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The Cyclicality of the User Cost of Labor with Search and Matching*

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Abstract

The user cost of labor captures the hiring wage and the expected effect of the economic conditions at the time of hiring on future wages. In search and matching models, I show that it is the user cost and not the wage that is weighed against the worker's marginal product at the time of hiring; thus, the user cost is the allocational variable. I construct its measure in the data and estimate that it is more procyclical than average wages or wages of newly-hired workers. I show that the textbook search and matching model cannot simultaneously generate the empirical elasticities of the vacancy-unemployment ratio and of the user cost of labor, irrespective of the surplus-division rule. (JEL: E24, E32, J30, J41, J63, J64.)

Key Words: User Cost of Labor. Cyclicality. Wage Rigidity. Search and Matching.

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I. Introduction

Macroeconomists have long been interested in the cyclicality of the real marginal cost of labor. Using wages as the measure of the cost, the standard conclusion is that there is little (pro)cyclicality, if any. However, the wage may not be a good measure of the labor's cost. Existing empirical evidence suggests that wages of newly hired workers are much more procyclical than wages of workers in ongoing employment relationships, i.e., that wages within existing employment relationships do not respond to changes in the labor market conditions to the extent that wages of newly hired workers do.[1] In such a case, neither average wage nor the wage of newly hired workers capture the labor's cost. In fact, Hall (1980) writes that "[T]o see what is happening today in the labor market, one should look at the implicit asset prices of labor contracts recently negotiated, not at the average rate of compensation paid to all workers." This paper provides a measure of the implicit asset prices of labor – the user cost of labor.

In this paper I argue that the user cost of labor and not wage is relevant for the hiring decision of a firm. The user cost of labor takes into account both the wage at the time of hiring and the effect of the economic conditions since the time of hiring on future wages. Formally, I define the user cost of labor as the difference between the expected present discounted value of wages paid to a worker hired in the current year and the expected present discounted value of wages to be paid to an identical worker hired next year.[2] If the labor market is a spot market, then this difference is the wage. If a worker is contracted for more than one period, then this difference need not be equal to the wage, as economic conditions at the time of hiring may have an impact on the future wages within the employment relationship. This impact is captured by the user cost.

I propose a measure of the user cost of labor and estimate its cyclicality in the data. The paper's main empirical finding is that the constructed user cost of labor is more than three times as cyclical as average wages and noticeably more cyclical than wages of newly hired workers. In particular, I find that the user cost of labor decreases by more than 4.5%

[1] See, for example, Bils (1985) and a survey in Pissarides (2009).

[2] The user cost of labor is analogous to the user cost of capital, which is the difference between the purchase price and the expected price that can be recovered from selling the un-depreciated part of the factor at the end of the period.

in response to a one percentage point increase in the unemployment rate, while the wages of newly hired workers decrease by 3% and average wages decrease by 1.5%.

I use the estimated cyclicality of the user cost of labor to examine how much volatility of the vacancy-unemployment ratio can be generated by the standard search and matching model (Pissarides (1985), Mortensen and Pissarides (1994)). I show that the model cannot simultaneously generate the empirical elasticities of the user cost of labor and of the vacancy-unemployment ratio. This conclusion holds irrespective of the surplus division rule at the beginning of the match or the individual wage dynamics within employment relationships.

This result shows that the unemployment volatility puzzle, described by Shimer (2005), cannot be solved by a wage formation. That is, Shimer (2005) showed that the standard search and matching model lacks amplification of the productivity shock to generate the empirical volatility of the vacancy-unemployment ratio. Theoretically, a possible amplification mechanism is the rigidity of the statistics from wages that are relevant for the job creation decision of a firm. I show that this mechanism works through making the user cost of labor rigid. Empirically, however, the user cost of labor is very procyclical, i.e., the data lack required rigidity.

I estimate the cyclicality of the user cost of labor using the NLSY79 data. Because the user cost is not directly observed in the data, I construct its empirical counterpart based on the behavior of individual wages and turnover. First, I estimate a projection of wages on a set of time dummies, each of which captures a year when the job starts and a contemporaneous year, and on the series of individual- and firm-specific controls. Next, using the estimates of wages and the empirical separation rate, I construct a series of the user cost of labor. In the construction, future payments are discounted to take into account the separation rates and the real interest rates. Finally, I project the constructed series of (the log of) the user cost of labor on the unemployment rate.

I find that the user cost of labor is substantially more procyclical than the average wages or even the wages of newly hired workers. The intuition behind the large cyclicality of the user cost of labor as compared to the cyclicality of wages is as follows. Consider a firm hiring when the unemployment rate is high. Since the unemployment rate is high, the wage of new hires is low. In addition, once a worker is hired, his wages do not respond as

much to the contemporaneous labor market conditions as the wages of new hires do. This is because empirical findings show that wages of newly hired workers are more procyclical than the wages of workers in ongoing relationships. Thus, by hiring currently, a firm locks in a worker to a stream of wages that is expected to be lower than the stream of wages to be paid to an identically productive worker hired when unemployment is lower. As a result, the user cost of labor is even lower than the already low hiring wage because the user cost also captures comparatively low future wages in this relationship. The opposite is true when a worker is hired at the peak of the cycle, when the unemployment rate is low but is expected to rise. Then the user cost of labor is higher than the hiring wage. Thus, the procyclical hiring wage and the lock-in effect contribute to the user cost of labor being more procyclical than the hiring wage.

In the search and matching model, the free entry condition ties the elasticity of the user cost of labor and the elasticity of the vacancy-unemployment ratio with respect to productivity. The estimated -4.5% cyclicality of the user cost with respect to unemployment translates into its elasticity with respect to productivity in excess of 1.5. Using the free entry condition, I show that the search and matching model cannot simultaneously accommodate the empirical elasticity of the vacancy-unemployment ratio and the empirical elasticity of the user cost of labor. This result does not depend on a particular wage formation in the model.

Additionally, I illustrate the allocational role of the user cost as opposed to wages, by examining the behavior of the user cost, wages of newly hired workers, and average wages in search and matching models with specific wage settings. I consider different wage formations that have been widely used in the literature: a model where wages are renegotiated each period by Nash bargaining, and models where wages are smoothed due to the implicit contracts between a worker and a firm. In particular, I consider self-enforcing insurance contracts between a risk-averse worker and a risk-neutral firm distinguishing full commitment contracts, one-sided lack of commitment contracts, and two-sided lack of commitment contracts. In the quantitative investigation I hit the four economies with the same series of productivity shocks and calculate the cyclicalities from the simulated data. First, I find that when wages are smoothed within employment relationships, the user cost of labor is

more cyclical than the wages of newly hired workers, which in turn are more procyclical than the wages of all workers. In a model with Nash bargaining in each period, all three are the same by construction. Second, when the cyclicality of the user cost of labor across economies with different wage formations is calibrated to the same target, then the volatility of the vacancy-unemployment ratio in all economies is approximately the same. However, the cyclicality of individual wages, including the cyclicality of wages of newly hired workers, varies significantly depending on the wage formation. This illustrates that the user cost of labor, not wage, is allocational for employment. Third, when the cyclicality of the user cost of labor is calibrated to its empirical estimate, none of the four models can generate the volatility of the vacancy-unemployment ratio as high as observed in the data. This illustrates the result described above.

This paper shows theoretically that the user cost of labor (rather than the current wage or the wage of new hires) is the relevant measure of wages for firms' hiring decisions in the Mortensen-Pissarides model, this paper also derives the joint restrictions implied by the model (independent of the wage formation) on the wage component of the user cost of labor and the vacancy-unemployment ratio. The main empirical contribution of the paper is to construct the user cost of labor in the data and provide an estimate of its cyclicality. A large empirical literature exists that studies the behavior of individual wages over the business cycle. However, one crucial aspect of the existing empirical literature is that it provides evidence on the cyclical behavior of the current wage, but not on the cyclical behavior of the expected present discounted value of future wages within a match formed in the current period. This paper, to my knowledge, is the first attempt to directly measure the cyclicality of the price of labor taking into account the effect of economic conditions at the time of hiring on future wages. The contribution of the paper to the theoretical literature consists of showing that the solution to the unemployment volatility puzzle cannot be explained by a wage formation. This is so because any wage formation that is designed to solve the unemployment volatility puzzle should also generate the empirical cyclicality of the user cost of labor. However, as the paper shows, the free entry condition of the model imposes the restrictions that cannot simultaneously accommodate the empirical volatility of the vacancy-unemployment ratio and of the user cost of labor.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the user cost of labor and presents the main analytical result using the model's free entry condition. Section 4 presents the main empirical results. Section 5 illustrates implications of the estimated elasticity of the wage component of the user cost of labor for the free entry condition using an example. Section 6 presents a quantitative investigation of the cyclicality of the user cost of labor in models with specific wage settings. Section 7 concludes.

II. Related Literature

The idea that wage is not allocational for employment if there is history dependence in wages goes back to Barro (1977), Hall (1980), Kydland and Prescott (1980) and, most recently, Shimer (2004). They argue that what matters to a firm is the value of wages to be paid during the course of a firm-worker relationship. Barro calls sticky wages just a "façade" of the implication of the long-term labor contracts to short-term macro fluctuations.

Empirical studies of the cyclicality of individual wages provide both direct and indirect evidence that wages diverge from worker's contemporaneous marginal product but depend on the history of labor market conditions from the start of the job. Indirect evidence comes from the differences in the cyclicality of wages of newly hired workers and workers in ongoing relationships. Using NLS data, 1966-80, Bils (1985) concludes that the wages of newly hired workers are substantially more procyclical than the wages of workers in ongoing relationships. Numerous studies since, using different data sets, also find that the cyclicality of job changers is substantially higher than that of job stayers (among them Solon, Barsky, and Parker (1994); Shin (1994); and Carneiro, Guimaraes, and Portugal (2009)). The empirical evidence on the cyclicality of individual wages is summarized in Pissarides (2009). He reports that the general consensus in the literature on the cyclicality of the wages of newly hired workers is -3.0%, while the cyclicality of the wages of job stayers is approximately -1%. This evidence suggests that the wages of newly hired workers are adjusted to reflect the economic conditions at the time of hiring. Within employment relationships, however, wages are smoothed and respond only weakly to changes in economic conditions. More direct evidence of the history dependence in wages is presented in Beaudry and DiNardo (1991). They find that the

minimum unemployment rate experienced by a worker from the start of the job has a larger impact on wages than the contemporaneous unemployment rate. Beaudry and DiNardo (1991) conclude that a contractual wage model is more consistent with the formation of wages than a spot market model.[3]

These empirical findings lead to the following conclusions: (1) wages exhibit dependence on the past history of unemployment, and (2) wages of newly hired workers are more pro-cyclical than wages of workers in ongoing employment relationships. In turn, these results imply that wage alone does not summarize the wage commitment a firm takes upon hiring a worker. The relevant measure of the price of labor to a firm should take into account both the wage at the time of hiring and the effect of the economic conditions at the time of hiring on future wages. This paper constructs such a measure and estimates its cyclicality.

In addition to providing the estimate of the cyclicality of the user cost of labor, this paper contributes to the ongoing discussion on the quantitative behavior of the textbook search and matching model. In the job creation decision, a firm takes into account not only the initial wage in a newly formed match but also the entire expected stream of future wages to be paid in the match. Thus, job creation in frictional labor markets places the focus on the cyclical behavior of the expected present discounted value of wages. As Shimer (2004) emphasizes, what is relevant for the volatility of job creation, and, thus, of unemployment, is the rigidity of the present discounted value of wages that at the time of hiring a firm expects to pay to a worker over the course of the employment relationship. The rigid wage mechanism generated vast interest in the literature and gave rise to developments of the alternative wage formations in a search and matching model. However, to gauge whether the wage data exhibit enough rigidity to amplify fluctuations in the standard search and matching model, the empirical estimates from wage data must be contrasted with the statistics obtained from the model.

Only recently has literature turned to contrasting the wage dynamics in the model with the data. This paper takes the focus on the cyclical behavior of the expected present

[3]Recently, Gertler and Trigari (2009) and Hagedorn and Manovskii (2010) emphasize the importance of controlling for match quality in estimating the response of wages to unemployment. I re-estimate Beaudry and DiNardo (1991) regressions using job dummies as a most general measure of match quality. The Beaudry and DiNardo (1991) conclusions carry through. These results are available from the author.

discounted value of wages seriously and provides the relevant calibration target. I calibrate the elasticity of the wage component of the user cost of labor in the model to its empirical estimate and examine how much volatility of vacancies and unemployment can be generated by the calibrated model.

Pissarides (2009) and Haefke, Sonntag, and van Rens (2009) are two related papers that contrast the behavior of the model with the data while emphasizing that the volatility of the average individual wage is not the relevant statistic. Pissarides (2009) compares the elasticity of wages from the standard search and matching model using common parameter values to the elasticity of wages of newly hired workers in the data. He finds that the elasticity of wages of new hires with respect to productivity in the data is close to 1 and that this is consistent with the elasticity of wages generated by the model. Thus, a model with more rigid wages will not be able to match the data. Haefke, Sonntag, and van Rens (2009) emphasize the role of the present discounted value of wages as opposed to just the current wage. They argue for the importance of the elasticity of the expected present discounted value of wages with respect to the expected present discounted value of productivity in newly formed matches, which they refer to as permanent values of wages and productivity, respectively. Using model simulations, they conclude that the elasticity of the current period wage of newly hired workers with respect to current period productivity "constitutes a good proxy for the elasticity of the permanent wage with respect to permanent productivity for the case of instantaneously rebargained wages" and "can be seen as a lower bound for [the elasticity of the permanent wage with respect to permanent productivity] in the case of wage rigidity on the job."

This paper differs from Pissarides (2009) and Haefke, Sonntag, and van Rens (2009) in that I directly calculate the present discounted values of wages from the individual wage data; construct the user cost of labor, which has a model counterpart; and empirically estimate its cyclicality. Because the user cost of labor is more cyclical than wages of newly hired workers, the results in this paper further push away the rigidity of wages as a candidate solution for the unemployment volatility puzzle.

The concept of the user cost was introduced by Keynes and clarified in Scott (1953). Later, Jorgenson (1963) applies the term to define the 'shadow' price of capital and Rosen

(1969) adopts the term for labor's cost.[4] Despite a long history, the user cost has not been employed in the recent literature. While the studies acknowledge that what matters for job creation is the present value of wages, the literature usually proceeds by contrasting models with the average wage in the data. However, the empirical evidence points to the importance of the effect of the economic conditions at the time of hiring for future wages.

III. The User Cost of Labor

In this section, I introduce the user cost of labor in a discrete time search and matching model, and show that the user cost, not wages, is the key factor in the job creation decision of firms.

A. Environment

The economy is populated by a continuum of infinitely lived profit-maximizing homogeneous firms and a continuum of measure 1 of homogeneous infinitely lived workers. Workers maximize the present discounted value of utility, $u(c)$, with $u'(c) > 0$, $u''(c) \leq 0$. They do not have access to credit markets and cannot save or borrow. Firms and workers discount the future with a common discount factor β, $0 < \beta < 1$.

A firm can choose to remain inactive or to start production. Production requires only labor input. To start production, a firm must enter the labor market and hire a worker. Upon entering the labor market, a firm opens vacancies and searches for a worker. There is free entry; however, a firm must pay a per vacancy cost, c, measured in units of the consumption good. Workers in the economy can be employed or unemployed. An unemployed worker receives a per period unemployment benefit, b, and costlessly searches for a job. Given the number of unemployed workers, u, and the number of vacancies, v, the number of newly created matches in the economy is determined by a matching function, $m(u, v) = Ku^{\alpha}v^{1-\alpha}$, where $\alpha \in [0, 1]$ (Petrongolo and Pissarides (2001)) and K is a positive constant. Given $\theta \equiv \frac{v}{u}$, the labor market tightness, the probability of filling a vacancy for a firm is $q(\theta) \equiv K\theta^{\alpha}$

[4] In Rosen (1969) the user cost of labor refers to the required return to cover the real interest and turnover costs. In addition to Rosen's components, the concept used here encompasses the worker's wage and the effect of the economic conditions at the time of hiring on future wages.

and the probability of finding a job for an unemployed worker is $\mu(\theta) \equiv K\theta^{1-\alpha}$. While matched, each firm-worker match produces per period output z. The stochastic process for z is governed by a stationary first-order Markov process.

