Accounting for Unemployment: The Long and Short of It

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Abstract

Shimer (2012) accounts for the volatility of unemployment based on a model of homogeneous unemployment. Using data on short-term unemployment he finds that most of unemployment volatility is accounted for by variations in the exit rate from unemployment. The assumption of homogeneous exit rates is inconsistent with the observed negative duration dependence of unemployment exit rates for the U.S. labor market. We construct a simple model of heterogeneous unemployment with short-term and long-term unemployed, and use data on the duration distribution of unemployment to account for entry to and exit from the unemployment pool. This alternative account continues to attribute most of unemployment volatility to variations of exit rates from unemployment, but it also suggests that most of unemployment volatility is due to the volatility of long-term unemployment rather than short-term unemployment. We also show that once one allows for heterogeneous unemployment, the expected value of income losses from unemployment increases substantially, and unemployment volatility implied by a simple matching model increases.

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From 2008 to 2010 the unemployment rate in the United States more than doubled from about 4 percent to more than 10 percent. At the same time the share of long-term unemployed, that is, those unemployed for more than 26 weeks, more than doubled from less than 20 percent to more than 40 percent. This comovement between the unemployment rate and the share of long-term unemployment is not unusual: in every previous recession the share of long-term unemployed has increased with total unemployment, Figure 1. Two responses to this observation are common. First, long-term unemployed are seen as different from the rest of the unemployed in that they presumably have a lower chance of exiting unemployment. Furthermore, the high unemployment rate is attributed to the presence of these long-term unemployed. Second, long-term unemployment feeds on itself and creates future unemployment in that the chance that an unemployed worker will exit the unemployment state declines with the duration of being unemployed.

Figure 1. Unemployment Rate and Long-Term Unemployed

We use a simple unemployment accounting framework to provide some perspective on these two interpretations of long-term unemployment. First, we point out that the positive correlation of the unemployment rate and the share of long-term unemployed does not necessarily imply that the long-term unemployed are different from the rest of the unemployed. A simple model of unemployment that assumes homogeneity among the unemployed cannot, however, account quantitatively for the observed increase in long-term unemployment. We then extend the accounting framework slightly and assume that there are two types of unemployed: short-term unemployed with a high exit rate from unemployment, and long-term unemployed with a low exit rate. This simple extension allows us to account for the duration distribution of unemployment, and it sheds new light on the sources of unemployment. In particular, we find that variations in the entry and exit rates of long-term unemployed account for most of unemployment volatility.

Suppose that all unemployed workers are identical in their chances of exiting unemployment. Even in this model with homogeneous unemployment the unemployment rate and the share of long-term unemployed will be positively correlated if changes in the unemployment rate are mainly due to changes in the exit rate from unemployment. Simply put, if it gets harder to find a job then relatively more unemployed will be around for a long time. Shimer (2012) argues that most of the variation in the unemployment rate is indeed driven by variations in the exit rate.

Even though variations in a common exit rate can account qualitatively for the correlation between the unemployment rate and the share of long-term unemployment, this framework cannot account quantitatively for changes in the overall duration distribution of unemployment. This failure is associated with the observed negative duration dependence

of unemployment, that is, observed exit rates from unemployment appear to decline with the duration of unemployment. Observed duration dependence can be due to 'true duration dependence,' that is, the exit rates for all unemployed simply decline with unemployment duration, but it does not have to be. An alternative interpretation of observed duration dependence is 'unobserved heterogeneity' among the unemployed. In this case, unemployed are assumed to differ in their exit rates from the time they become unemployed. Even if an individual's exit rate is not changing over time, the composition of the pool of unemployed with the same unemployment duration is changing over time. In particular, the share of unemployed with low exit rates is increasing over time, and the average exit rate of the pool is declining with the time members have been unemployed.

We propose a simple model of unobserved heterogeneity and use it to account for the contributions of long-term unemployment to overall unemployment. There are two types of unemployed: 'short-term' (ST) unemployed with a high exit rate from unemployment, and 'long-term' (LT) unemployed with a low exit rate from unemployment. Newly unemployed can enter the unemployment pool as either of the two types, and while unemployed ST unemployed can switch type and become LT unemployed. This model contains two special cases: ex-ante heterogeneity only and ex-post heterogeneity only. With ex-ante heterogeneity ST unemployed do not switch type while being unemployed, and with ex-post heterogeneity only ST unemployed enter the unemployment pool. Ex-ante heterogeneity corresponds to the 'unobserved heterogeneity' explanation of duration dependence, and ex-post heterogeneity corresponds to the 'true duration dependence' explanation of duration dependence.

Ex-ante heterogeneity has an interpretation that is consistent with unemployment reflecting the degree of 'mismatch' in the economy. A worker may lose his job for some idiosyncratic reason related to the employer, or a worker's job loss may be due to structural change and represent a permanent loss of human capital. We would expect that the worker's exit rate from unemployment will be relatively higher in the first case. A relatively bigger inflow of LT unemployed would then indicate relatively more 'mismatch' in the economy. We find that fluctuations in the entry and exit rate of LT unemployed workers account for more than half of unemployment rate volatility, and that transitions from ST to LT unemployment are not a quantitatively important source of unemployment volatility. Indeed, the results for the general framework are quite close to the model with ex-ante heterogeneity only. One should note, however, that exit rates of ST unemployed and LT unemployed tend to be positively correlated and volatility of the combined exit rates also accounts for more than half of unemployment volatility.

The fact that the LT unemployed account for most of total unemployment suggests that standard estimates of income risk that are based on transition rates derived from short-term unemployment data will understate actual income risk. We provide some suggestive

calculations on how the expected present value of being (un)employed is affected by variations of the estimated transition rates. These calculations indicate that accounting for unobserved heterogeneity and using the information in the unemployment duration distribution can increase the estimated income loss in recessions by a factor of ten relative to standard measures.

Accounting separately for ST and LT unemployed affects our interpretation of the 'quality' of the pool of unemployed workers. Since in a recession the exit rate of LT unemployed declines relatively more, the share of LT unemployed increases in a recession. Suppose that LT unemployed have lower exit rates because whenever they match with a potential employer they are less likely to be a productive match. If in a matching framework employers cannot distinguish ex-ante between ST and LT unemployed then an increasing share of long-term unemployed reduces the expected quality of a match and thereby the incentive to post vacancies. This negative correlation between the unemployment rate and the average quality of the pool of unemployed has then the potential to amplify the volatility of unemployment exit rates, Shimer (2004). We show that accounting for changes in the relative matching efficiency of LT unemployed jointly with changes in labor productivity can account for about half of unemployment volatility.

Existing empirical work on long-term unemployment and duration dependence estimates functional forms for exit rates from unemployment using micro data. The standard approach is to estimate a Multiplicative Proportional Hazard (MPH) model for exit rates, that is, the exit rate is the product of terms that depend on (1) observable individual characteristics other than unemployment duration, (2) a calendar time effect, (3) the duration of unemployment, and (4) an unobserved individual fixed effect. Heckman and Singer (1984) provide an early survey for this approach, and Machin and Manning (1999) provide a more recent survey with an emphasis on long-term unemployment and negative duration dependence in Europe. Recently van den Berg and van der Klaauw (2001) have estimated the MPH model using both micro data and aggregate data on unemployment duration distributions. Abbring, van den Berg and van Ours (2002) is closest to our contribution in that they use only aggregate data to estimate an MPH model with pure duration dependence and unobserved heterogeneity. Our work differs from theirs in that we explicitly model the continuous time aspect of the unemployment inflows and outflows. Within this continuous time framework we replace the assumption of time-invariant fixed effects for unobserved heterogeneity by the assumption that agents transition among an ordered set of unemployment exit rate types that takes them from high exit rates to low exit rates. The transition between exit rate types is governed by a Poisson process and replaces the assumption of a hazard rate that declines with duration.

Our work is related to the recent literature originating with Shimer (2012) on the relative importance of unemployment inflow and outflow rates for the determination of aggregate un-

employment. Shimer (2012) using data on short-term unemployment argues that most of unemployment volatility is due to counter-cyclical variations of unemployment exit rates. Others such as Elsby, Michaels, and Solon (2009) and Fujita and Ramey (2009) have argued that variations in unemployment entry rates are also clearly counter-cyclical and make a significant contribution to unemployment volatility. None of the empirical work that we are aware of has evaluated the contributions of time-varying entry and exit rates to overall unemployment in a framework with unobserved heterogeneity in unemployment. An early precursor of our approach is Darby, Haltiwanger and Plant (1985) who speculate on the possibility that changes in the relative inflow rates of ST and LT unemployed account for variations of total unemployment and unemployment duration. Abbring et al (2003) argue that within their framework of unobserved heterogeneity exit rates are clearly more volatile and more closely correlated with the unemployment rate than are entry rates. They take this as evidence against the Darby et al (1985) hypothesis, but they do not provide a quantitative assessment of the relative contributions of entry and exit rates. Using the information contained in the overall duration distribution of unemployment and not just short-term unemployment we find that the contribution coming from unemployment entry rates increases, but variations in exit rates continue to account for most of unemployment volatility.

One could argue that the unobserved heterogeneity for aggregate unemployment reflects differential changes of unemployment among observable demographic groups. Baker (2002) and Shimer (2012) argue that correcting for composition effects based on observable characteristics is not important for the measured cyclicality of exit rates and unemployment duration. For the 2007-09 recession Aaronson, Mazumder, and Schechter (2010) and Elsby, Hobijn, and Sahin (2010) also find that changes in the observed demographic composition of unemployment have had limited impact on the aggregate unemployment duration. Along these lines we show that our approach yields evidence for unobserved heterogeneity among the unemployed even among identifiable demographic groups.

In section 1 we review Shimer's (2012) accounting framework with homogeneous unemployment and show that it cannot account for the duration distribution of unemployment. In section 2 we describe our model with heterogeneous unemployment, how it can be used to estimate entry rates to and exit rates from unemployment, and how variations in these transition rates contribute to overall unemployment volatility. In section 3 we confirm that most of the results we obtain for aggregate unemployment also apply to more narrowly defined demographic groups: male workers of different ages, and workers in different industries and occupations. In section 4 we discuss the robustness of our results to measurement error concerning the possible misclassification of unemployed and the misreporting of unemployment duration. In section 5 we provide some estimates on how unemployment heterogeneity might affect estimates of income volatility and the matching model's ability to generate significant

volatility of unemployment exit rates. Section 6 concludes.

1. Long-term unemployment with homogeneous job seekers

Shimer (2012) proposes a simple framework that uses observations on total unemployment and short-term unemployment to account for the dynamics of unemployment. We now review a simplified version of this accounting scheme and show that total unemployment and long-term unemployment are positively correlated, even though all unemployed workers have the same chance of finding a job. This result obtains because changes in the exit rate from unemployment are the main source of changes in total unemployment. Based on the measured transition rates we then calculate the implied duration distribution for an homogeneous pool of unemployed. We find that this framework does not capture the duration distribution well: the model significantly under-predicts the number of long-term unemployed.

1.1. A simple framework for unemployment accounting

Consider the following simple model of total unemployment in continuous time. All unemployed are homogeneous, newly unemployed enter the unemployment pool at a rate $f(s)$, and the current unemployed exit the pool according to a Poisson process with arrival rate $\lambda(s)$. The differential equation for total unemployment, $u(s)$, is

$$\dot{u}(s) = f(s) - \lambda(s)\, u(s). \tag{1.1}$$

Suppose that the entry and exit rates are constant, then the steady state measure of unemployment is

$$u = \frac{f}{\lambda}. \tag{1.2}$$

Given a constant exit rate, at any point in time the average duration of an unemployment spell is $\bar{D} = 1/\lambda$, and the share of unemployed that have been unemployed for longer than duration D is $\exp(-\lambda D)$. Thus if changes in unemployment are mainly driven by the exit rate from unemployment, then higher unemployment will be associated with a higher average duration of unemployment and a shift in the duration distribution towards longer unemployment spells.

Assume that the instantaneous entry and exit rates remain constant during a unit of time, that is, $\lambda(s) = \lambda_t$ and $f(s) = f_t$ for $s \in (t-1, t]$ where t denotes the end of a unit time period and is an integer. In the following we will interpret a unit time period as a month. The dynamics of total unemployment u_t^m and short-term unemployment $u_{t,1}^m$, i.e.

the number of unemployed that have been unemployed for less than one month is then given by

$$u_t^m = \int_0^1 f_t e^{-\lambda_t s} ds + e^{-\lambda_t} u_{t-1}^m = f_t \frac{1 - e^{-\lambda_t}}{\lambda_t} + e^{-\lambda_t} u_{t-1}^m = u_{t,1}^m + \left(1 - \bar{\lambda}_t\right) u_{t-1}^m. \tag{1.3}$$

Note that the measured unit inflow rate, $u_{t,1}^m$, combines the effects of the underlying instantaneous inflow rates and exit rates. We use data on total unemployment and short-term unemployment, $\left\{u_t^m, u_{t,1}^m\right\}$, together with the unemployment transition equation (1.3), to recover the instantaneous entry and exit rates, $\{f_t, \lambda_t\}$.

Assume that workers are either employed or unemployed, that is, we are not considering movements in and out of the labor force. Then our definition of the inflow rate is not truly exogenous relative to the outflow rate. There can be workers that cycle through repeated unemployment-employment spells within a month. Shimer (2012) takes this possibility into account and estimates job separation rates that remain constant during the month, $\sigma(s) = \sigma_t$ for $s \in (t-1, t]$. Assuming a constant labor force during the month, $l_t = u(s) + n(s)$, this procedure implies the following law of motion for employment

$$\dot{n}(s) = \lambda_t u(s) - \sigma_t n(s) = \lambda_t u(s) - f(s), \tag{1.4}$$

and the entry rate to unemployment is time-varying.[1] For the U.S. labor market employment is large relative to unemployment and the implied exit rate from employment is small relative to the exit rate from unemployment, such that the number of workers who go through repeated unemployment-employment spells within a month is quite small. We do not report the numbers here, but for all practical purposes the employment exit rate from the Shimer (2012) procedure and the normalized unemployment entry rate are indistinguishable, $\sigma_t \simeq f_t / n_t$. In the following we will use our simplified approach to account for unemployment inflows since it allows for a closed form solution of the unemployment accounting for heterogeneous workers.[2]

We construct (normalized) entry and exit rates for unemployment using monthly observations on (un)employment from the Monthly Household Data part of the BLS Employment and Earnings survey: Employment, Table A-3, and Duration of Unemployment, Table A-12.

