Labor Market Power

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The views expressed herein are those of the authors and not those of the Census.

Introduction

1. How has labor market power in U.S. changed over time?
2. Implications for (i) welfare, (ii) labor share, (iii) minimum wage policy?

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- Develop tractable general equilibrium oligopsony framework
- Wage is firm-specific markdown on MRPL

$$\text{Labor market power} := \text{Markdown}$$

- Estimate key model parameters using Census LBD data

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- Estimate key model parameters using Census LBD data

- Validate (i) Pass-through rates, (ii) Concentration distribution

- Extend (i) Mergers, (ii) Cross-region empirics

Findings

1. Welfare

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- Between 1976 and 2014, this measure declined significantly
- Contribution to labor share: +2.89 ppt

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- Closed-form link to a measure of local labor market concentration
- Between 1976 and 2014, this measure declined significantly
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3. Minimum wage policy

- Optimal minimum wage binds for 5%, raises welfare by 0.07%.
- Larger minimum wages reduce employment, increase concentration

Literature

1. **Theory**

Oilgopsony	Robinson (1933), Hotelling (1929), Salop (1979), Bhaskar, Manning & To (2002)
Frictional	Burdett Mortensen (1998), Flinn (2010), Manning (2003, 2006)
Competitive	Card Cardoso Heining Kline (2018), Lamadon Mogstad Setzler (2018)

New - (i) Quantitative framework for Census data
(ii) Strategic interaction, (iii) Map discreteness to data

2. **Empirics**

Concentration	Benmelech et al (2018), Azar et al (2018), Rinz (2018)
	Hershbein, Macaluso (2018), Rossi-Hansberg et al (2018)
Wage pass-through	Card, Cardoso, Heining, Kline ('16), Kline, Petkova, Williams, Zidar ('18)
Pass-through	Atkeson, Burstein (2008), Amiti, Itskhoki, Konings (2014, 2016)
Corp. Taxes	Suarez Serrato, Zidar (2016), Giroud and Rauh (2017)

New - (i) Model-relevant concentration statistics, map to welfare
(ii) Identification: Within-firm, across-market variation

MODEL

Environment

Representative family

- Disutility of supplying workers across firms
- Continuum of labor markets $j \in [0, 1]$
- Labor market j has a fixed number of firms $M_j \in [1, 2, \ldots, \infty)$

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Firms

- Firm i has idiosyncratic productivity z_{ijt}
- Hire workers n_{ijt}, rent capital k_{ijt} to produce identical final good
- Profits rebated lump sum to family

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Markets

- Local, Cournot competition for labor
- National, Walrasian markets for output and capital

Household

Preferences

$$U_0 = \max_{\{n_{ijt}, c_{ijt}, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u\left(\mathbf{C}_t - \frac{1}{\overline{\varphi}^{\frac{1}{\varphi}}} \frac{\mathbf{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) \quad , \quad \beta \in (0,1) \quad , \quad \varphi > 0$$

Disutility of labor supply

$$\mathbf{N}_t := \left[\int_0^1 \mathbf{N}_{jt}^{\frac{\theta+1}{\theta}} \, dj\right]^{\frac{\theta}{\theta+1}} \quad , \quad \theta > 0$$

$$\mathbf{N}_{jt} := \left[n_{1jt}^{\frac{\eta+1}{\eta}} + \cdots + n_{M_j jt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} \quad , \quad \eta > \theta$$

Budget constraint

$$\mathbf{C}_t + \left[K_{t+1} - (1-\delta)K_t\right] = \int_0^1 \left[w_{1jt} n_{1jt} + \cdots + w_{M_j jt} n_{M_j jt}\right] dj + R_t K_t + \Pi_t,$$

$$\mathbf{C}_t := \int_0^1 \left[c_{1jt} + \cdots + c_{M_j jt}\right] dj.$$

Example - Maximum labor market power

1. Markets are perfect complements

$$\mathbf{N}_t := \left[\int_0^1 \mathbf{N}_{jt}^{\frac{\theta+1}{\theta}}\, dj\right]^{\frac{\theta}{\theta+1}} = \min_j \left\{ \mathbf{N}_{jt} \right\}$$

