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Firm Fragmentation and Urban Patterns

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Firm Fragmentation and Urban Patterns*

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Abstract

We document several empirical regularities regarding the evolution of urban structure in the largest U.S. metropolitan areas over the period 1980-1990. These regularities relate to changes in resident population, employment, occupations, as well as the number and size of establishments in different sections of the metropolitan area. We then propose a theory of urban structure that emphasizes the location and integration decisions of firms. In particular, firms can decide to locate their headquarters and operation plants in different regions of the city. Given that cities experienced positive population growth throughout the 1980s, we show that our theory accounts for the diverse facts documented in the paper.

JEL: R12, R14

Keywords: Population Growth, City Structure, Multiple Plants, Firm Integration

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1. INTRODUCTION

The internal structure of U.S. metropolitan areas has evolved dramatically over the last three decades. This evolution exhibits striking patterns that hold for a wide range of cities. If one divides metropolitan areas into a center county and edge counties, employment and residential population have increased both at the center and at the edge. However, over this period, we also observe an important increase in the share of city residents, employment, and establishments at the edge. This shift in economic activity to the edge of the city is more pronounced for non-management occupations than for managers. In addition, the size of establishments decreased in both areas throughout this period. The first part of this paper is devoted to documenting these changes in U.S. cities.

What accounts for the evolution of U.S. urban structure over this period? Much of the urban literature attributes the migration of residents to the edge to decreases in transport costs. Explanations of this type, however, are generally not consistent with the migration of workers and firms to the edge.[1] Furthermore, there exists a more fundamental problem with all available explanations for subsets of these phenomena. Specifically, such explanations rely on mechanisms that decrease agglomeration forces at the center thereby explaining how the *share* of economic activity can increase at the periphery, but not the simultaneous increase in the *level* of economic activity at the center. These theories are also silent on the issues of functional (management versus non-management) and establishment shifts.[2]

This paper proposes a theory aimed at addressing the full set of facts we have just described. The key concept we emphasize relates to firms' ability to break down their production process into headquarters and production plants, where either can locate in different sections of the city. Given this margin, we show that increases in population lead to changes in organizational structure such that a larger proportion of firms choose not to integrate their operations. In particular, standard agglomeration forces motivate firms to keep only those

[1] See Anas, Arnott and Small (1998), and Glaeser and Kahn (2003), for a general review of this literature, and Burchfield et. al. 2004 for a recent empirical study of urban sprawl in the US.

[2] In theories developed by Fujita and Ogawa (1982), or Lucas and Rossi-Hansberg (2002), a decline in transport costs tends to disentangle the location of business and residential areas and can lead to employment concentration at the center. Because these theories incorporate a richer spatial dimension, the implications of commuting costs, or changes in externality parameters, depend on exactly how one defines the city center. These theories do not incorporate occupational choices or firms' integration decisions.

workers at the center who benefit from interactions in downtown locations (i.e. those arising from knowledge spillovers or, more generally, production externalities). Consequently, as city population increases, employment rises at the center but this rise is driven primarily by increases in managerial population and, therefore, establishments. In addition, since each manager at a headquarter supervises several workers, and the production plants of these new firms are located more remotely, employment growth is even more pronounced at the periphery. These changes immediately translate into a decline in the share and a simultaneous increase in the level of employment in the central region of the city.

Because land rents are lower nearer the city's outskirts, population growth implies that more firms will integrate their operations away from the center. Therefore, increases in population also lead to an increase in managerial employment at the periphery, reenforcing the decline in the share of center employment. Ultimately, however, the combined set of changes resulting from a rise in city population implies a concentration of managers at the center. Furthermore, consistent with the data, rising urban population also leads to a decrease in the share of establishments at the center, and a decrease in establishment sizes as more establishments become non-integrated firms.

One interpretation of the theory we present, and the empirical evidence more broadly, is that with firms sending their larger and more routine operations to the periphery, city centers are steadily becoming management or administrative hubs. Examples of firms breaking up their operations geographically within a given metropolitan area are ubiquitous across industries. For instance, the Federal Reserve Bank of New York moved its cash and check processing center to neighboring East Rutherford, NJ in 1992; the Washington Post moved its printing operations away from its headquarters in downtown Washington to neighboring Springfield, VA in 1999; the tire manufacturer Michelin, headquartered in Greenville, SC, located its rubber production operations in nearby Anderson County in 2000; and Home Depot is currently moving a distribution center to McDonough Georgia outside Atlanta, the location of its corporate headquarters.

In a related paper, Chatterjee and Carlino (2001) seek to explain systems of cities and argue that the deconcentration of U.S. metropolitan employment stems from an increase in aggregate employment. The theory they present is one where an aggregate increase in employment raises densities faster in small metropolitan areas than in larger ones, since large metropolitan areas already have high employment densities and cannot accommodate the new workers cheaply enough. Our paper shares with Chatterjee and Carlino (2001) the focus on population growth as the main engine driving the structural change of U.S. cities. In contrast to their work, however, we model a representative city, rather than the interaction between cities, and focus on its internal structure. That is, we study the spatial allocation of employment within a city. We also analyze the location decisions of establishments and agents in different occupations. The new set of facts we uncover leads us to emphasize firms' integration decisions, which we argue can rationalize observed changes in city structure as a result of population growth alone.

Along a different line of work, Duranton and Puga (2004) argue that cities have moved from being sectorally specialized to becoming functionally specialized. They contend that decreases in the cost of communication between headquarters and plants have lead to the location of headquarters in cities and the location of production plants in smaller towns. Our view of the changes in city structure shares many elements with their work. In particular, Duranton and Puga (2004) also model explicitly the firm's decision to integrate its headquarters and production plants. In their view, this integration decision has implications across metropolitan areas. In fact, we argue that firms' ability to separate headquarters and plants is also key in explaining changes in the internal structure of cities. Our paper differs from Duranton and Puga (2004) in that we do not view changes in communication technology as the force underlying changes in urban structure, but instead show that the latter changes emerge simply from rising population. Further, our framework has implications for the share of establishments located in different sections of the city that are consistent with the data. Davis and Henderson (2004) provide evidence that complements our findings. They observe that firms take advantage of services and production externalities (which decline with distance) at the city's business sectors by locating their headquarters at the center and their operation plants elsewhere in the city.

The rest of the paper is organized as follows. Section 2 presents our data organized in nine different stylized facts. Section 3 presents a simple urban framework that incorporates the firm's integration decision. Section 4 shows that increases in population lead to changes in city structure consistent with the diverse stylized facts we present, and Section 5 concludes.

2. SOME FACTS ON THE EVOLUTION OF CITY STRUCTURE

This section documents a set of regularities in the evolution of city structure that we care to rationalize with the simple theory proposed in this paper. We document these facts for the decade spanning the 1980s, although most of the empirical regularities hold from 1970 to 2000. The reason is that for some of these regularities, in particular the ones that involve the location of agents with different occupations, we do not have data covering a longer period. Thus, we chose to homogenize the time period and document our stylized facts over the same decade.

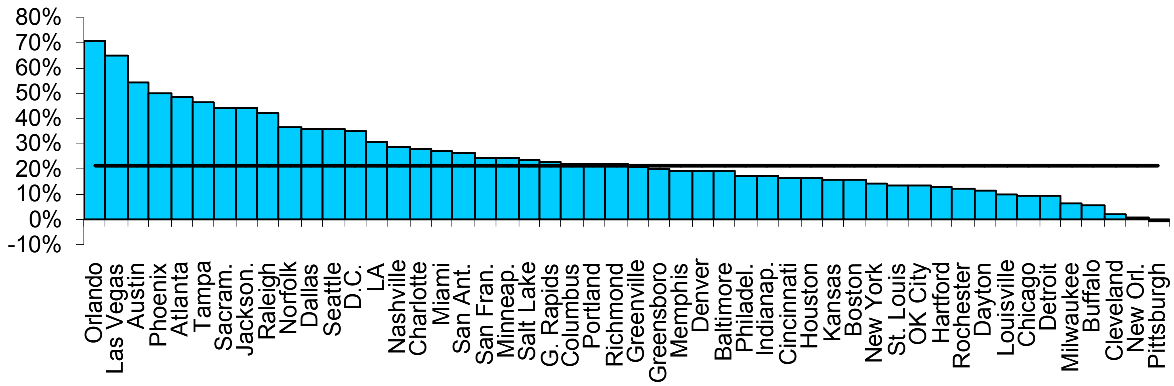
Given our focus on the structure of cities, we separate the city into two locations: center and edge. The center is the area encompassed by the central county of the city. The edge is the set of counties that surround the central county. The central county always includes the central business district of the city, or the downtown area, and is generally much larger than just the downtown area. Our study relies on the 50 largest U.S. Metropolitan Areas according to their 1999 population. In particular, for each city, we use the most extensive definition of metropolitan area available, either Metropolitan Statistical Area (MSA) or Consolidated Metropolitan Statistical Area (CMSA), as defined by the Office of Management and Budget.

Our data originates from four sources: the Census Bureau "Commute to Work" data, the Census County Business Patterns, the Housing and Urban Development State of the Cities database, and the Bureau of Economic Analysis (BEA) Regional Economic Information System. Since the theory we propose is that of a city rather than a system of cities, and we wish to abstract from idiosyncratic city characteristics, our analysis focuses only on time changes in city structure during the 1980s, and not on the state of city structure across U.S. cities at a given point in time.

2.1 Changes in Absolute Population Levels

The first set of facts we mention are well known. As shown in Figure 1, overall population increased throughout the 1980s in all but one city in our sample. City population growth averaged 21.3% over that period, while Pittsburgh's population contracted by 0.38%. All averages presented in this section are weighted averages using population shares, and are shown as horizontal lines in the bar graphs and circles in the scatter plots.

Figure 1: Population Growth, 1980-1990



Moreover, population in most cities increased both at the center and at the edge. To illustrate this point, Figure 2 plots population changes in these areas from 1980 to 1990. In almost all cities, changes are positive both at the center and at the edge and, for some cities, very large. The population of Las Vegas, for example, grew by more than 80% in the edge counties and more than 50% at the center. A 1% increase in population at the edge is associated with a 0.6% increase in population at the center. The correlation between changes in population at the center and at the edge is 0.69. All 50 cities in our sample grew in terms of edge population, and only 7 declined in terms of population at the center. In the latter cases, this decline is always small except for New Orleans whose population fell by 12.8% at the center, but increased by 9.7% at the edge.

Figures 1 and 2 are indicative of overall city population growth, and the fact that this population locates both at the center and in peripheral counties. A question which then arises is: has city population growth led to a change in the link between employment location and residential location? As cities become larger, one might expect residential sprawl (i.e. residents locating at the boundary of the city) and employment concentration at the center (see Fujita and Ogawa [1982] and Lucas and Rossi-Hansberg [2002]). In general, our view of the data is that this phenomenon did not dominate in the U.S.: changes in employment are in general paralleled by changes in the number of residents in both areas of the city.

A fact consistent with this view is that net commuting between the center and the edge, as a percentage of total population, hardly changed throughout the 1980s. Net commuting represented 8.98% of total MSA population in 1980 and 8.38% in 1990. Average residential growth in the 1980s amounted to 15.4% at the center and 25.7% at the edge. Similarly, average employment growth in the 1980s was 14.8% at the center and 28.3% at the edge. This similarity in the size of changes in employment and residents across city areas is surprising, and suggests that the link between employment and residential location is a key component of urban structure. In addition, this evidence suggests that commuting costs did not decline in any significant way during the 1980s.

