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**MONETARY CONTROL UNDER ALTERNATIVE OPERATING PROCEDURES**

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## 1. Introduction

This paper provides a detailed examination of various money stock control procedures in a rational expectations environment. The analysis investigates the relative efficiency of controlling a monetary aggregate through the use of an interest rate instrument or through various reserve measures under both lagged and contemporaneous reserve requirements. A major result is that borrowed reserve targeting is not necessarily equivalent to a noisy interest rate instrument. Further, it is possible that borrowed reserves can represent a more efficient control procedure than an interest rate instrument. However, total reserve targeting under contemporaneous reserve requirements generally provides the most efficient control of money and the lowest variability of prices and output. However, total reserve targeting also implies higher interest rate volatility, which may be one reason why the Fed has never adopted this procedure.

The conclusion that borrowed reserve targeting is not unambiguously worse than an interest rate instrument is different from that derived by McCallum and Hoehn (1983) and results in part from the incomplete description of the market for reserves used in their paper. Their paper does, however, provide the basic framework employed below, and represents a useful first attempt to analyze the effects of various monetary control procedures in a well posed rational expectations macro model. By incorporating a more developed model of the reserve market, this paper shows that their conclusion only occur for a specific stochastic specification of shocks and a particular information structure. In a more general setting that includes a detailed treatment of bank behavior in the reserve market their results may not be robust. Specifically, if monetary disturbances have some permanence, and

banks trade in the federal funds market based on information contained in the current funds rate and private observations on their own portfolio, then borrowed reserve targeting may not be inferior to an interest rate instrument. Using an interest rate instrument destroys an important source of information, which adversely affects the variances of money, output, and prices around their expected value. The effects of the information loss are analogous to the results found in Dotsey and King (1983, 1986) in slightly different contexts. The use of borrowed reserve targeting may therefore involve a tradeoff between the more efficient procedure of total reserve targeting and a concern for the volatility of interest rates.

The structure of the paper is as follows. In Section 2 the basic macroeconomy is described. Section 3 analyzes the various operating procedures and compares the variances of money, interest rates, the price level, and output under alternative procedures. Since attention is commonly given to the monetary control properties of operating procedures, the paper emphasizes this aspect. Section 4 presents a numerical comparison of the various variances and Section 5 briefly summarizes the paper.

## 2. The Macroeconomic Model

The macroeconomic model used in this paper is essentially that of McCallum and Hoehn (1983) with the exception that lagged output is omitted from the aggregate supply relationship. This omission simplifies some of the algebra without affecting the qualitative nature of the results. All variables, with the exception of the nominal interest rate  $r_t$ , are in logarithms.

The log of aggregate supply,  $y_t^s$ , and aggregate demand,  $y_t^d$ , are given by:

$$(1) \quad y_t^s = a_1^s(p_t - E_{t-1}^+ p_t) + u_t$$

$$(2) \quad y_t^d = a_0 - a_1^d(r_t + p_t - E_{t-1}^+ p_{t+1}) + w_t$$

where  $p_t$  is the log of the price level,  $r_t$  is the nominal interest rate, and  $E_{t-1}^+$  represents the expectations operator conditional on the information set  $I_{t-1}^+$ . This set is assumed to contain all information pertaining to variables dated  $t-1$  and earlier, including past values of all disturbances. This notation is used because it will be important to distinguish between the information sets  $I_t$  and  $I_t^+$  where  $I_t = I_{t-1}^+$  plus the current interest rate, and information specific to individual banks.  $I_t$  does not include knowledge of the aggregate disturbances. The disturbances  $u$  and  $w$  are serially and mutually uncorrelated normally distributed random variables with mean zero and variances  $\sigma_u^2$  and  $\sigma_w^2$  respectively.

The real side of the model given by (1) and (2) represents a basic and fairly popular macroeconomic representation of an economy possessing natural rate properties. It will therefore only be discussed briefly. Equation (1) indicates that aggregate supply responds positively to unanticipated price level movements while equation (2) implies that aggregate demand is negatively related to the expected real rate of interest. For simplicity, the definition of a period corresponds to the length of the reserve maintenance period.<sup>1</sup>

The log of the real demand for money,  $m_t$ , depends positively on income and negatively on the interest rate. It is also affected by a

disturbance term  $v_t = \rho v_{t-1} + e_t$  where  $e_t$  is an independently normally distributed random variable with mean zero and variance  $\sigma_e^2$ . Therefore, the money demand disturbance has a degree of permanence, a property that is often emphasized in Federal Reserve literature. The permanence in the money demand shock may be due to technology type shocks to cash management. As indicated later in the paper some avenue for generating persistence is necessary for the existence of a nontrivial comparison between operating procedures. Putting persistence in the money demand shock is merely the simplest way to proceed. Formally,

$$(3) \quad m_t - p_t = c_0 - c_1 r_t + c_2 y_t + v_t.$$

Equations (1), (2) and (3) summarize the economy. As in McCallum and Hoehn they are assumed to be invariant to the form of the policy rules considered in this paper.

### 3. The Effects of Various Operating Policies

Three basic policies will be analyzed: an interest rate instrument, a borrowed reserve target, and a total reserve target. It will also be shown that a non-borrowed reserve target under lagged reserve requirements amounts to a noisy borrowed reserve target. The effects of the first two policies are independent of the reserve accounting regime while the results for targeting total reserves is not. Therefore, total reserve targeting regime will be analysed under both lagged reserve requirements (LRR) and contemporaneous reserve requirements (CRR). The investigation emphasizes the variability of money and interest rates, but a complete description of the results which includes output and price level variability is contained in Table 1.

