

KALDOR AND PIKETTY'S FACTS: THE RISE OF MONOPOLY POWER IN THE UNITED STATES*

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Abstract

The macroeconomic data of the last thirty years has overturned at least two of Kaldor's famous stylized growth facts: constant interest rates, and a constant labor share. At the same time, the research of Piketty and others has introduced several new and surprising facts: an increase in the financial wealth-to-output ratio in the US, an increase in measured Tobin's Q, and a divergence between the marginal and the average return on capital. In this paper, we argue that these trends can be explained by an increase in market power and pure profits in the US economy, i.e., the emergence of a non-zero-rent economy, along with forces that have led to a persistent long term decline in real interest rates. We make three parsimonious modifications to the standard neoclassical model to explain these trends. Using recent estimates of the increase in markups and the decrease in real interest rates, we show that our model can quantitatively match these new stylized macroeconomic facts.

Keywords: Monopoly power, secular stagnation, labor share, Tobin's Q
JEL Classification: D42, E43, E22, E21

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1 Introduction

The goal of this paper is to give a unified explanation of five puzzling trends in the US macroeconomic data. We pursue the hypothesis that an increase in monopoly profits and a decline in the natural rate of interest have been key drivers of these trends. Towards that goal, we build a quantitative model of the US economy that includes imperfect competition, barriers to entry, the trading of pure profits, and realistic asset pricing. We then explore how the economy responds to changes in market power and interest rates, and find that our model is able to quantitatively match all of these trends.

First, we briefly describe the five macroeconomic trends that motivate our research, leaving a full discussion to section 2. We term these “puzzling facts” (or simply ‘puzzles’ for short), following the tradition in the macroeconomic literature which describes as puzzles basic features of the data that are inconsistent with the canonical neoclassical model.¹ Throughout the paper we will reference the five puzzling facts by their number in bold.

(P1) *An increase in the financial wealth-to-income ratio despite low savings rates, with a stagnating capital-to-income ratio.* To paraphrase Thomas Piketty, wealth is back in the United States. Household financial wealth, measured by the market value of housing and business assets, has increased from 250% of income in 1970 to 400% in 2015. The new wealth was not accumulated by traditional savings. In fact, savings rates have generally decreased since the 1970s. The financial wealth is not embodied in new productive capital goods. The replacement value of capital goods to output has remained relatively constant over the period. This is puzzling because in the perfect-competition neoclassical model wealth is accumulated by savings, and the market value of wealth cannot diverge in the long run from the replacement value of the capital stock.

(P2) *An increase in Tobin’s Q to a level permanently above 1.* Rather than through savings, wealth was largely accumulated through capital gains in the stock and housing markets. The value of the S&P 500 index increased by 8% per year from 1970 to 2015, while the replacement value of its productive capital stock did not increase nearly as fast. This has led to an increase in Tobin’s Q , the ratio of the market value of corporations to the replacement cost of their capital stock, of public corporations from approximately 1 in 1970 to 1.75 in 2015. This is a puzzling fact because the standard neoclassical model implies that Tobin’s Q should be 1 in the long run. There has also been a boom in housing prices. The market value of all housing (plus land), relative to the replacement cost of housing, increased from 103% in 1970 to 128% in 2015.

(P3) *A decrease in the real rate of interest, while the measured average return on capital is relatively constant.* There has been a substantial decrease in real interest rates since the 1970s. However, this lower return is not mirrored in the average return on corporate capital, a measure of the profitability of firms, which has stayed constant. This is a puzzling fact because in the neoclassical model,

¹See, e.g., the “equity premium puzzle”, “international portfolio puzzle”.

the interest rate and the return on capital should move in tandem.

(P4) *A decrease in both the labor share and the capital share.* Over the same time period, there is a growing body of evidence that suggests a marked decrease in competition, an increase in concentration (see Dorn et al. (2017) and Grullon, Larkin and Michaely (2016), for example), a decline in business dynamism,² and an increase in profits.³ A growing literature has connected these trends to a decline in the labor share.⁴ In our analysis, following Barkai (2016), Karabarounis and Neiman (2014), and Rognlie (2016), we also directly estimate the capital share of income, and find this share has been declining as well. The large and growing residual share of income we will interpret as the ‘pure profit share’.⁵ This is a puzzling fact because in the standard neoclassical model, there is no residual income share, let alone a growing residual share.

(P5) *A decrease in investment-to-output, even given historically low borrowing costs and a high value of empirical Tobin’s Q.* The lower interest rates (P3) have not led to a boom in investment; in fact, investment-to-output has been sluggish for two decades, despite the high levels of Tobin’s Q (P2).⁶ This is a puzzling fact because in the standard model, low interest rates and a high level of Tobin’s Q should lead to a boom in investment.

These are not your father’s growth facts. The changes over the past 40 years have overturned at least two of Kaldor’s famous stylized facts: constant interest rates, and constant labor share.⁷ With Kaldor’s constants in doubt, it is necessary to give a thorough reexamination of the usefulness of the perfect competition neoclassical model for explaining the macroeconomic data of the modern economy; recall that the main triumph of the neoclassical model is its ability to generate Kaldor’s growth facts. More broadly, the neoclassical growth model cannot address many of the changes that have occurred over the past 40 years, discrepancies which have become increasingly clear with the work of Piketty and others, and through a careful study of the Integrated National Accounts, which was first published in 2006.

As already noted, we hypothesize that two forces are driving these broad macro-trends: (i) an increase in monopoly profits, and (ii) a decrease in the natural rate of interest. We model these forces through parsimonious modifications

²See Decker et al. (2016).

³See, for example, Barkai (2016), Barkai (2018), and Chen, Karabarounis and Neiman (2017).

⁴This decline is documented in Karabarounis and Neiman (2014) and Elsby, Hobijn and Şahin (2013).

⁵For a different interpretation of the residual share, see Karabarounis and Neiman (2018), who call the residual share “factorless income”. We discuss this paper in section 6 below.

⁶See Gutiérrez and Philippon (2017) and Gutiérrez and Philippon (2016).

⁷Some of Kaldor’s other facts also seem more tenuous: (i) a constant growth rate in output per capita (US output growth has slowed considerably) (ii) constant capital-to-output ratio (whether this holds depends on whether you take into account the relative price of investment) (iii) a wage rate that grows at a constant rate equal to the growth rate of output (median wages have been sluggish relative to output growth) (iv) constant hours worked per capita (hours per capita have declined).

of the neoclassical model. We depart from perfect competition, and posit that market power allows firms to make pure profits. There are barriers to entry, which prevent competition from driving these profits to zero. Crucially, claims to the (nonzero) pure profits of firms are traded and priced, and the ratio of the market value of firms (which includes the rights to pure profits) to the replacement value of the productive capital stock (Tobin's Q) is permanently above one. We model a decline in the natural rate of interest through a time varying utility wedge. This is a reduced form way of modeling factors that affect the natural rate of interest in an OLG setting, such as changes in fertility and mortality. In previous work (Eggertsson, Mehrotra and Robbins (2017)) we modeled these in detail in the context of a large scale quantitative OLG model; for simplicity, in this paper we work with a representative agent model.

We briefly sketch how our modifications to the neoclassical model solve the five puzzles. An increase in market power leads to an increase in pure profits, thus an increase in stock prices (**P1**). This leads to an increase in Tobin's Q (**P2**) and financial wealth. With an increase in markups, the pure profit share increases, and the labor and capital share both decrease (**P4**). Markups will also lead to an increase in the wedge between the marginal product of capital and the rental rate, decreasing investment (**P5**).

An increase in pure profits will tend to drive up the average return on capital. To generate a constant average return, as we see in the data, we need a decline in interest rates, which pushes down the average return on capital. In our model, the two forces roughly cancel each other out and lead to a constant average return (**P3**). The decline in interest rates also increases the present discounted value of profits, which further contributes to an asset price boom, and an increase in wealth-to-income and Tobin's Q (**P1** and **P2**).

Both changes in markups and interest rates are necessary parts of our explanation of the five puzzles. By itself, a secular decline in real interest rates could account for a higher wealth-to-income ratio (**P1**). The key challenge, however, is that a fall in real interest rates should then trigger an investment boom (**P5**), which did not happen, and should furthermore lead to a fall in the overall return to capital, which similarly is not seen in the data (**P3**). A secular increase in market power works as a counterforce in a straightforward way to rationalize these developments. An increase in market power is also necessary in order to explain the divergence between wealth and capital, and to generate a level of Tobin's Q permanently above 1.

The assumption of pure profits in our model is in alignment with the field of business strategy and managerial economics, in which the existence of market power is emphasized, and indeed one of the main purposes of the firm to gain and secure this market power: the celebrated "sustainable competitive advantage".⁸ In our model, there is non-perfect competition. There are barriers to entry, which give market power to firms which is non-reproducible in the short run. This allows the market value of firms to permanently diverge from the re-

⁸See for example, Porter (1996), Barney (1991), and Oberholzer-Gee and Yao (2013).

placement value of the capital stock. Our paper thus follows a long tradition in the business and finance literature by using empirical Tobin's Q as a measure of firm market power.⁹ Our model is also in alignment with the endogenous growth and endogenous entry literature, in which the profit share of the economy is positive even in the long run.

We contrast our model with non-reproducible assets to those that assume either perfect competition, or monopolistic competition in which fixed costs generally drive profits to zero. All assets in these economies are fully reproducible, either the productive capital which can be accumulated through investment, or the monopoly profits, which can be gained by paying a fixed entry cost. The market value of firms can diverge from the replacement cost of capital, but only due to the presence of adjustment costs. In the long run, as stated by Tobin and Brainard (1976), "the increase in stock brings market value into line with replacement cost, lowering the former and/or raising the later". The canonical view of this position is found in Hall (2001).

A key contribution of this paper is to clarify the measurement and theory of a number of important concepts. With our model in hand, we can make subtle but important distinctions: between financial wealth and productive capital, measured labor share (which includes bargaining rents) and economic labor share, productive and nonproductive intangible investment, empirical Tobin's Q and theoretical Q, the capital and the profit share, the equity premium on profits and the equity premium on capital, and the marginal and average return. An important aspect of the paper is our integration of wealth and balance sheet data from the Integrated Financial Accounts into quantitative macroeconomics. In this paper, we are interested in matching historical movements of financial moments such as wealth-to-income and Tobin's Q, as well as the traditional quantity moments such as productive capital and investment. We further integrate advances from the finance literature into our model to match key moments such as the equity-premium, connecting to the recent growing literature on macro-finance as in Bansal and Yaron (2004), Barro (2006), Nakamura et al. (2013), and Croce (2014).

We quantitatively test the extent to which our model can match the changes seen in US data. To do so, we estimate the change in markups in the US over the past forty years from aggregate macroeconomic data, using methods from a burgeoning literature that estimates markups. Using our estimated change in markups from 1970 to the present, along with a decline in the equilibrium real rate of 2 percentage points, our model is able to quantitatively match the five puzzling facts described above. We explore the robustness of our results to other estimates of markups from this growing literature.

The primary goal of this paper is to pursue the hypothesis that changes in monopoly profits and interest rates have been the main driver of a variety of macroeconomic changes over the past forty years. We do not, however, claim

⁹See, for example, Lindenberg and Ross (1981). Smirlock, Gilligan and Marshall (1984). and Salinger (1984).

that ours is the only explanation for the five stylized facts seen in the US over the past forty years and that other forces are not at work. There are several compelling competing hypotheses that have been put forward: (i) an increase in the risk premium of capital (ii) unmeasured intangible capital (iii) an increase in the capital-to-output ratio, along with an elasticity of substitution greater than one (iv) a decrease in the bargaining power of labor. We will find credence to some of these other explanations: indeed, we argue that several of them are complementary. Our model, for example, generates an endogenous increase in the equity premium. In addition, many forms of intangible capital, such as investments in brand equity, advertising, and marketing, are closely related to the rise in economic rents we document.

Our work has important welfare implications. Higher pure profits will tend to increase income and wealth inequality, since equity holders (who receive the pure profits) tend to be in the upper portion of the distribution of income and wealth. Higher monopoly power may also be inefficient, and could decrease GDP through a lower capital stock and labor supply. The increase in monopoly power has likely contributed to these trends in the US over the past three decades.

In order to draw policy conclusions, it is necessary to study in more detail the reasons underlying the increase in monopoly power in the US. This is important so as to assess whether this increase in market power is a “malignant” or “benign” development, for example if it is due to benign technological change, or instead due to lax antitrust enforcement. Such exploration is beyond the scope of our paper. Instead we focus purely on the macroeconomic effects of an increase in monopoly power, and do not take a stand on the underlying cause of this increase. An important takeaway is that the rise in monopoly power is needed to explain a host of macroeconomic developments in recent decades which otherwise may appear puzzling. Accordingly, the stakes are high in understanding the underlying forces behind this macroeconomic phenomenon, which has important policy implications in terms of capital taxation, antitrust enforcement, and optimal redistributive policies.

1.1 Previous literature

For each of the five puzzling facts listed above, there is a growing literature documenting them, as well as proposed explanations for the patterns in the data.

Piketty (2014) and Piketty and Zucman (2014) document evidence that the wealth-to-income ratio has risen in many developed economies over the past forty years, along with an increase in Tobin’s Q in the United States. Piketty et al. decompose the increase in wealth into two components, a savings component and a valuation component. The savings component can be described by the equation $\beta = \frac{s}{g}$, where β is the ratio of wealth-to-income, s is the savings rate, and g the growth rate of output. Piketty argues that this fundamental relationship governs the evolution of the wealth-to-income ratio in the long run, i.e. over the course of centuries. In the short run, i.e. several years to several decades, there can be valuation effects, in which the price of capital goods (Tobin’s Q)

increases.¹⁰ Nevertheless, he does find substantial capital gains from 1970-2010. He proposes that changes in capital policies over the time period lead to the asset price gains. Our paper will propose a different explanation for the growth in Tobin's Q and wealth-to-income.

Gomme, Ravikumar and Rupert (2011) and Gomme, Ravikumar and Rupert (2015) document that the average return on productive capital has stayed relatively constant over the past 30 years, even as interest rates have decreased. Our model will provide an explanation for this divergence.

Karabarbounis and Neiman (2014) and Elsby, Hobijn and Şahin (2013) have provided evidence that there has been a global decline in the labor share, and Barkai (2016) has provided intriguing evidence that the pure capital share has decreased and the pure profit share has increased. The work of Barkai (2016) follows work by Karabarbounis and Neiman (2014) and Rognlie (2016), who provide evidence that the capital share for the US does not offset the decline in the labor share. As in Barkai, our model will use markups to explain a decline in the labor and capital share, and an increase in the pure profit share: we will also use a similar method as Barkai in order to back out markups from the profit share.

Gutiérrez and Philippon (2016) show that investment is weak relative to measures of profitability and valuation, particular Tobin's Q. They find support for the hypothesis that weak investment is being driven by a decrease in competition at the industry level. Our model will incorporate this explanation: a decrease in competition will lead to a stagnation in investment, even in spite of historically low interest rates.

A few recent studies have also made connections between several of the above facts. Stiglitz (2016) and Stiglitz (2015*b*) are concerned with many of the same macroeconomic data as this paper, such as the divergence between financial wealth and capital, and connect this to an increase in rents which are capitalized in financial assets. Caballero, Farhi and Gourinchas (2017) note several of the above facts, and try to explain them in an accounting framework through a combination of rents, increases in risk-premia, and technical change. Auclert and Rognlie (2016) explore how changes in markups affect the supply of assets and aggregate demand. In a series of important papers that are closest to ours, Gonzalez and Trivin (2016) and Brun and González (2017) explore many of the same facts, and connect them to an increase in Tobin's Q and monopoly rents, however the modeling approaches and implications are different: (i) Brun and González (2017) emphasizes the evolution of capital taxation and its impact on Tobin's Q and profits (ii) our model has a representative agent framework with long-run risk, while Brun and González (2017) have a Bewley-Aiyagari heterogeneous agent model.

¹⁰In the long run, however, Piketty believes there can be no significant divergence between "the price of consumption and capital goods". Because of this long run relationship, Piketty is comfortable using the terms 'wealth' and 'capital' interchangeably. We disagree with this: a major goal of this paper is to explore empirically and theoretically the difference between wealth and capital.

Our paper distinguishes itself from this literature conceptually, theoretically, quantitatively, and empirically. Conceptually, our paper is distinct in that it presents a unified explanation of five macroeconomic trends, spanning several literatures in macroeconomics, finance, and growth. Other papers generally focus on one or two of our five facts: Gutiérrez and Philippon (2016) focuses on (P5) and (P2), Barkai (2016) on (P3) and (P4), Caballero, Farhi and Gourinchas (2017) on (P3) and (P4), Piketty and Zucman (2014) on (P1), Autor et al. (2017) on the fall in the labor share, Gomme, Ravikumar and Rupert (2015) on (P3), De Loecker and Eeckhout (2017) on the rise of markups. In addition, this paper's key theoretical results are crystalized with analytic propositions and proofs.

Theoretically, our model includes a novel combination of (i) a general equilibrium model with long-run productivity risk (ii) the pricing of the rights to pure profits, which leads to a value of Tobin's Q permanently above 1. Our model produces a results that, to the best of our knowledge, has not been discussed before: a risk premium on pure profits that is greater than the risk premium on productive capital. With the pricing of pure profits, it becomes easy to match the equity premium in a full production model, even with relatively low values for the coefficient of risk aversion and capital adjustment costs. The addition of entry-and exit firm dynamics allows us to closely tie our model to the data on the market valuation of equities.

Our model quantitatively matches a wide range of moments in the data that other models cannot match or do not attempt to match, such as the wealth-to-income ratio, the average return on capital, and a level of Tobin's Q commensurate with the data. Simultaneously, we can also match the traditional macro and finance moments, such as the investment-to-output ratio, the real interest rate, the labor share, and the equity premium. Other papers in this literature do not have long run risk or other elements that allow them to match the equity premium, which is important for our quantitative results. Without the equity premium, it is difficult to simultaneously match the real interest rate, the average return, and Tobin's Q.

2 Five Macroeconomic Puzzles

Puzzle 1: Increasing wealth-to-output There has been a large increase in the wealth-to-output ratio in the US, rising from around 250% of output in 1980 to almost 400% in 2015, as illustrated in the left panel of figure 1. Household wealth is taken from the distributional national accounts (see Piketty, Saez and Zucman (2017)).¹¹

The puzzle lies in the fact that while wealth-to-output has increased, the ratio of capital-to-output has been stagnant, as seen in the dotted line in figure 1; in the standard neoclassical model, wealth and capital are equal.¹² Capital refers to

¹¹It consists of housing wealth, equities, fixed income assets, business assets and pensions, less liabilities. For computational details, see Piketty, Saez and Zucman (2017).

