Optimal State-Contingent Unemployment Insurance*

Juan M. Sánchez[†]

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Abstract

Since the probability of finding a job is affected not only by individual effort but also by the aggregate state of the economy, designing unemployment insurance payments conditional on the business cycles could be valuable. This paper answers a fundamental question related to this issue: How should the payments vary with the aggregate state of the economy?

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[†]Department of Economics, University of Rochester. Email: sncz@troi.cc.rochester.edu. Phone: 585-256-3471. Address: 216 University Park, Rochester, NY, 14620.

1 Introduction

The effort dedicated to job search affects the probability of success. Because effort is not observable, research on *optimal unemployment insurance*, such as that of Shavell and Weiss (1979), Wang and Williamson (1996) and Hopenhayn and Nicolini (1997), studies this issue as a repeated moral hazard problem. In addition to individual searching effort, aggregate conditions of the economy –business cycles– determine the probability of finding a job. During the last 50 years, the U.S. monthly job finding rate was below 20 percent in the most severe recessions and above 40 percent in times of economic prosperity (see Figure 1). Existing research on *optimal unemployment insurance* ignores this fact and considers a setup without business cycles.[1] Incorporating aggregate fluctuations to the previous framework this paper provides three important contributions. First, it analyzes the robustness of previous results to the incorporation of business cycles. Second, it characterizes the *optimal state-contingent unemployment insurance*. Finally, it provides a contract that can be used for a comparison with the current U.S. unemployment insurance system, which varies the duration of payments on the business cycles.

The main difference with the previous literature is the presence of aggregate fluctuations in the model. However, this paper also differs as it incorporates moral hazard during employment.[2] With this feature in the model, "after tax wages" may vary during employment to provide incentives.

The insurance problem presented here can be characterized using standard recursive contracts techniques. In particular, it can be analyzed using the approach

[1] Although it does not include aggregate fluctuations in the model, Kiley (2003) analyzes the relationship between unemployment insurance and business cycles comparing numerical results for different calibrations of the job finding probability. Its conclusions are in line with the theoretical results derived in this paper.

[2] Zhao (2000) and Wang and Williamson (2002) also incorporate this feature. However, none of them characterize the dynamic of "after tax wages" during employment.

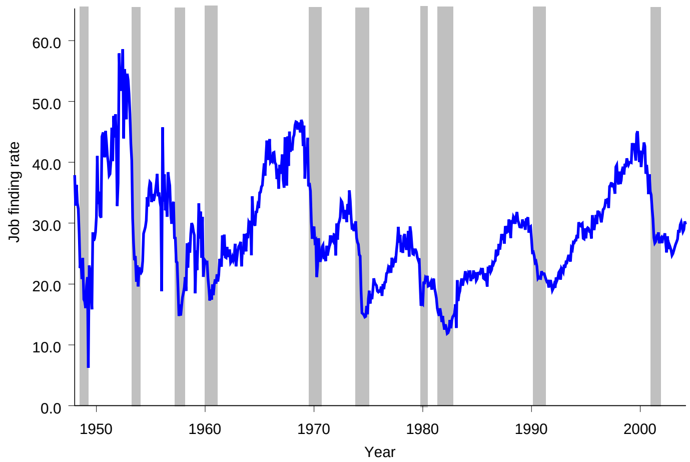

Figure 1: **Probability of finding a job and business cycles in the U.S.** Shadow areas are recessions according to the NBER. The job finding rate was obtained from Hall (2005).

developed by Spear and Srivastava (1987), Abreu, Pearce, and Stacchetti (1990), Phelan and Townsend (1991) and Atkeson and Lucas (1995). The analysis provides two key results. First, business cycles do not alter the most important findings in this literature. That is, unemployment insurance payments decrease with unemployment duration and the taxes financing this system decrease with tenure. Second, under reasonable restrictions on the probability functions, unemployment payments should decrease faster during economic booms than during recessions.

2 The Model

This section presents the infinitely repeated moral hazard problem used to characterize the *optimal state-contingent unemployment insurance*. First, all the elements needed to write this problem formally are described. And second, the problem is carefully stated using standard recursive contract techniques. Its sequential representation is described in the appendix.