3a. An Interest Rate Instrument

The effective use of an interest rate instrument in a rational expectations model was first examined by McCallum (1981), and the following analysis is based on his work. In order to hit some money target  $m_t^*$ , the monetary authority at the end of period  $t-1$  decides on an interest rate that is expected to yield  $m_t^*$ .<sup>2</sup> Rearranging (3) gives the targeted interest rate  $r_t^*$  as

$$(4) \quad r_t^* = \frac{1}{c_1} (c_0 + E_{t-1}^+ p_t + c_2 E_{t-1}^+ y_t + \rho v_{t-1} - m_t^*)$$

where  $E_{t-1}^+ y_t = 0$ . Pegging  $r_t$  at  $r_t^*$  implies that  $E_{t-1}^+ m_t = m_t^*$ . The actual realization of the monetary aggregate will not in general equal  $m_t^*$  due to the contemporaneous disturbance terms in the model. Using (1) and (2) to solve for  $p_t$  and  $y_t$ , (see appendix), it can be shown that

$$p_t - E_{t-1}^+ p_t = (1/a_1)(u_t - w_t) \text{ and } y_t - E_{t-1}^+ y_t = (1/a_1)(a_1^s u_t + a_1^d w_t) \text{ where}$$

$$a_1 \equiv a_1^s + a_1^d. \text{ Substituting into (3) yields}$$

$$(5) \quad m_t - m_t^* = m_1 w_t - m_2 u_t + e_t$$

and that

$$(6) \quad E (m_t - m_t^*)^2 = m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2$$

$$\text{where } m_1 \equiv (1+c_2 a_1^s)/a_1 \text{ and } m_2 \equiv (1-c_2 a_1^d)/a_1.$$

One notes that this procedure of monetary control circumvents any particular specification of the market for reserves making it insensitive to both the accounting regime (LRR or CRR) and discount window operations.

### 3b. Borrowed Reserve Targeting

The implementation of borrowed reserve targeting also works off the interest rate, but in a less direct manner than an interest rate instrument. The monetary authority estimates the expected level of borrowing that is consistent with achieving the interest rate,  $r_t^*$ , in (4). This implies that the targeted level of borrowing is consistent with achieving an expected value of money of  $m_t^*$ .

The demand for borrowed reserves is therefore an important consideration in designing a borrowed reserve targeting scheme. Based on the work of Goodfriend (1983) a log linear approximation that captures the essential characteristics of his demand for borrowed reserves equation can be written as:

$$(7) \quad br_t(z) = b_0 + b_1 r_t - b_2 E_{zt} r_{t+1} - b_3 br_{t-1}(z) + b_t(z)$$

where  $br_t(z)$  is the log of borrowed reserves demanded by bank  $z$ ,  $b_t(z)$  is an idiosyncratic shock to borrowing equal to an economy wide average shock to borrowing,  $b_t$ , and a bank specific shock,  $\beta_t(z)$ . Both  $b_t$  and  $\beta_t(z)$  are independently normally distributed random variables with means zero and variances  $\sigma_b^2$  and  $\sigma_\beta^2$  respectively. The conditional expectations operator  $E_{zt}$  is based on the beginning of time  $t$  information held by banks trading in the federal funds market and is assumed to include  $I_{t-1}^+$  and  $r_t$  in addition to

observations on  $b_t(z)$  and movements in the bank's own deposits. It is therefore assumed that banks observe  $m_t(z) = m_t + \varepsilon_t(z)$  where  $\varepsilon_t(z)$  is bank specific demand for money disturbance that is independently normally distributed with mean zero and variance  $\sigma_\varepsilon^2$ .

The demand for borrowed reserves by banks is based on the non-price rationing scheme invoked at the discount window. Implicit in the specification of (7) is the assumption of a constant below market discount rate.<sup>3</sup> Banks are only allowed to borrow a certain number of times per quarter and face non-pecuniary costs based on their history of borrowing. Therefore, if banks have borrowed a lot in the past their demand for current borrowing will be less. Also, if the current interest rate is high, implying a large discount window subsidy, desired borrowing will rise, while if the expected future rate is high banks will postpone borrowing in order to take advantage of the future subsidy.

Aggregating (7) across banks yields:

$$(8) \quad br_t = b_0 + b_1 r_t - b_2 \overline{E_{zt}} r_{t+1} - b_3 br_{t-1} + b_t$$

where the bar over  $E_{zt}$  indicates the average of all banks expectations. Consistent with  $E_{t-1}^+ r_t = r_t^*$  and therefore  $E_{t-1}^+ m_t = m^*$  is an aggregate borrowing level of:

$$(9) \quad br_t^* = b_0 + b_1 r_t^* - b_2 E_{t-1}^+ r_{t+1} - b_3 br_{t-1}$$

Therefore, the monetary authority will allow the current interest rate to fluctuate in order for actual borrowing to equal  $br_t^*$ . Equating (8) and (9) gives the interest rate  $r_t$  that results from this procedure.

$$(10) \quad r_t = r_t^* + \frac{b_2}{b_1} (\overline{E_{zt} r_{t+1}} - E_{t-1}^+ r_{t+1}) - \frac{1}{b_1} b_t$$

The nature of this policy has led to the observation that borrowed reserve targeting constitutes a noisy interest rate peg. The fluctuations in the interest rate in some sense allow the monetary authority to claim that it does not control interest rates, and that interest rates are market determined. This is no doubt one of the political reasons that makes this type of policy attractive. However, the introduction of noise in the interest rate does not necessarily imply an inferior monetary control procedure, nor does it imply greater variability in output or prices.

The expected value of the squared deviation of money from its target is given by:

$$(11) \quad E(m_t - m_t^*)^2 = [1 - (c_1 + a_1^d m_1) (b_2/b_1)^\theta \frac{\rho(1-\rho)}{1+c_1(1-\rho)}]^2 [m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2] \\ + [(c_1 + a_1^d m_1) (\theta/b_1 (\theta-\phi))]^2 \sigma_b^2$$

where  $0 < \phi < \theta < 1$  reflect weighted averages of the various variances  $\sigma_e^2$ ,  $\sigma_\epsilon^2$ ,  $\sigma_b^2$  and  $\sigma_\beta^2$  arising from the signal extraction problem faced by banks in the federal funds market (see appendix).

