Entry, Exit, Firm Dynamics, and Aggregate Fluctuations

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Setting the stage

- Heterogeneity in macroeconomic models
  - Lots of work on the household side
  - Much less done on the firm side
- Entry & exit selection is likely to be a bigger issue for firms
- Hopenhayn & Jovanovic
- Recent trade literature – see work by Kim Ruhl & Jonathan Willis
- Most work on business cycle models with firm heterogeneity abstracts from entry & exit – See Veracierto (AER 2002), Khan & Thomas (AER 2007) and Bachman & Bayer
- There are important exceptions – hold on for a couple of minutes

Main questions

- Can we build a (relatively) simple model of firm dynamics that is consistent with basic facts on investment behavior and has data–conforming implications for the unconditional means and the cyclicality of
  - entry rate,
  - exit rate,
  - average size of entrants,
  - average size of exiters?
- What is the cost of disregarding entry and exit when modeling aggregate fluctuations?

Methodology

1. Start with Hopenhayn's partial equilibrium model of industry dynamics
2. Model capital accumulation
3. Introduce aggregate productivity shocks
4. Calibrate to a set of *unconditional* cross–sectional moments
5. Figure out the correlations of those cross–sectional moments with industry output
6. Study impulse responses
7. Gauge the importance of entry and exit for macroeconomics

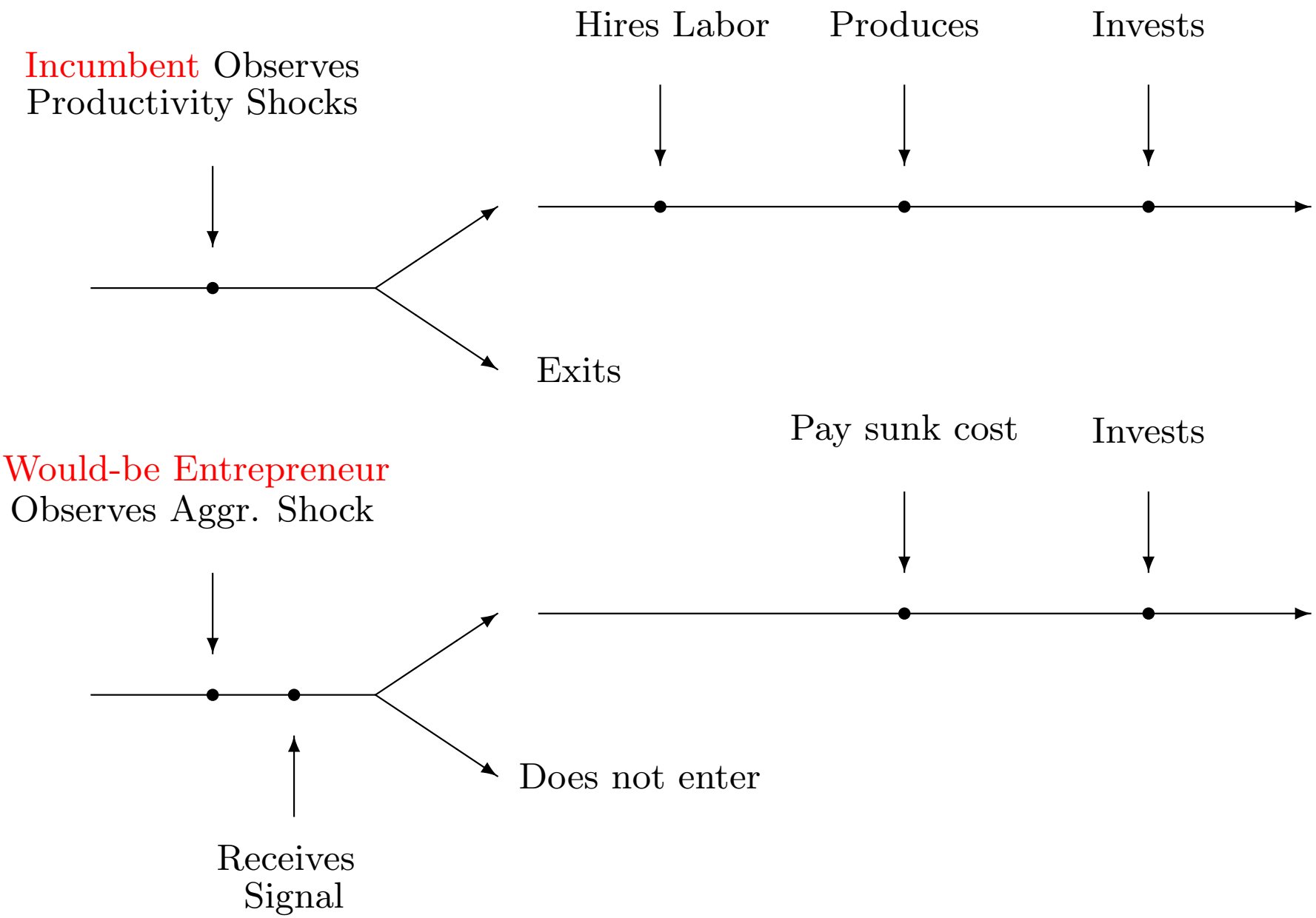
Related Literature

- Jeff Campbell (RED 1998)
- Samaniego (RED 2008), (AER 2009)
- Mukoyama & Lee

Model

- Discrete time, Infinite horizon
- A continuum of price–taking firms producing an homogenous consumption good by means of capital and labor
- Partial equilibrium
- Demand for the consumption good and supply of capital are infinitely elastic
- Supply of labor is: $L_t^s = w_t^\gamma$, $\gamma > 0$

Timing


Incumbent Observes
Productivity Shocks
Hires Labor
Produces
Invests
Exits
Would-be Entrepreneur
Observes Aggr. Shock
Pay sunk cost
Invests
Does not enter
Receives
Signal

Model

- Production function: $y_t = (z_t s_t)(k_t^{\alpha} l_t^{1-\alpha})^{\theta}$, $\alpha, \theta \in (0,1)$
- Productivity shocks
  - Idiosyncratic component (independently distributed across firms and independent from the aggregate component):

$$\log(s_t) = \rho_s \log(s_{t-1}) + \sigma_s \varepsilon_{s,t}$$
$$\text{with } \rho_s \in (0,1), \;\; \sigma_s > 0, \;\; \varepsilon_{s,t} \sim N(0,1)$$

  - Aggregate component:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \sigma_z \varepsilon_{z,t}$$
$$\text{with } \rho_z \in (0,1), \;\; \sigma_z > 0, \;\; \varepsilon_{z,t} \sim N(0,1)$$

- Capital adjustment costs: $g(x,k)$
- Fixed cost of operation: $c_f \geq 0$
- Discount factor: $1/R$, $R > 1$

Model – Incumbent

- Optimal labor choice

$$\pi(\omega, k, s) = \max_{l} \; e^{s+z}[k^{\alpha} l^{1-\alpha}]^{\theta} - wl$$

- Dynamic optimization program

$$\begin{aligned} V(\omega, k, s) = \max \Big[0, \max_{x} \; & \pi(\omega, k, s) - x - g(x, k) - c_f + \\ & \frac{1}{R} \int_{\Omega} \int_{\Re} V(\omega', k', s') dH(s'|s) dJ(\omega'|\omega) \Big], \\ \text{s.t. } \; & k' = k(1-\delta) + x \end{aligned}$$

Model – Entry

- In every period, there is a mass M of prospective entrants
- Each prospective entrant receives a signal about the idiosyncratic productivity component $q \sim Q(q)$
- Firm value conditional on signal q:

$$V_e(\omega, q) = \max_{k'} -k' + \frac{1}{R} \int V(\omega', k', s') dH(s'|q) dJ(\omega'|\omega)$$

