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Technical Appendix for "Frictional Wage Dispersion in Search Models: A Quantitative Assessment" *

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Abstract

In this Technical Appendix to Hornstein, Krusell, and Violante (2006) (HKV, 2006, hereafter) we provide a detailed characterization of the search model with (1) wage shocks during employment and (2) on-the-job search outlined in Sections 6 and 7 of that paper, and we derive all of the results that are only stated in HKV (2006). In particular, we derive the expressions for our preferred measure of frictional wage inequality: the ratio of average wages to the reservation wage, or, the 'mean-min' wage ratio.

Keywords: labor market, wage inequality, search frictions, job search
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1 Wage shocks during employment

In this section, we characterize the equilibrium of the search model with wage shocks while employed when wage shocks when employed and unemployed come from the same distribution. We first state the discrete-time approximation of the search model for a fixed and finite length of the time period. We then derive the continuous-time representation as the limit of the discrete-time approximation when the length of the time period becomes arbitrarily small. We show that the Bellman equations for the employment and unemployment values for the continuous- and discrete-time version are the same. Our derivation of the mean-min ratio for wage inequality is therefore independent of the time representation. We then show that in the discrete-time version the first-order autocorrelation coefficient of wages is one minus the arrival rate of wage changes. Finally, we consider a variation of the baseline model where wage shocks when employed come from a different distribution than wage shocks when unemployed. In particular, we study the Mortensen-Pissarides (1994) environment where unemployed workers on meeting a job always receive the highest wage.

We describe the parameters of the environment in terms of the continuous-time framework. The economy is populated by ex-ante equal, risk-neutral, infinitely lived individuals who discount the future at rate r. Unemployed agents receive job offers at the instantaneous rate λ_u, and the wage of employed agents changes at the instantaneous rate δ. Conditionally on receiving a wage change, the wage is drawn from a well-behaved distribution function $F(w)$ with upper support $w^{\max}$. Draws are i.i.d. over time and across agents. Note that employed and unemployed agents sample from the same wage distribution. If a job offer w is accepted, the worker is paid a wage w, until the next wage-change event occurs. While unemployed, the worker receives a utility flow b which includes unemployment benefits and a value of leisure and home production, net of search costs. We only consider steady-state allocations.

1.1 Discrete time versus continuous time

The discrete-time approximations of the Bellman equations for the value of employment, $W(w)$, and unemployment, U, are

$$\begin{aligned} W(w) &= w\Delta + e^{-r\Delta}\left\{\delta\Delta\int \max\{W(z), U\}\,dF(z) + (1-\delta\Delta)\,W(w)\right\}, \qquad (1)\\ U &= b\Delta + e^{-r\Delta}\left\{\lambda_u\Delta\int \max\{W(z), U\}\,dF(z) + (1-\lambda_u\Delta)\,U\right\}, \qquad (2)\end{aligned}$$

where Δ is the length of the time interval, and $\lambda_u\Delta$ ($\delta\Delta$) is the probability that an (un)employed worker receives a wage offer at the end of the interval Δ. Using the definition of the reservation wage, $W(w^*) = U$, and rearranging terms the value equations can be rewritten as

$$\begin{aligned} \left(1-e^{-r\Delta}\right)W(w) &= w\Delta + e^{-r\Delta}\left\{\delta\Delta\int_{w^*}^{w^{\max}}[W(z)-W(w)]\,dF(z) - \delta\Delta F(w^*)\,[W(w)-U]\right\},\\ \left(1-e^{-r\Delta}\right)U &= b\Delta + e^{-r\Delta}\left\{\lambda_u\Delta\int_{w^*}^{w^{\max}}[W(z)-U]\,dF(z)\right\}\end{aligned}$$

Dividing by the length of the time interval and taking the limit as $\Delta \to 0$ we get the continuous-time Bellman equations

$$\begin{aligned} rW(w) &= w + \delta\int_{w^*}^{w^{\max}}[W(z)-W(w)]\,dF(z) - \delta F(w^*)\,[W(w)-U], \qquad (3)\\ rU &= b + \lambda_u\int_{w^*}^{w^{\max}}[W(z)-U]\,dF(z). \qquad (4)\end{aligned}$$