¹²Output is Gross Domestic Product, BEA Account Code: A191RC

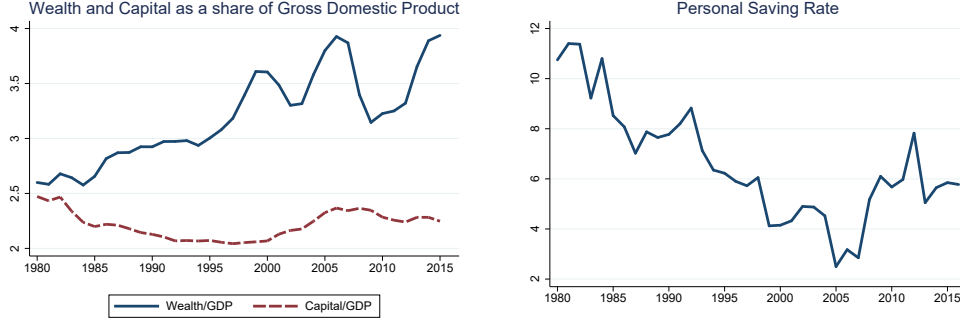


Figure 1: Wealth and capital as a share of GDP.

the value of private capital as measured by the BEA in the fixed asset tables.¹³ In addition, this increase in wealth has taken place despite a decline in the savings rate, as illustrated in the right panel of figure 1.¹⁴ Since the wealth was not accumulated by savings, it must have been accumulated by capital gains, as we indeed see in the data. But this does not solve the puzzle, it simply shifts it: what is the source of these capital gains?

The trends in wealth and capital are not being driven by housing. In appendix figure A.6, we decompose the increase in wealth share into six components. Although the value of housing-to-income has indeed increased, from 1.32 in 1980 to 1.48 in 2015, most of the increase in wealth has come from other assets.

Puzzle 2: Increasing Tobin's Q There has been a large increase in empirical measures of Tobin's Q since 1980. Tobin's Q measures the market value of corporations relative to the replacement cost of their capital. We follow Hall (2001) and Gutierrez and Philippon (2016) in calculating Tobin's Q according to formula (1). V^e denotes the value of equities, as measured in the financial accounts.¹⁵ Corporate liabilities L and financial assets FA are also from the financial accounts.¹⁶ Inventories and the value of corporate capital $P_K K$ are taken from the BEA.¹⁷ We follow Gutierrez and Philippon (2016) in also calculating an alternative Tobin's Q which excludes miscellaneous liabilities and financial assets, as given by formula (2).

$$Q_1 = \frac{V^e + (L - FA) - Inventories}{P_K K} \quad (1)$$

$$Q_2 = Q_1 + \frac{A^{misc} - L^{misc}}{P_K K} \quad (2)$$

¹³Capital is the Current-Cost Net Stock of Fixed Assets: Private (K1PTOTL1ES000)

¹⁴The saving rate is the Personal Saving Rate, BEA Account Code: A072RC

¹⁵The value of equities source series id is LM103164103.Q.

¹⁶The source id numbers are FL104190005.Q and FL104090005.Q respectively

¹⁷Nonfinancial corporate capital: k1pnofi2es000

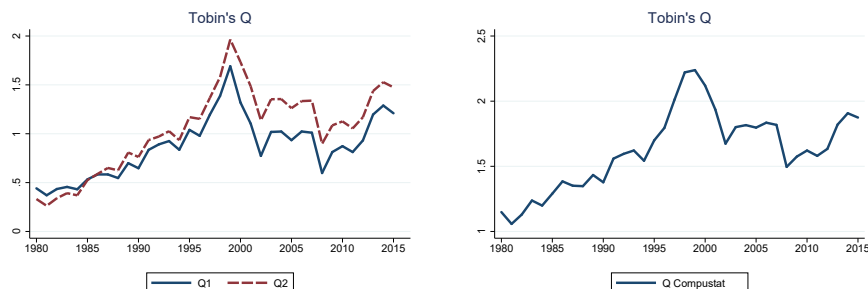


Figure 2: Tobin's Q. Left: financial accounts and BEA data. Right: Compustat

Figure 2 depicts our measures of Tobin's Q, and shows a striking upwards trend. The two alternative measures provide qualitatively similar results. Even using our most conservative estimate, Tobin's Q has increased almost threefold since 1980, implying a large increase in the market value of capital relative to the replacement cost of capital.¹⁸

Measures of Tobin's Q using micro data find qualitatively similar results. The right panel of figure 2 depicts the mean Tobin's Q of publicly listed corporations using Compustat data. There has been a large increase in this measure of Tobin's Q since 1980, and Q has been substantially above 1 for most of the period. This large increase in Tobin's Q is a puzzle from the standpoint of standard capital adjustment costs models, which predicts that the value of Tobin's Q should be 1 in the long run.

Puzzle 3: Declining corporate bond rates, constant return on business capital There has been a substantial decrease in the real interest rate of corporate bonds over the past thirty years. Figure 3 shows the real return for two corporate bond series, an index of AAA bonds and an index of BAA bonds.¹⁹ Both series show a substantial decrease over the time period. Other measures of corporate borrowing costs such as bank interest rates have also decreased.²⁰ Corporate borrowing costs are a measure of the marginal return on capital, since economic theory tells us that corporations should invest until the marginal product on capital is equal to the interest rate.

The puzzle is that while there has been a significant decline in the marginal return of capital as measured by corporate bond markets, measures of the prof-

¹⁸Tobin's Q as calculated using macro data is often found to be below one for extensive periods of time, as is also the case in figure 2. This implies that firms are valued at less than the replacement cost of their capital, which is at odds with economic theory, and may be driven by mis-measurement in macro data. See, for example, the discussion on this point in Piketty and Zucman (2014).

¹⁹Moody's Seasoned Aaa Corporate Bond Yield and Moody's Seasoned Baa Corporate Bond Yield are available from FRED, Federal Reserve Bank of St. Louis. The returns are deflated using a 5 year moving average of the Gross National Product: Implicit Price Deflator (A001RI1)

²⁰See Federal Reserve data series H.15, selected interest rates. <https://www.federalreserve.gov/releases/h15/>.

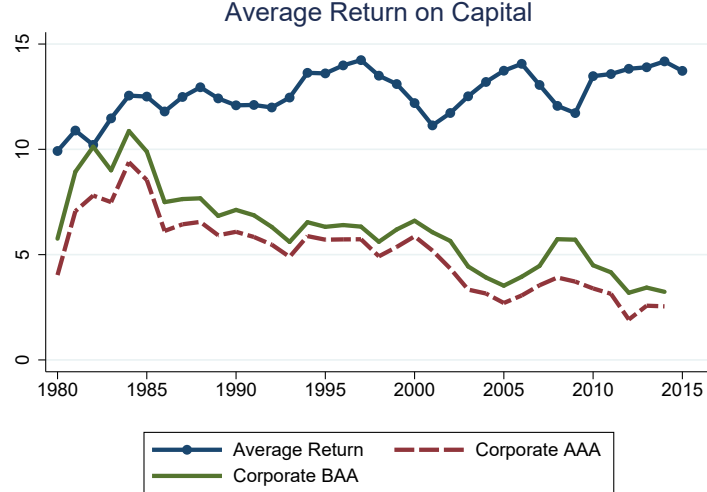


Figure 3: Average return on capital

itability of firms that correspond to the business return of capital have not declined; in the standard neoclassical model, they should be equal. We follow Gomme, Ravikumar and Rupert (2011) and Gomme, Ravikumar and Rupert (2015) in calculating the business return on capital, which we will term the ‘average return’. The exact definition is given in equation (3). We use data from the corporate sector of the national accounts. The numerator is a measure of corporate profits: we start with gross corporate value added, and subtract payments to labor, depreciation, and taxes.²¹ The denominator is a measure of the value of the corporate capital stock, using data from the BEA.

$$AR = \frac{GVA - wL - \delta P^K K - Tax}{P^K K} \quad (3)$$

The average return on capital is depicted by the blue connected line in figure 3. The trend of the average return has not followed decrease seen in corporate bond rates: if anything, there is an upward trend in the average return.

Puzzle 4: Decreasing labor share and capital share It is a well known fact that the labor share has decreased over the past 30 years in the US, and our analysis of the data confirms this. Figure 4, using data from the national accounts, shows that the labor share was around 64 percent in the early 1980s, and has fallen to around 57 percent by the end of our sample.²² The decline in the labor share in the US is not due solely to housing.²³ If we restrict our attention to simply the corporate sector the decline is seen as well. Nor is the decline

²¹The numerator is measured as net value added (BEA Account Code: A457RC) less labor expenditures (BEA Account Code: A460RC)

²²The labor share is measured as compensation to labor (BEA Account Code: A460RC) relative to gross value added (BEA Account Code: A455RC).

²³The story in Europe is slightly different. See Rognlie (2016) and Gutierrez (2017).

entirely explained by an increase in intellectual property capital. As we will discuss in section 8, even with the revisions to the national accounts to include intangible capital, the income share of invested capital has not increased over the time period. The decline in the labor share is further robust to measures which take into account the rise of S-Corporations.²⁴

Most analyses of factor shares only separate income into two factors, labor and capital. Since labor income is observed in the data while capital income is not, the capital share is usually calculated as the residual of the labor share. Computing the capital share this way shows an increase over the time period.

Rather than calculating the capital share indirectly as a residual, we can try to calculate it directly. To do so we first need to estimate total capital income, the value of the capital stock times the rental rate of capital. In this procedure, we follow the work of Barkai (2016). Since most firms own rather than rent their capital, we cannot observe a rental rate of capital. Instead, we must estimate it. Economic theory gives an arbitrage condition for the rental rate r^K , given by equation (4). The rental rate must equal the risk free rate r^{rf} plus the risk premium r^{rp} , less expected inflation $E(\pi)$ and plus the capital depreciation rate. In addition, if the price of capital is not equal to the price of output, the rental rate on capital must account for expected price gains of holding capital $E(g_{PK})$.

$$r_{PG}^K = P^K (r^{rf} + r^{rp} - E(\pi) + \delta + (1 - \delta)E(g_{PK})) \quad (4)$$

We use the return on the three month treasury bill to proxy for the risk free rate. As our measure of the risk premium we use the spread between the rate of return on corporate BAA bonds and long term treasury bonds. Expected inflation is approximated by a five-year moving average of realized inflation. Finally, the relative price of capital P^K is calculated using the price deflator for investment and the price deflator for consumption. To proxy for expected capital price gains we calculate the five-year moving average of realized price gains. We calculate the rental rate under four different assumptions: BAA risk premium and no capital price gain, BAA risk premium with capital price gains, no risk premium and no capital price gain and no risk premium with capital price gains. Our baseline estimate is an average over these four estimates.

Using the rental rate on capital, we calculate the capital share as capital income over total income, the labor share as labor income over total income, and the tax share as taxes on production over income. Finally, there is a residual share of income which is not accounted for by labor, capital, or taxes. Figure 4 shows that, along with the labor share, the capital share is also declining over time, from 27 percent in the 1980s to 15 percent in the present. The main driver of this decline has been a decrease in the risk free interest rate, which has driven down the rental rate of capital; the capital-to-output ratio has been relatively stable (see figure 1).

In 1980 there was very little residual factor income — labor, capital, and taxes accounted for virtually all of national income. However, because of the

²⁴See Zidar et al. (2017).

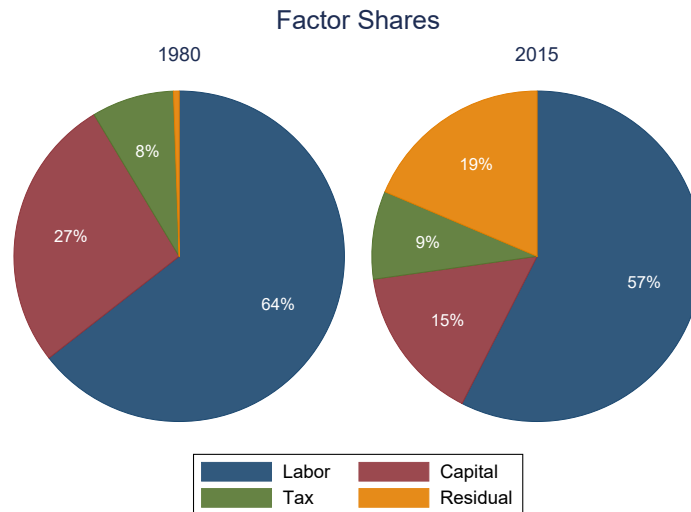


Figure 4: Factor shares

decline in the labor and capital share, the residual share increased to 19% by 2015. In other words, there is now a “missing” factor of income that does not go to labor or capital. As we will argue that this missing factor is pure profits, we refer to this residual share as the profit share.²⁵

Puzzle 5: Low investment despite low r , high Q As the work of Gutiérrez and Philippon (2016) has shown, several measures of investment have been trending down since the 1980s. In figure 5 we plot net investment as a share of net operating surplus, a measure of business operating income.²⁶ While 50% of net operating revenue was invested in the early 1980s, this has declined to about 25% in the present. Net investment as a share of GDP shows a similar decline. This decrease in investment is somewhat puzzling, given historically low interest rates and high levels of Tobin’s Q , both of which should lead to higher investment.²⁷

3 Model

The core of our model is a DSGE economy with long-run productivity risk. To this base, we add three elements which allow us to explain the five puzzling facts. We focus our exposition on these modifications of the standard neoclassical model, leaving the full description of the model’s equations and first order conditions to the appendix.

²⁵For a different interpretation of the residual share, see Karabarbounis and Neiman (2018), who call the residual share “factorless income”.

²⁶We use Net Fixed Investment: Nonresidential (BEA Account Code: A593RC1)

²⁷See, for example, Hayashi (1982).

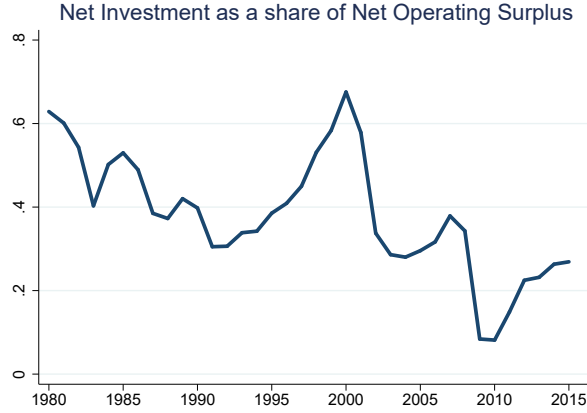


Figure 5: Net investment as a share of net operating surplus

3.1 Market structure

We hypothesize that an increase in pure profits is a key force behind the five puzzles. We introduce profits into our model economy in the simplest possible manner, through Dixit-Stiglitz monopolistic competition, even though the key results are more general and hold for a number of other market structures.²⁸ This is the first key departure of our model from neoclassical theory. The pure profits are distributed to the holders of securities which hold the rights to the pure profits: the value of these securities form the wedge between financial wealth and productive capital, and will help explain **P1** and **P2**.

There is a unit mass of monopolistically competitive final goods firms that differentiate an intermediate good and resell it to consumers. The final good composite is the CES aggregate of these differentiated final goods, which are indexed by i :

$$Y_t = \left[\int_0^1 y_t^f(i)^{\frac{\Lambda_t-1}{\Lambda_t}} di \right]^{\frac{\Lambda_t}{\Lambda_t-1}}.$$

Final goods firms set prices in each period, and face a demand curve of the form $y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\Lambda_t}$, where P_t is the nominal price index of the final

²⁸Although our exposition uses monopolistic competition to generate pure profits, the results of our model are more general. The reason is simple: under a wide variety of micro-founded models, market structure primarily serves to determine the level of markups μ_t in the economy. Thus different market structures with the same markups μ_t will give at least qualitatively similar results. To see this directly, consider equation (6). The value of securities depends only on the exit rate of firms and the level of markup in the economy. Next, examining equations (A.29) and (A.30), we see that there is a wedge between the marginal product of labor and the wage, as well as the marginal product of capital and the implicit rental rate of capital, however independent of this market structure, the wedge is equal to the markup.

good aggregate and Λ_t is a time-varying measure of a firm's market power.²⁹ An increase in Λ_t decreases a firm's market power and lowers equilibrium markups.

Each final goods producer uses y_t^m of intermediate goods to produce output, according to a linear technology function $y_t^f = y_t^m$. A final goods firm chooses real prices $\frac{p_t(i)}{P_t}$ and $y_t^f(i)$ to maximize real profits, subject to the production constraint. The marginal cost of producing a unit of final good is the price of the intermediate good $\frac{p_t^{int}}{P_t}$.

The optimality condition for the real price of the firm's good is a time-varying markup μ_t over marginal cost (which is the price of the intermediate good): $\frac{p_t(i)}{P_t} = \frac{\Lambda_t}{\Lambda_t - 1} \frac{p_t^{int}}{P_t} = \mu_t \frac{p_t^{int}}{P_t}$. Since the price of the intermediate good is the same, all final goods firms make the same pricing decisions, and thus $p_t(i) = P_t$, yielding $\frac{p_t^{int}}{P_t} = \frac{1}{\mu_t}$.

Market power in our model is determined by the CES elasticity Λ_t , which determines the level of markups in our economy. We posit that $\frac{\Lambda_t}{\Lambda_t - 1}$, i.e. markups μ_t , follow an AR(1) process, given by

$$\ln(\mu_t) = (1 - \rho_\mu)\ln(\bar{\mu}) + \rho_\mu \ln(\mu_{t-1}) + \epsilon_t^\mu. \quad (5)$$

The long run level of markups (determined by the long-run level of Λ) in the economy is given by $\bar{\mu}$. An increase in the long-run level of markups will allow us to explain an increase in the profit share, and a decrease in the labor and capital share (**P4**).

The second key element of our model is that there are barriers to entry for final goods firms. There are two types of barriers to entry in our model. First, final goods firms hold the unique rights to the specific variety they produce. This ensures that entry of new firms does not drive price equal to marginal cost, and thus profits to zero. Second, we assume that the supply of final goods firms is fixed in the short run. Individuals cannot create new final goods firms, and thus these firms are non-reproducible. The next section will show that, in the long run, the barriers to entry are not permanent, and firms will eventually go out of business.

Due to these barriers to entry, there can be nonzero pure profits in the economy; final goods firms make aggregate profits Π_t equal to $\Pi_t = \frac{\mu_t - 1}{\mu_t} Y_t$. The profit share of income in the economy is thus given by $PS_t = \frac{\mu_t - 1}{\mu_t}$.

Pure profits are distributed to shareholders of final goods firms, who own the rights to the economic rents, as dividends. Aggregate dividends distributed to shareholders at time t are thus given by $d_t^f = \Pi_t$. Since all firms make identical profits, each firm receives an equal fraction of aggregate dividends. As will be seen, the securities that hold the rights to the pure profits of final goods firms will be priced and traded, which will allow the value of wealth to diverge from that of productive capital (**P1** & **P2**).