- Enter the market and invest if and only if $V_e(\omega, q) \geq c_e$

Equilibrium

For given Γ_0, a competitive recursive equilibrium consists of

- value functions $V(\omega, k, s)$ and $V_e(\omega, q)$,
- policy functions $x(\omega, k, s)$, $l(\omega, k, s)$, $k'(\omega, q)$
- bounded sequences of wages $\{w_t\}_{t=0}^{\infty}$, of incumbents' measures $\{\Gamma_t\}_{t=1}^{\infty}$, and of entrants'measures $\{\mathcal{E}_t\}_{t=0}^{\infty}$

such that

1. $V(\omega, k, s)$, $x(\omega, k, s)$, and $l(\omega, k, s)$ solve the incumbent's problem
2. $V_e(\omega, q)$ and $k'(\omega, q)$ solve the entrant's problem
3. The labor market clears: $\int l(\omega_t, k, s) d\Gamma_t(k, s) = L^s(w_t)$ $\forall \ t \geq 0$,

Equilibrium – Continued

4. For all Borel sets $\mathcal{S} \times \mathcal{K} \in \Re \times \Re^+$ and $\forall \ t \geq 0$,

$$\mathcal{E}_{t+1}(\mathcal{S} \times \mathcal{K}) = \int_{\mathcal{S}} \int_{\mathcal{B}_e(\mathcal{K},\omega_t)} dQ(q) dH(s'|q),$$

where $\mathcal{B}_e(\mathcal{K}, \omega_t) = \{q \text{ s.t. } k'(\omega_t, q) \in \mathcal{K} \text{ and } V_e(\omega_t, q) \geq c_e\}$

5. For all Borel sets $\mathcal{S} \times \mathcal{K} \in \Re \times \Re^+$ and $\forall \ t \geq 0$,

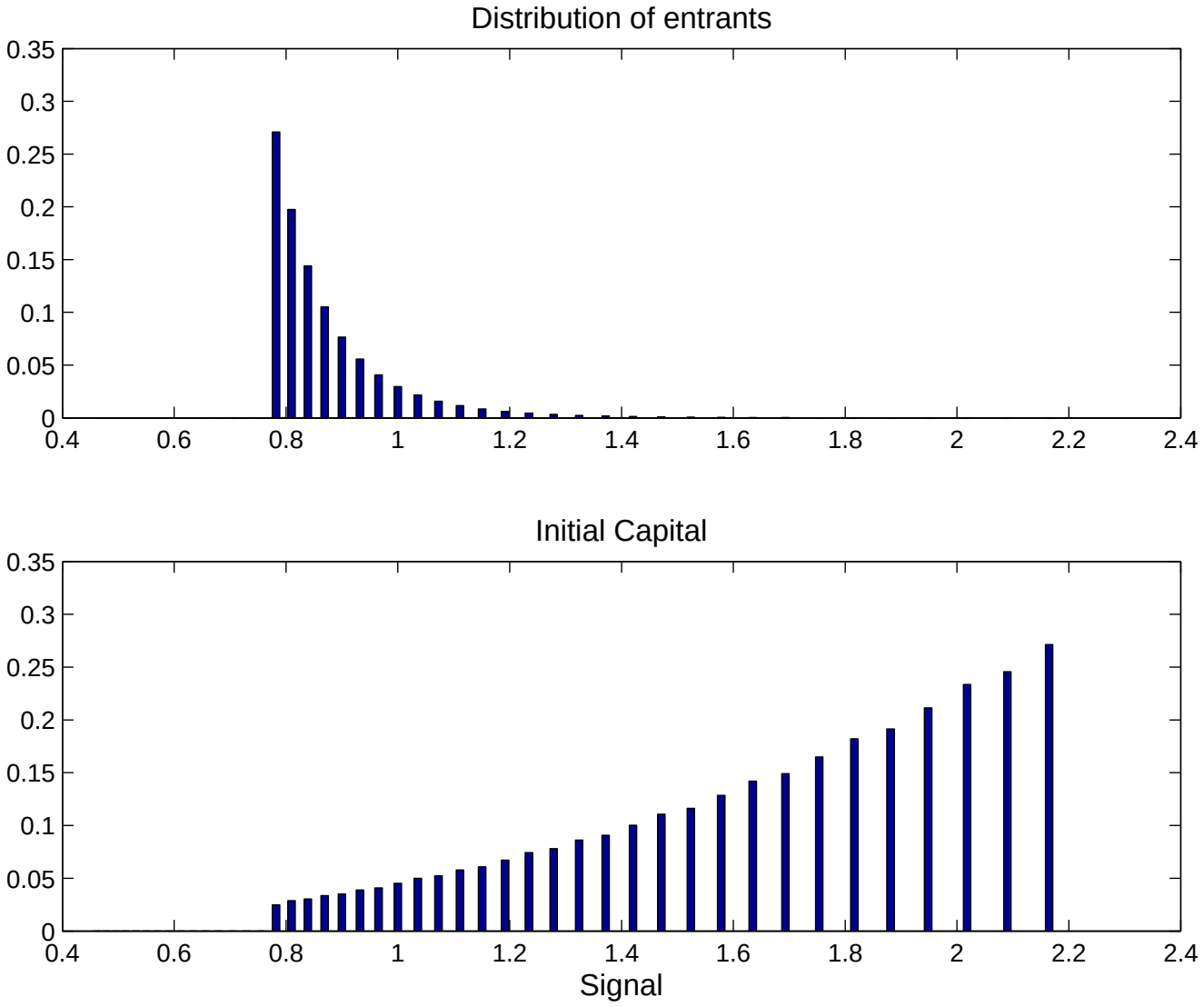
$$\Gamma_{t+1}(\mathcal{S} \times \mathcal{K}) = \int_{\mathcal{S}} \int_{\mathcal{B}(\mathcal{K},\omega_t)} d\Gamma_t(k, s) dH(s'|s) + \mathcal{E}_{t+1}(\mathcal{S} \times \mathcal{K}),$$

where $\mathcal{B}(\mathcal{K}, \omega_t) = \{(k, s) \text{ s.t. } V(\omega_t, k, s) > 0 \text{ and } k(1-\delta) + x(\omega_t, k, s) \in \mathcal{K}\}$.

Functional Forms

- Adjustment cost: $g(x,k) = c_0 k + c_1 \left(\frac{x}{k}\right)^2 k$, $c_0, c_1 \geq 0$
- Distribution of signals: $Q(q) = \left(\frac{q}{\underline{q}}\right)^{\xi}$, $\underline{q} \geq 0$, $\xi \in \mathcal{N}$

Entry Mechanism



Stationary Case

(No aggregate shocks)

Parameter Values

Description	Symbol	Value
Capital share	α	0.3
Span of control	θ	0.8
Depreciation rate	δ	0.1
Interest rate	R	1.04
Labor supply elasticity	γ	5.0
Mass of entrants	M	4,890.44
Persist. idiosync. shock	ρ_s	0.55
Variance idiosync. shock	σ_s	0.215
Fixed cost of operation	c_f	0.00532
Fixed cost of investment	c_0	0.0002
Variable cost of investment	c_1	0.036
Pareto exponent	ξ	8.0
Entry cost	c_e	0.00

Stationary Case – Calibration Targets

Statistic	Model	Data
Mean investment rate	0.137	0.122
Std. Dev. investment rate	0.303	0.337
Investment autocorrelation	0.062	0.058
Inaction rate	0.105	0.081
Entry rate	0.061	0.062
Entrants' relative size	0.60	0.60
Exiters' relative size	0.22	0.49

Results – Exit Hazard Rate

- The exit hazard rate declines with age ▸ pic
- Heterogeneity of exit probability, conditional on size

$$V(\omega,k,s) = \max\Big[0, \max_x \ \pi(\omega,k,s) - x - g(x,k) - c_f + \frac{1}{R}\int_\Omega \int_\Re V(\omega',k',s')dH(s'|s)dJ(\omega'|\omega)\Big],$$
$$\text{s.t. } k' = k(1-\delta) + x$$

$$V(\omega,k,s) = \max\left[0,\ \pi(\omega,k,s) + k(1-\delta) - g(-k(1-\delta),k) - c_f\right]$$