2.1 Environment

Time is discrete and denoted by $t = 0, 1, 2, ...$ There is a single perishable good whose consumption is denoted by c. *Agents* are infinitely-lived and risk averse. At each period t, an agent's *employment status* $\varsigma \in \mathbf{S} = \{e, u\}$ takes one of two states: employed, e or unemployed, u. The *aggregate state of the economy* is indexed by $i \in \mathbf{N} = \{1, ..., N\}$. Thus, the relevant state variable for each individual is $s_t = (\varsigma, i) \in \mathbf{S} \times \mathbf{N}$. Let $s^t = \{s_1, s_2, ..., s_t\} \in (\mathbf{S} \times \mathbf{N})^t$ denotes the history of s_t up to date t.

The agent's output depends just on employment status.[3] Formally,

$$\varpi(s_t) = \begin{cases} \omega \text{ if } s_t = (e, i) \text{ for each } i \\ 0 \text{ if } s_t = (u, i) \text{ for each } i. \end{cases} \tag{1}$$

An agent that is employed at period t stays employed at $t + 1$ with probability $q(h)$, where $h \in \mathbb{R}_+$ is the *level of effort* that the agent exerts in period t. Similarly, an unemployed agent finds a job next period with probability $p_i(h)$.[4] The functions $p_i : \mathbb{R}_+ \to [0, 1]$ and $q : \mathbb{R}_+ \to [0, 1]$ are strictly increasing, strictly concave and twice differentiable in $h \in \mathbb{R}_+$ for each $i \in \mathbf{N}$ and satisfy standard Inada conditions so that the optimal effort h is interior. Also, p_i is increasing in i for each h. That is, higher values of i mean that aggregate conditions are better –it is easier to find a job.

The aggregate state of the economy follows a discrete-time Markov Chain. Let π_{ij} denotes the probability of transition from the aggregate state i to j, where $i, j \in$

[3] This is a simplification. If output ϖ depends on the state of the economy, but the agent cannot affect it, all results still hold. If the agent can affect output, results on the dynamics of unemployment insurance payments do not change. The last case is analyzed by Zhao (2000).

[4] Notice that aggregate conditions of the economy do not affect the job-keeping probability q while they do affect the job-finding probability p. This assumption is supported by empirical evidence: Hall (2005) and Shimer (2005) find that fluctuations in the separation rate over the business cycles are very small.

N. The agent discounts the future by the factor $\beta \in (0, 1)$. The utility function for consumption is represented by $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ and is strictly increasing, strictly concave and differentiable with $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. Likewise, the cost of effort function is represented by $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ and is strictly increasing, convex and differentiable with $\lim_{h \to 0} v'(h) = 0$ and $\lim_{h \to \infty} v'(h) = \infty$.

In the next section, the recursive formulation of the optimal contract problem is carefully stated.

2.2 Recursive problem

The planner's goal is to maximize *surplus*, given that the agent gets certain level of life-time utility.[5] The optimal policies of this problem depend on the complete employment history of the individual. However, it can be simplified following Spear and Srivastava (1987) and including life-time utility as a state variable. This variable is sufficient to summarize information about an individual total employment history.

Additional assumptions are standard. The agent is not allowed to save. The planner can borrow or lend resources without affecting the interest rate r, which satisfies $(1 + r)\beta = 1$. He can observe the agent's employment status history but he cannot observe the agent's effort.

Let w denotes *net wages* and b *unemployment insurance payments*. Since they are not allowed to save, agents' consumption is b or w depending if they are unemployed or employed, respectively. Finally, *promised life-time utility* for the state $s = (\varsigma, j)$ is U_j^ς. Thus, a recursive contract specifies: consumption (b or w), effort (h) and next period promised life-time utilities U_j^ς for each employment state (ς) and for each aggregate state of the economy (j).

[5] This problem is the dual of the maximization of life-time utility subject to an intertemporal budget constraint.