²⁹The price index for the final aggregate is given by $P_t = \left(\int_0^1 p_t(i)^{1-\Lambda_t} di \right)^{\frac{1}{1-\Lambda_t}}$.

3.1.1 Firm entry and exit dynamics

Although there are strong barriers to entry for final goods firms, these barriers are not permanent: all final goods firms in our economy will eventually go out of business and be replaced by new firms. We include exogenous firm exit as in Melitz (2003). Firm exit in our model affects the value of the securities which hold the rights to the pure profits of firms. Having firm exit will allow us to better match the level of financial wealth in the economy, and thus quantitatively explain **P1** and **P2**.

We assume that between period $t - 1$ and t a final good firm has a probability Δ_t of exiting the market. Thus, of all the firms that are extant in time $t - 1$, only $(1 - \Delta_t)$ will survive to period t , a smaller fraction $(1 - \Delta_t) \cdot (1 - \Delta_{t+1})$ will survive to period $t + 1$, and an ever diminishing percent $\prod_{n=t}^s (1 - \Delta_n)$ will survive to period s . When a firm exits, shareholders who hold the rights to its profits are wiped out.³⁰ The information on whether a firm goes out of business is revealed at the end of period $t - 1$, before asset decisions are made for the next period.

Entry in our model is also exogenous. Each period a mass Δ_t of new firms enters, replacing the firms that have exited. New “IPO” securities are issued at the end of time $t - 1$ that give the rights to these firms’ profits; this corresponds to, for example, when Google was floated for the first time on the stock exchange.

3.1.2 Asset pricing

The third key element of our model is that the rights to the pure profits of final goods firms are traded. There are security markets in which the rights to the future profits of final goods firms are bought and sold. The trading of these securities leads to a divergence between financial wealth and productive capital (**P1**), and allows Tobin’s Q to diverge permanently away from 1 (**P2**).

At the end of period $t - 1$, securities S_t^f are traded for each firm, which give the rights to all future dividends d_t^f of these firms for as long as they survive. Individuals do not buy shares of individual final goods firms, which are infinitesimally small. They instead buy positive fractions of the continuum of firms. Since the continuum spans from 0 to 1, every period there is a single share of securities S_t^f outstanding. Due to the law of large numbers, for every mass of shares an individual purchases, exactly Δ_{t+1} percent of the underlying firms will go out of business between period t and $t + 1$.

The value of these securities S_t^f is given by the present discounted value of the dividends the shares receive:

$$X_{t-1}^f = E_{t-1} \left[\sum_{s=t}^{\infty} m_s d_s^f \prod_{n=t+1}^s (1 - \Delta_n) \right]. \quad (6)$$

³⁰From the point of view of an investor in these securities, the churning of firms is somewhat analogous to the depreciation of capital. An individual buys securities, receives a return, and some of the security “depreciates” through the bankruptcy of the underlying firms.

In this equation, m_t is the stochastic discount factor that is determined in equilibrium by the optimal asset choice of households. Note that future dividends are discounted by two factors: by ‘ $(1 - \Delta)$ ’, taking into account that some firms will go out of business, and the stochastic discount factor, taking into account that future dividends must be discounted by the riskiness of these profits and the time value of money.³¹

3.1.3 Intermediate goods firms

Production is fairly standard; the only twist from a purely neoclassical model is the presence of the markup wedge μ_t between the marginal product of capital and the rental rate; an increase in this wedge will be our main explanation for why investment has decreased **(P5)**.

A representative intermediate goods firm uses labor L_t and productive capital K_t to produce output Y_t^m according to the production function

$$Y_t^m = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (7)$$

where σ is the production elasticity of substitution and A_t the level of labor augmenting productivity. “Productive capital” K_t includes both tangible and intangible capital that contributes to the production of goods and services, which is broadly in line with the BEA.³² Here forward, we will simply use the term “capital” when referring to “productive capital”.

Intermediate goods firms distribute the excess of its profits over retained investment as a dividend $d_t^i S_t^i = \frac{1}{\mu_t} Y_t^m - w_t L_t - I_t$. Investment increases the firm’s future stock of capital according to

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1 - \delta)K_t, \quad (8)$$

where δ is the rate of depreciation and adjustment costs $\Phi(\cdot)$ are a positive concave function. Following Jermann (1998), we use an adjustment cost function given by $\Phi(I_t/K_t) = \frac{a_1}{1-\xi} \left(\frac{I_t}{K_t} \right)^{1-\xi} + a_2$.³³

The first order conditions from the firm’s problem are derived in section A.4. Note that markups μ_t create a wedge between the marginal product of capital and the rental rate. As markups increase, the increased wedge will cause investment to decrease **(P5)**.

³¹Note that although we have motivated the discounting of the value of shares $(1 - \Delta)$ through the notion of firm entry and exit, there are several other interpretations that can be given to this discount rate. First, it can be thought of as an additional, not modeled, risk premium on pure profits. Second, it can be thought of as a reduced form way to model the fact that not all firms are publicly traded in the US, and thus their pure profits are not capitalized.

³²We do not include in this term capitalized spending by businesses on economic competencies such as advertising, marketing research, and branding. Because this spending does not serve to increase the production of goods and services, we term this spending “non-productive intangible investment”. We discuss this distinction further in section 8.2.

³³For a full discussion of the adjustment cost function, see appendix A.4.

3.2 Long Run Risk

The value of securities S_t^f (and thus financial wealth, **P1** & **P2**) depends upon the rate at which pure profits are discounted, and thus the equity premium. In order to match the equity premium in the data, we follow the macro-finance literature and include long-run productivity risk in our model, as in Bansal and Yaron (2004) and Croce (2014).³⁴ There are two sources of uncertainty in productivity growth: an i.i.d short-run shock that is standard in RBC models (ϵ_a), and a long-run component (ϵ_x) that leads to small but persistent movement in long-run growth. Let A_t denote the level of labor augmenting productivity, and lowercase letters denote log-units. The growth rate of productivity is given by:

$$\Delta a_{t+1} = \zeta + x_t + \sigma_a \epsilon_{a,t+1} \quad (9)$$

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t} \quad (10)$$

$$\begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} \sim iid N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right) \quad (11)$$

Here x_t is the “long run risk” of productivity growth, and $\epsilon_{a,t+1}$ is the short run risk.

3.3 Household preferences

Preferences are fairly standard: representative households have preferences of the Epstein and Zin (1989) variety, which will allow us to more easily match the equity premium. The one nonstandard element is a time-varying wedge in the utility between time periods. Changes in the wedge will allow us to match the path of the real interest rate in a representative agent setting, and thus match **P3**. We use this wedge as a reduced form way of modeling factors that affect the real interest rate in an OLG model, such as changes in demographics, which we have modeled in detail in previous work (see Eggertsson, Mehrotra and Robbins (2017)). While an OLG structure would allow us to model the forces affecting the natural rate of interest rate in greater detail (and we have done simulation with this variation of the model) we have chosen here to work with a representative agent model. The more detailed OLG structure has not added substantive insights beyond those in Eggertsson, Mehrotra and Robbins (2017) in the experiments we have considered, but makes the exposition more complex.

The economy consists of a unit mass of identical infinitely lived agents. Utility is given by

$$V_t = \left[(1 - \beta) (c_t^\nu (A_{t-1}(1 - L_t))^{1-\nu})^{\frac{1-\gamma}{\theta}} + \beta D_t (E_t V_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (12)$$

³⁴The exact way we match the equity premium is not important for our theoretical or quantitative results: our analysis would be similar if we had generated the equity premium through consumption habit formation or rare disaster risks.

where the time discount factor is β , ν is a weight determining the average share of total hours worked, γ is the risk aversion parameter, and θ is a parameter defined as $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. In this expression, ψ is the elasticity of intertemporal substitution. The main advantage of using Epstein-Zin utility is that there is no longer a link between the intertemporal elasticity of substitution and the coefficient of risk aversion, which makes it easier to match the equity-risk premium (and quantitatively explain **P1** & **P2**). If $\gamma = 1/\psi$, the utility collapses to the CRRA variety. As in Croce (2014), leisure utility is scaled by productivity.

The term D_t is an additional wedge between utility in period $t+1$ and period t , beyond the time discount rate of β . It represents additional unmodeled factors, such as population growth and mortality, that might effect the supply of loanable funds (and thus the real interest rate) in an OLG model, but have no effect in a representative agent model. D_t is a lever we will use in our quantitative exercises to decrease the long-run real interest rate, and thus match **P3**.³⁵

Individuals maximize this utility subject to a series of budget constraints,

$$\begin{aligned} c_t + X_t^i S_{t+1}^i + X_t^f S_{t+1}^f &= w_t L_t + d_t^i S_t^i + d_t^f S_t^f \\ &+ \Delta_{t+1} X_t^f + (1 - \Delta_t) X_t^f S_t^f + X_t^i S_t^i. \end{aligned} \quad (13)$$

On the left hand side of the budget constraint, individuals use their income to purchase either consumption, shares of intermediate good firms ($X_t^i S_{t+1}^i$), or shares of final goods firms ($X_t^f S_{t+1}^f$). On the right hand side of the budget equation, agents receive income from a variety of sources: labor income $w_t L_t$, dividends from intermediate goods firms $d_t^i S_t^i$, dividends from final goods firms $d_t^f S_t^f$, IPO issued securities of final goods firms $\Delta_{t+1} X_t^f$, remaining share value of final goods firms $(1 - \Delta_t) X_t^f S_t^f$, and remaining share value of intermediate goods firms, $X_t^i S_t^i$.

3.4 Equilibrium and solution

An equilibrium is a set of quantities and prices such that individuals maximize utility subject to budget constraints, intermediate and final goods firms maximize profits subject to production constraints, and markets clear. While this model is not stationary, we can make it so by applying a standard transformation: divide all quantities (except labor) by A_{t-1} , as well as wages and the price of securities X_t^i and X_t^f . Appendix A contains the full definition of equilibrium and lists all of the equations of the model. A steady state equilibrium is the same as the above equilibrium definition, except that all variables are constant rather than subscripted by time. Appendix A contains the steady state equations of the model.

We solve the model using a 2nd order perturbation methods, using the software Dynare (Adjemian et al. (2011)); a period in our model is a month. We

³⁵De Groot et al. (2017) show how to model a preference shock with Epstein-Zin preferences.

solve the model around the nonstochastic steady state. The work of Caldara et al. (2012) has shown that perturbation methods are well suited to solving DSGE models with recursive preferences and long run risk, with accuracy that is competitive with Chebyshev polynomials and value function iteration. Once we have solved the model, we will compare the theoretical moments generated by the model to data moments.

4 Characterizing the puzzles

In this section, we derive model statistics that correspond to the data moments of our five puzzles: financial wealth, average return, Tobin's Q, etc. These statistics will be the main objects of analysis for our theoretical as well as quantitative results, and will also illustrate how our theory differs in key ways from the neoclassical model. For each macro-statistic, we will also derive steady-state equations, which will be useful in the comparative statics analysis.

P1: Increasing financial wealth-to-output ratio, constant capital-to-output ratio We measure financial wealth in our model in a way that corresponds to the measurement in the US Financial Accounts. In the Financial Accounts, wealth is defined as the market value of stocks and bonds, thus we define financial wealth W_t as equal to the combined value of securities and capital, $W_t = X_t^f + q_t K_t$. We immediately see from this equation a key difference between our model and the neoclassical: when there are non-zero pure profits in the economy, the value of financial wealth may diverge from the value of the capital stock.

We now derive the steady state security price X^f as a percent of output. In a balanced growth equilibrium, the ratio of dividends to output is given by $\frac{d}{Y} = PS$. In addition, dividends grow at the same rate as output, thus future dividends must be adjusted for productivity growth. Thus we have

$$\frac{X^f}{Y} = \sum_{s=0}^{\infty} \frac{PS(\exp(\zeta))^s (1 - \Delta)^s}{(1 + r)^s} = \frac{PS}{1 - \frac{\exp(\zeta)(1 - \Delta)}{1 + r}}, \quad (14)$$

using the formula for the sum of a geometric series. The 'r' in this expression is the steady state real interest rate.

Given the steady state real interest rate, it is easy to characterize the capital-to-output ratio. The f.o.c. of intermediate goods firms yields $r = \frac{F_K}{\mu} - \delta$, which allows us to calculate steady-state capital-to-labor; combining this with steady state labor-to-output (equation A.76) yields steady state capital-to-output.

P2: increasing Tobin's Q Empirical Tobin's Q is defined both in the financial accounts and in our model as the ratio of the market value of wealth to the value of capital goods:

$$Q = \frac{W_t}{K_t} = \frac{X_t^f + q_t K_t}{K_t}. \quad (15)$$

We immediately see from this equation a second way in which our model differs from the neoclassical: with nonzero pure profits, Tobin's Q can be permanently above 1.

P3: declining interest rate, constant average return The average return on capital is measured in the data as GDP minus depreciation minus payments to labor, divided by the replacement value of capital: $AR_t = \frac{Y_t - w_t L_t - \delta K_t}{K_t}$. In a steady state, the average return is equal to

$$AR = r + \frac{\Pi}{K} = r + \frac{PS}{\frac{K}{Y}} \quad (16)$$

We see from this equation that with pure profits, there can be a divergence between the average and the marginal return on capital. In a neoclassical model, they are one and the same.

It is simple to characterize the steady state real interest. From equation (A.17), the stationary discount rate is

$$m = \beta D \cdot \exp\left(\frac{-1}{\psi}\zeta\right). \quad (17)$$

The risk free interest rate is $r = \frac{1}{m} - 1$.

P4: increasing profit share, declining labor share We measure labor share as payments to labor, $LS_t = \frac{w_t L_t}{Y_t}$, capital share as the return on productive capital, $KS_t = \frac{r_t^K q_t K_t}{Y_t}$, and profit share as the residual, $PS_t = 1 - LS_t - KS_t$. In the neoclassical model, the profit share is zero.

P5: declining investment In a stationary equilibrium the capital to output ratio is constant, and thus investment must be exactly enough so that capital grows with output, i.e. at the rate of productivity growth plus depreciation. Thus in a steady state investment to output equals

$$\frac{I}{Y} = [\exp(\zeta) - 1 + \delta] \frac{K}{Y}. \quad (18)$$

Finally, we characterize the equity premium in our model, because our quantitative results will produce interesting endogenous movements in this moment. Since there are two assets in our model (intermediate and final goods firms), we will calculate two equity premiums, as well as the equity premium on the return from holding both types of assets together, in proportion to their values. The returns of our model are calculated as follows. The risk free return is given by $R_t = \frac{1}{E_t[m_{t+1}]}$, the expected return on the final goods firm is given by

$E_t[R_{t+1}^f] = E_t \left[\frac{d_{t+1}^f + (1 - \Delta_{t+1})X_{t+1}^f}{X_t^f} \right]$, and the return on intermediate good firm is given by $E_t[R_{t+1}^i] = E_t \left[\frac{d_{t+1}^i + q_{t+1}K_{t+2}}{q_t K_{t+1}} \right]$. The combined return is the total return from holding the total value of intermediate and final good firms,

$$E_t[R_{t+1}^c] = E_t \left[\frac{d_{t+1}^f + (1 - \Delta_{t+1})X_{t+1}^f + d_{t+1}^i + q_{t+1}K_{t+2}}{X_t^f + q_t K_{t+1}} \right]. \quad (19)$$

The equity premium for all three of these assets is then calculated as the expected return minus the risk free rate, adjusted for leverage. In line with Croce (2014), we use a leverage ratio of 2.

We are not aware of any previous work that distinguishes between the equity premium on profits and the equity premium on productive capital, although the distinction is important for our results. Our empirical results will show that the equity premium on securities is substantially higher than the equity premium on productive capital. As discussed in detail in section 7.6, because there are barriers to entry, final goods firms are nonreproducible assets, and thus their price can have large fluctuations in response to changes in pure profits. In contrast, productive capital is reproducible, and thus the price of capital q has only moderate fluctuations (due to the presence of adjustment costs). As a direct result, in moving from an economy in which the value of securities is high relative to the value of the capital stock the equity premium will increase.³⁶

5 A qualitative solution to the puzzles

We hypothesize that a permanent increase in markups, along with a decrease in the real interest rate, can explain the five puzzles. We now examine the impact of both changes on our five stylized facts of interest. We will compare analytic comparative statics in the steady state of our model, which allows us to unambiguously sign the derivatives. Our results will show that the proposed mechanisms can *qualitatively* solve the five puzzles that are the subject of this paper: in the succeeding sections, we will then test whether our mechanisms can *quantitatively* solve them.

5.1 An increase in markups

We examine the change in steady state model moments to small changes in the steady state level of markups, $\bar{\mu}$.

Proposition 1. *The following comparative static results hold:*

³⁶This result should be tempered somewhat by the fact that we may be missing a key element. As Larry Summers has pointed out (see Summers (2017)), as equity prices rise, firms become less levered.

1. $\frac{\partial \bar{K}}{\partial \bar{\mu}} < 0$, $\frac{\partial \bar{Y}}{\partial \bar{\mu}} < 0$. (**P1**) An increase in steady state markups will lead to a decrease in the capital-to-labor ratio and the capital-to-output ratio.
2. $\frac{\partial Q}{\partial \bar{\mu}} > 0$. (**P2**) An increase in steady state markups will lead to an increase in empirical Tobin's Q .
3. $\frac{\partial AR}{\partial \bar{\mu}} > 0$. (**P3**) An increase in steady state markups will lead to an increase in the average return on capital.
4. $\frac{\partial PS}{\partial \bar{\mu}} > 0$. (**P4**) An increase in steady state markups will lead to an increase in the pure profit share.
5. $\frac{\partial LS}{\partial \bar{\mu}} < 0$, if $\sigma \leq 1$. (**P4**) An increase in steady state markups will lead to a decrease in the labor share if production is Cobb-Douglas, or the production elasticity of substitution is less than one.
6. $\frac{\partial \bar{I}}{\partial \bar{\mu}} < 0$. (**P5**) An increase in steady state markups will lead to a decrease in the investment-to-output ratio.

Proof. See appendix B. □

Proposition 1 shows that an increase in markups can potentially explain many of the five puzzles. We give proofs of the above results in appendix B, and discuss here the intuition.

An increase in markups will increase profits of final goods firms, and thus dividends to the owners of final-goods firms securities. This will increase the value of these securities, leading to an increase in wealth-to-output (**P1**) and Tobin's Q (**P2**).

Because of the increased profit share, there is also an increase in the measured average return on capital – note that this goes against **P3**, the constant average return seen in the data. However, the increase in markups leads to an increase in the wedge between the average return on capital and the risk free interest rate (see equation (16)), which is supportive of **P3**.