Figure 2: Population Growth, 1980-1990

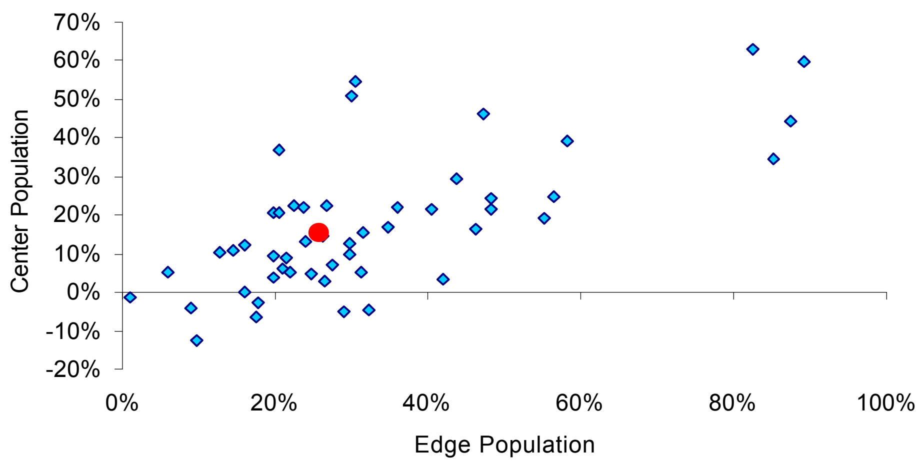


Figure 3: Employment Growth, 1980-1990

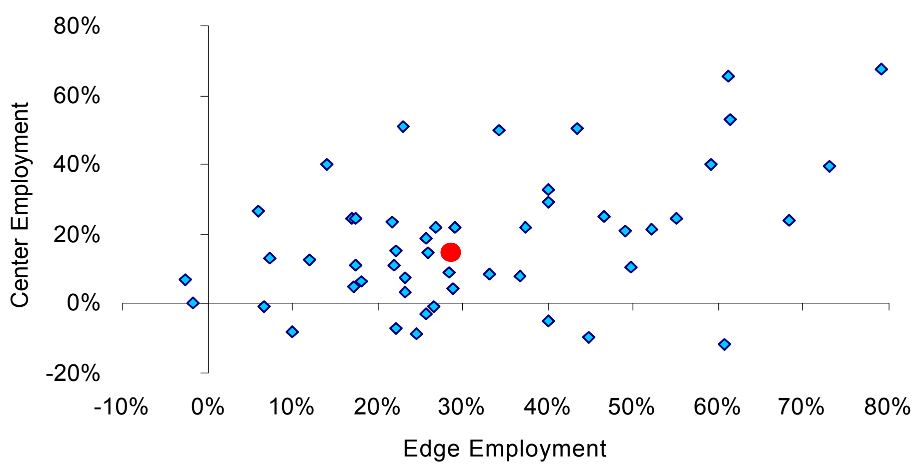
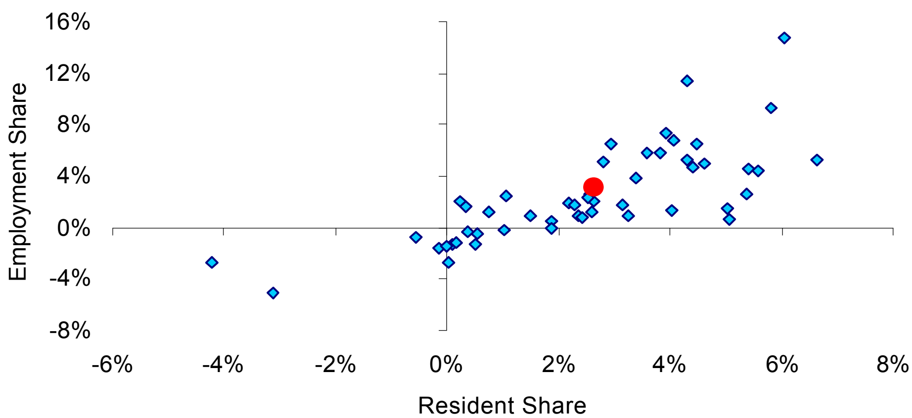


Figure 3 depicts employment growth across all cities in our sample (resident growth is identical to population growth, which is presented in Figure 2). The figures indicate that residential population and employment grew in the majority of cities during this period. More specifically, resident and employment growth is positive at the edge in all cities. At the center, employment growth is negative in only nine cities while resident growth is negative in only seven cities.

2.2 Changes in Population Shares Across City Areas

The facts we have just presented relate to absolute quantities of employment and resident growth. We now address changes in the shares of residents and employment at the edge during the 1980s. Our data show that the shares of both residents and employment have generally increased at the edge. That is, while levels grew everywhere, the share of population has shifted from the center to the edge of U.S. cities. The average increase in the share of employment at the edge during the 1980s was 3.11% while the average increase in the share of residents at the edge was 2.64%. Put simply, economic activity in the U.S. is moving to the periphery. Figure 4 depicts changes in resident and employment shares at the edge during this period. It is clear from the graph that in most U.S. cities, the share of individuals who both reside and work at the edge increased.

Figure 4: Change in Resident and Employment Share at the Edge, 1980-1990



Given the shift in employment shares towards the periphery, one might wonder whether the increase in edge employment was driven by particular industries. In other words, the facts above could have resulted from specific industries moving away from city centers while other industries, perhaps less labor and land intensive, moved to the city center. This does not appear to be the case. The average employment share at the center declined from 0.42 to 0.38 in manufacturing and from 0.47 to 0.43 in services. That is, average employment shares decline by about the same percentage in both sectors.

2.3 Changes in the Location of Managers

To gain further insight into the change in employment shares at the center, we examine the change in employment across occupations. In particular, we divide employment into two classes: management and non-management occupations. The first class includes managers and professional workers. The second includes what the Housing and Urban Development State of the Cities database classifies as non-management workers.[3] The latter category includes technicians, sales, administrative support, precision workers, laborers and machine operators, and service workers.

The share of managers at the center declined in all but 3 cities in our sample. The average change in the manager share during the 1980s was -4.71%. The share of non-managers at the center also declined in all but a handful of cities during this period, with an average decline of -5.63%. Although these facts are informative about changes in city structure, comparisons related to the *relative* location of managers and non-managers are also telling. Figure 5a presents the difference between the fall in manager and non-manager shares across cities. Note that at the center, manager shares fell less rapidly than non-manager shares in approximately 75% of the cities throughout the 1980s. There are only 12 cities (out of 50) where the reverse is true. Even accounting for the fact that some of these exceptions are among the largest U.S. cities, including New York and Chicago, the weighted average in Figure 5a remains positive at 0.23%. Figure 5b compares the change in the ratio of managers to non-managers at the center with the change in the same ratio at the edge: an alternative way of looking at the same phenomenon.

[3] The definition of center and edge in the HUD database differs somewhat from our previous breakdown in terms of counties. These data divide the city into a business center (defined by the geographical city boundary) and the CMSA/MSA area outside of it. For consistency, we extend the business center area to that of the central county, using the assumption that densities of managers and non-managers are constant within each area.

Figure 5a: Change in Management Share less the Change in Non-Management Share at the Center, 1980-1990

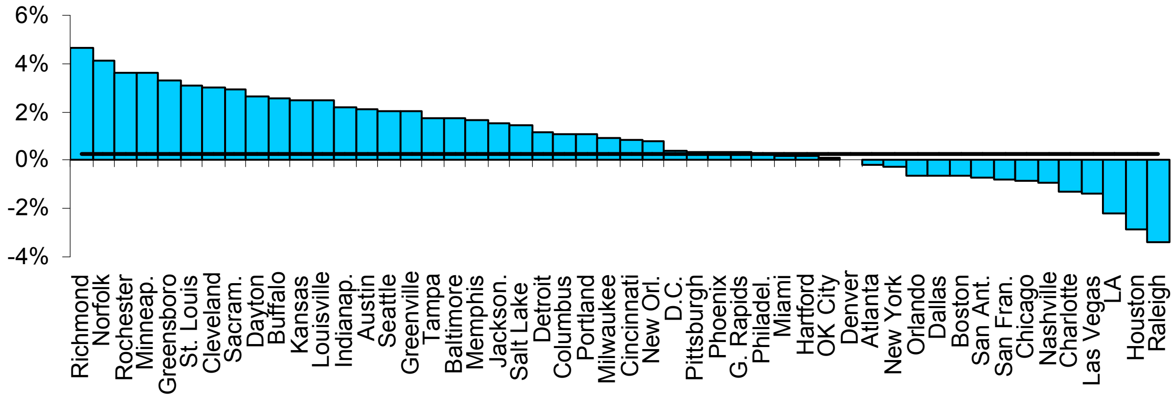
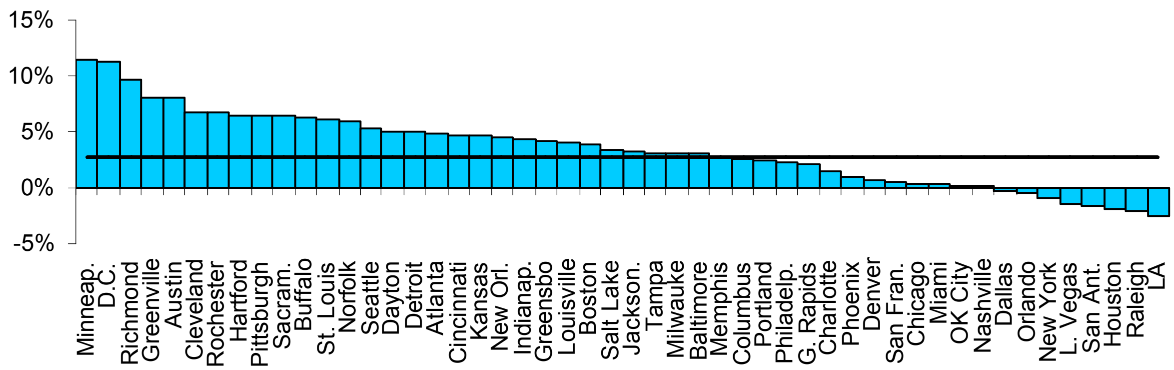


Figure 5b: Change in Management : Non-Management Ratio at the Center Relative to the Edge, 1980-1990

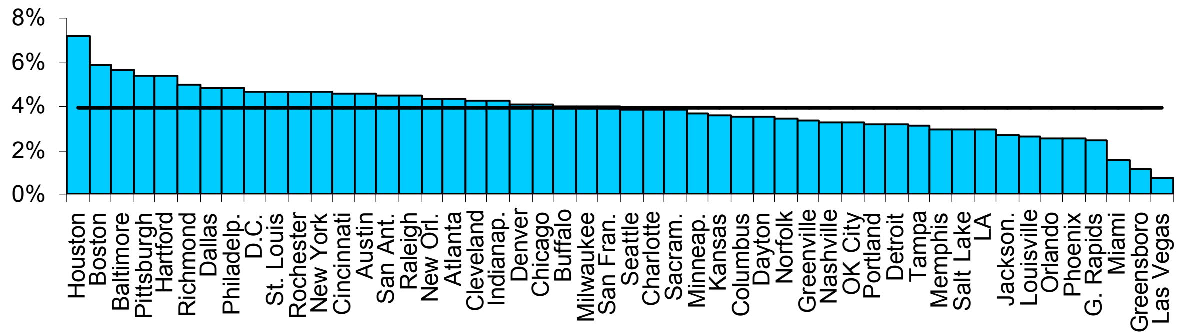


Consistent with the observation we have just made, Figure 5b shows that the ratio of managers to non-managers rose more rapidly at the center than at the edge in 86% of the cities in our sample.[4] The average difference in the change of these ratios was 2.82%. In general, therefore, our interpretation of the data is that city centers are becoming management or administrative hubs, with managers heading operation plants at the boundary of the city where land is cheap. In fact, we shall use this interpretation of Figures 5a and 5b in the model we develop below. Observe that although managers were generally more concentrated in city centers throughout the 1980s, every single city over this period saw an increase in the overall ratio of managers to population. In other words, cities as a whole are also evolving towards administrative functions. Figure 5c illustrates the change in the ratio of management to population, with a mean of approximately 3.89% across all cities. The

[4] The ratio of managers to non-managers increased both at the center and at the edge in every city.

evidence presented in Figure 5c only reinforces the findings in Duranton and Puga (2004) who argue for the functional specialization of cities.