$\theta \to 0$: Same allocation of $\mathbf{N}_{jt}$ to all markets

2. Firms within markets are perfect substitutes

$$\mathbf{N}_{jt} := \left[n_{1jt}^{\frac{\eta+1}{\eta}} + \cdots + n_{M_j jt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} = \sum_{i=1}^{M_j} n_{ijt}$$

$\eta \to \infty$: All workers to highest productivity firm

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Equivalence result - Nested logit individual choice model

Firms - Cournot competition

$$\pi_{ijt} = \max_{n_{ijt},k_{ijt}} \overline{Z} z_{ijt} \left(k_{ijt}^{1-\gamma} n_{ijt}^{\gamma} \right)^{\alpha} - R_t k_{ijt} - w_{ijt} n_{ijt} \quad , \quad \alpha > 0$$

s.t.

$$w_{ijt} = \overline{\varphi}^{-\frac{1}{\varphi}} \left(\frac{n_{ijt}}{\mathbf{N}_{jt}} \right)^{\frac{1}{\eta}} \left(\frac{\mathbf{N}_{jt}}{\mathbf{N}_t} \right)^{\frac{1}{\theta}} \mathbf{N}_t^{\frac{1}{\varphi}}$$

$$\mathbf{N}_{jt} = \left[n_{1jt}^{\frac{\eta+1}{\eta}} + \ldots n_{ijt}^{\frac{\eta+1}{\eta}} + \ldots n_{M_j jt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

Firms - Nash equilibrium

Wages

$$
\begin{aligned}
w_{ijt} &= \mu_{ijt} MRPL_{ijt} \\
MRPL_{ijt} &= \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}-1}
\end{aligned}
$$

Firms - Nash equilibrium

Wages

$$w_{ijt} = \mu_{ijt} MRPL_{ijt}$$

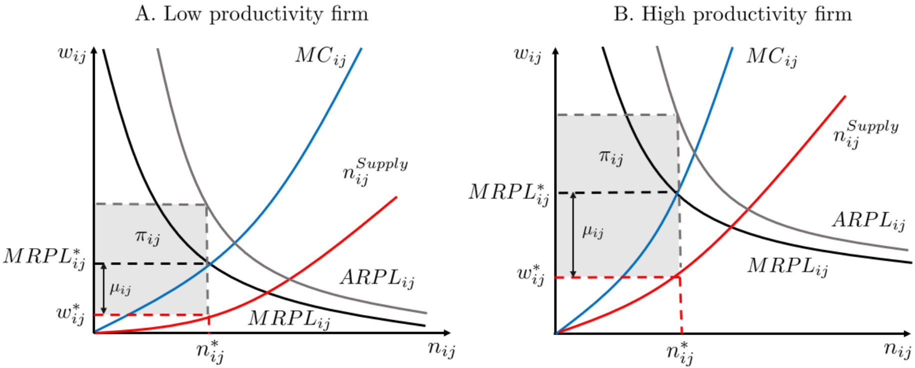
$$MRPL_{ijt} = \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}-1}$$

Markdown

$$\underbrace{\mu_{ijt} = \frac{\varepsilon_{ijt}}{\varepsilon_{ijt}+1}}_{\text{Markdown}} \quad , \quad \underbrace{s_{ijt}^{wn} = \frac{w_{ijt} n_{ijt}}{\sum_{k\in j} w_{kjt} n_{kjt}}}_{\text{Wage bill share}} \quad , \quad \underbrace{\varepsilon_{ijt} = \left[s_{ijt}^{wn} \frac{1}{\theta} + \left(1 - s_{ijt}^{wn}\right) \frac{1}{\eta} \right]^{-1}}_{\text{Elasticity } \varepsilon_{ijt} := \frac{d \log n_{ijt}}{d \log w_{ijt}}}$$

▸ Uncompensated individual labor supply elasticities   ▸ Product market competition

Firms - Nash equilibrium



- Larger firms face lower labor supply elasticity, $\frac{\partial \varepsilon_{ij}}{\partial s_{ij}^{wn}} < 0$
- Have greater mark-downs, $\frac{\partial \mu_{ij}}{\partial s_{ij}^{wn}} < 0$