Workers matched with firms earn wages and cannot search while employed. Generally, one can think of the wage as the result of a surplus division agreement between the firm and the worker, which may or may not entail history dependence in wages. In this section I do not assume a particular surplus division rule either at the beginning of or within the employment relationship.

The economy operates according to the following time-line:[5] 1) At the beginning of a period, a firm decides whether to create a job or to stay inactive; if the decision is to create a job, the firm posts vacancies and incurs the vacancy posting cost. Also, workers who were unemployed for at least one period costlessly search for jobs. 2) When firms with open vacancies meet unemployed workers, new matches are created. 3) Production takes place in both newly created matches and matches that were carried over from the previous period; employed workers receive wages and unemployed workers receive unemployment benefit, b. 4) At the end of a period, a fraction δ of productive matches is randomly selected and exogenously destroyed: the workers who were employed in those matches become unemployed and the firms who operated those matches return to the pool of inactive firms. 5) Surviving matches are carried over to the next period.

B. The User Cost of Labor

The only nontrivial economic decision in this environment is a firm's decision to create a productive match with a worker in the current period versus to postpone the creation until the next period. The costs of such a decision are summarized by the user cost of labor: they are all expenses associated with creating a match and maintaining it starting from the current period that can be avoided if the creation is postponed until the next period. Therefore, the user cost does not includes the total payments associated with creation of a productive match in the current period; it includes only the part that is expected to be in excess of what a firm will need to pay the next period.

[5] The value functions in the economy are summarized in the Appendix.

Thus, the user cost of labor can be calculated as the expected present discounted value of the costs associated with creating and maintaining a productive match with a worker that starts in period t (i.e., hiring costs and wage payments throughout the duration of a match) less the expected present discounted value of the costs of replacing the worker in period $t+1$. In the model there are two sources of costs associated with creating a productive match: costs associated with vacancy openings and wage payments to a worker. Thus, the user cost of labor, UC_t, can be decomposed into its two components: the vacancy component, UC_t^V, and the wage component, UC_t^W:

$$UC_t = UC_t^V + UC_t^W.$$

The vacancy component is associated with fixed cost on vacancy opening, c, and the probabilities of filling a vacancy in t, q_t, and in $t+1$, q_{t+1}. Suppose, for example, that a worker is always available for hire, i.e., $q_t = q_{t+1} = 1$. Then the vacancy component is $\beta\delta c$, reflecting a worker turnover and the real interest rate associated with paying cost c in t instead of delaying it until $t+1$. The turnover cost is due to the possibility of separation in period t, which decreases the expected number of matches surviving until period $t+1$. However, with search and matching frictions the probability of filling a vacancy may differ between t and $t+1$. Given that to create one match in period t, a firm opens $1/q_t$ vacancies, each at cost c, the vacancy component, UC_t^V, is

$$UC_t^V = \frac{c}{q_t} - \beta(1-\delta)E_t\frac{c}{q_{t+1}}.$$

The wage component of the user cost of labor is associated with the wage payments to a worker. If wages are renegotiated every period in all matches and do not depend on the history of economic conditions from the start of the job, then the wage component is simply the hiring wage. However, if wages depend on the history of economic conditions from the time a worker is hired, then the wage component should take into account the hiring wage and the effect of the economic conditions at the time of hiring on future wages within the employment relationship. The wage component, UC_t^W, can be calculated as the difference between the expected present discounted value of wages paid to a worker hired in t and the

expected present discounted value of wages to be paid to an identical worker hired in $t+1$:

$$UC_t^W = PDV_t - \beta(1-\delta)E_t PDV_{t+1}, \tag{1}$$

where $PDV_t = w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t w_{t,\tau}$.

Substituting expression for PDV_t in (1) yields the following expression for the wage component of the user cost:

$$UC_t^W = w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t(w_{t,\tau} - w_{t+1,\tau}). \tag{2}$$

The wage component at time t consists of two parts: the hiring wage at time t and the expected present discounted value of the differences between wages paid from the next period onward in the employment relationship that starts in period t and the employment relationship that starts in period $t+1$. Unless the second term is 0, it is not equal to the wage at the time of hiring.

Consider the conditions under which the second term in (2) vanishes. One such example is the case where the wage is bargained each period and is not history-dependent. Then, if the contemporaneous productivity in two matches is the same, these matches pay the same wages, independent when they were formed. In such a case, the wage component of the user cost of labor is just the wage at the time of hiring. (Nash bargaining in the textbook search and matching model is an example of such wage setting.) Alternatively, the wage component of the user cost is equal to the wage at the time of hiring if the wage is rigid and is not responsive to changes in economic conditions. Finally, if the aggregate shock in the economy is perfectly autocorrelated, then there is no expected change in productivity. In such case, depending on the nature of the wage contract, the expected wages may remain constant due to the unchanged economic conditions.

In general, if wages depend on the history of the economic conditions from the start of the job, the wage component of the user cost of labor does not equal wage. Such history dependence can arise, for example, if workers are risk-averse and cannot save and firms provide insurance against fluctuations in productivity. In this case, the wage at the time of

hiring is a part of a contractual scheme and does not necessarily equal the worker's marginal product.

It is an empirical question whether wages depend on the history of economic conditions from the start of the job or only on the contemporaneous market conditions. The literature summarized in Section 2 provides empirical evidence in favor of the former case.

C. Free Entry and the User Cost of Labor

The key equilibrium condition in a search and matching model is the free entry condition for firms, which implies that firms enter the labor market and create vacancies until the value of vacancy is driven to 0. Given this condition, the following proposition obtains.[6]

Proposition 1. *Given the free entry condition for firms, the marginal productivity of a firm-worker match equals the user cost of labor, $z_t = UC_t$ $\forall t$:*

(3)

$$z_t = \left(\frac{c}{q(u_t, v_t)} - \beta(1-\delta) E_t \frac{c}{q(u_{t+1}, v_{t+1})} \right) + \left(w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} E_t (w_{t,\tau} - w_{t+1,\tau}) \right).$$

Equation (3) is intuitive: firms create jobs in period t as long as the marginal benefit from adding a worker exceeds the user cost of labor. With free entry, the firms will enter the labor market until the net benefit is driven to 0. At that point, the decision to add a worker exactly equates the current benefit from a worker with the current cost and the present value of the expected future cost resulting from the current decision.

Equation (3) is crucial to understanding the concept of allocational price of labor. Given the dynamics of the wage component of the user cost, the dynamics of individual wages do not have a direct impact on the dynamics of vacancies and unemployment. The dynamics of the wage component of the user cost are what matters for the dynamics of firms' job creation activity.

The result of Proposition 1 illustrates the restrictions imposed by a search and matching model. It allows bringing together the data on the unemployment-vacancy ratio and the statistics from wage data that are relevant for the job creation decision. To examine these restrictions, I rewrite equation (3) in terms of elasticities with respect to productivity.

[6] See the Appendix for a proof.

First, consider a steady state. Total differentiation of (3) and rearrangement yields

$$1 = \varepsilon_{UC^V,z}\frac{UC^V}{z} + \varepsilon_{UC^W,z}\left(1 - \frac{UC^V}{z}\right),$$

where $\varepsilon_{UC^V,z}$ and $\varepsilon_{UC^W,z}$ are the elasticities of the vacancy component and the wage component, respectively, with respect to productivity evaluated at steady state values. Then, the vacancy component share in productivity is

$$\frac{UC^V}{z} = \frac{1 - \varepsilon_{UC^W,z}}{\varepsilon_{UC^V,z} - \varepsilon_{UC^W,z}}. \tag{4}$$

In the steady state, $UC^V = \frac{c}{K\theta^{-\alpha}}(1-\beta(1-\delta))$. Then, the elasticity of the vacancy component with respect to productivity equals

$$\varepsilon_{UC^V,z} = \alpha\varepsilon_{\theta,z},$$

where $\varepsilon_{\theta,z}$ is the elasticity of the vacancy-unemployment ratio with respect to productivity.

Since in steady state $UC^V > 0$, $UC^W > 0$ and $z = UC^V + UC^W$, it must be that $0 < \frac{UC^V}{z} < 1$. Then, using $\varepsilon_{UC^V,z} = \alpha\varepsilon_{\theta,z}$, the following must hold from (4):

$$0 < \frac{1 - \varepsilon_{UC^W,z}}{\alpha\varepsilon_{\theta,z} - \varepsilon_{UC^W,z}} < 1. \tag{5}$$

Condition (5) holds if 1) either $\varepsilon_{UC^W,z} < 1 < \alpha\varepsilon_{\theta,z}$, or 2) $\alpha\varepsilon_{\theta,z} < 1 < \varepsilon_{UC^W,z}$. Given the value of the elasticity of the vacancy-unemployment ratio, $\varepsilon_{\theta,z}$, of 7.56 (see, for example, Rudanko (2009) and Pissarides (2009)) and a range of values for α that can be found in the literature, $[0.235, 0.72]$, one obtains $\alpha\varepsilon_{\theta,z} > 1$. Thus, for (5) to hold, one should have $\varepsilon_{UC^W,z} < 1$. In Section 4, I provide an estimate of $\varepsilon_{UC^W,z}$.

The analogous argument carries over to the stochastic case.[7] Specifically, assume that z_t follows the $AR(1)$ process in logs with autocorrelation coefficient ρ and normal innovations. Then it can be shown that the elasticity of the vacancy-unemployment ratio with respect to

[7] It can be shown that $\Pr(UC_t^V > 0) > 0.99$, $\Pr(UC_t^W > 0) > 0.99$, which implies $0 < \frac{UC_t^V}{z_t} < 1$. See the appendix for the details and the derivation of the expression for $\varepsilon_{UC_t^V,z_t}$, given the empirical volatility and autocorrelation of θ.

productivity takes the form $\varepsilon_{UC_t^V,z_t} = \alpha\varepsilon_{\theta,z}x_t$,where $x_t > 1$ provided $\rho < \alpha\varepsilon_{\theta,z}$.[8]

Similarly as above, I obtain

$$0 < \frac{1 - \varepsilon_{UC^W,z}}{\alpha\varepsilon_{\theta,z}x_t - \varepsilon_{UC^W,z}} < 1. \tag{6}$$

Condition (6) holds if 1) either $\varepsilon_{UC^W,z} < 1 < \alpha\varepsilon_{\theta,z}x_t$, or 2) $\alpha\varepsilon_{\theta,z}x_t < 1 < \varepsilon_{UC^W,z}$. Since $x_t > 1$ and $\alpha\varepsilon_{\theta,z} > 1$, for (6) to hold, it should be the case that $\varepsilon_{UC^W,z} < 1$.

Equations (5) and (6) demonstrate a trade-off between the elasticity of the wage component of the user cost of labor and the elasticity of the vacancy-unemployment ratio imposed by the free entry condition of the model. Because both the wage component and the vacancy component of the user cost of labor covary positively with productivity, there is a trade-off in the degree of the response of UC_t^W and UC_t^V to changes in productivity. Notice that equations (5) and (6) are derived without evoking a particular surplus division rule or wage setting. These restrictions allow examining whether the model can potentially generate empirical elasticities $\varepsilon_{\theta,z}$ and $\varepsilon_{UC^W,z}$. Since, as mentioned above, the conventional values for α and empirical estimates of $\varepsilon_{\theta,z}$ deliver the value of $\alpha\varepsilon_{\theta,z}$ that exceeds 1, the answer depends on the value for $\varepsilon_{UC^W,z}$.

In the next section, I find that the estimate of the elasticity of the wage component of the user cost of labor with respect to productivity, $\varepsilon_{UC^W,z}$, is above 1.5. Thus, since both $\alpha\varepsilon_{\theta,z}$ and $\varepsilon_{UC^W,z}$ exceed 1, the model cannot generate both the empirical elasticities of the vacancy-unemployment ratio and the wage component of the user cost of labor. This leads to the conclusion that if the model is to match the volatility of quantities (vacancies and unemployment) and the relevant measure of prices (the wage component of the user cost of labor), then the solution for the unemployment volatility puzzle cannot be explained by a wage formation. This is so because any wage formation should be able to generate the elasticity of the wage component of the user cost of labor observed in the data, and, as the estimates in the next section show, the wage component of the user cost of labor is not rigid.

Additionally, it is worth noting that using the wage component of the user cost of labor as a calibration target as opposed to the average wage or the wage of the newly hired workers

[8] See the Appendix, which derives the expression for $\varepsilon_{UC_t^V,z_t}$.

helps to isolate the quantitative test of the search and matching framework from the issue of the wage formation in the model. Consider, for example, a search and matching model in which wages are rebargained by Nash bargaining period by period. With such a wage formation, in the model the wage component of the user cost of labor equals average wages and wages of newly hired workers. When bringing the dynamics of wages from such a model to the data, a researcher faces three calibration targets: the dynamics of individual wages of all workers, the dynamics of wages of newly hired workers, and the dynamics of the wage component of the user cost of labor. The choice would not be crucial for the test if in the data the dynamics of the three statistics were the same. However, this is not the case. As the results in the next section reveal, the wage component of the user cost of labor is economically and statistically more procyclical than the wages of newly hired workers, and wages of newly hired workers are more procyclical than wages of all workers. Calibration of the dynamics of wages from such a model to the dynamics of individual wages in the data is the joint test of the wage setting and search and matching framework and, thus, may lead to inaccurate conclusions about the quantitative performance of the search and matching framework. With wage bargaining period by period, the dynamics of wages from the model should be calibrated to the dynamics of the wage component of the user cost in the data.

IV. Cyclicality of the Wage Component of the User Cost of Labor in the Data

This section contains the main empirical result of the paper. I construct the counterpart of the wage component of the user cost of labor, UC_t^W, and measure its behavior over the business cycle.

A. Estimation Procedure

Given the constant separation rate, δ, and the discount factor, β, the wage component of the user cost of labor, UC_t^W, is (just rewriting equation (2) here)

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} (\beta (1-\delta))^{\tau-t} (w_{t,\tau} - w_{t+1,\tau}), \tag{7}$$

where w_{t_1,t_2} is a wage in period t_2 at the job that started in t_1.

The cyclicality of the wage component of the user cost of labor is the expected proportional change in the wage component, UC_t^W, in response to a unit change in the unemployment rate, U_t, i.e., the semielasticity of UC_t^W with respect to U_t. This is the measure typically used in the literature on the cyclicality of wages (for example, see a review in Pissarides (2009)) and, thus, it facilitates a comparison of the cyclicality of the wage component of the user cost of labor with the cyclicality of wages.[9] It can be measured as the projection of $\ln UC_t^W$ on U_t:

$$\gamma = \frac{cov(\ln UC_t^W, U_t)}{var(U_t)}. \tag{8}$$

Let UC_t^{WR} be the realized, ex post value of the wage component of the user cost of labor. Then

$$UC_t^W = E_t(UC_t^{WR}).$$

Given the standard rational expectations argument, the cyclicality of the wage component can be calculated as[10]

$$\gamma = \frac{cov(\ln UC_t^{WR}, U_t)}{var(U_t)}. \tag{9}$$

Now the task is to construct an empirical counterpart of UC_t^{WR} and to estimate the cyclicality in (9). The wage component is not directly observed in the data; hence, I construct an empirical counterpart of UC_t^{WR}, $\widehat{UC_t^{WR}}$, from individual wages and turnover.

Calculation of the wage component requires two sequences of wages for each t in the sample: a sequence of wages to be paid to a worker hired in t and a sequence of wages to be paid to the same worker or an identically productive worker hired in $t+1$. The existing individual panel data sets allow constructing the sequence of wages for workers hired in

[9] In the following section, analogously to Pissarides (2009), I use the estimated elasticity with respect to unemployment to obtain the elasticity with respect to productivity and then use the latter as a calibration target in the standard search and matching model.

