}^N$. This forecast step places us back at the beginning of Proposition 1, but one period ahead in our conditioning.

The following pseudocode summarizes the description of the algorithm:

Step 0, Initialization: `Set` $t \rightsquigarrow 1$. `Sample` N `values` $\left\{\sigma^i_{tb,0|0}\right\}_{i=1}^N$ from $p\left(\sigma_{tb,0};\Psi\right)$.

Step 1, Prediction: `Sample` N `values` $\left\{\sigma^i_{tb,t|t-1}\right\}_{i=1}^N$ `using` $\left\{\sigma^i_{tb,t-1|t-1}\right\}_{i=1}^N$, `the law of motion for states and the distribution of shocks` $u_{\sigma_{tb},t}$.

Step 2, Filtering: `Assign to each draw` $\left(\sigma^i_{tb,t|t-1}\right)$ `the weight` ω^i_t `in Proposition 1.`

Step 3, Sampling: `Sample` N `times with replacement from` $\left\{\sigma^i_{tb,t|t-1}\right\}_{i=1}^N$ `using the probabilities` $\{\omega^i_t\}_{i=1}^N$. `Call each draw` $\left(\sigma^i_{tb,t|t}\right)$. `If` $t < T$ `set` $t \rightsquigarrow t+1$ `and go to step 1. Otherwise stop.`

With the output of the algorithm, we just substitute into our formula

$$\begin{aligned} p\left(\varepsilon^T_{tb};\Psi\right) \simeq\ & \frac{1}{N}\sum_{i=1}^N \frac{1}{(2\pi)^{0.5}}\exp\left[-\frac{1}{2}\left(\frac{\varepsilon_{tb,1}-\rho_{tb}\varepsilon_{tb,0}}{e^{\sigma^i_{tb,0|0}}}\right)^2\right] * \\ & * \prod_{t=2}^T \frac{1}{N}\sum_{i=1}^N \frac{1}{(2\pi)^{0.5}}\exp\left[-\frac{1}{2}\left(\frac{\varepsilon_{tb,t}-\rho_{tb}\varepsilon_{tb,t-1}}{e^{\sigma_{tb,t|t-1}}}\right)^2\right] \end{aligned} \tag{17}$$

and we obtain the estimate of the likelihood. Del Moral and Jacod (2002) and Künsch (2005) provide weak conditions under which the right-hand side of the previous equation is a consistent estimator of $p\left(\varepsilon^T_{tb};\Psi\right)$ and a central limit theorem applies.

9.3. Computation

In the main part of the paper, we argued that a third-order approximation was important if we wanted to evaluate the effects of volatility shocks independently of real interest rate shocks. In this appendix, we provide some evidence that the effects on allocations of the higher order terms are non-trivial.

We simulate the Argentinian economy for 500 periods (after a period of burn-in to eliminate the effect of initial conditions) at the benchmark calibration parameter values and we follow the results for the deviations of consumption, investment, output, labor, and debt with respect to the steady state when we have a first-, second-, and a third-order approximation. The interest rate evolution was kept the same in all three simulations. We plot the results in figure A1. We see how, even if the general pattern of behavior is similar, there are non-trivial differences, in particular in investment, debt, and consumption. The differences are particularly salient between, on the one hand, the first-order approximation, and the other hand, the second- and third-order approximations. The presence of constants in the higher order approximations that reflects precautionary behavior are largely responsible for the permanent differences in levels that we see, for example, in the consumption series.