Example - Shares, Markdowns, Wages, Employment

Concentration and the labor share

Payroll Herfindahl index

$$HHI_j^{wn} := \sum_{i \in j} (s_{ij}^{wn})^2 \qquad , \qquad \widetilde{IHI}^{wn} := \left[\int_0^1 s_j^{wn} HHI_j^{wn} dj \right]^{-1}$$

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Labor share

$$LS = \frac{\tilde{\alpha} \widetilde{IHI}^{wn}}{\left(\frac{\eta+1}{\eta}\right) \widetilde{IHI}^{wn} + \left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta}\right)} \tag{1}$$

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Two results

1. If $\theta < \eta$ then labor share is increasing in $\widetilde{IHI}^{wn}$
2. If $\theta = \eta$ then labor share is independent of $\{HHI_j^{wn}\}$ distribution

▸ Labor share algebra ▸ Bias of employment HHI relative to wage bill HHI

Concentration, 1976 and 2014

Data: LBD tradeable firms, Market: NAICS3 $\times$ Commuting Zone

Concentration, 1976 and 2014

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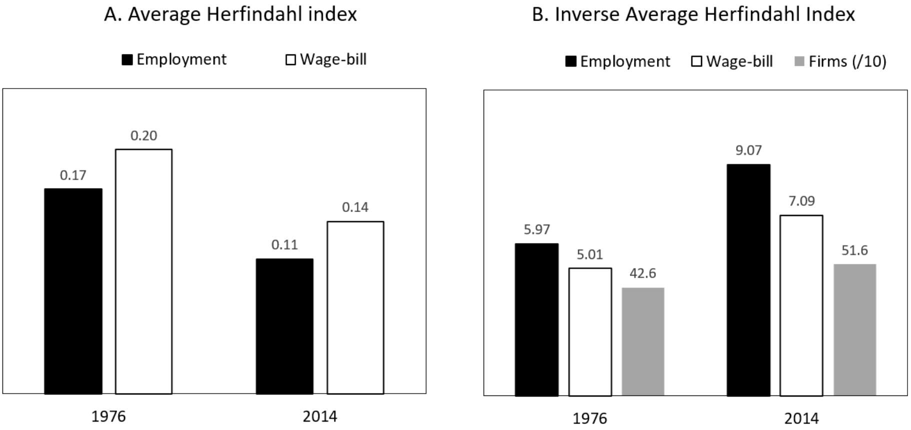
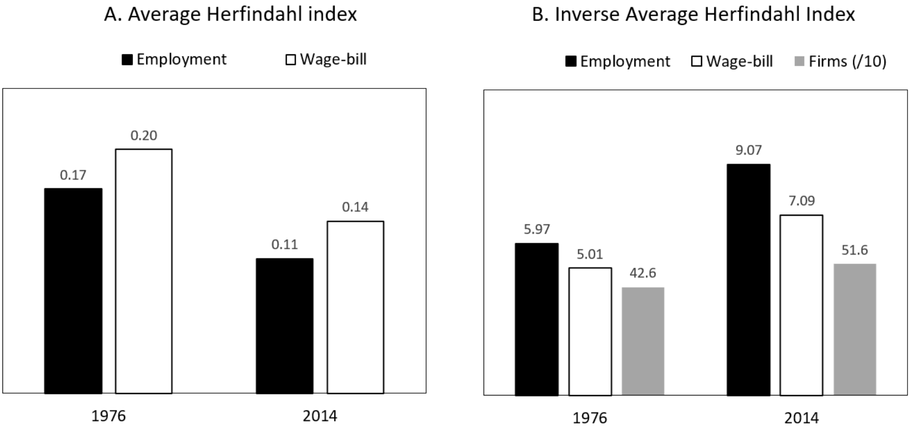


Concentration, 1976 and 2014

Data: LBD tradeable firms, Market: NAICS3 $\times$ Commuting Zone



At estimated $\{\eta, \theta, \tilde{\alpha}\}$ (*next*) falling HHI added 2.89 ppt to Labor Share

CALIBRATION

Identifying θ and η

- Large firm ($s_{ijt}^{wn,L}$) with labor supply elasticity (ε_{ijt}^{L})
- Small firm ($s_{ijt}^{wn,S}$) with labor supply elasticity (ε_{ijt}^{S})
- Exact identification:

$$\varepsilon_{ijt}^{L} = \left[s_{ijt}^{wn,L} \frac{1}{\theta} + \left(1 - s_{ijt}^{wn,L}\right) \frac{1}{\eta} \right]^{-1} \quad , \quad \varepsilon_{ijt}^{S} = \left[s_{ijt}^{wn,S} \frac{1}{\theta} + \left(1 - s_{ijt}^{wn,S}\right) \frac{1}{\eta} \right]^{-1}$$

