Privately Efficient Bargaining between Workers and Large Firms

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Motivation (1)

Question: how is the output of a firm divided between n workers and their employer?

- Competitive environment: $w = y'(n)$
- Environment with search frictions: ex-post quasi-rent to split
  - Bargaining models
  - Directed search models

Motivation (2)

Why do we care?

- Positive
  - Competitive model has difficulty accounting for variation in hours worked, particularly cyclically. Can a model with frictions do better?
- Normative
  - Is there room for policy to improve on equilibrium outcomes?

Mortensen-Pissarides

- One worker, one firm matches (or constant returns)
- Free entry; costly vacancy posting.
- Matching through exogenous matching function $M(u, v)$ with constant returns, so if $\theta = v/u$
  - Firms meet workers at Poisson rate $M(u/v)/v \equiv M(u/v, 1) \equiv q(\theta)$
  - Workers meet firms at Poisson rate $M(u/v)/u \equiv v/u \times M(u/v, 1) \equiv \theta q(\theta)$
- Notation: output y, wage w, vacancy posting cost γ, exogenous job destruction rate δ, discount rate r, Bellman values of filled job J, unemployed worker V^u, employed worker V^e

Mortensen-Pissarides (2)

Bellman equations:

$$
\begin{aligned}
0 &= -\gamma + q(\theta)\left[J - 0\right] \\
rJ &= y - w + \delta\left[0 - J\right] \\
rV^u &= b + \theta q(\theta)\left[V^e - V^u\right] \\
rV^e &= w + \delta\left[V^u - V^e\right]
\end{aligned}
$$

- Any wage $w \in [b, y]$ is individually rational for matched firm-worker pair.
- Which w is expected affects equilibrium:

$$
-\gamma + q(\underbrace{\theta}_{\theta=\theta(w)})\frac{y-w}{r+\delta} = 0.
$$

Mortensen-Pissarides (3)

- Nash bargaining assumption: $\eta J = (1-\eta)\left[V - V^u\right]$
- $\Rightarrow w = (1-\eta)b + \eta(y + \theta\gamma)$
- Hold-up problems and search externalities
- Hosios (1989): equilibrium is constrained efficient if $\eta = M_U U/M$
  (although cf: Acemoglu and Shimer, 1999)
- Shimer (2005), Hagedorn and Manovskii (2008), etc: can match cyclical volatility of unemployment only if match surplus $y - b$ is very small

Decreasing returns to labor

- Natural generalization if fixed factors (at cyclical frequencies)
  - Lucas span of control
  - Intellectual property
  - Lumpy adjustment of physical capital
- Question: how are wages set for marginal and inframarginal workers?

Stole and Zwiebel

- Benchmark assumption:

$$\eta\left[J(n) - J(n-1)\right] = (1-\eta)\left[V(n) - V^u\right]$$

- Stole and Zwiebel (1996) provide microfoundation
  - Firm bargains sequentially with workers (Rubinstein, 1982)
  - Outside options: for firm, start bargaining game again with $n-1$ workers; for worker, unemployment

Stole and Zwiebel (2)

Static example

- Continuous employment variant:
  $\eta J'(n) = (1-\eta)\left[V(n) - V^u\right]$
- Firm with n workers; $\eta = 1/2$
  - $J(n) = y(n) - nw(n)$ and $V(n) = w(n)$
  - $\Rightarrow y'(n) - nw'(n) - w(n) = w(n) - b$
  - $\Rightarrow w(n) = \frac{1}{2}\frac{\int_0^n \nu y'(\nu)\,d\nu}{\int_0^n \nu\,d\nu} + \frac{1}{2}b$
- Firm chooses n to maximize $y(n) - nw(n)$; at optimum $w(n) = b$, but $y'(n) < b$.

Surplus division and control rights

- Surplus: $y(n) - nb = J(n) + n\left[V(n) - V^u\right]$
- Marginal surplus from hiring last worker: $y'(n) - b$, split as
  - Firm gets $J'(n)$
  - Newly-hired worker gets $V(n) - V^u$
  - Each existing worker gets $V'(n)$
- Question: who gets to choose hiring?
  - Stole and Zwiebel: the firm
  - Hold-up problems
    1. New worker extracts rent from (firm and existing workers)
    2. Firm extracts rent from existing workers

Private efficiency

- After firm and workers meet, can commit to act in privately efficient manner in future
  - Hold-up problem 2 is not inevitable
  - Barro (1977)
  - Sign contract at hiring to maximize surplus

$$S(n) = J(n) + n\left[V(n) - V^u\right]$$

- Firm and workers cannot commit to pre-interaction behavior (search, entry, ex ante investment)
  - Hold-up problem 1 is inevitable
  - Caballero and Hammour (1996): ‘fundamental transformation’
  - Surplus-sharing: $\eta S'(n) = V(n) - V^u$

This paper

$$S(n) = J(n) + n\left[V(n) - V^u\right]$$
$$\eta S'(n) = V(n) - V^u$$

Privately efficient decision-making by firm-worker group

- What does equilibrium in an economy with such a bargaining rule look like?
  - In steady state?
  - With aggregate shocks?
- How does it differ from the Stole-Zwiebel benchmark in the literature?