[10] Define a random variable ε_t such that $UC_t^{WR} = UC_t^W \varepsilon_t$, where ε_t is independent of the variables in the information set of a firm in t. Then $cov(\ln UC_t^{WR}, U_t) = cov(\ln UC_t^W, U_t) + cov(\ln \varepsilon_t, U_t)$. Because the information set of a firm in t contains the contemporaneous unemployment rate, U_t, the last term is 0. Then $cov(\ln UC_t^{WR}, U_t) = cov(\ln UC_t^W, U_t)$. This yields expression (9) for the cyclicality of the wage component of the user cost of labor.

different years. However, for each worker hired in t we observe only the sequence of wages in the employment relationship that starts in t. We do not observe what the sequence of wages would have been if an identical worker had been hired the following year. Thus, before using the individual wages in constructing the series of the wage component of the user cost, I need to clean the wage data from the observable and unobservable individual-specific effects.

To obtain a series of wages free of the observable and unobservable individual-specific effects, I estimate the following wage equation using the panel data set on individual wages that covers period from year 1 to year T:

$$\ln w_{j,t_1,t_2} = const_w + \alpha_j + \rho t_2 + \Psi X_{j,t_2} + \sum_{\tau=1}^{T}\sum_{k=\tau}^{T} \gamma_{\tau k} D_{\tau k} + \varepsilon_{j,t_2}, . \tag{10}$$

where w_{j,t_1,t_2} is the wages of worker j in year t_2 hired in period t_1, X_{j,t_2} is a vector of individual- and job-specific characteristics, α_j is a worker-specific individual fixed effect and ε_{j,t_2} is an error term such that $\varepsilon_{j,t_2} \sim N(0, \sigma_\varepsilon^2)$. $D_{\tau k}$ is a dummy variable that takes value 1 if $\tau = t_1$ and $k = t_2$ and 0 otherwise. That is, $D_{t_1 t_2}$ takes value 1 for all wage observations in year t_2 that are paid in employment relationships that start in year t_1, where $t_1 \in [1, T]$ and $t_2 \in [t_1, T]$ in the sample of length T.

The estimates of the coefficients $\{\gamma_{t_1 t_2}\}$ allow constructing the expected wage for each $\{t_1, t_2\}$ combination in the sample period, conditional on worker characteristics. Importantly, equation (10) does not impose any particular structure of wage formation on the wage data. In equation (10) wages are allowed to vary depending on the year when the job starts and on the contemporaneous year; however, there is no restriction imposed on that variation.

Detailed steps that I use to construct the wage component of the user cost of labor and calculate its cyclicality are as follows.

Step 1.

I estimate equation (10) using the NLSY79 data that span a period from 1978 to 2004; the details of the sample construction are described below. The vector of individual- and job-specific characteristics, X_{j,t_2}, consists of education, a quadratic in tenure, and a quadratic in potential labor market experience. I estimate equation (10) using OLS, weighting each observation by sampling weights and controlling for worker-specific fixed effects. In the

estimation I cluster the standard errors by time.

Step 2.

Using the coefficient estimates on the set of dummies, $\{\gamma_{t_1 t_2}\}$, I calculate the fitted values for wages, $\widehat{w_{t_1 t_2}}$, for all t_1 and $t_2 : t_1, t_2 = \{1, T; t_1 \leq t_2\}$, i.e.,

$$\widehat{w_{t_1 t_2}} = \exp\left(\widehat{const_w} + \widehat{\rho}\bar{t} + \widehat{\Psi}\overline{X} + \widehat{\gamma_{t_1 t_2}}\right),$$

where $\bar{t}$ and $\overline{X}$ are the sample means. Note that $E_t(\widehat{w_{t_1 t_2}}) = w_{t_1 t_2} / \exp \frac{\sigma_\xi^2}{2}$. Assuming that $\sigma_\xi^2 = const$ and $\overline{X}$ is uncorrelated with the contemporaneous unemployment rate, the cyclicality of the wage component of the user cost of labor does not depend on the actual values of $\bar{t}$, $\overline{X}$ and σ_ξ^2.[11]

Step 3.

The expression for the wage component of the user cost of labor, (7), assumes infinitely lived firms and workers, so the calculations involve infinite sums while the data allow a sample of a finite size. Thus, I truncate the calculations of the sum at different time horizons and check the sensitivity of the estimated cyclicality to the truncation horizon. In the benchmark case I truncate the horizon at 7 years.[12] Truncation of the time horizon for calculation of UC_t^W can be justified by two considerations. First, the discount factor, which includes the turnover rate and the real interest rate, increases. This decreases the weight of the terms far in the future. Second, if, for example, the model behind the dependence of wages on the history of unemployment rates is as in Thomas and Worrall (1988) and the unemployment rate follows the mean-reverting process, then wages in the employment relationships that started in different years but that have lasted long enough to experience similar episodes of minimum and maximum unemployment rates will be the same. In that case, the terms in brackets in (7) will be equal to 0 for all τ higher than some τ'.

Step 4.

[11] This assumption is consistent with the representative agent model in Section 3, in which in every period all matches are identically productive.

[12] Given the truncation period of 7 years and the sample period from 1978 to 2004, the wage component of the user cost of labor can be calculated for 20 years, from 1978 to 1997. This number of observation is typical for the papers on the cyclicality of wages that employ a two-step estimation procedure (Solon, Barsky and Parker (1994), Devereux (2001)). For example, Devereux (2001) reports 22 observations in the second-stage regression.

I set a discount factor, β, at $1/(1+0.045)$ and annual separation rate, δ, at 0.295, which is calculated from the monthly separation rate of 0.029. Then I calculate an empirical counterpart of the realized wage component using the constructed series $\widehat{w_{t_1,t_2}}$ and truncating the calculation at 7 years.

Step 5.

To obtain the cyclicality of the wage component, I regress the logarithm of the constructed realized wage component of the user cost of labor on the unemployment rate and a time trend. The cyclicality is the coefficient on the unemployment rate multiplied by 100%.

B. Data

The data in the study come from the NLSY79, 1978–2004. The survey collects information on work histories of a nationally representative sample of young individuals who were between 14 and 21 years of age in 1979 when the first interview was taken.

I focus on the cross-sectional sample that represents the non-institutionalized civilian population and further restrict my analysis to males. This restriction is typical in other empirical studies of wage cyclicality (see, for example, Beaudry and DiNardo (1991) and Shin (1994)). Hence, I work with the following sub-samples, as defined in the NLSY: 1 = cross-sectional white males, 3 = cross-sectional black males, and 4 = cross-sectional Hispanic males.

The data set is particularly suited for the purposes of this study because it separately records wages and other job characteristics for up to five jobs that an individual might hold between two consecutive interviews. By tracking individuals over the years, I can isolate the individual-specific fixed effects. In addition, if a worker simultaneously held more than one job, the NLSY79 kept a separate record for each job, as opposed to PSID data that report the average wage in such cases. However, the data contain information on individuals at the early stages of their labor market experience. Because jobs taken at the early stages of an individual's labor experience may be predominantly seasonal or temporary, these job changers may disproportionately affect the wage cyclicality. To alleviate this problem, I restrict observations included in the wage equation to the observations of individuals who started a job at the age of 16 and older, and who were 21 years old and older at the time

of the observation. Because I use workers' fixed effects in the estimation, the sample is restricted to the workers having more than one observation.

Wage is an hourly pay variable constructed by the NLSY. I deflate wages using the annual CPI index of the year the observation refers to. Unemployment rate is the annual, national, civilian unemployment rate for ages 16 and older obtained from the Bureau of Labor Statistics. The contemporaneous unemployment rate is the annual unemployment rate of the calendar year to which the wage observation corresponds to.

Before estimating the cyclicality of the wage component of the user cost of labor, I estimate the response of the average wages and wages of newly hired workers to the unemployment rate. In particular, I regress the natural logarithm of wages on the contemporaneous unemployment rate and the individual- and job-specific controls that are typically used in estimating the cyclicality of wages. These controls are the number of years of education, a quadratic in tenure, a quadratic in experience, a dummy for union status, and a linear time trend. The regression is estimated using OLS, controlling for worker's fixed effects and clustering standard errors by time.

Column 1 of Table 1 shows the results from the regression estimated from the full sample. Column 2 of Table 1 shows the results from the regression estimated on the sample of newly hired workers, who are defined as workers with tenure less than 52 weeks. Column 3 of Table 1 shows the results from the regression similar to the one estimated by Beaudry and DiNardo (1991). In particular, in addition to the contemporaneous unemployment rate I include the unemployment rate at the start of the job and the minimum unemployment rate from the start of the job.

Consistent with earlier studies (for example, Bils (1985) and Solon, Barsky, and Parker (1994)), I find that wages on newly hired workers respond much more to the contemporaneous unemployment rate as compared to wages of all workers. Column 3 shows that once I include the unemployment rate at the start of the job and the minimum unemployment rate experienced by a worker from the start of the job, the effect of the contemporaneous unemployment on wages becomes almost 0. The minimum unemployment rate has the most sizable and statistically significant effect on wages. In particular, the coefficient on the minimum unemployment rate shows that a decline in the minimum unemployment rate of

one percentage point is associated with an average increase of wages in the full sample by 3.32%.[13] These findings are consistent with the evidence of Beaudry and DiNardo (1991).

These results support the conjecture that, on average, wages are not described by a spot market wage setting. Consequently, average wage or wage at the start of the job do not capture per period user cost of labor. I proceed to construct the wage component of the user cost of labor as described above and estimate its cyclicality.

C. Main Empirical Result

The main empirical results are presented in Table 2. The first row presents the estimates of the cyclicality of the wage component of the user cost of labor constructed using the constant separation rate. The estimates of -5.10% show that when the unemployment rate goes up by a one percentage point, the wage component of the user cost of labor, on average, goes down by 5.10%.

For comparison, Row 2 of Table 2 contains the cyclicality of wages of newly hired workers and Row 3 of Table 2 contains the cyclicality of wages of all workers in the sample. These estimates are the estimates of the coefficient on the unemployment rate in columns 1 and 2, respectively, of Table 1. The results show that a one percentage point increase in the unemployment rate is associated with a 3% decrease in wages of newly hired workers and a 1.78% decrease in wages of all workers. Thus, the results of the estimation show that the wage component of the user cost of labor, wages of new hires, and average wages are procyclical. However, the wage component of the user cost of labor is substantially more procyclical than average wages, and also more procyclical than wages of newly hired workers.

The intuition behind the large cyclicality of the wage component of the user cost of labor as compared to the cyclicality of wages is as follows. Consider a firm that hires a worker when the unemployment rate is high, i.e., in recession, as opposed to hiring later, i.e., during a recovery period, when the unemployment rate is expected to return to its lower level. Empirical findings show that wages of newly hired workers are procyclical. Hence,

[13] To examine whether the history dependence in wages is driven by a set of industries, I re-estimate specifications in Columns $1-3$ of Table 1 with the controls for 14 industry dummies. The results of the estimation are presented in columns $4-6$ of Table 1, respectively. The sample size is restricted to observations with non-missing industry data. The estimation from columns 1- 3 results carry through.

when hiring currently, a firm pays a comparatively lower hiring wage. Once a worker is hired, his wages are shielded from the effect of contemporaneous labor market conditions and bear the effect of the past unemployment rates. This is so because empirical evidence shows that wages of newly hired workers are more cyclical than the wages of workers in ongoing relationships. Thus, by hiring when the unemployment rate is high, a firm locks in a worker to a stream of wages that is expected to be lower than the stream of wages to be paid to an identically productive worker hired when unemployment is lower. As a result, the wage component of the user cost of labor is even lower than the already low hiring wage because the wage component also reflects comparatively low future expected wages. The opposite is true when a worker is hired at the peak of the cycle, when the unemployment rate is low but is expected to rise. Then the wage component is higher than the hiring wage. Thus, the procyclical hiring wage and the lock-in cause the wage component to be more procyclical than the hiring wage.

D. Robustness: Estimation with Time-Varying Separation Rates

The wage component of the user cost of labor in equation (7) is based on the assumption of the constant separation rate, which is consistent with the textbook search and matching model described in Section 3. However, in the data the separation rate may vary with the contemporaneous labor market conditions or the labor market conditions at the start of the job. Bowlus (1995) provides evidence that jobs started when the unemployment rate is high usually last longer. Thus, I proceed by estimating the cyclicality of the user cost of labor allowing for the time-varying separation rates.

To understand the effect that the time-varying separation rates might have on the cyclicality of the wage component of the user cost of labor, suppose that the separation rates depend positively on the unemployment rate at the time of hiring. Then, the workers who are hired when the unemployment rate is high tend to have shorter tenures. Once a worker is separated, a firm must hire a new one to fill the position. But if the labor market conditions have improved, a new worker is offered a new present discounted value of wages that is expected to be higher than the value paid to the previous employee. Thus, higher separation rates might weaken the lock-in to the initial labor market conditions. To estimate

whether this effect is quantitatively important for the cyclicality of the wage component of the user cost, I examine the cyclicality of the wage component that allows the separation rates to depend on the history of unemployment rates from the start of the job.

Following the definition of the user cost of labor in Section 3, if the separation rate depends on the contemporaneous time period but does not depend on the period when the job started, the wage component of the user cost of labor can be calculated as the difference between the expected present discounted value of wages paid to a worker hired in t and the expected present discounted value of wages to be paid to an identical worker hired in $t+1$:

$$UC_t^W = PDV_t - \beta(1-\delta_t)E_t PDV_{t+1},$$

where $PDV_t = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} \left(\beta^{\tau-t} \prod_{k=0}^{\tau-t}(1-\delta_{t+k})\right) w_{t,\tau}$ and δ_τ is the separation rate in period τ. Substituting an expression for PDV_t yields

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} \left(\beta^{\tau-t} \prod_{k=0}^{\tau-t}(1-\delta_{t+k})\right)(w_{t,\tau} - w_{t+1,\tau}), \tag{11}$$

Equation (11) is the expected difference in the costs between two alternatives: hiring a worker this period, or hiring a worker next period with probability $(1-\delta_t)$. These two options differ only in how many workers the firm employs in period t; they give the same expected employment levels in all future periods. Therefore, the difference between them gives the implicit price of the services of one worker this period.

If the separation rate depends on the contemporaneous period and on the period when the job started, equation (11) should be modified to ensure that in every period after t, the employment level in the relationship that starts in t and the employment level in the relationship that starts in $t+1$ are equal. We can define the wage component of the user cost of labor in period t as the difference between the expected present discounted value of wages paid at the position opened in period t and $t+1$, i.e., whenever a worker separates, a firm must rehire a worker to replace the separated one at a new wage agreement. These two options give the same expected employment level – one – in all future periods. Therefore, the difference between them gives the implicit price of the services of one worker during period

t. The exact expression for the wage component of the user cost of labor with time-varying separation rates is given in the appendix.

To estimate the cyclicality of the wage component with time-varying separation rates, I construct the realized wage component of the user cost of labor using the procedure similar to the one described above, except that I use the estimated series of the separation rates from the NLSY79 data instead of the constant separation rate. To obtain the series of separation rates that depend only on the contemporaneous period, I proceed in two steps. First, I detrend the monthly separation rates. For that purpose, I estimate a linear probability model with the dependant variable taking value 1 if a worker does not work at the same job in the following month and 0 otherwise. The explanatory variables are a quartic in the monthly trend. Then, I subtract the value of a quartic in the trend multiplied by the estimated coefficients from the dependent variable and add the value of a quartic of a trend calculated at the mean multiplied by the estimated coefficients. Second, I estimate a linear probability model of the detrended monthly separation rates with a set of contemporaneous time dummies as explanatory variables. Then, using the coefficient estimates on the set of contemporaneous dummies, I obtain fitted projections, $\widehat{\delta_t}$, for all $t : t = \{1, 324\}$.[14] To obtain the series of the separation rates that depend on the contemporaneous period and on the period when the job starts, I estimate a linear probability model of the detrended monthly separation rates with two sets of time dummies as explanatory variables: one set of time dummies corresponds to the year the job starts and another set of dummies corresponds to the contemporaneous year. Then, I use monthly fitted projections to obtain annual separation rates. I proceed to estimate the cyclicality as described in the previous subsection.