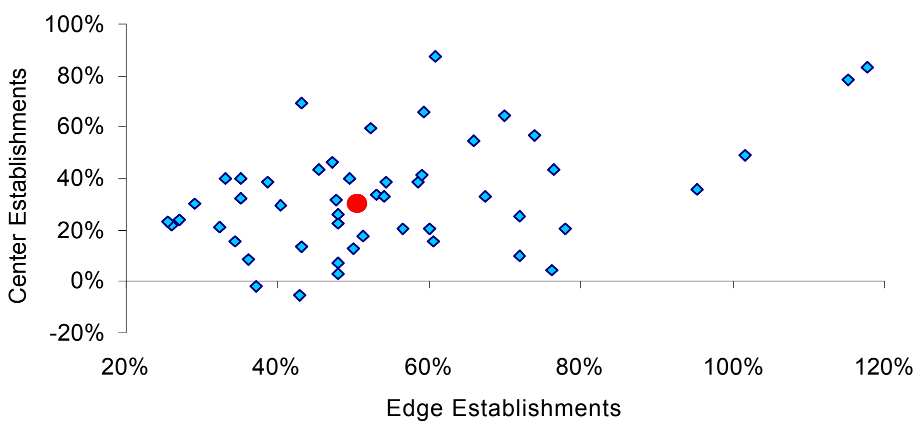
Figure 5c: Change in Total Management : Population Ratio, 1980-1990



2.4 Changes in the Location and Characteristics of Establishments

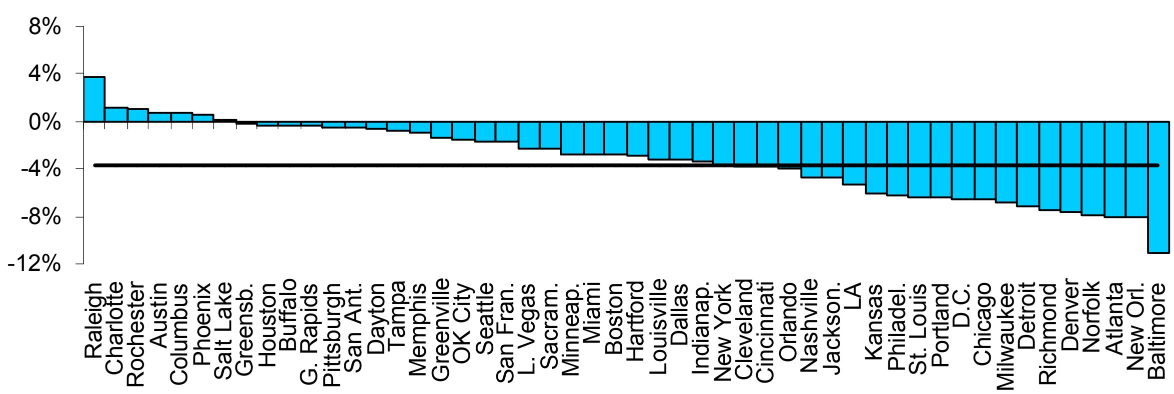
Having set out some facts regarding changes in population location within U.S. cities, we now turn to the location of establishments in different parts of the city. Consistent with overall population and employment growth throughout the 1980s, the number of establishments also increased in all but three U.S. cities, both at the center and at the edge.

Figure 6: Establishment Growth at the Center and at the Edge, 1980-1990



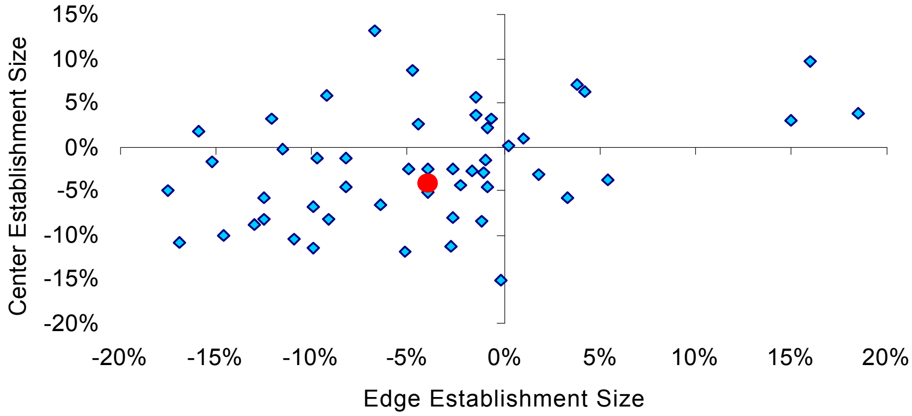
The number of establishments grew on average by 30.1% at the center and by 50.5% at the boundary. Hence, while the net entry of firms or plants is more pronounced at the city edge, firm entry is also substantial at the center. The correlation between establishment net entry at the center and edge is 0.45. Figure 6 illustrates these changes. Note that in some cities, net entry of establishments at the edge exceeds 80% over our sample period and, in some cases, reaches as high as 110%. Although central counties experienced significant net firm entry throughout the 1980s, more establishments located at the boundary over that period. Indeed, the change in the share of establishments at the center is negative in all but a few cities, as Figure 7 illustrates. The average change in the share of establishments located at the center is -3.65%. In 64% of the cities, the share of establishments was larger at the center than at the edge in 1980, with an average establishment share of 54.9% at the center. Therefore, while more than one half of the establishments were located at the center in 1980, many new firms chose to locate near the city's outskirts during the subsequent decade which lead to a significant decrease in the share of firms residing at the center.

Figure 7: Change in Establishment Share at the Center, 1980-1990



How were establishment sizes, measured in number of employees, affected during this period? In general, we find that establishment sizes declined over the 1980s. This finding is consistent with other evidence in the literature regarding the average size of firms in the U.S. (see Garicano and Rossi-Hansberg [2003]). Establishment sizes declined on average by 4.08% at the center and 3.92% at the edge. Figure 8 shows changes in establishment sizes both at the center and at the edge. We can see that for most cities, establishment sizes fell in both regions. This finding, however, does not hold for all cities.

Figure 8: Change in Establishment Size at the Center and at the Edge, 1980-1990



About half of the cities in our sample experienced a decline in average establishment size in both regions simultaneously. Establishments tend to be larger at the center than at the boundary, with 21.4 employees per establishment at the center versus 17.1 employees per establishment at the edge in 1980. The last characteristic of urban economic activity that we wish to establish in this section concerns the relationship between changes in the number of managers and the number of establishments across cities. Figures 9a and 9b illustrate this relationship. Observe that the number of establishments and managers are highly correlated: the correlation is 0.81 at the center and 0.62 at the edge. At the center, a 1% increase in the number of managers is associated with a 0.89% increase in the number of establishments, where the fit is characterized by a surprisingly large R^2 statistic, 0.65. In contrast, at the edge, a 1% increase in the number of managers is associated with only 0.57% more establishments, with the relationship having a much lower R^2. This evidence suggests that the number of establishment per manager at the center is larger than at the edge, and that changes in the number of establishments are more tightly related to changes in managers at the center than at the edge. Although our measure of managers also includes professional workers, the fact that managers and establishment are so closely linked gives us some confidence that many of these agents are in fact performing administrative or managerial roles.

Figure 9a: Change in Managers and Change in Establishments at the Center, 1980-1990

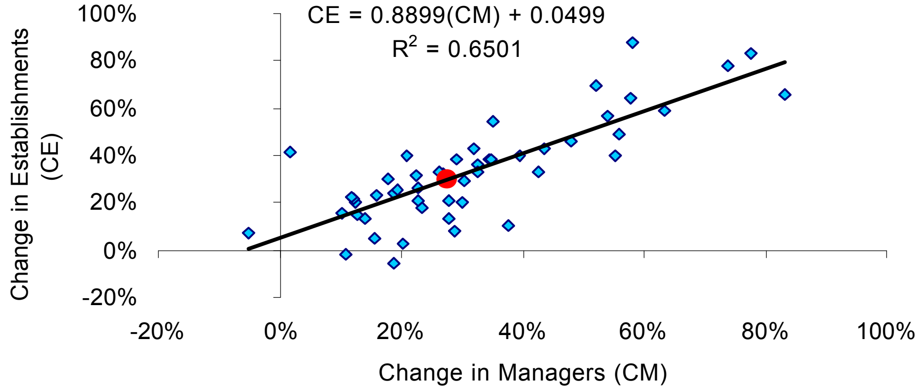
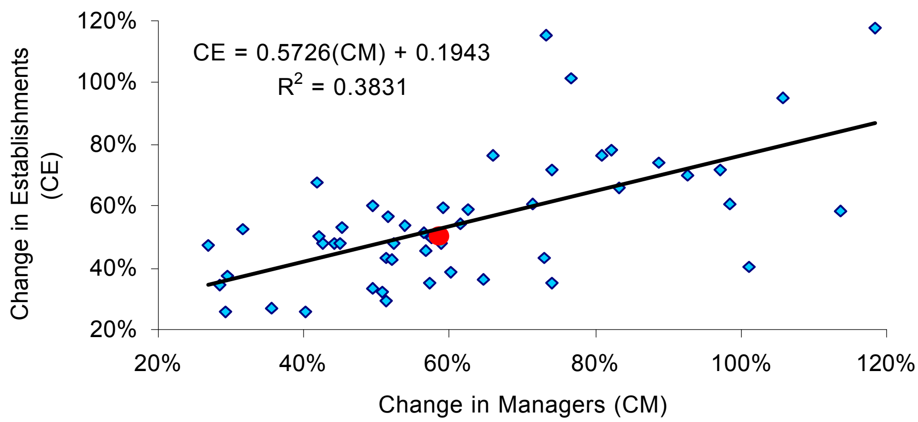


Figure 9b: Change in Managers and Change in Establishments at the Edge, 1980-1990



2.5 Stylized Facts on Urban Structural Change

We now summarize the set of stylized facts presented for the fifty largest U.S. cities over the 1980-1990 decade. Throughout the paper, we shall refer to these stylized facts by the number we assign to each below.

1. Overall population growth.
2. Resident population growth at the center and at the edge of cities.
3. Employment growth at the center and at the edge of cities.

4. A similar reduction in resident and employment shares at the center.
    (a) Present both in services and manufacturing.
5. City centers increasingly becoming management or administrative hubs:
    (a) An increase in the share of managers compared to the share of non-managers at the center,
    (b) An increase in the ratio of managers to non-managers at the center compared to the same ratio at the edge,
    (c) An increase in the total number of managers relative to total employment.
6. An increase in the number of establishments at the center and at the edge of cities.
7. A decline in the establishment share at the center.
8. A decline in establishment size both at the center and at the edge of cities.
9. The number of establishments and managers are more tightly related at the center than at the edge.

In the next section, we propose a simple model of city structure and explain how Fact 1 alone can lead to stylized Facts 2 through 9. Put another way, it is possible to think of the changes in residents, employment, occupations, and establishments described above as the result of urban population growth. At the heart of the theory we present lies a decision on the part of firms to either integrate their operations in one location or separate them into headquarters and production plants. We argue that adding this additional margin to urban models is crucial in explaining the diverse set of changes observed in the internal structure of U.S. cities.

3. THE MODEL

This section presents a theory of the internal structure of cities simple enough to remain analytically tractable yet rich enough to address the diverse set of facts outlined in the previous section. Since our goal is to illustrate the main forces that lead to these empirical regularities, we model cities as consisting of only two areas: the center of the city and what we call the edge. We think of these two areas as the model equivalent of the central and edge counties in the data. Given this parallel, we assume that the central area of the city is given exogenously by $L_c > 0$. The edge area is endogenous, and we denote it by $L_b > 0$, where b stands for the city boundary. We assume that residential land rents at the edge are given by an agricultural land rent, $\overline{R} \geq 0$, that represents its opportunity cost. Land rents at the center of the city are endogenous and determined in equilibrium. Total city population, P, is exogenously given. In fact, we shall argue below that the theory we develop can rationalize the set of stylized facts presented above simply as the result of population growth. Our theory, therefore, is a partial equilibrium theory that takes as given two key elements from national economic behavior, namely, agricultural land rents and city population sizes. Any theory of city structure must take a stand on the variables to be determined at the aggregate rather than the city level, and our choice is driven by the set of facts that we seek to explain.

The key insight of our model is that allowing firms to separate their location into headquarters and production plants implies that city growth leads to a set of empirical regularities regarding urban structure. Headquarters develop knowledge and, therefore, experience external effects from other headquarters. Knowledge transactions are carried out in headquarters which tend to agglomerate in high rent areas of the city. Production plants, in contrast, carry out more routine tasks that do not lead to knowledge spillovers and, consequently, tend to locate in areas where land rents are low. Production plants in our framework can be interpreted as either manufacturing plants, retail stores, or other production facilities.