Implementation through payments

- Conditional on assumption of privately efficient actions, actual time path of payments doesn't matter
  - PDV of wage payments affects firm's profit, which determines entry
  - Arrangement over time is indeterminate
- One possible implementation: when new worker is hired, make transfer to existing workers and afterwards pay all workers the same
  - Transfer ≥ 0 (compensation for loss of bargaining power with falling marginal product)

Static example redux

- Production stage (S):
  - $S(n) = [y(n) - nw(n)] + n\,[w(n) - b] = y(n) - nb$
  - $\eta\,[y'(n) - b] = w(n) - b$
- Hiring stage ($\hat{S}$):
  - One firm, many workers
  - $\Rightarrow \hat{S}(n) = S(n^*)$ for all $n \leq n^*$; no surplus generated by hiring (but worker anticipates $w(n)$ rather than b)
  - $\Rightarrow$ hired worker pays $w(n) - b$ to firm and existing workers.
  - Surplus from hiring an additional worker beyond n^* would be $y'(n^*) - b$
  - $\Rightarrow n^*$ is efficient (and $w(n^*) = b$)

Search frictions

- Model as before; frictional hiring. Pay cost of γ to hire a worker in hiring stage
- For $n \leq n^*$, $\hat{S}(n) = \hat{S}(n^*) - \frac{1}{1-\eta}(n^* - n)\gamma$.
- $\Rightarrow (1-\eta)\hat{S}'(n) = \gamma$
- $\Rightarrow$ payment by hired worker to (firm and existing workers) is reduced by $\frac{\eta}{1-\eta}\gamma$
- Equilibrium:

$$y'(n^*) = b + \frac{1}{1-\eta}\gamma$$

$$w(n^*) = b + \frac{\eta}{1-\eta}\gamma$$

- Efficient?

Plan of talk

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Benchmark model

- Continuous time; common discount rate r
- Mass 1 of workers; risk-neutral
- Unemployment income $b > 0$
- Large mass of potential firms; pay k to become active
- Active firms post vacancies; flow marginal cost γ
- Flow of matches $M(u, v)$; assume CRS, let $\theta = v/u$, and $q(\theta) = M(u, v)/v = M(u/v, 1)$. Then
  - Vacancy yield $1/q(\theta)$
  - Job finding rate $f(\theta) \equiv \theta q(\theta)$
- Flow output of firm with n workers $y(n)$, $y' > 0$, $y'' < 0$
- Firm destruction rate δ

Optimal vacancy posting

- Posting v vacancies for time dt yields $v\,dt/q(\theta)$ hires and costs $v\gamma$.
- So does posting ϕv vacancies for time dt/ϕ.
- Optimal to have $\phi \to \infty$; hire n^* workers instantly at cost $n^* \frac{\gamma}{q(\theta)}$

Constrained efficiency

$$\max_{e,n^*,\theta,U} \int_0^\infty e^{-rt} \left(\left[\frac{y(n^*)}{r+\delta} - n^* \frac{\gamma}{q(\theta)} - k \right] e + bU \right) dt$$

subject to: $n^* e = \theta q(\theta) U$

$$\dot{U} = \delta(1-U) - \theta q(\theta) U \qquad [\lambda]$$

First order conditions:

$$y'(n^*) = \frac{1}{n^*} \left[y(n^*) - (r+\delta)k \right] \qquad [n^*]$$

$$r\lambda - \dot{\lambda} = \left[\frac{y(n^*)}{r+\delta} - n^* \frac{\gamma}{q} - k \right] \frac{\theta q}{n^*} + b - \lambda(\delta + \theta q) \qquad [U]$$

$$0 = \left[\frac{y(n^*)}{r+\delta} - n^* \frac{\gamma}{q} - k \right] \frac{U}{n^*} \frac{d\theta q}{d\theta} - e n^* \gamma \frac{d\frac{1}{q}}{d\theta} - \lambda \frac{d\theta q}{d\theta} U \qquad [\theta]$$