Comparing (11) to (6), one observes that the relative efficiency of monetary control between the interest rate instrument and a borrowed reserve target depends on the size of the coefficient multiplying  $m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2$  and the variability of the demand for borrowed reserves. This coefficient is less than one and reflects the value of the information contained in the interest rate under borrowed reserve targeting. Similar terms appear in the expressions for price level and output variance in Table 1. Using reasonable parameter values, the analysis of Section 4 indicates that borrowed reserve targeting is approximately equivalent to an interest rate instrument in terms of the efficiency of monetary control.

Theoretically, there is a possibility for borrowed reserve targeting to improve monetary control if the effect that a shock has on the interest rate partially offsets the direct effect that the shock has on the demand for money. For this to happen, there must be a positive covariance between  $(r_t - E_{t-1}^+ r_t)$  and  $e_t$ ,  $w_t$ , and  $-u_t$ , which is the case.

Some intuition can be obtained by examining the case of a money demand disturbance. Because there is some persistence to the money demand disturbance ( $0 < \rho < 1$ ), next period's interest rate is influenced by the current disturbance. Therefore, when agents in the funds market have sufficient information to partially discern the disturbance, today's demand for borrowing will be influenced by the current money demand shock. In the case of a partially perceived positive shock, banks will expect next period's interest rate to rise and reduce their demand for current borrowing. In order to induce banks to borrow  $br_t^*$  the current interest rate must rise. The rise in the interest rate will reduce the demand for money and generally result in  $m_t$  deviating less from  $m_t^*$ .

One observes that for  $\rho$  equal to 0 or 1 that the coefficients on  $\sigma_w^2$ ,  $\sigma_u^2$ , and  $\sigma_e^2$  in (11) are identical to the values given in (6), and that (11) is, therefore, equal to (6) with an additional term involving  $\sigma_b^2$ . In these two cases the borrowed reserve targeting approach is unambiguously worse than the interest rate peg, which is basically the result derived by McCallum and Hoehn (1983). The reason is that in both these cases the money demand disturbance does not influence the interest rate and there is no offsetting effects arising from the covariance between  $r_t - r_t^*$  and the various shocks on  $E(m_t - m_t^*)^2$ .

In the case where  $\rho=0$ , this period's money demand disturbance will not affect  $r_{t+1}^*$  and hence will not affect  $E_{zt} r_{t+1}^*$ . This implies that current shocks (other than  $b_t$ ) will have no effect on today's demand for discount window borrowing and therefore no effect on  $r_t$ .

The result for  $\rho=1$  is a little less straightforward and arises because with  $\rho=1$  past money demand disturbances have no effect on the value of  $r_t^*$ . This is because the movement in  $E_{t-1}^+ p_t$  due to  $v_{t-1}$  exactly offsets the direct effect that  $v_{t-1}$  has on  $r_t^*$ . Hence, the current money demand disturbance does not change  $r_{t+1}^*$  and does not shift the current demand for borrowed reserves. This implies that there is no change in  $r_t$ . That this is the correct solution can also be seen by examining equilibrium in the output market. Past permanent positive shocks to the demand for money,  $v_{t-1}$ , cause the entire path of the price level to shift down one for one. Thus, no change in the inflation rate occurs. Since  $v_{t-1}$  is part of the information contained in  $I_{t-1}^+$ , aggregate supply is unaffected. Because there is no change in inflation, aggregate demand will be unaffected only if  $r_t$  is unaffected. Therefore, for the goods market to clear past money demand disturbance cannot influence the interest rate.

The condition that  $0 < \rho < 1$  is necessary for a money demand disturbance to have any effect in the above case is somewhat particular to this model. More generally, it is only required that money demand disturbances affect output and that these disturbances be propagated through time. A propagation mechanism such as the addition of a lagged output term in the aggregate supply equation would not be sufficient in this model since a white noise money demand shock would not affect output when the interest rate is either directly or indirectly targeted. In a model such as Barro (1980), an interest rate peg, and therefore a borrowed reserve targeting procedure, does not remove the effects of a white noise money demand disturbances on output (see Dotsey 1986). This means that in such a model one would only need heterogeneity of information among banks and some general propagation mechanism for borrowed reserve targeting to potentially outperform an interest rate instrument.

Further, in the model employed above, even when  $0 < \rho < 1$  banks must possess some idiosyncratic information regarding the money demand disturbance for economic disturbances to cause any movement in the interest rate. This occurs because disturbances only affect the current interest rate through the expectational channel  $(\overline{E}_{z_t} r_{t+1} - E_{t-1}^+ r_{t+1})$ . If banks have no direct observation of  $e_t$  through  $m_t(z)$ , then the interest rate will be unaffected by  $e_t$  and will not communicate any information regarding  $e_t$ . This is because with no heterogeneity of information it is clear from (10) that observing the interest rate will only convey this period's borrowed reserve demand shock. Thus, the two conditions of  $0 < \rho < 1$  and some direct (although perhaps imperfect) observation of  $e_t$  are needed for borrowed reserve targeting to outperform an interest rate instrument. These two conditions, however, do not seem

unreasonable since banks can probably infer something about  $e_t$  by examining movements in their own deposits and most empirical studies on money demand indicate that the disturbance term is positively autocorrelated.

### 3c. Non-borrowed Reserve Targeting (LRR)

Under lagged reserve requirements non-borrowed reserve targeting can be shown to be inferior to borrowed reserve targeting and essentially reduces to a noisy borrowed reserve targeting approach. The targeted level of non-borrowed reserves is set so that it plus the expected level of borrowed reserves is equal to the level of required reserves plus the expected level of excess reserves consistent with  $r_t^*$ . That is  $NBR_t^* + BR_t^* = \lambda M_{t-1} + E_{t-1}^+ ER_t$  where  $NBR_t^*$ ,  $BR_t^*$ ,  $ER_t$ , and  $M_{t-1}$  are the unlogged levels of targeted non-borrowed reserves, targeted borrowed reserves, excess reserves, and last periods money balances, while  $\lambda$  is the required reserve ratio. Taking logs and linearizing around the unconditional means,  $\overline{NBR}$ ,  $\overline{BR}$ ,  $\overline{ER}$ , and  $\overline{M}$ , yields:

$$(12) \quad nbr_t^* = d_0 + d_1 m_{t-1} + d_2 E_{t-1}^+ er_t - d_3 br_t^*$$

where small letters indicate the logarithm of the relevant variable and

$$d_1 = \lambda \overline{M} / \overline{NBR}, \quad d_2 = \overline{ER} / \overline{NBR}, \quad d_3 = \overline{BR} / \overline{NBR} \text{ and}$$

$$d_0 = \log(\overline{NBR}) - d_1 \log(\overline{M}) - d_2 \log(\overline{ER}) + d_3 \log(\overline{BR})$$