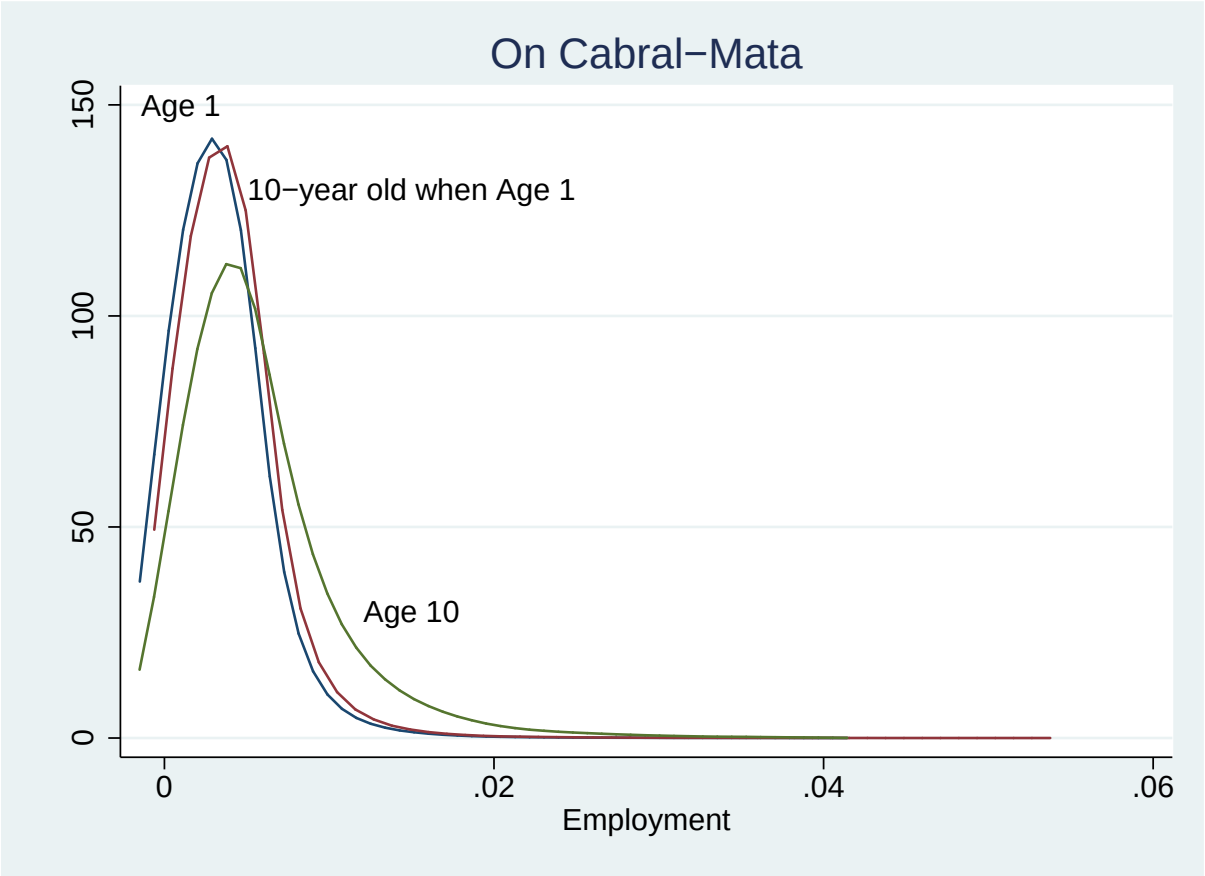
Results – Firm Growth

- The employment growth rate declines with age, both unconditionally and conditional on size
- The employment growth rate declines with size, both unconditionally and conditional on age ▸ pic

Results – The Size Distribution

- The size distribution displays positive skewness
- The skewness declines with cohort age ▸ pic
- Selection can account for the evidence by Cabral & Mata

More on the Evolution of the Size Distribution



Aggregate Fluctuations

State Space

- The market clearing wage is

$$\log w_t = \frac{\log[(1-\alpha)\theta z_t]}{1+\gamma[1-(1-\alpha)\theta]} + \frac{1-(1-\alpha)\theta}{1+\gamma[1-(1-\alpha)\theta]} G_t,$$

with $G_t = \log\left[\int \left(sk^{\alpha\theta}\right)^{\frac{1}{1-(1-\alpha)\theta}} d\Gamma_t(k,s)\right]$.

- The aggregate state is $\omega_t = \{z_t, \Gamma_t\}$

State Space – Continued

- We posit that G_{t+1} is an affine function of G_t and $\log(z_{t+1})$
- It follows that the law of motion for the wage is

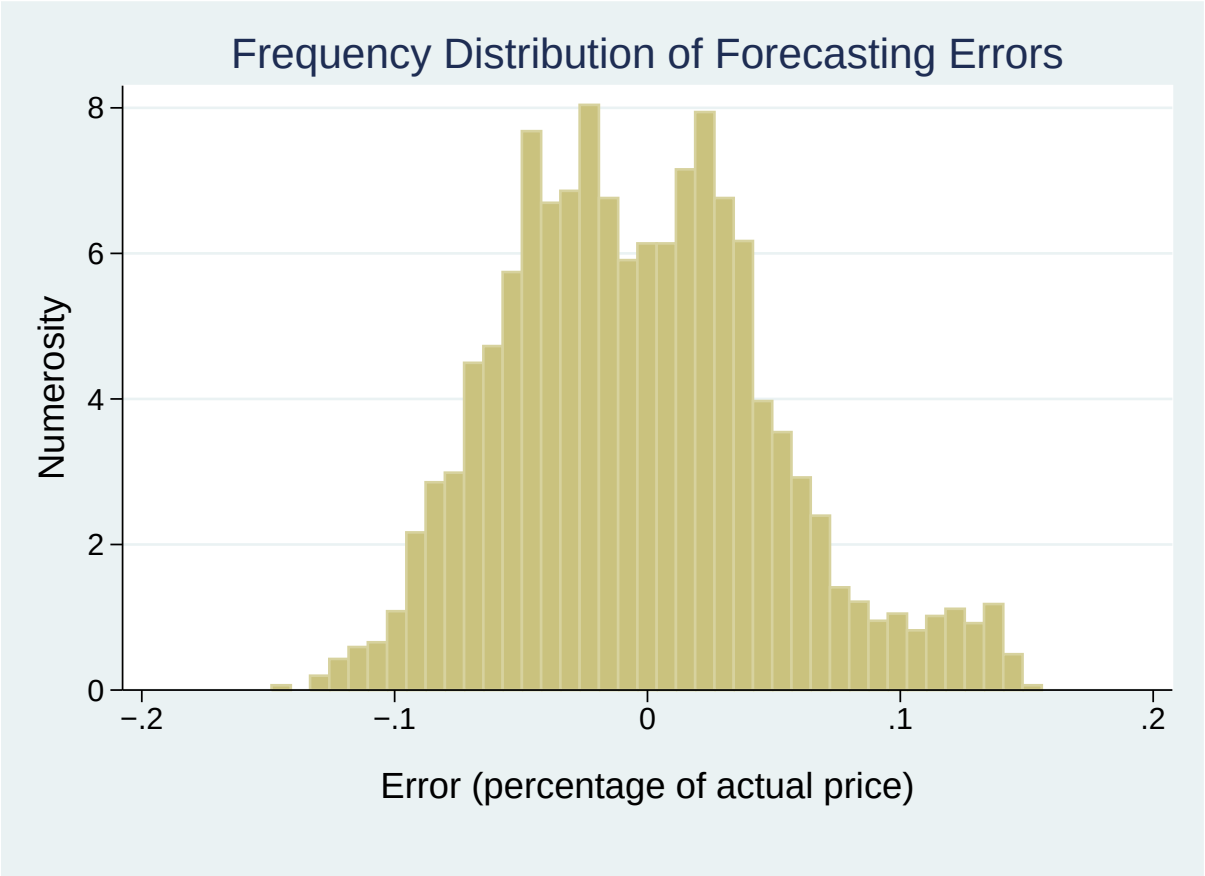
$$\log w_{t+1} = \beta_0 + \beta_1 \log w_t + \beta_2 \log z_{t+1} + \beta_3 \log z_t + \varepsilon_{t+1}.$$