PE bargaining: firm size

- For $n = n^*$, $S(n^*) = \frac{y(n^*) - n^* r V^u}{r+\delta}$
- For $n < n^*$, $S(n) = -(n^* - n)\frac{1}{1-\eta}\frac{\gamma}{q} + S(n^*)$
- For $n > n^*$, $S(n) = \frac{y(n) - n r V^u}{r+\delta}$
- FOC for n^*:

$$\frac{y'(n^*) - rV^u}{r+\delta} = \frac{1}{1-\eta}\frac{\gamma}{q}$$

$$\Rightarrow \qquad \forall n \leq n^*, \quad V(n) - V^u = \eta S'(n) = \frac{\eta}{1-\eta}\frac{\gamma}{q}$$

PE bargaining: Bellman equations

$$
\begin{aligned}
rV^u &= b + \theta q \int_n h(n)\left[V(n) - V^u\right] dh(n) \\
&= b + \frac{\eta}{1-\eta}\gamma\theta \\
rV(n) &= w(n) + \delta\left[V^u - V(n)\right]
\end{aligned}
$$

$\Rightarrow$

- $V(n) \equiv V^e$ and $w(n) \equiv w$
- $w = b + \frac{\eta}{1-\eta}\frac{\gamma}{q}(r + \delta + \theta q)$.

PE bargaining: closing the model

Free entry:

$$k = S(0) = \frac{y(n^*) - n^* r V^u}{r + \delta} - n^* \frac{1}{1-\eta} \frac{\gamma}{q}$$

Optimal n^*:

$$\frac{y'(n^*) - r V^u}{r + \delta} = \frac{1}{1-\eta} \frac{\gamma}{q}$$

Equilibrium value of unemployment:

$$r V^u = b + \frac{\eta}{1-\eta} \gamma \theta$$

Key endogenous variables: θ, rV^u, n^*

PE bargaining: determining n^*

Substitute for rV^u into first two equations:

$$\frac{y'(n^*) - b - \frac{\eta}{1-\eta}\gamma\theta}{r+\delta} = \frac{1}{1-\eta}\frac{\gamma}{q}$$

$$\frac{y(n^*) - n^* b}{r+\delta} - \frac{\gamma n^*}{(1-\eta)q}\left(1 + \frac{\eta\theta q}{r+\delta}\right) = k$$

Subtract: $$y'(n^*) = \frac{1}{n^*}\left[y(n^*) - (r+\delta)k\right]$$

Decentralized equilibrium always generates the efficient firm size

PE bargaining: determining entry

- After some algebra, can also show

$$\Rightarrow \qquad \theta = \theta^* \text{ iff } \eta = -\frac{\theta q'(\theta)}{q(\theta)} = \frac{M_U U}{M}$$

- Efficient entry iff η satisfies the Hosios condition.
  - Rent-extraction by workers balances out the fact that firms ignore their congestion externalities on other firms
- (Same as in Mortensen-Pissarides benchmark.)

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- Benchmark model
- Efficient allocations
- Equilibrium allocations with PE *and SZ bargaining*
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SZ bargaining: wage determination

$$\eta J'(n) = (1-\eta)\left[V(n) - V^u\right]$$

Assume 'hiring opportunities' arrive at Poisson rate α.

$$(r+\delta)J(n) = y(n) - nw(n) + \alpha\left[J(n^*) - J(n) - (n^* - n)\frac{\gamma}{q}\right]$$

$$rV(n) = w(n) + \delta\left[V^u - V(n)\right] + \alpha\left[V(n^*) - V(n)\right]$$

$$\Rightarrow \qquad w(n) = (1-\eta)rV^u + n^{-\frac{1}{\eta}}\int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu$$

Note: this does not depend on α, so can return to the case $\alpha \to +\infty$.

SZ bargaining: key equations

Wages: $$w(n) + \eta n w'(n) = \eta y'(n) + (1-\eta) r V^u$$

$$w(n) = (1-\eta) r V^u + n^{-\frac{1}{\eta}} \int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu$$

FOC for optimal n^*:

$$\frac{\gamma}{q} = J'(n^*) = \frac{y'(n^*) - w(n^*) - n^* w'(n^*)}{r+\delta}$$

Worker Bellman equations:

$$r V(n^*) = w(n^*) + \delta \left[V^u - V(n^*) \right]$$

$$r V^u = b + \theta q \left[V(n^*) - V^u \right]$$

Free entry: $$k = \frac{y(n^*) - n^* w(n^*)}{r+\delta} - n^* \frac{\gamma}{q}$$

SZ bargaining: efficiency of hiring

Free entry: $\frac{1}{n^*}\left[y(n^*) - (r+\delta)k\right] = w(n^*) + (r+\delta)\frac{\gamma}{q}$

Efficient hiring: $\frac{1}{n^*}\left[y(n^*) - (r+\delta)k\right] = y'(n^*)$

$$\Rightarrow \qquad \frac{y'(n^*) - w(n^*)}{r+\delta} = \frac{\gamma}{q}$$

FOC for equilibrium n^*:

$$\frac{y'(n^*) - w(n^*) - n^* w'(n^*)}{r+\delta} = \frac{\gamma}{q} \qquad \Rightarrow w'(n^*) = 0$$

which requires that $-\int_0^{n^*} n^{\frac{1}{\eta}} y''(\nu)\, d\nu = 0$.
Impossible if y is strictly concave.