Rather than studying the continuous-time limit of the search model with wage changes, we can just use the discrete-time approximation and consider a unit length interval, $\Delta = 1$. In this case we get the following expressions for the value functions of being employed and unemployed:

$$\begin{aligned} (1-\beta)\,W(w) &= w + \beta\left\{\delta\int_{w^*}^{w^{\max}}[W(z)-W(w)]\,dF(z) - \delta F(w^*)\,[W(w)-U]\right\}\\ (1-\beta)\,U &= b + \beta\lambda_u\int_{w^*}^{w^{\max}}[W(z)-U]\,dF(z),\end{aligned}$$

where $\beta \equiv e^{-r} \equiv 1/(1+\bar{r})$. Note that we can rewrite these discrete-time value equations as

$$\begin{aligned} \bar{r}\bar{W}(w) &= w + \delta\int_{w^*}^{w^{\max}}\left[\bar{W}(z)-\bar{W}(w)\right]dF(z) - \delta F(w^*)\left[\bar{W}(w)-\bar{U}\right] \qquad (5)\\ \bar{r}\bar{U} &= b + \lambda_u\int_{w^*}^{w^{\max}}\left[\bar{W}(z)-\bar{U}\right]dF(z), \qquad (6)\end{aligned}$$

where $\bar{W} \equiv W/(1+\bar{r})$ and $\bar{U} \equiv U/(1+\bar{r})$. Expressions (5) and (6) are formally equivalent to the expressions (3) and (4) for the continuous-time value functions. Therefore, the results for the mean-min ratio also apply for the discrete-time version of the paper.

1.2 The reservation wage

As a first step towards deriving the mean-min wage ratio for the continuous-time model we characterize the reservation wage. For this purpose, evaluate the employment value expression (3) at w^* and use the definition of the reservation wage, $W(w^*) = U$, to get

$$rU = w^* + \delta \int_{w^*}^{w^{\max}} [W(z) - W(w^*)]\, dF(z).$$

Now substitute the unemployment value expression (4) for the left-hand side and solve for the reservation wage

$$w^* = b + (\lambda_u - \delta) \int_{w^*}^{w^{\max}} [W(z) - W(w^*)]\, dF(z). \tag{7}$$

Integration by parts on the right-hand side yields

$$\begin{aligned}
\int_{w^*}^{w^{\max}} [W(z) - W(w^*)]\, dF(z) &= [(W(z) - W(w^*))\, F(z)]_{w^*}^{w^{\max}} - \int_{w^*}^{w^{\max}} W'(z)\, F(z)\, dz \\
&= W(w^{\max}) - W(w^*) - \int_{w^*}^{w^{\max}} W'(z)\, F(z)\, dz \\
&= \int_{w^*}^{w^{\max}} W'(z)\, [1 - F(z)]\, dz. \qquad (8)
\end{aligned}$$

From the employment value expression (3) it follows that

$$W'(w) = \frac{1}{r+\delta}. \tag{9}$$

Hence the reservation wage expression is

$$\begin{aligned}
w^* &= b + \frac{\lambda_u - \delta}{r+\delta} \int_{w^*}^{w^{\max}} [1 - F(z)]\, dz \\
&= b + \frac{(\lambda_u - \delta)[1 - F^*]}{r+\delta} \int_{w^*}^{w^{\max}} \left[\frac{1}{1-F^*} - \frac{F(z)}{1-F^*}\right] dz \qquad (10)
\end{aligned}$$

with $F^* = F(w^*)$.

