Operational Risk Management: Preventive vs. Corrective Control

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Research Questions

- How to manage operational risk?
- How does the management strategy depend on market or firm environment?
- How much performance improvement can we achieve?

Problem Formulation

Firm value process V_t satisfies the stochastic differential equation (SDE):

$$dV_t = r(t)V_t dt + \sigma(t)V_t dB_t, \qquad V_0 = v > 0$$

Or equivalently,

$$V_t = v \; exp\left\{\int_0^t \left(r(s) - \frac{1}{2}\sigma^2(s)\right) ds + \int_0^t \sigma(s) dB_s\right\},$$

- $r(t): \mathbb{R}^+ \to \mathbb{R}$ the natural logarithmic growth rate of the firm value at time t
- $\sigma(t) : \mathbb{R}^+ \to \mathbb{R}^+$ the value volatility caused by market uncertainty at time t
- B_t: the standard Brownian motion process that starts at zero at time zero

Problem Formulation

Following Jarrow (2008), we consider operational risk process follows a jump process, and thus

$$V_t = v\ exp\left\{\int_0^t \left(r(s) - \frac{1}{2}\sigma^2(s)\right) ds + \int_0^t \sigma(s) dB_s - J_t\right\},$$

where

$$J_t = \sum_{i=1}^{N_t} Y_i$$

- Y_i (Severity Distribution): i.i.d $\mathbb{R}^+$valued random variables with PDF $f(y), y > 0$
- N_t(Frequency Distribution): a standard Poisson process with intensity rate $\lambda(t) > 0$

Controls

Preventive Control

Corrective Control

Controls

Preventive Control: a mechanism to keep errors or irregularities from occurring in the first place.

- Prevents events from happening
- Affects risk *frequency*

Corrective Control: a mechanism to mitigate damage once an operational risk event has occurred.

- Reduces losses after an event has happened
- Affects risk *severity*

Controls

Preventive Control

Preventive Control

Preventive control $u(t)$ on the frequency function:

$$\tilde{\lambda}(t) = G\big(t, u(t), \lambda(t)\big),$$

where $G(t, u(t), \lambda(t))$ is

- Positive, continuously differentiable, and decreasing convexly in $u(t)$
- $G(t, u(t), \lambda(t)) \to 0$ as $u(t) \to +\infty$
- $G\big(t, u(t), \lambda(t)\big) \leq \lambda(t)$, $G(t, 0, \lambda(t)) = \lambda(t)$, and $G(t, u(t), \lambda(t))$ increases in $\lambda(t)$

$X_t = logV_t$ satisfies:

$$dX_t = \left(r(t) - \frac{1}{2}\sigma^2(t) - u(t)\right)dt + \sigma(t)dB_t - dJ_t^G,$$

where $X_0 = x = \log\ v$, $J_t^G = \sum_{i=1}^{N_t^G} Y_i$, and N_t^G is a simple point process with

$$\tilde{\lambda}(t) = G\big(t, u(t), \lambda(t)\big) > 0.$$

Controls

Corrective Control

Corrective Control

Corrective control $v(t)$ on the severity function:

$$\widetilde{Y}_i = K(\tau_i, v(\tau_i), Y_i),$$

where $K(\tau_i, v(\tau_i), Y_i)$ is

- Positive, continuously differentiable, and decreasing convexly in $v(\tau_i)$
- $K(\tau_i, v(\tau_i), Y_i)$ could go to 0 with proper $v(\tau_i)$
- $K(\tau_i, v(\tau_i), Y_i) \leq Y_i, K(\tau_i, 0, Y_i) = Y_i$, and $K(\tau_i, v(\tau_i), Y_i)$ increases in Y_i

$X_t = logV_t$ satisfies:

$$dX_t = \left(r(t) - \frac{1}{2}\sigma^2(t) - v(t)\right)dt + \sigma(t)dB_t - dJ_t^K,$$

where $X_0 = x = logv,\ \ J_t^K = \sum_{i:\tau_i \leq t}^{N_t} K(\tau_i, v(\tau_i), Y_i).$

Joint Controls

Preventive Control

Corrective Control

Joint Controls

The process $X_t = logV_t$ now satisfies the dynamics:

$$dX_t = \left(r(t) - \frac{1}{2}\sigma^2(t) - u(t) - v(t)\right)dt + \sigma(t)dB_t - dJ_t^{G,K},$$

with $X_0 = x = logv$, and

$$J_t^{G,K} = \sum_{i:\tau_i \le t}^{N_t^G} K(\tau_i, v(\tau_i), Y_i),$$

where N_t^G is a simple point process with intensity $G\big(t, u(t), \lambda(t)\big)$

at any given time t.