Two conditions restrict the planner's maximization. The *incentive compatibility constraint* assures that the agent cannot be better off by deviating from the recommended sequence of effort. The *promise keeping constraint* guarantees a given level of life-time utility to the agent. Both constraints require analyzing the agent's problem given a contract. Therefore, consider the problem of an unemployed agent in the aggregate state i given b, U_j^u and U_j^e

$$\max_{h \geq 0} \left\{ u(b) - v(h) + \beta \sum_{j=1}^{N} \pi_{ij} \left[p_j(h) \left(U_j^e - U_j^u \right) + U_j^u \right] \right\}. \tag{2}$$

The first order condition characterizing effort is

$$\beta \sum_{j=1}^{N} \pi_{ij} \left[p_j'(h) \left(U_j^e - U_j^u \right) \right] = v'(h). \tag{3}$$

Equation (3) is the *incentive compatibility constraint* of the planner's problem with an agent starting the contract unemployed. Similarly, consider the problem of an employed agent in the aggregate state i given w, U_j^u and U_j^e

$$\max_{h \geq 0} \left\{ u(w) - v(h) + \beta \sum_{j=1}^{N} \pi_{ij} \left[q(h) \left(U_j^e - U_j^u \right) + U_j^u \right] \right\}. \tag{4}$$

In this case, the first order condition,

$$\beta \sum_{j=1}^{N} \pi_{ij} \left[q'(h) \left(U_j^e - U_j^u \right) \right] = v'(h), \tag{5}$$

is the *incentive compatibility constraint* of the planner's problem with an agent starting the contract employed.

Using first order conditions as incentive compatibility constraints simplifies the analysis. However, the validity of this approach is questioned in some frameworks.[6]

[6] Kocherlakota (2004) shows that this approach fails for the case of optimal unemployment insurance with hidden saving if the cost of effort function has sufficiently low curvature.

This is not troublesome here. The concavity of the utility and the probability functions and the convexity of the cost of effort function guarantee that first order conditions are necessary and sufficient.

The *discounted life-time surplus of a planner* is $B : \mathbb{R} \times \mathbf{N} \to \mathbb{R}$ if the agent starts the contract unemployed and $W : \mathbb{R} \times \mathbf{N} \to \mathbb{R}$ if the agent starts the contract employed. Thus, life-time surplus of a planner providing insurance to an agent starting the contract unemployed, with promised life-time utility U, when the aggregate state is i, is represented by

$$\begin{aligned} B_i(U) = & \max_{h \geq 0, b \geq 0, \left\{U_j^u, U_j^e\right\}} -b \\ & + \tfrac{1}{1+r} \sum_{j=1}^{N} \pi_{ij} \left\{ p_j(h) \left[W_j(U_j^e) - B_j(U_j^u) \right] + B_j(U_j^u) \right\}, \end{aligned} \tag{6}$$

subject to the incentive compatibility constraint (3) and the promise keeping constraint

$$u(b) - v(h) + \beta \sum_{j=1}^{N} \pi_{ij} \left[p_j(h) \left(U_j^e - U_j^u \right) + U_j^u \right] \geq U. \tag{7}$$

Likewise, the discounted life-time surplus of a planner providing insurance to an agent starting the contract employed, with promise utility U, when the aggregate state is i, is represented by

$$\begin{aligned} W_i(U) = & \max_{h \geq 0, w \geq 0, \left\{U_j^e, U_j^u\right\}} \omega - w \\ & + \tfrac{1}{1+r} \sum_{j=1}^{N} \pi_{ij} \left\{ q(h) \left[W_j(U_j^e) - B_j(U_j^u) \right] + B_j(U_j^u) \right\}, \end{aligned} \tag{8}$$

subject to the incentive compatibility constraint (5) and the promise keeping constraint

$$u(w) - v(h) + \beta \sum_{j=1}^{N} \pi_{ij} \left[q(h) \left(U_j^e - U_j^u \right) + U_j^u \right] \geq U. \tag{9}$$

3 How should the payments vary?

The optimal contract presented in the previous section prescribes payments for unemployed agents –unemployment payments– and for employed agents –net wages. The main remaining question is how should these payments vary. To answer this question, the first order conditions of the planner's problems are intensively used.[7] Subindexes indicating the current state of the economy, i, are dropped to simplify notation. All the results here hold for each $i \in \mathbf{N}$.