1.3 The equilibrium wage distribution

We now construct the equilibrium wage distribution $G(w)$ implied by the interaction of the wage-offer distribution and the reservation wage. The measure of agents with wage below w is $(1-u)\,G(w)$. Agents leave this stock because their wage changes and their new wage is either less than the reservation wage or higher than the current wage. Agents enter this stock if they were unemployed and receive an acceptable wage offer below w, or if they were employed at wage above w and are forced to accept a lower wage; hence

$$\begin{aligned} & (1-u)\,G(w)\,\delta\,\{F(w^*)+[1-F(w)]\} \\ = \; & \{u\lambda_u+(1-u)\,[1-G(w)]\,\delta\}\,[F(w)-F(w^*)]\,. \end{aligned} \tag{11}$$

We can solve this expression for the equilibrium wage distribution as a function of the wage offer distribution:

$$G(w)=\left[\frac{\lambda_u u}{\delta\,(1-u)}+1\right][F(w)-F(w^*)]\,. \tag{12}$$

In steady state, the inflows and outflows from employment balance:

$$(1-u)\,\delta F(w^*)=u\lambda_u\,[1-F(w^*)]\,.$$

Using the expression for steady-state employment in (12) we get

$$G(w)=\frac{F(w)-F(w^*)}{1-F(w^*)}. \tag{13}$$

Thus, the equilibrium wage distribution with and without wage shocks during employment are the same, namely the wage-offer distribution truncated at the reservation wage.

1.4 The mean-min ratio

Based on the equilibrium wage distribution we can calculate the average wage of employed workers as

$$\bar{w}=\int_{w^*}^{w^{\max}} w\frac{dF(w)}{1-F^*}=\frac{w^{\max}-w^*F^*}{1-F^*}-\int_{w^*}^{w^{\max}}\frac{F(z)}{1-F^*}dz. \tag{14}$$

Solving the average-wage expression (14) for the right-hand side integral term and substituting this term for the corresponding integral in the reservation-wage expression (10) yields

$$\begin{aligned} w^* &= b + \frac{(\lambda_u - \delta)[1 - F^*]}{r + \delta} \left[\int_{w^*}^{w^{\max}} \frac{1}{1 - F^*} dz + \bar{w} - \frac{w^{\max} - w^* F^*}{1 - F^*} \right] \\ &= b + \frac{(\lambda_u - \delta)[1 - F^*]}{r + \delta} [\bar{w} - w^*] . \end{aligned} \tag{15}$$

Using the definition of the replacement rate, $b = \rho\bar{w}$, we can solve equation (15) for the reservation wage and obtain an expression for the mean-min ratio, that is, the ratio of average wages to the reservation wage,

$$\frac{\bar{w}}{w^*} = \frac{\frac{(\lambda_u - \delta)[1 - F(w^*)]}{r+\delta} + 1}{\frac{(\lambda_u - \delta)[1 - F(w^*)]}{r+\delta} + \rho} = \frac{\frac{\lambda_u^* - \delta + \sigma^*}{r+\delta} + 1}{\frac{\lambda_u^* - \delta + \sigma^*}{r+\delta} + \rho}, \tag{16}$$

with $\lambda_u^* \equiv (1 - F^*)\lambda_u$ and $\sigma^* \equiv \delta F^*$. This is equation (13) in Section 6 of HKV (2006). Note that as δ goes to infinity $\frac{\bar{w}}{w^*}$ goes to $1/\rho$.

1.5 Wage persistence in the discrete time model

It is straightforward to show that the equilibrium wage distribution for the discrete-time and the continuous-time versions of the model are the same; we now work with the former. We need the expected value of the cross-product of today's and tomorrow's wage, conditional on being employed in both periods, to calculate the autocorrelation coefficient. We proceed in two steps: first, we obtain the value conditional on today's wage, and then we integrate over today's wage to get the unconditional expectation.

$$\begin{aligned} E[w'w|w] &= (1 - \delta) w^2 + \delta w E[\tilde{w}|\tilde{w} \geq w^*] \\ &= (1 - \delta) w^2 + \delta w \bar{w} \\ E[w'w] &= (1 - \delta) E\left[w^2\right] + \delta \bar{w}^2 \end{aligned}$$

We can now define the first-order autocorrelation coefficient as

$$\begin{aligned} \rho &= \frac{(1 - \delta) E[w^2] + \delta \bar{w}^2 - \bar{w}^2}{Var(w)} \\ &= \frac{(1 - \delta)(E[w^2] - \bar{w}^2)}{Var(w)} \\ &= 1 - \delta \end{aligned}$$

1.6 The mean-min ratio for a Mortensen-Pissarides (1994) environment

Suppose now that an unemployed worker who receives a wage offer always receives the highest wage $w^{\max}$, whereas wage changes of employed workers continue to be drawn from the distribution F. We will show that the mean-min ratio that we previously derived for our baseline model, equation (16), represents an upper bound for the mean-min ratio in this Mortensen-Pissarides (1994) environment.