The derived demand for non-borrowed reserves is

$$(13) \quad nbr_t^d = d_0 + d_1 m_{t-1} + d_2 er_t^d - d_3 br_t^d.$$

Assuming that there is some white noise control error,  $n_t$ , in supplying non-borrowed reserves that is due to unanticipated changes in other sources or uses such as float or treasury balances, the supply of non-borrowed reserves will be  $nbr_t^* + n_t$ . Equating the supply and demand for non-borrowed reserves implies that

$$(14) \quad br_t = br_t^* + (d_2/d_3) (er_t - E_{t-1}^+ er_t) - (1/d_3) n_t$$

If excess reserve demand is interest insensitive (which empirically seems to be the case under LRR and subsidized discount window borrowing), then (14) reduces to borrowed reserve targeting plus some additional variability.<sup>4</sup>

### 3d. Total Reserve Targeting (LRR)

The case for total reserve targeting under lagged reserve requirements essentially amounts to excess reserve targeting. The demand for total reserves is equal to the sum of predetermined required reserves plus the demand for excess reserves,  $\lambda M_{t-1} + ER_t^d$ . The Fed will set its target for total reserves at a level that is expected to clear the reserve market at  $r_t^*$ . Hence  $TR_t^* = \lambda M_{t-1} + ER_t^*$ , where  $ER_t^*$  is the expected demand for excess reserves consistent with  $r_t^*$  and  $m_t^*$ . With no control error affecting the supply of total reserves, the interest rate in the current period will be that rate which yields  $ER_t^d = ER_t^*$ . In the presence of a proportional total reserve control error,  $\tau_t$  (where  $\tau_t$  is in percentage terms and is independent normally distributed with variance  $\sigma_\tau^2$ ), then the interest rate will be determined from the equilibrium condition

$$(15) \quad er_t^d = er_t^* + (1/\epsilon_t) \tau_t$$

where  $\epsilon_t \equiv \overline{ER}/(\lambda M_{t-1} + \overline{ER})$  and small letters refer to natural logarithms.<sup>5</sup>

If excess reserves are interest insensitive, then there is no conceivable way of implementing total reserve targeting with LRR, since current interest rate movements have no effect on the demand for total reserves. Alternatively if the demand for excess reserves was expressed as  $ER_t^d = e(r_t)M_t$ , as is done in the money multiplier approach to money supply, where  $e(r)$  is a general functional form, then a log linear approximation of excess reserve demand implies that

$$(16) \quad er_t^d = e_o - e_1 r_t + x_t + m_t$$

where  $x_t$  is an independent normally distributed random disturbance with variance  $\sigma_x^2$  that affects the demand for excess reserves. The setting of  $er_t^*$  will then be

$$(17) \quad er_t^* = e_o - e_1 r_t^* + m_t^*$$

and reserve market equilibrium implies that

$$(18) \quad r_t = r_t^* + \frac{1}{e_1} (m_t - m_t^*) + \frac{1}{e_1} (x_t - (1/\epsilon_t) \tau_t)$$

Using equation (1), (2), and (3) along with (18) implies that

$$(19) \quad E(m_t - m_t^*)^2 = (e_1/\delta)^2 [m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2] + [(c_1 + a_1^d m_1)/\delta]^2 [\sigma_x^2 + (1/\epsilon_t)^2 \sigma_\tau^2]$$

where  $\delta \equiv e_1 + c_1 + a_1^d m_1$ . In general, comparing the variance of money control errors under total reserve control, the peg, and borrowed reserve control will involve relative sizes of parameters and variances. However, one observes that  $(1/\varepsilon_t)$  is a rather large number (generally larger than 40) and that the proportional variation in excess reserves is large relative to the other variances in the model. Therefore, total reserve targeting under lagged reserve requirements will be an inefficient method of monetary control and will produce excessive volatility in prices, output, and interest rates.

The results given in (19) do not match up with those of McCallum and Hoehn. However, if the demand for excess reserves is assumed to be of the form  $ER_t^d = e(r_t)M_{t-1}$  then their results can be obtained. Under this assumption  $er_t^d = e_0 - e_1 r_t + x_t + m_{t-1}$  and  $er_t^* = e_0 - e_1 r_t^* + m_{t-1}$ . This implies that  $r_t = r_t^* + (1/e_1) (x_t - (1/\varepsilon_t) \tau_t)$  and that

$$(20) \quad E(m_t - m_t^*)^2 = m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2 + \left[ \frac{c_1 + a_1^d m_1}{e_1} \right]^2 (\sigma_x^2 + (1/\varepsilon_t)^2 \sigma_\tau^2)$$

which is essentially their result.

### 3e. Total Reserve Targeting Under (CRR)

The ability to control the money supply under total reserve targeting is sensitive to the reserve accounting regime in use. It will be shown that total reserve targeting is likely to produce better control of money and lower output and price level variability.

Under total reserve targeting and the assumption that  $ER_t^d = e(r_t)M_t$ , the monetary authority will attempt to supply total reserves,  $TR_t^*$ , equal to

$(\lambda + e(r_t^*))M_t^*$ . In reality, reserve control is not perfect and total reserve supply,  $TR_t^S$ , will be  $TR_t^S = (1 + \tau_t)TR_t^*$  where  $\tau_t$  is a proportional error affecting the supply of total reserves. Taking a log linearization of total reserve supply implies that

$$(21) \quad tr_t^S = e_0 - \varepsilon e_1 r_t^* + m_t^* + \tau_t$$

where  $\varepsilon = (\overline{ER}/\overline{TR})$  and  $tr_t^S$  is the log of total reserves supplied.