3.1 Firms

The city produces and consumes one good, the price of which we normalize to one. A firm is made up of a manager who hires workers to produce. The manager and her workers can locate at either the center or the edge of the city. We refer to the location of the manager as the firm's headquarters and the location of her workers as the firm's production plant. The number of workers a manager can hire is determined by whether the firm is located in only one location (an integrated firm), or whether the headquarters and production plants reside in different locations. In the former case, the manager finds it less costly to communicate and interact with workers that are located close by so that she can oversee a larger set of workers, $n_{cc} = n_{bb} = \alpha\overline{\delta} > 1$, where n_{ij} denotes the number of employees of a firm with headquarters in area i and production plant in area j. In contrast, if the manager decides to set up a non-integrated firm, she needs to spend additional resources to monitor and interact with her workers, who are physically removed, and her span of control is given by $n_{cb} = n_{bc} = \alpha\underline{\delta} > 1$, where $\underline{\delta} < \overline{\delta}$. This assumption is motivated by Fact 9 above. In other words, in the data, changes in the number of managers are clearly positively correlated with changes in the number of establishments both at the center and at the edge. In fact, the correlation between changes in managers and changes in the number of establishments is significantly higher at the center than at the edge in the 1980s, which is consistent with a constant number of managers per firm (abstracting from composition effects which are not present at the center).[5]

In our model, the location of a firm's headquarters matters significantly in that it determines its productivity. In particular, firm productivity depends on the number of managers located in the area of the city where the firm's headquarters are also located: a production externality. Total output of a firm with headquarters in area i and production plant in area j is given by $AE_i n_{ij}$, where E_i denotes the number of managers in i and A is a city-wide productivity parameter. A firm has to pay labor costs given by a city-wide wage $w > 0$ (since all agents in the city are assumed identical) times the number of workers it hires, n_{ij}, as well as land rents. We assume that the firm needs to hire one unit of land per worker in order to operate, so that total land rent paid by this firm is given by $R_j n_{ij}$.[6] Firms are owned by managers whose earnings are given by firms' profits. It follows that a manager who owns the firm we have just described earns

$$F_{ij} = (AE_i - w - R_j)n_{ij}. \tag{1}$$

The problem of a manager is then to choose the location of the firm's headquarter and

[5] See Garicano (2000), and Garicano and Rossi-Hansberg (2003), for organizational models that yield smaller team sizes as communication costs increase.

[6] Note that managers do not require land. This assumption allows for a much simpler analysis of the model without driving any of our central results.

production plant to solve

$$\begin{aligned} \overline{F} &= \max_{ij} \{F_{ij}\} \text{ for } i,j \in \{c,b\} \text{ and subject to} \\ n_{ij} &= \begin{cases} \alpha\overline{\delta} \text{ if } i=j \\ \alpha\underline{\delta} \text{ if } i \neq j \end{cases}. \end{aligned}$$

Put alternatively, managers decide whether to locate at the center or at the edge of the city and, from that location, whether to operate integrated or separate production facilities.

3.2 Individuals

A population P of identical agents lives and works in the city. Agents consume the only good produced in the city and they live where they work. The latter assumption is justified by the fact that in the data, employment and residents in both areas of the city have moved to the edge in similar proportions, as summarized in Fact 4. Recall also that the share of net commuters between the center and edge stayed remarkably constant throughout our sample period.

Consumers order consumption according to a linear utility function. Therefore, given that the price of consumption goods is normalized to one, they solve

$$U = \max\left\{\overline{F}, w\right\}. \tag{2}$$

Since all agents are identical and, in equilibrium, some agents become managers while others become workers, $\overline{F} = w$. Furthermore, the fact that all agents have the option to set up integrated or non-integrated firms in any set of locations yields, in equilibrium, $\overline{F} = F_{ij}$ for all operating firms with headquarters in i and production plants in j.[7]

3.3 Equilibrium

We denote by E_{ij} the number of managers operating firms with headquarters in i and operation plants in j. Hence, the total number of managers at location i, E_i, is given by

$$E_i = E_{ii} + E_{ij}. \tag{3}$$

[7] We abstract from differences in agents' human capital or ability that may lead to differences in wages or managerial rents. We could introduce these difference only to reproduce our findings in terms of efficiency units of labor.

Since the number of units of land at the center is exogenously given by L_c, and firms rent one unit of land per worker, the number of workers at the center is given by

$$L_c = E_{cc}n_{cc} + E_{bc}n_{bc}. \tag{4}$$

Analogously, the number of workers at the edge is also given by the number of units of land used at the boundary,

$$L_b = E_{cb}n_{cb} + E_{bb}n_{bb}. \tag{5}$$

Land use at the edge, however, is endogenous and the area occupied by the city expands or contracts as the economic environment changes. It follows that the total number of workers in the city is given by $W = L_c + L_b$. Total city population is given by these workers plus those individuals who become managers of firms with headquarters at the center and at the edge. Therefore, labor market equilibrium requires that

$$E_c + E_b + W = P. \tag{6}$$

We are now ready to define a competitive equilibrium for this city:

A competitive city equilibrium is a set of scalars $\{E_c, E_b, E_{cc}, E_{bb}, E_{cb}, E_{bc}, L_b, R_c, w, \overline{F}, F_{cc}, F_{bb}, F_{cb}, F_{bc}\}$ *such that:*

1. *Agents solve (2), managers solve (1), and* $w = \overline{F} = F_{ij}$ *for all firms of type* ij *that operate in location,* $i, j \in \{c, b\}$.

2. *Equilibrium conditions (3), (4), (5), and (6) are satisfied.*

3. *Population size is given by* P*, land available at the center by* L_c*, and land rates at the boundary by* $R_b = \overline{R}$.

The number of establishments at the center is given by

$$S_c = \underbrace{E_{cc} + E_{cb}}_{E_c} + E_{bc}, \tag{7}$$

where S_c counts integrated production units at the center, E_{cc}, headquarters at the center used by managers who operate plants in the periphery, E_{cb}, and production plants at the center run by managers residing at the boundary, E_{bc}. Similarly, the number of establishments at the edge is defined as

$$S_b = \underbrace{E_{bb} + E_{bc}}_{E_b} + E_{cb}. \tag{8}$$

Within the framework of our model, average establishment sizes at the center and at the edge are given by $(L_c + E_c)/S_c$ and $(L_b + E_b)/S_b$ respectively. Establishments are of only three sizes: size one in the case of headquarters of non-integrated firms, size $\alpha\underline{\delta}$ in the case of the operation plant of a non-integrated firm, and size $1 + \alpha\overline{\delta}$ in the case of integrated firms.

The model we have just laid out potentially yields different types of equilibria. These types correspond to different sets of firms (i.e. integrated or not) operating in different areas of the city. The fact that spans of control differ across integrated and non-integrated firms rules out equilibria where all types of firms coexist. In essence, if differences in spans of control are such that a firm finds worth it to locate its headquarters at the center and its production plant at the edge, then the reverse cannot be true. We formalize this result in the next proposition. Proofs of all propositions are included in the Appendix.

Proposition 1 *There are no equilibria where both integrated and non-integrated firms operate at all locations.*

Of the remaining cases, the one corresponding to the type of city encountered in the data has most headquarters locating at the center (which in fact defines what we call the center and what is defined as a central county in the data). We now show that this case exists as an equilibrium of our model under a mild parameter restriction. In this equilibrium, there are no firms whose headquarters are at the edge but whose production plants reside at the center. Thus, the equilibrium we have just described is such that

$$\overline{F} = F_{cc} = F_{cb} = F_{bb} = w \text{ and } F_{bc} < \overline{F}, \tag{9}$$

so that $E_{cc}, E_{cb}, E_{bb} > 0$ and $E_{bc} = 0$. Because land rents are much lower at the edge than at the center in the data, firms that have operation plants at the center and headquarters at the edge are indeed very rare. Perhaps the most compelling reason to focus on this type of equilibrium is Fact 9. This fact shows that the number of managers is very tightly connected to the number of establishments, especially at the center where the number of managers and establishments move almost one for one in Figure 9a. In fact, in the model equilibrium with

$E_{bc} = 0$, the number of establishments and managers does move one for one at the center since $S_c = E_c$ from (7). This will not be the case at the edge, however, as $S_b = E_b + E_{cb}$ in (8), where E_{cb} captures establishments that are stand-alone operation plants. As in Fact 9, therefore, this equilibrium of the model implies that changes in managers are less closely related to changes in establishments at the edge than at the center. Thus, we prove all results below only for this case.

3.4 Equilibrium Allocation

Given the restriction $E_{bc} = 0$, we now construct an equilibrium allocation for our model. From (3), we know that $E_b = E_{bb}$ since $E_{bc} = 0$, and that $E_{cb} = E_c - E_{cc}$ where, by equation (4), $E_{cc} = L_c / \left(\alpha\overline{\delta}\right)$. Then, the number of workers in the city is given by

$$L_c + L_b = W$$

so that

$$L_c \left(1 - \frac{\underline{\delta}}{\overline{\delta}}\right) + E_c \alpha \underline{\delta} + E_b \alpha \overline{\delta} = W. \tag{10}$$

Condition (9) implies that integrated and non-integrated firms at the center earn equal profits,

$$(AE_c - w - R_c)\alpha\overline{\delta} = (AE_c - w - \overline{R})\alpha\underline{\delta}, \tag{11}$$

as do integrated firms across locations,

$$(AE_c - w - R_c)\alpha\overline{\delta} = (AE_b - w - \overline{R})\alpha\overline{\delta}.$$

These relations implicitly link the number of managers working at the center and boundary according to

$$E_b = E_c + \frac{\overline{R} - R_c}{A}. \tag{12}$$

Equality between rents and wages then implies that

$$(AE_c - w - R_c)\alpha\overline{\delta} = w. \tag{13}$$

From equations (11) and (13), we further have that

$$(AE_c - w - R_c)\alpha\overline{\delta} = w = (AE_c - w - \overline{R})\alpha\underline{\delta}.$$

Consequently,

$$\overline{R} - R_c = \left(\frac{1}{\alpha\overline{\delta}} - \frac{1}{\alpha\underline{\delta}}\right) w = -\underbrace{\left(\frac{\overline{\delta} - \underline{\delta}}{\alpha\overline{\delta}\underline{\delta}}\right)}_{\Lambda<0} w, \tag{14}$$

so that $R_c > \overline{R}$ under our maintained assumption regarding the span of control, $\overline{\delta} > \underline{\delta}$. That is, land rents are larger at the center than at the edge, an implication which follows from the assumption that $E_{bc} = 0$. The fact that most cities see land rents decrease as one moves away from the center is well known and reinforces our focus on an equilibrium with this feature.

From equation (13), and substituting for R_c using (14), we obtain, after some manipulations,

$$E_c = \frac{1 + \frac{1}{\alpha\overline{\delta}}}{A} w + \frac{R_c}{A}, = \frac{1 + \frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A} \tag{15}$$

which implies $E_c'(w) > 0$; the number of managers at the center increases with city wages. Using (14) and equation (12), we can solve for the set of managers at the edge as a function of wages,

$$E_b = E_c + \left(\frac{\Lambda}{A}\right) w = \frac{1 + \frac{1}{\alpha\overline{\delta}}}{A} w + \frac{\overline{R}}{A}. \tag{16}$$

Therefore, the number of managers at the edge also increases with wages, but at a slower rate since the rent differential decreases with wages and reduces the incentives to locate at the boundary.

Now consider the market clearing equation (6) given by,

$$L_c \left(1 - \frac{\delta}{\overline{\overline{\delta}}}\right) + E_c\alpha\underline{\delta} + E_b\alpha\overline{\delta} = W = P - E_c - E_b.$$

Substituting for the number of managers in both regions, the demand for workers becomes

$$\begin{aligned} W &= L_c \left(1 - \frac{\delta}{\overline{\overline{\delta}}}\right) + E_c\alpha\underline{\delta} + E_b\alpha\overline{\delta} && (17) \\ &= L_c \left(1 - \frac{\delta}{\overline{\overline{\delta}}}\right) + \left(\frac{\alpha\underline{\delta} + \alpha\overline{\delta}}{A}\right)\overline{R} + \frac{2 + \alpha\underline{\delta} + \alpha\overline{\delta}}{A} w && (18) \end{aligned}$$

which is linear and increasing in profits or wages (w). Since higher profits resulting from greater externalities are associated with more managers in both areas of the city, the demand for workers increases as profits rise. The supply of workers is given by

$$P - E_c - E_b = \left(P - \frac{2\overline{R}}{A}\right) - \frac{2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}}{A} w, \tag{19}$$

which again is linear but decreasing in profits or wages (w). Because larger profits motivate more agents to become managers, the supply of workers decreases with profits. The equilibrium wage can then be found by equating (17) and (19). That is

$$AP - AL_c\left(1 - \frac{\underline{\delta}}{\overline{\overline{\delta}}}\right) - \left(2 + \alpha\underline{\delta} + \alpha\overline{\delta}\right)\overline{R} = \left(4 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\overline{\delta}}}\right) w$$

so that city wages are given by

$$w = \frac{AP - AL_c\left(1 - \frac{\underline{\delta}}{\overline{\overline{\delta}}}\right) - \left(2 + \alpha\underline{\delta} + \alpha\overline{\delta}\right)\overline{R}}{4 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\overline{\delta}}}}. \tag{20}$$

With the equilibrium wage in hand, we can solve for all equilibrium variables of the model. For the comparative statics in the next section, it is helpful to denote the denominator of (20) as $D > 0$. An equilibrium of the type in which we are interested exists only if

$$\frac{P}{L_c} > \left(1 - \frac{\underline{\delta}}{\overline{\overline{\delta}}}\right) + \frac{\left(2 + \alpha\underline{\delta} + \alpha\overline{\delta}\right)\overline{R}}{AL_c}, \tag{21}$$

which ensures that $w > 0$. This restriction essentially requires city population densities that are large enough to make the creation of a city profitable given land rents at the edge. Population densities must also be large enough so that agglomeration effects guarantee that some non-integrated firms' headquarters choose to locate at the center. The following proposition obtains directly from the equilibrium wage derived above.