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1. State-corporate tax changes $\tau_{s(j)t}$ map to $MRPL_{ijt}$ shocks
2. Recover $\varepsilon(s_{ij}^{wn})$ with share-dependent responses

$$\varepsilon(s_{ijt}^{wn}) := \frac{\partial \log n_{ijt}(s_{ijt}^{wn})}{\partial \log w_{ijt}(s_{ijt}^{wn})} \approx \frac{\frac{\mathrm{d} \log n_{ijt}(s_{ijt}^{wn})}{\mathrm{d}\tau}}{\frac{\mathrm{d} \log w_{ijt}(s_{ijt}^{wn})}{\mathrm{d}\tau}}$$

▸ Details - How corporate taxes map to $MRPL_{ijt}$ through Accounting vs. Economic profits

1. Share-dependent tax pass-through

State corporate taxes

- **Sample** Tradeable C-corps operating in 2 mkts in state s, '02-'12
- **Variation** Within firm-state is, across markets $j \in s$

Specification

$$\log n_{ijt+1} = \mu_t + \alpha_{is(j)} + \psi s_{ijt}^{wn} + \beta_n \tau_{s(j)t} + \gamma_n \left(s_{ijt}^{wn} \times \tau_{s(j)t} \right) + e_{ijt}$$

$$dlogn_{ijt+1} = \left[\hat{\beta}_n + \hat{\gamma}_n s_{ijt}^{wn} \right] d\tau_{s(j)t}$$

▸ Sample summary statistics ▸ Regression table

2. Recovering θ and η

Data

$$\widehat{\varepsilon}\left(s_{ijt}^{wn}\right) \approx \frac{d\log n_{ijt}/d\tau_{s(j)t}}{d\log w_{ijt}/d\tau_{s(j)t}} = \frac{-0.00321 + 0.0172 s_{ijt}^{wn}}{-0.000913 + 0.00373 s_{ijt}^{wn}}$$

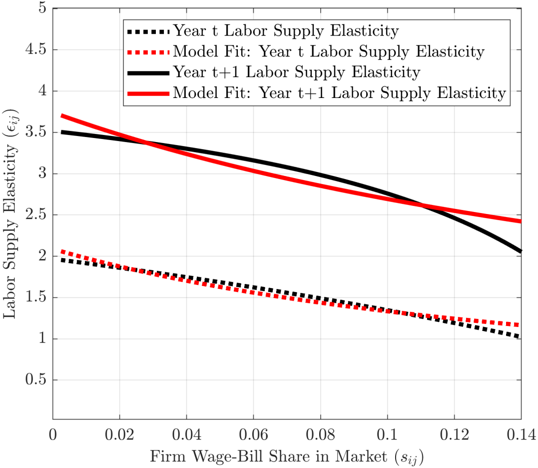
Model

$$\widehat{\varepsilon}\left(s_{ijt}^{wn}\right) = \left[s_{ijt}^{wn}\frac{1}{\theta} + \left(1 - s_{ijt}^{wn}\right)\frac{1}{\eta}\right]^{-1}$$

- Draw s_{ijt}^{wn} from empirical distribution and compute $\widehat{\varepsilon}\left(s_{ijt}^{wn}\right)$
- Use NLLS to recover $\theta = 0.76$, $\eta = 3.74$:

Details - Non-linear least-squares

Model fit



$\theta = 0.76, \eta = 3.74$ (Year $t+1$)

Fig - Distribution of elasticities ε_{ij} and markdowns μ_{ij}

Calibration - Annual, 2014

- $J = 5,000$ markets in model ($15,000$ in LBD)
- M_j mixture of Paretos, mass pt. at $M_j = 1$ [1 firm in 15% mkts, LBD]

Parameters

- Frisch elasticity: $\varphi = 0.5$
- Log-normal productivity: $\log \widetilde{z}_{ij} \sim^{iid} N(1, \sigma_{\widetilde{z}})$