An increase in markups will directly lead to an increase in the profit share (**P4**). With an increase in the profit share, there will be a decrease in the combined labor and capital share. Whether there will be an unambiguous decrease in the labor share depends on the value of σ , which determines the elasticity of the ratio of the labor to capital share to a change in markups. Under Cobb-Douglas, there is no response in the relative labor-capital share to a change in markups. If $\sigma < 1$, the relative labor share will decline with an increase in markups (**P4**). Although the relative capital share increases, in our simulations this is never enough to lead to an overall increase in the capital share: although with an increase in markups capital gets a bigger slice over the overall labor + capital share pie, the shrinkage of the pie due to an increased profit share outweighs this effect.

The higher markups will increase the wedge between the marginal product of capital and the rental rate of capital (see equation (A.29)). However, since the

Table 1: Effect of an increase in markups

<i>Model statistic</i>	<i>Symbol</i>	<i>Effect</i>
Capital-to-output (P1)	K_t/Y_t	↓
Tobin's Q (P2)	Q_t	↑
Average return (P3)	AR_t	↑
Profit share (P4)	PS_t	↑
Labor share (P4)	LS_t	↓ if $\sigma \leq 1$
Investment-to-output (P5)	I_t/Y_t	↓

interest rate does not change, neither does the rental rate of capital. Thus there must be an increase in the marginal product of capital, and hence a decrease in the capital-to-output ratio. This will also lead to lower investment (**P5**, see equation (18)).

All of the effects of an increase in markups are summarized in Table 1, which shows that an increase in markups can go a long way towards explaining the five puzzles. One remaining challenge is that an increase in markups would tend to increase the average return on capital, while in the data it is relatively stable; the next section will suggest a possible solution to this challenge.

5.2 A decrease in interest rates

We now study the comparative static effects of a lower real interest rate. We model the decrease in interest rates in a reduced form way, through an increase in the utility wedge D . Two key examples we have in mind are a decrease in population growth and a decrease in the mortality rate.³⁷

Proposition 2. *The following comparative static results hold:*

1. $\frac{\partial W}{\partial D} > 0$. (**P1**) *An increase in D will lead to an increase in the wealth-to-output ratio.*
2. $\frac{\partial K}{\partial D} > 0$, $\frac{\partial L}{\partial D} > 0$. (**P1**) *An increase in D will lead to an increase in the capital-to-output ratio and the capital-to-labor ratio.*
3. $\frac{\partial AR}{\partial D} < 0$. (**P3**) *An increase in D will lead to a decrease in the average return on capital.*
4. $\frac{\partial LS}{\partial D} > 0$ if $\sigma < 1$. (**P4**) *An increase in D will lead to an increase in the labor share if $\sigma < 1$. If $\sigma > 1$, $\frac{\partial LS}{\partial D} < 0$.*
5. $\frac{\partial KS}{\partial D} < 0$ if $\sigma < 1$. (**P4**) *An increase in D will lead to a decrease in the capital share if $\sigma < 1$. If $\sigma > 1$, $\frac{\partial KS}{\partial D} > 0$.*

³⁷Although in a representative agent model slower population growth will not push down the real interest rate, in most OLG models it will have this effect. Eggertsson, Mehrotra and Robbins (2017), in an OLG context, discuss a variety of forces that have driven down interest rates since 1970.

Table 2: Effect of an increase in D

<i>Model statistic</i>	<i>Symbol</i>	<i>Effect</i>
Wealth-to-income (P1)	W_t/Y_t	↑
Capital-to-output (P1)	K_t/Y_t	↑
Tobin's Q (P2)	Q_t	?
Average return (P3)	AR_t	↓
Profit share (P4)	PS_t	0
Labor share (P4)	LS_t	↑ if $\sigma < 1$
Capital share (P4)	KS_t	↓ if $\sigma < 1$
Investment-to-output (P5)	I_t/Y_t	↑

6. $\frac{\partial I}{\partial D} > 0$. (**P5**) An increase in D will lead to an increase in the investment-to-output ratio.

Proof. See Appendix B. □

Once again, we leave the proofs to appendix B, and discuss the intuition for these results.

A decrease in r would tend to decrease the marginal product of capital (see equation (A.29)), and thus increase the capital-to-output ratio. In addition, a lower r means that the rights to future dividends are discounted at a lower rate, which would tend to raise the value of securities, X_t^f , relative to output. Since both capital-to-output and security value-to-output increase, the wealth-to-output ratio will increase as well (**P1**). The total effect on Tobin's Q is ambiguous, however, since both the numerator and denominator of this ratio will increase.

As seen in equation (16), a decrease in the real interest rate would tend to decrease the measured average return on capital (**P3**). Note that a decrease in interest rates thus has the opposite effect of an increase in markups. If we consider both changes simultaneously, there are two opposing forces on the average return on capital: the increase in profits would tend to raise the AR , while the decrease in r would tend to decrease it.

Since a lower r means a higher capital-to-output ratio, this will tend to increase the labor share and decrease the capital share (**P4**), assuming, as most evidence suggests, that the production elasticity of substitution $\sigma < 1$.

Because a decrease in r leads to an increase in the capital-to-output ratio, there will be also be an increase in the investment-to-output ratio (**P5**, see equation (18)). Thus once again, the force of a lower r tends to counteract the force of higher profits. The effects of changes in r on the capital-to-output ratio and investment-to-output ratio are strongly dependent on whether capital and labor are complements or substitutions, i.e. the elasticity of substitution σ is less than or greater than one. The lower is σ , the smaller the effects of changes in the interest rate on capital and investment.

5.3 A decrease in productivity

We also examine the effect of a decrease in the permanent growth rate of productivity, ζ . From equation 17, we see a decrease in ζ will lead to an increase in the discount factor, and thus a decrease in the steady state real interest rate r . Depending on the elasticity of intertemporal substitution, ψ , the change in interest rates will either be greater than the change in ζ ($\psi < 1$), less than the change in ζ ($\psi > 1$), or of the same magnitude ($\psi = 1$). In our baseline calibration we have $\psi > 1$, and thus the change in interest rates will be smaller in magnitude than the change in productivity growth. Then, from equation 14, we see that when productivity growth decreases, the security value-to-output ratio will decrease. The explanation for this is fairly straightforward. A slowdown in productivity means a slowdown in the growth rate of the economy, and thus of dividends, leading to a decline in the value of securities, and with $\psi > 1$ this outweighs the effect of a lower discount rate of the dividends (r) on the value of securities.

Proposition 1 and 2 have analytically opened the possibility to explaining the five puzzles. Whether they can do so quantitatively is the subject of the rest of the paper.

6 Estimating changes in markups and interest rates

The comparative statics results above show that an increase in markups, combined with a decrease in the equilibrium interest rate, can potentially explain the five trends. To test whether these qualitative predictions hold up in a quantitative sense, we need a measure of how these variables have changed in the US data. To do so, we rely upon a growing literature which estimates secular changes in markups. We estimate markups using aggregate macro data, and find there has been a moderate increase from 1970 to the present, a result roughly in the midpoint of this recent literature on markups. We use existing findings from the literature on the decline in the natural rate of interest, and use a conservative decline of 2 percentage points over our sample period. We emphasize that this paper is primarily concerned with explaining long term secular trends, and thus for our purposes it is the *change* in markups and interest rates that is important rather than their cyclicalities.³⁸

6.1 Markups

We estimate markups using aggregate macroeconomic data, exploiting the fact that under constant returns to scale (CRS) production, markups are proportional

³⁸Our stylized model does not include the many features and complications that the RBC literature has showed is necessary to match the co-movements and cyclicalities of the major business cycle moments. See, for example, Christiano, Eichenbaum and Evans (2005).

to the profit share of the economy: in this technique we follow the work of Barkai (2016). In particular, under CRS markups equal the inverse of the share of production not accounted for by pure profits:

$$PS = \frac{\mu - 1}{\mu} \implies \mu = \frac{1}{1 - PS}. \quad (20)$$

As discussed in section 2, we estimate the profit share from the data by taking the residual share of output after subtracting labor and capital income.³⁹ We use the different versions of the capital rental rate previously discussed, and average over the different estimates. Under these assumptions, the markup increases significantly over the time period, from 1.11 in 1970 to around 1.23 in 2015.

Whereas prior to 2016 it was difficult to find an extended time series for markups, there has been a recent bumper crop of papers estimating markups. Three separate strands have emerged, each estimating markups using a different technique, described in detail in appendix C: (i) a method taken from the New Keynesian literature (ii) a method using techniques from IO and firm level data (iii) a method backing out markups from the profit share. Table 3 displays the various estimates, and shows that our results (ERW) fall into the midpoint of this growing literature. In our quantitative analyses we will use these alternative markup estimates as robustness checks to our baseline results.

In a recent paper, Karabarbounis and Neiman (2018) has extended the calculation of factor shares back to 1960, using a similar formula to the Hall-Jorgensen formula for the rental rate of capital used in section 2 and in Barkai (2016). Karabarbounis and Neiman (2018) finds that “factorless income”, income which accrues neither to labor nor to capital, is not only high in 2015, as is emphasized in this paper and in Barkai (2016), but also high in 1960. In appendix figure A.5, we extend our markup calculation back to 1960, and find a similar U shape: markups are moderately high in 1960, before declining to a nadir in 1980, after which they increase to their maximum in the present. Interestingly, papers using separate methods to calculate markups have also found a U-shape from 1960 to the present; see Traina (2018), De Loecker and Eeckhout (2017), and Gutierrez (2017). This growing literature has made it clear that pure profits are not a new thing, although this paper focuses on the evolution of profits over the past half-century.

Karabarbounis and Neiman (2018) raise an important objection to using the Hall-Jorgensen method to estimate the rental rate of capital; since the formula includes the real interest rate, this mechanically creates a tight negative correlation between real interest rates and the profit share at short and medium frequencies.

³⁹The estimates of capital income are dependent on factors which are difficult to measure or uncertain, in particular the risk premium on capital investments and the expected change in the relative price of capital. For this reason, we calculate four different versions of our markup series, each under different assumptions about the risk premium and the relative price of capital; these series all show a similar trend and are reported in appendix figure A.5. The four series differ along two dimensions: whether (i) risk premium and/or (ii) the change in the relative price of capital are included in the calculation of the rental rate of capital. See appendix figure A.5.

Thus in periods where there is a dramatic rise in the real interest rate, like in the early 1980s, there is a corresponding fall in the profit share. Like many returns of financial assets, real interest rates can be volatile in the short and medium term, and measures of the factor shares using the Hall-Jorgensen formula will inherit this volatility. Does this mean we should give up trying to measure the rental rate of capital using the interest rate? We argue that although there indeed volatility in interest rates, over the long horizon there has been clear downward trend in interest rates, and thus a decline in the rental rate of capital. Thus, for explaining long-run trends at least, declining interest rates suggest a lower rental rate of capital, even if we are not able to fully account for short-term movements in real interest rates.

Table 3: Markup Estimates

<i>Estimate</i>	Year Range	μ_{pre}	μ_{post}	$\Delta \mu$
Barkai (2017)	1984-2015	1.025	1.21	.19
Nakarda & Ramey (2013)	1970-2014	.95	1.07	.12
De Loecker & Eeckhout (2017)	1980-2014	1.18	1.67	.49
Traina (2018)	1980-2014	1.10	1.15	.05
Gutierrez (2017)	1980-2014	1.15	1.21	.06
Hall (2018)	1988-2015	1.12	1.27	.15
ERW (2018)	1970-2015	1.11	1.23	.12

6.2 Interest rates

Table 4: Natural Rate Estimates

<i>Estimate</i>	r^*_{1970}	r^*_{2015}
Holston, Laubach, and Williams (2017)	3.91	.43
Del Negro et. al. (2017)	2.5	1.5
5-Year MA Real Federal Funds	2.25	-1.55

We use a conservative estimate of the decline in the natural rate of interest; results in the literature vary somewhat widely. Holston, Laubach and Williams (2017), using the Laubach-Williams model, estimate a decline of ≈ 3.5 percentage points, from 3.91 in 1970 to .43% in 2015. Del Negro et al. (2017), using DSGE and time series analysis, estimate a decline of 1-1.5 percentage points, from 2-2.5% to 1-1.5%. A simple five year moving average of the real federal funds rate yields a decline of 3.8 percentage points, from 2.25% to -1.55%. For our baseline analysis, we use a decline in the natural rate of interest of 2 percentage points, from 3% in 1970 to 1% in 1970.

7 A quantitative solution to the puzzles

We now test the hypothesis that changes in markups and interest rates can quantitatively explain the five puzzles outlined in the introduction. While the analytic comparative statics of section 5 suggested our model can qualitatively match the puzzles, we now investigate the magnitudes. To do so, we cannot rely purely on steady state results, since in a steady state there is no equity premium. Instead, we will compare moments of the stochastic economy. Our results will show that changes in markups and interest rates can indeed quantitatively account for the five puzzles.

The five puzzles all involve *changes* in macroeconomic quantities. For this reason, we will judge our quantitative success by whether changes in our model moments can match changes in the data moments. We first calibrate our model to US data in 1970 (matching levels only). We then solve the model, and compute moments of interest by calculating theoretical moments from the second-order approximation of the model's solution (see section 4). We then change the long run level of markups from their 1970 to their 2015 level, then interest rates, and then both together, calculating the model's moments for each change.⁴⁰ We will then compare changes in our model moments between the two time periods to changes in the data moments.

We emphasize that our goal in this exercise is not to understand short-run quarter to quarter movements in the economic variables of interest. It is well known that in order to explain short run co-movements in output, consumption, investment, and the real interest rate, a wide variety of frictions are needed (see, for example, Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005)). In this paper we are interested in long-run changes in our variables of interest over the past half century, and abstract from questions about the short and medium term.

7.1 Quantitative calibration

We calibrate the model to the US economy in 1970. The goal of the calibration is to match the level of the model's moments to the 1970 data moments: once again, we do not target changes in moments over the time period, as this will be the main outcome and test of our analyses. We focus on matching in particular the moments which correspond to the five macroeconomic puzzles of interest. The calibration results will show our model moments closely matching the data

⁴⁰Our choice of 1970 as the initial steady state is largely guided by the fact that this corresponds to the period before the onset of the great inflation, which was a period of great volatility for inflation and the real interest rate. The real interest rate fell in the early 1970s on account of a monetary policy expansion, before increasing dramatically during the Volker disinflation of the early 1980s. By choosing 1970 as a comparison point we estimate a moderate decline in interest rates of 2% over the time period, and thus avoid exaggerating the movement on the real interest rate. Our choice of 1970 vs 1980 also affects our estimated increase in markups, as markups reached their nadir in 1980.

Table 5: Parameters taken from the data and related literature

<i>Panel A: Data</i>	<i>Symbol</i>	<i>Value</i>	<i>Source</i>
Productivity growth (/yr)	ζ	2.02%	Fernald (2012)
<i>Panel B: Related literature</i>			
Long run risk persistence	ρ	.98	Croce (2014)
Long run risk std. dev.	σ_x	.0010	Croce (2014)
Short run risk std. dev.	σ_a	.01	Croce (2014)
Depreciation rate (/yr)	δ	6%	Jorgensen (1996)
Adjustment costs	ξ	.12	Croce (2014)

moments in levels for 1970, with parameter estimates within the range of values reported in the existing literature.

There are three categories of parameters: (i) the long run level of markups and interest rates estimated in section 6⁴¹ (ii) parameters taken from the data and literature, and (iii) parameters chosen to match 1970 data moments, through the minimization of an objective function.

Parameters from category (i) are based upon our estimation in section 6: long-run markups $\bar{\mu}$ are 1.11 in 1970, and the real interest rate is 3%. Parameters from category (ii) are displayed in table 5. The rate of productivity growth ζ we take from Fernald (2012). We use the estimates of Croce (2014) to calibrate our long-run and short-run productivity risks. The depreciation rate comes from Jorgenson (1996). Following Croce (2014), we choose ξ to match the variability of investment to output, and find $\xi = .12$.

We choose the remaining parameters (with the exception of the AR(1) parameters, which are estimated separately) to match six key moments of the US economy as of 1970. All data moments are calculated through a 5-year moving average around 1970, using yearly data. The first four moments are standard in the DSGE literature: a real interest rate of 3.0%, an investment-to-output ratio of 16.2%, a labor share of 71.5%, and a share of hours worked of 18%. In light of our above theoretical and empirical analysis, we add a new moment to the calibration: a wealth-to-output ratio of 2.66. Finally, in consideration of our goal of matching financial moments, we calibrate our model to match an equity premium of 4.71%. The parameters chosen this way are the rate of time preference β , the capital production coefficient α , the production elasticity of substitution σ , the firm exit rate Δ , the labor supply coefficient ν , and the risk aversion parameter γ .

There is no one-to-one mapping between these remaining parameters and the targets, hence we jointly choose all parameters to match the model output to the targets. Nevertheless, each of the parameters above correspond relatively closely with one of the key moments we are trying to match. The rate of time preference β is closely related to the real interest rate; as β increases, the real interest

⁴¹In appendix E we report robustness results using alternative measures of markups.

Table 6: Calibrated parameter results

<i>Parameters chosen to match targets</i>	<i>Symbol</i>	<i>Value</i>
Capital production elasticity	α	0.30
Production elasticity	σ	0.90
Firm exit rate	Δ	0.0048
Rate of time preference	β	0.9957
Risk Aversion	γ	8.11
Hours supplied	ν	0.21
AR(1) persistence	ρ_μ	0.97
AR(1) error variance	σ_μ	.005

rate falls. The capital share parameter α along with the production elasticity σ determine the labor share as well as the investment-to-output ratio. The rate of firm exit Δ has a large effect on the wealth-to-output ratio. The parameter ν determines the number of hours worked. Finally, the risk aversion parameter γ is the most important determinant of the equity premium.

We minimize a weighted sum of the squared difference of our model moments and the 1970 data moments. Table 6 displays the parameter values, while 7 shows the resulting model moments and compares them with the data. With the parameters selected by our minimization, the model moments come very close to matching those in the data.

We separately estimate the AR(1) parameters for markups, ρ_μ and σ_μ (the standard deviation of the error ϵ_t^μ). To do so, we detrend our baseline markup series and estimate $\rho_\mu = .97$ and $\sigma_\mu = .005$ via OLS.

The parameters in table 6 fall directly within the range of parameter values in the macro-finance literature. We now compare our parameters with previous calibrations/estimations of DSGE models with recursive preferences and/or long-run productivity risk. Our β of 0.9957 is quite close to the value of .9957 in Croce (2014), .997 in Van Binsbergen et al. (2012), and .9606 in Rudebusch and Swanson (2012). While previous literature has generally used Cobb-Douglas production, we use CES, and find $\sigma = 0.90$.⁴² Our capital elasticity parameter α , 0.30, is smaller than the capital share parameter in the existing literature (.34 in Croce (2014), .3 in Van Binsbergen et al. (2012), .33 in Rudebusch and Swanson (2012)): in order to match the labor share with a higher level of markups, α must be smaller. Our estimated coefficient of risk aversion, $\gamma = 8.11$, is in line with Croce (2014), as well as the estimates of Vissing-Jørgensen and Attanasio (2003). We are able to match the equity premium with such a small coefficient of risk aversion because of the presence of long-run risk in our model: in models without this feature, the estimated coefficients are much higher.