- In today's calibration

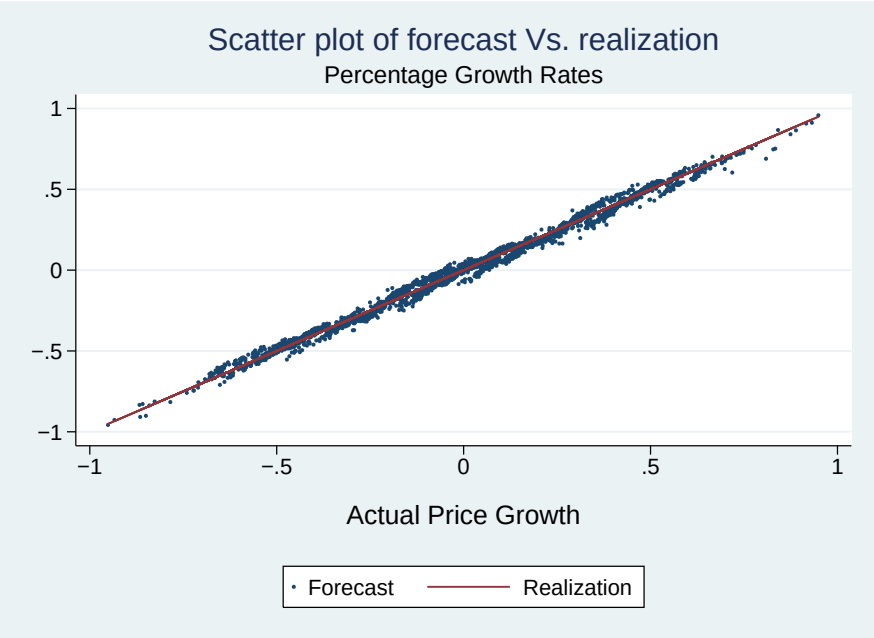
$$\log w_{t+1} = 0.286 + 0.735 \log w_t + 0.313 \log z_{t+1} - 0.116 \log z_t + \varepsilon_{t+1}.$$

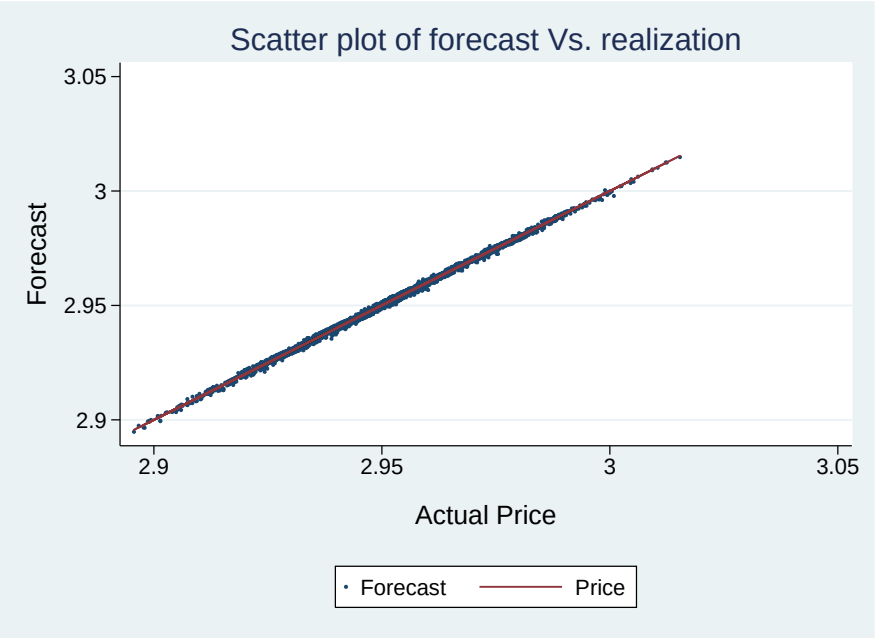
- The R^2 of the regression is .9981

Precision in Price Forecasting



Precision in Price Forecasting





More on Calibration

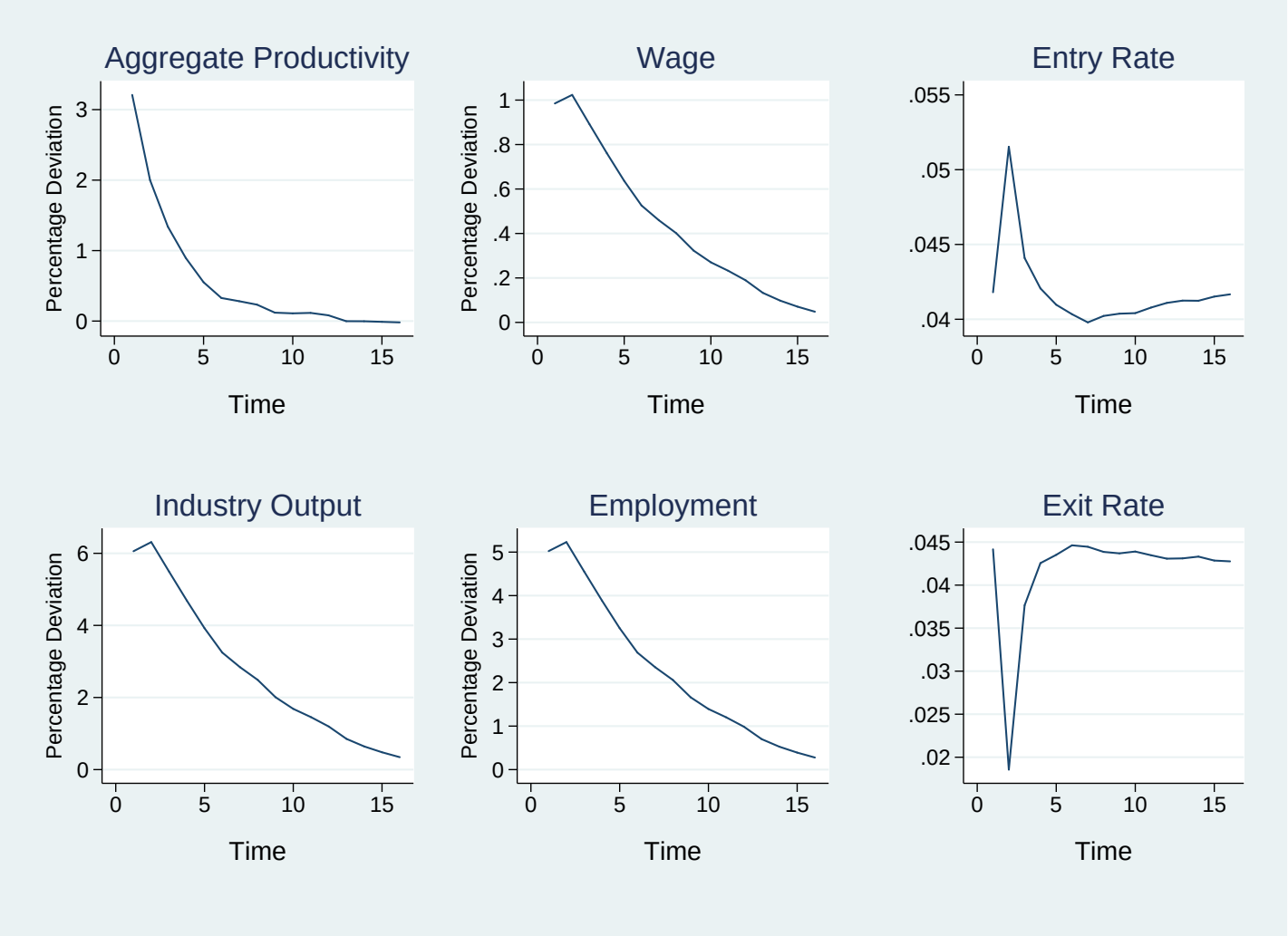
- $\rho_z = 0.65$, $\sigma_z = 0.008$, $\gamma = 5$