The results of the estimation are presented in Table 3. The rows of the table contain the estimated cyclicality of the wage component of the user cost of labor constructed using the constant separation rate and the time-varying separation rates. The columns of the table show the cyclicality for truncation horizons of 5, 7, and 9 years. As can be seen from the table, the cyclicality of the wage component of the user cost of labor is more than -4.5%. Thus, it is substantially higher than the cyclicality of individual wages of all workers and

[14] For the robustness check, I have also repeated this procedure with the probit in the first step instead of a linear probability model.

also noticeably higher than the cyclicality of wages of newly hired workers.

V. Implications for Free Entry

Following Pissarides (2009), I use the semi-elasticity of UC^W with respect to unemployment, $\frac{d \ln UC^W}{du}$, to calculate the elasticity of UC^W with respect to productivity as follows:

$$\frac{d \ln UC^W}{d \ln z} = \frac{d \ln UC^W}{du} \frac{du}{d \ln z},$$

where $\frac{du}{d \ln z}$ is the change in unemployment in response to a percentage change in productivity. Pissarides (2009) provides the following estimates of $\frac{du}{d \ln z}$: -0.34 for the period 1948-2006 and -0.49 for the period 1970-1993. Combining these estimates with the estimates of the semi-elasticity of the wage component of the user cost of labor of -4.5% above yields the elasticity of the wage component with respect to productivity of 1.530 and 2.205, respectively.

As shown in equation (6) of Section 3, for the free entry condition in the search and matching model to hold, provided that $\varepsilon_{\theta,z} = 7.56$, the elasticity of the wage component of the user cost of labor should be less than 1. As the results of the estimation show, $\varepsilon_{UC^W,z} > 1$. Thus, the restrictions imposed by the free entry condition in the model on the data do not hold.

A. Illustration with Elasticities

To illustrate the findings, I consider the search and matching model as described in Section 3 with two additional assumptions: 1) workers are risk neutral and 2) at the time the match is formed, the surplus between a worker and a firm is divided by a generalized Nash bargaining with constant bargaining shares. Note that there are different wage settings that encompass this surplus division rule at the beginning of the match. One example of such a wage setting is Nash bargaining period by period in all matches. Another example is a constant wage within the employment relationship.

It can easily be shown that, given linear utilities for a worker and a firm, models with different wage formations within the match but in which the surplus at the beginning of the match is divided using a constant shares Nash bargaining rule, deliver identically equal wage

components of the user cost of labor and, following Proposition 1, the same allocations, i.e., the vacancy-unemployment ratio. (This can also be seen from the results in Table 9.) Thus, to analyze the implications of Proposition 1 for a model with such a surplus division rule, it is sufficient to analyze a model with one of the wage formations with such a surplus division rule at the beginning of a match. A convenient model to analyze is the model in which wages are set by Nash bargaining in every period in all matches, which is widely used in the literature. Thus, in addition to the two additional assumptions above I add the following one: 3) every period the wage is set by Nash bargaining between a worker and a firm with a constant bargaining share of a worker equal η.

With Nash bargaining period by period, the wage depends only on the contemporaneous economic conditions, $w_{t_1,\tau} = w_{t_2,\tau} = w_\tau$ for all t_1, t_2, τ. Then, the last term in brackets in equation (2) is 0. It implies that with Nash bargaining period by period, the wage component of the user cost of labor equals wage, i.e., $UC_\tau^W = w_\tau$ for all τ. This conveniently allows deriving the closed-form expression for UC_τ^W in the model: first, I derive w_τ and, then, set $UC_\tau^W = w_\tau$.

It has been discussed that two parameters are crucial for the volatilities of the vacancy-unemployment ratio and wages: the unemployment benefit, b, and a worker's bargaining power, η (see, for example, Hornstein, Krusell and Violante (2005)). Without replicating the analysis here, I derive the expressions for b/z as a function of the elasticity of the vacancy-unemployment ratio with respect to productivity, $\varepsilon_{\theta z}$, and as a function of the elasticity of wages, ε_{wz}, respectively. I obtain the following equations:[15]

$$\frac{b}{z} = (1 - \frac{1}{\varepsilon_{\theta z}} \frac{1 - \beta(1 - \delta - \eta\mu)}{\alpha - \beta(\alpha - \delta\alpha - \eta\mu)}). \tag{12}$$

$$\begin{aligned} \frac{b}{z} &= \frac{\eta}{1-\eta} \frac{1}{\varepsilon_{wz}} \frac{1}{1-\beta(1-\delta)} ((1 - \beta(1 - \delta - \eta\mu))(1 - \varepsilon_{wz}) + \\ & \frac{(1-\eta)(1-\alpha)\beta\mu(1-\beta(1-\delta))}{\alpha - \beta(\alpha - \delta\alpha - \eta\mu)}) \text{ if } \frac{b}{z} \neq 0, \end{aligned} \tag{13}$$

[15] See the Appendix for the expressions for $\varepsilon_{\theta z}$ and ε_{wz}.

where μ is a steady state value of job finding rate.

Now I can plot two functions of $\frac{b}{z}$: $\frac{b}{z}(\eta|\varepsilon_{\theta z})$ and $\frac{b}{z}(\eta|\varepsilon_{wz})$ given values for $\varepsilon_{\theta z}$ and ε_{wz} and a set of parameters $(\alpha, \beta, \delta, \mu)$. The intersection of the two functions, if one exists, gives pairs of $(\frac{b}{z}, \eta)$ that deliver targeted values of $\varepsilon_{\theta z}$ and ε_{wz}.

I obtain the following parametrization for the quarterly model: $\beta = 1/(1 + 0.012); \delta = 0.10; \mu = 1.35$ (Hornstein, Krusell and Violante (2005) and Shimer (2005)). Since literature provides a range of values for the elasticity of the matching function with respect to unemployment, α, I provide results for three different values of α: $\alpha = 0.235$ (Hall 2005), $\alpha = 0.72$ (Shimer 2005), and $\alpha = 0.5$ (the value in the range proposed by Pissarides and Petrongolo (2000)). I set $\varepsilon_{\theta z} = 7.56$ (see Rudanko (2009) and Pissarides (2009)).

It remains to specify the value of ε_{wz}. As shown above, in the model with wage bargaining period by period, wages are equal across all matches in each period. Thus, the average wage equals wages of newly hired workers and equals the wage component of the user cost of labor. However, in the data those three statistics from wages are different. In particular, Rudanko (2009) and Hagedorn and Manovskii (2008) summarize the elasticity of averages wages with respect to productivity at 0.5 and 0.47, respectively. Pissarides (2009), based on the cyclicality of -3%, summarizes the implied elasticity of wages of newly hired workers between 1.02 and 1.47, depending on the value of $\frac{du}{d\ln z}$. In this paper, I find the elasticity of the wage component of the user cost of labor to be between 1.53 and 2.20, based on the cyclicality of -4.5%.

Thus, it matters how the calibration target for ε_{wz} is chosen in the data. Since in the model with Nash bargaining period by period all three responses above are the same, this wage setting cannot be used to describe the behavior of individual wages in the data. To sidestep the question of what the exact wage setting within employment relationships is, one can calibrate ε_{wz} in the model to the elasticity of the wage component of the user cost of labor.

In Figure 1, I plot $\frac{b}{z}(\eta|\varepsilon_{\theta z})$ and $\frac{b}{z}(\eta|\varepsilon_{wz})$ given $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 1.5$. The graphs illustrate two points. First, as stated in the conclusion reached above, given the specified targets for $\varepsilon_{\theta z}$ and ε_{wz} and a set of parameters $(\alpha, \beta, \delta, \mu)$ as described previously, two functions $\frac{b}{z}(\eta|\varepsilon_{\theta z} = 7.56)$ and $\frac{b}{z}(\eta|\varepsilon_{wz} = 1.5)$ do not have points in common. Thus, the

model cannot generate both the elasticity of the vacancy-unemployment ratio of 7.56 and the elasticity of the wage component of the user cost of labor in excess of 1. Second, given the wage setting, the model can generate the elasticity of the wage component of the user cost of labor equal to 1.5 for only a small set of parameter values. In particular, for $\alpha = 0.72$, there are no admissible values of the pair $(\frac{b}{z}, \eta)$ that can deliver $\varepsilon_{wz} = 1.5$, given the values for (β, δ, μ).

In addition, from Figure 1 one can also see that the elasticity of the vacancy-unemployment ratio is very sensitive to the value of $\frac{b}{z}$ and less sensitive to η; and that the empirical value of $\varepsilon_{\theta z}$ requires a high value of $\frac{b}{z}$, which is the conclusion reached in Hornstein, Krusell and Violante (2005) and Hagedorn and Manovskii (2008).

To illustrate the conclusions reached in Section 3, I plot $\frac{b}{z}(\eta|\varepsilon_{\theta z})$ and $\frac{b}{z}(\eta|\varepsilon_{wz})$ for $\varepsilon_{wz} < 1$. In particular, in Figure 2, I plot $\frac{b}{z}(\eta|\varepsilon_{wz})$ for $\varepsilon_{wz} = 0.5$, which is close to the targets used in Hagedorn and Manovskii (2008) and Rudanko (2009). As can be seen from Figure 2, there exists a pair of $(\frac{b}{z}, \eta)$ that can deliver $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 0.5$. However, as discussed in Section 3.C and above, this calibration assumes a particular wage setting mechanism that lacks support in the data.[16] This result illustrates that focusing on the cyclicality of individual wages might lead to a misleading assessment of the quantitative behavior of the model if the wage setting, which is not a central feature of the model, is specified incorrectly.

VI. Example: Cyclicality of the User Cost of Labor in Models with Specific Wage Settings

In this section, I examine the cyclicality of the components of the user cost of labor and wages in four search and matching models with alternative wage formations.

A. Description of the Models

Consider the environment as described in Section 3 with the assumption that workers are risk averse and an assumption about a wage formation. The first three models I consider are the models with implicit contracts as in Thomas and Worrall (1988). In these models,

[16] See also a review in Pissarides (2009) and a discussion in Martins, Solon and Thomas (2009).

individual wages may depend on the history of economic conditions from the start of the job. Consequently, under these wage formations the wage component of the user cost of labor in (2) has a non-zero second term. The fourth model is a model where wages are bargained every period and the wage component of the user cost of labor equals the wage at the time of hiring, which in turn equals average wage. See Rudanko (2009) for an excellent treatment of Thomas and Worrall (1988) contracts in the search and matching model.

In the first three models, wages are the outcome of the implicit self-enforcing contracts between a worker and firm. In the models, risk-neutral firms insure risk-averse workers, who do not have access to capital markets, against fluctuations in consumption due to fluctuations in earnings. Three types of contracts are distinguished based on different degrees of commitment: full commitment contracts, contracts with lack of commitment from the worker's side and full commitment from the firm's side, and contracts with lack of commitment from both the worker's and firm's sides. If there is a lack of commitment from any side of the contract, the contract should be self-enforcing for that side to prevent reneging. In the original Thomas and Worrall (1988) environment, workers who renege on the contract are prohibited from entering any contractual arrangements in the future and are bound to trade their labor services at the spot market wage. In the current environment, once unemployed, workers search and enter contractual arrangements as soon as they find a new match. Both firms and workers face search and matching frictions. These frictions influence the value of the outside option through the probability of finding a new match.

Firms open vacancies with associated employment contracts and workers direct their search to the contracts. The vacancies opened with the associated contract σ and the unemployed workers searching for contract σ constitute a labor market with a market tightness θ_σ. A contract is a state-contingent sequence of wages that delivers a promised value to the worker. Equilibrium contracts are limited to efficient optimal contracts. To ensure a unique contract in equilibrium, I follow Rudanko (2009) and impose the following equilibrium refinement: there does not exist an efficient self-enforcing contract $\sigma'(z)$ and an associated labor market with tightness $\theta_{\sigma'}(z)$ such that the net surpluses from search for a worker and a firm are at least as much as under $\sigma(z)$ and $\theta_\sigma(z)$ and, for one party, it is strictly more.

Thomas and Worrall (1988) and Rudanko (2009) show that in such an environment,

for any history (z^t, z_{t+1}), there exists a $w_{\min}(z_{t+1})$ and $w_{\max}(z_{t+1})$, $w_{\min}(z_{t+1}) \leq w_{\max}(z_{t+1})$, such that the contract wage at $t+1$ is:

1) in the contract with full commitment: $w(z^t, z_{t+1}) = w(z^t)$;

2) in the contract with lack of commitment from the worker and full commitment from the firm:

$$w(z^t, z_{t+1}) = \begin{cases} w(z^t) & \text{if } w_{\min}(z_{t+1}) \leq w(z^t) \\ w_{\min}(z_{t+1}) & \text{if } w(z^t) < w_{\min}(z_{t+1}) \end{cases};$$

3) in the contract with two-sided lack of commitment:

$$w(z^t, z_{t+1}) = \begin{cases} w_{\max}(z_{t+1}) & \text{if } w(z^t) > w_{\max}(z_{t+1}) \\ w(z^t) & \text{if } w_{\min}(z_{t+1}) \leq w(z^t) \leq w_{\max}(z_{t+1}) \\ w_{\min}(z_{t+1}) & \text{if } w(z^t) < w_{\min}(z_{t+1}) \end{cases}.$$

Thus, whenever possible, the optimal contract offers a constant wage. However, in the contracts with lack of commitment, if the value of the outside option exceeds the value under the contract, the wage is adjusted to prevent reneging.

In the fourth model, wages are determined period by period in new and existing matches by the following rule: $\frac{W(z)-U(z)}{u'(w(z))}/J(z) = \frac{\eta}{1-\eta} \forall z \in Z$, where $W(z)$, $U(z)$ and $J(z)$ are values for an employed worker, an unemployed worker, and a firm with filled vacancy, respectively, and η is a worker's bargaining power. This rule is well known in the literature: the share of the surplus that the agent obtains from a match corresponds to his bargaining power. If workers are risk neutral, then it describes generalized Nash bargaining period by period over total surplus as in the canonical search and matching model (see, for example, Pissarides (1985)). In the appendix I specify the firm's optimization problem and define the equilibrium in the models described above.

B. Quantitative Results I

The parameters of the stochastic process for productivity shocks can be calibrated outside of the models.[17] Then, the only parameter that requires calibration within a model is

[17] To calibrate a stochastic process for productivity, I consider a three-state symmetric Markov process as in Rudanko (2009), $\mathbf{z} = [z_0 - \Delta, z_0, z_0 + \Delta]$, $\Delta > 0$, with the transition matrix (by row): $[\lambda, 1-\lambda, 0;$

the cost of posting a vacancy, c, which I calibrate to match the mean monthly job-finding rate, $E(\mu) = 0.45$. The model period is one month. The adopted parameters are reported in Table 4. The discount factor is 0.9960, which corresponds to the annual discount rate of 4.88%. The monthly separation rate is set to 0.034 (Shimer 2005). I set the bargaining power of workers to equal α to preserve the mathematical equivalence of the competitive search and random search equilibria (Rudanko 2009).

I obtain corresponding statistics for the models by simulating economies with each of the four different wage formations as follows. First, a common vector of aggregate shocks, z, is generated. For the panel of $10,000$ individuals, an initial employment status is drawn. Then, each period, the separation shock is drawn for each employed individual and his employment status is updated, and for each unemployed individual the job finding shock is drawn and his unemployment status is updated. Given the employment histories, individual wages are generated according to a model-specific wage setting. The first $4,000$ periods of the simulated series are discarded; the statistics are based on the series from the last 636 periods. The cyclicality of the series x is measured as $cov(\ln(x), u)/var(u) * 100$, which is the semi-elasticity of the series with respect to unemployment.

Table 5 reports the cyclicality of the individual wages of all workers and wages of newly hired workers, and the cyclicality of the components of the user cost of labor assuming log utility function for workers. The cyclicality of individual wages varies across economies, with the wages being only mildly procyclical in the implicit contract models and as cyclical as the wage component of the user cost in the period by period bargaining model. Importantly, in the models with contracts, the wage component of the user cost of labor is much more procyclical than the wages of newly hired workers. And the wages of newly hired workers are approximately 3 times as cyclical as the wages of all workers.[18]

$0.5(1-\lambda), \lambda, 0.5(1-\lambda); 0, 1-\lambda, \lambda]$. The variance of this process, σ_z^2, is $\frac{\Delta^2}{2}$ and the autocorrelation, ρ, is λ. $E(z)$ is normalized to 1. Δ and λ are calibrated to match the standard deviation, 0.02, and autocorrelation, 0.878, of productivity per worker, obtained from Shimer (2005), Table 1. To find Δ and λ, I draw the initial shock from a stationary distribution of z and, using the initial values for Δ and λ, generate monthly series of length $12T$, where T is the length of the time series in the data in years (from 1951 to 2003); aggregate by summing to obtain quarterly data; calculate the standard deviation and the autocorrelation of the logged quarterly series; and iterate until matching the targets.