Objective

Objective?

Risk-averse Utility Maximization

The utility function $U(V_t)$ of a risk-averse investor is given by

$$U(V_t) = V_t^{\beta},$$

where $\beta \in (0,1)$ is the *risk tolerance level*. The finite period T utility maximization problem is then given by

$$\sup_{u(\cdot)\in\mathcal{U},\, v(\cdot)\in\mathcal{V}} \mathbb{E}[U(V_T)] = \sup_{u(\cdot)\in\mathcal{U},\, v(\cdot)\in\mathcal{V}} \mathbb{E}\left[e^{\beta X_T}\right]$$

where $\mathcal{U}$ and $\mathcal{V}$ are the sets of admissible strategies.

Objective

Optimal Strategy

Preventive Control

THEOREM 1: *The optimal preventive control* $u^*(t)$ *for the optimization problem is, for all* $t \leq T$*, given by*

$$\boldsymbol{u^*(t) = H\left(t,\ -\beta/\left(1 - \mathcal{L}(\beta)\right), \lambda(t)\right)},$$

If $H\left(t,\ -\beta/\left(1 - \mathcal{L}(\beta)\right),\ \lambda(t)\right) > 0$, *otherwise,* $u^*(t) = 0$.

$H(\cdot)$ as the inverse function of $\partial G\big(t, u(t), \lambda(t)\big)/\partial u(t)$, i.e.,

$$\frac{\partial G\left(t, H\big(t, u(t), \lambda(t)\big), \lambda(t)\right)}{\partial u(t)} = u(t).$$

And

$$\mathcal{L}(\beta) = \int_0^\infty e^{-\beta y} f(y)dy$$

where $f(\cdot)$ is the probability density function of operational risk losses.

Preventive Control

PROPOSITION 1. *For any fixed time* t, $t \leq T, u^{*}(t)$ *decreases in* β.

--- The higher risk tolerance level, the lower investment.

Preventive Control

Denote the risk reduction efficiency of preventive control at given time t as

$$EF_t = \left|\frac{\partial G\big(t, u(t), \lambda(t)\big)}{\partial u(t)}\right|.$$

PROPOSITION 2. *If at any given time t, EF_t decreases (increases) in $\lambda(t)$, then $u^*(t)$ decreases (increases) in $\lambda(t)$.*

Corrective Control

THEOREM 2. *The optimal corrective control $v^*(t)$ for the utility maximization problem, for all $t \leq T$, is given by*

$$v^*(t) = \underset{v \geq 0}{arg\,max} \left\{ -v\beta + \lambda(t) \int_0^\infty e^{-\beta K(t,v,y)} f(y) dy \right\}$$

PROPOSITION 4. *For given time t, $t \leq T$, $v^*(t)$ decreases in β.*

PROPOSITION 5. *For given time t, $t \leq T$, $v^*(t)$ increases in $\lambda(t)$.*

Joint Control

THEOREM 3. *The optimal preventive control* $(u^*(t), v^*(t))$ *for the optimization problem (11) is, for all* $t \leq T$, *given by*

$$(u^*(t), v^*(t)) = arg\max_{u,v\geq 0}\left\{-(u+v)\beta + G\big(t,u,\lambda(t)\big)\left(1-\int_0^\infty e^{-\beta K(t,v,y)} f(y)dy\right)\right\}$$

Consider the special case

$$\tilde{\lambda}(t) = \lambda e^{-\delta_1 u(t)}, \qquad K(v,y) = ye^{-\delta_2 v(t)},$$

then $$v^{**}(t) = \frac{1}{\delta_2 E[Y]}\log\left(\frac{\beta}{\delta_2/\delta_1 - 1}\right),\ u^{**}(t) = \frac{1}{\delta_1}\log\left(\frac{\lambda\delta_1}{\beta}\left(1-\frac{\delta_1}{\delta_2}\right)\right)$$

if $\beta > \delta_2/\delta_1 - 1 > 0$ and $\lambda\delta_1(1-\delta_1/\delta_2) \geq \beta$;

otherwise, either $v^*(t) \equiv 0$ or $u^*(t) \equiv 0$, or $v^*(t) = u^*(t) \equiv 0$.

- **δ_1, δ_2: risk reduction efficiency rates.**

Substitution and Complementarity

PROPOSITION 10. *The* ***substitution and complementarity effects*** *in the Joint Investment Region can be characterized by the following scenarios:*

(i) If β increases, then u^* decreases and v^* increases.