[1] Substituting for employment in (1.4) defines a differential equation for unemployment.

[2] With some abuse of terminology we will occasionally refer to our normalized entry rates to unemployment as job separation rates and our unemployment exit rates as job finding rates. Strictly speaking this is not correct, since there are transitions of workers in and out of the labor force. Some of the measured inflows into unemployment come from out of the labor force and some of the measured outflows from unemployment go out of the labor force.

The data are seasonally adjusted and cover the period from January 1950 to March 2012.[3] The quarterly entry and exit rates for unemployment implied by our accounting procedure are displayed in Figure 2, panels A and B.[4] The panels display the monthly transition probabilities based on the quarterly averages of monthly flow transition rates. For example, the probability that a worker will exit unemployment within a month is $1 - \exp(-\lambda)$, where λ is the quarterly average of monthly exit rates.

The exit probabilities from unemployment vary between thirty and sixty percent with an average of about forty-five percent, and the normalized entry probabilities vary between two and five percent with an average of about three and a half percent. Thus unemployment is of short duration with an average of somewhat more than two months.[5] There is a downward trend in the exit probability from unemployment, and an upward trend in the entry probability to unemployment that is reversed in the 1990s. The declining unemployment exit probability is reflected in the increasing trend for the average duration of unemployment, whereas the trend reversal for the unemployment entry rate in the 1990s accounts for the decline in the average unemployment rate, Table 1.A. The 2007-09 recession is associated with a sharp decline of the unemployment exit rate and a transitory up tick of the unemployment inflow. Comparing Figures 2.A, B and C we can see that periods of high unemployment are associated with low exit rates from unemployment and high entry rates to unemployment.

Figure 2. Homogeneous Unemployment

1.2. Accounting for unemployment volatility

Shimer (2012) argues that the volatility of unemployment exit rates accounts for most of the cyclical fluctuations of the unemployment rate. He comes to this conclusion using a procedure that linearizes the unemployment rate process around its trend path. We now revisit this question using an alternative procedure to decompose unemployment rate fluctuations. Our alternative procedure confirms that for homogeneous unemployment variations in unemployment exit rates are the most important source of unemployment volatility. This alternative procedure will be quite useful for the analysis of the unemployment rate fluctuations when we allow for heterogeneous unemployment.

[3] The 1994 CPS redesign, see e.g. Polivka and Miller (1998), introduced a break into the data collection process such that short-term unemployment, that is, those that have been unemployed for less than 5 weeks, tends to be lower after January 1994. Shimer (2012) suggests to increase post-1994 short-term unemployment by ten percent to make up for the structural break. Accordingly we move a share of those who have been unemployed for 5 to 14 weeks to the short-term unemployed such that the latter group increases by ten percent. Elsby et al (2009) suggest a correction factor of fifteen percent.

[4] For reasons explained below, Section 2.1, we only discuss transition rates for the period ending December 2010.

[5] If we interpret the entry rate as the job separation rate then employment is of long duration with an average duration of about two and a half years.

We calculate the trend of a variable using a band-pass filter that eliminates fluctuations with periodicity less than twelve years, for example, Baxter and King (1999).[6] The dashed lines in Figure 2, panels A, B and C, display the trends of the unemployment exit and entry rates and the unemployment rate. We also define an alternative trend for the unemployment rate as follows. Given a time path for monthly instantaneous inflow rates and exit rates from unemployment, $x = (f, \lambda)$, equation (1.3) defines a mapping for the path of the unemployment rate, $u = G(x)$. We calculate the trend for each component of x using a band-pass filter, $x^T = \left(f^T, \lambda^T\right)$, and define the alternative trend unemployment rate as $u^T = G\left(x^T\right)$ and the deviation of unemployment from trend as $du^T = u - u^T = G(x) - G\left(x^T\right)$. Next, we calculate the contribution of a variable to the trend deviation of the unemployment rate as

$$du_i^T = G\left(x_i, x_{-i}^T\right) - G\left(x_i^T, x_{-i}^T\right).$$

If G is a linear mapping and the trend filter is linear, as the band-pass filter is, then this alternative procedure yields the same result as applying the trend filter directly to the unemployment rate. Furthermore, the sum of the individual variables' trend deviation contributions sum to the unemployment rate trend deviation. Indeed, for the model with homogeneous unemployment, there is almost no difference between the two definitions of the trend unemployment rate. On the other hand, for the models with heterogeneous unemployment that we consider below the mapping G can be sufficiently nonlinear, such that the residual term

$$r^T = du^T - \sum_i du_i^T$$

would become quantitatively important if we were to follow Shimer's (2012) original procedure.

In Figure 2.D we plot the trend deviations of the unemployment rate and the contributions of the exit rate from unemployment and the (normalized) inflow rate to unemployment. It is quite clear that even though spikes in the unemployment entry rate precede an increase of the unemployment rate, most of the unemployment rate increase is attributable to a decline in the exit rate from unemployment. For the most recent 2007-09 recession the drastic unemployment rate increase has to be attributed to an exceptional decline of the unemployment exit rate, whereas the up tick in the unemployment entry rate was not exceptional compared to past recessions.

The average contributions of different exit rate volatilities to the overall unemployment

[6] Shimer (2012) calculates a very smooth trend with a Hodrick-Prescott filter using a smoothing parameter 100,000 for monthly data. The fact that we use a different filter to calculate the trend has only a minor impact.

rate volatility are displayed in Table 1.B. We follow Shimer (2012) and write the variance components of the unemployment rate as

$$1 = \sum_i \frac{Cov\left(du^T, du_i^T\right)}{Var\left(du^T\right)} + \frac{Cov\left(du^T, r^T\right)}{Var\left(du^T\right)}.$$

For the full sample, 1950-2010, unemployment exit rate volatility accounts for eighty percent of unemployment rate volatility, and the contribution of the unemployment exit rate volatility increases steadily for more recent time periods. So far our results are essentially the same as in Shimer (2012).

1.3. Accounting for long-term unemployment

By construction our accounting procedure matches total and short-term unemployment, but not necessarily the overall duration distribution of unemployment. We now construct the duration distribution implied by the homogeneous agent model and compare that hypothetical distribution with the actual duration distribution. In addition to the data on short-term unemployment that we have used above, Table A.12 of the BLS Employment and Earnings survey also provides monthly data on the number of unemployed that have been unemployed between 5 and 14 weeks, $u_{t,2}^m$, between 15 and 26 weeks, $u_{t,3}^m$, and for 27 weeks or more, $u_{t,4}^m$.[7] The sequence of duration distributions is denoted $u^m = \left\{u_{t,k}^m : k = 1, \ldots, 4 \text{ and } t = 1, \ldots, T\right\}$. The duration distribution implied by the homogeneous exit model is obtained by simple iteration using the constructed entry and exit rates

$$u_{t,1} = u_{t,1}^m \tag{1.5}$$

$$u_{t,j} = \left(1 - \bar{\lambda}_t\right) u_{t-1,j-1} \text{ for } j = 2, \ldots, J \tag{1.6}$$

The duration distribution is truncated at a sufficiently large value J. For our calculations we use a maximum unemployment duration of four years, $J = 48$. We can then time-aggregate the monthly duration data and obtain the implied numbers of unemployed for the reported long-term unemployment bins, $\hat{u}_{t,j}^m$. These measurement equations are

$$\hat{u}_{t,2}^m = \sum_{j=2,3} u_{t,j},\ \hat{u}_{t,3}^m = \sum_{j=4,5,6} u_{t,j}, \text{ and } \hat{u}_{t,4}^m = \sum_{j=7,\ldots,J} u_{t,j}. \tag{1.7}$$

The duration distribution is displayed in the four panels of Figure 3. By construction the homogeneous exit rate model matches the very short-term unemployment, Panel A.

[7] We assume that an unemployment duration of 5 to 14 weeks represents 2 and 3 months, a duration of 15 to 26 weeks represents 4 to 6 months, and a duration of more than 26 weeks represents more than 6 months.

The model overstates short-term and medium-term unemployment, Panels B and C, and significantly understates long-term unemployment, Panel D. Even though the model captures the qualitative features of long-term unemployment, it fails to account for the magnitude of long-term unemployment. For almost all recessions the model accounts for only one third of long-term unemployment at its peak.

Figure 3. Duration Distribution of Unemployment

2. Unemployment accounting with heterogeneous unemployment

We now describe a simple model of unobserved heterogeneity among unemployed workers that accounts quite well for the duration distribution of unemployment. For this model we assume that there are two types of unemployed workers: 'short-term' (ST) unemployed with a relatively high exit rate and 'long-term' (LT) unemployed with a relatively low exit rate from unemployment. An unemployed worker may start out as being of either type. Furthermore, a ST unemployed worker that does not find work may over time make a transition to LT unemployment, but the reverse does not happen. On the one hand, this framework confirms Shimer's (2012) results that variations in exit rates from unemployment account for most of the unemployment rate volatility. On the other hand, this framework also suggests that even though unemployment tends to be short-term on average, unemployment rate volatility is mostly driven by variations in the entry and exit rates of the LT unemployed.

2.1. A simple model of long-term unemployment

Consider two types of unemployed workers, $i = 1, 2$, and type 2 has a lower exit rate from unemployment than does type 1, $\lambda^1(s) > \lambda^2(s)$ for all s. The transition equations for short-term and long-term unemployment are

$$\begin{aligned} \dot{u}^1(s) &= f^1(s) - \left[\lambda^1(s) + \gamma^1(s)\right] u^1(s) \\ \dot{u}^2(s) &= f^2(s) + \gamma^1(s)\, u^1(s) - \lambda^2(t)\, u^2(s). \end{aligned} \tag{2.1}$$

Similar to the model with homogeneous unemployment, we assume that the instantaneous entry and exit rates are constant for the monthly time intervals, $f_t^i = f^i(s)$, $\lambda_t^i = \lambda^i(s)$, and $\gamma_t^1 = \gamma^1(s)$ for $s \in (t-1, t]$.

Our framework captures the two explanations for negative duration dependence in unemployment data that have been proposed in the literature: 'true duration dependence' and 'unobserved heterogeneity,' for example, Machin and Manning (1999). The case of 'true duration dependence' is represented by the assumption that all new entrants to unemployment

are ST unemployed, $f^2(s) = 0$, and over time ST unemployed make a random transition to LT unemployment, $\gamma^1(s) \geq 0$. The case of 'unobserved heterogeneity' is represented by the assumption that at the time of entry into unemployment a worker's type is determined as either ST or LT, $f^i(s) \geq 0$, and the unemployed worker will not switch type before he exits the unemployment pool, $\gamma^1(s) = 0$. In the following we will refer to the 'unobserved heterogeneity' case as ex-ante heterogeneity and to the 'true duration dependence' case as ex-post heterogeneity.

Our model is a slight generalization of Darby et al (1985) who suggest that changes in the relative inflow rates of ST and LT unemployed could represent a major source of unemployment rate volatility. They interpret LT unemployed as those who have acquired significant employer-specific human capital. In normal times these workers are unlikely to lose their job, but once they have lost their job it will take a very long time for them to find another job, that is, they have a low exit rate from unemployment. They conjecture that periods of high unemployment occur because more LT unemployed enter the pool of unemployed. One might also think of the LT unemployed representing 'structural' unemployment. A worker may lose his job for some idiosyncratic reason related to the employer, or a worker's job loss may be due to structural change and represent a permanent loss of human capital. We would expect that the worker's exit rate from unemployment will be relatively higher in the first case. The 'structural' unemployment interpretation might be more appealing if periods of a relatively high entry rate of LT unemployed are also associated with a relatively low exit rate for LT unemployed.

2.2. Implementation

In the Appendix we derive for each type the expressions for the implied monthly net-entry to unemployment and the transition equations for end-of-period unemployment by duration,

$$\begin{aligned}
u_{t,1}^1 &= F^1\left(f_t^1, f_t^2, \lambda_t^1, \lambda_t^2, \gamma_t^1\right) \\
u_{t,1}^2 &= F^2\left(f_t^1, f_t^2, \lambda_t^1, \lambda_t^2, \gamma_t^1\right) \\
u_{t,j}^1 &= \left[1 - \Lambda^1\left(\lambda_t^1\right)\right]\left[1 - \Gamma^1\left(\gamma_{1t}\right)\right] u_{t-1,j-1}^1 \\
u_{t,j}^2 &= \left[1 - \Lambda^1\left(\lambda_t^1\right)\right]\left[u_{t-1,j-1}^2 + \Psi\left(\lambda_t^1, \gamma_t^1\right) u_{t-1,j-1}^1\right]
\end{aligned}$$

The measurement equations for the model are

$$\begin{aligned}
u_{t,1}^m &= \sum_{i=1,2} u_{t,1}^i,\ u_{t,2}^m = \sum_{j=2,3}\sum_{i=1,2} u_{t,j}^i \\
u_{t,3}^m &= \sum_{j=4,5,6}\sum_{i=1,2} u_{t,j}^i, \text{ and } u_{t,4}^m = \sum_{j=7,\ldots,J}\sum_{i=1,2} u_{t,j}^i.
\end{aligned}$$

We find the entry and exit rates for the two types by solving a nonlinear least-squares problem. For the algorithm we first specify (1) an initial distribution for unemployment by monthly duration, $x_1 = \left\{u^i_{1,j} : j = 1, \ldots, J, \text{ and } i = 1, 2\right\}$, and (2) a sequence of transition rates for both types, $x_2 = \left\{f^1_t, f^2_t, \lambda^1_t, \lambda^2_t, \gamma^1_t : t = 1, \ldots, T\right\}$.[8] This allows us to construct the sequence of end-of-period duration distributions for unemployment $\{u^i_{t,j} : i = 1, 2,\ j = 1, \ldots, J, \text{ and } t = 1, \ldots, T\}$, which we then time-aggregate using the measurement equations to get the implied measured duration distribution, $\hat{u}^m = \left\{\hat{u}^m_{t,k} : k = 1, \ldots, 4 \text{ and } t = 1, \ldots, T\right\}$. We choose the vector of unknowns, (x_1, x_2), to minimize the criterion function $\sum_{t=1}^{T}\sum_{k=1}^{4}\left(\hat{u}^m_{t,k} - u^m_{t,k}\right)^2$. We also impose a penalty on month to month changes in the transition rate from type 1 to type 2, and the relative exit rate of type 2 workers, $\kappa_t = \lambda^2_t/\lambda^1_t$. The smoothness restrictions are imposed for two reasons. First, the type-to-type transition rate tends to be excessively volatile without this restriction. Second, standard MPH model estimates of unemployment exit rates impose a constant relative hazard rate. We have experimented with two different initial conditions for the non-linear least squares problem, namely the solutions to the problem with ex-ante or ex-post heterogeneity only. For both initial conditions, the algorithm converges to the same terminal solution.