SZ model: efficiency of entry

- S-Z bargaining with n^* constrained at the efficient value decentralizes the efficient allocation with the Hosios η.
- If the government taxes firm output at rate τ and rebates the proceeds lump sum to firms, the equilibrium is efficient if n^* is efficient

$$(1-\eta)\left[\frac{1-\tau}{\eta}n^{*-\frac{1}{\eta}}\int_0^{n^*}\nu^{\frac{1}{\eta}-1}y'(\nu)\,d\nu - rV^u_{\text{eff}}\right] = (r+\delta)\frac{\gamma}{q}$$

PE bargaining, Cobb-Douglas production

Assume that $y(N) = An^{\alpha}$.
PE bargaining:

$$(r+\delta)k = A(1-\alpha)n^{*\alpha}$$

$$\underbrace{\frac{\gamma}{1-\eta}\left[\eta\theta + \frac{r+\delta}{q(\theta)}\right]}_{\text{increasing in } \theta} = A\alpha n^{*\alpha-1}$$

$$= \frac{A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{((r+\delta)k)^{\frac{1-\alpha}{\alpha}}} - b$$

Unique equilibrium; constrained efficient if Hosios condition holds.

SZ bargaining, Cobb-Douglas production

SZ bargaining: (with $\phi = (\alpha\eta + 1 - \eta)^{-1} > 1$),

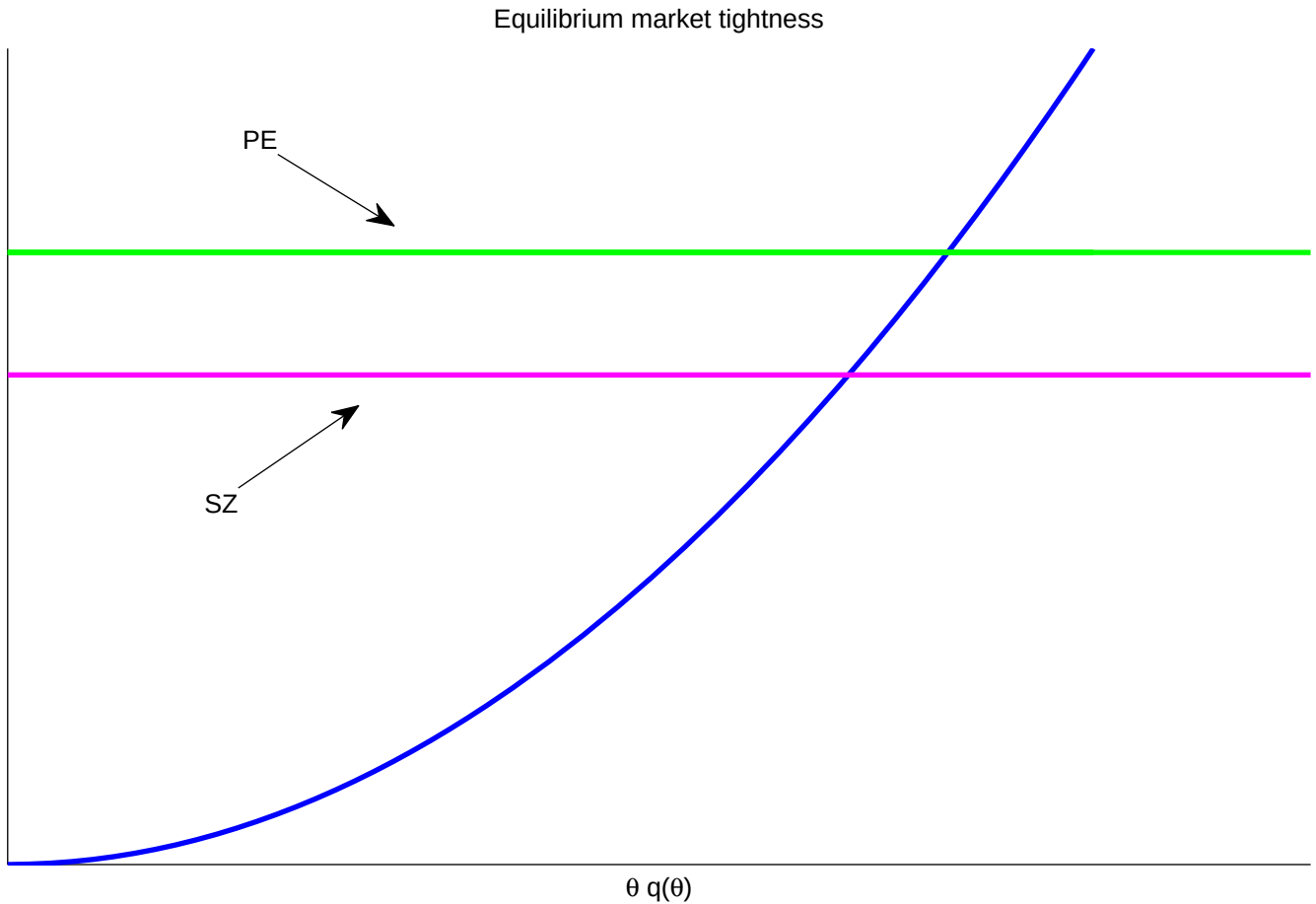
$$w(n) = (1-\eta)rV^u + \eta\phi A\alpha n^{\alpha-1}$$