3.1 Analysis

There are two key equations that can be obtained from the system of first order conditions of the planner's problem with an agent starting the contract unemployed.[8] The first one is obtained using the first order conditions with respect U_j^u and b, together with the envelope condition. The first order condition with respect to U_j^u is

$$\frac{1}{1+r}\pi_{ij}\left(1-p_j(h)\right)B_j'(U_j^u)+\lambda\beta\pi_{ij}\left(1-p_j(h)\right)-\mu\beta\pi_{ij}p_j'(h)=0. \tag{10}$$

Replacing the first order condition with respect to b and the envelope condition, it implies

$$-\frac{1}{1+r}\pi_{ij}(1-p_j(h))\frac{1}{u'(b_j')}+\frac{\beta}{u'(b)}\pi_{ij}(1-p_j(h))-\mu\beta\pi_{ij}p_j'(h)=0. \tag{11}$$

Finally, using $\beta(1+r)=1$ and reordering terms it transpires

$$\frac{1}{u'(b)}=\frac{1}{u'(b_j')}+\mu\frac{p_j'(h)}{(1-p_j(h))}, \tag{12}$$

[7]In particular, we use analogous equations to those used by Hopenhayn and Nicolini (1997) to prove that unemployment benefits decrease over time while the worker remains unemployed. Pavoni (2003), in a similar setup, proves that the interior solution to this problem can be characterized by its first order conditions.

[8]The complete system of first order equations is presented in the appendix.

where μ is the multiplier of the incentive compatibility constraint, which is always positive.[9] To find the second important equation the first order condition with respect to U_j^e is now considered. This gives

$$\frac{1}{u'(b)} = \frac{1}{u'(w_j')} - \mu \frac{p_j'(h)}{p_j(h)}. \tag{13}$$

Adding equations (12) and (13) and using Jensen's inequality implies

$$u'(b) < p_j(h)u'(w_j') + (1 - p_j(h))\, u'(b_j'). \tag{14}$$

Multiplying (14) by π_{ij} and summing across j yield

$$u'(b) < \sum_{j=1}^{N} \pi_{ij} \left[p_j(h)u'(w_j') + (1 - p_j(h))\, u'(b_j') \right]. \tag{15}$$

This inequality provides the intuition for the dynamics of benefits: the optimal way to provide incentives in the next period is to punish the agent so severely during unemployment that it would like to save in the current period.[10] Saving-constrained agents are willing to increase expected consumption for the next period. This could be achieved, in principle, by saving or by exerting higher effort. Since savings are not allowed, inequality (15) provides incentives for high effort.

3.2 Unemployment payments' dynamics

The optimal unemployment insurance system prescribes a decreasing function for payments over time. An analogous result was first provided by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) in similar frameworks but without business cycles.

Lemma 1 *For each $j \in \boldsymbol{N}$, unemployment insurance payments, b, decrease with unemployment spell.*

[9]Lemma 1 in Hopenhayn and Nicolini (1997) shows that μ is positive. The same argument holds here.

[10]This intuition is provided by Kocherlakota (2004) in a related framework.

Proof. By inspecting equation (12), it follows that $1/u'(b'_j) < 1/u'(b)$. Then, by strict concavity of the utility function it transpires that $b'_j < b$ for each j. That is, unemployment payments decrease from the current period to the next one for each possible state of the aggregate economy. ■

Before characterizing unemployment payments variations across the aggregate states of the economy, it is useful to define the *elasticity of the probability of staying unemployed with respect to effort*,

$$\xi_h^j \equiv \frac{\partial(1-p_j(h))}{\partial h}\frac{h}{(1-p_j(h))} = -\frac{p'_j(h)h}{(1-p_j(h))}. \tag{16}$$

The main result of this paper (proposition 1) uses the condition that this elasticity is decreasing with the aggregate state of the economy, i.e. $\xi_h^j > \xi_h^{j+1}$. This *decreasing elasticity condition* is intuitive and natural. If effort increases, the probability of staying unemployed decreases proportionally more during a boom than during a recession. Thus, this condition is natural if recessions are thought as periods when there is lower return to the effort exerted to looking for a job. Moreover, in terms of specific functional forms, notice that it is satisfied for reasonable specifications of $p_j(h)$. For example, the most common specification could be $p_j(h) = 1 - e^{-\rho_j h}$ with ρ_j increasing in j. In this case, $\xi_h^j = -\rho_j h$ clearly satisfies the decreasing elasticity condition. Technically, this condition requires

$$p_{j+1}(h) - p_j(h) \geq \frac{(p'_j(h) - p'_{j+1}(h))}{p'_j(h)}(1-p_j(h)), \tag{17}$$

which is clearly satisfied, for example, if $p'_{j+1}(h) \geq p'_j(h)$.