Statistic	Model	Data
Std. dev. output	0.048	0.040
Autocorrelation output. investment rate	0.895	0.782
Std. dev. employment (rel. to output)	0.832	0.923

Correlations with Industry Output

Entry Rate	0.2565
Exit Rate	-0.4184
Entrants' Relative Size	-0.4142
Exiters' Relative Size	-0.5967
Job Creation Rate	0.2368
Job Destruction Rate	-0.2456

Response to a positive productivity shock



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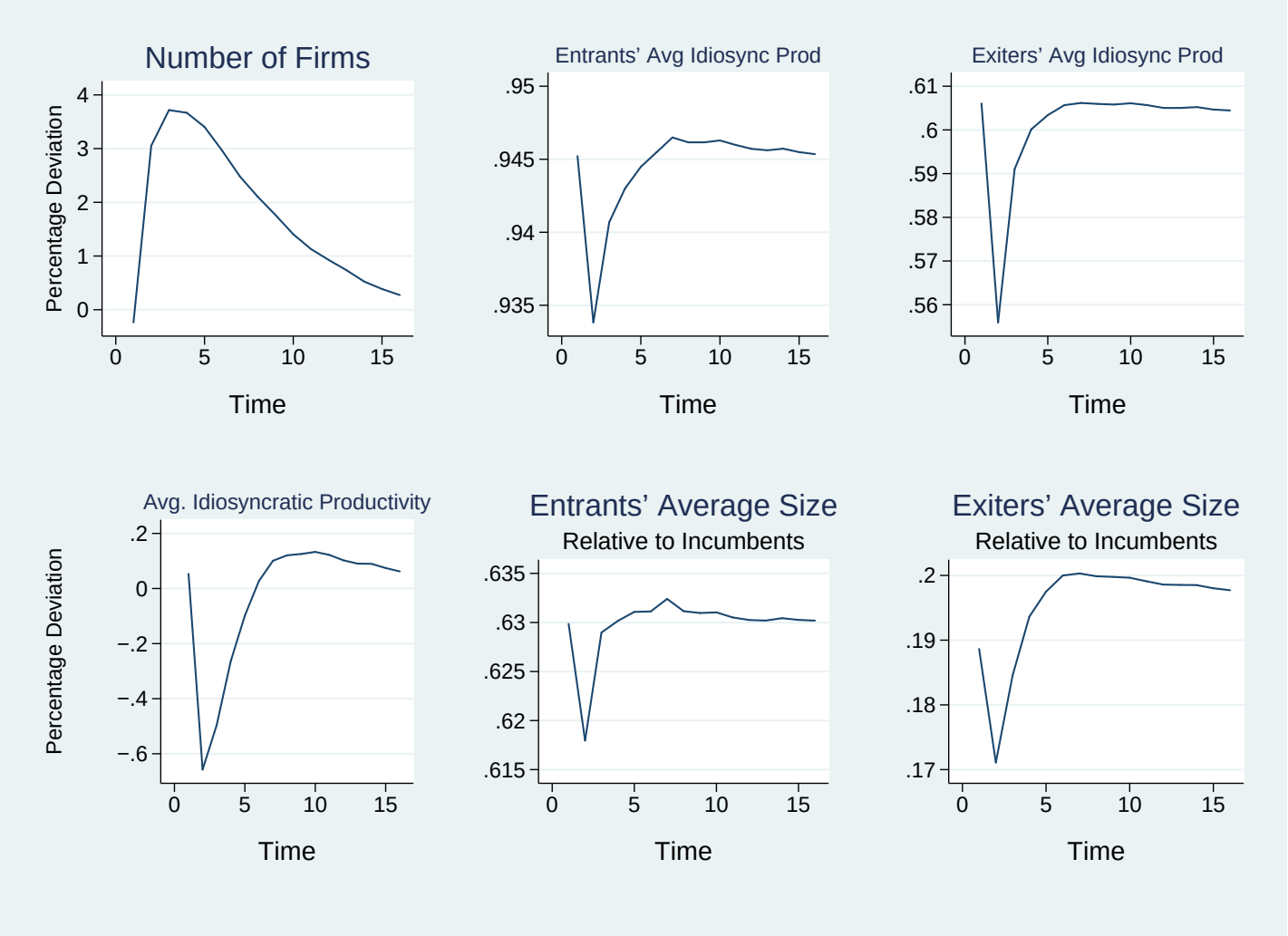

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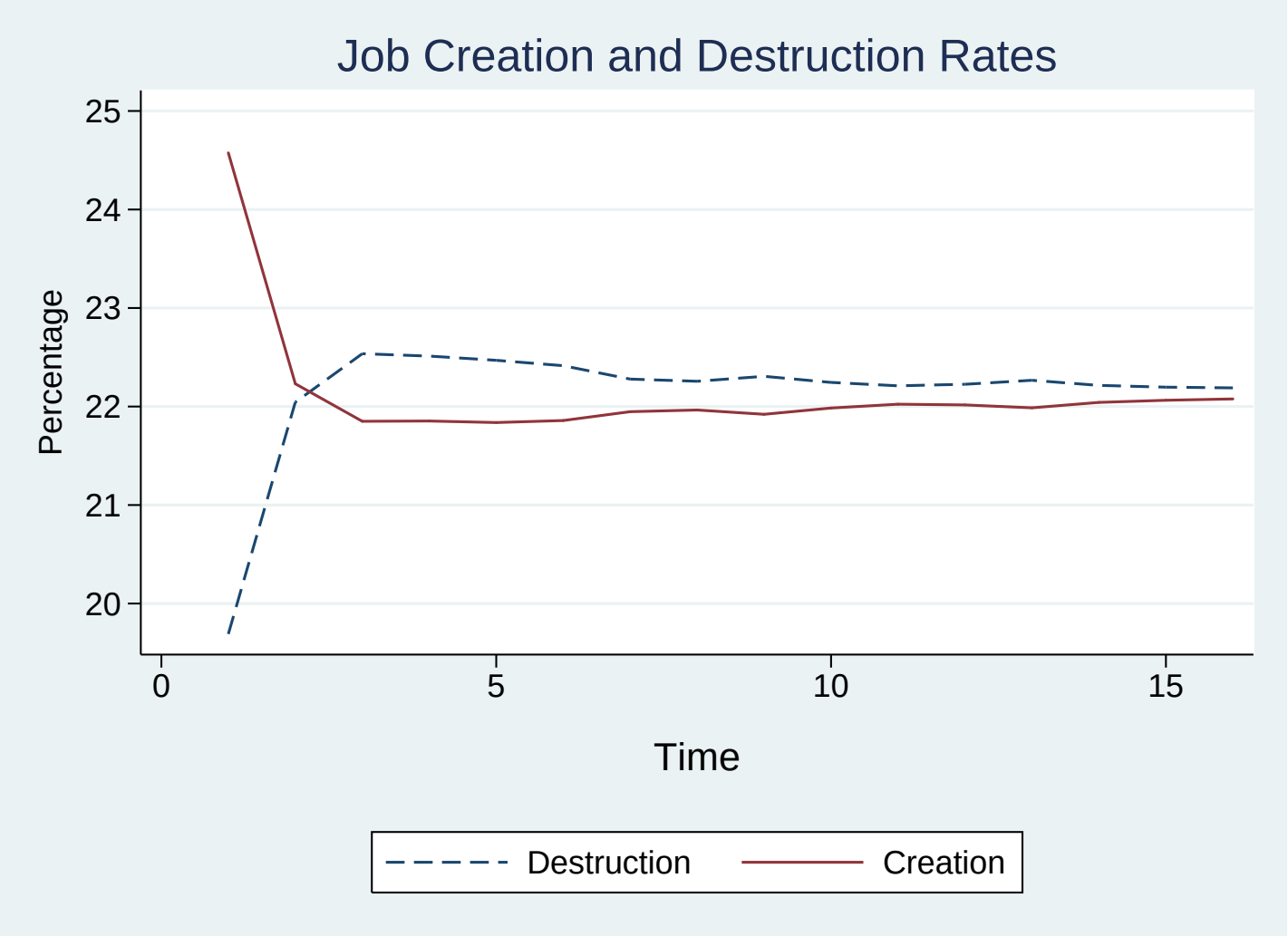