$$(r+\delta)k = \boxed{(1-\eta)\phi} A(1-\alpha)n^{*\alpha}$$

$$\frac{\gamma}{1-\eta}\left[\eta\theta + \frac{r+\delta}{q(\theta)}\right] = \boxed{\phi} A\alpha n^{*\alpha-1}$$

$$= \boxed{(1-\eta)^{\frac{1-\alpha}{\alpha}}\phi^{\frac{1}{\alpha}}} \frac{A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{((r+\delta)k)^{\frac{1-\alpha}{\alpha}}} - b$$

- $(1-\eta)\phi < 1$: so n^* is distorted upwards.
- $\phi > 1$: so θ is distorted upwards (conditional on n^*)
- $(1-\eta)^{\frac{1-\alpha}{\alpha}}\phi^{\frac{1}{\alpha}} < 1$: so on net, θ is distorted downwards


Equilibrium market tightness
PE
SZ
θ q(θ)

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Calibration strategy

Parameters: A, α, b, γ, r, δ, $M(u,v) = Zu^{\rho}v^{1-\rho}$, η
Targets:

- $r = 0.05$
- $\delta = 0.10$: separation rate (CPS: Shimer, 2005)
- $\rho = 0.72$: slope of Beveridge curve (CPS: Shimer 2005)
- Z to hit $\theta q = 1.355$ quarterly (CPS: Shimer 2005)
- $\eta = \rho$ (Hosios)
- Normalizations: $b = 1$, $q = 1$
- $n^* = 23.8$ (Davis et al, 2006): determines A/k
- $b/w = 0.4$ (Shimer, 2005): determines γ and A
- $\alpha = 0.67$ (i.e., η)

Distinguishing SZ and PE

Proposition: an econometrician with access only to the moments used in the calibration cannot determine whether bargaining is PE or SZ.

Intuition: the two models are identical if we set

$$(A/k)_{PE} = (1-\eta)\phi(A/k)_{SZ}$$

and

$$\left(\frac{A^{\frac{1}{\alpha}}}{k^{\frac{1-\alpha}{\alpha}}}\right)_{PE} = (1-\eta)^{\frac{1-\alpha}{\alpha}}\phi^{\frac{1}{\alpha}}\left(\frac{A^{\frac{1}{\alpha}}}{k^{\frac{1-\alpha}{\alpha}}}\right)_{SZ}$$

The SZ distortion reduces productive efficiency relative to PE: so A/k must be higher. Then adjust A (or k) to make the free entry condition hold for the same θ.

Factor shares

- Note that firms make zero profits in both models.
- But the share of output spent on entry costs is very different across the two models.
  - Under PE, $k = (1-\alpha)\frac{y(n^*)}{r+\delta}$.
  - Under SZ, $k > (1-\alpha)\frac{y(n^*)}{r+\delta}$.
  - For $\alpha = 0.67$, the two shares are 0.33 and 0.43.
- That is, an econometrician who can identify α and the return to the fixed factor can distinguish the two bargaining models.
- Weak evidence in favor of PE bargaining. . .