Proposition 1 *$b - b'_j$ is increasing in j if and only if ξ_h^j is decreasing in j.*

Proof. Taking b as given, it is required to show that $b'_{j+1} < b'_j$ occurs if and only if $\xi_h^j > \xi_h^{j+1}$. First notice that the latter condition holds if and only if

$$\frac{p'_{j+1}(h)}{1 - p_{j+1}(h)} > \frac{p'_j(h)}{1 - p_j(h)}. \tag{18}$$

Then, the first direction of the proposition is to show that $b'_{j+1} < b'_j$ holds if $\frac{p'_{j+1}(h)}{1-p_{j+1}(h)} > \frac{p'_j(h)}{1-p_j(h)}$. For this purpose, notice that equation (12) can also be written for $j+1$,

$$\frac{1}{u'(b)} = \frac{1}{u'(b'_{j+1})} + \mu \frac{p'_{j+1}(h)}{(1 - p_{j+1}(h))}. \tag{19}$$

Then, equations (12) and (19) imply

$$\frac{1}{u'(b'_j)} + \mu \frac{p'_j(h)}{(1 - p_j(h))} = \frac{1}{u'(b'_{j+1})} + \mu \frac{p'_{j+1}(h)}{(1 - p_{j+1}(h))}. \tag{20}$$

Therefore,

$$\frac{1}{u'(b'_j)} - \frac{1}{u'(b'_{j+1})} = \mu \left[\frac{p'_{j+1}(h)}{(1 - p_{j+1}(h))} - \frac{p'_j(h)}{(1 - p_j(h))} \right]. \tag{21}$$

Hence, if $\frac{p'_{j+1}(h)}{1-p_{j+1}(h)} > \frac{p'_j(h)}{1-p_j(h)}$, $\frac{1}{u'(b'_{j+1})} < \frac{1}{u'(b'_j)}$. Then, by strict concavity of the utility function, $b'_{j+1} < b'_j$.

For the proof in the other direction, it is enough to show $\frac{p'_{j+1}(h)}{1-p_{j+1}(h)} \leq \frac{p'_j(h)}{1-p_j(h)}$ implies $b'_{j+1} \geq b'_j$. Using equation (21) it is clear that $\frac{p'_{j+1}(h)}{1-p_{j+1}(h)} \leq \frac{p'_j(h)}{1-p_j(h)}$ implies $\frac{1}{u'(b'_{j+1})} \geq \frac{1}{u'(b'_j)}$. Then, by strict concavity of the utility function, $b'_{j+1} \geq b'_j$. ■

The intuition for the result in the last proposition is appealing. If an agent did not find a job during a boom, it is more likely that he was not exerting enough effort, compared to someone looking for a job during a recession. Therefore, to provide incentives, the former should receive a tougher punishment than the latter. This is in fact what occurs if payments decrease faster during booms than recessions. Similarly, this result can be interpreted in terms of investing effort in looking for a job. Job search effort during a boom has a higher return than during a recession. To take advantage of this higher return, the optimal contract gives incentives for higher effort during booms providing lower unemployment payments.

The previous results completely characterize the *optimal state-contingent unemployment insurance*. Payments decrease with unemployment spell. Moreover, the rate of decrease varies across the business cycles. In particular, unemployment payments decrease faster during booms than during recessions (see Figure 2).