The monthly firm exit rate of 0.48% yields a yearly rate of 5.71%. This is half the exit rate used in Bilbiie, Ghironi and Melitz (2012), but comparable in magnitude to the annual production destruction rate (in terms of share of prod-

⁴²This is line with the range of estimates in Antras (2004).

ucts and market share) of 8.8% from Bragdon, Redding and Schott (2010). We note, however, that the Δ parameter in our model corresponds to the notion of bankruptcy (in which the shareholders are wiped out) and/or the replacement of a firm’s products with that of a competitor’s (in which some portion of shareholder value is wiped out), not to the replacement of a product within a firm by the firm’s own product. Data from Decker et al. (2016) shows the average annual firm exit rate is around 9%. However, since young firms, which tend to be the smallest, go out of business at a much higher rate than old firms, the average exit rate weighted by employment is only around 2.75%. We note that, unlike the firm entry rate, there has not been a secular decline in firm exit rates since 1980.

Table 7: 1970 calibration results

<i>Targets</i>	<i>Model</i>	<i>Data</i>	<i>Source</i>
Real interest rate	2.99%	3.00%	Federal Reserve
Wealth-to-output ratio	2.66	2.66	Financial Accounts
Investment-to-output ratio	15.64%	16.15%	NIPA
Labor share	71.17%	71.49%	Elsby (2013)
Equity premium	4.64%	4.71%	Croce (2014)
Labor supply	0.18%	0.18%	Croce (2014)

We once again emphasize that we are choosing parameters only to match 1970 moments. In particular, we do not choose any parameters to try and match the change in moments from 1970 to the present. The success or failure of our exercise will be comparing changes in our model moments with changes in the data moments.

7.2 Quantitative results: overall hypothesis

We begin with a test of our overall hypothesis: whether changes in markups and interest rates can explain the five macroeconomic puzzles. Table 8 shows the combined effects of changing markups, productivity growth, and D from their steady steady values in 1970 to their 2015 values.⁴³ Overall, our model does a reasonably good job of explaining the five macroeconomic puzzles: the changes in the model moments which correspond to the puzzles are similar in magnitude to the data moments. The first column lists the macroeconomic moment of interest (with the corresponding puzzle in parentheses). The second column displays the calculated change in model moments between 1970 and 2015 that is the result of changing markups and interest rates. The third column lists the change in the data moments.

We discuss each puzzle result in turn. **(P1)** Our model produces, in line with the data, an increase in the wealth-to-output ratio, with a relatively stable capital-to-output ratio. Our model’s increase in the wealth-to-output ratio of 0.88 is quite close to the increase seen in the data, .95, while the increase

⁴³Section 7.4 describes our calibration procedure for the change in D .

Table 8: Quantitative results: changes in markups, productivity growth rates, interest rates

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	0.88	0.95
Capital-to-output ratio (P1)	0.21	0.31
Tobin's Q (P2)	0.17	0.40
Real interest rate (P3)	-2.00 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	0.96	0.64
Profit share (P4)	8.94 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	-6.85 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-2.09 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	-1.35 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	2.22 <i>pp</i>	0 - 2 <i>pp</i>

in the capital-to-output ratio is also comparable. (**P2**) Our model produces an increase in Tobin's Q, however the increase is somewhat smaller in our model: 0.17 compared to .40 in the data.

(**P3**) In line with the puzzle, our results show a moderate increase in the average return, with a declining real interest rate.⁴⁴ (**P4**) The model does a good job of matching the increase in the profit share, as well as the decline in the labor and the capital share; however, the ability of the model to exactly match the magnitude of the profit share decline should not be surprising, since the change in markups was estimated explicitly to match the movement in this moment. (**P5**) Our model produces a decline in investment, despite a high Tobin's Q and low real interest rate. The decline in investment is larger than seen in the data.

In summary, we have shown that the estimated increase in markups and decrease in the real interest rate can quantitatively account for the five facts described in the introduction. In the following two sections, we examine the impact of changes in markups and interest rates separately on our five puzzles of interest. We will see that both changes are needed in order to match the macroeconomic data.

7.3 Quantitative results: markups

Table 9 shows the results of increasing the long-run level of markups from our baseline estimated level in 1970, 1.11, to its estimated level in 2015, 1.23. The results are overall quite supportive of our hypothesis, however it is clear that an increase in markups alone cannot fully explain all five puzzles.

(**P1**) In line with puzzle 1, there is an increase in the wealth-to-output ratio, 0.50, however it is smaller than the .95 increase in the data. In addition, there is a moderate decrease in the capital-to-output ratio, compared with a moder-

⁴⁴It is not surprising that the change in interest rates exactly matches the decline: we choose the increase in D to exactly target a decrease in r of 2% (see section 7.4).

ate increase in the data. **(P2)** There is an increase in Tobin’s Q of 0.34, quite comparable to the .40 in the data. **(P3)** Due to the increase in profits there is a large increase in the average return – much larger than the increase seen in the data. There is no change in interest rates from the increase in markups: although there is an increase in the supply of assets due to the increase in security values, in a representative agent economy this is exactly counteracted by an increase in demand, and there is no change in interest rates. This result shows that although an increase in markups can explain the divergence between the average return and the interest rate, it cannot by itself generate a constant average return.

(P4) The increase in markups leads to an increase in the profit share of 8.94, and a corresponding decrease in both the labor and the capital share. **(P5)** Due to the increase in the wedge between the marginal product of capital and the rental rate, there is a moderate decline in investment of -1.36 p.p..

Table 9: Quantitative results: changes in markups only

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	0.50	0.95
Capital-to-output ratio (P1)	-0.17	0.31
Tobin’s Q (P2)	0.34	0.40
Real interest rate (P3)	-0.00 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	5.09	0.64
Profit share (P4)	8.94 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	-7.23 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-1.70 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	-1.36 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	2.34 <i>pp</i>	0 – 2 <i>pp</i>

Table 9 is strongly supportive of our hypothesis. Quantitatively, it shows that changes in markups alone can go a long way towards explaining several of the five puzzles that are the subject of this paper. The large increase in the average return predicted by the model is one exception to this; however, as we will see in the next section, a decline in interest rates can serve as a check to this force, driving down the average return.

7.4 Quantitative results: interest rates

We now further decompose the results shown in table 8. We consider two channels which lead to a decline in interest rates: (i) a decline in productivity growth and (ii) a reduced form increase in the utility wedge D . We decrease the productivity growth rate (the parameter ζ), from 2.02% per year in 1970 to its value of .65% in the present. We increase D such that the combined effect of the changes in μ , ζ , and D lead to a decrease in interest rates over the time period of 2%, which is in line with the data.

The results of increasing D are shown in table 10. With lower interest rates, there is a decrease in the average return **(P3)**, and an increase in investment

Table 10: Quantitative results: changes in D only

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	0.24	0.95
Capital-to-output ratio (P1)	0.21	0.31
Tobin's Q (P2)	-0.05	0.40
Real interest rate (P3)	-1.19 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	-1.89	0.64
Profit share (P4)	-0.00 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	0.24 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-0.24 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	1.69 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	0.25 <i>pp</i>	0 - 2 <i>pp</i>

(**P5**). There are also moderate changes in the labor and capital share, wealth-to-output, and capital-to-output. The results of changing productivity growth only are shown in table A.1. A decline in productivity growth leads to a decline in the real interest rate of -0.84— not enough to match the full decrease seen in the data. With slower productivity growth, there is also a decline in the average return on capital.

7.5 Other markup estimates

Appendix tables A.3-A.6 show our quantitative results for other estimates of markups from the literature – those of Nekarda and Ramey (2013), De Loecker and Eeckhout (2017), Gutierrez (2017), and Hall (2018). The results for the Nekarda and Ramey and Hall are quite similar to our baseline results, since the increase in markups is similar in magnitude.

The results for the De Loecker and Eeckhout estimates are much larger in magnitude than our baseline results. This is unsurprising, given that they estimate a massive increase in markups, from 18% in 1980 to 67% in the present. Table A.4 shows the combined results of changing markups and interest rates from their 1970 to their 2015 values. The results show a massive decline of -20.38 in the labor share, an increase in the pure profit share of 24.87, an increase in the average return of 12.89, and an increase in Tobin's Q of 0.99. These numbers are well out of line with the data. A natural question is then, is there anything in our model which could square these numbers with the data? The answer is, tentatively, yes. If the increase in markups was also accompanied by other forces that caused gross profits to decrease, this would tend to decrease the magnitudes shown in Table A.4. We discuss two potential mitigating factors in sections 8.2 and 8.3.

7.6 Excess returns of final goods firms

Table 9 displays a surprising result: an increase in markups causes an increase in the equity premium of 2.34%. As discussed in section 4, this is due to a composition effect. The risk premium on pure profits is greater than the risk premium on productive capital. Then as the value of the pure profits increases relative to that of capital, i.e., an increase in Tobin's Q , the combined equity premium on holding profits and capital will increase. There are only negligible changes in the returns on the different assets: risk free bonds, intermediate goods, and final goods firms.

We now further discuss the reason why the risk premium on final goods firms, ER^f , is higher than the risk premium on intermediate goods firms, ER^i . Excess returns for an asset, following Cochrane (2009), can be written as $E_t[r_{t+1} - r_t^f] \approx -cov_t(m_{t+1}, r_{t+1})$. Because $ER^f > ER^i$, ER^f must (negatively) covary more with m than ER^i . Assets must offer a higher equity premium when its return move in a direction opposite to that of the discount factor.

The returns of final goods firms show larger fluctuations than the returns on physical capital because the supply of final goods firms is fixed in the short run. A key assumption of our analysis is that there are significant barriers to entry for final-goods firms. Thus, in contrast to productive capital, the securities which hold the rights to pure profits are non-reproducible assets. When pure profits increase, there is thus a large increase in the price of these securities. When the productivity of capital increases, firms can simply produce new capital to take advantage of this increased productivity. We should thus expect the price of final goods firms to be more volatile than intermediate goods firms, and thus yield a greater risk premium.

It is well known since Jermann (1998) that capital adjustment costs are necessary in order to generate a realistic equity premium in production economies. In the absence of adjustment costs, agents optimally choose to smooth their consumption (especially with high risk aversion), which decreases the volatility of the stochastic discount factor. Relatedly, without capital adjustment costs, the price of capital q does not fluctuate, and thus the only source of fluctuations of returns are in the productivity of capital. The higher the adjustment costs, the higher the capital risk premium. The existing literature generally needs high levels of adjustment costs and high risk aversion to match the equity premium. With our addition of traded monopolistic final goods firms, we can have both lower adjustment costs and lower risk aversion.

8 Other explanations for the puzzles

While this paper pursues the hypothesis that changes in markups and interest rates are largely responsible for the macroeconomic changes seen over the past forty years, we do not argue that other forces are not at work. We see our hypothesis as complementary to three other explanations: a decrease in the bargaining

power of workers, an increase in risk premia, and an increase in intangible investment.

8.1 Neoclassical and zero-rent economy

In the neoclassical Ramsey/Koopmans/Cass model, there are no monopoly profits, and securities markets measure only the value of a firm's capital stock. In the basic version of this model, the value of wealth is always equal to the value of capital, Tobin's Q always equals one, and the average return moves in line with interest rates. This model is thus at odds with the divergence between wealth and capital in the data, the divergence between the average return and interest rates, and the increase in Tobin's Q to a level permanently above 1.

Robert Hall (2001) pursues the hypothesis that a firm's stock market price measures the value of its capital. As Hall notes, if there are no barriers to entry, in the longer run capital earns no rent because it is in perfectly elastic supply to the firm; he calls this the "zero-rent" economy. In a zero-rent economy, movements in Tobin's Q and wealth to output can only be due to two factors (i) adjustment costs (ii) unmeasured intangible capital. However, adjustment costs cannot explain a permanently elevated Tobin's Q , as we have seen in the data, unless the speed of adjustment of capital is unrealistically slow. Whether there is unmeasured intangible capital will be considered extensively below.

8.2 Unobserved intangible capital

Another possible explanation for the five facts we have observed is a large amount of capital that is unmeasured in the national accounts. If this capital was valued by investors, it would be reflected in the market price of firms, and thus explain a high level of wealth-to-capital and Tobin's Q . In addition, it would increase the measured average return on capital, since the unmeasured capital would generate income but would not be counted in the denominator of the average return equation.

For many years business spent heavily on certain "intangible" products, which, although from an economic point of view appear to be capital, were counted in the national income accounts as expenses: computer software, computer databases, scientific and non-scientific R&D, mineral exploration, spending to create artistic originals, and many more (see Corrado, Hulten and Sichel (2009) and Corrado, Hulten and Sichel (2005) for details). However, over the past 20 years the BEA has twice revised the national income series to include many types of intangible capital, such as software investment, R&D investment, and spending on artistic originals. All of our figures and statistics in this paper already include these forms of intangible capital.

Corrado, Hulten and Sichel (2005) classify intangible capital into three categories: (i) computerized information, which includes software and databases (ii) innovative property, which includes investment in R&D and artistic originals, and (iii) economic competencies, which include brand equity, advertising,

marketing, firm specific human capital, and worker training. While much of categories (i) and (ii) are already capitalized in the GDP accounts, category (iii) is largely uncapitalized, and is by far the largest category of intangible capital according to Corrado, Hulten and Sichel (2005). The BEA is considering adding a number of categories to intangible investment, including computer design services, architectural and engineering services, management consulting services, advertising, and marketing research.

A key question, however, is whether all of these activities should be considered as capital investment. In Corrado and Hulten’s framework, “any outlay that is intended to increase future rather than current consumption is treated as a capital investment.” From a firm’s point of view, spending money in period t to increase market share in period $t + 1$, is indeed investment, and would be classified as such by Corrado, Hulten and Sichel (2009). However, from a macroeconomic point of view, this “intangible capital” does not lead to future production or output, it is simply a shift in output from one firm to another. In fact, there is a very close correspondence between Corrado and Hulten’s third category of intangible capital and market power, branding, and product differentiation: in other words, the “unmeasured” intangible investment may have been an investment in creating the very “non-zero-rent” economy which is the subject of this paper.

We favor a different interpretation of Corrado and Hulten’s third category (spending on economic competencies), rather than interpreting it as intangible investment. Because spending on these categories does not, and will not, contribute to the production of goods, we argue they should not be counted as investment spending and accumulated into the capital stock. For this reason, we term this spending “non-productive intangible investment”. While from a firm’s point of view there may be little difference between investing in a marketing campaign and investing in a new factory, from an economic accounting point of view there is a difference: marketing spending will not ever lead to an increase in the productive capacity of a firm. This suggests that there must be a distinction between a firm’s financial accounting, where it makes a lot of sense to accumulate this intangible investment in capital, and in national economic accounting, in which we argue it makes less sense.

We can include this non-productive intangible (NPI) spending in our model. If we assume this NPI spending is done by final goods firms, aggregate profits would then be given by $\Pi_t = \frac{\mu_t - 1}{\mu_t} Y_t - NPI_t$. This non-productive spending is one way to make quantitative sense of the the large increases in markups estimated by De Loecker and Eeckhout (2017). If the increase in markups estimated by De Loecker and Eeckhout was also accompanied by an increase in the share of corporate income spent on non-productive intangibles, there would be a much smaller measured effect of these markups on Tobin’s Q, the average return, and the measured labor share.

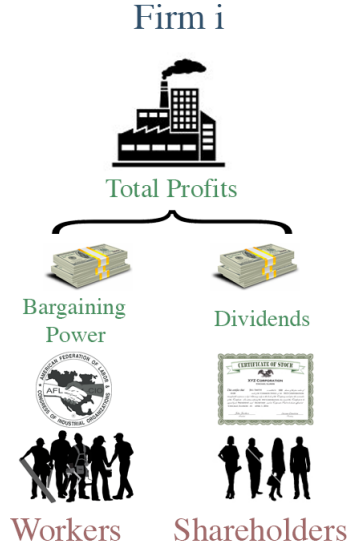


Figure 6: Profit distribution of final goods firms

8.3 Labor bargaining power

An alternative, and complimentary, hypothesis of what is driving the changes in the five macro trends is that there has been a decrease in labor bargaining power over the time period. In this story, the ability of firms to outsource production abroad or to contracting firms, the decline of labor unions, and the rise of dominant firms has lead to a reduction in the bargaining power of workers (see, for example, Weil (2014), Elsby, Hobijn and Şahin (2013), Dube and Kaplan (2010), Stiglitz (2015a) and Abdi and Danninger (2017)). To explore this hypothesis, we add a reduced form version of labor bargaining power. The pure profits of the firm, instead of being all being distributed to the shareholders, are now shared with the workers.

Figure (6) displays how the pure profits of final goods firms are distributed. Profits flow to two distinct groups, workers and shareholders of the final good firm. We assume workers have some level of bargaining power, ξ_t , which is taken as exogenous. This bargaining power may come about through a variety of channels: labor unions, powerful managers and executives who are able to bargain for their pay (Bebchuk and Fried (2004)), or matching frictions in the labor market (Rogerson, Shimer and Wright (2005)). Due to this bargaining power, workers receive a share ξ_t of aggregate profits.

The rest of the profits go to shareholders of final goods firms, who own the rights to the remaining pure profits, as dividends. Aggregate dividends distributed to shareholders at time t are thus given by

$$d_t = (1 - \xi_t)\Pi_t. \quad (21)$$

If labor's bargaining power ξ_t decreases, dividends to shareholders will increase. There will thus be an increase in the value of shares, wealth-to-output,

and empirical Tobin's Q . However, unlike in the case of an increase in pure profits, there will not be any changes in capital or investment, since there is no change in the wedge between the marginal product capital and its rental rate.⁴⁵ In addition, the share of income which is measured to go to labor would decrease, since this measured amount includes the rents from bargaining power, and the share of income measured to go to profits would increase.

8.4 Capital risk premium

Another factor that may be driving some of the five macro-trends is an increase in the capital risk premium. In an analysis that also includes changes in markups, the relative price of capital, and the capital share, Caballero, Farhi and Gourinchas (2017) find that an increase in the capital risk premium may be driving some of the trends, such as a stable average return on capital despite a decreasing risk free rate and a decrease in the labor share. This finding is not mutually exclusive with our hypothesis of increasing markups: in many specifications of their accounting exercise, Caballero, Farhi and Gourinchas (2017) find moderate increases in markups along with an increase in the capital risk premium.

We note that one of the results of our quantitative exercises is that there may be a distinction between the risk premium on productive capital and that on pure profits. In our baseline exercise, there is a large increase in the equity premium (analogous to the equity premium measured in the data), while there is not an increase in the capital risk premium.