Our algorithm estimates current transition rates based on their implications for current and future duration distributions. This means that at the end of the sample the restrictions imposed by data on the transition rates are quite loose. Estimating transition rates for truncated samples suggests that at least a half year of data is required to obtain transition rates that remain invariant to an extension of the truncated sample. In the following we therefore report only transition rates up to December 2010 even though we use data up to March 2012 to estimate the transition rates.

2.3. Exit and entry

The model with heterogeneous unemployment matches the duration distribution of unemployment quite well, Figure 3. The lines for the actual duration distribution (black) and the constructed duration distribution (red) are almost on top of each other. Most of the inflow into unemployment consists of ST unemployed who exit unemployment rapidly. Even though LT unemployed account for only a small share of unemployment inflows, due to their very low exit rate they constitute close to one half of total unemployment.

The unemployment exit rates for the model with heterogeneous unemployment bracket the exit rates from the homogeneous agent model, Figure 4.A. For the sample, the monthly exit probability for ST unemployed fluctuates between fifty and eighty percent, with no clear

[8] In fact we do not specify the complete initial distribution, but we use a lower-dimensional parametric representation of the distribution. The effects of the initial distribution for the remaining parameter estimates are limited and temporary.

trend and an average of about sixty-five percent. Thus the average duration of ST unemployment is less than one month, which is less than half the unemployment duration predicted by the homogeneous agent model. On the other hand, the monthly exit probability of the LT unemployed fluctuates between ten and thirty percent, with an average exit probability of fifteen percent and a clear downward trend. Thus for LT unemployed the average unemployment duration is about seven months, more than three times the average duration of unemployment in the model with homogeneous unemployment.

Figure 4. Heterogeneous Unemployment

The unemployment entry probabilities of LT and ST unemployed roughly sum to the entry probabilities of the model with homogeneous unemployment. Most of unemployment entry is ST, and LT unemployed contribute a relatively small share to the unemployment inflow, between ten and twenty percent, Figure 4.B and C. Because of their low exit rate LT unemployed account, however, for a substantial share of total unemployment, between thirty and sixty percent, Figure 4.C. Furthermore, LT unemployed make up essentially all of measured long-term unemployed that is, those unemployed for more than 26 weeks. In this sense, long-term unemployed are indeed different from the overall pool of unemployed.

'Pure duration dependence' appears to play a rather limited role in the determination of unemployment. The monthly transition probability from ST to LT unemployed fluctuates around 1.5 percent, Figure 4.B. Given the high exit rates from unemployment for ST unemployed the probability that such a worker makes the transition to type LT before finding a job is negligible, about 1.5 percent. The low type transition rate in the general model reflects that the general model is actually quite close to the special case with ex-ante heterogeneity only.

Prior to the 2007-09 recession unemployment declined, but the share of workers that were unemployed for more than 26 weeks stayed higher than in previous expansion phases, Figure 1. The model accounts for this secular increase in the share of long-term unemployment through a decline of the inflow rates for ST unemployed and relatively constant inflow of LT unemployed, Figure 4.B. The apparent trend increase in the inflow share of LT unemployed started already in the 1990s. Whereas the inflow of LT unemployed almost never contributed more than 20 percent to total inflow before the mid-1990s, LT unemployed have been contributing close to twenty percent or more to total inflow since the mid-1990s.

The empirical labor literature on duration dependence usually estimates a multiplicative proportional hazard model (MPH) for unemployment exit rates, for example Machin and Manning (1999) or Abbring et al (2002). In the MPH model the exit rate from unemployment is the product of a function of known demographic characteristics, a function

of observed unemployment duration ('true duration dependence'), and a fixed effect ('unobserved heterogeneity'). This multiplicative structure then implies that the relative exit rates of workers with different fixed effects are constant. Our simple two-type model of unobserved heterogeneity does not impose constant relative exit rates. As we can see from Figure 4.D, even though the relative exit rate of LT unemployed workers is quite stable it nevertheless exhibits a downward trend over the sample period. Furthermore, the exit rate of LT unemployed appears to decline more in recessions than does the exit rate of ST unemployed, especially in the 2007-09 recession. Another way to see that the relative exit rate is not constant is to look at the cross-correlation between the trend deviations of the two unemployment exit rates. For the full sample that cross correlation is 0.66 and it increases to 0.73 for later periods.

2.4. Contributions to unemployment rate volatility

The model with heterogeneous unemployment suggests a reassessment of the sources of unemployment rate volatility. Associated with the heterogeneity of unemployment we find that the LT unemployed alone account for three fourths of unemployment rate volatility. Similar to Shimer (2012), we find that overall exit rate volatility of ST and LT unemployed accounts for most of unemployment rate volatility.

In Figure 5 we plot the contributions of different transition rates to the trend deviations of the unemployment rate. The contributions are calculated as described in Section 1.2. We can see that a decline of the unemployment exit rate for LT unemployed workers makes a substantial contribution to every increase of the unemployment rate. Furthermore, increased inflow of LT unemployed and/or a reduction of the exit rate of ST unemployed make substantial contributions to most increases of the unemployment rate. Fluctuations of the entry rate of ST unemployed and the type transition rate make only small contributions to unemployment rate volatility. The visual impression is confirmed by the variance decomposition in Table 1.C. For the full sample, 1950-2010, exit and entry rate volatility of the LT unemployed accounts for about one third each of overall unemployment rate volatility, and exit rate volatility of ST unemployed accounts for another one fourth of unemployment rate volatility. There is no big difference between more recent sample periods, except for an increased contribution to volatility coming from the LT unemployment exit rate.

The overall contribution of entry rate volatility with unobserved heterogeneity increases relative to the model with homogeneous unemployment. For the full sample, the overall contribution of entry rate volatility more than doubles from 15 percent to more than 30 percent once one allows for unobserved heterogeneity. In part this seems to be the case because the inflows of ST and LT unemployed often move in opposite directions. This is

similar to the observation of Elsby et al (2009) who note the opposing unemployment inflow movements from job losers and job quitters in the seventies and early eighties. The inflow rate of LT (ST) unemployed in our framework appears to behave like the inflow rate of job losers (quitters) in Elsby et al (2009).

Even though accounting for heterogeneous unemployment increases the contribution of entry rate volatility to unemployment rate volatility it does not confirm the conjecture of Darby et al (1985) that unemployment rate volatility is mainly driven by changes in the relative entry rate of the LT unemployed. Variations in the entry rate of LT unemployed are important, in that they account for almost all of the entry rate volatility, but their contribution is limited to about one third of overall volatility. On the other hand, relative declines in the exit rate of LT unemployed make another large contribution to unemployment rate volatility, and the entry and exit rate volatility of the LT unemployed accounts for two thirds of unemployment rate volatility.

Figure 5. Contributions to Unemployment

The model with heterogeneous unemployment allows a characterization of the unemployment rate increase in the 2007-09 recession that is consistent with structural reallocation as an important source of unemployment. Previously we suggested that unemployment due to structural change in the economy is likely to show up as an increased inflow and reduced exit rate of LT unemployment. As we can see from Figure 5, increased entry and reduced exit of LT unemployment are indeed the major drivers of the unemployment increase in 2009. With respect to the behavior of LT unemployment, the 2007-09 recession is similar to other previous recessions in 1957-58, 1981-82, and 2001. Exceptions to this pattern are the recessions of 1953-54, 1969-70, 1973-75 and 1990-91 where declining exit rates for both, ST and LT unemployment, are important sources of increased unemployment. We should note that our results on LT unemployment as an important driver of the unemployment rate depend on the fact that we do not restrict the relative exit rates from unemployment. As noted above the cross-correlation between the ST and LT unemployment exit rates is positive, but not large, about 0.65 for the full sample. If we impose a constant relative exit rate from unemployment, then the model will attribute the unemployment increase in the 2007-09 recession to a general decline in exit rates from unemployment.

3. Unobserved heterogeneity for demographic groups

The character of unemployment can differ substantially across identifiable demographic groups. For example, the average unemployment rate among college graduates is less than one third the unemployment rate among workers with less than a high school degree, and the

average unemployment duration of older male workers is about twice that of younger male workers. Applying our accounting framework to aggregate unemployment may then mistakenly attribute changes in the composition of the unemployment pool to unobserved heterogeneity. To evaluate this possibility we now perform our accounting exercise for different age groups of unemployed males, and for industry and occupation classifications of unemployed workers. We find that the results we obtain for aggregate unemployment are broadly consistent with the results from more narrowly defined demographic groups. This result should not be too surprising, after all studies of micro data have found significant evidence for duration dependence of unemployment exit rates, for example, Machin and Manning (1999), and composition effects have not found to be important for aggregate unemployment rates and average unemployment duration, for example, Baker (1992) or Aaronson et al (2010).

3.1. Unemployment for male age groups

Unemployment rates for older workers tend to be lower than for younger workers, yet once they are unemployed older workers tend to remain unemployed for longer than younger workers, Table 2.A. Despite these differences between older and younger workers, the driving sources of unemployment in terms of exit and entry rates for ST and LT unemployment are broadly comparable with those for aggregate unemployment. Unemployment rate volatility for each age group is mainly driven by variations in the entry and exit rates from LT unemployment.

We use monthly data from Table A.35 of the BLS Employment and Earnings Survey for male workers from 1976 to March 2011 to estimate (un)employment transition rates for the age groups 20-24, 25-34, 35-44, 45-44, 55-64 and 65 years and older. Unemployment duration distribution data are available for five bins: less than 5 weeks, 5 to 14 weeks, 15 to 26 weeks, 26 weeks to 51 weeks, and 51 weeks or more.[9] For each age group we estimate transition rates for the model with unobserved heterogeneity, merging the last two bins.

The results of the accounting exercise are displayed in Table 2.B For each age group we display the average sample values for transition rates in the first column and the contributions of transition rates to unemployment rate volatility in the second column. The model accounts for the increasing average unemployment duration for older workers mostly through a declining exit rate from unemployment for both types. An exception is the age group of 45 to 54 year old males where the average exit rate declines because relatively more LT unemployed enter the unemployment pool. Unemployment rates are declining with age, despite the average unemployment duration being increasing with age, because entry rates to unemployment decline with age. For almost all age groups variations in the LT transition

[9] The data are not seasonally adjusted. We use Watson's (1996) version of X-11 to deseasonalize the data.

rates remain the most important source of unemployment variations, and variations in exit rates from unemployment tend to be more important than variations of entry rates.[10] Only for workers aged 34 or younger are variations in the entry rate of LT unemployed relatively less important. On the other hand, for workers older than 45 years the role of variations in the ST exit rate as a source of unemployment volatility is significantly diminished. For no age group, except the workers aged 55 to 64 years do variations in the transition rate from ST to LT unemployment play a role for unemployment volatility, and even for this group the contribution is small.

3.2. Unemployment across industries and occupations

The data for aggregate unemployment and the different age groups of male unemployment indicate that the volatility of exit rates rather than entry rates and the volatility of LT transition rates rather than ST transition rates are the major sources of unemployment rate volatility. Unemployment data by industry and occupation confirm the latter, and the former for most but not all industries and occupations. We use monthly data from Table A.36 of the BLS Employment and Earnings Survey for industries and occupations from 2000 to March 2011 to estimate unemployment transition rates for the model with unobserved heterogeneity using unemployment duration distribution data for four bins: less than 5 weeks, 5 to 14 weeks, 15 to 26 weeks, and 27 weeks or more.

The results for industry data are displayed in Table 3. For the majority of industries variations in the unemployment exit rates are the main source of unemployment rate volatility, but there are four industries–construction, the production of durable and nondurable goods, and leisure and hospitality–for which variations in the unemployment entry rates are the main source of unemployment rate volatility. In either case, variations in the transition rates for LT unemployed make the biggest contribution to unemployment volatility. Variations in the type transition rate tend to have a negligible effect on unemployment volatility, with the exception of nondurable goods production and public administration. But even for these two industries the contribution of transition rate volatility is small.