Proposition 2 *The equilibrium wage, w, is an increasing function of population size, P, and city-wide productivity, A. It is a decreasing function of the supply of land at the center, L_c, the span of control parameter, α, and edge land rents, $\overline{R}$.*

Note that the wage increases with population size. While this finding matches the fact that wages are generally higher in larger cities, it nevertheless seemingly conflicts with the standard intuition that wages fall as the supply of workers increases. There are two characteristics of our set up that contribute to overturning this intuition. First, the wage is the compensation of workers, but since all agents in the city are identical, it also represents profits of entrepreneurs or managers. Hence, since an increase in overall population creates new managers, the demand for workers also increases. Second, the production externality in our framework implies that the more managers operate in a given location, the higher the productivity (output per worker) of all firms in that location. As population grows, and more firms operate in the city, this effect contributes to raising manager rents and worker wages. As we have just mentioned, the prediction regarding wages and city size can be directly contrasted with data. In 2003, for instance, the average wage in the largest five cities in our sample was $42,976, as compared to just $34,340 for the five smallest cities.[8]

The effect of the model's other parameters on wages are more standard. Wages increase with city-wide productivity and decrease with the amount of land available at the center. The latter result reflects the fact that more firms at the center become integrated as L_c rises. This effect reduces the number of managers per worker at the center and, therefore, externalities in that location and manager profits (wages). As rents at the boundary increase, the advantage of setting up a non-integrated firm falls, which again reduces externalities at the center and wages. Finally, as firms' span of control, α, increases, firms become larger, less agents become managers, and externalities fall along with wages and managerial rents.

4. ADDRESSING THE STYLIZED FACTS

This section shows that the model we have developed naturally leads to the changes in city structure laid out in Section 2. From Fact 1, we know that population growth was positive in virtually all cities in our sample. Recall that the first set of facts referred to population size both at the center and at the edge. Since, in our model, agents live and work in the same location, the model's predictions concerning the growth in residents and employment are identical. Therefore, if we can show that as population grows, employment increases both at the center and at the edge (Fact 3), then the model will immediately satisfy Fact 2.

To address Fact 3, observe that two effects emerge as population grows. First, the number of managers at the center increases, as a result of the rise in agglomeration effects, which raises population at the center (recall that the worker population at the center is pinned down at level L_c by assumption). Second, since managerial population also increases at the boundary with urban growth, again as a result of larger externalities, so does the number of workers given the fixed span of control. Therefore, total employment must also increase

[8] See Lee (2005) for recent evidence on the urban wage premium.

at the edge. A portion of the additional workers at the boundary will work for managers that head firms with headquarters at the center. Ultimately, this reasoning implies that the model is consistent with Fact 2.

Total employment at the center and at the edge is given by $E_c + L_c$ and $E_b + L_b$ respectively. Simple differentiation then leads to the following proposition, consistent with Facts 2 and 3. All proofs in this section are relegated to the Appendix.

Proposition 3 *An increase in population implies an increase in total employment at the center, $E_c + L_c$, and at the edge, $E_b + L_b$.*

The proposition above provides conclusions regarding the level of employment in both areas of the city. However, we are equally interested in the share of employment in each area. We have already argued that as population grows, managerial population increases at the center. All new center managers, however, lead non-integrated firms. The reason is that the number of workers at the center cannot expand given the fixed amount of land and the technology that requires one unit of land per worker. Managers choose non-integrated rather than integrated firms because rents at the boundary do not grow with population, since they are pinned down by the opportunity cost of land, $\overline{R}$. At the same time, since the center land area of the city is constant, the price of land increases at the center which again motivates some managers to send their operation plants to the edge. In fact, rents at the center rise until the number of center managers who choose to operate integrated firms reverts back to its initial equilibrium. We can show that given a large enough population share at the center, the increase in center employment (caused solely by the increase in managerial population) is always smaller than the increase in managers and workers at the edge. This result implies that the employment share at the center falls with population growth, consistent with Fact 4. We formalize this reasoning in the next proposition.

Proposition 4 *An increase in population implies a decrease in the employment share at the center, $\frac{E_c+L_c}{P}$, if and only if*

$$\frac{E_c + L_c}{P} > \frac{1 + \frac{1}{\alpha\underline{\delta}}}{\left(1 + \frac{1}{\alpha\underline{\delta}}\right) + 3 + \alpha\overline{\delta} + \alpha\underline{\delta} + \frac{1}{\alpha\overline{\delta}}}.$$

The lower bound on the share of population working at the center amounts to restrictions on α and $\overline{\delta}/\underline{\delta}$ that turn out to be very mild (the proof of the proposition in the Appendix includes a parallel restriction expressed only in terms of the exogenous parameters). To illustrate this point, observe that in our framework, $\alpha\overline{\delta}$ determines the size of integrated firms which are the largest in the economy. As pointed out earlier, the average employment size of establishments in 1980 was 21.4 at the center and 17.1 at the edge. Therefore, we can conjecture that $1 + \alpha\overline{\delta} > 21.4$ and, furthermore, that $\overline{\delta}/\underline{\delta} > 20.4/17.1 = 1.19$ since $\overline{\delta}/\underline{\delta}$ represents the ratio of the largest to the smallest operation plant. Note also that the restriction set out in Proposition 4 becomes more severe as $\alpha\overline{\delta}$ and $\overline{\delta}/\underline{\delta}$ fall. Thus, suppose that we very conservatively set $\alpha\overline{\delta} = 10$ and $\alpha\overline{\delta} = 12$. Then Proposition 4 indicates that the share of employment at the center decreases with overall population whenever the share of center employment exceeds 4.2%, a condition that is easily met by all cities in our sample.[9]

To summarize thus far, an increase in overall city population leads to findings for the levels and shares of employment in different areas of the city that are consistent with the data. In particular, population growth leads to an increase in employment levels everywhere, but also to a shift in employment from the center to the edge in shares. These results follow directly from firms having the opportunity to break up their operations geographically.

It is difficult to understand how observed changes in levels and shares of population in different sections of the city could be the result of forces that are not related to an overall expansion in size (i.e. population growth). If other forces were responsible for these changes, and since one needs to introduce scale effects in order to generate cities, reductions in the share of employment at the center will generally lead to reductions in the level of employment as well. Hence, it seems that two elements are needed to obtain models that can reproduce these dimensions of the data. First, one needs models where these changes are the result of city growth. Second, these models should also allow for endogenous employment densities. Evidently, if densities were not endogenous, given that the center county area has not changed in the data, employment at the center would necessarily be predicted to remain constant.

The advantage of writing down a model in which agents' occupations are explicitly consid-

[9] If one actually uses $\alpha\underline{\delta}$= 17.1 and $\alpha\overline{\delta} = 21.4$, the lower bound required in Proposition 4 becomes even less stringent at 2.5 percent.

ered is that it has predictions for the effect of changes in exogenous variables on the location of different occupations within cities. In Section 2, we presented a set of facts that are related to the locations of agents working in different occupations. Specifically, we showed that the changes in manager shares at the center were generally larger than those in non-manager shares (Fact 5a). We also showed that the change in the manager to non-manager ratio was larger at the center than at the edge (Fact 5b). Both these facts imply that managers are increasingly concentrated at the center relative to the boundary. These facts were expressed in terms of differences in shares and ratios between the center and the edge, and not in term of levels, shares, or ratios directly.[10] Fact 5c shows that the fraction of urban population in management positions increased during the 1980s. While our model is consistent with this fact, other forces working at a more aggregate level – such as changing transport and communication costs across cities, or between cities and rural areas – may have helped increase the total number of managers in the city beyond that implied by population growth alone (as in Duranton and Puga [2004]). Hence, we concentrate primarily on the predictions of our model for changes in the difference between the center and edge management shares, as well as manager to non-manager ratios, driven by city population growth (Facts 5a and 5b).

Consider first the difference in the share of managers and non-managers at the center. The analysis we just carried out suggests that population growth leads to an increase in the number of managers at the center. In fact, it also leads to an increase in the number of managers at the boundary. Some of the new managers establish themselves at the center because of the production externalities generated by managers in that section of the city. Others take advantage of the larger spans of control, as well as lower rents, at the boundary. However, the share of managers at the center increases unambiguously since managers do not use land and, therefore, do not drive up land rents. As we argued earlier, all employees under the supervision of new entrepreneurs at the center work in operation plants located at the periphery. In addition, all new managers at the boundary head integrated firms. As a result, the share of workers at the center must decrease and, given the rise in the share of managers at the center, the difference between manager and non-manager shares must

[10] Since our model focuses on only one city and not a system of cities, it is silent on level differences across cities. In our framework, these cross-sectional differences would in principle stem from different values of L_c, $\overline{R}$, α, or A across cities.

increase following population growth.

It should be remarked that the increase in the share of managers at the center is actually a counter-factual implication of our model. Specifically, the theory over-emphasizes the concentration of managers at the center. This implication can be attenuated by requiring that managers rent land at the headquarter's location. This extension, however, would come at the cost of a much more complicated setup. In addition, we view the decrease in manager share at the center as resulting partly from the overall increase in the number of agents in management occupations, as in Fact 5c (i.e. $E_c/\left(E_c+E_b\right)$ tends to fall as E_c+E_b increases).

Our model also predicts that the ratio of managers to non-managers will increase more rapidly at the center than at the edge with population growth. Since the number of managers at the center increases and the number of workers is fixed, it is clear that this ratio increases at the center. At the edge, since the number of workers increases in part because of the increase in managers of non-integrated firms at the center, the ratio always decreases. Together these results directly lead to Fact 5b. In addition, as population increases, all new managers at the center run non-integrated firms. Given that non-integrated firms have fewer workers per manager (smaller spans of control), this leads to an increase in the share of city residents that become managers, as in Fact 5c. These results are stated formally in the next proposition, consistent with Fact 5.

Proposition 5 *An increase in population implies an increase in:*

- *The difference between manager and non-manager shares at the center,* $\frac{E_c}{E_c+E_b}-\frac{L_c}{L_c+L_b}$.
- *The difference in the ratio of managers to non-managers between the center and the edge,* $\frac{E_c}{L_c}-\frac{E_b}{L_b}$.
- *The number of managers per resident,* $\frac{E_c+E_b}{P}$.

Our model evidently has predictions for the number of establishments at the center and at the edge of the city. First, recall that at the center, the number of establishments is equal to the number of managers since establishments at that location are either headquarters or non-integrated firms, but under our assumptions never just operation plants. It follows immediately that the number of establishments at the center increases with population growth.

Furthermore, since new managers at the center operate only non-integrated firms, the boundary also sees an increase in operation plants. The latter two findings are consistent with Fact 6. Because every new manager at the center is associated with an additional operation plant at the edge, and the edge also experiences entry of new integrated firms following population growth, the number of establishments at the edge must increase and, in fact, must increase by more than the increase in center establishments. This implies that the share of establishment at the center must fall, as in Fact 7.

Proposition 6 *An increase in population implies:*

- *An increase in the number of establishments located at the center and at the edge,* S_c *and* S_b*, with the number of establishments increasing more rapidly at the edge than at the center.*
- *A decrease in the share of establishments at the center,* $\frac{S_c}{S_c+S_b}$.