In our reading, the data on the capital risk premium and the equity premium is somewhat mixed. Measures of the return on capital based on corporate bond rates (such as AAA, BAA, and BBB bonds) have decreased over the time along with the risk-free rate. However, there has been an increase in bond-spreads since the financial crisis. Furthermore, some measures of the equity risk premium have increased. Duarte and Rosa (2015) compile 20 different measures of the equity risk premium using a variety of different models. The first-principle component of these measures has increased substantially since 2000. The longer-term trend of the first principal component, however, is less apparent: since 1980 it does not appear there has been a change in this measure of the ERP, or in other combined measures such as the cross-sectional mean of the different models.

One counterfactual implication of an increase in the capital-risk premium is the effect on the wealth-to-output ratios. With an increase in capital risk premium stock prices should decrease (the profits are discounted at a higher rate), and thus we would predict a decrease in the wealth-to-output ratio instead of the large increase we have seen. Of course, if there are also contemporaneous

⁴⁵This is a simplification. In many search and matching model, labor bargaining power has a negative effect on investment, depending on the contract structure (see, for example, Grout (1984), Arnsperger and Croix (1990), and Acemoglu and Shimer (1999)). In these models, a decrease in the labor bargaining power would lead to an increase in investment – exactly the opposite of the effect of an increase in markups.

increases in dividends (perhaps through higher markups), this could offset this tendency.

9 Conclusion

The analysis of this paper relies heavily on our estimates of the level of pure profits and markups in the US economy. Unfortunately, there is a great deal of uncertainty around these estimates. To estimate pure profits, we need a good measure of both the capital and the risk premium in the economy. To estimate markups directly, we need an estimate of marginal cost, which has always been a difficult proposition. With these caveats in mind, we discuss some implications of our results.

There are important welfare implications to an increase in market power. Markups create distortions in product markets, and recent research suggests the welfare costs of markups are high, on the order of 7.5%-40% of GDP (see Edmond, Midrigan and Xu (2018) and Baqaee and Farhi (2017)). These theoretical results are in contrast to the classic estimates of Harberger (1995), who suggested the cost of monopolistic distortions was only .1% of GDP. In addition, an increase in markups can lead to lower investment. Markups are a wedge between the marginal product of labor and the wage, and the marginal product of capital and its rental rate. With higher labor wedges, the wage is lower and thus labor supply is lower, decreasing output. With higher capital wedges there is lower investment and a lower capital to output ratio, which also lowers output. There are also important indirect costs of monopoly power. As emphasized by Tullock (1967) and Krueger (1974), if there is competition for these monopoly profits through rent seeking behavior, in general the welfare loss is larger than in the absence of competition. If the value of pure profits is indeed in the order of 20% of output, there is an enormous incentive for firms to compete to take, maintain, or extend these profits. In this paper we have emphasized one particular aspect of “rent seeking” behavior, namely firms’ expenditures on product differentiation, branding, and advertising in order to maintain market share, which the literature has often called intangible investment. Of course there is also the more traditional political rent seeking.

The increase in market power also has implications for income inequality. With higher pure profits, workers receive a lower share of output and capitalists a higher share. Since individuals with higher incomes receive a larger percentage of their income as capital income, and the poorest individuals generally do not hold financial assets, this mechanism will tend to increase income inequality. There is also a potential interaction of inequality with changes in labor bargaining power. Although it is unclear that the overall level of labor bargaining power has decreased, there is some evidence that with the fall of unionization and the rise of outsourcing the bargaining power of the poorest workers has decreased. Meanwhile, the bargaining power of the highest educated superstars and CEOs may have increased. If CEOs are taking a larger portion of the profits and work-

ers a smaller portion, this would also tend to increase income inequality. An increase in monopoly rents also has implications for wealth inequality. We have seen how an increase in pure profits leads to a boom in stock prices, since equities hold the residual rights to corporate profits. Since those with the highest level of wealth tend to hold a greater fraction in equities, and those with lower wealth tend to hold a greater portion in housing, an increase in monopoly rents would tend to increase wealth inequality.

The increase in market power has important implications for corporate tax policy. Standard economic theory tells us that taxing pure profits is generally a good idea (Guo and Lansing (1999)), while taxing capital income may not be a good idea (Judd (1985)). As shown by Guo and Lansing (1999), the optimal tax on corporate profits depends on the relative size of capital income to pure profits. Even in an economy with a moderate profit share (on the order of 8%), relatively high levels of corporate income taxes are optimal, especially with interest deductibility and accelerated depreciation. Traditionally, it has been thought that the level of pure profits in the economy was small – a common citation is Basu and Fernald (1997)’s estimate of 3%. With a profit share of 15-20%, as our analysis suggests, high level of corporate taxes may be optimal.

Our model is also a theory of wealth accumulation in the US, and thus can directly address some of the questions raised in Piketty’s “Capital in the 21st Century”. A major criticism of Piketty’s theory of wealth-accumulation is that the return on capital, the ‘ r ’ in $r - g$, is assumed to be constant even as capital increases. With a standard, neoclassical production function, however, as capital increases, the marginal product of capital and the return on capital must decrease. Thus there would seem to be a natural force which would tend to counteract the increase in wealth. The analysis in this paper suggests a possible reconciliation between the view of Piketty and the views of the neoclassicals. With an increase in monopoly profits, there can be an increase in wealth, without a corresponding decrease in the average return on capital.

Finally, our work has implications for national macroeconomic income and wealth accounting. Measuring the level of pure profits in the US economy is of first order importance, and should be considered by national statistical agencies. Non-productive intangible investment should not be capitalized.

The analysis in this paper has a number of testable implications beyond the macro data on which we have formulated our theory. A direct implication of our work is that firms in industries with higher markups should have a lower labor share, higher Tobin’s Q , a higher average return on capital, and lower investment.

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Online Appendix for *Kaldor and Piketty's Facts: the Rise of Monopoly Power in the United States*

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A Full equations of model

A.1 Final goods firms

There is a unit mass of monopolistically competitive final goods firms that differentiate an intermediate good and resell it to consumers. The final good composite is the CES aggregate of these differentiated final goods, which are indexed by i :

$$Y_t = \left[\int_0^1 y_t^f(i)^{\frac{\Lambda_t-1}{\Lambda_t}} di \right]^{\frac{\Lambda_t}{\Lambda_t-1}}.$$

Final goods firms set prices in each period, and face a demand curve that takes the following form: $y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\Lambda_t}$, where Λ_t is a time-varying measure of a firm's market power. An increase in Λ_t decreases a firm's market power and lowers equilibrium markups. The nominal price index is defined as

$$P_t = \left(\int_0^1 p_t(i)^{1-\Lambda_t} di \right)^{\frac{1}{1-\Lambda_t}}.$$

Each final goods producer uses y_t^m of intermediate goods to produce output, according to a linear technology function $y_t^f = y_t^m$. A final goods firm chooses real prices $\frac{p_t(i)}{P_t}$ and $y_t^f(i)$ to maximize real profits, subject to the production and demand constraints:

$$\begin{aligned} & \max \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}}{P_t} y_t^f(i) \\ & \text{subject to } y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\Lambda_t}, \end{aligned}$$

where $\frac{p_t^{int}}{P_t}$ is the price of the intermediate good taken as given by the firm.

The optimality condition for the real price of the firm's good is a time-varying markup over the price of the intermediate good:

$$\frac{p_t(i)}{P_t} = \frac{\Lambda_t}{\Lambda_t - 1} \frac{p_t^{int}}{P_t} = \mu_t \frac{p_t^{int}}{P_t}, \quad (\text{A.1})$$

where μ_t is the optimal markup of the firm.

Since the price of the intermediate good is the same, all final goods firms make the same pricing decisions, and thus $p_t(i) = P_t$, yielding $\frac{p_t^{int}}{P_t} = \frac{1}{\mu_t}$.

Final goods firms have market power which allows them to set prices above marginal costs. Market power is determined by the CES elasticity Λ_t , which determines markups. We posit that $\frac{\Lambda_t}{\Lambda_{t-1}}$, i.e. markups μ_t , follow an AR(1) process, given by

$$\ln(\mu_t) = (1 - \rho_\mu)\ln(\bar{\mu}) + \rho_\mu\ln(\mu_{t-1}) + \epsilon_t^\mu. \quad (\text{A.2})$$

The long run level of markups (determined by the long-run level of Λ) in the economy is given by $\bar{\mu}$. Final goods firms make aggregate profits equal to

$$\Pi_t = \frac{\mu_t - 1}{\mu_t} Y_t. \quad (\text{A.3})$$

Pure profits go to shareholders of final goods firms, who own the rights to the economic rents, as dividends. Aggregate dividends distributed to shareholders at time t are thus given by

$$d_t^f = \Pi_t. \quad (\text{A.4})$$

Since all firms make identical profits, each firm receives an equal fraction of aggregate dividends.

A.2 Long Run Risk

Let A_t denote the level of productivity, and lowercase letter denote log-units. The growth rate or productivity is given by:

$$\Delta a_{t+1} = \zeta + x_t + \sigma_a \epsilon_{a,t+1} \quad (\text{A.5})$$

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t} \quad (\text{A.6})$$

$$\begin{bmatrix} \epsilon_{a,t+1} \\ \epsilon_{x,t+1} \end{bmatrix} \sim iid N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xa} \\ \rho_{xa} & 1 \end{bmatrix} \right) \quad (\text{A.7})$$

Here x_t is the ‘‘long run risk’’ of productivity growth, and $\epsilon_{a,t+1}$ is the short run risk.

A.3 Consumer’s problem

The model mainly follows Caldara et al. (2012) and Croce (2014). An infinitely lived individual has Epstein-Zin utility given by

$$V_t = \left[(1 - \beta) (c_t^\nu (A_{t-1}(1 - L_t))^{1-\nu})^{\frac{1-\gamma}{\theta}} + \beta D (E_t V_{t+1}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (\text{A.8})$$

where the time discount factor is β , the labor supply coefficient is ν , γ is the risk aversion parameter, and θ is defined as

$$\theta = \frac{1 - \gamma}{1 - \frac{1}{\psi}}. \quad (\text{A.9})$$

In this last expression, ψ is the elasticity of intertemporal substitution. The term D_t is an additional wedge between utility in period $t + 1$ and period t , beyond the time discount rate of β . Individuals maximize utility subject a series of budget constraints,

$$\begin{aligned} c_t + X_t^i S_{t+1}^i + X_t^f S_{t+1}^f &= w_t L_t + d_t^i S_t^i + d_t^f S_t^f \\ &+ \Delta_{t+1} X_t^f + (1 - \Delta_t) X_t^f S_t^f + X_t^i S_t^i. \end{aligned} \quad (\text{A.10})$$

On the left hand side of the budget constraint, individuals use their income to purchase either consumption, shares of intermediate good firms ($X_t^i S_{t+1}^i$), or shares of final goods firms ($X_t^f S_{t+1}^f$). On the right hand side of the budget equation, agents receive income from a variety of sources:

1. Labor income $w_t L_t$
2. Dividends from intermediate goods firms $d_t^i S_t^i$
3. Dividends from final goods firms $d_t^f S_t^f$
4. IPO issued securities of final goods firms $\Delta_{t+1} X_t^f$
5. Remaining share value of final goods firms. Agents come into the period holding S_t^f shares of the security. Because of firm exit, a fraction Δ_t of shares lose their value, and thus the value remaining is $(1 - \Delta_t) X_t^f S_t^f$.
6. Remaining share value of intermediate goods firms, $X_t^i S_t^i$

Due to the nature of the recursive utility, we can write the optimal solution as a recursive function

$$\begin{aligned} V_t(S_t^i, S_t^f) &= \max_{c_t, L_t, S_{t+1}^i, S_{t+1}^f} \left[(1 - \beta) (c_t^\nu (A_{t-1}(1 - L_t))^{1-\nu})^{\frac{1-\gamma}{\theta}} + \right. \\ &\quad \left. \beta D \left(E_t V_{t+1}(S_{t+1}^i, S_{t+1}^f)^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (\text{A.11}) \\ \text{s.t. } c_t + X_t^i S_{t+1}^i + X_t^f S_{t+1}^f &= w_t L_t + d_t^i S_t^i + d_t^f S_t^f \\ &+ \Delta_{t+1} X_t^f + (1 - \Delta_t) X_t^f S_t^f + X_t^i S_t^i. \end{aligned}$$

From this equation, we can derive the first order conditions for optimization. Setting up the Lagrangean and differentiating with respect to c_t , we have

$$\frac{\partial \mathcal{L}}{\partial c_t} : (1 - \beta) V_t^{1 - \frac{1-\gamma}{\theta}} (c_t^\nu (A_{t-1}(1 - L_t))^{1-\nu})^{\frac{1-\gamma}{\theta}} \nu \frac{1}{c_t} = \lambda_t. \quad (\text{A.12})$$

Taking first order conditions with respect to L_t , we have

$$\frac{\partial \mathcal{L}}{\partial c_t} : (1 - \beta)V_t^{1-\frac{1-\gamma}{\theta}} (c_t^\nu (A_{t-1}(1 - L_t))^{1-\nu})^{\frac{1-\gamma}{\theta}} (1 - \nu) \frac{1}{(1 - L_t)} = w_t \lambda_t. \quad (\text{A.13})$$

Combining the first order conditions with respect to labor and with respect to consumption, we have

$$\frac{(1 - \nu)}{\nu} \frac{c_t}{(1 - L_t)} = w_t, \quad (\text{A.14})$$

and taking the first order condition with respect to S_{t+1}^f , we have

$$\frac{\partial \mathcal{L}}{\partial S_{t+1}^f} : X_t^f \lambda_t = \beta E_t[\lambda_{t+1}((1 - \Delta_{t+1})X_{t+1}^f + d_{t+1}^f)]. \quad (\text{A.15})$$

Now, taking the first order condition with respect to c_{t+1} , we have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_{t+1}} : & V_t^{1-\frac{1-\gamma}{\theta}} \beta D (E_t V_{t+1}^{1-\gamma})^{\frac{1}{\theta}-1} \times \\ & E_t \left[V_{t+1}^{-\gamma} (1 - \beta) V_{t+1}^{1-\frac{1-\gamma}{\theta}} (c_{t+1}^\nu (A_t(1 - L_{t+1}))^{1-\nu})^{\frac{1-\gamma}{\theta}} \nu \frac{1}{c_{t+1}} \right], \end{aligned} \quad (\text{A.16})$$

where in the last step we make a substitution by forwarding $\frac{\partial}{\partial c_t}$ one period. Canceling redundant terms, we get

$$m_{t+1} = \frac{\partial V_t / \partial c_{t+1}}{\partial V_t / \partial c_t} = \beta D \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\nu(1-\gamma)}{\theta}-1} \left(\frac{A_t(1 - L_{t+1})}{A_{t-1}(1 - L_t)} \right)^{\frac{(1-\nu)(1-\gamma)}{\theta}} \left(\frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} \quad (\text{A.17})$$

A.4 Intermediate Goods Firm's Problem

Representative intermediate goods firms use labor L_t and capital K_t to produce intermediate goods Y_t^m according to the production function

$$Y_t^m = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (\text{A.18})$$

where σ is the production elasticity of substitution. $\sigma = 1$ corresponds to Cobb-Douglas. The firm finances part of its investment I_t through retained earnings RE_t and issues shares to cover the remaining part,

$$I_t = X_t^i (S_{t+1}^i - S_t^i) + RE_t. \quad (\text{A.19})$$

It distributes the excess of its profits over retained earnings to its shareholders as a dividend,

$$d_t^i S_t^i = \frac{1}{\mu_t} Y_t^m - w_t L_t - R E_t. \quad (\text{A.20})$$

Since $Y_t = Y_t^m$, we have

$$d_t^i S_t^i = \frac{1}{\mu_t} Y_t - w_t L_t - R E_t. \quad (\text{A.21})$$

Investment increases the firm's future stock of capital according to

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1 - \delta)K_t, \quad (\text{A.22})$$

where δ is the rate of depreciation and adjustment costs $\Phi(\cdot)$ are a positive concave function. The adjustment costs function is given by

$$\Phi(I_t/K_t) = \frac{a_1}{1 - \xi} \left(\frac{I_t}{K_t} \right)^{1-\xi} + a_2. \quad (\text{A.23})$$

Following Jermann (1998), we will choose the adjustment costs parameters so that the steady state ratio of investment to capital is not affected. In a model with productivity growth, the investment to capital ratio is given by $\frac{I}{K} = (\delta + e^\zeta - 1)$. From equation (A.22), in the steady state with productivity growth we have that $K(\delta + e^\zeta - 1) = \Phi(I/K)K$, thus we must have in the steady state $\Phi(I/K) = (\delta + e^\zeta - 1)$. Note that this also ensures that if a firm replaces depreciation and accounts for growth, adjustment costs are zero.

In addition, we also need q to be 1 in the steady state, and thus $\Phi'(I/K) = 1$. These conditions imply the following two conditions:

$$\frac{a_1}{1 - \xi} (\delta + e^\zeta - 1)^{1-\xi} + a_2 = (\delta + e^\zeta - 1) \quad (\text{A.24})$$

$$a_1(\delta + e^\zeta - 1)^{-\xi} = 1. \quad (\text{A.25})$$

Thus we have that $a_1 = (\delta + e^\zeta - 1)^\xi$, and $a_2 = (1 - \frac{1}{(1-\xi)})(\delta + e^\zeta - 1)$.