The value function equation for an employed worker, (3), remains unchanged, but the value function equation for an unemployed worker is now

$$rU = b + \lambda_u \left[W(w^{\max}) - U \right]. \tag{17}$$

Following the same steps as in Section 1.2 we derive the modified expression for the reservation wage:

$$w^* = b + \frac{\lambda_u - \delta}{r + \delta} \left[w^{\max} - w^* \right] + \frac{\delta}{r + \delta} \int_{w^*}^{w^{\max}} F(z)\, dz. \tag{18}$$

The modified steady-state expression characterizing the equilibrium wage distribution is now

$$\begin{aligned} & (1-u)\, G(w)\, \delta \left\{ F(w^*) + \left[1 - F(w) \right] \right\} \qquad (19) \\ = \;& (1-u) \left[1 - G(w) \right] \delta \left[F(w) - F(w^*) \right] \text{ for } w < w^{\max}. \end{aligned}$$

Note that there are no inflows from the pool of unemployed since all unemployed workers who receive wage offers receive the highest wage. Thus, the equilibrium wage distribution for $w < w^{\max}$ is

$$G(w) = F(w) - F(w^*). \tag{20}$$

Since all unemployed workers receive the highest wage with probability one there is now a mass point at $w = w^{\max}$, that is, the cumulative density function is discontinuous at $w^{\max}$.

Integrating the wage with respect to the equilibrium wage distribution then yields the average wage

$$\bar{w} = \int_{w^*}^{w^{\max}} w dF(w) + w^{\max} F(w^*) = w^{\max} + F(w^*)\left(w^{\max} - w^* \right) - \int_{w^*}^{w^{\max}} F(w)\, dw. \tag{21}$$

Solving the average-wage expression (21) for the right-hand side integral term and substituting this term for the corresponding integral in the reservation-wage expression (18) yields

$$\begin{aligned}\left[1+\frac{\lambda_u-(1-F^*)\,\delta}{r+\delta}\right]w^* &= \left(\rho-\frac{\delta}{r+\delta}\right)\bar{w}+\frac{\lambda_u+\delta F^*}{r+\delta}w^{\max} \\ &> \left(\rho+\frac{\lambda_u-\delta\,(1-F^*)}{r+\delta}\right)\bar{w}. \end{aligned}\tag{22}$$

Note that the last inequality implies that the mean-min ratio for this setup is bounded above by that of the baseline economy with wage shocks given in (16).

2 On-the-job search

We describe a general version of the on-the-job search model that includes forced job-to-job mobility and wage shocks (the models in Section 7 of HKV, 2006). The Bellman equations for the employment and unemployment values are

$$\begin{aligned} rW(w) &= w+\lambda_w\int\max\{W(z)-W(w),0\}\,dF(z)-\sigma\,[W(w)-U] \\ &\quad +\phi\int[\max\{W(z),U\}-W(w)]\,dF(z) \\ rU &= b+\lambda_u\int\max\{W(z)-U,0\}\,dF(z) \end{aligned}$$

The basic on-the-job search model in HKV (2006), Section 7.1, assumes that a worker receives outside wage offers at a rate λ_w. Without loss of generality, and motivated by what equilibrium firm behavior would dictate, we assume that the wage-offer distribution is such that unemployed workers accept all wage offers: $F(w^*)=0$. A worker can always reject a wage offer and keep the current wage. The worker may lose the current job at an exogenous separation rate, σ.

On-the-job search with forced job-to-job mobility in HKV (2006), Section 7.2, assumes that for some wage offers, the worker just has to take the offer, even if the new job pays less than the current job. Forced job mobility is reflected in the parameter $\phi>0$. In the basic on-the-job search model $\phi=0$. On-the-job search with wage shocks in HKV (2006), Section 7.2, is essentially the same as forced job-to-job mobility. The only difference is in its implications for observable transitions. The arrival of a type ϕ wage offer now results in a wage cut on the existing job, rather than a separation and transfer to a lower-paying job.