The results for occupational data are displayed in Table 4. The results for occupational unemployment are very similar to the ones for industry unemployment. With the exception of production and professional occupations, variations in exit rates from unemployment are the most important source of unemployment rate volatility, and variations in the transition

[10] For the different male age groups the residual part of unemployment rate volatility is quite large at 15 percent. Thus, relative to overall unemployment the accounting framework does not as nicely separate out the contributions of the different transition rates to unemployment rate volatility. Part of this seems due to the fact that our X-11 procedure does not completely remove all the seasonal components in the duration distribution of unemployment.

rates for LT unemployed are more important than those for ST unemployed.[11]

4. Measurement issues

Statistics on overall unemployment in the United States are based on self-reporting by households in the CPS. A household reports on its members' labor market states, and if a member is unemployed the household reports how long that member has been looking for work. There are well known measurement problems for unemployment statistics which may affect our estimates of short-term and long-term unemployment. For example, Elsby, Hobijn, Sahin, and Valletta (2011) point out that as of the fall of 2011 more than 50 percent of those who state that they have become unemployed in the current month also report more than one month's unemployment duration and 25 percent report even more than six months duration. That reported unemployment durations are subject to considerable measurement error was pointed out more than 25 years ago by Poterba and Summers (1984) who showed that less than 40 percent of households that reported being unemployed in two consecutive months would 'correctly' increase their reported unemployment duration by one month. We now try to address these measurement issues. Since to our knowledge there is only limited empirical evidence on the structure and magnitude of the measurement error our approach will necessarily be somewhat speculative.

4.1. Misclassification of labor market states

One would not necessarily expect that a household member report an unemployment duration exceeding one month if that member was employed or out of the labor force in the previous month. Yet Elsby et al (2011, Figure 4) show that in 2008 the number of unemployed who do exactly that increased dramatically. One possible reason for this observation is that an unemployed household reports on how long he or she has been looking for work, not for how long he or she has been unemployed. Thus recently unemployed workers that started their job search in anticipation of a potential lay-off may very well report a job search duration of more than one month. Another potential problem concerns the misclassification of a household's labor market state. There is an established literature that argues that a significant share of unemployed are misclassified as out of the labor force (OLF), e.g., Abowd and Zellner (1985) or Poterba and Summers (1986). In this case some of the households that were classified as being OLF in the previous month were actually unemployed, and if they remain unemployed and are correctly classified in the current month they will report more

[11] The only exception to this pattern is the farming related occupation, but for this occupation the framework we use for the decomposition of unemployment volatility performs quite badly. The main reason is again that our X-11 procedure does not remove the seasonal components completely.

than one month's duration of unemployment. We will show now that a stable pattern of misclassification could account for the observation of Elsby et al (2011).

The CPS has a rotating panel structure in that a household will be in the sample for four consecutive months, out of the sample for eight consecutive months, and then in the sample one more time for another four consecutive months. Thus for three fourths of the households that are in a rotation group we potentially have information on the current and previous month's employment state and the current month's reported unemployment duration.[12] In Figure 6 we graph the distribution of currently unemployed workers according to their labor market state in the previous month. We distinguish among unemployed according to the unemployment duration they report in the current month. For example, in Panel A we consider those that are currently unemployed and that report an unemployment duration of less than 5 weeks, and we plot the share of those that reported being unemployed (U), employed (E), and OLF in the previous month. First, note the pronounced structural break for these short-term unemployed in 1994 which can be attributed to the 1994 CPS redesign, see for example, Shimer (2012). After 1995 the majority of these short-term unemployed were reported being employed or OLF in the previous month. This observation is consistent with a basic understanding of labor market transitions, but we should note that there is also a small fraction of unemployed that were already reported as being unemployed in the previous month, and yet they report less than 5 weeks unemployment duration. The pattern for those reporting an unemployment duration of more than 26 weeks, Panel D, suggests a possible explanation of the Elsby et al (2011) observation. About twenty percent of these currently unemployed were classified as OLF in the previous month, and a substantial part of them may have been misclassified.

Figure 6. Sources of Unemployment by Duration

The most interesting aspect of Figure 6 is the remarkable stability of the shares for the different reported unemployment durations. This stability suggests the following data correction where we use average inflow shares for duration cells. Let α_s^E respectively α_s^{OLF} denote the average share of those that are currently unemployed in duration cell s and that reported being employed, respectively OLF in the previous month. First, for all those that were employed in the previous month we only count the unemployment duration, not the time they spent looking for work in anticipation of a possible lay-off. We implement this by moving a constant fraction α_s^E of the unemployed that report a duration of more than 5 weeks, $s > 1$, to the unemployed with less than 5 weeks of unemployment, $s = 1$. Second, we assume that

[12] For various reasons not all households that are in an ongoing rotation group can be matched. I would like to thank Marianna Kudlyak for making available to me a panel of matched households from the CPS.

all inflows from OLF that report more than 5 weeks of unemployment duration in the current month were unemployed in the previous month. We therefore increase unemployment in the previous month in the cells with more than 5 weeks duration by the OLF share α_s^{OLF} and we allocate these unemployed as follows: (1) for current durations of 5-14 and 15-25 weeks assign 1/3 to the preceding cell and 2/3 to the current cell; (2) for 26-52 weeks assign 1/6 to the preceding cell and 5/6 to the current cell; and (3) for those unemployed more than 52 weeks assign all to that cell in the preceding month. Formally the 'corrected' unemployment duration distribution is defined as

$$\begin{aligned}
u_{t,1}^c &= u_{t,1} + \sum_{s>2} \alpha_s^E u_{t,s} + \frac{1}{3}\alpha_2^{OLF} u_{t+1,2} \\
u_{t,2}^c &= \left(1-\alpha_2^E\right) u_{t,2} + \frac{2}{3}\alpha_2^{OLF} u_{t+1,2} + \frac{1}{3}\alpha_3^{OLF} u_{t+1,3} \\
u_{t,3}^c &= \left(1-\alpha_3^E\right) u_{t,3} + \frac{2}{3}\alpha_3^{OLF} u_{t+1,3} + \frac{1}{6}\alpha_4^{OLF} u_{t+1,4} \\
u_{t,4}^c &= \left(1-\alpha_4^E\right) u_{t,4} + \frac{5}{6}\alpha_4^{OLF} u_{t+1,4} \\
u_{t,5}^c &= \left(1-\alpha_5^E\right) u_{t,5} + \alpha_5^{OLF} u_{t+1,5}
\end{aligned}$$

The proposed misclassification correction has a minor impact on the overall duration distribution of unemployment. In Figure 7 we display the measured duration distribution and the corrected duration distribution of unemployment. The corrected version allocates somewhat more unemployment towards short-term unemployment of less than 5 weeks, but the distribution for more than 5 weeks of unemployment is hardly affected. Estimating transition rates for the corrected duration distribution hardly affects the results obtained so far.

Figure 7. Misclassification Corrected Duration Distribution

4.2. Misreporting unemployment durations

Prior to the 1994 redesign of the CPS every household that reported being unemployed was asked how long he or she had been looking for work. Since households are interviewed in the CPS for four consecutive months the reported duration of job search for a household that reported being unemployed in two consecutive months should in principle increase by one month. Poterba and Summers (1984) show that this was the exception rather than the rule, in that less than 40 percent of households that were unemployed for two consecutive months reported that their time of unemployment increased by one month. As part of the 1994 CPS redesign households that report two consecutive months of unemployment are no longer asked about their unemployment duration, rather the reported duration from the previous month is incremented by one month. Inspired by the work of Poterba and Summers (1984) we now

propose a simple model of misreporting for unemployment duration. For pre-1994 data our model with heterogeneous unemployment cannot match the observed/reported duration distribution well. Essentially, the proposed measurement model distorts the reported duration distribution for short-term unemployment too much. On the other hand, for the post-1994 data the impact of the proposed measurement model for the reported duration distribution of short-term unemployment is limited, and the model with heterogeneous unemployment can match the observed/reported duration distribution quite well.

Poterba and Summers (1984) study a sub-sample of households from the May and June 1976 CPS that reported (1) being unemployed for more than 5 weeks in May 1976 and (2) being unemployed in June 1976. Table 5 below represents a stylized version of Table 2 from Poterba and Summers (1984). In the full sample only about one third of the households report that from one month to the next their unemployment duration increased by one month. The results are marginally better for households with a shorter unemployment duration. Polivka and Rothgeb (1993) and Polivka and Miller (1998) also show that self-reported unemployment duration in the CPS before 1994 are subject to significant measurement error. After the 1994 CPS redesign we can longer verify how accurate households' self-reported unemployment durations are since the duration of a household that is unemployed in two consecutive months is automatically incremented by one month.

Table 5. Reported Duration Changes

We now consider a simple random walk model for reported duration to evaluate how measurement error might affect our estimates of ST and LT unemployment. Suppose that households that have become unemployed in the current month report their duration correctly, but that households in an ongoing unemployment spell randomly increment their reported duration. In particular, a household that reported r_{-1} months of unemployment in the previous month will with probability α_i report $r = r_{-i} + i$ months in the current month with $i = -1, 0, 1, 2, 3$. We recursively construct a sequence of conditional probabilities $g(r|s)$ denoting the probability that an unemployed household in an ongoing unemployment spell of duration s will report duration r. For the period before 1994 this defines the mapping from the true unemployment duration distribution to the measured unemployment duration distribution. For the period after 1994 we have to make an adjustment since in the redesigned CPS misreporting should only apply to households that enter the panel unemployed. The details are in Appendix B.

Superimposing these two measurement models on the model with heterogeneous unemployment we re-estimate the transition rates for total unemployment for the pre-1994 sample and the post-1994 sample separately. For both samples the reported unemployment overstates the estimated true unemployment for the very short durations, less than 5 weeks, and

understates true estimated unemployment for the five to fourteen week cell. For the post-1994 sample the model's ability to match the reported duration distribution is still quite good, but it does deteriorate noticeably relative to the model without measurement error. For the pre-1994 period the model no longer provides a good fit for the duration distribution: the fitted reported unemployment for the cell with less than 5 weeks (five to fourteen weeks) of unemployment substantially overstates (understates) measured unemployment for the cell.[13]

Unconditional exit rates from unemployment provide another reason in favor of the model without reporting errors. Elsby et al (2011) estimate the average exit probability from unemployment by duration for the period 2010-2011. They find that the exit rate declines rapidly for the first six months of unemployment and then stabilizes. In Figure 8 we display the annual averages of duration contingent exit probabilities from unemployment for 2007 and 2009 implied by the model with heterogeneous unemployment. These unconditional exit probabilities are the weighted averages of the exit probabilities for the two types where the weights are the shares of each type for the given duration. The solid (dashed) lines indicate the exit probabilities for the model without (with) reporting error. The onset of the Great Recession results in a downward shift of the unemployment exit probabilities for both models. For the model without measurement error most of the decline in exit probabilities takes place during the first six months, whereas the decline is more drawn out for the model with measurement error.

Figure 8. Exit Probabilities from Unemployment

5. Why long-term unemployment matters

Unobserved unemployment exit rate differences of the magnitude observed for the U.S. economy can affect how we view the workings of the U.S. labor market. First, if a substantial fraction of the unemployed, the data suggest up to one half of all unemployed, are on average unemployed for more than half a year then the income losses associated with unemployment can be significant. In this section we provide a back of the envelope calculation that suggests that income losses can be ten times higher than what is estimated based on average unemployment durations between two and three months. Second, changes in the relative unemployment exit rates of LT unemployed might be a significant source of unemployment volatility. Below we show that in a simple matching model with unobserved heterogene-

[13] We do not provide an account for the sources of unemployment volatility here since for the pre-1994 sample the model with measurement error does not fit well the measured distribution and for the post-1994 sample the results with and without measurement error are qualitatively quite similar.

ity that is calibrated to the U.S. labor market changes in relative matching efficiency can generate substantial unemployment rate volatility.

5.1. Welfare effects of long-term unemployment

Most quantitative macro-theory work on the search and matching model of the labor market relies on a characterization of labor market transitions that emphasizes the short-term nature of unemployment. Allowing for unobserved unemployment heterogeneity does suggest, however, that it is mainly the volatility of long-term unemployment that contributes to overall unemployment volatility. The following 'back of the envelope' calculation shows that this feature might have consequences for the welfare costs of income fluctuations associated with unemployment.

Suppose that all employed workers get a wage, normalized at one, $w = 1$, and that the flow value of unemployment is sixty percent of the wage, $b = 0.6$. Suppose that workers are risk-neutral and discount the future at a constant annual interest rate, $r = 0.05$. This is a standard calibration for quantitative macro models of the labor market. We now calculate the capital values of being (un)employed for the estimated (un)employment transition rates of the model with homogeneous and heterogeneous unemployment. For example, we define the capital value equation for being employed as

$$\begin{aligned} rW_t &= w + \sigma_{1,t}\left(U_{1,t} - W_t\right) + \sigma_{2,t}\left(U_{2,t} - W_t\right) + \theta\left(\bar{W} - W_t\right) \\ r\bar{W} &= w + \bar{\sigma}_1\left(\bar{U}_1 - \bar{W}\right) + \bar{\sigma}_2\left(\bar{U}_2 - \bar{W}\right) \end{aligned}$$

The first equation defines the return on being employed in period t, given the current exit rates from employment, and the possibility that the transition rates change to the sample average transition rates at a rate θ. The second equation is the converse of the first equation if the current state is defined by the sample average transition rates. If we set the aggregate transition rate to zero, $\theta = 0$, we get the steady state capital values if the current transition rates were to persist forever. The larger is θ the less variation there is in capital values. The capital value equations for current unemployment are defined as

$$\begin{aligned} rU_{1,t} &= b + \lambda_{1,t}\left(W_t - U_{1,t}\right) + \gamma_{1,t}\left(U_{2,t} - U_{1,t}\right) + \theta\left(\bar{U}_1 - U_{1,t}\right) \\ rU_{2,t} &= b + \lambda_{2,t}\left(W_t - U_{2,t}\right) + \theta\left(\bar{U}_2 - U_{2,t}\right) \end{aligned}$$

with corresponding capital value equations for steady state unemployment with sample average transition rates. Implicit in these capital value definitions is that an employed worker may end up as ST or LT unemployed on separation, and that the unemployment type affects the rate at which the worker finds new employment, but it does not affect the type of

employment.