As pointed out earlier, establishments are of three different sizes in our set up: Headquarters of size one, operation plants of size $\alpha\underline{\delta}$, and integrated firms of size $1+\alpha\overline{\delta}$. Hence, the specific combination of firms of each type in a given region of the city determines average establishment size in that region. Since there are no operation plants at the center, firms can only be of size one or $1+\alpha\overline{\delta}$ in that section of the city. As population increases, the number of managers at the center increases and so does the share of establishments of size one. Therefore, population growth contributes to a decrease in average firm size in the center region, as in Fact 8. In contrast, the boundary comprises only establishments of size $\alpha\underline{\delta}$ and $1+\alpha\overline{\delta}$. First, this implies that the average size of establishments is larger at the edge than at the center, unless most firms are production plants and many integrated firms reside at the center. The latter case would make our model consistent with the larger establishment sizes at the center observed in 1980. As population grows, the set of establishments that are production plants increases at the edge, and so does the set of integrated firms. In the next proposition, we show that the increase in the number of production plants dominates, thereby leading to a decrease in firm size at the edge, consistent with Fact 8.

Proposition 7 *An increase in population implies a decrease in the average size of establishments at the city center and edge,* $\frac{E_c+L_c}{S_c}$ *and* $\frac{E_b+L_b}{S_b}$*, respectively.*

Thus, our model predicts that population growth reduces establishment sizes in both regions following simple composition effects. The data, however, shows that changes in establishment size are negative in both regions for slightly more than half of the cities in our sample. What factors may explain the behavior of establishment size in the remaining cities? Given the theory we have just laid out, a possible answer is that lower communication costs have led to larger spans of control (an increase in α) and, therefore, larger firms throughout the city. The evidence suggests that this phenomenon did not dominate changes in average firm size in most cities in the 1980s, but may nevertheless be significant in more recent time periods for several cities (as argued by Garicano and Rossi-Hansberg [2003] for the late 1990s).

5. ROBUSTNESS OF THE THEORY: LABOR MOBILITY

Our analysis to this point has focused on a theory where land rents at the boundary are given by some alternative country-wide non-urban land use. Wages and profits in the city were therefore endogenously determined. In a model of a system of cities, this would be equivalent to assuming a perfectly elastic supply of urban land at $\overline{R}$ and high moving costs that impede the mobility of workers between cities. One might instead imagine an alternative construct where wages are fixed at some economy wide level, w, exogenous to the city. Heterogeneity in the quality of land in different regions would then attract a certain population which in turn would determine all land prices in the city. This section establishes that all propositions derived in the previous sections continue to hold using this alternative interpretation, provided a mild restriction on parameters.

The model remains as in Section 4, except that condition (20) now determines land rents at the edge instead of wages,

$$\overline{R} = \frac{AP - AL_c(1-\frac{\underline{\delta}}{\overline{\delta}}) - w(4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}})}{2+\alpha\underline{\delta}+\alpha\overline{\delta}}.$$

It is straightforward to check that $\partial\overline{R}/\partial P > 0$, $\partial\overline{R}/\partial L_c < 0$, and $\partial\overline{R}/\partial w < 0$, consistent with Proposition 2. We can then prove that results replicating Proposition 3 through 7 continue to hold. The derivations for a subset of these results requires that the number of managers per establishment at the boundary be greater than one half, $E_b/S_b > 1/2$.[11] Figure 9b suggests that this is likely the case in the data, where a given percentage change in the number of establishments is associated with approximately twice that percentage change in the number of managers. These results are formalized in the next proposition.

Proposition 8 *With exogenous wages, w, and endogenous land rents at the edge, $\bar{R}$, an increase in population implies:*

- *i) An increase in total employment at the center, $E_c + L_c$, and at the edge, $E_b + L_b$.*
- *ii) A decrease in the employment share at the center, $\frac{E_c+L_c}{P}$.*
- *iii) An increase in: a) the difference between manager and non-manager shares at the center, $\frac{E_c}{E_c+E_b} - \frac{L_c}{L_c+L_b}$, b) the difference in the ratio of managers to non-managers between the center and the edge, $\frac{E_c}{L_c} - \frac{E_b}{L_b}$ if $\frac{E_b}{S_b} > \frac{1}{2}$, and c) the number of managers per resident, $\frac{E_c+E_b}{P}$.*
- *iv) An increase in the number of establishments located at the center and at the edge, S_c and S_b respectively, and a decrease in the share of establishments at the center, $\frac{S_c}{S_c+S_b}$.*
- *v) A decrease in the average size of establishments located at the center and at the edge, $\frac{E_c+L_c}{S_c}$ and $\frac{E_b+L_b}{S_b}$ respectively, if $\frac{E_b}{S_b} > \frac{1}{2}$.*

6. CHANGES IN CITY STRUCTURE AND POPULATION GROWTH

In Section 2, we presented a set of stylized facts on the evolution of city structure. We then argued in the three subsequent sections that observed increases in population alone could help rationalize those facts. At this stage, therefore, it is natural to ask whether one could establish the implications of our model more directly in the data? As a first pass, we can use the data to assess whether the changes in city structure presented in

[11] However, since a different price is now taken as given from the city's standpoint, the restriction on the share of center population in Section 3, Proposition 4, is no longer necessary.

Facts 2 through 9 are in fact correlated with population growth. However, one needs to be cautious in the interpretation of such an empirical exercise. First, the theory predicts that the effect of changes in population size should lead to Facts 2 through 9 only if cross-sectional characteristics of cities are properly controlled for. In particular, the theory has predictions for the sign of these correlations after controlling for center county land sizes (L_c), land rents at the boundary ($\overline{R}$), spans of control (α, $\overline{\delta}$ and $\underline{\delta}$), and productivity (A), as well as any changes in these variables during the 1980s. At this point, we do not have residential land rents at the boundary for 1980, or a suitable proxy. However, we can control for land area L_c and, to some degree, for productivity as well as spans of control using the 1980 level of per capita income and ratio of managers to population respectively. Since our theory also abstracts from available city infrastructure and other idiosyncratic city characteristics, we take city age into consideration by using the decade in which the city became one of the largest 50 cities in the U.S. This last variable helps but cannot obviously capture all cross-sectional characteristics omitted from the model. Because of the size of our sample, we do not control for changes in any of these variables. Finally, a key problem with calculating simple correlations is that our theory does not predict a linear response of city structure to population changes. Despite these caveats regarding the mapping between these correlations and our theoretical results, Table 1 presents encouraging results that are consistent with the framework introduced in this paper.

Table 1 presents correlations between population growth and the residuals obtained from running an OLS regression of the various changes in city structure laid out in Section 2 against the controls discussed above. Observe that all but the last two correlations, the ones related to establishment size, have the sign predicted by our theory. Some of these correlations are admittedly low. Nevertheless, our framework does surprisingly well given that increases in only one independent variable, namely population growth, are shown to be consistent with eleven diverse changes in the internal structure of cities.[12] The incorrect sign on the correlation between center/edge establishment sizes and population growth is somewhat disappointing, and indeed the model does not contain a force that would lead to

[12] If we eliminate from the sample the four fastest growing cities (Orlando, Las Vegas, Austin, and Phoenix) correlations related to occupations increase substantially (both become larger than 0.22). This result is consistent with the non-linear response of these changes to population growth in our model.

larger firms in larger cities. Here, firms are larger only if they decide to integrate but there are no differences across integrated firms. In practice, larger cities have larger firms partly because demand for firms' varieties is larger, a dimension from which we have abstracted.

Table 1

Correlations with Population Growth

	Population Growth
Center Population Growth	0.511
Edge Population Growth	0.558
Center Employment Growth	0.496
Edge Employment Growth	0.538
Change in Edge Population Share	0.127
Change in Edge Employment Share	0.155
Change in Management - Non-Management Shares	0.133
Change in Management over Non-Management Ratio	0.128
Change in Center Establishments	0.381
Change in Edge Establishments	0.572
Change in Center Establishment Share	-0.188
Change in Center Establishment Size	0.467
Change in Edge Establishment Size	0.129

6. CONCLUSIONS

This paper makes three distinct contributions. First we document a set of facts regarding changes in urban structure experienced by U.S. cities in the 1980s. These facts include overall population growth; an increase in residents and employment at the center and city boundaries; a reduction in the share of employment and residents in the center region of cities; a concentration of managers relative to non-managers at the center; an increase in establishments in both areas of the city but a decrease in establishment shares at the center; and a decline in establishment size both at the center and at the edge of cities. Second, we propose a theory that incorporates firms' location and integration decisions and characterize the implications of such a theory for urban structure. Third, we show that population growth alone is consistent with the set of changes observed in the 1980s, thereby highlighting the effects of population growth on urban structure.

The theory we present has urban policy implications that differ from more standard models of urban structure. In particular, we provide a framework that could potentially be used to analyze the kinds of policies aimed at "reviving city centers" that have been put in practice in many cities across the U.S. Given that our framework includes agglomeration forces in the form of externalities, some urban policies may improve equilibrium allocations in our setup. However, the specific type of policy in place is critical. For example, Au and Henderson (2004) show that restrictions on urban migration have had important efficiency costs in China. For now, the question of whether zoning restrictions or location subsidies, as in Rossi-Hansberg (2004), are optimal in our setup, and in general the design of these policies, is left to future research.

In order to underscore the importance of firms' location decisions, as well as their decision regarding whether or not to integrate their operations, our model abstracts from important elements of cities typically emphasized in the urban literature. One such element is a spatial setup in which multiple sub-centers may arise (see Fujita and Ogawa [1982], and Lucas and Rossi-Hansberg [2002]). Other dimensions, such as the effect of durable housing structures, as in Glaeser and Gyourko (2004), and urbanization patterns in a system of cities, as in Henderson (2003), and Henderson and Wang (2004), are undoubtedly important. In addition, our theory focuses on one particular type of agglomeration force. However, as argued by Rosenthal and Strange (2003), different agglomeration forces interact in metropolitan areas. One could, in principle, study any of these forces along with the firm's location and integration decisions we emphasize. The explanatory power gained by incorporating these firms' decisions with respect to the facts we document in this paper will, hopefully, push the urban literature to add these margins to the rich set of frameworks available.

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APPENDIX

Proof of Proposition 1

The proof proceeds by contradiction. Suppose that both integrated and non-integrated firms exist in both areas of the city. Then by (2), it must be the case that

$$F_{bb} = F_{cc} = F_{cb} = F_{bc}. \tag{22}$$

The fact that $F_{cc} = F_{bb}$ implies that

$$\begin{aligned} (AE_c - w - R_c)\alpha\overline{\delta} &= (AE_b - w - \overline{R})\alpha\overline{\delta} \text{ or} \\ AE_c - AE_b &= R_c - \overline{R}. \end{aligned} \tag{23}$$

Similarly, the fact that $F_{cb} = F_{bc}$ implies that

$$\begin{aligned} (AE_c - w - \overline{R})\alpha\underline{\delta} &= (AE_b - w - R_c)\alpha\underline{\delta} \text{ or} \\ AE_c - AE_b &= \overline{R} - R_c. \end{aligned} \tag{24}$$

Equations (23) and (24) can only hold if $R_c = \overline{R} = R$, in which case $E_c = E_b = E$. It follows that profits for an integrated firm are $(AE - w - R)\alpha\overline{\delta}$ while those of a non-integrated firm are $(AE - w - R)\alpha\underline{\delta}$. That is, $F_{cc} = F_{bb} > F_{cb} = F_{bc}$ which contradicts (22).

Proof of Proposition 2

Simple partial derivatives imply that

$$\frac{\partial w}{\partial P} = \frac{A}{D} > 0, \qquad \frac{\partial w}{\partial A} = \frac{P - L_c\left(1 - \frac{\underline{\delta}}{\overline{\delta}}\right)}{D} > 0,$$
$$\frac{\partial w}{\partial L_c} = -\frac{A}{D}\left(1 - \frac{\underline{\delta}}{\overline{\delta}}\right) < 0, \qquad \frac{\partial w}{\partial \overline{R}} = \frac{-\left(2 + \alpha\underline{\delta} + \alpha\overline{\delta}\right)}{D} < 0,$$

and

$$\frac{\partial w}{\partial \alpha} = -\frac{\left(\underline{\delta} + \overline{\delta}\right)\overline{R}}{D^2} - w\frac{\underline{\delta} + \overline{\delta} - \frac{1}{\alpha^2\underline{\delta}} - \frac{1}{\alpha^2\overline{\delta}}}{D^2} < 0$$

The sign of the last term is guaranteed since the span of control of integrated and non-integrated firms is greater than $\alpha\overline{\delta} > \alpha\underline{\delta} > 1$.

Proof of Proposition 3

Employment in the center of the city is given by

$$E_c + L_c = \frac{1 + \frac{1}{\alpha\underline{\delta}}}{A}w + \frac{\overline{R}}{A} + L_c.$$

Differentiating with respect to P, we obtain

$$\frac{\partial\left(E_c+L_c\right)}{\partial P}=\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\frac{\partial w}{\partial P}>0,$$

which captures the increase in population at the center.