[18] The cyclicality of individual wages in the models with contracts also depends on the number of states of the productivity process. However, it does not have an impact on the main results.

The cyclicality of the wages in the model with implicit contracts and full two-sided commitment obtains due to new hires entering employment relationships, constant separation rates and a positive autocorrelation in the productivity process. In the model with full commitment on the firm's side and lack of commitment on the worker's side, in addition to the composition effect, the wages in the existing employment relationships are bid up whenever the worker's outside option value becomes more attractive than the value from the contract. In the model with lack of commitment on both the firm's and the worker's sides, the wages can also be bid down whenever the value from the match for a firm falls below 0. The appendix contains the results for different values of b and for different utility functions. All conclusions carry through.

To understand why the cyclicality of the wage component of the user cost in the contract models is higher than the cyclicality of wages at the time of hiring, recall the workings of this wage setting. The implicit contracts offer wages that are rigid during the employment relationship to insure workers against fluctuations in consumption. The wages of new hires adjust to reflect the worker's outside option value. Consequently, the wages of newly hired workers are more cyclical than the wages of all workers. For example, when the job finding rate is low, the hiring wage is relatively low. In addition, the wages in all subsequent periods in the employment relationship are relatively lower than the wages in the contracts, initiated under more favorable economic conditions. The wage component of the user cost takes into account both the lower hiring wage and lower future wage payments. Hence, it is more procyclical than the wages of newly hired workers.

C. Quantitative Results II

Note from Table 5 that given $b = 0.70$, the implicit contract model generate a standard deviation of the vacancy-unemployment ratio of approximately 0.0620 and only slightly higher in the model with period by period bargaining, while the empirical counterpart is 0.382 (Shimer 2005). Next, I use b to calibrate the models to match the cyclicality of the wage component of the user cost estimated in Section 4 and check how much volatility of the vacancy-unemployment ratio the models generate.

In Table 9, c and b are calibrated to match the expected job finding rate and the

cyclicality of the wage component of the user cost, $cov(\ln(UC^W), u)/var(u) = -0.045$. As can be seen from the table, regardless of the wage setting, the models generate approximately $1/3 - 1/2$ of the empirical volatility of the vacancy-unemployment ratio, 0.382. The strong procyclicality of the wage component of the user cost dampens the response of the job creation to changes in productivity. Alternatively, when the models are calibrated to match the empirical volatility of the vacancy-unemployment ratio, the wage component of the user cost is too rigid as compared to its empirical counterpart (Table 11).

As the results in Table 9 show, once the cyclicality of the wage component of the user cost is calibrated across different models to its empirical counterpart, the economies that are hit by the same sequence of productivity shocks generate very similar dynamics of vacancies and unemployment, regardless of the wage formation. However, there are substantial differences in the cyclicality of wages of newly hired workers and the cyclicality of average wages across the wage formations.

It is instructive to examine the case when both firms and workers are risk neutral.[19] In such a case, when the elasticity of the matching function, α, equals the worker's bargaining power, η, the individual wage formation does not have any impact on job creation. The economies with different wage formations that are hit with the same productivity shock are observationally equivalent except for the dynamics of individual wages. Hence, the allocations across four economies are identical: the job finding rate and the components of the user cost are equal state by state across the economies. All four economies share the same cyclicality of the wage component of the user cost. However, individual wages, and, hence, the cyclicality of the individual wages and even the wages of newly hired workers differ substantially across all four economies.

The magnitudes of the cyclicality of individual wages in Table 9 provide some insight into the relevance of implicit contracts for modelling individual wage dynamics. In particular, as discussed above the empirical studies report the cyclicality with respect to unemployment in the range -1 to -1.5% for wages of all workers and -3% for wages of newly hired workers. In Table 9, the model with implicit contracts and two-sided lack of commitment generates individual wage dynamics with the cyclicality comparable to their empirical counterpart.

[19] The implicit contracts do not have a micro-foundation in this context.

Note that in this table I do not calibrate the cyclicality of the individual wages (wages of newly hired workers or average wages) but only the cyclicality of the wage component of the user cost of labor.

The results demonstrate that, when wages depend on the history from the start of the job, individual wages or wages of newly hired workers are not allocational for employment. With wage smoothing, the dynamics of wages are not directly related to the dynamics of the wage component of the user cost. In this case, a weak procyclicality of hiring wages can conceal a substantial procyclicality of the wage component of the user cost.[20]

VII. Conclusion

This paper contributes to the broad and long-standing debate in macroeconomics on the allocational price of labor and its cyclicality (Barro (1977), Hall (1980), Kydland and Prescott (1980)). In the paper, I show that it is the wage component of the user cost of labor, which captures the hiring wage and the effect of the economic conditions at the time of hiring on future wages, that is weighed against the marginal revenue product of a worker at the time of hiring. I propose a procedure to estimate its cyclicality and find that it is more procyclical than average wages or even wages of newly hired workers.

The empirical contribution of the paper is to provide an estimate of the cyclicality of the wage component of the user cost of labor. The importance of the empirical result consists of showing that the wage component of the user cost of labor is much more procyclical than the wages of newly hired workers. Using the estimate, I analyze the quantitative behavior of the textbook search and matching model. The contribution of the paper to the theoretical literature consists of showing that the solution to the unemployment volatility puzzle in the model cannot be explained by a wage formation alone. This is because the model's free entry condition imposes a trade-off between the elasticity of the wage component of the user cost of labor and the elasticity of the vacancy-unemployment ratio, while in the data these elasticities are higher than the trade-off in the model allows.

The user cost of labor equals the expected present discounted value of the hiring costs and wage payments in a firm-worker match that starts in the current period less the expected

[20] The appendix contains additional results for different values of α and b. All results carry through.

present discounted value of the costs of replacing the worker in the subsequent period. In a model with search and matching, the user cost of labor can be decomposed into two components: the vacancy component and the wage component. The wage component of the user cost of labor summarizes the hiring wage as well as the current value of the expected future savings or losses associated with hiring a worker.

With free entry of firms, the marginal productivity of a worker equals the user cost of labor, the sum of the vacancy component and the wage component. This condition allows for testing the quantitative behavior of the search and matching model. The test examines the model's ability to jointly replicate the elasticity of the vacancy-unemployment ratio and the elasticity of the wage component of the user cost of labor observed in the data in response to productivity shocks. To perform the test requires an estimate of the cyclicality of the wage component of the user cost of labor. In the empirical part of the paper I construct such an estimate, which, to my knowledge, is new in the literature.

I estimate the cyclicality of the wage component of the user cost from the NLSY data. Because it is not directly observed in the data, I construct the wage component of the user cost based on the behavior of individual wages and turnover. I find that a one percentage point increase in unemployment generates more than 4.5% decrease in the wage component of the user cost. This cyclicality is three times higher than the cyclicality of individual wages and also noticeably higher than the cyclicality of wages of newly hired workers.

The cyclicality of the wage component of the user cost of labor translates into elasticity with respect to productivity of above 1.5. Using the free entry condition, I show that the search and matching model cannot simultaneously generate empirical elasticities of the vacancy-unemployment ratio and the wage component of the user cost of labor. This conclusion does not depend on a surplus division rule at the beginning of the match or individual wage dynamics within employment relationships.

To illustrate the cyclicality of the user cost of labor in the search and matching model, I consider economies with different wage settings: implicit contracts and wage bargaining period by period. The simulation results from the models show that in the presence of contracts, a weak cyclicality of individual wages can conceal a substantial cyclicality of the wage component of the user cost. The results also show that the wage component of

the user cost of labor, rather than individual wages or wages of newly hired workers, is allocational for employment. In particular, once the cyclicality of the wage component of the user cost is calibrated to be the same across the models with different wage settings, the models generate very similar volatility of the vacancy-unemployment ratio. However, the cyclicality of individual wages (and the wages of newly hired workers) is different. When the models match the estimated cyclicality of the wage component of the user cost, the generated volatility of the vacancy-unemployment ratio is less than half of its empirical counterpart.

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Figure 1. Pairs of $(\frac{b}{z}, \eta)$ that generate $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 1.5$

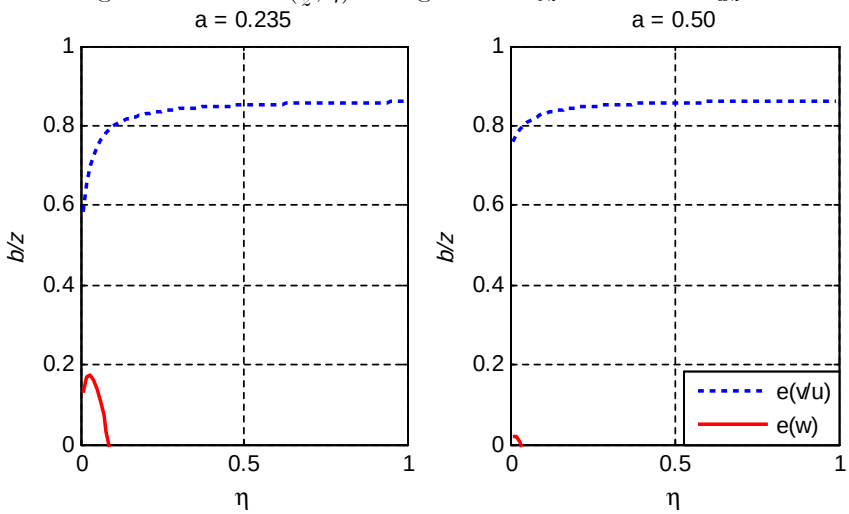

Figure 2. Pairs of $(\frac{b}{z}, \eta)$ that generate $\varepsilon_{\theta z} = 7.56$ and $\varepsilon_{wz} = 0.5$

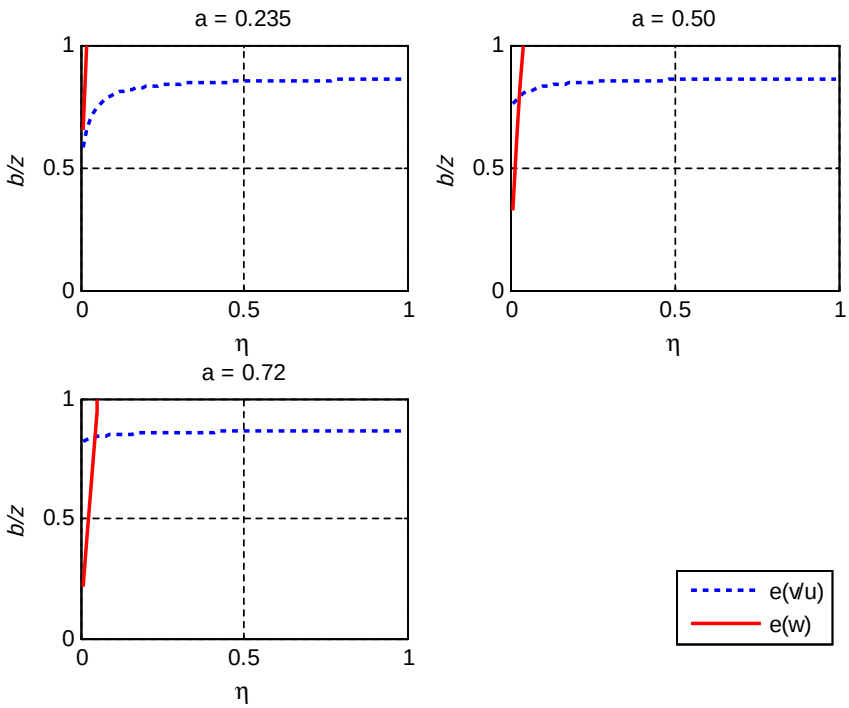

Table 1: The Effect of Past and Current Unemployment Rates on Wages

	1978 - 2004			1978 - 2002		
	Full sample	New hires	Full sample	Full sample	New hires	Full sample
	(1)	(2)	(3)	(4)	(5)	(6)
$u_{current}$	-1.78**	-3.00***	0.09	-2.02**	-2.99***	-0.33
	(0.72)	(0.78)	(0.96)	(0.93)	(0.92)	(1.26)
u_{start}	x	x	-0.79**	x	x	-0.89**
			(0.37)			(0.41)
$u_{\min}$	x	x	-3.34***	x	x	-3.02**
			(0.98)			(1.26)
Grade	7.98***	12.52***	7.86***	7.42***	11.67***	7.22***
	(1.52)	(1.65)	(1.49)	(1.55)	(1.98)	(1.46)
Experience	4.22**	8.28***	4.16**	3.71*	7.77***	3.57**
	(1.66)	(1.75)	(1.54)	(1.84)	(2.10)	(1.60)
Experience^2	-0.13***	-0.14***	-0.13***	-0.14***	-0.15***	-0.14***
	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)
Tenure	3.55***	4.02	3.14***	3.71***	7.57	3.34***
	(0.23)	(4.60)	(0.27)	(0.29)	(4.95)	(0.43)
Tenure^2	-0.11***	3.29	-0.09***	-0.13***	-0.29	-0.11***
	(0.02)	(4.30)	(0.02)	(0.02)	(4.64)	(0.03)
Trend	1.03	-3.52**	0.87	1.55	-2.95	1.48
	(1.74)	(1.70)	(1.65)	(1.86)	(2.04)	(1.68)
Union dummy	0.19***	0.17***	0.19***	0.16***	0.15***	0.16***
	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)
Industry dummies	x	x	x	yes	yes	yes
Observations	52593	19406	52593	46753	16963	46753
R-squared	0.529	0.472	0.530	0.558	0.507	0.560

Note – The data are from NLSY79, men only. The sample of new hires is restricted to observations with tenure less than 1 year. The dependent variable is the natural logarithm of real hourly wage. All regressions are estimated with fixed effects using sampling weights. Unemployment rate is the annual unemployment rate. Columns 4 – 6 include controls for 14 industries. The estimated standard errors are in parentheses, clustered by time. The coefficients and standard errors are multiplied by 100. P-values: ***p<0.01, ** p<0.05, * p<0.1.

Table 2: Cyclicality of the Wage Component of the User Cost of Labor

	Semi-Elasticity with respect to Unemployment	
	Estimate	95% Confidence Interval
Wage component of the user cost of labor, UC_t^W	-5.20***	-6.69...-3.71
	(0.76)	
Wage of new hires	-3.00***	-4.61...-1.40
	(0.78)	
Average wage	-1.78**	-3.26...-0.30
	(0.72)	

Note – The semi-elasticity is the coefficient on the unemployment rate from the regression of the (natural logarithm of the) respective series on the contemporaneous unemployment rate and other controls. The estimates for the average wage and for the wage of newly hired workers are from the regressions in Columns 1 and 2 of Table 1, respectively. The estimates for the wage component of the user cost of labor are from the regression of the (natural logarithm of the) wage component of the user cost of labor on the unemployment rate and a time trend (annual). There are 20 observations in the regression of the wage component of the user cost - from 1978 to 1997. The bootsrapped standard errors are in parentheses (1000 replications); p-values: ***p<0.01, ** p<0.05, * p<0.1. All coefficients and standard errors are multiplied by 100.

Table 3: Robustness Results on the Cyclicality of the Wage Component of the User Cost of Labor

	# of years in calculating UC_t^W		
	5 years	7 years	9 years
UC_t^W, $\delta_t = const$	-5.03	-5.24	-5.33
	(0.77)	(0.81)	(0.83)
UC_t^W, δ_t	-5.02	-5.19	-5.27
	(0.80)	(0.76)	(0.81)
UC_t^W, $\delta_{t_0,t}$	-4.79	-4.91	-4.89
	(0.16)	(0.59)	(0.70)

Note - The estimates are from the regression of the natural logarithm of the wage component of the user cost of labor on the unemployment rate and a time trend (annual). There are 18 observations in each regression - from 1978 to 1995. The bootstrapped standard errors are in parentheses (1000 replications). All coefficients and standard errors are multiplied by 100. The three rows reflect different ways of treating the separation rates in the construction of the wage component of the user cost of labor: 1) constant separation rate, $\delta_t = const$; 2) separation rate that depends on the current period, δ_t; and 3) separation rate that depends both on the current period and the period when the job started, $\delta_{t_0,t}$.