Finally, we analyze the on-the-job search model of Christensen et al. (2005) where search effort is endogenous, as discussed in HKV (2006), Section 7.1.

2.1 The reservation wage

Workers continue to follow reservation-wage strategies and the Bellman equations can be rewritten as

$$\begin{aligned} rW(w) &= w + \lambda_w \int_w^{w^{\max}} [W(z) - W(w)]\, dF(z) - \sigma [W(w) - U] \\ &\quad + \phi \int_{w^*}^{w^{\max}} [W(z) - W(w)]\, dF(z) \\ &= w + (\lambda_w + \phi) \int_w^{w^{\max}} [W(z) - W(w)]\, dF(z) - \sigma [W(w) - U] \\ &\quad + \phi \int_{w^*}^{w} [W(w) - W(z)]\, dF(z) \qquad (23) \\ rU &= b + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - U]\, dF(z) \qquad (24) \end{aligned}$$

Evaluate the employment value equation (23) at w^*, using the reservation wage property, $W(w^*) = U$, and the unemployment value expression (24) to obtain

$$\begin{aligned} rU &= w^* + (\lambda_w + \phi) \int_{w^*}^{w^{\max}} [W(z) - W(w^*)]\, dF(z) \\ &= b + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - W(w^*)]\, dF(z). \end{aligned}$$

We can solve this expression for the reservation wage

$$w^* = b + (\lambda_u - \lambda_w - \phi) \int_{w^*}^{w^{\max}} [W(z) - W(w^*)]\, dF(z). \qquad (25)$$

As with equation (7), we can integrate the right-hand side integral by parts, as in (8), and get the reservation-wage equation

$$w^* = b + (\lambda_u - \lambda_w - \phi) \int_{w^*}^{w^{\max}} W'(z) [1 - F(z)]\, dz. \qquad (26)$$

Note that differentiating the employment value equation (23) with respect to the current wage yields

$$W'(w) = \frac{1}{r + \sigma + \phi + \lambda_w [1 - F(w)]}. \qquad (27)$$

Substituting (27) in (26) we can rewrite the reservation wage as

$$w^* = b + (\lambda_u - \lambda_w - \phi) \int_{w^*}^{w^{\max}} \frac{1 - F(z)}{r + \sigma + \phi + \lambda_w [1 - F(z)]} dz. \tag{28}$$

2.2 The equilibrium wage distribution

We now construct the equilibrium wage distribution $G(w)$ implied by the interaction of the wage-offer distribution and the reservation wage.

The measure of agents with wage below w is $(1-u) G(w)$. Agents leave this stock because (1) they get separated at rate σ, (2) they receive an outside offer which they accept at rate $\lambda_w [1 - F(w)]$, or (3) they are forced to leave at rate ϕ but are lucky enough to get an offer above w. Workers enter this stock if (1) they were unemployed and receive a wage offer below w, or (2) they were employed at wage above w and are forced to accept a lower wage. In a steady state the inflows and outflows balance:

$$\begin{aligned} & (1-u) G(w) \{\sigma + (\lambda_w + \phi) [1 - F(w)]\} \qquad (29) \\ = \ & u\lambda_u F(w) + \phi (1-u) [1 - G(w)] F(w). \end{aligned}$$

We can solve this expression for the equilibrium wage distribution as a function of the wage offer distribution:

$$G(w) = \frac{\lambda_u u + \phi (1-u)}{1-u} \cdot \frac{F(w)}{\sigma + \phi + \lambda_w [1 - F(w)]} \tag{30}$$

In steady state, if all the job offers are above w^* so that $F(w^*) = 0$,

$$u\lambda_u = (1-u)\sigma.$$

Hence

$$G(w) = \frac{(\sigma + \phi) F(w)}{\sigma + \phi + \lambda_w [1 - F(w)]} \tag{31}$$

and

$$\begin{aligned} 1 - G(w) &= \frac{\sigma + \phi + \lambda_w}{\sigma + \phi + \lambda_w [1 - F(w)]} [1 - F(w)] \\ &\simeq \frac{r + \sigma + \phi + \lambda_w}{r + \sigma + \phi + \lambda_w [1 - F(w)]} [1 - F(w)] \qquad (32) \end{aligned}$$