Match

$\overline{\varphi}$	Average earnings per worker
$\widetilde{Z}$	Average firm size
$\widetilde{\alpha}$	Labor share
$\sigma_{\widetilde{z}}$	Payroll weighted wage-bill Herfindahl

Calibration - Annual, 2014

Parameter	Description	Value
Assigned		
φ	Aggregate Frisch elasticity	0.50
θ	Across market substitutability	0.76
η	Within market substitutability	3.74
J	Number of markets	5,000
r	Risk free rate	0.04
δ	Depreciation rate	0.10
γ	Cobb-Douglas labor exponent	0.818
$G(M_j)$	Firms per mkt, mixtures of Paretos w/ mass pt at 1	$\{$15% mkts have 1 firm, Loc1=2, Sh1=0.67, Sc1=5.7, Loc2=2, Sh2=0.67, Sc2=35.6$\}$
Estimated		
$\widetilde{\alpha}$	DRS parameter	0.984
$\sigma_{\widetilde{z}}$	Productivity dispersion	0.391
$\widetilde{Z}$	Productivity shifter	23,570
$\overline{\varphi}$	Labor disutility shifter	6.904

▸ Figure - Distribution of M_j, and calibration of $G(M_j)$

Next

1. Validation
    (i) Weighted and unweighted concentration distribution
    (ii) Pass-through (Kline, Petkova, Williams, Zidar, QJE 2019)
2. Welfare
3. Labor share
4. Minimum wage

1. Non-targeted concentration measures

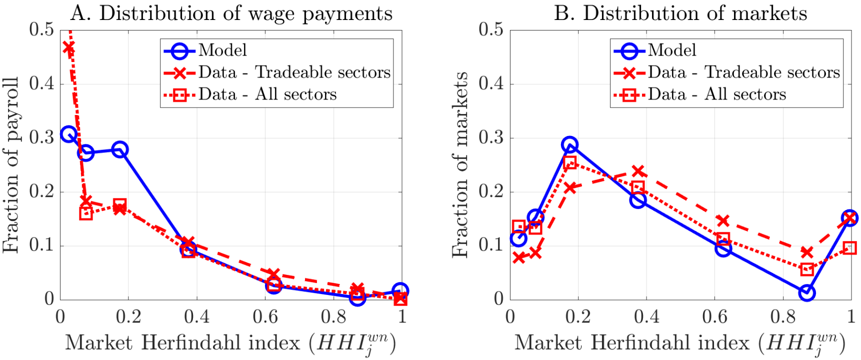
Fact Unweighted HHI^{wn} is 2 times larger than weighted HHI^{wn}

Fact Concentrated markets are small

Wage bill herfindahl	Model	Data
Unweighted average	0.35	0.45
Payroll weighted average	0.14	0.14
Correlation with market employment	-0.75	-0.21

▸ Additional concentration measures

1. Non-targeted concentration measures



- Matches both weighted and unweighted across market distributions
- Similar concentration distribution outside tradeables

2. Pass-through

- Replicate patent experiment in Kline et al (2018)
- Same sample properties & increase in average labor productivity

	Benchmark (1)	**Kline et al (2018)** (2)	**Cardoso et al (2018)** (3)	**Competitive** (4)
Pass-through	0.423	0.317	0.327	0.984
Dependent var.	w_{ij}	Labor compensation per worker	Hourly wage	w_{ij}
Independent var.	y_{ij} / n_{ij}	Labor compensation plus earnings (EBITDA) per worker	Value added per worker (IV with avg. sales per worker)	y_{ij} / n_{ij}

- Increase VA per worker by $\$1$, wages per worker by $\$0.42$

▸ Experiment details ▸ Further validation: Size-wage premium

Welfare

Welfare cost of labor market power

- How much would consumption have to increase to be indifferent between U.S. labor market and a competitive labor market?