Using the accounting identity that total reserve demand equals  $\lambda M_t^d + ER_t^d$  implies that

$$(22) \quad tr_t^d = e_0 - \varepsilon e_1 r_t + \varepsilon x_t + m_t^d.$$

Equilibrium in the reserve market yields

$$(23) \quad m_t^d = m_t^* + \varepsilon e_1 (r_t - r_t^*) - \varepsilon x_t + \tau_t.$$

Employing (3) and the fact that  $m_t^* = E_{t-1}^+ m_t$  and  $r_t^* = E_{t-1}^+ r_t$  gives

$$(24) \quad r_t = r_t + (1/\hat{\delta})[m_1 w_t - m_2 u_t + e_t - \tau_t + \varepsilon x_t]$$

where  $\hat{\delta} = c_1 + \varepsilon e_1 + a_1^d m_1$ . In deriving (24) use was made of the fact that

$$p_t - E_{t-1}^+ p_t = -(a_1^d/a_1)(r_t - r_t^*) + (w_t - u_t)/a_1 \quad \text{and} \quad y_t - E_{t-1}^+ y_t = (a_1^d a_1^s/a_1)$$

$$(r_t - r_t^*) + (a_1^s/a_1)w_t + (a_1^d/a_1)u_t \quad \text{which implies that} \quad m_t^d = m_t^* - (c_1 + a_1^d m_1)$$

$$(r_t - r_t^*) + m_1 w_t - m_2 u_t + e_t.$$

Using (23) and (24) yields the efficiency expression for monetary control under total reserve targeting.

$$(25) \quad E(m_t - m_t^*)^2 = \left( \frac{\varepsilon e_1}{\varepsilon e_1 + c_1 + a_1^d m_1} \right)^2 [\sigma_e^2 + m_1 \sigma_w^2 + m_2 \sigma_u^2] + \left( \frac{c_1 + a_1^d m_1}{\varepsilon e_1 + c_1 + a_1^d m_1} \right)^2 (\varepsilon^2 \sigma_x^2 + \sigma_\tau^2).$$

This expression is similar to the expression given in McCallum and Hoehn (1983) and implies that total reserve targeting is likely to be the most efficient procedure. The first term captures the money supply miss that is due to a proportional shift in excess reserve demand initiated by disturbances that effect the demand for money. Since excess reserves are fairly insignificant ( $\varepsilon$  is close to zero), control errors from this source will not be important. The last term reflects the consequences of proportional shocks to total reserves supplied and these shocks essentially have a one for one effect on monetary control misses. Since the Fed should be fairly good at hitting its total reserve target this approach to monetary control will be the most efficient. Along with better control of money, total reserve targeting is also likely to reduce output and price level variability.

TABLE 1

	$\frac{E(m_c - m_c^*)^2}{E(m_c - m_c^*)^2}$	$\frac{E(r_c - r_c^*)^2}{E(r_c - r_c^*)^2}$	$\frac{E(p_c - E_c^+ - 1 - p_c)^2}{E(p_c - E_c^+ - 1 - p_c)^2}$	$\frac{E(y_c - E_c^+ - 1 - y_c)^2}{E(y_c - E_c^+ - 1 - y_c)^2}$
Interest rate instrument	$\frac{m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2}{m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2}$	0	$(1/a_1)^2 (\sigma_w^2 + \sigma_u^2)$	$(a_1^S/a_1)^2 (\sigma_w^2 + (a_1^d/a_1)^2 \sigma_u^2)$
borrowed reserve instrument	$(1-\eta)^2 (m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2)$	$\left(\frac{b}{b_1}\right)^2 \theta y_2^2 (m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2)$	$(1/a_1)^2 \left[ \left(1 - \frac{m_1 b_2 a_1^d \theta y_2}{b_1}\right)^2 \sigma_w^2 + \left(1 - \frac{m_2 b_2 a_1^d \theta y_2}{b_1}\right)^2 \sigma_u^2 \right]$	$(1/a_1^2) \left[ a_1^S - \frac{a_1^d a_1^S m_1 b_2 \theta y_2}{b_1} \right]^2 \sigma_w^2 + (a_1^d/a_1)^2 \left[ a_1^S + \frac{a_1^d a_1^S m_2 b_2 \theta y_2}{b_1} \right]^2 \sigma_u^2$
	$+ (c_1 + a_1 m_1)^2 [(s/b_1)(\theta - \phi)]^2 \sigma_b^2$	$+ [\theta/b_1 (\theta - \phi)]^2 \sigma_b^2$	$+ \left(\frac{a_1^d b_2 \theta y_2}{b_1}\right)^2 \sigma_e^2 + \left(\frac{a_1^d \theta}{b_1 (\theta - \phi)}\right)^2 \sigma_b^2$	$+ (a_1^d/a_1 + \frac{a_1^d a_1^S m_2 b_2 \theta y_2}{b_1})^2 \sigma_e^2 + \left(\frac{a_1^d a_1^S b_2 \theta y_2}{b_1}\right)^2 \sigma_b^2 + \left(\frac{a_1^d a_1^S \theta}{b_1 (\theta - \phi)}\right)^2 \sigma_b^2$
total reserve instrument (LRR)	$(e/\delta)^2 (m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2)$	$(1/\delta)^2 (m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2)$	$(1/\delta)^2 \left[ \left(\frac{e_1 + c_1}{a_1}\right)^2 \sigma_w^2 + \left(\frac{e_1 + c_1 + a_1 \theta}{a_1}\right)^2 \sigma_u^2 \right]$	$(1/\delta)^2 \left[ \left(\frac{a_1^S (e_1 + c_1)}{a_1}\right)^2 \sigma_w^2 + \left(\frac{a_1^d (1 + e_1 + c_1)}{a_1}\right)^2 \sigma_u^2 \right]$
	$+ (c_1 + a_1 m_1/\delta)^2 [\sigma_x^2 + (1/\epsilon_c)^2 \sigma_r^2]$	$+ (1/\delta)^2 [\sigma_x^2 + (1/\epsilon_c)^2 \sigma_r^2]$	$+ (a_1^d/a_1 \delta)^2 [\sigma_e^2 + \sigma_x^2 + (1/\epsilon_c)^2 \sigma_r^2]$	$+ (a_1^d a_1/a_1 \delta)^2 [\sigma_e^2 + \sigma_x^2 + 1/\epsilon_c)^2 \sigma_r^2]$
total reserve instrument (CRR)	$(e e_1/\delta)^2 (m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2)$	$(1/\delta)^2 (m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2)$	$(1/\delta)^2 \left[ \left(\frac{e e_1 + c_1}{a_1}\right)^2 \sigma_w^2 + \left(\frac{e e_1 + c_1 + a_1^d c_2}{a_1}\right)^2 \sigma_u^2 \right]$	$(1/\delta)^2 \left[ \left(\frac{a_1^S (e e_1 + c_1)}{a_1}\right)^2 \sigma_w^2 + \left(\frac{a_1^d (1 + e e_1 + c_1)}{a_1}\right)^2 \sigma_u^2 \right]$
	$+ (c_1 + a_1 m_1/\delta)^2 [\sigma_x^2 + \sigma_r^2]$	$+ (1/\delta)^2 (\sigma_x^2 + \sigma_r^2)$	$+ (a_1^d/a_1 \delta)^2 [\sigma_e^2 + \sigma_x^2 + \sigma_r^2]$	$+ (a_1^d a_1/a_1 \delta)^2 [\sigma_e^2 + \sigma_x^2 + \sigma_r^2]$
	where $\delta = e_1 + c_1 + a_1 m_1$	$m_1 = (1/a_1) (1 + c_2 a_1^S)$	$y_2 = \frac{\rho (1 - \rho)}{1 + c_1 (1 - \rho)}$	$e = \overline{ER}/\overline{TR}$
	$\hat{\delta} = e e_1 + c_1 + a_1 m_1$	$m_2 = (1/a_1) (1 - c_2 a_1^d)$	$\eta = (c_1 + a_1 m_1) \left(\frac{b}{b_1}\right) \theta y_2$	$\epsilon_c = \overline{ER}/(\lambda M_{c-1} + \overline{ER})$