A.4.1 Computation of the intermediate good firm's value

Intermediate goods maximize the expected value of cash flow to the shareholders, discounted by the stochastic discount factor of individuals. Defining cash flow, $CF_t^i = \frac{1}{\mu_t} Y_t - w_t L_t - I_t$, the value of the intermediate good firm is given by

$$V_t^i = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} D^{s-t} \frac{\lambda_s}{\lambda_t} CF_s^i \right]. \quad (\text{A.26})$$

Firms maximize (A.26) subject to (A.22). The first order conditions are given by:

$$\frac{\partial}{\partial I_t} : \frac{1}{\Phi'(I_t/K_t)} = q_t, \quad (\text{A.27})$$

where q_t is the the Lagrange multiplier of the maximization problem. Continuing, we have

$$\frac{\partial}{\partial K_{t+1}} : -q_t + E_t \left[\beta D \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{1}{\mu_{t+1}} f'(K_{t+1}) + q_{t+1} \left(-\Phi' \left(\frac{I_{t+1}}{K_{t+1}} \right) \left(\frac{I_{t+1}}{K_{t+1}} \right) + \Phi \left(\frac{I_{t+1}}{K_{t+1}} \right) + (1 - \delta) \right) \right] \right].$$

Then, using the fact that $\Phi'(I_{t+1}/K_{t+1}) = \frac{1}{q_{t+1}}$ and

$$F'(K_{t+1}) = \left(\alpha(K_{t+1})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(A_{t+1}L_{t+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha K_{t+1}^{\frac{-1}{\sigma}}, \quad (\text{A.28})$$

we have that

$$q_t = E_t \left[\beta D \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{1}{\mu_{t+1}} \left(\alpha(K_{t+1})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(A_{t+1}L_{t+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha K_{t+1}^{\frac{-1}{\sigma}} - \left(\frac{I_{t+1}}{K_{t+1}} \right) + q_{t+1} \left(\Phi \left(\frac{I_{t+1}}{K_{t+1}} \right) + (1 - \delta) \right) \right] \right]. \quad (\text{A.29})$$

Finally, we have

$$\frac{\partial}{\partial L_t} : \frac{1}{\mu_t} \left(\alpha(K_t)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} (1 - \alpha) L_t^{\frac{-1}{\sigma}} = w_t. \quad (\text{A.30})$$

A.5 Asset pricing implications

As usual, the return on the risk free rate is given by

$$R_t = \frac{1}{E_t[m_{t+1}]} \quad (\text{A.31})$$

The return for investing in intermediate goods firms is given by

$$R_{t+1}^i = \frac{d_{t+1}^i S_{t+1}^i + X_{t+1}^i S_{t+1}^i}{X_t^i S_{t+1}^i} = \frac{d_{t+1}^i + X_{t+1}^i}{X_t^i}. \quad (\text{A.32})$$

Now, using the fact that $q_t K_{t+1} = X_t S_{t+1}^i$, we have

$$\begin{aligned}
 R_{t+1}^i &= \frac{\frac{1}{\mu_t} Y_{t+1}^m - w_{t+1} L_{t+1} - RE_{t+1} + X_{t+1}^i S_{t+1}^i}{q_t K_{t+1}} \\
 &= \frac{\frac{1}{\mu_t} Y_{t+1} - w_{t+1} L_{t+1} - I_{t+1} + X_{t+1}^i (S_{t+2}^i - S_{t+1}^i) + X_{t+1}^i S_{t+1}^i}{q_t K_{t+1}} \quad (\text{A.33}) \\
 &= \frac{\frac{1}{\mu_t} Y_{t+1} - w_{t+1} L_{t+1} - I_{t+1} + q_{t+1} K_{t+2}}{q_t K_{t+1}}.
 \end{aligned}$$

As usual, we have the asset pricing equation

$$1 = E_t[m_{t+1} R_{t+1}^i]. \quad (\text{A.34})$$

The return for investing in final goods firms is given by

$$R_{t+1}^f = \frac{d_{t+1}^f S_{t+1}^f + (1 - \Delta_{t+1}) S_{t+1}^f X_{t+1}^f}{S_{t+1}^f X_t^f} = \frac{\frac{\mu_{t+1}-1}{\mu_{t+1}} Y_{t+1} + (1 - \Delta_{t+1}) X_{t+1}^f}{X_t^f} \quad (\text{A.35})$$

A.6 Equilibrium

An equilibrium is a set of quantities: $\{c_t, K_t, L_t, I_t, Y_t, Y_t^m, d_t^i, d_t^f\}_{t=0}^\infty$, a set of prices $\{w_t, X_t^i, X_t^f, q_t, m_t\}_{t=0}^\infty$, and a set of exogenous processes $\{\mu_t, A_t, x_t, \Delta_t\}_{t=0}^\infty$ that jointly satisfy:

1. Consumption maximizes (A.8) subject to (A.10)
2. The stochastic discount factor is given by (A.17)
3. Intermediate firms maximize (A.26) subject to (A.22)
4. Intermediate good production is given by (A.18), and final good production is given by $Y_t = Y_t^m$
5. Aggregate profits of final goods firms are given by (A.3), and aggregate dividends are given by (A.4)
6. The price of securities satisfies (A.15)
7. The wage is given by (A.30)
8. The stochastic processes for μ_t , A_t , and x_t are given by (A.2), (A.5), and (A.6)
9. The paths for Δ_t is exogenously specified

While this model is not stationary, we can make it so by applying a standard transformation: divide all quantities (except labor) by A_{t-1} , as well as wages and the price of securities X_t^i and X_t^f .

A.7 Full Equations of Model

Now, collecting the equations of the model, we have

$$V_t = \left[(1 - \beta) (c_t^\nu (A_{t-1}(1 - L_t))^{1-\nu})^{\frac{1-\gamma}{\theta}} + \beta D (E_t(V_{t+1})^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (\text{A.36})$$

$$m_{t+1} = \beta D \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\nu(1-\gamma)}{\theta} - 1} \left(\frac{A_t(1 - L_{t+1})}{A_{t-1}(1 - L_t)} \right)^{\frac{(1-\nu)(1-\gamma)}{\theta}} \left(\frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{1 - \frac{1}{\theta}} \quad (\text{A.37})$$

$$\frac{(1 - \nu)}{\nu} \frac{c_t}{(1 - L_t)} = w_t \quad (\text{A.38})$$

$$\frac{1}{\Phi'(I_t/K_t)} = q_t, \quad (\text{A.39})$$

$$q_t = E_t \left[m_{t+1} \left[\frac{1}{\mu_{t+1}} \left(\alpha (K_{t+1})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) (A_{t+1} L_{t+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha K_{t+1}^{\frac{-1}{\sigma}} - \left(\frac{I_{t+1}}{K_{t+1}} \right) + q_{t+1} \left(\Phi \left(\frac{I_{t+1}}{K_{t+1}} \right) + (1 - \delta) \right) \right] \right] \quad (\text{A.40})$$

$$\frac{1}{\mu_t} \left(\alpha (K_t)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) (A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} (1 - \alpha) A_t^{\frac{\sigma-1}{\sigma}} L_t^{\frac{-1}{\sigma}} = w_t \quad (\text{A.41})$$

$$Y_t^m = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) (A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (\text{A.42})$$

$$Y_t = Y_t^m \quad (\text{A.43})$$

$$Y_t = c_t + I_t \quad (\text{A.44})$$

$$K_{t+1} = \Phi(I_t/K_t) K_t + (1 - \delta) K_t, \quad (\text{A.45})$$

$$\Delta a_{t+1} = \zeta + x_t + \sigma_a \epsilon_{a,t+1} \quad (\text{A.46})$$

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t} \quad (\text{A.47})$$

$$\ln(\mu_t) = (1 - \rho_\mu) \ln(\bar{\mu}) + \rho_\mu \ln(\mu_{t-1}) + \epsilon_t^\mu. \quad (\text{A.48})$$

$$R_t = \frac{1}{E_t[m_{t+1}]} \quad (\text{A.49})$$

$$R_{t+1}^i = \frac{\frac{1}{\mu_t} Y_{t+1} - w_{t+1} L_{t+1} - I_{t+1} + q_{t+1} K_{t+2}}{q_t K_{t+1}}. \quad (\text{A.50})$$

$$d_t^f = \frac{\mu_t - 1}{\mu_t} Y_t \quad (\text{A.51})$$

$$X_t^f = E_t[m_{t+1}((1 - \Delta_{t+1})X_{t+1}^f + d_{t+1}^f)]. \quad (\text{A.52})$$

$$R_{t+1}^f = \frac{\frac{\mu_{t+1}-1}{\mu_{t+1}} Y_{t+1} + (1 - \Delta_{t+1}) X_{t+1}^f}{X_t^f} \quad (\text{A.53})$$

The variables are $V, Y, Y^m, c, L, m, w, q, K, I, a, x, \mu, R, R^i, R^f, d_t^f, X^f$. Thus there are 18 equations and 18 variables.

A.8 Making the model stationary

We now make a standard transformation by dividing the following variables by A_{t-1} : $V, Y, Y^m, c, w, K, I, X^f, X^i, d^f, d^i$. We then have the following set of equations:

$$V_t = \left[(1 - \beta) (c_t^\nu ((1 - L_t))^{1-\nu})^{\frac{1-\gamma}{\theta}} + e^{(\frac{1-\gamma}{\theta})\Delta a_t} \beta D (E_t(V_{t+1})^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (\text{A.54})$$

$$m_{t+1} = \beta D \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\nu(1-\gamma)}{\theta}-1} \left(\frac{(1 - L_{t+1})}{(1 - L_t)} \right)^{\frac{(1-\nu)(1-\gamma)}{\theta}} \left(\frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\theta}} e^{(\frac{1-\gamma}{\theta}-1)\Delta a_t} \quad (\text{A.55})$$

$$\frac{(1 - \nu)}{\nu} \frac{c_t}{(1 - L_t)} = w_t \quad (\text{A.56})$$

$$\frac{1}{\Phi'(I_t/K_t)} = q_t, \quad (\text{A.57})$$

$$q_t = E_t \left[m_{t+1} \left[\frac{1}{\mu_{t+1}} \left(\alpha (K_{t+1})^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) e^{\frac{\sigma-1}{\sigma} \Delta a_{t+1}} (L_{t+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha K_{t+1}^{\frac{-1}{\sigma}} \right. \right. \quad (\text{A.58})$$

$$\left. \left. - \left(\frac{I_{t+1}}{K_{t+1}} \right) + q_{t+1} \left(\Phi \left(\frac{I_{t+1}}{K_{t+1}} \right) + (1 - \delta) \right) \right] \right]$$

$$\frac{1}{\mu_t} \left(\alpha(K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\Delta a_t} (L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} (1-\alpha)e^{\frac{\sigma-1}{\sigma}\Delta a_t} L_t^{\frac{-1}{\sigma}} = w_t \quad (\text{A.59})$$

$$Y_t^m = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\Delta a_t} (L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (\text{A.60})$$

$$Y_t = Y_t^m \quad (\text{A.61})$$

$$Y_t = c_t + I_t \quad (\text{A.62})$$

$$K_{t+1}e^{\Delta a_t} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \quad (\text{A.63})$$

$$\Delta a_{t+1} = \zeta + x_t + \sigma_a \epsilon_{a,t+1} \quad (\text{A.64})$$

$$x_t = \rho x_{t-1} + \sigma_x \epsilon_{x,t} \quad (\text{A.65})$$

$$\ln(\mu_t) = (1-\rho_\mu)\ln(\bar{\mu}) + \rho_\mu \ln(\mu_{t-1}) + \epsilon_t^\mu. \quad (\text{A.66})$$

$$R_t = \frac{1}{E_t[m_{t+1}]} \quad (\text{A.67})$$

$$R_{t+1}^i = \frac{\frac{1}{\mu_t} Y_{t+1} - w_{t+1} L_{t+1} - I_{t+1} + q_{t+1} K_{t+2} e^{\Delta a_{t+1}}}{q_t K_{t+1}}. \quad (\text{A.68})$$

$$d_t^f = \frac{\mu_t - 1}{\mu_t} Y_t \quad (\text{A.69})$$

$$X_t^f = E_t[m_{t+1}((1-\Delta_{t+1})X_{t+1}^f e^{\Delta a_t} + d_{t+1}^f e^{\Delta a_t})]. \quad (\text{A.70})$$

$$R_{t+1}^f = \frac{d_{t+1}^f e^{\Delta a_t} + (1-\Delta_{t+1})X_{t+1}^f e^{\Delta a_t}}{X_t^f} \quad (\text{A.71})$$

A.9 Steady State

In the steady state, all transformed variables are constant.

We begin by finding steady state investment. From equation (A.63), and using the assumed properties of the $\Phi(\cdot)$ function, in particular that in the steady state $\Phi(\cdot) = \delta + e^\zeta + 1$ we have that $\frac{I}{K} = \delta + e^\zeta - 1$. Next, using the fact that

$$\bar{m} = \beta D e^{\left(\frac{1-\gamma}{\theta} - 1\right)\zeta}, \quad (\text{A.72})$$

from equation (A.58) we have that

$$1 = \bar{m} \left[\frac{1}{\bar{\mu}} \left(\alpha \left(\frac{\bar{K}}{\bar{L}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta} \right)^{\frac{1}{\sigma-1}} \alpha \left(\frac{\bar{K}}{\bar{L}} \right)^{\frac{-1}{\sigma}} + 1 - \delta \right]. \quad (\text{A.73})$$

Rearranging, we have

$$\left(\frac{\bar{K}}{\bar{L}} \right) = \left[\frac{(1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta}}{\left[\left(\frac{1}{\bar{m}} - 1 + \delta \right) \frac{\bar{\mu}}{\alpha} \right]^{\sigma-1} - \alpha} \right]^{\frac{\sigma}{\sigma-1}}. \quad (\text{A.74})$$

Continuing, from (A.60) and (A.62) we have that

$$\frac{\bar{c}}{\bar{L}} = \frac{\bar{Y}}{\bar{L}} - (e^\zeta + \delta - 1) \frac{\bar{K}}{\bar{L}}, \quad (\text{A.75})$$

where

$$\frac{\bar{Y}}{\bar{L}} = \left(\alpha \left(\frac{\bar{K}}{\bar{L}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta} \right)^{\frac{\sigma}{\sigma-1}}. \quad (\text{A.76})$$

Now, combining the definition for the wage with the first order condition for labor, and rearranging, we have

$$\left(\frac{1}{\bar{\mu}} \right) M\bar{P}L = \frac{(1-\nu)}{\nu} \frac{\bar{c}}{(1-\bar{L})}, \quad (\text{A.77})$$

where

$$M\bar{P}L = (1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta} \left(\alpha \left(\frac{\bar{K}}{\bar{L}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta} \right)^{\frac{1}{\sigma-1}}. \quad (\text{A.78})$$

Rearranging, we have

$$\frac{\bar{c}}{\bar{L}} = \frac{\nu}{1-\nu} \frac{(1-\bar{L})}{\bar{L}} \left(\frac{1}{\bar{\mu}} \right) M\bar{P}L. \quad (\text{A.79})$$

Now combining (A.75) and (A.79), we have

$$\frac{\bar{Y}}{\bar{L}} - (e^\zeta + \delta - 1) \frac{\bar{K}}{\bar{L}} = \frac{\nu}{1-\nu} \frac{(1-\bar{L})}{\bar{L}} \left(\frac{1}{\bar{\mu}} \right) M\bar{P}L. \quad (\text{A.80})$$

Solving for \bar{L} , we have

$$\bar{L} = \frac{\frac{\nu}{1-\nu} \left(\frac{1}{\bar{\mu}} \right) M\bar{P}L}{\frac{\bar{Y}}{\bar{L}} - (e^\zeta + \delta - 1) \frac{\bar{K}}{\bar{L}} + \frac{\nu}{1-\nu} \left(\frac{1}{\bar{\mu}} \right) M\bar{P}L}. \quad (\text{A.81})$$

Turning now to the value function, we have

$$\bar{V} = \left(\frac{(1 - \beta) (\bar{c}^\nu ((1 - \bar{L}))^{1-\nu})^{\frac{1-\gamma}{\theta}}}{1 - e^{(\frac{1-\gamma}{\theta})\zeta}\beta} \right)^{\frac{\theta}{1-\gamma}}. \quad (\text{A.82})$$

B Proofs

B.1 Comparative statics of markups

Proposition 3. $\frac{\partial \bar{K}/\bar{L}}{\partial \bar{\mu}} < 0$. *If steady-state markups $\bar{\mu}$ increase, the steady state capital-to-labor ratio \bar{K}/\bar{L} will decrease.*

Proof. Equation A.74 gives the steady state level of the capital-to-labor ratio. Differentiating with respect to $\bar{\mu}$, we have

$$\frac{\partial \bar{K}/\bar{L}}{\partial \bar{\mu}} = \left[\frac{\left[\left(\frac{1}{\bar{m}} - 1 + \delta \right) \frac{\bar{\mu}}{\alpha} \right]^{\sigma-1} - \alpha}{(1 - \alpha) e^{\frac{\sigma-1}{\sigma}\zeta}} \right]^{\frac{2\sigma-1}{1-\sigma}} \frac{\sigma}{1 - \sigma} (\sigma - 1) \quad (\text{A.83})$$

$$\left[\left(\frac{1}{\bar{m}} - 1 + \delta \right) \frac{1}{\alpha} \right]^{\sigma-1} \bar{\mu}^{\sigma-2} \frac{1}{(1 - \alpha) e^{\frac{\sigma-1}{\sigma}\zeta}}.$$

This term is negative, since either $(\sigma - 1)$ or $\frac{\sigma}{1-\sigma}$ is negative, but not both. Note that both bracketed terms are positive: the first, because the capital-to-labor ratio is positive; the second, because the steady state real interest rate cannot be negative. \square

Corollary 1. $\frac{\partial \bar{K}/\bar{Y}}{\partial \bar{\mu}} < 0$. *If steady-state markups $\bar{\mu}$ increase, the capital-to-output ratio \bar{K}/\bar{Y} will decrease.*

Proof. We have $\bar{K}/\bar{Y} = \bar{K}/\bar{L} \cdot \bar{L}/\bar{Y}$. We can write the elasticity of the capital-to-output ratio with respect to markups as

$$\epsilon_{\bar{K}/\bar{Y}, \bar{\mu}} \equiv \frac{\partial \bar{K}/\bar{Y}}{\partial \bar{\mu}} \cdot \frac{\bar{\mu}}{\bar{K}/\bar{Y}} = \frac{\partial \bar{K}/\bar{L}}{\partial \bar{\mu}} \cdot \frac{\bar{\mu}}{\bar{K}/\bar{L}} + \frac{\partial \bar{L}/\bar{Y}}{\partial \bar{\mu}} \cdot \frac{\bar{\mu}}{\bar{L}/\bar{Y}}. \quad (\text{A.84})$$

From equation A.76, we see that $\frac{\partial \bar{L}/\bar{Y}}{\partial \bar{\mu}} = \frac{\partial \bar{L}/\bar{Y}}{\partial \bar{K}/\bar{L}} \frac{\partial \bar{K}/\bar{L}}{\partial \bar{\mu}}$. We thus have

$$\epsilon_{\bar{K}/\bar{Y}, \bar{\mu}} = \frac{\partial \bar{K}/\bar{L}}{\partial \bar{\mu}} \cdot \frac{\bar{\mu}}{\bar{K}/\bar{L}} \left[1 + \frac{\partial \bar{L}/\bar{Y}}{\partial \bar{K}/\bar{L}} \frac{\bar{K}/\bar{L}}{\bar{L}/\bar{Y}} \right]. \quad (\text{A.85})$$

From equation A.76, we have that

$$\frac{\partial \frac{\bar{L}}{\bar{Y}} \frac{\bar{K}}{\bar{L}}}{\partial \frac{\bar{K}}{\bar{L}} \frac{\bar{L}}{\bar{Y}}} = \frac{-\alpha \left(\frac{\bar{K}}{\bar{L}} \right)^{\frac{\sigma-1}{\sigma}}}{\left(\alpha \left(\frac{\bar{K}}{\bar{L}} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)e^{\frac{\sigma-1}{\sigma}\zeta} \right)}, \quad (\text{A.86})$$

which is negative and less than 1 in absolute value, thus the term in square brackets in equation A.85 is positive. Since we have already shown $\frac{\partial \frac{\bar{K}}{\bar{L}}}{\partial \bar{\mu}} < 0$, $\epsilon_{\frac{\bar{K}}{\bar{Y}}, \bar{\mu}} < 0$. \square