Figure 9. Capital Value of Employment

In Figure 9 we plot the percentage deviation of the capital value of employment from its mean for the two models of unemployment when $1/\theta = 3$ years. The qualitative features of the capital values of employment are very similar across the two models: they exhibit sharp declines in recessions. The capital values do differ in the magnitude of their decline during and after recessions. For the model with homogeneous unemployment matched to data on short-term unemployment the declines in the capital value of employment are relatively small: the biggest declines represent about one percent of the mean sample value and occur in the early 1980s and after the 2007-09 recession. The capital value of employment declines much more with heterogeneous unemployment, especially following the 2007-09 recession. For this model the employment value declines by ten percent relative to its mean. The capital values of unemployment are closely correlated with the capital values of employment, but more volatile with the volatility of ST unemployment being lower than for LT unemployment.

5.2. Changes in relative matching efficiency and the volatility of unemployment

Changes in the relative matching efficiency of the LT unemployed are a potentially important source of unemployment rate volatility. For the standard matching model of unemployment, Shimer (2005) has argued that it cannot account for the observed volatility of unemployment given job separation shocks and productivity shocks as standard sources of volatility. Job separation shocks are discounted for two reasons. First, variations in unemployment inflows do not contribute much to unemployment volatility. [14] Second, separation shocks tend to generate a positive comovement between the vacancies and unemployment, which is inconsistent with the Beveridge curve, that is, the observation that unemployment and vacancies are negatively correlated. Shimer (2005) then shows that if the standard matching model with Nash bargaining is calibrated to the U.S. economy labor productivity shocks can account for at most one-tenth of the observed unemployment rate volatility.[15] We now show that changes in the relative match efficiency of LT unemployed in a model with pooled matching can result in significant unemployment rate volatility.

[14] The relative contribution of unemployment inflow and outflow rates to the volatility of unemployment continues to be debated, e.g. the literature mentioned in footnote . The results in this paper are consistent with a limited contribution coming from inflow rate variations.

[15] Alternative specifications of the matching model have been proposed that provide a better account for the observed unemployment rate volatility, e.g. Hall (2005) or Hagedorn and Manovskii (2008). For a survey see Hornstein, Krusell, and Violante (2005).

We first review the relevant business cycle properties of the estimated transition rates. In Table 6 we display the means, standard deviations, and cross-correlations with unemployment for the detrended variables of interest.[16] The cyclicality of a variable is defined relative to the total unemployment rate, that is, we will call a variable pro-cyclical if it is positively correlated with the unemployment rate. Table 6 quantifies some of the information provided in Figure 5 on the contributions of transition rates to unemployment volatility.

Table 6. Business Cycle Statistics, 1950-2010

First, entry rates are pro-cyclical, slightly leading the unemployment rate, and the inflow of ST unemployed is about seven times that of the inflow of LT unemployed. The inflow rates are somewhat more pro-cyclical if normalized with respect to employment, otherwise the normalization does not matter much. Second, exit rates are counter-cyclical with exit rates for LT unemployed slightly leading the unemployment rate, and the average exit rate for ST unemployed is about five times that of LT unemployed. Third, type transition rates are small and essentially acyclical. Fourth, the relative exit rate of LT unemployed is counter-cyclical and the relative entry rate of LT unemployed is pro-cyclical, with the relative exit rate leading the unemployment rate. Finally, the share of LT unemployed in total unemployment is pro-cyclical. We will now describe a simple matching model where declines in the relative exit rate of LT unemployed that are of a magnitude consistent with observations generate a substantial increase of the unemployment rate.

Consider the standard matching model of unemployment, but assume that there are two types of workers and both match with the same pool of vacancies posted by firms. We restrict attention to steady states. Assume that there is a measure one of workers, and let ϕ_i denote the measure of type $i = 1, 2$ workers, $\phi_1 + \phi_2 = 1$. A type i worker exits unemployment at rate λ_i and separates from a job at rate σ_i, for i. As before we assume that type 2 workers are LT unemployed, $\lambda_2 < \lambda_1$. To simplify the model and since measured transition rates from ST to LT unemployed are small we ignore transitions between types. The steady state unemployment rate of type i workers is

$$u_i = \frac{\sigma_i}{\sigma_i + \lambda_i},$$

and total unemployment is $u = \phi_1 u_1 + \phi_2 u_2$. All unemployed workers match in a common pool with vacancies v, and the matching rate is determined by a standard Cobb-Douglas matching function, $m = Av^{1-\alpha}u^{\alpha}$ with $0 < \alpha < 1$. Thus the rate at which an unemployed

[16] All variables are detrended using a Baxter-King bandpass filter that eliminates frequency components with a periodicity of more than twelve years..

worker meets a vacancy is $\lambda = m/u$, and the rate at which a vacancy meets a worker is $\lambda^f = m/v$.

The unemployment exit rates of worker types differ because not every match between a vacancy and a LT unemployed is productive. Assume that each match that involves a type one worker is productive, but that only with probability κ matched type two workers are suitable for the vacancy they have met. Then the unemployment exit rates for the two types are $\lambda_1 = \lambda$ and $\lambda_2 = \kappa\lambda$. We also allow for the possibility that the two worker types differ in terms of how productive they are in a successful match, p_i, and in terms of their unemployment flow value, b_i. For a successful match between a worker and a vacancy, the surplus from the match, S_i, is shared through the Nash bargaining solution with the bargaining share for workers β. Workers and firms are risk-neutral and discount the future at the common rate r. The surplus value of a successful match depends on the type of the worker

$$S_i = \frac{p_i - b_i}{r + \sigma_i + \beta\lambda_i}.$$

This model is a straightforward modification of the standard matching model, and the crucial difference is in the specification of the free-entry condition for vacancies. Free entry drives the value of posting a vacancy to zero such that the flow cost of posting a vacancy, c, is equated with the expected value from meeting a worker

$$c = (1-\beta)\,\lambda^f \left(\frac{\phi_1 u_1}{u} S_1 + \kappa \frac{\phi_2 u_2}{u} S_2 \right),$$

which depends on the composition of the unemployment pool. We now calibrate the model to evaluate the impact of changes in the relative exit rate on the unemployment rate.

We take a standard calibration of the model with homogeneous unemployment as our starting point, e.g. Hornstein et al (2005). The time unit is one month and the discount rate is $r = 0.05/12$, the unemployment elasticity of the matching function is $\alpha = 0.72$, the Hosios condition for efficient matching is satisfied, $\alpha = \beta$, and the production flow is normalized at one, $p = 1$. We consider a range of the ratio of the flow values of unemployment and employment, $b \in \{0.5, 0.6, 0.7, 0.8\}$. The remaining parameters (A, σ, c) are determined by matching steady state unemployment $u = 0.057$, the exit rate from unemployment, $\lambda = 0.6$, and normalizing market tightness, $\theta = 1$.

For the model with heterogeneous unemployment we obtain type-specific parameters by matching the relative transition rates and shares estimated for the U.S. economy in addition to the aggregate statistics. The relative exit rate for LT is $\lambda_2/\lambda_1 = 1/5$, the relative inflow rate of LT is $f_2/f_1 = 1/6$, and the share or LT unemployed in total unemployment is $\phi_2 u_2/u = 0.5$. The implied unemployment rates for ST and LT unemployed are $u_1 = 0.042$

and $u_2 = 0.1$, and the population share of LT unemployed is $\phi_2 = 0.26$. Production of type one agents is normalized at one, $p_1 = 1$. Since we do not have any information on the (un)employment flow values of the two types, we consider a range of steady states, indexed by the vector $(p_2, S_2/S_1, b/p)$. We assume that LT unemployed are not only less likely to form a successful match, but they are also less productive, $p_2 \leq p_1$. Furthermore, we want to study an equilibrium where an increased share of LT unemployed makes it less attractive to post vacancies. Since the value of a filled vacancy is proportional to the match surplus, we only consider steady states with $S_2 \leq S_1$. Finally, we match the ratio of the average flow values of unemployment and employment

$$\frac{b}{p} = \frac{\sum_i \frac{\phi_i u_i}{u} b_i}{\sum_i \frac{\phi_i (1-u_i)}{1-u} p_i}.$$

Given these considerations we consider steady states indexed by $p_2, S_2/S_1 \in [0.5, 1]$, and $b/p \in \{0.5, 0.6, 0.7, 0.8\}$.

We display the steady state response of unemployment and vacancies to a change in aggregate productivity and relative matching efficiency for the model with homogeneous and heterogeneous unemployment in Figure 9 below. We consider the impact of a one standard deviation increase of labor productivity and relative matching efficiency on unemployment and vacancies. For the U.S. economy this corresponds to a two percent increase in labor productivity p and a fourteen percent increase of the relative matching efficiency κ.[17] Panel A plots the percentage response of unemployment and vacancies. The scatter plot on the right, marked "1 Type: p only," denotes the response to increased labor productivity in the baseline model with homogeneous unemployment for different values of the unemployment flow value b. Higher values of b imply movements to the north-west with a bigger absolute response of unemployment and vacancies. The next scatter plot marked "2 Types: p only" denotes the response of unemployment and vacancies to a common productivity increase in the model with heterogeneous unemployment. Different colors indicate different values of the aggregate b/p ratio, and higher values of the b/p ratio again imply movements to the north-west. The scatter plot marked "2 Types: κ only" denotes the response to an increase in relative matching efficiency. Finally, the left most scatter plot marked "2 Types: p and κ" denotes the response to a joint increase in productivity and relative matching efficiency.

Figure 9. Unemployment Volatility with Pro-Cyclical Relative Matching Efficiency

It is well known that for the baseline calibration of the model with homogeneous unemployment productivity shocks have a small impact on unemployment. Increased productivity

[17] For labor productivity see Hornstein et al (2005) and for the relative match productivity see Table 6.

raises the surplus value of a match and to maintain the free-entry condition, the rate at which vacancies meet workers declines, that is, market tightness increases. This in turn increases the unemployment exit rate and the unemployment rate declines. The increased unemployment exit rate creates an additional reduction of the surplus value, thus limiting the required inflow of vacancies. In our baseline calibration with an unemployment flow value of $b = 0.5$ a two percent increase of productivity reduces the unemployment rate by about one percent and increases the vacancy rate by about three percent. In other words, if productivity changes were the sole source of shocks then unemployment would be about half as volatile as productivity and vacancies would be about three times as volatile as unemployment. Contrary to this for the U.S. labor market the standard deviation of unemployment is about ten times that of labor productivity, and vacancies are only slightly more volatile than unemployment, e.g. Hornstein et al (2005). Unemployment becomes more responsive to productivity changes if unemployment flow benefits increase relative to productivity, that is, the b/p ratio increases. Essentially, with a larger b/p ratio the same percentage change in productivity p represents a much bigger change in the net flow value of employment, $p - b$, which determines the value of a match. The values for the b/p ratio considered here are, however, not big enough to generate a twenty percent increase of the unemployment rate. In order to get that unemployment response one needs values of the b/p ratio closer to one, cf. Hagedorn and Manovskii (2008).

Heterogeneous unemployment alone does not change much the impact of a productivity increase on unemployment. On the one hand, for the same average b/p-ratio, heterogeneity increases the elasticity of the expected surplus with respect to productivity. On the other hand, the share of LT unemployment increases with a higher matching rate, which reduces the expected surplus value. The first effect comes about because the elasticity of the net flow value of employment is a convex function of the b/p-ratio. Suppose that both types are equally productive. Since we have assumed that ST unemployed also have a higher match surplus, the net flow value of employment for ST unemployed must be larger, that is, their b/p-ratio must be smaller than the average b/p-ratio. Conversely, the b/p-ratio for LT unemployed is larger than the average ratio. Since the elasticity of the net flow value is a convex function of the b/p-ratio, the average elasticity with heterogeneous unemployment is larger than the elasticity in the homogeneous unemployment model with the corresponding average b/p-ratio. The second effect comes about because the unemployment rate of LT unemployed is higher. Therefore, the same increase in the aggregate matching rate reduces the unemployment rate of LT unemployed relatively less than among ST unemployed, and the unemployment share of LT unemployed increases. Since the match surplus for LT unemployed is assumed to be less than for ST unemployed, the expected value from a match declines which limits the increase of market tightness that is required to maintain the free-

entry condition.

An increase of the relative match efficiency for LT unemployed substantially reduces unemployment. Other things equal, the higher effective exit rate for LT unemployed reduces their unemployment rate, and thereby their share in overall unemployment. This means that not only does the probability for a successful match with a LT unemployed increase, but also that the probability of meeting a LT unemployed with associated lower surplus value declines. Thus the expected surplus from a match increases and to maintain the free entry condition market tightness has to increase. The direct reduction of unemployment from the increased relative matching efficiency is, however, so large that it already created a substantial increase in market tightness, and in equilibrium vacancies are actually falling. Thus changes in the relative matching efficiency only would generate a positively sloped Beveridge curve.

Finally, a combined one standard deviation increase of aggregate productivity and the relative matching efficiency can reduce the unemployment rate by more than seven percent and up to ten percent for values of the b/p-ratio of 0.8. In this case, the LT unemployment share declines because the increased relative matching efficiency more than compensates for the overall increase in matching efficiency. For larger values of the b/p-ratio the responses of vacancies and unemployment are of similar magnitude. Thus, compared with a standard matching model of homogenous unemployment, in the model with heterogeneous unemployment productivity changes in conjunction with changes of relative matching efficiency generate substantially more unemployment and vacancy volatility that is substantially closer to what is observed in the date

6. Conclusion

Relative to other OECD countries the labor market of the United States is usually characterized as being subject to a high degree of turnover, for example, Elsby, Hobijn and Sahin (2008). Workers in the U.S. economy are more likely to lose their job, but they are also more likely to find a job. Shimer (2012) provides an unemployment accounting scheme that focuses on this feature of short-term unemployment. Shimer's characterization of transition rates, in particular, the importance of high and volatile exit rates from unemployment has become very influential in the way macroeconomists quantitatively account for labor markets in search and matching models of unemployment.