#### 4. A Numerical Comparison

As indicated in the discussion in Section 3, the consequences of adopting different operating procedures depends on the parameter values and the variances of the disturbance terms. In this section the variances of money, interest rates, the price level, and output are analyzed for plausible parameter values and relative sizes of the various variances that conform with casual empirical evidence. The results of this experiment are that total reserve control under CRR produces the lowest variability in money, prices, and output, but leads to relatively high interest rate volatility. Also, the difference between an interest rate instrument and borrowed reserve targeting appears to be insignificant.

Following the evidence in Hall (1986) regarding the intertemporal elasticity of consumption, the elasticity of aggregate demand is assumed to be small with  $a_1^d = .20$ . Using evidence in King and Plosser (1986),  $a_1^s$  is set at 1.0. The interest elasticity of the demand for money is taken to be .10, implying that  $c_1 = 1.0$  for interest rates that average ten percent, while the income elasticity of money demand is assumed to be 1.0. The first order regressive coefficient in the money demand disturbance is set at .5. Banks are assumed to be twice as responsive as the public in seeking funds. Coupled with the fact that the correct interest elasticity is in terms of the interest rate--discount rate spread implies that  $b_1 = 10.0$ .<sup>6</sup> It is further assumed that banks have a greater response to the current spread than they do the expected future spread and  $b_2/b_1$  is set at .75. With respect to excess reserve demand  $e_1 = 1.0$  (the same as money demand) and  $\epsilon = \overline{ER}/\overline{TR} = .01$ . The qualitative results, also, do not appear to be overly sensitive to the parameter values, especially with respect to  $b_2$ ,  $e_1$ , and  $\epsilon$ . Values of  $a_1^s$  as high as  $\infty$  (no price level

movements within a period), and  $a_1^d$  as high as 1 did not affect the results, although I am sure that given unlimited freedom one could derive a different ranking of the operating procedures.<sup>7</sup>

Approximating the size of the relative variances is less firm than postulating reasonable parameter values and the sensitivity of some of the assumptions will be discussed when interpreting the result. With little evidence regarding the comparative variability of output supply, output demand, and money demand disturbances, all of these variances were set equal to a common value,  $\sigma^2$ . To rank the relative sizes of  $\sigma_e^2$ ,  $\sigma_b^2$ ,  $\sigma_r^2$ ,  $\sigma_x^2$ , it was assumed that 95% of all money disturbances were no larger than 1% of the money stock, implying that 95% of all unexpected movements in M1 would be less than approximately \$7 billion in absolute value. Regarding the other disturbance, 95% of the disturbances to borrowing were expected to be less than 5% in absolute value of the average level of borrowing, 95% of the excess reserve disturbances were assumed to be less in absolute value than 50% of the average value of excess reserves, and 95% of the total reserve control errors were assumed to be less in absolute value than 1% of total reserves.<sup>8</sup> The assumption of normality implies that  $\sigma_b^2 = 25\sigma_e^2$ ,  $\sigma_x^2 = 2500\sigma_e^2$ , and  $\sigma_r^2 = \sigma_e^2$ . In deriving  $\theta$ , and  $\phi$  the assumptions that  $\sigma_\epsilon^2 = \sigma_e^2$  and  $\sigma_\beta^2 = \sigma_b^2$  were used. Although there is a great deal of latitude in the values that can be assumed for the relative variances, total reserve control remained the most efficient procedure under a wide range of variances, while the ranking between borrowed reserve targeting and an interest rate instrument is sensitive to the relative sizes of  $\sigma_b^2$  and  $\sigma_e^2$ .

Under the above assumptions,  $\theta_3 = .192$  and  $(\theta - \phi)/\phi = 1.0$ . The comparative values of the relevant variances are depicted in Table 2. Given the relatively large value of  $\sigma_x^2$ , the values for a total reserve instrument under lagged reserve accounting are omitted.