2.3 The mean-min ratio

The average wage is

$$\begin{aligned}\bar{w} &= \int_{w^*}^{w^{\max}} w dG(z) = [wG(w)]_{w^*}^{w^{\max}} - \int_{w^*}^{w^{\max}} G(z)\, dz \\ &= w^{\max} - \int_{w^*}^{w^{\max}} G(z)\, dz \\ &= [w^{\max} - w^*] + w^* - \int_{w^*}^{w^{\max}} G(z)\, dz \\ &= w^* + \int_{w^*}^{w^{\max}} [1 - G(z)]\, dz. \end{aligned} \tag{33}$$

Solve the wage distribution expression (32) for $1 - F$ and use it in the reservation-wage expression (28) to get

$$w^* \simeq b + \frac{\lambda_u - \lambda_w - \phi}{r + \sigma + \phi + \lambda_w} \int_{w^*}^{w^{\max}} [1 - G(z)]\, dz.$$

Finally substituting for the integral term from the average-wage equation (33) we can solve for the mean-min ratio:

$$\begin{aligned} w^* &\simeq \rho \bar{w} + \frac{\lambda_u - \lambda_w - \phi}{r + \sigma + \phi + \lambda_w} (\bar{w} - w^*) \quad \Rightarrow \\ Mm &\simeq \frac{\frac{\lambda_u - \lambda_w - \phi}{r+\sigma+\phi+\lambda_w} + 1}{\frac{\lambda_u - \lambda_w - \phi}{r+\sigma+\phi+\lambda_w} + \rho}. \end{aligned} \tag{34}$$

Equation (34) corresponds to equations (17) and (20) in Section 7 of HKV (2006).

2.4 Turnover rates in the basic on-the-job search model

In the basic on-the-job search model there are exogenous separations, $\sigma > 0$, but no forced wage changes, $\phi = 0$.

The average *completed job tenure* in the model is

$$\tau = \int_{w^*}^{w^{\max}} \frac{dG(w)}{\sigma + \lambda_w [1 - F(w)]}. \tag{35}$$

Recall the equilibrium wage distribution from (31) and solve that expression for the wage-offer distribution

$$1 - F(w) = \frac{\sigma [1 - G(w)]}{\sigma + \lambda_w G(w)}.$$

Substitute this expression in the expression for average job tenure (35) and we get

$$\begin{aligned}\tau &= \int_{w^*}^{w^{\max}} \frac{dG(w)}{\sigma + \lambda_w \frac{\sigma[1-G(w)]}{\sigma+\lambda_w G(w)}} = \int_{w^*}^{w^{\max}} \frac{[\sigma + \lambda_w G(w)]\, dG(w)}{\sigma(\lambda_w + \sigma)} \\ &= \frac{1}{\sigma(\lambda_w + \sigma)} \left[\sigma + \lambda_w \int_{w^*}^{w^{\max}} G(w)\, dG(w)\right]. \end{aligned} \tag{36}$$

Note that

$$\int_{w^*}^{w^{\max}} G(w)\, dG(w) = \left[G^2(w)\right]_{w^*}^{w^{\max}} - \int_{w^*}^{w^{\max}} G(w)\, dG(w).$$

Thus

$$\int_{w^*}^{w^{\max}} G(w)\, dG(w) = 1/2.$$

Hence, the average job tenure is

$$\tau = \frac{\sigma + \lambda_w/2}{\sigma(\lambda_w + \sigma)} \in \left(\frac{1}{2\sigma}, \frac{1}{\sigma}\right). \tag{37}$$

The (total) *separation rate* for employed workers, the sum of exogenous separations and job-to-job transitions, is

$$sep = \sigma + \lambda_w \int_{w^*}^{w^{\max}} [1 - F(w)]\, dG(w). \tag{38}$$

Hence, the share of separations attributable to job-to-job separations is

$$\frac{jjsep}{sep} = \frac{\lambda_w \int_{w^*}^{w^{\max}} [1 - F(w)]\, dG(w)}{\sigma + \lambda_w \int_{w^*}^{w^{\max}} [1 - F(w)]\, dG(w)}. \tag{39}$$