In view of the increasing share of long-term unemployment since the 2007-09 recession we provide an extension of the Shimer (2012) unemployment accounting scheme that allows for both, short-term and long-term unemployment, and that uses data on the duration distribution of unemployment. We find that the distinction between short-term and long-term

unemployment is important, and that the share of long-term unemployment has increased since the 1990s. Whereas before 1990 short-term unemployed workers with an average unemployment duration of less than one month accounted for the majority of total unemployment, after 1990 long-term unemployed with an average unemployment duration of eight months account for the majority of unemployment. Furthermore, it appears that most of unemployment volatility is not due to variations in the entry and exit rate of short-term unemployed but due to variations in the entry and exit rates of long-term unemployed. Finally, based on the results from our accounting scheme we provide two examples that suggest the distinction between short-term and long-term unemployment is potentially important for our models of unemployment. First, we show that distinguishing between short-term and long-term unemployment has the potential to significantly increase our estimates of the costs associated with job loss. Second, we show that once one distinguishes between short-term and long-term unemployment search and matching models of unemployment provide a better account of unemployment rate volatility.

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Appendix

A. A hybrid model with ex-ante and ex-post unobserved heterogeneity

This Appendix develops the continuous time version of the model with two types of unemployed agents, $i = 1, 2$. Let $u^i(t, \tau)$ denote the number of type i agents that at time t have been unemployed for duration τ, both t and τ are potentially real valued. Agents become newly unemployed at the instantaneous inflow rates f^i, unemployed agents exit the unemployment state at the instantaneous rate λ^i, and type one agents make the transition to type two at the instantaneous rate γ^1. Let the unit of time be one month. We observe the number of unemployed agents at discrete time intervals. We assume that the instantaneous transition rates are constant for a given time interval, e.g. $\lambda^i(s) = \lambda_t^i$ for $s \in (t-1, t]$ where t is integer valued. Unemployed workers make the following transitions between unit time intervals:

$$\begin{aligned} u^1(t,\tau) &\rightarrow u^1(t+1,\tau+1), u^2(t+1,\tau+1), \text{ or employed} \\ u^2(t,\tau) &\rightarrow u^2(t+1,\tau+1) \text{ or employed} \end{aligned}$$

Between discrete time intervals the measures of agents that have been unemployed for longer than one unit period, that is, for $\tau > 1$, evolve as follows. For type one agents, the measure of agents that remain unemployed is

$$\begin{aligned} u^1(t+1,\tau+1) &= u^1(t,\tau)\, e^{-\left(\lambda_{t+1}^1+\gamma_{t+1}^1\right)} \\ &= u^1(t,\tau)\left(1-\bar{\lambda}_{t+1}^1\right)\left(1-\bar{\gamma}_{t+1}^1\right) \end{aligned}$$

Note that the law of motion is independent of the duration the agent has been unemployed. Similarly, for type two agents the measure of agents that remain unemployed evolves according to

$$u^2(t+1,\tau+1) = u^2(t,\tau)\, e^{-\lambda_{t+1}^2} + u^1(t,\tau)\int_0^1 e^{-\lambda_{t+1}^1 s}\left[\gamma_{t+1}^1 e^{-\gamma_{t+1}^1 s}\right] e^{-\lambda_{t+1}^2(1-s)} ds$$

where the product in the integral represents the probability that a type one agent does not exit unemployment within duration s from time t, makes a transition to type two at duration s, and does not exit unemployment in the remaining time up to $t+1$. This law of motion

can be rewritten as

$$\begin{aligned}
u^2(t+1,\tau+1) &= u^2(t,\tau)\,e^{-\lambda^2_{t+1}} + u^1(t,\tau)\,e^{-\lambda^2_{t+1}}\gamma^1_{t+1}\int_0^1 e^{-\left(\lambda^1_{t+1}+\gamma^1_{t+1}-\lambda^2_{t+1}\right)s}ds \\
&= u^2(t,\tau)\,e^{-\lambda^2_{t+1}} + u^1(t,\tau)\,e^{-\lambda^2_{t+1}}\frac{\gamma^1_{t+1}}{\gamma^1_{t+1}+\lambda^1_{t+1}-\lambda^2_{t+1}}\left[1-e^{-\left(\gamma^1_{t+1}+\lambda^1_{t+1}-\lambda^2_{t+1}\right)}\right] \\
&= \left(1-\bar{\lambda}^2_{t+1}\right)\left\{u^2(t,\tau)+u^1(t,\tau)\,\bar{\psi}^1_{t+1}\right\}
\end{aligned}$$

The number of type i unemployed workers that have been unemployed for s weeks at time t is

$$u^i_{t,s} \equiv \int_s^{s+1} u^i(t,\tau)\,d\tau.$$

Using the above defined transition equations for $u^i(t,\tau)$ we get the following expressions for the law of motion of the measured unemployment stocks

$$\begin{aligned}
u^1_{t+1,s+1} &= \left(1-\bar{\lambda}^1_{t+1}\right)\left(1-\bar{\gamma}^1_{t+1}\right)u^1_{t,s} \\
u^2_{t+1,s+1} &= \left(1-\bar{\lambda}^2_{t+1}\right)\left[u^2_{t,s}+u^1_{t,s}\bar{\psi}^1_{t+1}\right]
\end{aligned}$$

The measures of agents that have been unemployed for less than unit of time are defined as follows. For type one agents, the number of unemployed agents at time t is the cumulative entry into unemployment in the preceding time interval that has not exited again

$$u^1_{t,1} = f^1_t\int_0^1 e^{-\left(\lambda^1_t+\gamma^1_t\right)\tau}d\tau = f^1_t\frac{1-e^{-\left(\lambda^1_t+\gamma^1_t\right)}}{\lambda^1_t+\gamma^1_t} = f^1_t\bar{\phi}^1_t$$

And for type two agents that measure is the cumulative entry into unemployment in the preceding time interval that has not exited again plus the entry of type one agents that have made the transition to type two and also have not exited unemployment

$$\begin{aligned}
u^2_{t,1} &= \int_0^1 f^2_t e^{-\lambda^2_t\tau}d\tau + \int_0^1 f^1_t\left[\int_\tau^1 e^{-\lambda^1_t(s-\tau)}\gamma^1_t e^{-\gamma^1_t(s-\tau)}e^{-\lambda^2_t(1-s)}ds\right]d\tau \\
&= f^2_t\frac{1-e^{-\lambda^2}}{\lambda^2} + f^1_t\frac{\gamma^1_t}{\lambda^1_t-\lambda^2_t+\gamma^1_t}\left\{\frac{1-e^{-\lambda^2_t}}{\lambda^2_t}-\frac{1-e^{-\left(\lambda^1_t+\gamma^1_t\right)}}{\lambda^1_t+\gamma^1_t}\right\} \\
&= f^2_t\bar{\phi}^2_t + f^1_t\frac{\gamma^1_t}{\lambda^1_t-\lambda^2_t+\gamma^1_t}\left(\bar{\phi}^2_t-\bar{\phi}^1_t\right)
\end{aligned}$$

B. A model of misreported unemployment duration

Suppose that the reported unemployment duration follows a 'random walk,' that is, there is a probability distribution over the incremental reported duration that is not contingent on the current reported duration. Thus as long as a worker remains unemployed the reported duration r is an increment i over the previously reported duration and

$$r = r_{-1} + i \text{ with probability } \alpha_i \text{ for } i = -1, 0, +1, +2, +3 \text{ and } r_{-1} > 1$$

For unemployed workers that do not exit unemployment the distribution for the reported duration is constructed as follows. A worker that has just become unemployed correctly reports the duration of one month,

$$\begin{aligned} g(1|1) &= 1 \\ g(r|1) &= 0 \text{ for } r > 1 \end{aligned}$$

A worker that has been unemployed for two months reports durations between one and four months with probabilities

$$\begin{aligned} g(1|2) &= (\alpha_{-1} + \alpha_0)\, g(1|1) \\ g(1+i|2) &= \alpha_i g(1|1) \text{ for } i = 1, 2, 3. \end{aligned}$$

Finally, the probability distribution for reported durations for workers that have been unemployed for more than two months, $s > 2$, is constructed recursively

$$\begin{aligned} g(r|s) &= \alpha_{-1} g(r+1|s-1) + \alpha_0 g(r|s-1) + \alpha_1 g(r-1|s-1) \text{ for } r = 1 \\ g(r|s) &= \alpha_{-1} g(r+1|s-1) + \alpha_0 g(r|s-1) + \alpha_1 g(r-1|s-1) + \alpha_2 g(r-2|s-1) \text{ for } r = 2 \\ g(r|s) &= \sum_{i=-1}^{+3} \alpha_i g(r-i|s-1) \text{ for } r = 3, \ldots, 1 + (s-1) \end{aligned}$$

Conditional on the true duration distribution, the reported duration distribution is simply

$$\hat{u}_t(r) = \sum_{s=1}^{S} g(r|s)\, u_t(s) \text{ or } \hat{u}_t = G u_t.$$

After the 1994 CPS redesign duration report errors should occur only at the time respondents enter the sample. As long as respondents are part of the survey an additional month of unemployment is correctly reflected in the durations. Suppose that the sample is representative for the actual duration distribution. One fourth of the sample is rotated in/out

every month. Workers who become unemployed in the month correctly report their duration, whether they were already part of the sample or whether they are rotated in. Workers who are unemployed for two months correctly report their duration if they are in the sample from the beginning, but one fourth are rotated in and may misreport their duration. Half of the unemployed with three months true duration were always in the sample, so their duration is correct, but one fourth were rotated in with initial true duration two or three and they may misreport. For unemployed with true duration s of four or more months, one fourth entered the sample 3 months ago and reported an initial duration r which was then incremented by one month for every period of unemployment, etc.

$$\begin{aligned}
\hat{u}_t(r,1) &= g(r|1)\,u_t(1) \\
\hat{u}_t(r,2) &= \frac{1}{4}\left[3g(r-1|1)+g(r|2)\right]u_t(2) \\
\hat{u}_t(r,3) &= \frac{1}{4}\left[2g(r-2|1)+g(r-1|2)+g(r|3)\right]u_t(3) \\
\hat{u}_t(r,s) &= \frac{1}{4}\left[g(r-3|s-3)+g(r-2|s-2)+g(r-1|s-1)+g(r|s)\right]u_t(s)
\end{aligned}$$

Thus we get the reported duration distribution as

$$\hat{u}_t(r) = \sum_{s=1}^{S} h(r|s)\,u_t(s) \text{ or } \hat{u}_t = Hu_t$$

Based on Table 5 we choose a probability distribution that is symmetric around the one-month increment

$$\alpha_{-1} = 0.1,\ \alpha_0 = 0.2,\ \alpha_1 = 0.4,\ \alpha_2 = 0.2,\ \alpha_3 = 0.1.$$

The probabilities for reported unemployment durations conditional on the true underlying durations for this parameterization of the reporting error are displayed in Table A.1. From the conditional probabilities we see that the reported durations associated with the post-94 CPS redesign are clustered more tightly around the true durations than they are for the pre-1994 CPS set-up.

Table A.1. Unemployment Duration: Actual and Reported

r	$F(r\|s)$						$G(r\|s)$					
	s						s					
	2	3	4	5	6	12	2	3	4	5	6	12
1	30.0	13.0	6.1	3.1	1.6	0.0	7.5	3.3	1.5	0.8	0.4	0.0
2	40.0	22.0	12.3	6.8	3.8	0.1	85.0	13.0	6.3	3.2	1.7	0.1
3	20.0	27.0	18.9	12.1	7.4	0.4	5.0	66.8	17.7	9.4	5.1	0.2
4	10.0	21.0	21.9	16.9	11.8	0.8	2.5	10.3	47.2	21.9	12.3	0.5
5	0.0	12.0	18.8	18.8	15.3	1.5	0.0	5.5	15.0	26.9	18.3	1.1
6	0.0	4.0	12.6	16.9	16.7	2.6	0.0	1.0	8.7	19.2	21.1	2.1
> 6	0.0	1.0	9.4	25.4	43.4	94.7	0.0	0.3	3.6	18.6	41.1	96.0

Note: Probability (in percent) that a household that has been unemployed for s months reports r months duration for reporting error distribution α.

Table 1. Accounting for Unemployment, January 1950 - December 2010

Sample	1950-2010	1967-2010	1976-2010	1987-2010
		A. Aggregate Statistics		
u	5.7	6.1	6.3	5.8
$\bar{D}$	14.2	14.9	16.2	16.8
		B. Homogeneous Unemployment		
σ	0.16	0.13	0.11	0.09
λ	0.82	0.87	0.89	0.93
Residual	0.02	0.00	-0.00	-0.02
		C. Heterogeneous Unemployment		
σ^1	0.03	0.02	0.00	-0.03
σ^2	0.33	0.32	0.34	0.35
λ^1	0.24	0.23	0.21	0.20
λ^2	0.35	0.40	0.43	0.48
γ^1	0.00	0.00	0.00	0.00
Residual	0.04	0.02	0.01	-0.01

Note: For each sample period, Part A displays the average unemployment rate in percent, u, and the average mean duration of unemployment in weeks, $\bar{D}$; Part B displays the contributions of unemployment entry rates, σ, and exit rates, λ, to unemployment rate volatility for the model with homogeneous unemployment; Part C displays the contributions of entry rates, exit rates, and transition rates, γ, for different types to unemployment rate volatility for the two-type model of unobserved heterogeneity.

Table 2. Male Age Groups, June 1976-Dec 2009

Age	20-24		25-34		35-44		45-54		55-64		65+	
						A. Aggregate Statistics						
u	10.6		5.9		4.4		4.0		3.9		3.5	
$\bar{D}$	14.2		17.1		19.8		22.6		24.0		22.5	
						B. Transition Probabilities						
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ^1	0.063	0.04	0.026	0.10	0.017	0.08	0.013	0.06	0.014	0.05	0.010	0.07
σ^2	0.009	0.17	0.005	0.18	0.003	0.33	0.004	0.37	0.004	0.30	0.006	0.09
λ^1	0.621	0.31	0.580	0.23	0.565	0.17	0.588	0.04	0.612	0.04	0.717	0.07
λ^2	0.168	0.33	0.150	0.34	0.130	0.26	0.132	0.34	0.126	0.40	0.257	0.63
γ^1	0.005	-0.02	0.004	0.00	0.010	0.00	0.005	0.01	0.021	0.06	0.005	0.00
Res		0.17		0.15		0.16		0.18		0.15		0.14

Note: For each age group Part A displays the average unemployment rate in percent, u, and the average mean duration of unemployment in weeks, $\bar{D}$; Part B displays the properties of transition rates for the model with unobserved heterogeneity and two types, where column (1) is the sample average for the monthly transition probability and column (2) is the average contribution of the transition rate to the unemployment rate volatility.