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Competitive equilibrium

Wages w_{ijt} and an allocation of workers n_{ijt} such that

1. Taking w_{ijt} as given, n_{ijt} solves each firm's optimization problem

$$n_{ijt} = \arg\max_{n_{ijt}} \; \widetilde{Z}\widetilde{z}_{ijt} n_{ijt}^{\widetilde{\alpha}} - w_{ijt} n_{ijt}$$

2. Taking w_{ijt} as given, n_{ijt} is the household's optimal labor supply

$$n_{ijt} = \overline{\varphi} \left(\frac{w_{ijt}}{\mathbf{W}_{jt}} \right)^{\eta} \left(\frac{\mathbf{W}_{jt}}{\mathbf{W}_{t}} \right)^{\theta} \mathbf{W}_t^{\varphi}$$

Welfare

Consumption equivalent gain

$$u\left((1+\lambda)\mathbf{C}_o - \frac{1}{\overline{\varphi}^{\frac{1}{\varphi}}}\frac{\mathbf{N}_o^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) = u\left(\mathbf{C}_c - \frac{1}{\overline{\varphi}^{\frac{1}{\varphi}}}\frac{\mathbf{N}_c^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$$

Welfare

Consumption equivalent gain

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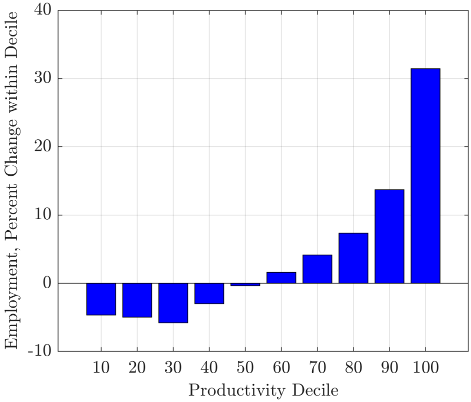
Frisch elasticity (φ)	Welfare gain (λ)	$\mathbf{N}_c/\mathbf{N}_o$
0.2	2.9	1.08
0.5	5.4	1.20
0.8	8.0	1.33

Interpretation

Households would need additional 5.4% of lifetime consumption to be indifferent across markets

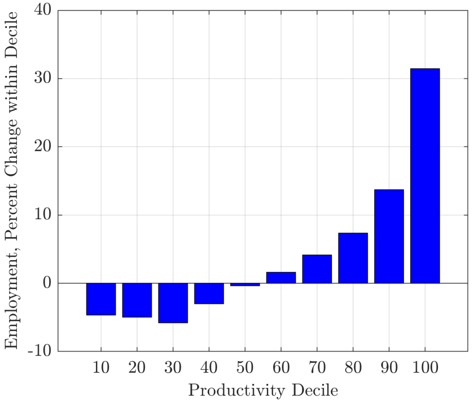
Welfare - Output decomposition



- Overall output gain: 21 percent
- *Share* due to reallocation: 26 percent

▸ Details - Reallocation vs. Scale decomposition

Welfare - Output decomposition



	Oligoposony	Competitive
Payroll weighted wage-bill HHI	0.14	0.27
Payroll weighted employment HHI	0.11	0.20

Details - Reallocation vs. Scale decomposition

Labor share

What are implications of declining concentration for labor share?

$$LS = \frac{\widetilde{\alpha}\widetilde{IHI}^{wn}}{\left(\frac{\eta+1}{\eta}\right)\widetilde{IHI}^{wn} + \left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta}\right)}$$

- $\widetilde{IHI}^{wn}$ increased from 5.01 in 1976 to 7.09 in 2014
- Use estimated parameters $\{\eta = 3.74, \theta = 0.76, \widetilde{\alpha} = 0.98\}$
- Contributed +2.89 ppt to Labor share
- Labor market concentration not driving declining Labor share

Minimum wage

Implications for Minimum wage

- (New) Tractable theory of minimum wage in oligopoly with DRS
- Simulate minimum wage hike to $12 per hour (binds for 10.4% of workers pre-min wage)
- Comparable increase studied in Germany, 2016
  e.g. Dustmann, Lindner, Schoenberg, Umkehrer, vom Berge (2018)
- Compare estimates of employment reallocation across firm sizes
- Use model to assess welfare and optimal policy