Steady-state elasticities

The response of labor-market variables to a permanent 1 percent rise in productivity:

- Unemployment falls from 6.87% to 6.83%
- Firm size falls from 23.8 to 23.44
- Wages rise by 1.49%
- Welfare of the unemployed rises by 1.47%

Modification (1)

Higher replacement rate: $b/w = 0.98$.
The response of labor-market variables to a permanent 1 percent rise in productivity:

- Unemployment falls from 6.87% to 5.93%
- Firm size falls from 23.8 to 23.44
- Wages rise by 1.47%
- Welfare of the unemployed rises by 1.39%

cf Hagedorn and Manovskii (2008)

Modification (2)

No extensive margin of adjustment (number of firms remains the same). $b/w = 0.4$

- Unemployment falls from 6.87% to 6.84%
- Firm size rises from 23.8 to 23.808
- Wages rise by 0.99%
- Welfare of the unemployed rises by 0.97%

Modification (3)

$\alpha = 0.98$, entry allowed, $b/w = 0.4$

- Unemployment falls from 6.87% to 6.84%
- Firm size falls from 23.8 to 23.56
- Wages rise by 1.00%
- Welfare of the unemployed rises by 0.99%

(Note: α crucially affects how large the distortion in the Stole and Zwiebel model is.)

Modification (4)

$\eta = 0.01$, entry allowed, $b/w = 0.4$, $\alpha = 0.67$

- Unemployment falls from 6.87% to 6.51%
- Firm size falls from 23.8 to 23.44
- Wages rise by 4.38%
- Welfare of the unemployed rises by 4.46%

(Note: low η means γ is very high; so small shocks to A generate a lot more hiring.)

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Aggregate shocks

Return to the case of PE bargaining henceforth.
For $n \geq n^*(p)$:

$$J_p(n) = y_p(n) - nw_p(n) + \beta(1-\delta)\sum_{p'} \lambda_{p,p'} \left[J_{p'}(n) - nt_\lambda(p,p',n)\right]$$

$$V_p(n) = w_p(n) + \beta(1-\delta)\sum_{p'} \lambda_{p,p'} \left[V_{p'}(n) + t_\lambda(p,p',n)\right]$$

$$S_p(n) = y_p(n) - n(1-\beta)V^u + \beta(1-\delta)\sum_{p'} \lambda_{p,p'} S_{p'}(n)$$

Aggregate shocks (2)

For $n < n^*(p)$:

$$J_p(n) = -\frac{1}{1-\eta}\frac{\gamma}{q_p}(n^*(p) - n) - nt_h(p,n) + J(p, n^*(p))$$

$$V_p(n) = t_h(p,n) + J(p, n^*(p))$$

$$S_p(n) = -\frac{1}{1-\eta}\frac{\gamma}{q_p}(n^*(p) - n) + S_p(n^*(p))$$

$$S_p'(n^*(p)) = \frac{1}{1-\eta}\frac{\gamma}{q_p}$$

$$V^u(p) = \theta_p q_p \left[V^u(p) + \frac{\eta}{1-\eta}\frac{\gamma}{q_p}\right] + (1 - \theta_p q_p)\left[b + \beta V^u(p)\right]$$

Aggregate shocks (3)

Look for solution in which $\theta = \theta_p$ depends only on p, and firms never wish to fire workers.
For $i \leq j$:

$$S_i(n_j^*) = y_i(n_j^*) - n_j^* \left[b + \frac{\eta}{1-\eta} \frac{\gamma \theta_i}{1 - \theta_i q_i} \right] + \beta(1-\delta) \sum_{i'} \lambda_{i,i'} S_{i'}(n_j^*)$$

$$S_i'(n_j^*) = y_i'(n_j^*) - \left[b + \frac{\eta}{1-\eta} \frac{\gamma \theta_i}{1 - \theta_i q_i} \right] + \beta(1-\delta) \sum_{i'} \lambda_{i,i'} S_{i'}'(n_j^*)$$

with $S_i(n_i^*) = k + n_i^* \frac{1}{1-\eta} \frac{\gamma}{q_i}$ and $S_i'(n_i^*) = \frac{1}{1-\eta} \frac{\gamma}{q_i}$.
Define $T_{ij} = S_i(n_j^*) - n_i^*(S_i'(n_j^*)$. Then $T_{ii} = k$ and

$$T_{ij} = y_i(n_j^*) - n_j^* y_i'(n_j^*) + \beta(1-\delta) \sum_{i'} \lambda_{i,i'} T_{i',j} \quad \Rightarrow \quad \{n_i^*\}$$

Aggregate shocks (4)

- Preliminary conclusion: aggregate shocks in full stochastic model have very similar effects to aggregate shocks.
- In fact, unemployment, firm size, and wages respond slightly less than in the steady state.
- Elsby and Michaels (2010): idiosyncratic shocks