The results depicted in Table 2 indicate that there is not much difference between borrowed reserve targeting and an interest rate instrument. As long as  $\sigma_b^2 > 14.8\sigma_e^2$ , borrowed reserve targeting will be a slightly inferior means of controlling money. If the assumption that  $\sigma_b^2 = 25\sigma_e^2$  is overstated, then it is likely that using borrowed reserves will improve monetary control and lower price level and output variability. However, borrowed reserve targeting does imply greater interest rate volatility. The results also show that, as expected, total reserve targeting represents the most efficient means of controlling money and leads to less volatility in output and prices. Unless interest rate volatility receives significant importance in the Fed's objective function, it is unclear why the Fed does not adopt this more efficient procedure.

TABLE 2

	$\frac{E(m_t - m_t^*)^2}{}$	$\frac{E(r_t - r_t^*)^2}{}$	$\frac{E(p_t - E_{t-1}^+ p_t)^2}{}$	$\frac{E(y_t - E_{t-1}^+ y_t)^2}{}$
interest rate instrument	$4.22\sigma^2$	0	$1.39\sigma^2$	$.72\sigma^2$
borrowed reserve target	$4.39\sigma^2$	$.43\sigma^2$	$1.38\sigma^2$	$.72\sigma^2$
total reserve target (CRR)	$1.25\sigma^2$	$3.08\sigma^2$	$.99\sigma^2$	$.49\sigma^2$

5. Summary and Conclusions

This paper analyzes various operating procedures and the interaction between these procedures and economic activity. The investigation builds on the work of McCallum and Hoehn (1983) by incorporating a market for reserves into the model. In doing so, one is able to analyze the effects of the information loss that occurs with an interest rate instrument and compare the efficiency of this procedure with the efficiency of borrowed reserve targeting. The conclusion is that the demand for borrowed reserves must display significantly greater variability than the demand for money for an interest rate instrument to be superior. Unfortunately, it is difficult to know the relative size of these two variances. The paper does indicate that nonborrowed reserve targeting in the presence of lagged reserve requirements is unambiguously worse than borrowed reserve targeting. Therefore, the Fed's shift to nonborrowed reserve targeting in October 1979 was a mistake.

Over a wide range of parameter values total reserve targeting with contemporaneous reserve requirements is the most efficient means of controlling money and results in lower price level and output variability. However, the Fed's concern for interest rate variability (see Goodfriend [1986], Dotsey [1987]), may make this procedure unappealing. The concern for smoothing interest rates is not costless, since output variability is likely to be directly related to economic welfare.

FOOTNOTES

1. This assumption does not affect the qualitative nature of the results. Similar results would occur in a model in which the reserve market met more frequently than the goods market.
2. The target  $m_t^*$  could be set according to some complicated feedback mechanism based on underlying policy goals. As in McCallum and Hoehn the results in this paper are not sensitive to how  $m_t^*$  is set. One does require that the target path for money is known. For simplicity,  $m_t^* = m^*$  is employed in the various solutions in the paper.
3. Including a variable discount rate would not affect the main results of the paper.
4. The result that non-borrowed reserve targeting under LRR will reduce to a noisy borrowed reserve target occurs even if excess reserves are interest sensitive as long as the interest sensitivity is relatively small.
5. Equation (15) can be easily derived by taking log linear approximations  $TR_t^d = \lambda M_{t-1} + ER_t^d$  and  $TR_t^* = \lambda M_{t-1} + ER_t^*$  and assuming the total reserve control error is of the form  $tr_t^s = tr_t^* + \tau_t$  (i.e.  $TR_t^s = (1+\tau_t)TR_t^*$  and  $\ln(1+\tau_t) \approx \tau_t$ ). Then  $tr_t^d = tr_t^s$  implies that  $\epsilon_t er_t^d = \epsilon_t er_t^* + \tau_t$ .

6. The coefficient  $c_1$  is independent of whether the demand for money is expressed in terms of the interest rate or the spread between the market rate and the rate paid on deposits,  $r_d$ , so long as  $r_d = (1-\lambda)r$ . However, with a fixed discount rate, or one that does not vary proportionately with  $r$ , then the elasticity of discount window borrowing with respect to the interest rate, is equal to  $\frac{r}{r-d}$ , where  $d$  is the discount rate, times the elasticity of discount window borrowing with respect to the spread. For  $r=10\%$ ,  $r-d=2\%$ , and assuming that banks are twice as interest sensitive as individuals gives  $b_1 = 10c_1 = 10$ .
7. It should also be noted that the elasticities used are largely based on empirical work performed on quarterly data. It is not known how time aggregation would affect the various coefficients.
8. These values for  $\sigma_x^2$  and  $\sigma_\tau^2$  were chosen based on observations of desk operations over the period October 1979 to June 1983. One notes how extremely volatile excess reserves are and that the desk is fairly competent at supplying its targeted level of reserves in any given maintenance period. Unfortunately, I know of no way to arrive at a good estimate of  $\sigma_b^2$ . Under strict borrowed reserves targeting equation (10) implies  $b_t = b_1(r_t - r_t^*) - b_2(\overline{E_{zt} r_{t+1}} - E_{t-1}^+ r_{t+1})$  which involves a combination of expectational errors. The Fed sometimes reports its best guess of the expected funds rate so that the first term might be approximated, but I know of no readily available data regarding the second term.