Table 3. Industry Groups, January 2001-December 2009

	MIN		CON		DUR		NDR		WRT		TRU	
	A. Aggregate Statistics											
u	4.9		7.4		5.7		5.7		5.4		4.0	
$\bar{D}$	17.0		15.1		20.2		20.4		17.4		18.4	
	B. Transition Probabilities											
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ^1	0.04	-0.03	0.04	-0.01	0.03	0.07	0.03	-0.01	0.03	-0.07	0.02	0.01
σ^2	0.01	-0.08	0.01	0.40	0.01	0.53	0.01	0.41	0.01	0.08	0.01	0.36
λ^1	0.78	0.36	0.70	0.05	0.73	0.00	0.74	0.04	0.67	0.28	0.77	0.24
λ^2	0.23	0.50	0.17	0.23	0.15	0.29	0.15	0.34	0.16	0.74	0.16	0.27
γ^1	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.07	0.00	0.00	0.01	-0.01
Res		0.24		0.33		0.11		0.15		-0.03		0.12

	IT		FAC		PBS		EHS		LHO		PAD	
	A. Aggregate Statistics											
u	5.0		3.2		5.9		2.8		7.8		2.1	
$\bar{D}$	21.0		19.0		17.9		16.9		15.1		20.3	
	B. Transition Probabilities											
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ_1	0.03	-0.05	0.01	-0.11	0.03	-0.05	0.02	-0.06	0.05	-0.14	0.01	0.00
σ_2	0.01	0.38	0.00	0.21	0.01	0.25	0.00	0.21	0.01	0.69	0.00	0.10
λ_1	0.79	0.21	0.66	0.27	0.68	0.25	0.67	0.23	0.71	0.06	0.71	0.20
λ_2	0.15	0.34	0.15	0.62	0.16	0.49	0.15	0.59	0.17	0.41	0.15	0.42
γ_1	0.04	-0.01	0.01	0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	0.01	0.05
Res		0.13		0.01		0.05		0.04		0.00		0.13

Note: Industries are Mining (MIN), Construction (CON), Durable Goods Manufacturing (DUR), Nondurable Goods Manufacturing (NDR), Wholesale and Retail Trade (WRT), Transportation and Utilities (TRU), Information (IT), Financial Activities(FAC), Professional and Business Services (PBS), Educational and Health Services (EHS), Leisure and Hospitality (LHO), Public Administration (PAD). Parts A and B are as defined in Table 2.

Table 4. Occupation Groups, January 2001 to December 2009

	MBFO		PR		SVC		S		OADM	
	A. Aggregate Statistics									
u	2.6		2.7		6.6		10.7		4.9	
$\bar{D}$	20.6		18.4		16.3		17.5		18.3	
	B. Transition Probabilities									
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ^1	0.01	-0.03	0.01	0.00	0.04	-0.12	0.05	-0.09	0.02	-0.07
σ^2	0.00	0.34	0.00	0.40	0.01	0.40	0.01	0.20	0.01	0.21
λ^1	0.67	0.11	0.68	0.12	0.69	0.15	0.68	0.13	0.69	0.10
λ^2	0.14	0.50	0.15	0.29	0.16	0.51	0.16	0.77	0.16	0.74
γ^1	0.01	-0.01	0.00	0.00	0.00	-0.02	0.00	0.01	0.00	0.00
Res		0.10		0.18		0.07		-0.02		0.02

	FFF		CE		IMR		PROD		TMM	
	A. Aggregate Statistics									
u	11.4		9.2		4.4		7.4		7.2	
$\bar{D}$	14.1		15.3		19.0		19.3		17.4	
	B. Transition Rates									
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
σ_1	0.09	0.00	0.06	0.01	0.03	-0.10	0.03	0.02	0.04	-0.08
σ_2	0.02	0.18	0.01	0.41	0.01	0.17	0.01	0.52	0.01	0.34
λ_1	0.76	0.13	0.71	0.06	0.77	0.15	0.72	-0.01	0.69	0.20
λ_2	0.23	0.10	0.16	0.21	0.16	0.52	0.14	0.35	0.15	0.34
γ_1	0.00	0.00	0.00	0.00	0.02	0.09	0.00	0.01	0.01	0.02
Res		0.58		0.31		0.16		0.11		0.19

Note: Occupations are Management, Business and Finance Operations (MBFO), Professional and related occupations (PR), Services (SVC), Sales and related (S), Office and administrative support (OADM), Farming, Forestry, and Fisheries (FFF): Construction and Extraction (CE), Installation, Maintenance, and Repair (IMR), Production (PROD), Transportation and Material Moving (TMM). Parts A and B are as defined in Table 2.

Table 5. Reported Duration Changes

Duration Change	Full sample	Initial Duration	
		less than 20 weeks	more than 20 weeks
(1)	(2)	(3)	(4)
less than 0 weeks	14	8	26
0-2 weeks	17	16	20
3-5 weeks	32	36	25
6-9 weeks	16	19	12
more than 10 weeks	21	21	17

Note: This table is based on Table 2 from Poterba and Summers (1984) for a sample of households that were unemployed for more than 5 weeks in May 1976 and remained unemployed in June 1976. Column (1) is the change of unemployment duration based on the self-reported durations in the two months; Column (2) displays the shares for the whole sample (in percent) that reported a change in unemployment duration as in Column (1); Column (3) respectively Column (4) display the shares for the reported change in unemployment duration for those that reported less than 20 weeks respectively more than 20 weeks of unemployment in the initial month.

Table 6. Business Cycle Statistics, 1950-2010

	Variable	Mean	St Dev	$Corr(x_{t+i}, u_t)$								
	x_t			-4	-3	-2	-1	0	1	2	3	4
1	f_1/l	0.032	0.0018	0.23	0.26	0.26	0.26	0.19	0.01	-0.15	-0.23	-0.25
2	f_2/l	0.004	0.0011	0.25	0.44	0.60	0.68	0.63	0.49	0.35	0.18	0.04
3	f_1/e	0.034	0.0020	0.27	0.33	0.37	0.41	0.35	0.17	-0.02	-0.12	-0.18
4	f_2/e	0.005	0.0012	0.26	0.45	0.62	0.70	0.66	0.52	0.37	0.20	0.05
5	λ_1	1.022	0.1057	-0.19	-0.36	-0.51	-0.62	-0.66	-0.55	-0.42	-0.28	-0.16
6	λ_2	0.198	0.0397	-0.32	-0.52	-0.63	-0.66	-0.62	-0.44	-0.28	-0.16	-0.05
7	γ	0.003	0.0036	-0.07	0.00	0.05	0.10	0.16	0.10	-0.01	-0.06	-0.08
8	λ_2/λ_1	0.193	0.0279	-0.34	-0.49	-0.54	-0.54	-0.45	-0.27	-0.12	-0.04	0.05
9	f_2/f_1	17.202	3.5327	0.15	0.32	0.47	0.53	0.50	0.42	0.34	0.22	0.11
10	u_2/u	42.441	5.7754	0.04	0.23	0.44	0.61	0.71	0.73	0.68	0.57	0.43

Note: This table reports a variable's mean, standard deviation, and correlation with the total unemployment rate. All variables are quarterly averages of monthly data from January 1950 to December 2010. The levels of all variables are detrended with the Baxter-King band-pass filter. Rows (1) and (2) denote entry rates to unemployment normalized by the labor force, and rows (3) and (4) denote entry rates normalized by employment. Rows (5) and (6) denote the exit rates from unemployment, and row (7) denotes the transition rate from STU to LTU Rows (8) and (9) are the ratios of the unemployment exit and entry rates, and row (10) denotes the relative share of LT unemployed in total unemployment.