Time to hire

$$(r+\delta)J(n) = y(n) - nw(n) - \gamma + (q - sn)(J'(n) - nt_n)$$

$$rV(n) = w(n) + (q - sn)\left[V'(n) + t_n\right] + (s+\delta)\left[V^u - V(n)\right]$$

$$\Rightarrow (r+\delta)S(n) = y(n) - \gamma + (q(1-\eta) - sn)S'(n)$$

Optimality: $S'(n^*) = \frac{\gamma}{1-\eta}$

Free entry: $S(0) = k$

Efficient bargaining: $\eta S'(n) = V(n) - V^u$

$$rV^u = b + \theta q \frac{\int_0^{n^*} h(n)[V(n) - V^u]\, dG(n)}{\int_0^{n^*} h(n)\, dG(n)}$$

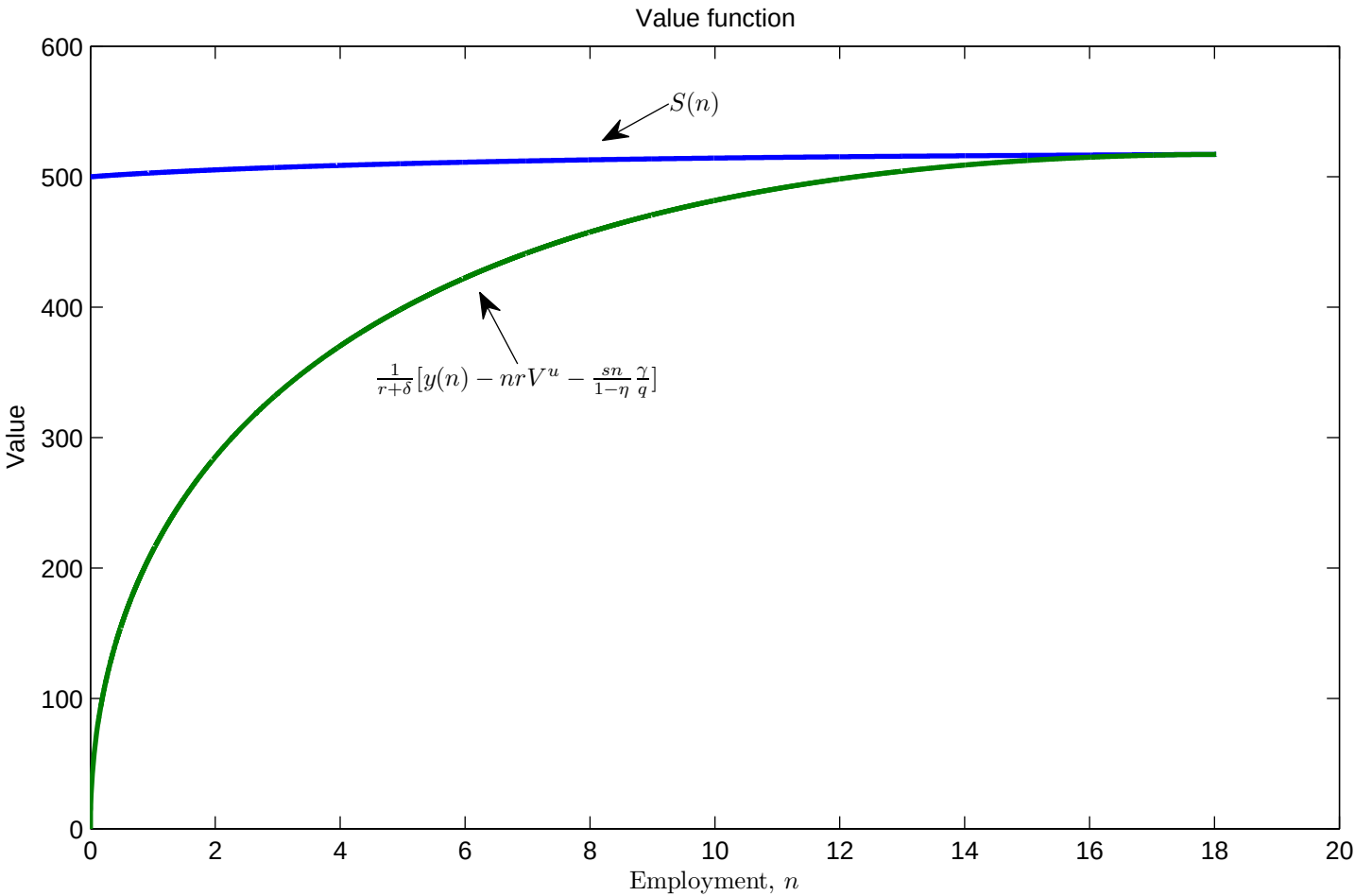
Steady-state flow equations for firm size distribution $G(n)$.