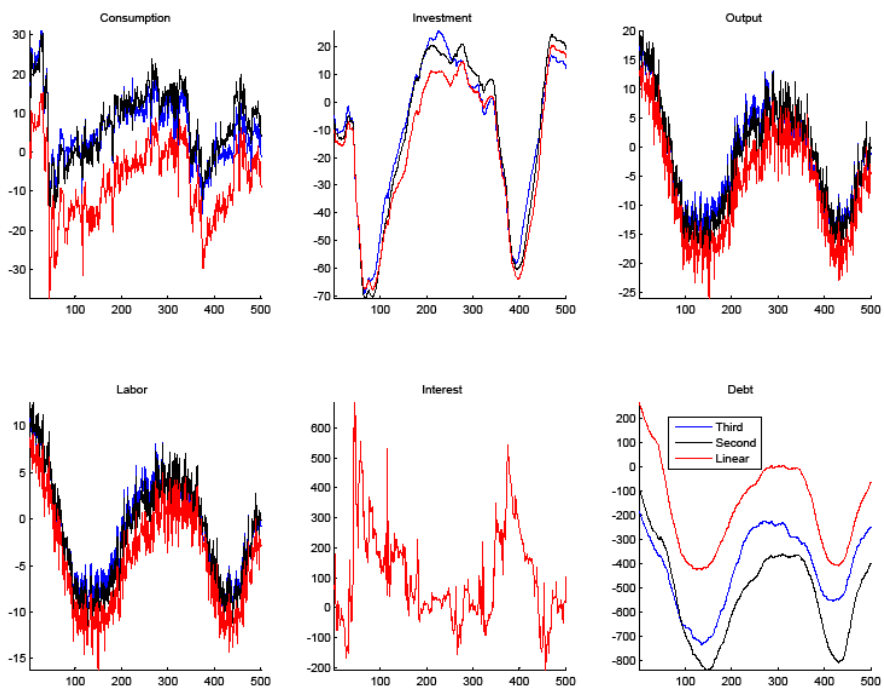

Figure A1: Simulation, different Approximations

Because the scale of figure A1 makes it difficult to appreciate our point, in figure A2 we

zoom in on a section of the simulation for investment in the center of the sample. We can see how around periods 30 to 40, in the first-order approximation, investment is stable around 10 percent above the steady state, in the second-order approximation, it is falling from around 20 percent above steady state to around 15 percent, and in the third-order approximation, investment is rising up to 25 percent. We could hardly have a clearer picture: as a response to the same real interest rate shocks, each level of approximation tells us a different history about the evolution of investment.

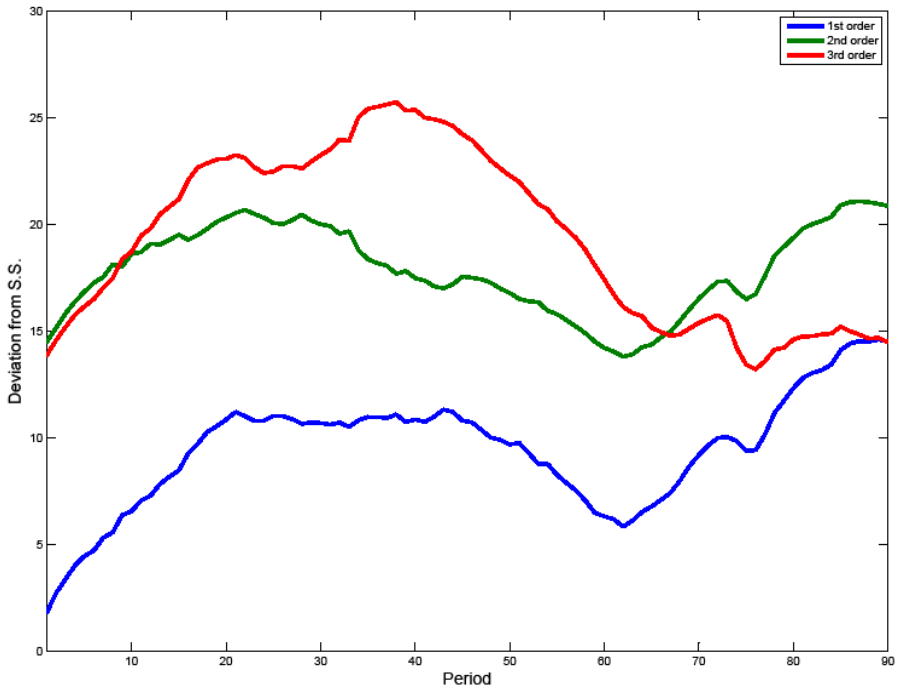

Figure A2: Evolution of Investment

9.4. Computing Impulse Responses

As argued in the main section, our higher order approximation makes the simulated paths of states and controls in the model move away from their steady-state values. Consequently, computing impulse responses as percentage deviations of the model's steady state is not informative. To compute the impulse responses reported in the paper, we proceed as follows:

1. We simulate the model, starting from its steady state, for 2096 periods. We disregard the first 2000 periods as a burn-in.

2. Based on the last 96 periods, we compute the mean of the ergodic distribution for each variable in our model. Adding more periods has essentially no impact on the mean.

3. Starting from the ergodic mean and in the absence of shocks, we hit the model with a one standard deviation shock to the volatility process $u_{\sigma,t}$.

4. We report the resulting impulse responses as percentage deviations from the variables' ergodic means.

In the context of a threshold model, Koop *et al.* (1996) have argued that the use of the standard impulse response functions may be misleading. These authors urge the use of the so-called generalized impulse response to overcome the drawbacks reported in their manuscript. We computed the generalized impulse response, but we essentially found no differences between this procedure and the one outline above. We choose to report the traditional impulse responses in the main body of the paper, since they are easier to interpret than the generalized impulse response functions.

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