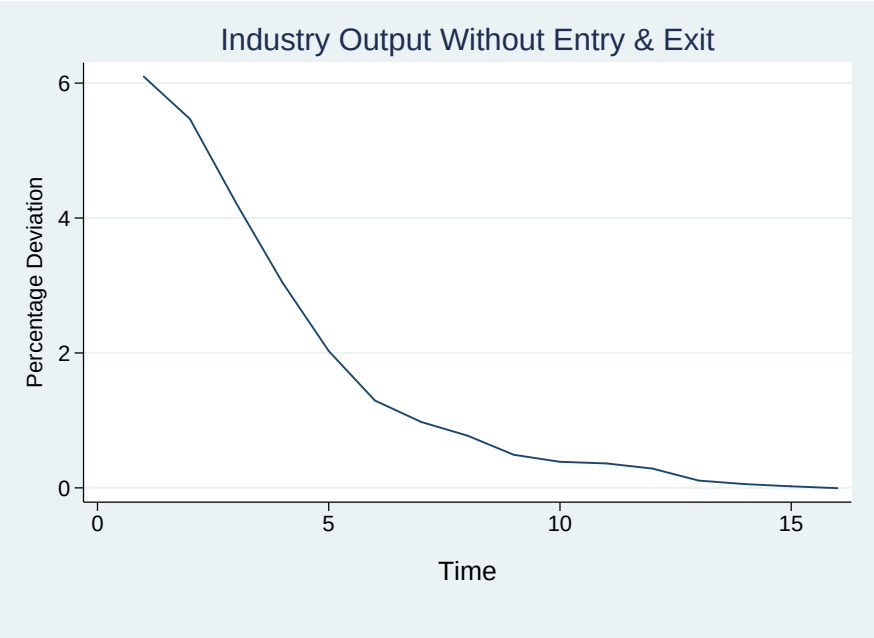
Empirical Evidence – Lee & Mukoyama

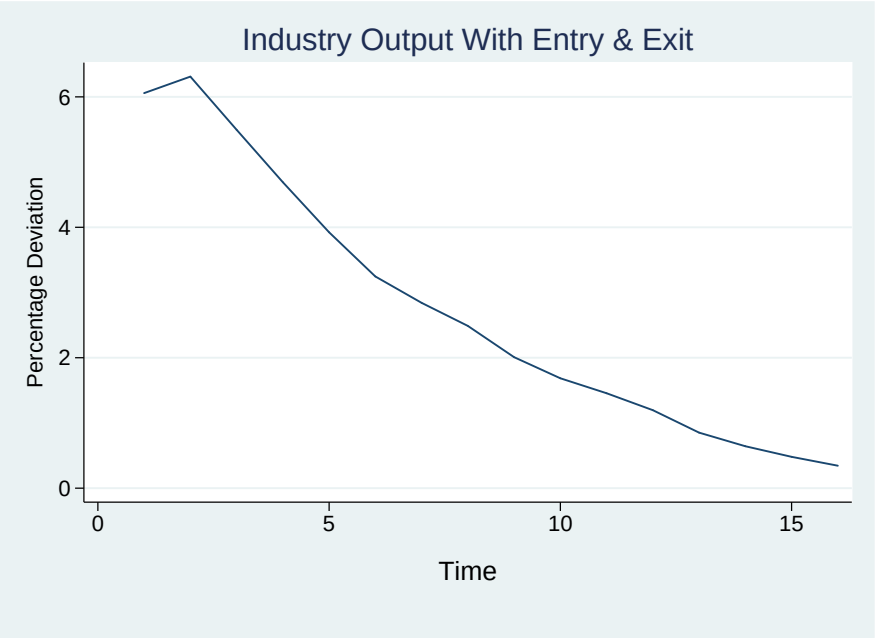
- Entry rate is strongly pro–cyclical (positively correlated with de–trended output)
- Exit rate is mildly pro–cyclical
- Relative size of entering firms is counter–cyclical
- Relative size of exiting firms is mildly pro–cyclical
- Relative productivity of entering firms is counter–cyclical
- Relative productivity of exiting firms is a–cyclical

Empirical Evidence – Jeff Campbell (RED 1998)

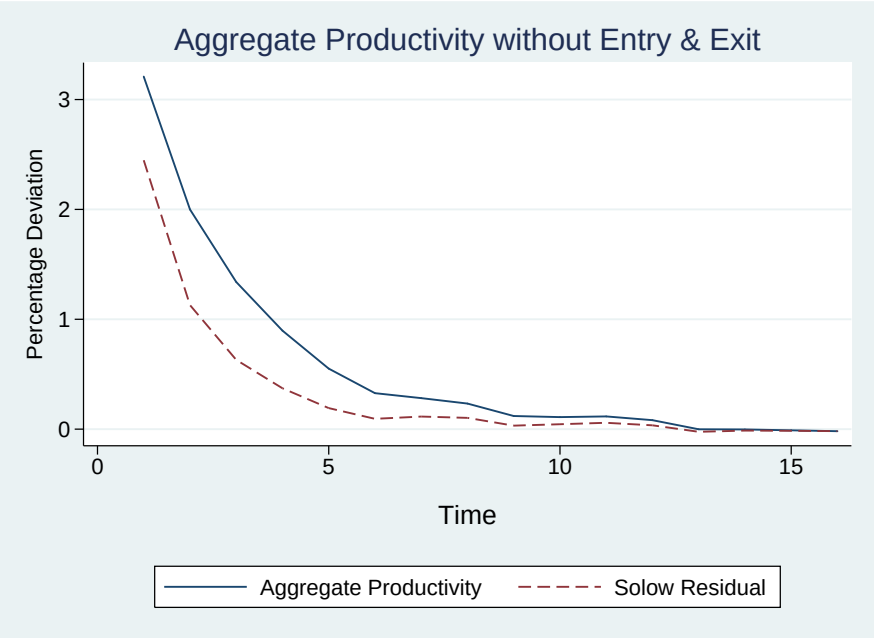
- Entry Rate
  - Pro–cyclical (positively correlated with output growth)
  - Positively correlated with lagged output growth
  - Maximum correlation is with output growth lagged one period
- Exit Rate
  - Counter–cyclical (negatively correlated with output growth)
  - Positively correlated with future output growth