Total employment at the edge is given by

$$\begin{aligned} E_b+L_b &= \left(E_c-\frac{L_c}{\alpha\overline{\delta}}\right)\alpha\underline{\delta}+E_b\left(1+\alpha\overline{\delta}\right) \\ &= \frac{\overline{R}}{A}\left(1+\alpha\underline{\delta}+\alpha\overline{\delta}\right)-L_c\frac{\underline{\delta}}{\overline{\overline{\delta}}}+\frac{3+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\overline{\overline{\delta}}}}{A}w \end{aligned}$$

so that

$$\frac{\partial\left(E_b+L_b\right)}{\partial P}=\frac{3+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\overline{\overline{\delta}}}}{D}>0,$$

which yields the increase in population at the boundary.

Proof of Proposition 4

The employment share at the center is given by

$$\frac{E_c+L_c}{P}=\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}w+\frac{\overline{R}}{A}+L_c\right)\frac{1}{P}.$$

Hence, the derivative with respect to population is

$$\frac{\partial\frac{E_c+L_c}{P}}{\partial P}=\frac{P\frac{\partial E_c}{\partial P}-\left(E_c+L_c\right)}{P^2},$$

where

$$\frac{\partial E_c}{\partial P}=\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}.$$

It follows that $\frac{\partial\frac{E_c+L_c}{P}}{\partial P}<0$ if and only if

$$\frac{\partial E_c}{\partial P}<\frac{E_c+L_c}{P},$$

or alternatively,

$$\frac{E_c+L_c}{P}>\frac{1+\frac{1}{\alpha\underline{\delta}}}{\left(1+\frac{1}{\alpha\underline{\delta}}\right)+3+\alpha\overline{\delta}+\alpha\underline{\delta}+\frac{1}{\alpha\overline{\overline{\delta}}}}.$$

This last condition can be re-written in terms of exogenous parameters only,

$$\left(3+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{2}{\alpha\overline{\overline{\delta}}}+\frac{\underline{\delta}}{\overline{\overline{\delta}}}\right)L_c>\overline{R}\left(\frac{\overline{\delta}}{\underline{\delta}}-1+\frac{1}{\alpha\underline{\delta}}-\frac{1}{\alpha\overline{\overline{\delta}}}\right),$$

in which case the share of employment at the center decreases with population growth.

Proof of Proposition 5

i) $\frac{E_c}{E_c+E_b} - \frac{L_c}{L_c+L_b}$ increases with P.

The share of managers at the center is given by,

$$\frac{E_c}{E_c+E_b} = \frac{E_c}{2E_c + \left(\frac{\Lambda}{A}\right) w} = \frac{(1+\frac{1}{\alpha\underline{\delta}})w + \overline{R}}{(2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}})w + 2\overline{R}}.$$

Hence,

$$\frac{\partial \frac{E_c}{E_c+E_b}}{\partial P} = \frac{\overline{R}\left[\frac{1}{\alpha\underline{\delta}} - \frac{1}{\alpha\overline{\delta}}\right]\frac{\partial w}{\partial P}}{\left[(2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}})w + 2\overline{R}\right]^2} > 0.$$

which captures the increase in manager share at the center. The share of workers at the center is given by

$$\frac{L_c}{L_c+L_b} = \frac{L_c}{W} = \frac{L_c}{L_c\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right) + \left(\frac{\alpha\underline{\delta}+\alpha\overline{\delta}}{A}\right)\overline{R} + \frac{2+\alpha\underline{\delta}+\alpha\overline{\delta}}{A}w}.$$

Therefore,

$$\frac{\partial \frac{L_c}{L_c+L_b}}{\partial P} = -\left(\frac{L_c}{W^2}\frac{2+\alpha\underline{\delta}+\alpha\overline{\delta}}{A}\right)\frac{\partial w}{\partial P} < 0$$

which captures the fall in worker share at the center. Thus,

$$\frac{\partial \frac{E_c}{E_c+E_b}}{\partial P} - \frac{\partial \frac{L_c}{L_c+L_b}}{\partial P} > 0.$$

ii) $\frac{E_c}{L_c} - \frac{E_b}{L_b}$ increases with P.

We analyze each term in turn. Since L_c is fixed and E_c increases with P, $\frac{E_c}{L_c}$ clearly increases with P,

$$\frac{\partial \frac{E_c}{L_c}}{\partial P} = \frac{1+\frac{1}{\alpha\underline{\delta}}}{L_c D} > 0.$$

To analyze the second term, note that E_b/L_b is given by

$$\begin{aligned}
\frac{E_b}{L_b} &= \frac{E_b}{\left(E_c - \frac{L_c}{\alpha\overline{\delta}}\right)\alpha\underline{\delta} + E_b\alpha\overline{\delta}} \\
&> \frac{\left(1+\frac{1}{\alpha\overline{\delta}}\right)w + \overline{R}}{\left(2+\alpha\overline{\delta}+\alpha\underline{\delta}\right)w + \alpha\left(\underline{\delta}+\overline{\delta}\right)\overline{R}}
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial \frac{E_b}{L_b}}{\partial P} &= \frac{1}{DL_b}\left(1+\frac{1}{\alpha\overline{\delta}} - \frac{E_b}{L_b}\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\right) \\
&< \frac{1}{DL_b}\left(1 - \frac{\overline{\delta}(2+\alpha\underline{\delta}+\alpha\overline{\delta})w + \overline{R}(\overline{\delta}-\underline{\delta}+\alpha\overline{\delta}\underline{\delta}+\alpha\overline{\delta}^2)}{\overline{\delta}[(2+\alpha\overline{\delta}+\alpha\underline{\delta})w + \alpha(\overline{\delta}+\underline{\delta})\overline{R}]}\right) < 0,
\end{aligned}$$

from which it immediately follows that that $\frac{\partial\left(\frac{E_c}{L_c}-\frac{E_b}{L_b}\right)}{\partial P}>0$.

iii) $\frac{(E_c+E_b)}{P}$ increases with P.

To see this, observe that

$$\frac{E_c}{P}=\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}w+\frac{\overline{R}}{A}\right)\frac{1}{P}.$$

Hence, the derivative with respect to population is

$$\begin{aligned}
\frac{E_c}{P} &= \frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\left(1-\frac{L_c}{P}\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)\right)+\left(1-\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\right)\frac{\overline{R}}{AP} \\
\frac{\partial\frac{E_c}{P}}{\partial P} &= \frac{1}{P^2}\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}L_c\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)-\left(1-\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\right)\frac{\overline{R}}{A}\right) \\
&= \frac{1}{P^2}\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}L_c\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)+\left(\frac{3+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{2}{\alpha\underline{\delta}}+\frac{\overline{\delta}}{\underline{\delta}}}{4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}-1\right)\frac{\overline{R}}{A}\right)>0
\end{aligned}$$

where the inequality follows from $\overline{\delta}>\underline{\delta}$. Moreover, note that

$$\begin{aligned}
\frac{E_b}{P} &= \frac{1+\frac{1}{\alpha\overline{\delta}}}{PA}w+\frac{\overline{R}}{PA} \\
&= \frac{1+\frac{1}{\alpha\overline{\delta}}}{D}\left(1-\frac{L_c}{P}\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)-\left(2+\alpha\underline{\delta}+\alpha\overline{\delta}\right)\frac{\overline{R}}{PA}\right)+\frac{\overline{R}}{PA}.
\end{aligned}$$

Hence,

$$\frac{\partial\frac{E_b}{P}}{\partial P}=\frac{1}{P^2}\left(\frac{\left(1+\frac{1}{\alpha\overline{\delta}}\right)\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)}{D}L_c+\left(\frac{3+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{2}{\alpha\overline{\delta}}+\frac{\underline{\delta}}{\overline{\delta}}}{4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}-1\right)\frac{\overline{R}}{A}\right),$$

so that

$$\begin{aligned}
\frac{\partial\frac{E_c+E_b}{P}}{\partial P} &= \frac{\left(2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}\right)\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)}{P^2D}L_c \\
&\quad+\left(\frac{6+2\alpha\underline{\delta}+2\alpha\overline{\delta}+\frac{2}{\alpha\underline{\delta}}+\frac{2}{\alpha\overline{\delta}}+\left(\frac{\overline{\delta}}{\underline{\delta}}+\frac{\underline{\delta}}{\overline{\delta}}\right)}{4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}-2\right)\frac{\overline{R}}{P^2A}
\end{aligned}$$

which is positive if

$$\frac{\overline{\delta}}{\underline{\delta}}+\frac{\underline{\delta}}{\overline{\delta}}>2,$$

or if

$$\overline{\delta}^2-2\overline{\delta}\underline{\delta}+\underline{\delta}^2=\left(\overline{\delta}-\underline{\delta}\right)^2>0$$

which holds trivially.

Proof of Proposition 6

The total number of establishments at the center is given by

$$S_c = E_c + E_{bc} = E_c = \frac{1+\frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A}.$$

It follows that

$$\frac{\partial S_c}{\partial P} = \frac{1+\frac{1}{\alpha\underline{\delta}}}{D} > 0,$$

which depicts the rise in establishments at the center. The number of establishments at the edge is given by

$$\begin{aligned} S_b &= E_b + E_{cb} = E_b + E_c - E_{cc} \\ &= \frac{2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}{A} w + 2\frac{\overline{R}}{A} - \frac{L_c}{\alpha\overline{\delta}}. \end{aligned}$$

Therefore

$$\begin{aligned} \frac{\partial S_b}{\partial P} &= \frac{2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}{A} \frac{\partial w}{\partial P} \\ &= \frac{2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}{D} > 0 \end{aligned}$$

which establishes the increase in establishments at the edge. It is also the case that

$$\frac{\partial S_c}{\partial P} = \frac{1+\frac{1}{\alpha\underline{\delta}}}{D} < \frac{2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}}{D} = \frac{\partial S_b}{\partial P},$$

so that the increase in the number of establishments is greater at the edge than at the center.

Finally, the change in the share of establishments with respect to population is given by

$$\begin{aligned} \frac{\partial \frac{S_c}{S_c+S_b}}{\partial P} &= \frac{(S_c+S_b)\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}\right)\frac{\partial w}{\partial P} - S_c \frac{1}{A}\left[3+\frac{1}{\alpha\overline{\delta}}+\frac{2}{\alpha\underline{\delta}}\right]\frac{\partial w}{\partial P}}{(S_c+S_b)^2} \\ &= \frac{\frac{1}{A}\frac{\partial w}{\partial P}\left[-S_c(2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}) + S_b(1+\frac{1}{\alpha\underline{\delta}})\right]}{(S_c+S_b)^2}. \end{aligned}$$

The above expression is negative whenever

$$\begin{aligned} S_b &< S_c \frac{(1+\frac{1}{\alpha\underline{\delta}}+1+\frac{1}{\alpha\overline{\delta}})}{(1+\frac{1}{\alpha\underline{\delta}})} \\ &= S_c\left(1+\frac{1+\frac{1}{\alpha\overline{\delta}}}{1+\frac{1}{\alpha\underline{\delta}}}\right) < 2S_c. \end{aligned}$$

Thus, $S_c > \frac{1}{2}S_b$ whenever

$$\frac{S_c}{S_c + S_b} > \frac{1}{3}.$$

This restriction, however, is always satisfied since

$$\frac{S_c}{S_c + S_b} = \frac{\left(\frac{1+\frac{1}{\alpha\underline{\delta}}}{A}\right) w + \frac{\overline{R}}{A}}{\frac{1}{A}\left[3 + \frac{1}{\alpha\overline{\delta}} + \frac{2}{\alpha\underline{\delta}}\right] w + 3\frac{\overline{R}}{A} - \frac{L_c}{\alpha\overline{\delta}}} > \frac{1}{3}$$

holds when

$$\frac{L_c}{\alpha\overline{\delta}} > \frac{w}{A}\left(\frac{1}{\alpha\overline{\delta}} - \frac{1}{\alpha\underline{\delta}}\right),$$

which is always the case since $\frac{1}{\alpha\overline{\delta}} - \frac{1}{\alpha\underline{\delta}} < 0$.