Table 4: Parameters

Parameter	Value	Comment
Discount rate, β	0.9960	
Separation rate, δ	0.0340	Shimer (2005)
Matching function elasticity ($Ku^{\alpha}v^{1-\alpha}$), α	0.5 - 0.7	Petrongolo and Pissarides (2001)
Matching function constant ($Ku^{\alpha}v^{1-\alpha}$), K	0.5	Normalization
Worker's bargaining power, η	α	Hosios (1990), Rudanko (2009)

Table 5: Cyclicality of the User Cost of Labor and Its Components

Log utility, $\alpha = 0.60$, $b = 0.70$					
		Commitment Models			Re-
		Full	1-sided lack of	2-sided lack of	bargain
			Cyclicality		
1	Individual wages (all)	-1.47	-1.47	-1.73	-9.47
2	Individual wages (new hires only)	-4.77	-4.77	-4.99	-9.41
3	Wage component of user cost	-11.15	-11.15	-11.07	-9.47
4	Vacancy component of user cost	-55.06	-55.06	-54.96	-55.14
5	User cost of labor	-11.89	-11.89	-11.82	-10.24
			θ statistics		
6	$\sigma_{\ln(\theta)}$, quarterly	0.0622	0.0622	0.0611	0.0704
			Calibrated parameter		
7	Vacancy creation cost, c	0.2675	0.2675	0.2676	0.2674

Results are from simulating the models with risk averse workers (log utility). The vacancy creation cost, c, is calibrated to match $E(\mu(\theta)) = 0.45$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component for the models are -11.15, -11.15, -11.08, and -9.4723, respectively.

Table 6: Cyclicality of the User Cost of Labor and Its Components and the Volatility of the V-U Ratio

	Linear utility				Logarithmic utility				CRRA 3 utility			
	Commitment Models			Re-	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
						Cyclicality						
Wages (all)	-0.58	-0.90	-1.65	-4.50	-0.58	-0.86	-1.56	-4.50	-0.58	-0.80	-1.36	-4.50
Wages (new hires only)	-1.94	-2.25	-2.85	-4.50	-1.94	-2.24	-2.77	-4.50	-1.94	-2.21	-2.59	-4.50
Wage component of UC		-4.50				-4.50				-4.50		
Vacancy component of UC		-57.26			-57.20	-57.29	-57.25	-57.02	-57.06	-57.32	-57.25	-56.53
User cost of labor		-4.81			-4.82	-4.83	-4.83	-4.85	-4.87	-4.88	-4.89	-4.99
						θ statistics						
$\sigma_{\ln(\theta)}$, quarterly		0.1480			0.1475	0.1473	0.1472	0.1468	0.1463	0.1458	0.1453	0.1431
						Calibrated parameters						
Consumption of un-ed, b		0.8702			0.8710	0.8698	0.8681	0.8616	0.8715	0.8683	0.8644	0.8381
Vacancy creation cost, c		0.0984			0.1041	0.1051	0.1069	0.1123	0.1176	0.1211	0.1262	0.1565

Note - Results from simulating the models with risk averse workers. c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and the cyclicality of the wage component of the user cost. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component for the models are equal to the ones reported in the table (to the decimal points reported).

Table 7: Cyclicality of the User Cost and Its Components given the Volatility of the V-U Ratio

	Linear utility				Logarithmic utility				CRRA 3 utility			
	Commitment Models			Re-	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
						Cyclicality						
Wages (all)	-0.21	-0.51	-1.09	-1.62	-0.21	-0.51	-1.06	-1.62	-0.21	-0.50	-1.01	-1.62
Wages (new hires only)	-0.70	-1.05	-1.39	-1.62	-0.70	-1.04	-1.38	-1.62	-0.70	-1.03	-1.35	-1.62
Wage component of UC		-1.62			-1.62	-1.62	-1.62	-1.62	-1.62	-1.62	-1.62	-1.62
Vacancy component of UC		-76.89			-76.76	-76.80	-76.57	-76.24	-76.47	-76.60	-75.98	-74.98
User cost of labor		-1.75			-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.76
						Calibration targets						
$\sigma_{\ln(\theta)}$, quarterly						0.3820						
$E(\mu(\theta))$						0.4500						
						Calibrated parameters						
Consumption of un-ed, b		0.9434			0.9438	0.9432	0.9428	0.9424	0.9444	0.9427	0.9417	0.9402
Vacancy creation cost, c		0.0415			0.0424	0.0427	0.0432	0.0435	0.0443	0.0454	0.0471	0.0481

Note - Results from simulating the models with workers with different utility functions as indicated. c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and $\sigma_{\ln(\theta)}= 0.0382$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component for the four models and different utility functions are equal with respect to the precision in the tables, except for the log utility, where the statistics are -1.63 for all four models.

APPENDIX

A Derivations and Proofs in Section III.

(a) Value Functions.

(b) Proof of Proposition 1.

(c) Proof of $\Pr(UC_t^V > 0)$.

(d) Proof of $\Pr(UC_t^W > 0)$.

(e) Derivation of $\varepsilon_{UC_t^V, z_t} = \alpha \varepsilon_{\theta,z} x_t$.

B Empirical Section IV.

(a) The Wage Component of the User Cost of Labor with Time-Varying Separation Rates.

(b) Estimation with Time-Varying Separation Rates.

(c) History Dependence in Wages.

C Expressions for $\varepsilon_{\theta z}$ and ε_{wz} in Section $V.A$.

D Quantitative Section VI.

(a) Models with Implicit Contracts.

(b) Model with Bargaining Period by Period.

E Additional Quantitative Results.

A Derivations and Proofs in Section *III*

A. Value Functions

The values in the economy that is described in Section 3, can be summarized by the following functions. Let Ω_t denote a vector of state variables at time t, including aggregate productivity z_t, and let $\Omega^t \equiv \{\Omega_\tau\}_{\tau=0}^t$. To save on notation, I suppress the dependence of the value functions on corresponding histories.

The option value of an inactive firm is assumed to be equal to 0. The value function of a firm with a worker at time t, given that a firm-worker match started at time t_0, is

$$J_{t_0,t} = z_t - w_{t_0,t} + \beta(1-\delta)E_t J_{t_0,t+1}. \tag{A1}$$

The value function of an opened vacancy at t is

$$V_t = -c + q_t J_{t,t} + \beta(1-q_t)E_t V_{t+1}. \tag{A2}$$

The value function of an employed worker at time t, given that a firm-worker match started at time t_0, $W_{t_0,t}$, is

$$W_{t_0,t} = u(w_{t_0,t}) + \beta E_t\left[(1-\delta)W_{t_0,t+1} + \delta U_{t+1}\right]. \tag{A3}$$

The value function of an unemployed worker at time t, U_t, is

$$U_t = u(b) + \beta E_t\left[\mu_{t+1}W_{t_{+1},t+1} + (1-\mu_{t+1})U_{t+1}\right]. \tag{A4}$$

In this set-up, wage may depend on the history of the labor market conditions from the start of the job. Thus, the wage is indexed by the contemporaneous period and the period a worker is hired.

B. Proof of Proposition 1

Proof.

Consider a value of a firm with a worker at time t given that the productive match starts at time t:

$$J_{t,t} = z_t - w_{t,t} + \beta(1-\delta)E_t J_{t,t+1} = z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t(z_\tau - w_{t,\tau}).$$

Then, the expected difference between the value of a firm at time t from the match that starts at time t and the expected present discounted value from the match at time $t+1$ that starts at $t+1$, is

$$\begin{aligned} J_{t,t} - \beta(1-\delta)E_t J_{t+1,t+1} &= z_t - \left[w_{t,t} + \sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}E_t(w_{t,\tau} - w_{t+1,\tau})\right] = \\ & z_t - UC_t^W. \end{aligned}$$

Substituting the free entry condition, $J_{t,t} = \frac{c}{q(\theta_t)}$, into the left-hand side of the above equation yields

$$\frac{c}{q(\theta_t)} - \beta(1-\delta)E_t\frac{c}{q(\theta_{t+1})} = UC_t^W.$$

Using $UC_t^V \equiv \frac{c}{q(\theta_t)} - \beta(1-\delta)E_t\frac{c}{q(\theta_{t+1})}$, the following equality obtains

$$z_t = UC_t^V + UC_t^W.$$

■

C. $\Pr(UC_t^V > 0)$

I show that $\Pr(UC_t^V > 0) > 0.99$, given the empirical volatility and autocorrelation of z_t.

$UC_t^V > 0$ can be rewritten as

$$\left(\frac{c}{q(\theta(z_t))} - \beta(1-\delta)E_t\frac{c}{q(\theta(z_{t+1}))}\right) > 0$$

or

$$\frac{\theta(z_t)^\alpha}{E_t(\theta(z_{t+1})^\alpha)} > \beta(1-\delta). \tag{A5}$$

Equation (A5) imposes restrictions on the volatility of the stochastic process of $\theta(z_{t+1})$ conditional on $\theta(z_t)$. One can check whether these restrictions hold in the data.

Since $0 < \alpha < 1$, by Jensen's inequality:

$$E_t\left(\theta(z_{t+1})^\alpha\right) \leq \left(E_t\theta(z_{t+1})\right)^\alpha.$$

It implies

$$\frac{\theta(z_t)^\alpha}{E_t(\theta(z_{t+1})^\alpha)} \geq \frac{\theta(z_t)^\alpha}{\left(E_t\theta(z_{t+1})\right)^\alpha}.$$

Thus, to show (A5), it is suffice to show

$$\frac{\theta(z_t)}{\left(E_t\theta(z_{t+1})\right)} > \left(\beta(1-\delta)\right)^{1/\alpha}. \tag{A6}$$

Log-linearization of $\theta(z_{t+1})$ around $\theta(z_t)$ yields

$$\theta(z_{t+1}) \simeq \theta(z_t)(1 + \varepsilon_{\theta(z_t),z_t}\ln\frac{z_{t+1}}{z_t}).$$

Then, (A6) can be rewritten as

$$\frac{1}{(1 + \varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t})} > \left(\beta(1-\delta)\right)^{1/\alpha}$$

or, noting that $1 + \varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t} > 0$ since $\theta(z_t), \theta(z_{t+1}) > 0$:

$$1 - \left(\beta(1-\delta)\right)^{1/\alpha} > \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}E_t\ln\frac{z_{t+1}}{z_t}. \tag{A7}$$

The stochastic process for z_{t+1} can be specified as

$$\ln z_{t+1} = (1-\rho)\ln\overline{z} + \rho\ln z_t + \iota_{t+1}, \tag{A8}$$

where $\iota_{t+1} \sim N(0, \sigma_\iota^2)$.

Then, inequality (A7) can be rewritten as

$$1 - \left(\beta(1-\delta)\right)^{1/\alpha} > \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}((1-\rho)\ln\overline{z} + \rho\ln z_t) - \left(\beta(1-\delta)\right)^{1/\alpha}\varepsilon_{\theta(z_t),z_t}\ln z_t,$$

which, given $\varepsilon_{\theta(z_t),z_t} > 0$, after simplification yields as

$$\ln\frac{z_t}{\overline{z}} > \frac{\left(\beta(1-\delta)\right)^{1/\alpha} - 1}{\left(\beta(1-\delta)\right)^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}}.$$

Given the stochastic process for z_t,(A8), quarterly values $\beta = 1/(1+0.012)$ and $\delta = 0.01$ (Shimer (2005), Hornstein, Krusell and Violante (2005)), $\rho_z = 0.878$ and $\sigma_z = 0.02$ for quarterly log deviations of z

from an HP trend (Shimer (2005)), and a high value of $\alpha = 0.72$ found in the literature, it yields

(A9)

$$\Pr\left(\ln\frac{z_t}{\bar{z}} > \frac{(\beta(1-\delta))^{1/\alpha}-1}{(\beta(1-\delta))^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}}\right) = \Pr\left(\frac{\ln\frac{z_t}{\bar{z}}}{\sigma_z} > \frac{0.89^{1/\alpha}-1}{0.89^{1/\alpha}(1-\rho)\varepsilon_{\theta(z_t),z_t}\sigma_z}\right) = 1-\Phi\left(\frac{-72.00}{\varepsilon_{\theta(z_t),z_t}}\right),$$

where $\Phi(.)$ is a c.d.f. of the standard normal distribution.

For $\varepsilon_{\theta,z} = 7.56$, the right hand side of (A9) is $1-\Phi(-9.52) >> 0.99$. When the value of $\varepsilon_{\theta(z_t),z_t}$ more than doubles, say, $\varepsilon_{\theta(z_t),z_t} = 20$, then $1-\Phi\left(\frac{-72.00}{\varepsilon_{\theta(z_t),z_t}}\right) = 1-\Phi(-3.6) > 0.99$. Thus, given $\rho_z = 0.878$, $\sigma_z = 0.02$, and $\varepsilon_{\theta,z} = 7.56$, $\Pr(UC_t^V > 0) > 0.99$.■

D. $\Pr(UC_t^W > 0)$

I show that $\Pr(UC_t^W > 0) > 0.99$, given the empirical volatility and autocorrelation of z_t.

$UC_t^W > 0$ can be rewritten:

$$PDV^W(z_t) - \beta(1-\delta)E_t PDV^W(z_{t+1}) > 0.$$

or

$$\frac{PDV^W(z_t)}{E_t PDV^W(z_{t+1})} > \beta(1-\delta). \tag{A10}$$

Log-linearization of $PDV(z_{t+1})$ around z_t yields

$$PDV^W(z_{t+1}) \simeq PDV^W(z_t)(1+\varepsilon_{PDV^W(z_t),z_t}\ln\frac{z_{t+1}}{z_t}),$$

where $\varepsilon_{PDV^W(z_t),z_t}$ is the elasticity of $PDV^W(z_t)$ at z_t. Note that $1-\varepsilon_{PDV^W(z_t),z_t}(1-\rho)\ln\frac{z_t}{\bar{z}} > 0$ because $PDV^W(z_{t+1}) > 0$, which holds true if all wages are non-negative and at least one is positive.

Equation (A10) can be rewritten:

$$\frac{1}{1-\varepsilon_{PDV^W(z_t),z_t}E_t\ln\frac{z_t}{\bar{z}}} > \beta(1-\delta). \tag{A11}$$

Using the stochastic process for z_t, (A8), inequality (A11) can be rewritten as follows:

$$\ln\frac{z_t}{\bar{z}} > \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)\varepsilon_{PDV^W(z_t),z_t}},$$

if $\varepsilon_{PDV^w(z_t),z_t} > 0$, and

$$\ln\frac{z_t}{\bar{z}} < \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)\varepsilon_{PDV^W(z_t),z_t}},$$

if $\varepsilon_{PDV^w(z_t),z_t} < 0$.

Given the quarterly parameter values discussed in the appendix above and the stochastic process for z_t, these two cases can be combined as follows:

$$\Pr\left(\ln\frac{z_t}{\bar{z}} > \frac{\beta(1-\delta)-1}{\beta(1-\delta)(1-\rho)|\varepsilon_{PDV^w(z_t),z_t}|}\right) = 1-\Phi\left(\frac{-50.65}{|\varepsilon_{PDV^w(z_t),z_t}|}\right). \tag{A12}$$

To obtain a bound on $\varepsilon_{PDV^w(z_t),z_t}$, consider free entry condition:

$$\frac{c}{\theta^{-\alpha}} = PDV^Z(z_t) - PDV^W(z_t).$$

Differentiating and rearranging yields

$$\alpha\varepsilon_{\theta(z_t),z_t}J(z_t) = \varepsilon_{PDV^Z(z_t),z_t}PDV^Z(z_t) - \varepsilon_{PDV^w(z_t),z_t}PDV^W(z_t),$$

where $J(z_t) \equiv PDV^Z(z_t) - PDV^W(z_t) \geq 0$, given free entry, and $\varepsilon_{PDV^Z(z_t),z_t} > 0$ (see below). Rearranging, it follows:

$$\varepsilon_{PDV^w(z_t),z_t} = \varepsilon_{PDV^Z(z_t),z_t}\frac{PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t)}{PDV^Z(z_t) - J(z_t)}. \tag{A13}$$

It can be shown that the following holds:

$$\left|\frac{PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t)}{PDV^Z(z_t) - J(z_t)}\right| < 1. \tag{A14}$$

To see this, note, that if $\varepsilon_{PDV^w(z_t),z_t} > 0$, then $PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t) > 0$ because $PDV^Z(z_t) - J(z_t) = PDV^W(z_t) > 0$. Then, equation (A14) can be rewritten: $PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t) < PDV^Z(z_t) - J(z_t)$, which holds when $\frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} > 1$.