Corollary 2. $\frac{\partial \frac{\bar{I}}{\bar{Y}}}{\partial \bar{\mu}} < 0$. *If steady-state markups $\bar{\mu}$ increase, the steady state investment-to-output ratio $\frac{\bar{I}}{\bar{Y}}$ will decrease.*

Proof. This follows from equation 18. Since investment-to-output is proportional to the capital-to-output ratio, if this ratio declines, the investment ratio will as well. \square

Proposition 4. $\frac{\partial PS}{\partial \bar{\mu}} > 0$. $\frac{\partial LS}{\partial \bar{\mu}} < 0$, if $\sigma \leq 1$. $\frac{\partial KS}{\partial \bar{\mu}} < 0$, if $\sigma \geq 1$. *An increase in steady state markups will increase the steady state profit share. If $\sigma \leq 1$, an increase in steady state markups will decrease the steady state labor share. if $\sigma \geq 1$, an increase in markups will decrease the capital share.*

Proof. Since the profit share is given by $PS = \frac{\bar{\mu}-1}{\bar{\mu}}$, an increase in markups will increase the profit share. Thus the combined labor and capital share will decrease. From the first order conditions of the firm, the ratio of the rental rate of capital to the wage is given by

$$\frac{r^k}{w} = \frac{\alpha}{1-\alpha} \left[\frac{\bar{K}}{\bar{L}} \right]^{\frac{-1}{\sigma}}. \quad (\text{A.87})$$

Multiplying both sides by $\frac{\bar{K}}{\bar{L}}$, the left hand side is the ratio of the capital share to the labor share, and thus we have

$$\frac{KS}{LS} = \frac{\alpha}{1-\alpha} \left[\frac{\bar{K}}{\bar{L}} \right]^{\frac{\sigma-1}{\sigma}}. \quad (\text{A.88})$$

Thus if $\sigma < 1$, a decrease in $\frac{\bar{K}}{\bar{L}}$ will lead to a decrease in the labor share relative to the capital share. When markups increase, there is a decrease in the combined labor and capital share. In addition, since there is a decrease in $\frac{\bar{K}}{\bar{L}}$, labor takes a smaller portion of this decreased overall share. Thus the labor share unambiguously declines. Similarly, when $\sigma > 1$, the capital share unambiguously declines with an increase in markups. If $\sigma = 1$, production is Cobb-Douglas, and the relative factor shares of capital and labor are unchanged by the capital-to-output ratio. In this case, both the capital and labor share unambiguously decline with an increase in markups. \square

Proposition 5. $\frac{\partial Q}{\partial \bar{\mu}} > 0$. *If markups increase, Tobin's Q will increase.*

Proof. We can write steady state Q as $Q = 1 + \frac{X^f}{K} = 1 + \frac{X^f Y}{Y K}$. From equation 14, if markups increase there will be an increase in $\frac{X^f}{Y}$, and since there will also be an increase in $\frac{Y}{K}$, there will be an increase in Tobin's Q . \square

Proposition 6. $\frac{\partial AR}{\partial \mu} > 0$. *If markups increase, the average return will increase.*

Proof. This follows from equation 16. When markups increase, the profit share increases and the capital-to-output ratio will decrease, thus the average return will increase. \square

B.2 Comparative statics of D

Proposition 7. $\frac{\partial AR}{\partial D} < 0$. *An increase in D will lead to a decrease in the average return.*

Proof. From equation A.72, an increase in D will increase \bar{m} , which will lower the steady state interest rate \bar{r} , since $\bar{r} = \frac{1}{\bar{m}}$. From equation A.74, a decrease in r will also increase the capital-to-output ratio. Then from equation 16, this will lower the average return. \square

Proposition 8. $\frac{\partial \frac{K}{Y}}{\partial D} > 0$, $\frac{\partial \frac{K}{L}}{\partial D} > 0$. *An increase in D will lead to an increase in the capital-to-output ratio and the capital-to-labor ratio.*

Proof. An increase in D will decrease r . From equation A.74 and the further derivations, a decrease in r will increase the capital-to-labor ratio and the capital-to-output ratio. \square

Corollary 3. $\frac{\partial \frac{I}{Y}}{\partial D} > 0$. *An increase in D will lead to an increase in the investment-to-output ratio.*

Proof. This follows from equation 18. Since investment-to-output is proportional to the capital-to-output ratio, if this ratio increases, the investment ratio will as well. \square

Proposition 9. $\frac{\partial LS}{\partial D} > 0$ if $\sigma < 1$. *An increase in D will lead to an increase in the labor share if $\sigma < 1$. If $\sigma > 1$, $\frac{\partial LS}{\partial D} < 0$.*

Proposition 10. $\frac{\partial KS}{\partial D} < 0$ if $\sigma < 1$. *An increase in D will lead to a decrease in the capital share if $\sigma < 1$. If $\sigma > 1$, $\frac{\partial KS}{\partial D} > 0$.*

Proof. This follows directly from equation A.88. If $\sigma < 1$, an increase in $\frac{\bar{K}}{\bar{L}}$ will lead to a increase in the labor share relative to the capital share. When D increases, there is an increase in the capital-to-labor ratio, and the labor share relative to the capital share increases. Since there is no change in the combined labor and capital share, the labor share unambiguously increases, and the capital share declines. Similarly, when $\sigma > 1$, the capital share unambiguously increases with an increase in D , and the labor share decreases. If $\sigma = 1$, production is Cobb-Douglas, and the relative factor shares of capital and labor are unchanged by the capital-to-output ratio. In this case, both the capital and labor share do not respond to a change in D . \square

Proposition 11. $\frac{\partial \frac{W}{Y}}{\partial D} > 0$. An increase in D will lead to an increase in wealth-to-output.

Proof. From equation 14, a decrease in interest rates will lead to an increase in the ratio of the security value to output. This result, combined with an increase in the capital-to-output ratio, leads to an increase in the wealth-to-output ratio. \square

C Markup estimation techniques

Table 3 displays markup estimates, which fall into three categories. We discuss each in turn.

The first method is taken from the New Keynesian literature, which has historically estimated markups in order to test their cyclicalities. With the assumption that the production function is CES, it can be shown (see Nekarda and Ramey (2013)) that markups are equal to

$$M^{CES} = \frac{\alpha}{s} \left(\frac{Y}{ZhN} \right)^{\frac{1}{\sigma}-1}, \quad (\text{A.89})$$

Where α is the coefficient on labor in the production function, Z is the level of labor productivity, hN the labor input, and σ the production CES parameter.

The idea behind the New Keynesian estimate of markups is that given constant production parameters α and σ and a constant ratio of output to labor inputs, any decrease in the labor share must be because markups are higher. While this may be useful for estimating cyclicalities of markups, there are a few reasons to believe why they are less useful for estimating long run changes in markups. First, over the long run it may be possible that aggregate production parameters can change, if, for example, there have been a shift from manufacturing to services. Second, the estimates are highly reliant on a specific production function and estimates of the production parameters, as well as on labor productivity and labor inputs that may be imprecisely estimated.

The second category uses techniques from IO and firm level data to estimate markups. The key insight of this, as described in De Loecker and Warzynski (2012), is that the output elasticity of a variable factor of production is only equal to its expenditure share under perfect competition, thus in order to estimate markups at the firm or industry level all that is needed is an estimate of production elasticities. This method is related to that of Hall (1988), who also uses a production-side approach to measure marginal cost. De Loecker and Eeckhout (2017) use this method to estimate a new time series of markups and find a massive increase in markups, from 18% in 1980 to 67% in the present. This paper has been criticized by Traina (2018) and Gutiérrez and Philippon (2017), who note that the estimates are sensitive to the definition of variable input. If total expenses are used in the estimation instead of the cost of goods sold, there are more moderate increases in markups, on the order of 3-5 percentage points.

The third category estimates markups using aggregate macroeconomic data in the spirit of Barkai (2016), exploiting the fact that under constant returns to scale production, markups are proportional to the profit share of the economy. In particular, under CRS markups equal the inverse of the share of production not accounted for by pure profits:

$$PS = \frac{\mu - 1}{\mu} \implies \mu = \frac{1}{1 - PS}. \quad (\text{A.90})$$

D Transition dynamics

In this section we discuss the dynamics of our model economy. Although the main purpose of this paper is to explain long term trends in the macroeconomic data, a separate (and important) question is to try and explain the transition dynamics of the economy over the past forty years. As the data of section 2 shows, although many of the data series show a fairly smooth transition from 1970 to the present, there are exceptions, such as the spike in Tobin's Q in the late 1990s. The biggest challenge in modeling these changes is in determining the stochastic process under which they were generated. Are these increases in markups a one-time occurrence? Or is there a stochastic process under which the permanent component of markups can change? A separate, but related, challenge in modeling dynamics is to determine the *timing* when agents are aware that there has been a shift in the economy. If agents realized starting in 1980 that profits would increase in the future, then stock prices (and Q) should jump in 1980. If instead there was a slow process or learning, we would see a gradual increase in Tobin's Q.

We consider four types of transition dynamics, leaving more detailed models of the transition to future work. First, we will compute impulse response functions of our economy to a shock to markups, in order to examine the dynamics of the model and the speed of convergence. Second, we solve the model around its 2015 steady state, and then simulate the model, taking as initial conditions the steady state value of all variables in 1970. Third, we feed in a smooth series of markup shocks, so that the level of markups moves from its empirical level in 1970 to its level in 2015. Finally, we will feed in a series of markup shocks so that the time series for markups matches the time series of markups in the data.

We examine the impulse response function of the model to a one-time shock to markups, ϵ_t^μ . Figure A.1 shows the model's response to a one standard deviation increase in markups, which is an increase of .5%. The model responds as we would expect: there is an increase in the profit share of .44 p.p., and a corresponding combined decrease in the combined labor and capital share of .44 p.p.. There is an increase in the wealth-to-output ratio of .014 (.52%), an increase of .006 (.5%) in Tobin's Q, and an increase in the average return of .2 p.p.. Finally, there is a decrease in the investment-to-output ratio of .2 p.p. (1.4%). The transition to the new steady state is relatively fast: after a temporary increase in markups, the effects have largely dissipated after 20 months.

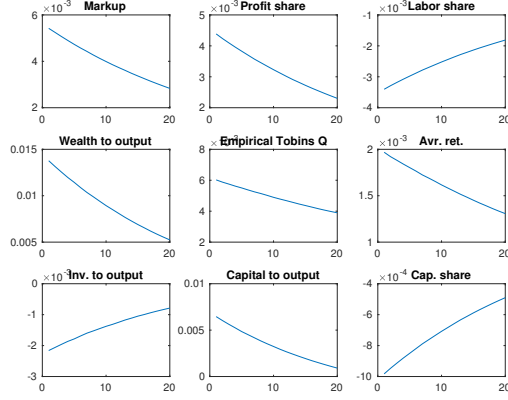


Figure A.1: Impulse response to increase in markups of 1 s.d. ($\approx .5\%$)

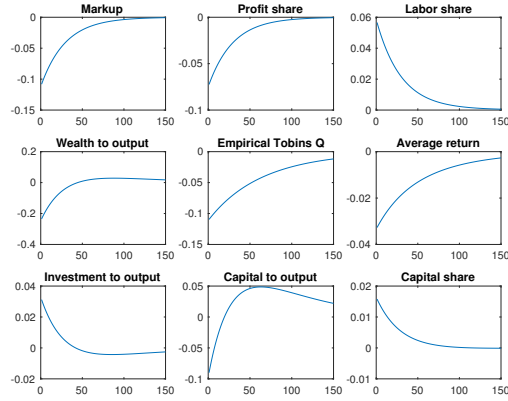


Figure A.2: Transition from 1970 markups to 2015 markups, one time shock

In the second exercise, we solve the policy and transition functions of the model around its 2015 steady state. We then simulate the model, taking as the initial condition the steady state value of all variables in 1970, in particular, the 1970 level of markups and capital-to-labor ratio. Figure A.2 shows the results of this exercise. The transition between steady states is slower: it takes around 7 years for most variables to reach the 2015 steady state, starting at 1970 steady state values. There are significant differences between some of the magnitudes in the transition IRFs and the magnitudes when comparing stochastic stationary states. The IRFs show an increase in the wealth-to-output ratio of .24 during the transition, compared to an increase of .5 in the comparative statics exercise, and an increase Tobin’s Q of .09, compared to an increase of .34 in the comparative statics exercise. It is not surprising that the magnitudes are smaller in the transition exercise, for changes in markups are not permanent, and are assumed to return to their steady state. The changes in the factor shares in the transition are similar in magnitude to the changes in the comparative statics exercise.

In the third exercise, we solve the policy and transition functions of the model around its 1970 steady state. We then feed in a smooth series of shocks, chosen so that markups increase from their steady state level in 1970 to their level in



Figure A.3: Transition from 1970 to 2015 steady state, smooth series of shocks

2015. Figure A.3 shows these results. The results of this transition exercise show linear dynamics, with a few exceptions: the transition path for wealth-to-output and investment are non-linear. The magnitudes of the changes, once again, show significant differences to that of the comparative statics exercise. In the fourth exercise, we feed into the model a series of shocks such that markups follow the exact empirical pattern of the data, from 1970 to 2015. Figure A.4 shows these results. Although there is an increase in markups over the period, the increase is not smooth: during the early 1980s there is a significant decrease in markups. This is due to an increase in the capital share in the data. Overall, the magnitudes of the changes seen in figure A.4 are similar to those in figure A.3.

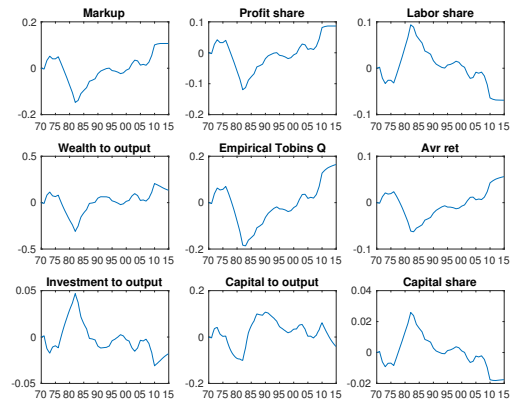


Figure A.4: IRF for full path of markups, 1970-2015

E Additional tables

Table A.1: Quantitative results: changes in productivity growth only

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	0.13	0.95
Capital-to-output ratio (P1)	0.17	0.31
Tobin's Q (P2)	-0.04	0.40
Real interest rate (P3)	-0.84 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	-1.17	0.64
Profit share (P4)	-0.00 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	0.17 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-0.17 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	-1.65 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	-0.39 <i>pp</i>	0 - 2 <i>pp</i>

Table A.2: Quantitative results: changes in D and productivity growth

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	0.41	0.95
Capital-to-output ratio (P1)	0.42	0.31
Tobin's Q (P2)	-0.08	0.40
Real interest rate (P3)	-2.00 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	-3.04	0.64
Profit share (P4)	-0.00 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	0.42 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-0.42 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	0.00 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	-0.22 <i>pp</i>	0 - 2 <i>pp</i>

Table A.3: Ramey quantitative results: changes in markups, productivity growth rates, interest rates

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	0.90	0.95
Capital-to-output ratio (P1)	0.22	0.31
Tobin's Q (P2)	0.05	0.40
Real interest rate (P3)	-2.00 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	1.32	0.64
Profit share (P4)	9.77 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	-7.06 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-2.71 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	-1.78 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	3.93 <i>pp</i>	0 - 2 <i>pp</i>

Table A.4: De Loecker quantitative results: changes in markups, productivity growth rates, interest rates

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	1.62	0.95
Capital-to-output ratio (P1)	-0.17	0.31
Tobin's Q (P2)	0.99	0.40
Real interest rate (P3)	-2.00 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	12.89	0.64
Profit share (P4)	24.87 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	-20.38 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-4.49 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	-3.41 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	2.95 <i>pp</i>	0 - 2 <i>pp</i>

Table A.5: Gutierrez quantitative results: changes in markups, productivity growth rates, interest rates

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	0.62	0.95
Capital-to-output ratio (P1)	0.31	0.31
Tobin's Q (P2)	0.02	0.40
Real interest rate (P3)	-2.00 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	-1.54	0.64
Profit share (P4)	4.31 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	-3.29 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-1.02 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	-0.46 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	0.75 <i>pp</i>	0 - 2 <i>pp</i>

Table A.6: Hall quantitative results: changes in markups, productivity growth rates, interest rates

<i>Moments</i>	Δ <i>Model</i>	Δ <i>Data</i>
Wealth-to-output ratio (P1)	0.96	0.95
Capital-to-output ratio (P1)	0.17	0.31
Tobin's Q (P2)	0.23	0.40
Real interest rate (P3)	-2.00 <i>pp</i>	-2.00 <i>pp</i>
Average return (P3)	1.78	0.64
Profit share (P4)	10.55 <i>pp</i>	8.94 <i>pp</i>
Labor share (P4)	-8.21 <i>pp</i>	-6.82 <i>pp</i>
Capital share (P4)	-2.33 <i>pp</i>	-2.12 <i>pp</i>
Investment-to-output (P5)	-1.55 <i>pp</i>	-0.19 <i>pp</i>
Equity Premium	2.36 <i>pp</i>	0 - 2 <i>pp</i>

F Additional figures

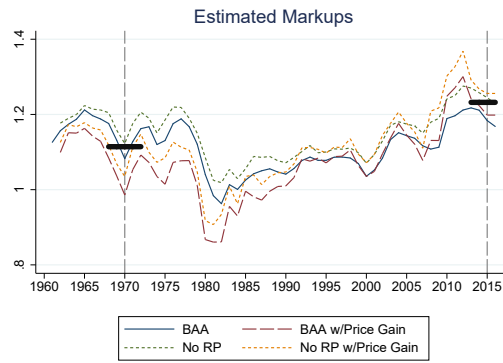


Figure A.5: Markups, 1960-2015

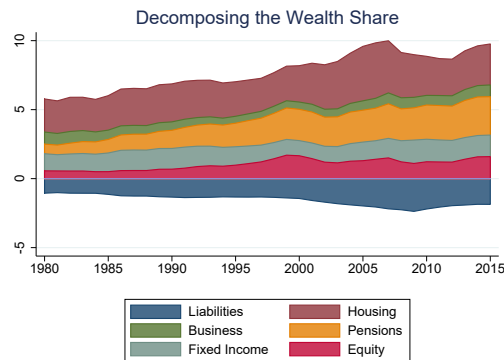


Figure A.6: Wealth and Capital as a share of Gross Value Added.