Proof of Proposition 7

Average establishment size at the center is given by

$$\frac{L_c + E_c}{S_c} = \frac{L_c + E_c}{E_c} = \frac{L_c + \frac{1+\frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A}}{\frac{1+\frac{1}{\alpha\underline{\delta}}}{A} w + \frac{\overline{R}}{A}},$$

so that

$$\frac{\partial \frac{L_c+E_c}{E_c}}{\partial P} = -\frac{\partial E_c}{\partial P}\frac{L_c}{(E_c)^2} = -\frac{1+\frac{1}{\alpha\underline{\delta}}}{D}\frac{L_c}{(E_c)^2} < 0.$$

Hence, average firm size at the center decreases with population growth. Average establishment size at the edge is analogously given by

$$\frac{E_b + L_b}{S_b} = \frac{\overline{R}\left(1 + \alpha\underline{\delta} + \alpha\overline{\delta}\right) - AL_c\frac{\underline{\delta}}{\overline{\delta}} + \left(3 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\overline{\delta}}\right) w}{\left(2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}\right) w + 2\overline{R} - A\frac{L_c}{\alpha\overline{\delta}}}.$$

It follows that

$$\frac{\partial \frac{E_b+L_b}{S_b}}{\partial P} = \frac{1}{S_b}\frac{A}{D}\left(\left(3 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\overline{\delta}}\right) - \frac{E_b + L_b}{S_b}\left(2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}\right)\right) < 0$$

when

$$\frac{3 + \alpha\underline{\delta} + \alpha\overline{\delta} + \frac{1}{\alpha\overline{\delta}}}{2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}} < \frac{E_b + L_b}{S_b},$$

or

$$\frac{AL_c}{\overline{\delta}\alpha} > -\left(\frac{\overline{\delta} - \underline{\delta} + \alpha\left(\overline{\delta} - \underline{\delta}\right)^2}{1 + \overline{\delta}^2\alpha^2 - \underline{\delta}\overline{\delta}\alpha^2 + 2\overline{\delta}\alpha - \underline{\delta}\alpha}\right)\frac{\overline{R}}{\underline{\delta}},$$

which always holds since $1 + \overline{\delta}^2\alpha^2 - \underline{\delta}\overline{\delta}\alpha^2 + 2\overline{\delta}\alpha - \underline{\delta}\alpha > 0$. Consequently, average firm size at the edge also falls.

Proofs of Proposition 8

i) At the center, we have

$$\frac{\partial(E_c+L_c)}{\partial P}=\frac{1}{A}\frac{\partial\overline{R}}{\partial P}=\frac{1}{D'}>0,$$

where $D'=2+\alpha\underline{\delta}+\alpha\overline{\delta}$. With respect to the edge, we have

$$E_b+L_b=\left(1+\alpha\overline{\delta}\right)E_b+\alpha\underline{\delta}E_c-\alpha\underline{\delta}E_{cc}.$$

Therefore,

$$\frac{\partial(E_b+L_b)}{\partial P}=\left(1+\alpha\overline{\delta}\right)\frac{\partial E_b}{\partial P}+\alpha\underline{\delta}\frac{\partial E_c}{\partial P}=\frac{1}{D'}\left(1+\alpha\overline{\delta}+\alpha\underline{\delta}\right)>0$$

ii) Taking the derivative of $\frac{E_c+L_c}{P}$ with respect to P yields

$$\begin{aligned}\frac{\partial\frac{E_c+L_c}{P}}{\partial P} &= -\frac{1}{AP^2}\left(\left(1+\frac{1}{\alpha\underline{\delta}}\right)w+\overline{R}+AL_c\right)+\frac{1}{PD'}\\ &= -\frac{1}{P}\left[\frac{\left(\left(1+\frac{1}{\alpha\underline{\delta}}\right)w+\overline{R}+AL_c\right)D'-AP}{APD'}\right],\end{aligned}$$

which is strictly negative when

$$\left(\left(1+\frac{1}{\alpha\underline{\delta}}\right)w+\overline{R}+AL_c\right)D'-AP>0.$$

The latter condition holds if and only if

$$w\left[\left(1+\frac{1}{\alpha\underline{\delta}}\right)D'-\left(4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}\right)\right]+AL_c\left[D'-\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right)\right]>0,$$

or alternatively if

$$\left(1+\frac{1}{\alpha\underline{\delta}}\right)D'>4+\alpha\underline{\delta}+\alpha\overline{\delta}+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}$$

and

$$D'>\left(1-\frac{\underline{\delta}}{\overline{\delta}}\right),$$

which both hold since $\overline{\delta}>\underline{\delta}$.

iii)

a) $\frac{E_c}{E_c+E_b}-\frac{L_c}{L_c+L_b}$ increases with P.

Simple derivation gives

$$\frac{\partial\frac{E_c}{E_c+E_b}}{\partial P}=\frac{\left(1+\frac{1}{\alpha\overline{\delta}}\right)w\frac{\partial\overline{R}}{\partial P}}{\left[\left(2+\frac{1}{\alpha\underline{\delta}}+\frac{1}{\alpha\overline{\delta}}\right)w+2\overline{R}\right]^2}>0$$

and

$$\frac{\partial \frac{L_c}{L_c+L_b}}{\partial P} = -\frac{L_c}{W}\left(\frac{\alpha\underline{\delta}+\alpha\overline{\delta}}{A}\right)\frac{\partial \overline{R}}{\partial P} < 0.$$

Therefore,

$$\frac{\partial \frac{E_c}{E_c+E_b}}{\partial P} - \frac{\partial \frac{L_c}{L_c+L_b}}{\partial P} > 0.$$

b) $E_c/L_c - E_b/L_b$ increases in P.

We have that

$$\frac{\partial\left(\frac{E_c}{L_c} - \frac{E_b}{L_b}\right)}{\partial P} = \frac{1}{L_c}\frac{\partial E_c}{\partial P} - \frac{1}{L_b^2}\left(\frac{\partial E_b}{\partial P}L_b - \frac{\partial L_b}{\partial P}E_b\right).$$

But since $\partial E_c/\partial P > 0$, it is sufficient to show that

$$\frac{\partial E_b}{\partial P}L_b - \frac{\partial L_b}{\partial P}E_b < 0. \tag{25}$$

Note that $L_b = (E_c - E_{cc})\,\alpha\underline{\delta} + E_b\alpha\overline{\delta}$ so that $\partial L_b/\partial P = \alpha\underline{\delta}\partial E_c/\partial P + \alpha\overline{\delta}\partial E_b/\partial P$. Then the LHS of (25) becomes

$$\begin{aligned}
& \frac{\partial E_b}{\partial P}\left(\alpha\underline{\delta}E_c - \alpha\underline{\delta}E_{cc} + \alpha\overline{\delta}E_b\right) - E_b\left(\alpha\underline{\delta}\frac{\partial E_c}{\partial P} + \alpha\overline{\delta}\frac{\partial E_b}{\partial P}\right) \\
= \;& \alpha\underline{\delta}\left(\frac{\partial E_b}{\partial P}E_c - \frac{\partial E_b}{\partial P}E_{cc} - \frac{\partial E_c}{\partial P}E_b\right) \\
= \;& \frac{\alpha\underline{\delta}}{D'}\left(E_c - E_{cc} - E_b\right)
\end{aligned}$$

since $\partial E_c/\partial P = \partial E_b/\partial P = 1/D' > 0$. It then follows that (25) holds if $E_c - E_{cc} - E_b < 0$, or equivalently, $E_{cb} < E_b$, which holds whenever

$$\frac{E_b}{S_b} > \frac{1}{2},$$

since $S_b - E_b = E_{cb}$.

c) $\frac{E_c+E_b}{P}$ increases with P.

$$\frac{E_c+E_b}{P} = \frac{1}{AP}\left(\left(2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}\right)w + 2\overline{R}\right).$$

Thus,

$$\begin{aligned}
\frac{\partial \frac{E_c+E_b}{P}}{\partial P} &= \frac{2}{PD'} - \frac{1}{AP^2}\left(\left(2 + \frac{1}{\alpha\underline{\delta}} + \frac{1}{\alpha\overline{\delta}}\right)w + 2\overline{R}\right) \\
&= \frac{1}{P}\left(\frac{2}{2+\alpha\underline{\delta}+\alpha\overline{\delta}} - \frac{E_c+E_b}{P}\right).
\end{aligned}$$

If more than half of the firms in the economy are integrated, $(E_c + E_b)/P < 1/(1 + \alpha\underline{\delta}/2 + \alpha\overline{\delta}/2)$. Hence the above expression is positive if

$$\frac{2}{2 + \alpha\underline{\delta} + \alpha\overline{\delta}} \geq \frac{1}{1 + \alpha\underline{\delta}/2 + \alpha\overline{\delta}/2}$$

which is trivially satisfied with equality. Observe also that when $E_b/S_b > 1/2$, and since $E_c = S_c$, then

$$E_b > \frac{1}{2}S_b \Leftrightarrow E_b + E_c > \frac{1}{2}(S_b + S_c) + \frac{1}{2}S_c$$

so that

$$\frac{E_b + E_c}{S_b + S_c} > \frac{1}{2} + \frac{1}{2}\frac{S_c}{S_b + S_c} > \frac{1}{2}.$$

iv). The total number of establishments at the center is given by $S_c = E_c + E_{bc} = E_c$. Thus, we have

$$S_c = \frac{1 + \frac{1}{\alpha\underline{\delta}}}{A}w + \frac{\overline{R}}{A}. \tag{26}$$

Similarly, the total number of establishments at the edge is

$$S_b = E_b + E_{cb} = \frac{1}{A}\left[2 + \frac{1}{\alpha\overline{\delta}} + \frac{1}{\alpha\underline{\delta}}\right]w + \frac{2}{A}\overline{R} - \frac{L_c}{\alpha\overline{\delta}}. \tag{27}$$

It follows that

$$\begin{aligned} \frac{\partial S_c}{\partial P} &= \frac{\partial}{\partial P}\left(\frac{\overline{R}}{A}\right) = \frac{1}{D'} > 0, \\ &\text{and} \\ \frac{\partial S_b}{\partial P} &= \frac{\partial}{\partial P}\left(\frac{2}{A}\overline{R}\right) = \frac{2}{D'} > 0, \end{aligned}$$

which also implies that $\partial S_b/\partial P > \partial S_c/\partial P$. To show that the share of establishments in the center declines with population, observe that

$$\frac{\partial \frac{S_c}{S_c + S_b}}{\partial P} = \frac{\frac{\partial \overline{R}}{\partial P}(S_c + S_b) - 3\frac{\partial \overline{R}}{\partial P}S_c}{A(S_c + S_b)^2} = \frac{S_c + S_b - 3S_c}{D'(S_c + S_b)^2} < 0$$

when

$$\frac{S_c}{S_c + S_b} > \frac{1}{3},$$

which is always the case as shown in Proposition 6.

v) The average size of establishment at the center is given by

$$\frac{E_c + L_c}{S_c} = 1 + \frac{L_c}{E_c}.$$

Therefore,

$$\frac{\partial\left(1 + \frac{L_c}{E_c}\right)}{\partial P} = -\left(\frac{L_c/D'}{E_c^2}\right) < 0.$$

The average size of establishment at the edge is given by

$$\frac{E_b + L_b}{S_b} = \frac{(1 + \alpha\overline{\delta})E_b + \alpha\underline{\delta}E_{cb}}{E_b + E_{cb}}.$$

Differentiating with respect to P yields

$$\begin{aligned}
\frac{\partial \frac{E_b + L_b}{S_b}}{\partial P} &= \frac{\left((1 + \alpha\overline{\delta})\frac{\partial E_b}{\partial P} + \alpha\underline{\delta}\frac{\partial E_{cb}}{\partial P}\right)(E_b + E_{cb})}{S_b^2} \\
&\quad - \frac{\left(\frac{\partial E_b}{\partial P} + \frac{\partial E_{cb}}{\partial P}\right)\left((1 + \alpha\overline{\delta})E_b + \alpha\underline{\delta}E_{cb}\right)}{S_b^2} \\
&= \frac{\frac{\partial E_b}{\partial P}\left[(1 + \alpha\overline{\delta})E_{cb} - \alpha\underline{\delta}E_{cb}\right] - \frac{\partial E_{cb}}{\partial P}\left[(1 + \alpha\overline{\delta})E_b - \alpha\underline{\delta}E_b\right]}{S_b^2}
\end{aligned}$$

which is negative when

$$\frac{\partial E_b}{\partial P}E_{cb} < \frac{\partial E_{cb}}{\partial P}E_b, \tag{28}$$

where

$$\frac{\partial E_b}{\partial P} = \frac{\partial E_{cb}}{\partial P} = \frac{1}{D'}.$$

Therefore (28) reduces to $E_{cb} < E_b$, which holds when $E_b/S_b > 1/2$.