Alternatively, if $\varepsilon_{PDV^w(z_t),z_t} < 0$, then $\frac{PDV^Z(z_t)}{J(z_t)} < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$. Then equation (A14) can be rewritten: $-(PDV^Z(z_t) - \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}J(z_t)) < PDV^Z(z_t) - J(z_t)$, which can be rewritten as

$$\frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} + 1 < 2\frac{PDV^Z(z_t)}{J(z_t)}. \tag{A15}$$

Since $\frac{PDV^Z(z_t)}{J(z_t)} < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$, equation (A15) holds if $1 < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$.

Thus, $\varepsilon_{PDV^W(z_t),z_t} = \varepsilon_{PDV^Z(z_t),z_t}x_t$,where $|x_t| < 1$ if $1 < \frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}}$.

Given the stochastic process for z_t, $PDV^Z(z_t)$ can be written:

$$PDV^Z(z_t) = z_t + \sum_{\tau=t+1}(\beta(1-\delta))^{\tau-t}\exp\left((1-\rho)\sum_{k=0}^{\tau-t}\rho^k\ln\overline{z} + \rho^{\tau-t}\ln z_t + \frac{\sigma_i^2}{2}\sum_{k=0}^{\tau-t-1}\rho^k\right).$$

Note the following:

$$\varepsilon_{PDV^Z(z_t),z_t} = \frac{dPDV^Z(z_t), z_t}{dz_t}\frac{z_t}{PDV^Z(z_t), z_t} = \frac{z_t + \sum_{\tau=t+1}(\rho\beta(1-\delta))^{\tau-t}E_t z_\tau}{z_t + \sum_{\tau=t+1}(\beta(1-\delta))^{\tau-t}E_t z_\tau},$$

which delivers $0 < \varepsilon_{PDV^Z(z_t),z_t} < 1$ since $0 < \rho < 1$ and $z_t + \sum_{\tau=t+1}(\beta(1-\delta))^{\tau-t}E_t z_\tau \equiv PDV^Z(z_t) > 0$.

Note that $\alpha\varepsilon_{\theta(z_t),z_t} > 1$ given the values for α and $\varepsilon_{\theta(z_t),z_t}$ as described in Section 3.C. Thus, from $\alpha\varepsilon_{\theta(z_t),z_t} > 1$ and $0 < \varepsilon_{PDV^Z(z_t),z_t} < 1$, it follows that $\frac{\alpha\varepsilon_{\theta(z_t),z_t}}{\varepsilon_{PDV^Z(z_t),z_t}} > 1$. Hence, $|\varepsilon_{PDV^w(z_t),z_t}| = |\varepsilon_{PDV^w(z_t),z_t}x_t| < 1$.

Using $|\varepsilon_{PDV^w(z_t),z_t}| < 1$ in expression (A12) delivers $\Pr\left(UC^W > 0\right) > 0.99$.■

E. Derivation of $\varepsilon_{UC_t^V,z_t} = \alpha\varepsilon_{\theta,z}x_t$

Using (A8), the probability density function for z_{t+1} given z_t is:

$$f(z_{t+1}|z_t) = \frac{1}{z_{t+1}\sigma_{\ln z_{t+1}}\sqrt{2\pi}}\exp(-\frac{\ln(z_{t+1}) - ((1-\rho)\ln\overline{z} + \rho\ln z_t)}{2\sigma^2_{\ln z_{t+1}}}).$$

The elasticity of the vacancy component of the user cost of labor with respect to productivity is:

$$\varepsilon_{UC_t^V,z_t} = \frac{d\left(\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)\int \frac{c}{K\theta_{t+1}^{-\alpha}} f(z_{t+1}|z_t)dz_{t+1}\right)}{dz_t} \frac{z_t}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} =$$

$$\frac{\alpha\varepsilon_{\theta,z}\frac{c}{K\theta_t^{-\alpha}} - \rho\beta(1-\delta)\int \frac{c}{K\theta_{t+1}^{-\alpha}} f(z_{t+1}|z_t)dz_{t+1}}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} =$$

$$\frac{\alpha\varepsilon_{\theta,z}\left(\frac{c}{K\theta_t^{-\alpha}} - \frac{\rho}{\alpha\varepsilon_{\theta,z}}\beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}\right)}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}} = \alpha\varepsilon_{\theta,z}x_t,$$

where $x_t = \frac{\frac{c}{K\theta_t^{-\alpha}} - \frac{\rho}{\alpha\varepsilon_{\theta,z}}\beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}}{\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}}}$. Since $\frac{c}{K\theta_t^{-\alpha}} - \beta(1-\delta)E_t\frac{c}{K\theta_{t+1}^{-\alpha}} > 0$ (see proof above) and $\rho < \alpha\varepsilon_{\theta,z}$ (for $\rho < 1$, $\varepsilon_{\theta,z} = 7.56$ and $\alpha \in [0.235; 0.72]$), one obtains $x_t > 1$.■

B Empirical Section *IV*

A. The Wage Component of the User Cost of Labor with Time-Varying Separation Rates

To define the wage component of the user cost of labor with time-varying separation rates, consider the following thought experiment. A firm hires a worker in period t. Assume that a worker is always available for hire, and the only costs associated with hiring a worker are wage payments. A firm pays according to the wage schedule agreed upon when the worker is hired. Every period, a nonzero probability exists that a worker will exogenously separate from the position. Separation probability, $\delta_{t,\tau}$, may depend on the history of labor market conditions a worker experiences from the time of hiring. After separation, a firm hires a new worker to replace the separated one. A new firm-worker relationship is likely to start with a new wage agreement. In this thought experiment, if a firm hires a worker in some period t, it maintains the number of workers at 1 from that period on by re-hiring in case the worker hired in τ separates. Thus, hiring a worker in t can be thought of as creating a position in period t that will be filled with probability 1 onwards. Then, the expected present discounted value of wages paid to create a position in t onwards is given by

$$\begin{aligned} PDV_t' = w_{t,t} + E_t[\beta((1-\delta_{t,t})w_{t,t+1} + \delta_{t,t}w_{t+1,t+1}) + \\ \beta^2((1-\delta_{t,t})(1-\delta_{t,t+1})w_{t,t+2} + \delta_{t,t}(1-\delta_{t+1,t+1})w_{t+1,t+2} + \\ ((1-\delta_{t,t})\delta_{t,t+1} + \delta_{t,t}(1-\delta_{t+1,t+1}))w_{t+2,t+2} + ...] = \\ w_{t,t} + E_t[\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\sum_{k=t}^{\tau-1}(\Lambda_{t,k,\tau-1}w_{k+1,\tau})], \end{aligned} \tag{B1}$$

where w_{t_1,t_2} is a wage paid in t_2 to a worker hired in t_1; δ_{t_1,t_2} is a separation rate at the end of t_2 for a worker hired in t_1, conditional that there is no separation between $t_1$and t_2; and $\Lambda_{t,k,\tau}$ is a probability that a separation takes place at the end of period k at the position that a firm opened in t and a new worker is hired in $k+1$ and continues working on that position in τ; and $E_t = E(.|I_t)$ where I_t is the firm's information set at time t. Both wage payments and separation rates are allowed to depend on the history of the labor market conditions from the period a worker is hired.

Equation (B1) states that a worker hired in period t is paid a wage $w_{t,t}$. With probability $1-\delta_{t,t}$ the firm-worker relationship survives until the period $t+1$ and the worker is paid wage $w_{t,t+1}$. With probability $\delta_{t,t}$ the relationship is terminated and the firm hires a new worker at a wage $w_{t+1,t+1}$ to fill the position. By analogy, in period $t+2$ a firm retains a worker hired in period t with probability $(1-\delta_{t,t})(1-\delta_{t,t+1})$ and pays a wage $w_{t,t+2}$. With probability $(1-\delta_{t,t})\delta_{t,t+1}$ that worker is separated and the firm replaces the worker with another at wage $w_{t+2,t+2}$. Also, in period $t+2$ a worker hired in $t+1$ is retained with probability $\delta_{t,t}(1-\delta_{t+1,t+1})$ and receives wage $w_{t+1,t+2}$. In case of separation, with probability $\delta_{t,t}\delta_{t,t+1}$ this worker is

replaced with a new one at wage $w_{t+2,t+2}$.

The wage component of the user cost of labor in period t is the difference between the expected present discounted value of wages paid at the position opened in period t and $t+1$:

$$UC_t^W = PDV_t' - \beta E_t PDV_{t+1}',$$

Substituting from (B1), I obtain the following expression for the wage component of the user cost of labor:

$$UC_t^W = w_{t,t} + E_t[\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(w_{t,\tau}\prod_{k=t}^{\tau-1}(1-\delta_{t,k}) - w_{t+1,\tau}(1-\delta_{t,t})\prod_{k=t+1}^{\tau-1}(1-\delta_{t+1,k}))+$$

$$\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(\sum_{k=t}^{\tau-1}(\Lambda_{t,k,\tau-1} - (1-\lambda_{t,t})\Lambda_{t+1,k,\tau-1})w_{k,\tau})]. \tag{B2}$$

If separation depends only on the contemporaneous labor market conditions, $\delta_{t_0,t} = \delta_t$ for all t and t_0, then (B2) simplifies to the following expression:

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}(\prod_{k=t}^{\tau-1}(1-\delta_k))(w_{t,\tau} - w_{t+1,\tau}). \tag{B3}$$

If the separation rate is constant, $\delta_{t_0,t} = \delta$, equation (B2) simplifies to

$$UC_t^W = w_{t,t} + E_t \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t}(w_{t,\tau} - w_{t+1,\tau}).$$

B. Estimation with Time-Varying Separation Rates

To obtain the series of separation rates I proceed in two steps. First, I detrend the monthly separation rates. To do that, I estimate the linear probability model with a dependant variable taking value 1 if a worker does not work for the same job in the next month and 0 otherwise. The explanatory variables are the quartic in the monthly trend. I subtract the value of a quartic in the trend multiplied by the estimated coefficients from the dependent variable and add the value a quartic of a trend calculated at the mean multiplied by the estimated coefficients. In the second step, I estimate a linear probability model of the detrended monthly separation rates with two sets of time dummies as explanatory variables: one set of time dummies corresponds to the year the job starts and another set of dummies corresponds to the contemporaneous year. Then, I use monthly fitted projections to obtain annual separation rates, $\widehat{\delta^A_{t_1,t_2}}$. Annual separation rates are calculated as follows: for all t_1 and t_2 : $t_1, t_2 = \{1978, 2004\}, t_1 < t_2$: $\widehat{\delta^A_{t_1,t_2}} = 1 - \frac{\sum_{\tau^{t1}=1}^{12}\left(\prod_{k^{t2}=1}^{12}(1-\widehat{\delta_{\tau^{t1},k^{t2}}})\right)}{12}$, where $\widehat{\delta_{\tau^{t1},k^{t2}}}$ is a fitted monthly separation rate in a calendar month k of year t_2 at the job that started in a calendar month τ of year t_1. In a similar manner I calculate annual separation rates $\widehat{\delta^A_{t_1,t_2}}$ for $t_1 = t_2$, annualizing monthly separations.

As a robustness check, I also construct the wage component of the user cost of labor as in equation (B3), where the separation rate depends on the contemporaneous period only. In this case, in the empirical model of the separation rates I use only one set of time dummies – the contemporaneous period dummies. Then I use monthly fitted projections to obtain annual separation rates, $\widehat{\delta^A_t}$. For all $t : t = \{1978, 2004\}$: $\widehat{\delta^A_t} = 1 - \prod_{\tau=1}^{12}(1-\widehat{\delta_{\tau^t}})$,where $\widehat{\delta_{\tau^t}}$ is a fitted monthly separation rate in a calendar month τ of year t.

C Expressions for $\varepsilon_{\theta z}$ and ε_{wz} in Section $V.A$

In a steady state the elasticity of the vacancy-unemployment ratio with respect to productivity is

$$\varepsilon_{\theta z} = \frac{1}{1-b/z}\frac{1-\beta(1-\delta-\eta\mu)}{\alpha-\beta(\alpha-\delta\alpha-\eta\mu)}$$

and the elasticity of wages is

$$\begin{aligned}\varepsilon_{wz} &= \frac{\eta}{\eta(1-\beta(1-\delta-\mu))+(1-\eta)\frac{b}{z}(1-\beta(1-\delta))}\cdot \\ &\quad ((1-\beta(1-\delta-\mu))+\frac{(1-\eta)(1-\alpha)\beta\mu(1-\beta(1-\delta))}{\alpha-\beta(\alpha-\delta\alpha-\eta\mu)}),\end{aligned}$$

where η is a worker's bargaining power, b is the unemployment benefit, and μ is a steady state value of job-finding rate.

D Quantitative Section

A. Models with Implicit Contracts

The value an employed worker receives in period t from a contract that started in period t_0, $W_\sigma(t_0, z^t)$, is

$$W_\sigma(t_0,z^t) = u(w_\sigma(t_0,z^t)) + E_t\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}(1-\delta)^{\tau-(t+1)}\left[(1-\delta)u(w_\sigma(t_0,\{z^{\tau-1},z_\tau\}))+\delta U(z_\tau)\right].$$

The value of a newly unemployed worker or a worker who did not find a match in the current period is a sum of the current utility, obtained from consuming an unemployment benefit, b, and the expected discounted value from searching:

$$U(z_t) = u(b) + \beta E_t\left[\mu(\theta_\sigma(\{z^{t+1},z_t\}))W_\sigma(t+1,\{z^t,z_{t+1}\}) + (1-\mu(\theta_\sigma(\{z^{t+1},z_t\})))U(z_{t+1})\right].$$

The value a firm obtains in period t given the aggregate state z_t from a contract σ that started in period t_0 is

$$J_\sigma(t_0,z^t) = z_t - w_\sigma(t_0,z^t) + E_t\sum_{\tau=t+1}^{\infty}(\beta(1-\delta))^{\tau-t}(z_\tau - w_\sigma(t_0,\{z^{\tau-1},z_\tau\})).$$

Equilibrium contracts are limited to efficient optimal contracts. A contract is efficient if there exists no other contract that offers each party at least as much expected utility and one party strictly more. A contract is optimal if it maximizes the total welfare given the initial promise of a value to one of the parties. An efficient contract cannot be Pareto dominated after any history. Hence, after any history it can be rewritten as a maximization problem. The Pareto frontier is traced by varying the value promised by the contract to the worker and maximizing the value of the firm given the worker's promised value. As in Thomas and Worrall (1988), the history of the productivity realizations from the start of the match can be summarized by the worker's promised value. Given the assumption that z_t follows a first order Markov process, it is sufficient to keep track of the current value of z to determine the expectations. In the presentation that follows the time subscripts are suppressed: z denotes the current value of productivity and z' denotes the value next period.