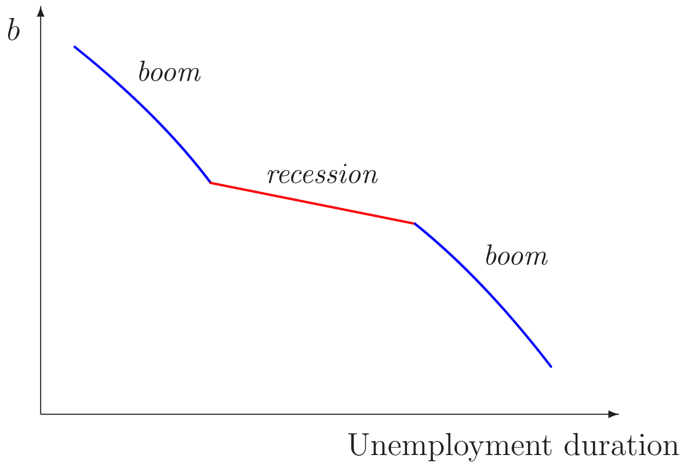

Figure 2: **State-contingent unemployment payments and business cycles**. This figure illustrates the dynamics of payments studied in lemma 1 and proposition 1.

3.3 Net wages' dynamics

The following two lemmas characterize the net wages' dynamics. First, lemma 2 shows that net wages are increasing with tenure.

Lemma 2 *For each $j \in \boldsymbol{N}$, net wages, w, increase with tenure.*

Proof. This proof uses the analogous equation to (12) for the case of the planner's problem with an agent starting the contract employed. It can be obtained taking first order conditions of the planner's surplus with an agent starting the contract employed.

In particular, the first order conditions with respect to w and U_j^e, together with the envelope condition. This equation is

$$\frac{1}{u'(w)} = \frac{1}{u'(w_j')} - \mu_e \frac{q'(h)}{q(h)}, \tag{22}$$

where μ_e is the multiplier of the incentive compatibility constraint, which is always positive.

Inspecting (22) it is clear that $1/u'(w_j') > 1/u'(w)$. Thus, by strict concavity of the utility function, it transpires $w_j' > w$ for each j. That is, the optimal way to give incentives is to increase net wages (decrease taxes) of employed agents that keep their jobs for the next period. ■

The next result shows that the increase in net wages during employment duration does not depend on the aggregate state of the economy. This result depends on the assumption that the function q does not depend on j. As it was previously mentioned, this assumption was motivated by empirical evidence on fluctuations in the separation rate.

Lemma 3 *$w_j' - w$ does not dependent on j.*

Proof. To prove this lemma it is enough to show that $w_j' = w_{j+1}'$ for each $j, j+1 \in \mathbf{N}$. First, notice that equation (22) can be also obtained for $j+1$. It is

$$\frac{1}{u'(w)} = \frac{1}{u'(w_{j+1}')} - \mu_e \frac{q'(h)}{q(h)}. \tag{23}$$

Now, subtracting equation (22) from (23) we obtain

$$\frac{1}{u'(w_{j+1}')} - \frac{1}{u'(w_j')} = 0. \tag{24}$$

Using strict concavity of the utility function equation (24) implies $w_j' = w_{j+1}'$ and therefore the difference $w_j' - w$ does not depend on j. ■

4 The optimal contract and the current system

Previous sections describe how the optimal unemployment insurance responds to business cycles. But how does the current system reacts to aggregate fluctuations? And more importantly, how does the current system variations relate to those prescribed by the optimal contract?

The current unemployment insurance system in the U.S. contains triggers for the extension of payments during recessions. In particular, it extends payments from 26 to 39 weeks during periods of high unemployment.[11] Moreover, in economic downturns, the federal government has traditionally supplemented regular state unemployment insurance with additional payments. According to the United States Congress (2004), before enactment of a permanent Extended Benefits Program, Congress authorized two temporary programs, during 1958 and 1959 and again in 1961 and 1962. During the 1970s and 1980s, temporary programs provided supplemental benefits to unemployment compensation recipients who had exhausted both their regular and extended benefits during periods of high unemployment. The *Emergency Unemployment Compensation* (EUC) following the 1990-91 recession was in effect from November 1991 through February 1994. The program following the 2001 recession, *Temporary Extended Unemployment Compensation* (TEUC), was implemented from March 2002 through December 2003.[12]

At first sight, the current system might not seem to be related with the *optimal state-contingent unemployment insurance.* However, this is not the case if we consider governments constrained to use step functions for payments.[13] If this is the case, to

[11] These payments are usually called *extended benefits*. For a better description see the United States Congress (2004).

[12] Some of these periods are slightly lagged with respect to the business cycles defined by the NBER. However, they are closely related to the cycles in the job finding rate depicted in Figure 1.