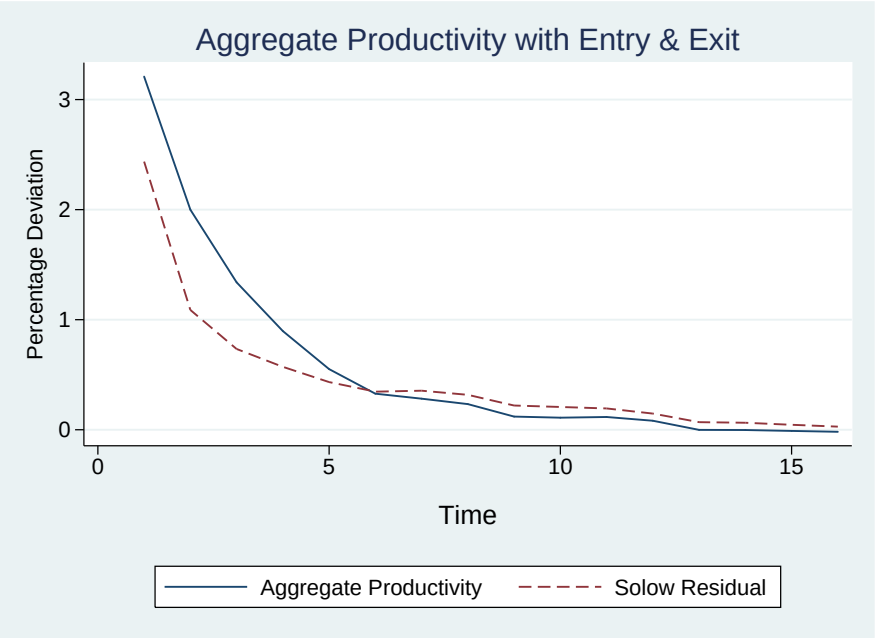
The cost of disregarding entry & exit



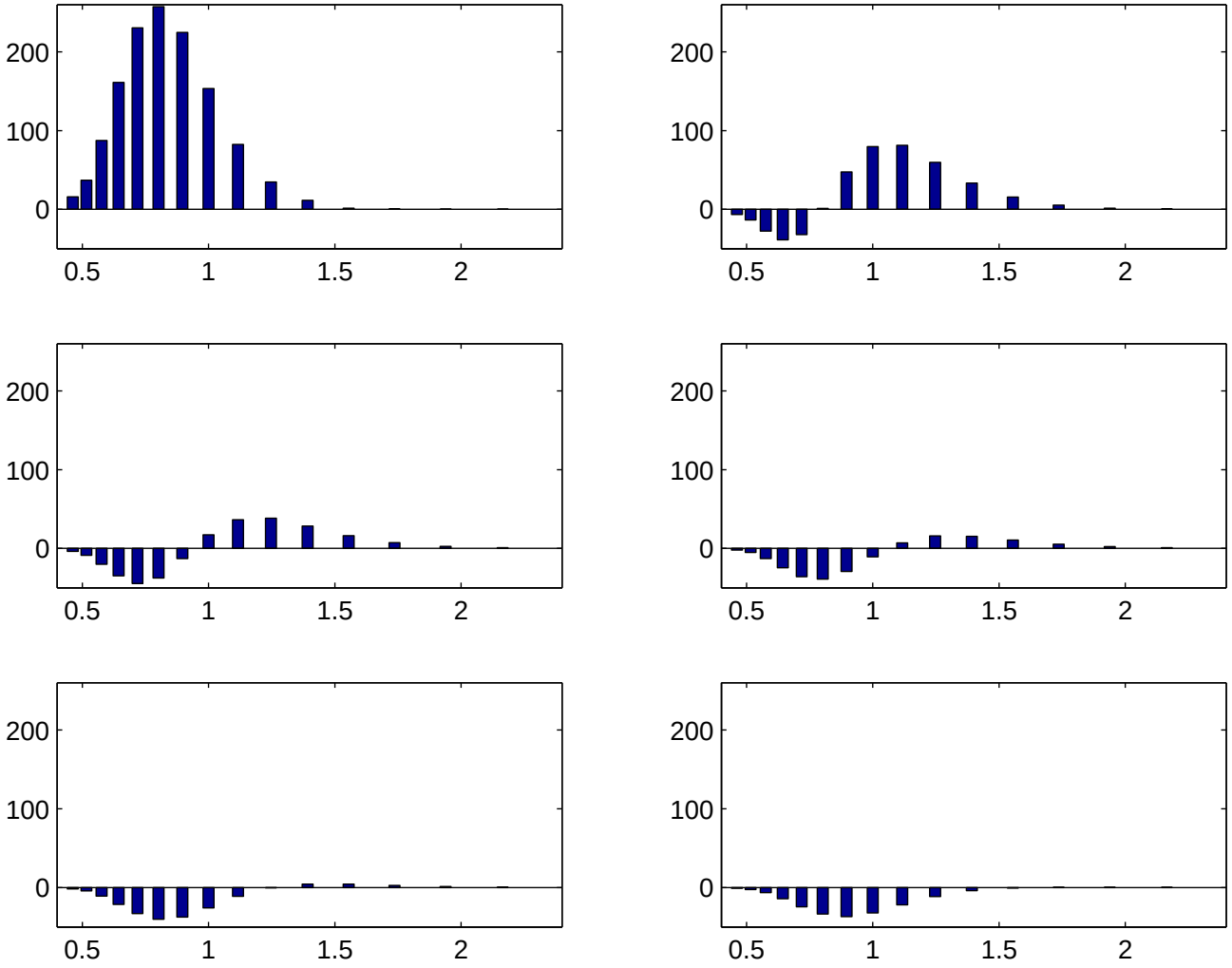


The cost of disregarding entry & exit





The cost of disregarding entry & exit



More on the importance of entry & exit

- Bachman & Bayer estimate that the cross–sectional variation in idiosyncratic productivity growth (firm-level Solow residual) is counter–cyclical
- In their model without entry or exit, they assume that the standard deviation of firm–level shock (σ_s) is negatively correlated with the aggregate shock
- In our model, σ_s is time–invariant, but the cross–sectional standard deviations of all variables are time–varying
- The standard deviation of firm–level Solow residual is counter–cyclical
- The std. dev. of firm–level productivity growth is pro–cyclical
- The std dev of expected firm–level productivity growth is pro–cyclical

Lee & Mukoyama

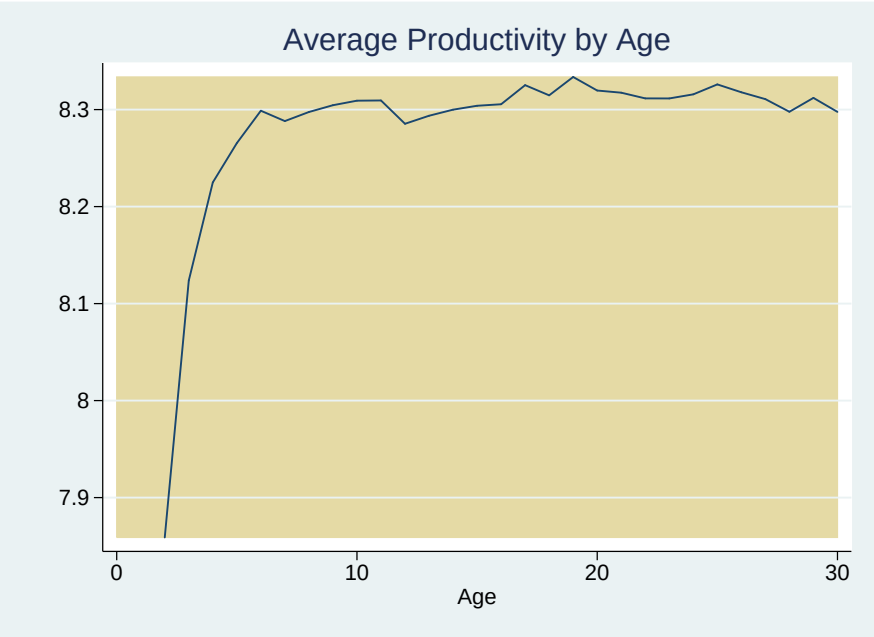
- Main difference: they have no capital accumulation. There is only on dimension of heterogeneity (productivity)
- They need stochastic exit shocks to generate exit patterns consistent with the data
- They need pro–cyclical idea costs in order to generate entry selection
  - To get positive entry at all times:

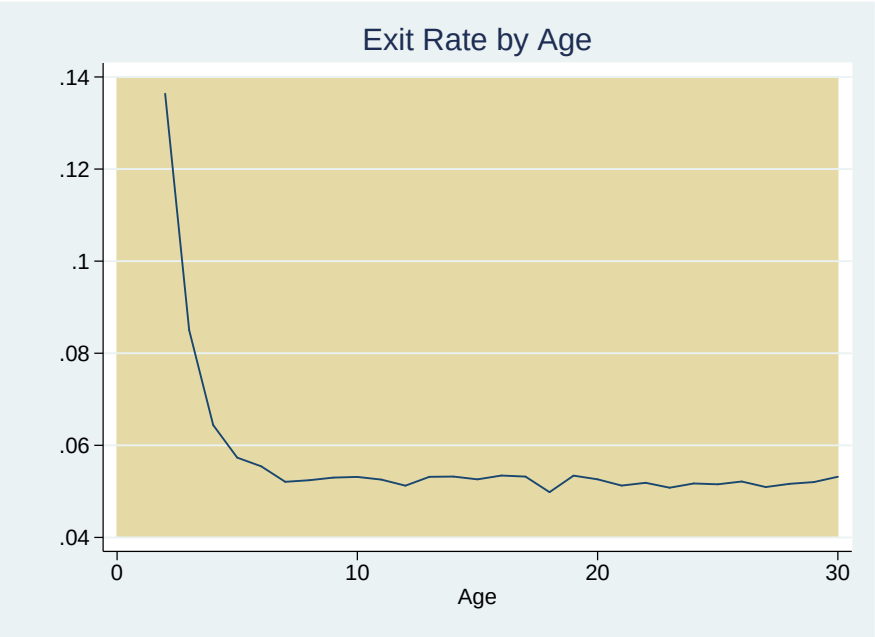
$$V^p(z_t, w_t; c_e) = c_q$$

  - Productivity and size of the entrants are pinned down by

$$V^e(q_t, z_t; w_t) = c_e$$

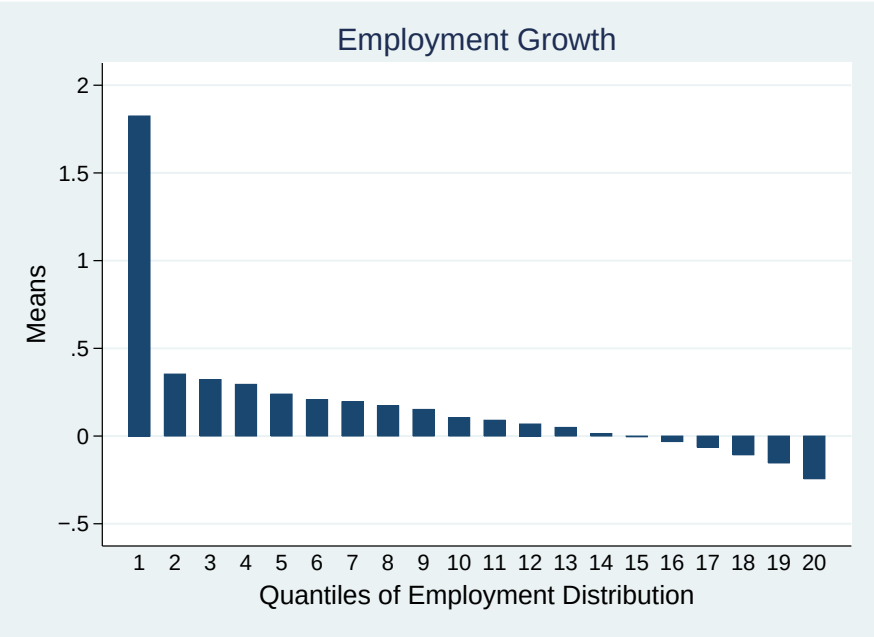
Exit Hazard Rate

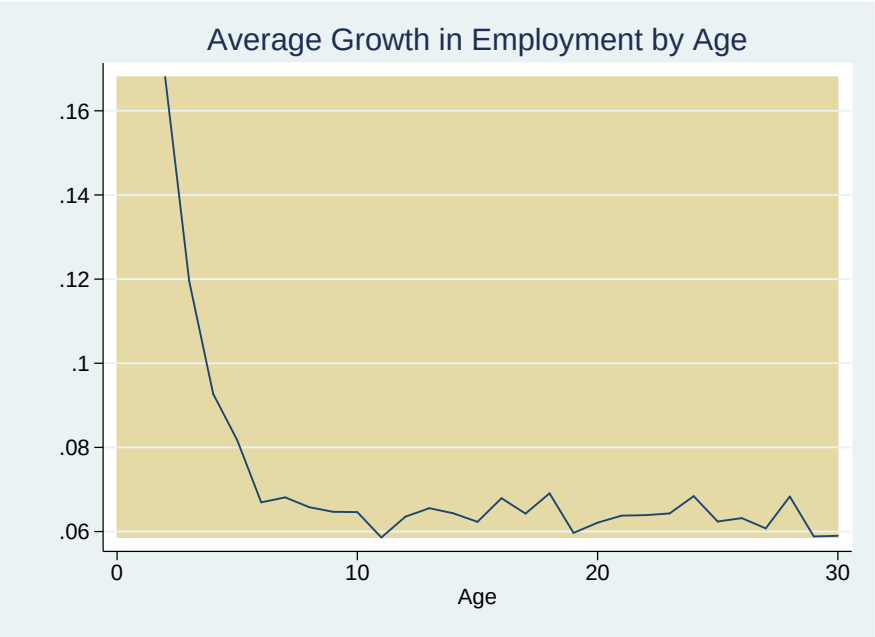




Return

Growth, Age, and Size





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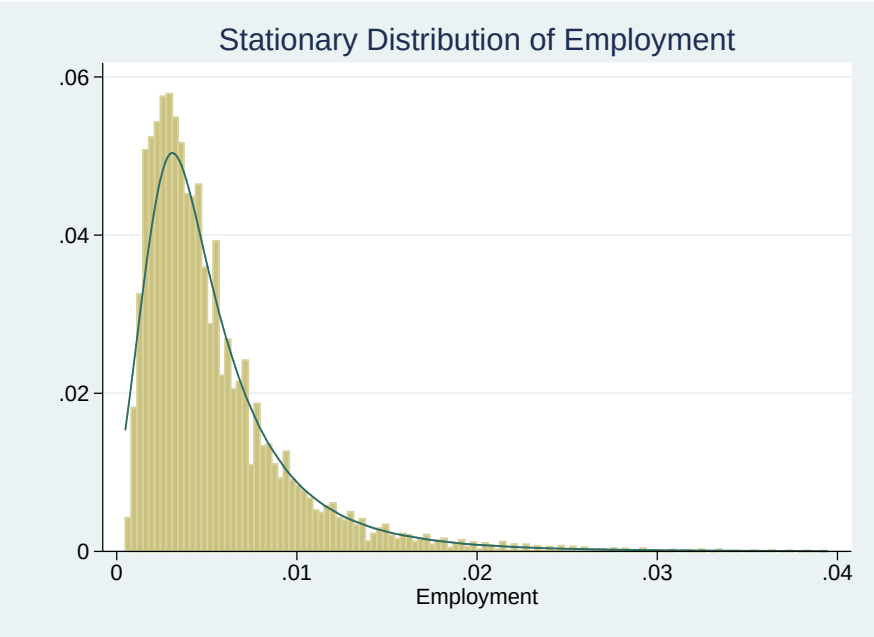
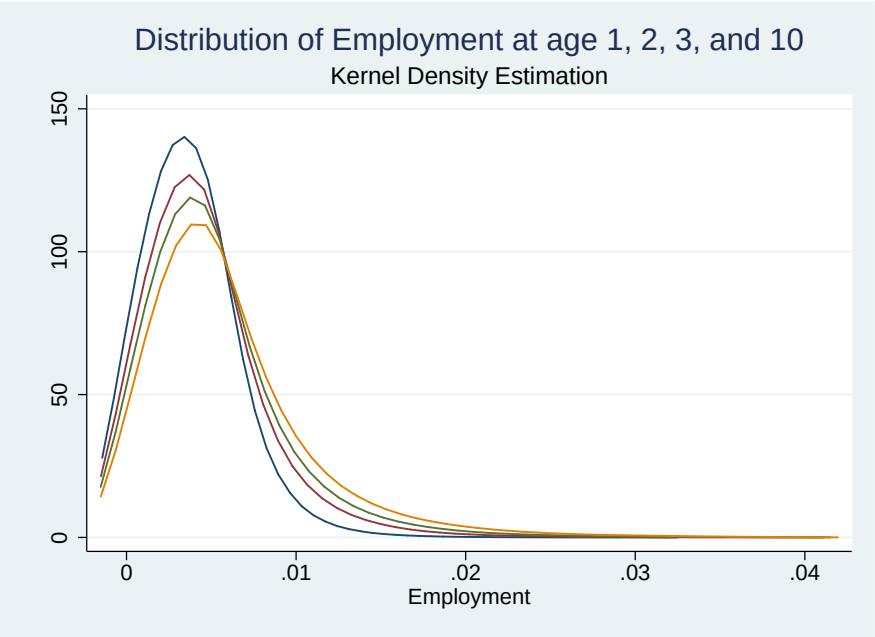
Skewness of the Size Distribution





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