Let W be the value promised to a worker under the contract. Let $U(z)$ be the value of an unemployed worker given aggregate state z and let $f(z,W,U(z))$ denote a value of a firm from a contract on a Pareto frontier given z, W, $U(z)$, and the evolution of $U(z)$. Then $f(z,W,U(z))$ solves the following dynamic programming optimization problem for all $z \in Z$:

$$\text{(D1)} \qquad f(z,W,U(z)) = \max_{w,\{W(z')\}_{z'\in Z}} z - w + \beta E_z(1-\delta)f(z',W(z'),U(z'))$$

s. t.

$$W = \varphi(w) + \beta E_z \left[(1-\delta)W(z') + \delta U(z')\right] \tag{D2}$$

$$W(z') \geq U(z') \ \forall \ z' \in Z \tag{D3}$$

$$f(z', W(z'), U(z')) \geq 0 \ \forall \ z' \in Z. \tag{D4}$$

An efficient contract maximizes the value of a firm, f, given the aggregate state, z, the promised value for the worker, W, and the worker's outside option, $U(z)$. The first constraint is a promise-keeping constraint that specifies that a worker gets exactly value W from the contract that pays wage w and promises values $W(z')$ for all states $z' \in Z$ where there is no exogenous separation. The second and third constraints are self-enforcing constraints for the worker and the firm, respectively. By omitting self-enforcing constraints, contracts with different degrees of commitment are obtained: 1) full commitment (by omitting (D3) and (D4)); 2) lack of commitment from the worker's side and full commitment from the firm's side (by omitting (D4)); and 3) two-sided lack of commitment (when both (D3) and (D4) are present).

I study equilibria of this economy which consist of a contract $\sigma(z)$, value functions for the firm from a contract $\sigma(z)$, f_σ, values promised to the worker at the time of hiring, $W_{h,\sigma}(z)$, values of an unemployed worker, $U(z)$, and a market tightness, $\theta_\sigma(z)$, associated with the contract $\sigma(z)$ for each $z \in Z$, such that

1. (Optimization) Given a vector U, the list of functions $f(z, W_{h,\sigma}(z), U(z))$ solves the dynamic programming problem (D1)-(D4).

2. (Free entry) Firms enter a labor market and post vacancies with the associated contract σ until the value of posting a vacancy is driven to 0:

$$q(\theta_\sigma(z)) f(z, W_{h,\sigma}(z), U(z)) = c. \tag{D5}$$

3. The value of an unemployed worker evolves according to the following rule:

$$U(z) = u(b) + \beta E_z \left[\mu(\theta_\sigma(z'))W_{h,\sigma}(z') + (1-\mu(\theta_\sigma(z')))U(z')\right]. \tag{D6}$$

In addition, I impose the following equilibrium refinement:

4. (Pareto efficiency) There does not exist an efficient self-enforcing contract $\sigma'(z)$ and an associated labor market with tightness $\theta_{\sigma'}(z)$ such that the net surpluses from search for a worker, $\mu(\theta_{\sigma'}(z))(W_{h,\sigma'}(z) - U(z))$,and for a firm, $-c + q(\theta_\sigma(z)) f(z, W_{h,\sigma'}(z), U(z))$, are at least as much as under $\sigma(z)$ and $\theta_\sigma(z)$ and for one party it is strictly more.

This refinement of the set of equilibrium contracts follows Rudanko (2009), who motivates it from the competitive search, in which competitive market-makers specify the set of the efficient self-enforcing contracts that can be posted in the economy. Each contract is offered in a separate market with an associated labor-market tightness, and in equilibrium each market must offer the same surplus from search for firms and the same surplus for workers. Because of competition between market-makers, only markets in which the offered contract is on the Pareto frontier will be opened in equilibrium. Condition 2 combined with Condition 3 determines equilibrium values of the promised value for the worker at the time of hiring, $W_{h,\sigma}(z)$, and an equilibrium value of the market tightness in the market with σ, $\theta_\sigma(z)$.

In this economy unemployment evolves according to the following law, given $u(z_{t_0})$:

$$u(\{z^t, z_{t+1}\}) = u(z^t) + (1-u(z^t))\delta - \mu(\theta(\{z^t, z_{t+1}\}))u(z^t).$$

The pool of unemployed in the current period consists of unemployed workers from the previous period and those who became unemployed because of the exogenous separations in the previous period, net of the unemployed workers who find jobs in the current period.

Thomas and Worrall (1988) and Rudanko (2009) prove that the optimization problem described above is a concave problem, so the first-order conditions are necessary and sufficient. The first-order conditions for

an arbitrary z read:

$$-\lambda_z = -\frac{1}{\varphi'(w)}. \tag{D7}$$

$$-\lambda_z = (1+\zeta(z'))f_V(z', W(z'), U(z')) + \phi(z') \ \forall z' \in Z, \tag{D8}$$

where λ_z is the Langrange multiplier on the promise-keeping constraint; $\beta\pi(z'|z)\phi(z')$, are Langrange multipliers on the self-enforcing constraints for a worker, and $\beta\pi(z'|z)\zeta(z')$ are Langrange multipliers on self-enforcing constraints for a firm $\forall z' \in Z$. Complimentary slackness conditions: $\lambda_z \geq 0$, $\zeta(z'), \phi(z') \geq 0 \ \forall z'$, and (D3) and (D4). The envelope condition:

$$f_V(z, W(z), U(z)) = -\lambda_z. \tag{D9}$$

Combining the envelope condition, (D9), with the first order conditions gives the following condition, which links current and next period wage:

$$\frac{1}{\varphi'(w(z, W, U(z)))} = (1+\zeta(z'))\frac{1}{\varphi'(w(z', W(z'), U(z')))} + \phi(z') \ \forall z' \in Z$$

Because of free entry and Pareto optimality, $W_h(z)$ and $\theta(z)$ solve the following maximization problem given $V_u(z)$:[21]

$$\max_{\{\theta(z)\},\{V_h(z)\}} \left\{ \ \mu(\theta(z))(W_h(z) - U(z)) \ \right\} \tag{D10}$$
$$\text{s.t. } q(\theta(z)f(z, W_h(z), U(z)) = c$$

Combining the first order condition for Pareto optimality problem, (D10), the free entry condition, (D5), the envelope condition, (D9), the first order condition for wages, (D7), and the law of motion for the value of unemployed workers, the following system of equations characterizes the equilibrium objects f, $U(z)$, $W_h(z)$ and $\theta(z)$ $\forall z \in Z$, given the optimal contract.

$$\frac{\alpha}{1-\alpha} f(z, W_h(z), U(z)) = \frac{W_h(z) - U(z)}{\varphi'(w(z, W_h(z), U(z)))}. \tag{D11}$$

$$\theta(z) = \left(\frac{c}{f(z, W_h(z), U(z))}\frac{1}{K}\right)^{-\frac{1}{\alpha}}.$$

$$U(z) = u(b) + \beta E_z\left[\mu(\theta_\sigma(z'))W_h(z') + (1-\mu(\theta_\sigma(z')))U(z')\right].$$

B. Model with Bargaining Period by Period

An equilibrium in the economy with bargaining period by period consists of the set of the value functions for a firm, $J(z)$, (A1), and $V(z)$, (A2), and a worker, $W(z)$, (A3) and $U(z)$, (A4), and a market tightness $\theta(z)$, such that

1. (Free entry) The value of a vacancy is 0:

$$q(\theta(z))J(z) = c.$$

2. (Surplus division) Each period during an employment relationship, the firm and the worker bargaining over the match surplus. At the time of bargaining the outside option value for a worker is the value of unemployment, while the outside option for a firm is 0 (the value of an inactive firm). A matched worker-firm

[21] Rudanko (2009) proves that given fairly mild conditions there is a unique Pareto-efficient contract offered in equilibrium.

pair divides the total surplus from the match by solving the following maximization problem:

$$\max_{V_e(z)-V_u(z),J_F(z)} (W(z)-U(z))^{\eta} J(z)^{1-\eta}$$

$$\text{s.t. } \frac{W(z)-U(z)}{u'(w(z))} + J(z) = S(z) \tag{D12}$$

where η is a bargaining power of the worker, $u'(w)$ is the marginal utility of income, and $S(z)$ is a total surplus.

3. The value of an unemployed worker evolves according to the following rule:

$$U(z) = u(b) + \beta E_z \left[\mu(\theta(z'))W(z') + (1-\mu(\theta(z')))W(z')\right].$$

C. Additional Quantitative Results

Additional quantitative results are presented in Tables 8-12.

Table 8: Cyclicality of the User Cost of Labor and Its Components, Robustness

CRRA $\gamma = 3$ utility, $\alpha = 0.60$, $b = 0.70$					
		Commitment Models			Re-
		Full	1-sided lack of	2-sided lack of	bargain
			Cyclicality		
1	Individual wages (all)	-1.44	-1.44	-1.45	-7.18
2	Individual wages (new hires only)	-4.67	-4.67	-4.68	-7.18
3	Wage component of user cost	-10.92	-10.92	-10.90	-7.18
4	Vacancy component of user cost	-54.98	-54.98	-54.96	-55.23
5	User cost of labor	-11.95	-11.95	-11.93	-8.29
			θ statistics		
6	$\sigma_{\ln(\theta)}$, quarterly	0.0604	0.0604	0.0604	0.0738
			Calibrated parameters		
7	Vacancy creation cost, c	0.3700	0.3700	0.3701	0.3699

Note - Results from simulating the models with risk averse workers (CRRA coefficient $\gamma = 3$). c is calibrated to match $E(\mu(\theta)) = 0.45$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$, which is the semi-elasticity of x with respect to unemployment, u. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are equal to the ones reported in the table (to the decimal points reported).

Table 9: Cyclicality of the User Cost of Labor and Its Components and the Volatility of the V-U Ratio

Two calibrated targets: expectation of the job finding rate and the cyclicality of the wage component of the user cost

	Log utility											
	$a = 0.50$				$a = 0.60$				$a = 0.72$			
	Commitment Models			Re-	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
	Cyclicality											
Wages (all)	-0.59	-0.78	-0.95	-4.50	-0.58	-0.86	-1.56	-4.50	-0.58	-1.13	-2.24	-4.50
Wages (new hires only)	-1.94	-2.18	-2.34	-4.50	-1.94	-2.24	-2.77	-4.50	-1.94	-2.56	-3.51	-4.50
Wage component of UC	-4.50				-4.50				-4.50			
Vacancy component of UC	-37.09	-37.14	-37.07	-37.00	-57.20	-57.29	-57.25	-57.02	-107.64	-107.79	-107.57	-107.10
User cost of labor	-4.90	-4.90	-4.90	-4.94	-4.82	-4.83	-4.83	-4.85	-4.73	-4.73	-4.74	-4.74
	θ statistics											
$\sigma_{\ln(\theta)}$, quarterly	0.1164	0.1163	0.1163	0.1156	0.1475	0.1473	0.1472	0.1468	0.2140	0.2138	0.2136	0.2136
	Calibrated parameters											
Consum.-n of unempl., b	0.8350	0.8335	0.8327	0.8193	0.8710	0.8698	0.8681	0.8616	0.9100	0.9088	0.9075	0.9056
Vacancy creation cost, c	0.1883	0.1901	0.1912	0.2079	0.1041	0.1051	0.1069	0.1123	0.0471	0.0476	0.0486	0.0495

Note - Results from simulating the models with risk averse workers (log utility). c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and the cyclicality of the wage component of the user cost. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are equal to the ones reported in the table (to the decimal points reported).

Table 10: VOLATILITY OF THE V-U RATIO GIVEN THE CYCLICALITY OF THE USER COST AND ITS COMPONENTS, $a = 0.60$

Different Utility Functions

Two calibrated targets: expectation of the job finding rate and the cyclicality of the wage component of the user cost

	Linear utility				CRRA 3 utility			
	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
				Cyclicality				
Wages (all)	-0.58	-0.90	-1.65	-4.50	-0.58	-0.80	-1.36	-4.50
Wages (new hires only)	-1.94	-2.25	-2.85	-4.50	-1.94	-2.21	-2.59	-4.50
Wage component of UC		-4.50				-4.50		
Vacancy component of UC		-57.26			-57.06	-57.32	-57.25	-56.53
User cost of labor		-4.81			-4.87	-4.88	-4.89	-4.99
				θ statistics				
$\sigma_{\ln(\theta)}$, quarterly		0.1480			0.1463	0.1458	0.1453	0.1431
				Calibrated parameters				
Consum.-n of unempl., b		0.8702			0.8715	0.8683	0.8644	0.8381
Vacancy creation cost, c		0.0984			0.1176	0.1211	0.1262	0.1565

Note - Results from simulating the models with workers with utility functions as indicated. c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and the cyclicality of the wage component of the user cost. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are equal to the ones reported in the table (to the decimal points reported).

Table 11: Cyclicality of the User Cost and Its Components given the Volatility of the V-U Ratio

Two calibrated targets: expectation of the job finding rate and the volatility of the v-u ratio

	$\alpha = 0.60$											
	Linear utility				Logarithmic utility				CRRA 3 utility			
	Commitment Models			Re-	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
	Cyclicality											
Wages (all)	-0.21	-0.51	-1.09	-1.62	-0.21	-0.51	-1.06	-1.62	-0.21	-0.50	-1.01	-1.62
Wages (new hires only)	-0.70	-1.05	-1.39	-1.62	-0.70	-1.04	-1.38	-1.62	-0.70	-1.03	-1.35	-1.62
Wage component of UC	-1.62				-1.62	-1.62	-1.62	-1.62	-1.62	-1.62	-1.61	-1.62
Vacancy component of UC	-76.89				-76.76	-76.80	-76.57	-76.24	-76.47	-76.60	-75.98	-74.98
User cost of labor	-1.75				-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.76
	Calibration targets											
$\sigma_{\ln(\theta)}$, quarterly	0.3820											
$E(\mu(\theta))$	0.4500											
	Calibrated parameters											
Consum.-n of unempl., b	0.9434				0.9438	0.9432	0.9428	0.9424	0.9444	0.9427	0.9417	0.9402
Vacancy creation cost, c	0.0415				0.0424	0.0427	0.0432	0.0435	0.0443	0.0454	0.0471	0.0481

Note - Results from simulating the models with workers with different utility functions as indicated. c and b are calibrated to match $E(\mu(\theta)) = 0.45$ and $\sigma_{\ln(\theta)}= 0.0382$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the four models and different utility functions are equal with respect to the precision in the tables, except for the log utility, where the statistics are -1.63 for all four models.

Table 12: Cyclicality of the user cost of labor and its components. Comparison Across Different Values of the Consumption of the Unemployed

One calibrated target: expectation of the job finding rate

	Log utility, $\alpha = 0.50$											
	$b = 0.40$				$b = 0.70$				$b = 0.90$			
	Commitment Models			Re-	Commitment Models			Re-	Commitment Models			Re-
	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain	Full	1-sided lack of	2-sided lack of	bargain
	Cyclicality											
Wages (all)	-2.44	-2.44	-2.44	-13.49	-1.13	-1.13	-1.13	-7.29	-0.33	-0.61	-0.98	-2.42
Wages (new hires only)	-7.78	-7.78	-7.78	-13.49	-3.69	-3.69	-3.69	-7.29	-1.11	-1.39	-1.64	-2.42
Wage component of UC	-18.48	-18.48	-18.48	-13.49	-8.56	-8.56	-8.56	-7.29	-2.56	-2.53	-2.51	-2.42
Vacancy component of UC	-36.37	-36.37	-36.37	-36.39	-36.57	-36.57	-36.57	-36.59	-38.36	-38.43	-38.46	-38.44
User cost of labor	-19.59	-19.59	-19.59	-14.91	-9.25	-9.25	-9.25	-8.01	-2.80	-2.77	-2.75	-2.66
	θ statistics											
$\sigma_{\ln(\theta)}$, quarterly	0.0296	0.0296	0.0296	0.0388	0.0623	0.0623	0.0623	0.0719	0.1991	0.2015	0.2030	0.2093
	Calibrated parameters											
Vacancy creation cost, c	0.9131	0.9131	0.9131	0.9131	0.3688	0.3688	0.3688	0.3687	0.1099	0.1097	0.1100	0.1099

Note - Results from simulating the models with risk averse workers (log utility). c is calibrated to match $E(\mu(\theta)) = 0.45$. All statistics are calculated from the monthly series unless mentioned otherwise. The cyclicality is calculated as $100cov(\ln(x), u)/var(u)$. The corresponding quarterly statistics for the cyclicality of the wage component of the user cost for the models are equal to the ones reported in the table (to the decimal points reported). $\sigma_{\ln(\theta)}$ is a statistic from quarterly non-HP filtered series.