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APPENDIX

Equilibrium in the output market implies that

$$(A1) \quad p_t = \frac{1}{a_1} [a_0 - a_1^d r_t + a_1^s E_{t-1}^+ p_t + a_1^d E_{t-1}^+ p_{t+1} + w_t - u_t]$$

Substituting (A1) and  $E_{t-1}^+ p_t = a_0/a_1^d - E_{t-1}^+ r_t + E_{t-1}^+ p_{t+1}$  into either the output supply or demand equation yields

$$(A2) \quad y_t = -\frac{a_1^d a_1^s}{a_1} (r_t - E_{t-1}^+ r_t) + \frac{a_1^s}{a_1} w_t + \frac{a_1^d}{a_1} u_t.$$

Using the demand for money equation, the fact that  $E_{t-1}^+ m_t = m_t^*$ , that

$$p_t - E_{t-1}^+ p_t = (-a_1^d/a_1) (r_t - E_{t-1}^+ r_t) + (1/a_1) (w_t - u_t), \text{ and that } y_t - E_{t-1}^+ y_t = y_t$$

one derives

$$(A3) \quad m_t - m_t^* = -(c_1 + a_1^d m_1) (r_t - E_{t-1}^+ r_t) + m_1 w_t - m_2 u_t + e_t$$

For an interest rate peg  $r_t = E_{t-1}^+ r_t = r_t^*$  and

$$(A4) \quad m_t - m_t^* = m_1 w_t - m_2 u_t + e_t$$

For a borrowed reserve target

$$(A5) \quad r_t = r_t^* + (b_2/b_1) (\overline{E_{zt} r_{t+1}} - E_{t-1}^+ r_{t+1}) - (1/b_1) b_t$$

Postulate the following undetermined coefficients solution for  $p_t$  and  $r_t$ .

$$(A6) \quad p_t = \Pi_0 + \Pi_1 m_t^* + \Pi_2 v_{t-1} + \Pi_3 e_t + \Pi_4 w_t + \Pi_5 u_t + \Pi_6 b_t$$

$$(A7) \quad r_t = \psi_0 + \psi_1 m_t^* + \psi_2 v_{t-1} + \psi_3 e_t + \psi_4 w_t + \psi_5 u_t + \psi_6 b_t$$

where  $m_t^* = m_t^*$ .

(A2)

Given that agents observe  $s_{r,t} = r_t - E_{t-1}^+ r_t = \psi_3 e_t + \psi_4 w_t + \psi_5 u_t + \psi_6 b_t$ ,

$s_{m,t} = m_1 w_t - m_2 u_t + e_t + \varepsilon_t$  and  $s_{b,t} = b_t + \beta_t(z)$ ,  $E_{z_t} r_{t+1}$  can be expressed as

(A8)  $E_{z_t} r_{t+1} = E_{t-1}^+ r_{t+1} + \alpha_1 s_{r,t} + \alpha_2 s_{m,t} + \alpha_3 s_{b,t}$  where

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \text{var}(s_r) & \text{cov}(s_r, s_m) & \text{cov}(s_r, s_b) \\ \text{cov}(s_r, s_m) & \text{var}(s_m) & 0 \\ \text{cov}(s_r, s_b) & 0 & \text{var}(s_b) \end{bmatrix}^{-1} \begin{bmatrix} \psi_2 \psi_3 \sigma_e^2 \\ \psi_2 \sigma_e^2 \\ 0 \end{bmatrix}$$

and  $\text{var}(s_r) = \psi_3^2 \sigma_e^2 + \psi_4^2 \sigma_w^2 + \psi_5^2 \sigma_u^2 + \psi_6^2 \sigma_b^2$ ,  $\text{var}(s_m) = m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2 + \sigma_e^2 + \sigma_\varepsilon^2$ ,

$\text{var}(s_b) = \sigma_b^2 + \sigma_\beta^2$ ,  $\text{cov}(s_r, s_m) = \psi_3^2 \sigma_e^2 + \psi_4 m_1^2 \sigma_w^2 - \psi_5 m_2^2 \sigma_u^2$ , and  $\text{cov}(s_r, s_b) = \psi_6^2 \sigma_b^2$ .

Solving and making use of the fact that  $\psi_3 m_2 - \psi_4 = \psi_3 m_2 + \psi_5 = 0$  yields

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \frac{\psi_2}{\Delta} \begin{bmatrix} \psi_3^2 \sigma_e^2 \sigma_\varepsilon^2 (\sigma_b^2 + \sigma_\beta^2) \\ \psi_6^2 \sigma_e^2 \sigma_b^2 \sigma_\beta^2 \\ -\psi_3 \psi_6^2 \sigma_e^2 \sigma_\varepsilon^2 \sigma_b^2 \end{bmatrix}$$

where  $\Delta \equiv (\psi_3^2 \sigma_e^2 + \psi_6^2 \sigma_b^2) (\sigma_e^2 + m_1^2 \sigma_w^2 + m_2^2 \sigma_u^2) \sigma_b^2 + (\psi_3^2 \sigma_e^2 + \psi_4^2 \sigma_w^2 + \psi_5^2 \sigma_u^2 + \psi_6^2 \sigma_b^2) \sigma_\varepsilon^2 \sigma_\beta^2$

Defining  $\theta \equiv \frac{1}{\Delta} [(\psi_3^2 \sigma_e^2 + \psi_6^2 \sigma_b^2) \sigma_e^2 \sigma_b^2 + \psi_3^2 \sigma_e^2 \sigma_\varepsilon^2 \sigma_\beta^2]$  and  $\phi \equiv (1/\Delta) (\psi_3^2 \sigma_e^2 \sigma_\varepsilon^2 \sigma_\beta^2)$ , the

undetermined coefficients solutions are  $\psi_2 = \frac{\rho(1-\rho)}{1+c_1(1-\rho)}$ ,  $\psi_3 = \frac{b_2}{b_1} \theta \psi_2$ ,  $\psi_4 = m_1 \psi_3$ ,

$\psi_5 = -m_2 \psi_3$ , and  $\psi_6 = -\frac{\theta}{b_1(\theta-\phi)}$

(A3)

Substituting  $r_t - E_{t-1}^+ r_t$  into (A3) yields

$$(A9) \quad m_t - m_t^* = [1 - (c_1 + a_1^d m_1) \frac{b_2}{b_1} \theta \frac{\rho(1-\rho)}{1+c_1(1-\rho)}] (m_1 w_t - m_2 u_t + e_t) \\ + (c_1 + a_1^d m_1) \frac{\theta}{b_1(\theta-\phi)} b_t$$

Taking the expectation of (A9) squared yields equation (11) in the text.