▸ Theory and Algorithm   ▸ Minimum wage graph

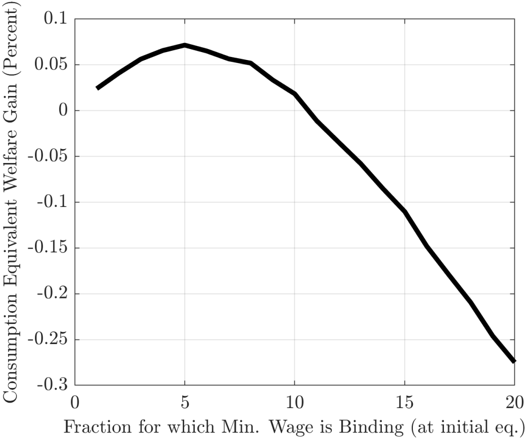
Minimum wage - Example of an increase

- Choose $\underline{w}$ such that 10.4% of workers initially receive $w < \underline{w}$

Moment	**Model**	**Data**
Post experiment min-median ratio (percent): w_{min} / w_{p50}	61	48
Δ log Ave firm size	0.01	0.12
Δ log Number of firms with $n_{ij} \leq 2$	-0.28	-0.15
Δ log Number of firms with $n_{ij} \geq 50$	0.000	0.120
Δ Share Emp at firms with $n_{ij} \leq 2$	-0.04	-0.03
Δ Share Emp at firms with $n_{ij} \geq 50$	0.00	0.01
Inequality	**Pre**	**Post**
$p50$-$p10$ (log difference)	0.50	0.49
$p90$-$p50$ (log difference)	0.43	0.43

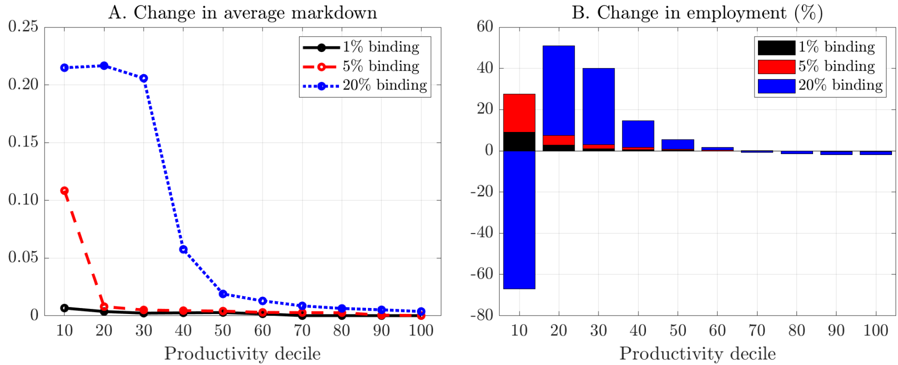
- Employment increases by $+1.07\%$
- Welfare gain of 0.064%

Minimum wage - Optimal minimum wage



- Optimum Minimum wage binds for 5% of workers (at initial eq.)
- Delivers 0.07% consumption equivalent welfare gain

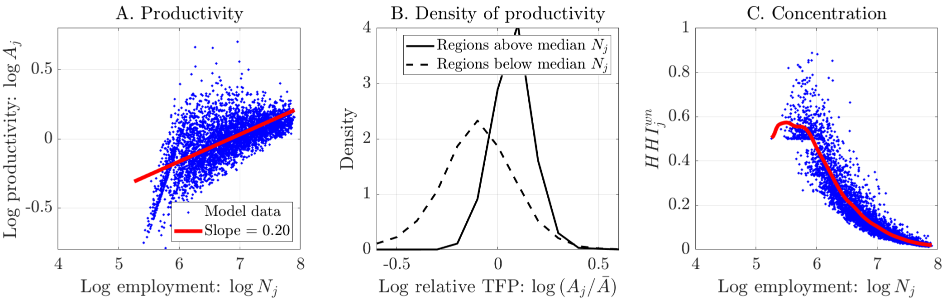
Minimum wage - Reallocation effects



- Reallocation effect is opposite of the competitive equilibrium
- Agg. N declines for large min. wage hikes, HHI^{wn} increases