Following Nagypal (2005), and integrating $jjsep$ by parts, yields

$$\begin{aligned} jjsep &= \lambda_w \int_{w^*}^{w^{\max}} [1 - F(w)]\, dG(w) \\ &= \lambda_w - \lambda_w \int_{w^*}^{w^{\max}} F(w)\, dG(w) \\ &= \lambda_w - \lambda_w [F(w)\, G(w)]_{w^*}^{w^{\max}} + \lambda_w \int_{w^*}^{w^{\max}} G(w)\, dF(w) \\ &= \lambda_w \int_{w^*}^{w^{\max}} G(w)\, dF(w) \\ &= \lambda_w \sigma \int_{w^*}^{w^{\max}} \frac{F(w)}{\sigma + \lambda_w [1 - F(w)]} dF(w). \end{aligned} \tag{40}$$

The last step uses equation (31) for G. Now change the variable of integration to $z = F(w)$, and we get

$$\begin{aligned}\int_0^1 \frac{z}{\sigma+\lambda_w[1-z]}dz &= -\frac{\lambda_w\{z\}_0^1+(\lambda_w+\sigma)\{\log[\sigma+\lambda_w(1-z)]\}_0^1}{\lambda_w^2} \\ &= -\frac{\lambda_w+(\lambda_w+\sigma)(\log\sigma-\log(\sigma+\lambda_w))}{\lambda_w^2} \\ &= \frac{(\lambda_w+\sigma)\log\left(\frac{\sigma+\lambda_w}{\sigma}\right)}{\lambda_w^2}-\frac{1}{\lambda_w}. \qquad (41)\end{aligned}$$

Hence, the job-to-job separation rate is

$$jjsep = \frac{\sigma(\lambda_w+\sigma)\log\left(\frac{\sigma+\lambda_w}{\sigma}\right)}{\lambda_w}-\sigma \qquad (42)$$

and the total separation rate is

$$sep = jjsep+\sigma = \frac{\sigma(\lambda_w+\sigma)\log\left(\frac{\sigma+\lambda_w}{\sigma}\right)}{\lambda_w}. \qquad (43)$$

The share of separations attributable to job-to-job transitions is then

$$\frac{jjsep}{sep} = 1-\frac{\lambda_w/(\sigma+\lambda_w)}{\log[(\sigma+\lambda_w)/\sigma]}. \qquad (44)$$

2.5 Wage cuts in the model with on-the-job wage changes

In this version of the model, not only does a worker receive outside wage offers at the rate λ_w, but the wage on the current job is also changing with arrival rate ϕ. If the new wage is less than the current wage, the worker has to accept a pay cut.

The average rate at which workers are forced to take a wage cut is

$$\begin{aligned}fcut &= \phi\int_{w^*}^{w^{\max}} F(w)\,dG(w) \qquad (45)\\ &= \phi[F(w)G(w)]_{w^*}^{w^{\max}}-\phi\int_{w^*}^{w^{\max}} G(w)\,dF(w) \\ &= \phi-\phi\int_{w^*}^{w^{\max}} G(w)\,dF(w).\end{aligned}$$

Now substitute for the equilibrium wage distribution from (31) and we get

$$fcut = \phi-\phi\hat{\sigma}\int_{w^*}^{w^{\max}} \frac{F(w)}{\hat{\sigma}+\lambda_w[1-F(w)]}dF(w), \qquad (46)$$

where $\hat{\sigma} \equiv \sigma + \phi$. We have previously derived the right-hand side integral: expressions (40) and (41). Use equation (41) for the integral and we get

$$\begin{aligned} fcut &= \phi - \phi\hat{\sigma}\left[\frac{(\lambda_w + \hat{\sigma})\log\left(\frac{\hat{\sigma}+\lambda_w}{\hat{\sigma}}\right)}{\lambda_w^2} - \frac{1}{\lambda_w}\right] \\ &= \phi\left[1 + \frac{\hat{\sigma}}{\lambda_w} - \frac{\hat{\sigma}\left(\lambda_w + \hat{\sigma}\right)\log\left(\frac{\hat{\sigma}+\lambda_w}{\hat{\sigma}}\right)}{\lambda_w^2}\right]. \end{aligned} \tag{47}$$

With $fcut$ being the average arrival rate of a wage cut, the fraction of employed workers that receive a wage cut in a unit time period is $1 - \exp(-fcut)$.