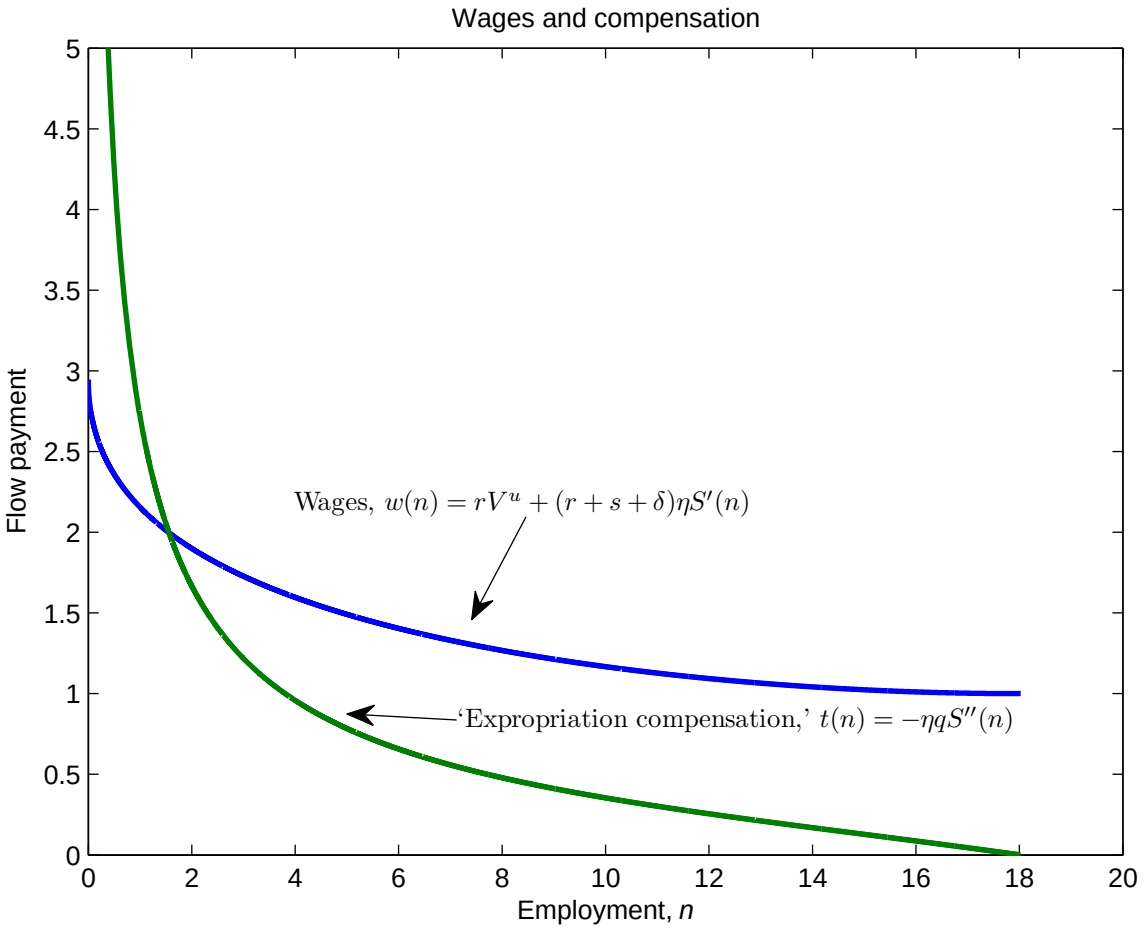
Wages

Long-term contracting: paths of $\{t_n, w_n\}$ indeterminate. One natural assumption: $t_n = -V'(n)$. Workers are then paid higher wages in growing firms as compensation for their loss of bargaining power as the firm grows.

Conclusion

- The benchmark model for bargaining in multi-worker firms should not be the benchmark model . . .
- Using worker-side data alone, the two models are difficult to distinguish . . .
- . . . but using factor shares may give some chance of distinguishing them.
- Clear policy implications in the steady state . . .
- . . . but adds little to our understanding of cyclical fluctuations relative to the MP benchmark.


Value function
600
500
400
300
200
100
0
0
2
4
6
8
10
12
14
16
18
20
Value
Employment, n
$S(n)$
$\frac{1}{r+\delta}[y(n) - nrV^u - \frac{sn}{1-\eta}\frac{\gamma}{q}]$


Wages and compensation
Flow payment
Employment, n
Wages, $w(n) = rV^u + (r+s+\delta)\eta S'(n)$
'Expropriation compensation,' $t(n) = -\eta q S''(n)$
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
2
4
6
8
10
12
14
16
18
20