(ii) If δ_2 increases, then u^* increases and v^* decreases.

(iii) If δ_1 increases, then v^* always increases and u^* increases (decreases) only if

$$\frac{1-\frac{2\delta_1}{\delta_2}}{1-\frac{\delta_1}{\delta_2}} > (<) \log\left(\frac{\lambda\delta_1}{\beta}\left(1-\frac{\delta_1}{\delta_2}\right)\right).$$

Industry Example

- In total 1441 operational risk events from 08/22/2013 till 04/30/2015.
- Retail bank with 50 branches in China with around 675 employees in total.

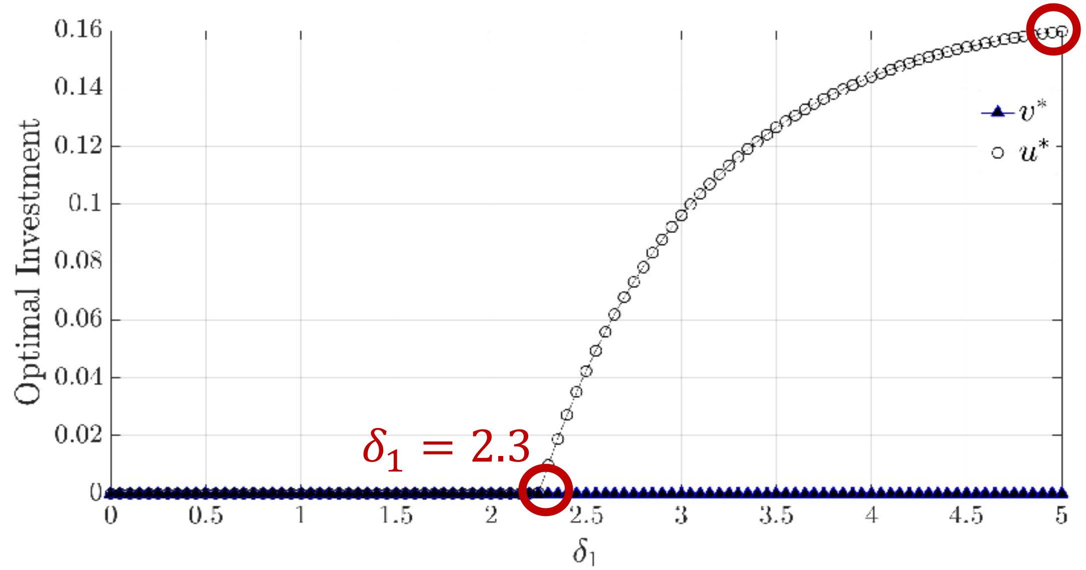
Parameters	Definition	Value
λ	Average number of risk events per month per branch.	1.692
$E[Y]$	Average event severity level (in amount of 100,000 RMB)	3.565
r	Average profit of each branch (in amount of million RMB)	3.785
σ	Volatility of the profit (in amount of million RMB)	4.412
T	investment horizon (in month)	12

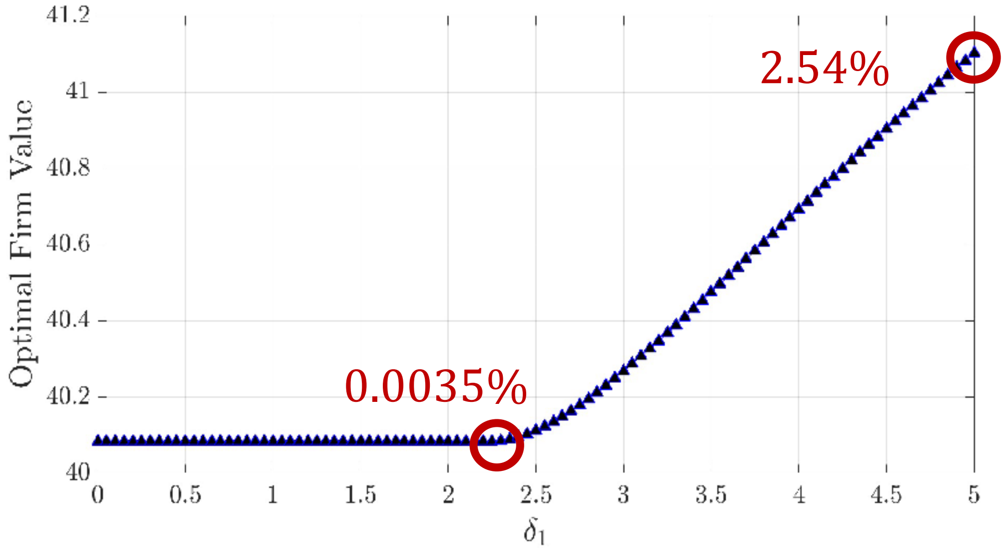
Industry Example

- Within a certain range of the frequency reduction efficiency, we can achieve up to 2.5% improvement in the expected bank revenue
- Within a certain range of the risk severity reduction efficiency, we can achieve up to 1.5% improvement in the expected bank revenue
- When the risk severity level increases, the performance improvement becomes even more significant