[13] This scenario is appealing because most of the governments providing unemployment insurance use step functions.

approximate different slopes the government is restricted to change the duration of each step –longer duration approximates flatter functions– or the distance between them –shorter distance approximates flatter functions. Thus, it seems reasonable to relate variations in the duration of unemployment payments, as those in the U.S. system, with changes in the rate at which they decrease, as those prescribed by the optimal contract. In addition, since the U.S. unemployment insurance system increases the duration of payments during recessions, it approximates the variation prescribed by the optimal contract over the business cycles.

But is the U.S. unemployment insurance *really* close to the optimal contract? To answer this question it is required to compute the optimal contract and to compare its cost with the current system given that they provide the same level of life-time utility. This quantitative exercise is beyond the scope of this paper.[14]

5 Concluding remarks

In Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), the effort dedicated to job search affects the probability of success. However, in addition to individual searching effort, business cycles determine the probability of finding a job. This paper presents a generalization of Hopenhayn and Nicolini (1997) allowing for fluctuations in the probability of finding a job over the business cycles and providing a characterization of the *optimal state-contingent unemployment insurance.*

The most important findings correspond to the key features of the model. First, standard results in the literature still hold: unemployment payments decrease with unemployment duration and net wages increase with tenure. Second, under specifications of the probability of finding a job satisfying an intuitive and natural condition,

[14]Obviously, schemes providing benefits according to a decreasing step function in which the duration and the level of each step vary over the business cycles could be more accurate to approximate the optimal contract.

unemployment payments should decrease faster during economic booms than during recessions.

This paper opens theoretical and quantitative questions for future research. On the theoretical ground, it could be valuable to analyze this issue in a model where the interest rate is determined in equilibrium. For example, working on the framework of Atkeson and Lucas (1995). In quantitative terms, it could be natural to ask whether the U.S. system is *really* close –in terms of the welfare associated with it– to the *optimal state-contingent unemployment insurance.*

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Appendix

Sequential planner's problem

The *optimal dynamic contract* is a history-dependent earnings scheme $\{c(s^t)\}_{t=1}^{\infty}$ and effort recommendations $\{h(s^{t-1})\}_{t=1}^{\infty}$ that maximize the ex-ante expected surplus of the planner, subject to *incentive compatibility constraint* and *promise keeping constraint*.

Let $h^t \in H^t$ be the history of effort level. Denote by $\mu(s^{t+1}; s_0, h^t)$ to the probability of being in the publicly observed node s^{t+1} given the initial state and the history of effort h^t. Then, the sequential incentive compatibility and promised keeping constraints can be formally stated as:

$$\begin{array}{l} \sum\limits_{t=\tau}^{\infty}\sum\limits_{s^t}\beta^t\mu(s^t;s_0,h^{t-1})\left[u\left(c(s^t)\right)-v\left(h(s^t)\right)\right]\geq \\ \sum\limits_{t=\tau}^{\infty}\sum\limits_{s^t}\beta^t\mu(s^t;s_0,\overline{h}^{t-1})\left[u\left(c(s^t)\right)-v\left(\overline{h}(s^t)\right)\right]\forall\tau,s^t,\overline{h}^t, \end{array} \tag{25}$$

and

$$\sum_{t=1}^{\infty}\sum_{s^t}\beta^t\mu(s^t;s_0,h^{t-1})\left[u\left(c(s^t)\right)-v\left(h(s^t)\right)\right]\geq U. \tag{26}$$

To simplify notation, call a *dynamic contract* $\{c(s^t), h(s^t)\}_{t=1}^{\infty} = (c, h)$. Then, the sequential problem is

$$\max_{(c,h)}\sum_{t=1}^{\infty}\sum_{s^t}\frac{\mu(s^t;s_0,h^{t-1})}{(1+r)^{t-1}}\left[\varpi(s^t)-c(s^t)\right], \tag{27}$$

subject to (25) and (26).