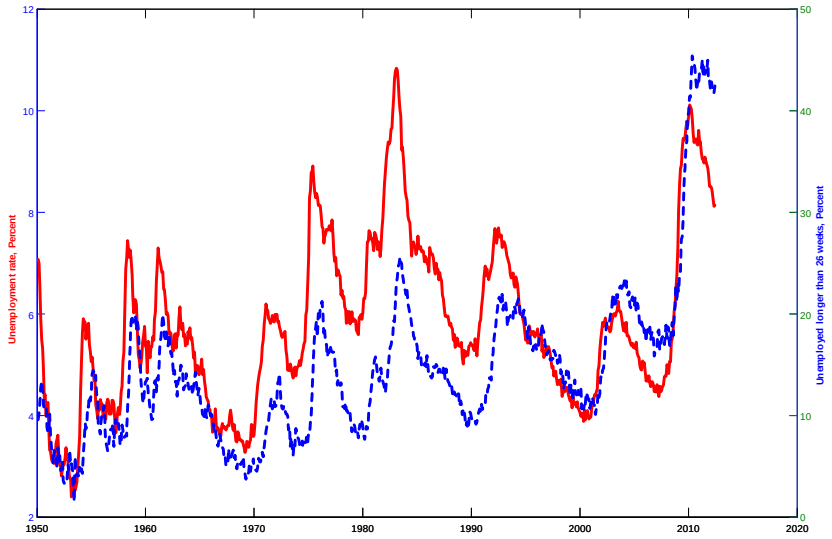

Figure 1. Long-Term Unemployment

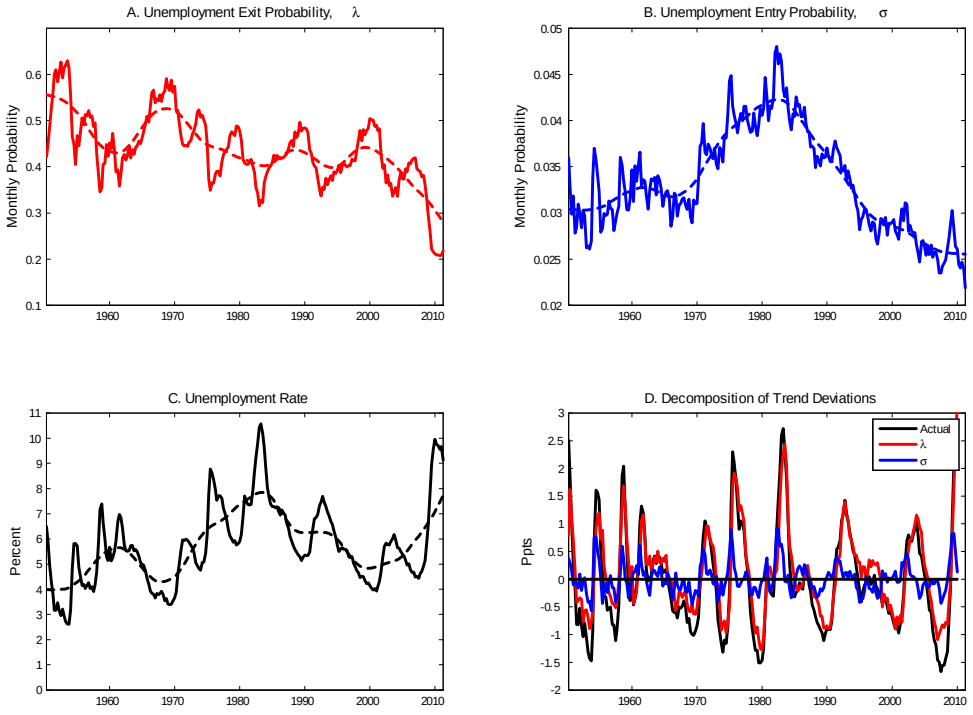

Figure 2. Homogeneous Unemployment

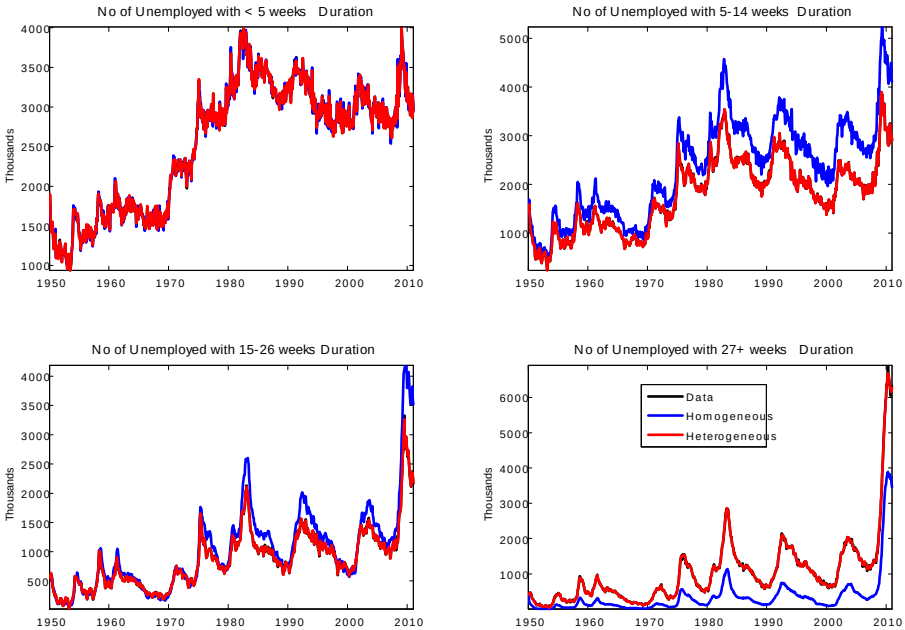

Figure 3. Duration Distribution of Unemployment

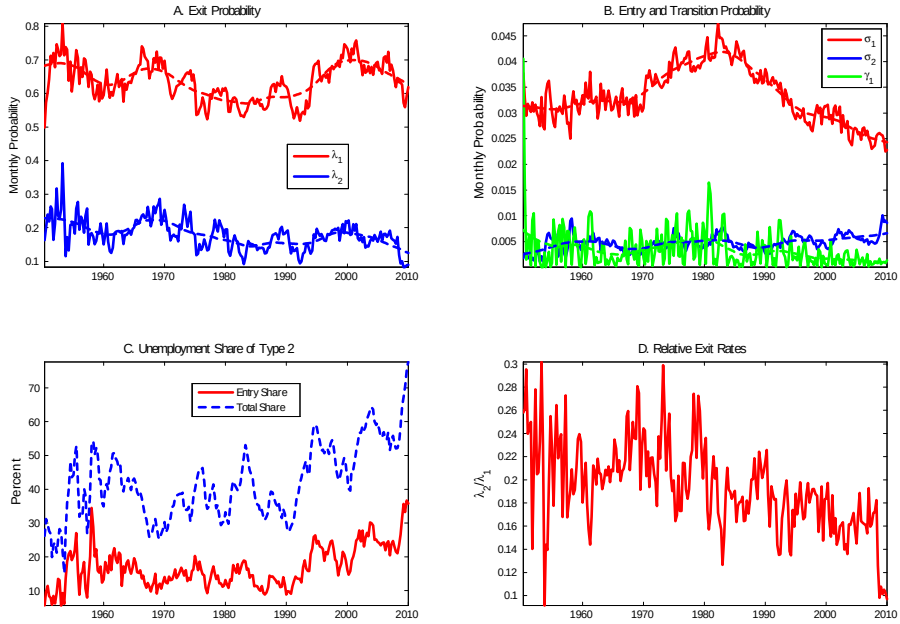

Figure 4. Heterogeneous Unemployment

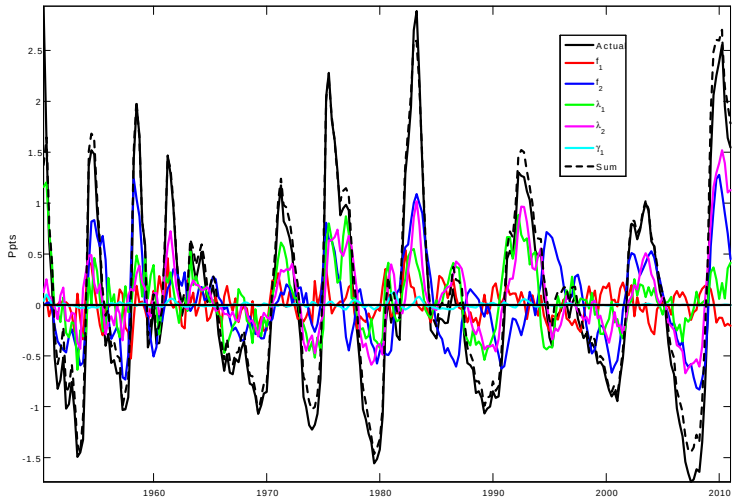

Figure 5. Contributions to Unemployment

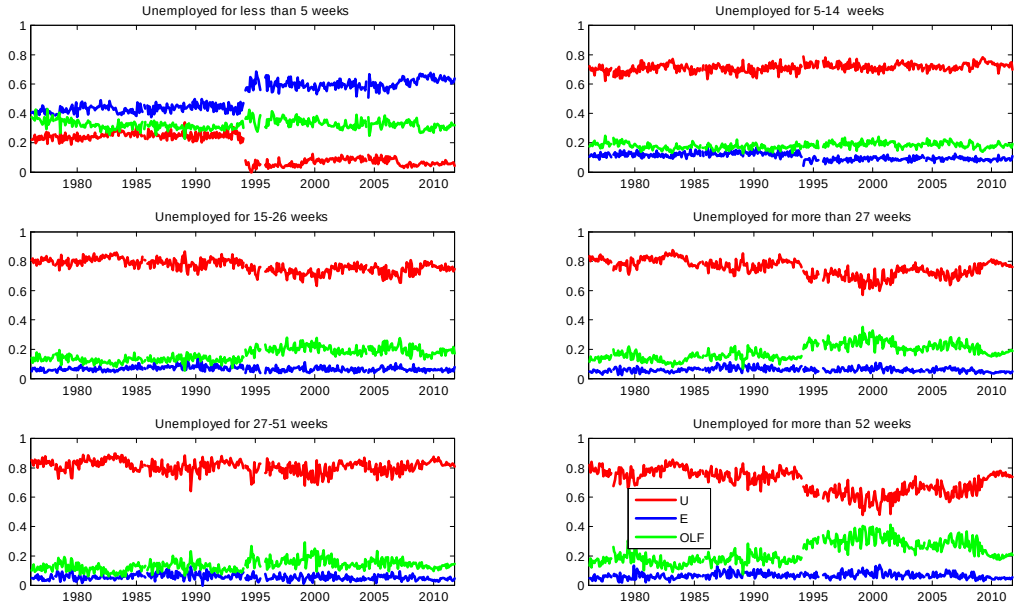

Figure 6. Sources of Unemployment by Duration

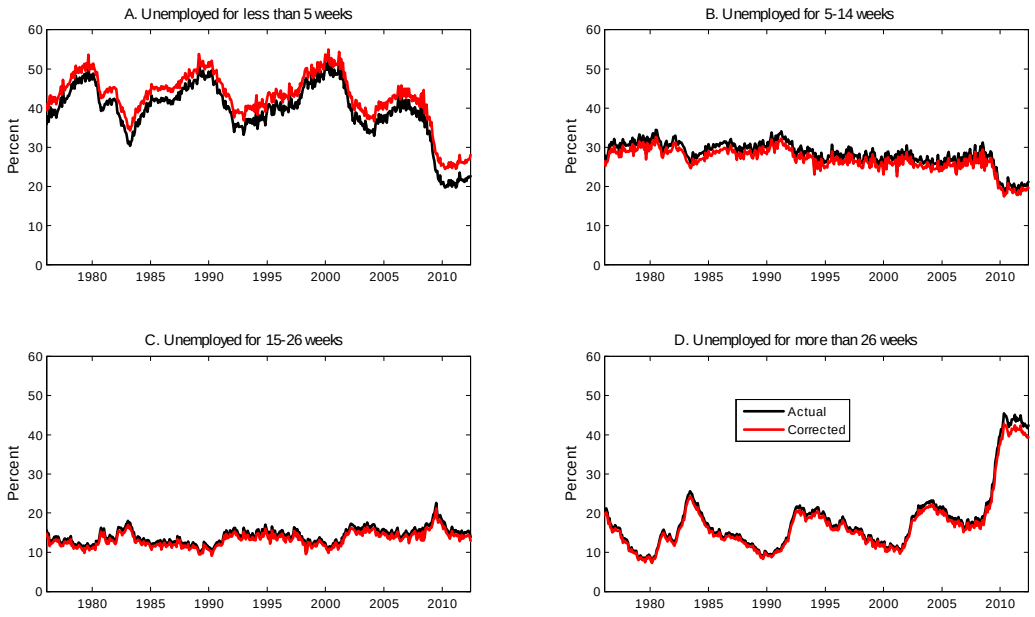

Figure 7. Misclassification Correction of Duration Distribution

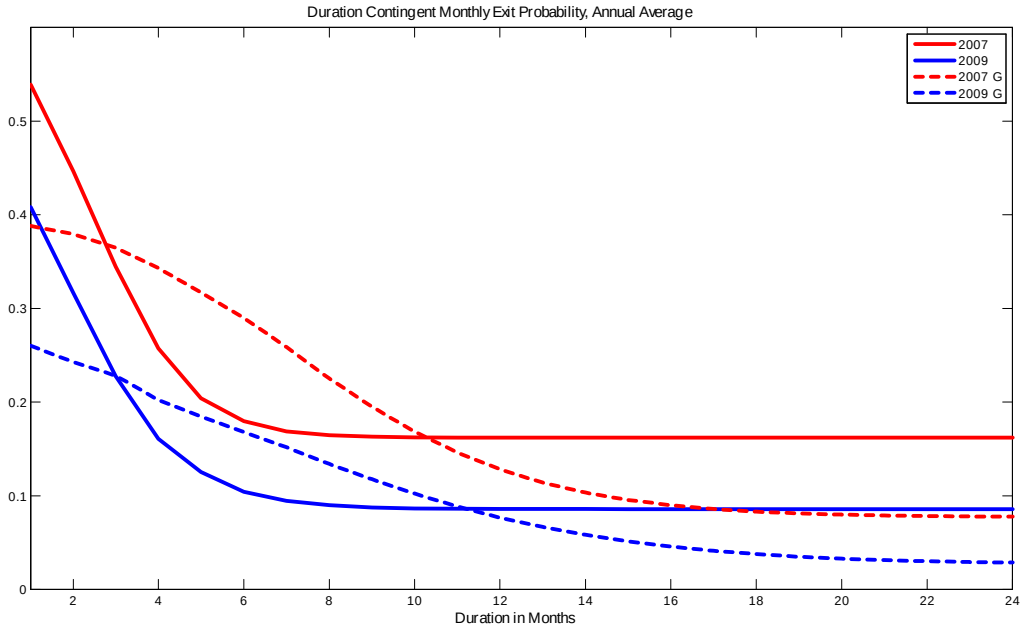

Figure 8. Exit Probabilities from Unemployment

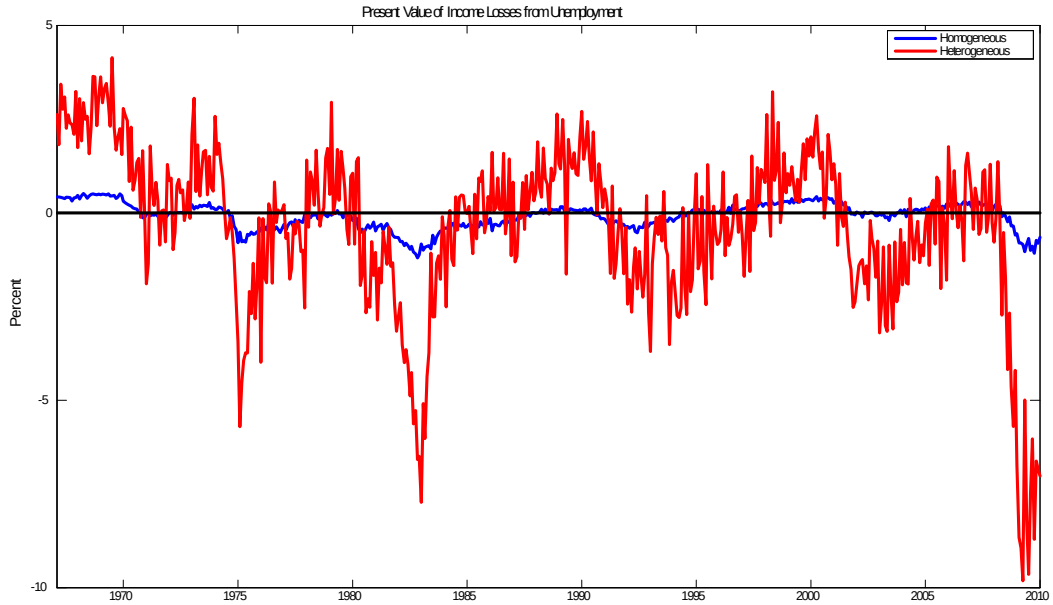

Figure 9. Capital Value of Employment

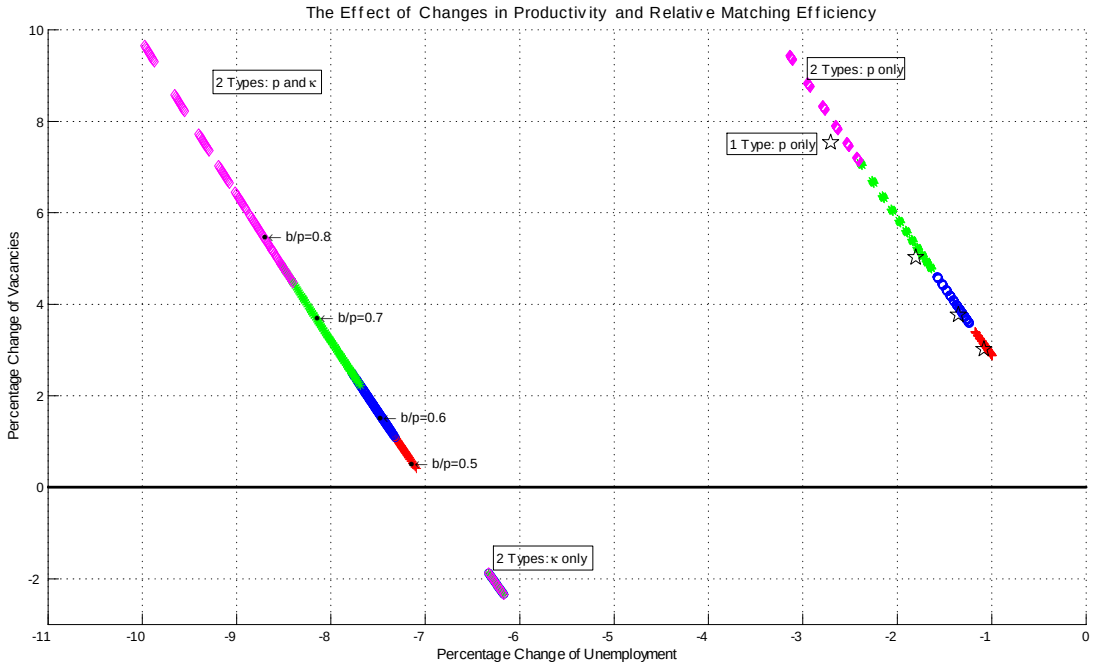

Figure 10. Unemployment Volatility with Pro-Cyclical Relative Matching Efficiency