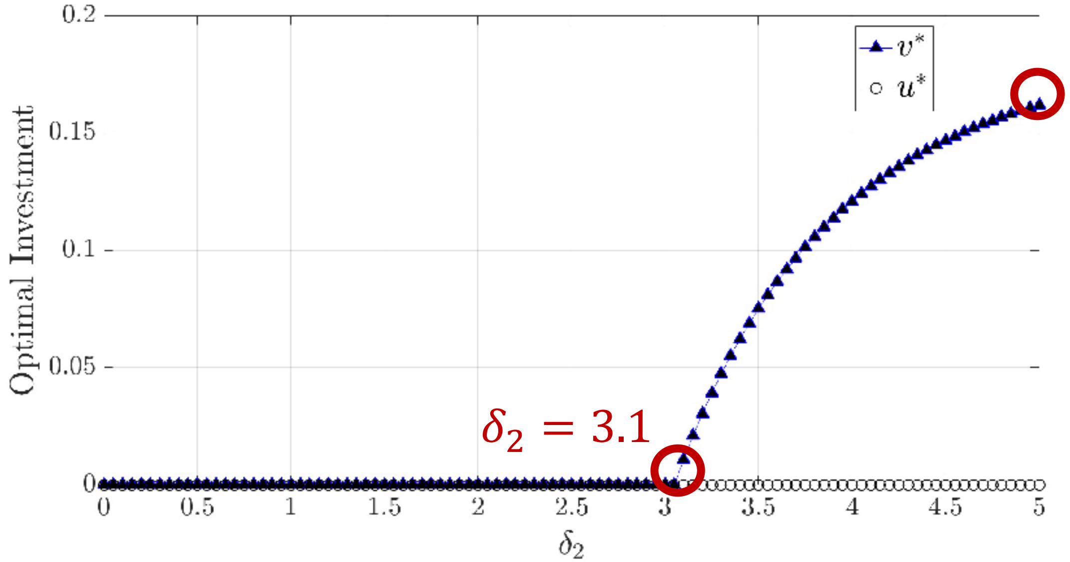
Real Bank Scenario

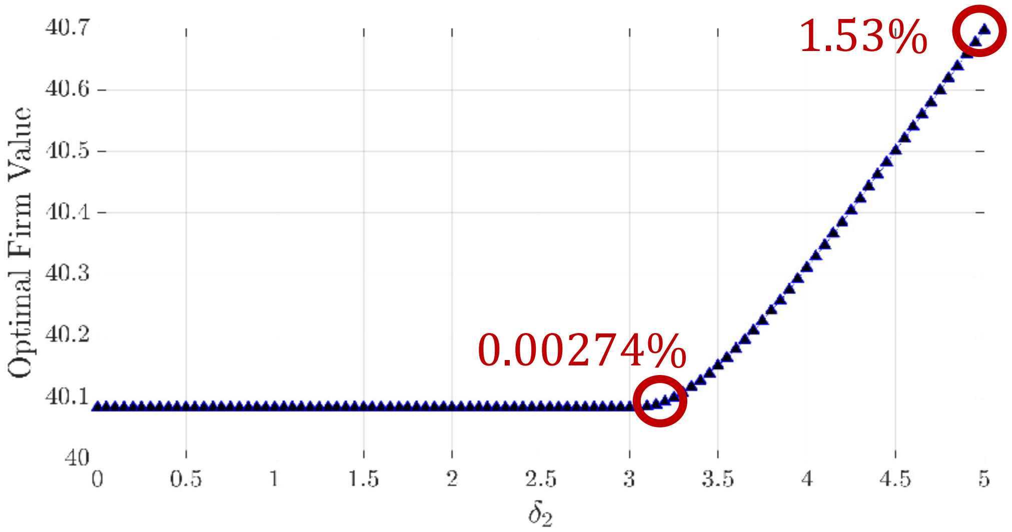
$\delta_2 = 1$



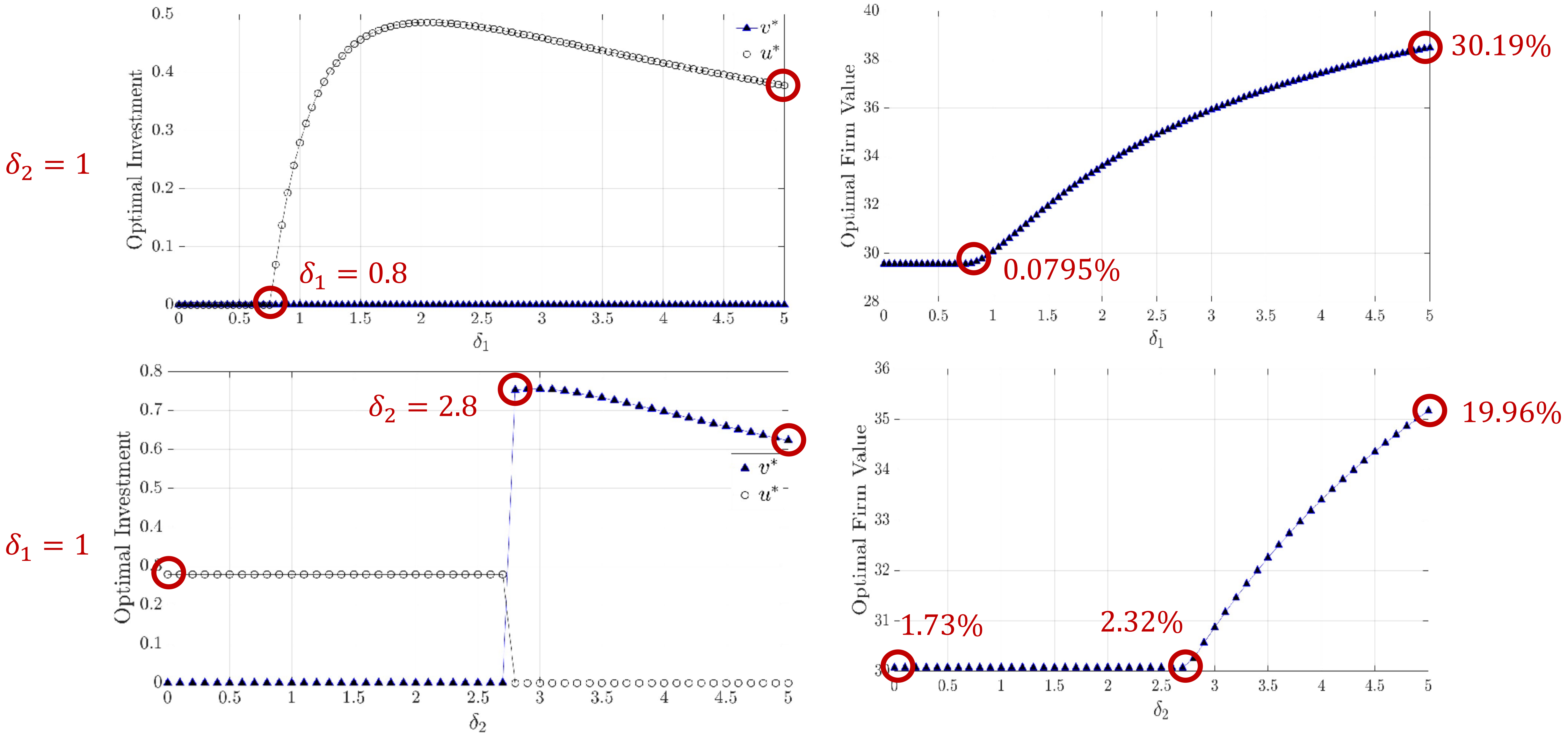


$\delta_1 = 1$





Severe Risk Events Scenario --- 10*E[Y]



Conclusion

- Proposed a general stochastic control framework for operational risk management.
- Characterized two types of controls: preventive vs. corrective control.
- Calculated performance improvement with real industry data.

The End

Preventive Control

PROPOSITION 3. *Assume that* $Y_1 \geq_{cx} Y_2$, and given time t, then $u_1^*(t) \leq u_2^*(t)$.

--- A risk-averse investor always prefers less stochastic variability.

Note: if $Y_1 \geq_{cx} Y_2$, then if $\mathbb{E}(Y_1) = \mathbb{E}(Y_2)$ we have $Var(Y_1) > Var(Y_2)$, see Ross (1983).