Planner's problems and their first order conditions

The planner's problems and their first order conditions are provided in this section. First, notice that planner's problem with an agent starting the contract unemployed is

$$\begin{array}{l} B_i(U)=\sup\limits_{h\geq 0,b,\left\{U_j^u,U_j^e\right\}}\inf\limits_{\lambda,\mu\geq 0}-b \\ +\frac{1}{1+r}\sum\limits_{j=1}^{N}\pi_{ij}\left\{p_j(h)\left[W_j(U_j^e)-B_j(U_j^u)\right]+B_j(U_j^u)\right\} \\ +\lambda\left\{u(b)-v(h)+\beta\sum\limits_{j=1}^{N}\pi_{ij}\left[p_j(h)\left(U_j^e-U_j^u\right)-U_j^u\right]-U\right\} \\ +\mu\left[\beta\sum\limits_{j=1}^{N}\pi_{ij}p_j'(h)\left(U_j^e-U_j^u\right)-v'(h)\right]. \end{array} \tag{28}$$

The interior solution of this problem can be described by its constraints and the following system of equations,

$$
\begin{aligned}
&(b:) \quad -1+\lambda u'(b)=0,\\
&(h:) \quad \tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{p_j'(h)\left[W_j(U_j^e)-B_j(U_j^u)\right]\right\}\\
&+\lambda\left\{-v'(h)+\beta\sum_{j=1}^{N}\pi_{ij}\left[p_j'(h)\left(U_j^e-U_j^u\right)\right]\right\}\\
&+\mu\left[\beta\sum_{j=1}^{N}\pi_{ij}p_j''(h)\left(U_j^e-U_j^u\right)-v''(h)\right]=0,\\
&(U_j^u:) \quad \tfrac{1}{1+r}\pi_{ij}\left(1-p_j(h)\right)B_j'(U_j^u)+\lambda\beta\pi_{ij}\left(1-p_j(h)\right)-\mu\beta\pi_{ij}p_j'(h)=0,\\
&(U_j^e:) \quad \tfrac{1}{1+r}\pi_{ij}p_j(h)W'(U_j^e)+\lambda\beta\pi_{ij}p_j(h)+\mu\beta\pi_{ij}p_j'(h)=0.
\end{aligned}
\tag{29}
$$

Likewise, the planner's problem with an agent starting the contract unemployed is

$$
\begin{aligned}
W_i(U) = &\sup_{h\geq 0, w, \left\{U_j^u, U_j^e\right\}} \inf_{\lambda,\mu_e\geq 0} \omega - w\\
&+\tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{q(h)\left[W_j(U_j^e)-B_j(U_j^u)\right]+B_j(U_j^u)\right\}\\
&+\lambda_e\left\{u(w)-v(h)+\beta\sum_{j=1}^{N}\pi_{ij}\left[q(h)\left(U_j^e-U_j^u\right)-U_j^u\right]-U\right\}\\
&+\mu_e\left[\beta\sum_{j=1}^{N}\pi_{ij}q'(h)\left(U_j^e-U_j^u\right)-v'(h)\right],
\end{aligned}
\tag{30}
$$

and its first order conditions are

$$
\begin{aligned}
&(w:) \quad -1+\lambda_e u'(w)=0,\\
&(h:) \quad \tfrac{1}{1+r}\sum_{j=1}^{N}\pi_{ij}\left\{q'(h)\left[W_j(U_j^e)-B_j(U_j^u)\right]\right\}\\
&+\lambda_e\left\{-v'(h)+\beta\sum_{j=1}^{N}\pi_{ij}\left[q'(h)\left(U_j^e-U_j^u\right)\right]\right\}\\
&+\mu_e\left[\beta\sum_{j=1}^{N}\pi_{ij}q''(h)\left(U_j^e-U_j^u\right)-v''(h)\right]=0,\\
&(U_j^u:) \quad \tfrac{1}{1+r}\pi_{ij}\left(1-q(h)\right)B_j'(U_j^u)+\lambda_e\beta\pi_{ij}\left(1-q(h)\right)-\mu_e\beta\pi_{ij}q'(h)=0,\\
&(U_j^e:) \quad \tfrac{1}{1+r}\pi_{ij}q(h)W_j'(U_j^e)+\lambda_e\beta\pi_{ij}q(h)+\mu_e\beta\pi_{ij}q'(h)=0.
\end{aligned}
\tag{31}
$$

These equations are used in the characterization of the *optimal state-contingent unemployment insurance.*