2.6 Endogenous effort choice in the model with on-the-job search

In the Christensen et al. (2005) model, the utility cost to attain a contact rate λ is $c(\lambda)$, with $c' > 0$ and $c'' > 0$. Thus, optimal search choice will deliver a policy function $\lambda(w)$, where we use the notation $\lambda^* \equiv \lambda(w^*)$.

The first-order condition at the reservation wage w^* reads

$$c'(\lambda^*) = \int_{w^*}^{w^{\max}} \frac{1 - F(z)}{r + \sigma + \lambda(z)(1 - F(z))} dz. \tag{48}$$

In steady state, flows for the workers employed at wage w or lower satisfy

$$(1-u)G(w)\sigma + (1-u)\left[1 - F(w)\right]\int_{w^*}^{w} \lambda(z)dG(z) = u\lambda^* F(w) \tag{49}$$

and for the unemployment pool the flows satisfy the relation

$$u\lambda^* = (1-u)\sigma. \tag{50}$$

Combining the last two equations we obtain

$$\sigma F(w) = G(w)\sigma + \left[1 - F(w)\right]\int_{w^*}^{w} \lambda(z)dG(z).$$

Because $\lambda(z)$ is decreasing, we have

$$\sigma F(w) \leq G(w)\left[\sigma + \lambda^*(1 - F(w))\right],$$

so

$$G(w) \geq \frac{\sigma F(w)}{\sigma + \lambda^*(1 - F(w))},$$

or

$$1 - G(w) \leq \frac{(\sigma + \lambda^*)(1 - F(w))}{\sigma + \lambda^*(1 - F(w))}.$$

This implies

$$\frac{1 - F(w)}{r + \sigma + \lambda^*(1 - F(w))} \geq \frac{1 - G(w)}{r + \sigma + \lambda^*(1 - F(w))} \frac{\sigma + \lambda^*(1 - F(w))}{\sigma + \lambda^*}. \tag{51}$$

Since r is small relative to σ, we have

$$\frac{\sigma + \lambda^*(1 - F(w))}{\sigma + \lambda^*} \simeq \frac{r + \sigma + \lambda^*(1 - F(w))}{r + \sigma + \lambda^*},$$

and hence, from the inequality in (51)

$$\frac{1 - F(w)}{r + \sigma + \lambda^*(1 - F(w))} \geq \frac{1 - G(w)}{r + \sigma + \lambda^*} \tag{52}$$

almost holds, for all w. Going back to the FOC for effort (48), using the fact that $\lambda(z)$ is decreasing, we have

$$c'(\lambda^*) \geq \int_{w^*}^{w^{\max}} \frac{1 - F(z)}{r + \sigma + \lambda^*(1 - F(z))} dz,$$

and then, using the inequality in (52), we obtain that an approximate inequality for $c'(\lambda^*)$ is given by

$$c'(\lambda^*) \geq \int_{w^*}^{w^{\max}} \frac{1 - G(z)}{r + \sigma + \lambda^*} dz = \frac{\int_{w^*}^{w^{\max}} (1 - G(z)) dz}{r + \sigma + \lambda^*} = \frac{\bar{w} - w^*}{r + \sigma + \lambda^*}.$$

Assuming, as done in Christensen et al. (2005), that $c(\lambda) = \lambda^\gamma$, we have that $c'(\lambda) = \gamma c(\lambda)/\lambda$ so that

$$c(\lambda^*) \geq \frac{\lambda^*(\bar{w} - w^*)}{\gamma(r + \sigma + \lambda^*)}$$

which yields the lower bound for search effort during unemployment discussed in the main text.

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