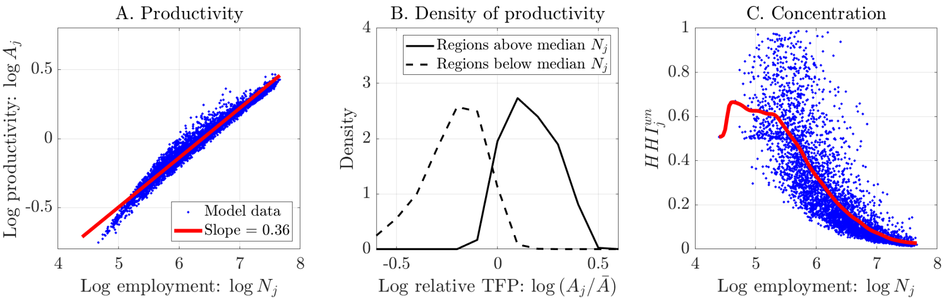
▸ Wages, employment, concentration

Productivity, Employment, Concentration



- Oligopsony economy
- Replicates Figure 1 of Combes et al (ECTA 2012)
  - *Productivity Advantage of Large Cities: Agglomeration vs. Selection*

Productivity, Employment, Concentration



- Recalibrated competitive economy $(\downarrow \tilde{\alpha}, \uparrow \sigma_z)$
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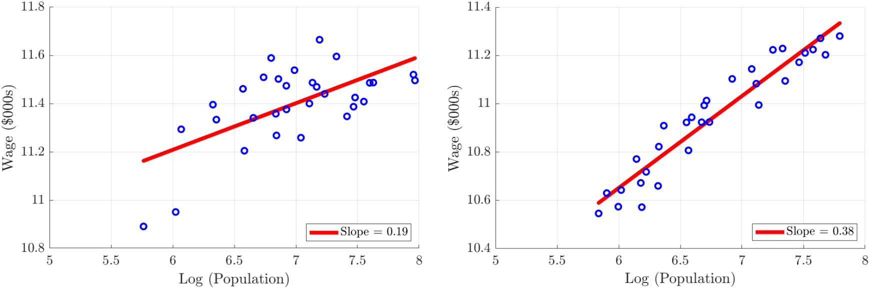
Productivity, Employment, Concentration



- Left: Oligopsony economy
- Right: Recalibrated competitive economy $(\downarrow \tilde{\alpha}, \uparrow \sigma_z)$
- Replicates Figure 1 of Glaeser Mare (2001) - *Cities and skills*

Productivity, Employment, Concentration

Independent variable Dependent variable		$\log HHI_j^{wn}$ $\log W_j$	HHI_j^{wn} $\log W_j$	$\log N_j$ $\log A_j$	IHI_j^{wn} $100 \times \log A_j$
1. Benchmark	$\hat{\beta}$	-0.187	-1.042	0.195	0.482
2. Recalibrated	$\hat{\beta}$	-0.205	-0.755	0.358	1.473
3. Minimum wage	$\hat{\beta}$	-0.149	-0.731	0.200	0.494

Conclusion

Five contributions

1. Develop and validate general equilibrium oligopsony model
2. Estimate using size dependent corporate tax response in LBD
3. Welfare losses from labor market power are large: 2.9% to 8.0%
4. Model relevant concentration measure is wage-bill Herfindahl
5. Declining wage-bill Herfindahls between 1976 and 2014 implies a +2.89 ppt labor share rise

THANK YOU!

APPENDIX

Representation - Logit model

- Workers $m \in [0,1]$ with committed income $y_m \sim F(y)$
- Minimize total labor disutility of attaining y_m

$$\min_{ij} \ \log h_m - \xi_{ij} \qquad \text{s.t.} \qquad w_{ij} h_m = y_m$$

- Random labor disutility

$$F\left(\xi_{11}, \ldots, \xi_{ij}, \ldots \xi_{NJ}\right) = \exp\left[-\sum_{j=1}^{J}\left(\sum_{i=1}^{M_j} e^{-(1+\eta)\xi_{ij}}\right)^{\frac{1+\theta}{1+\eta}}\right]$$

- Labor supply

$$n_{ij} = \frac{w_{ij}^{\eta}}{\sum_{i=1}^{M_j} w_{ij}^{1+\eta}} \frac{\left[\sum_{i=1}^{M_j} w_{ij}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}}{\sum_{l=1}^{J}\left[\sum_{k=1}^{M_l} w_{kl}^{1+\eta}\right]^{\frac{1+\theta}{1+\eta}}} Y. \tag{2}$$

- Result Delivers same supply system as rep. agent CES Back

Firms - Notation

- The '*tilde*' variables are defined as follows:

$$
\begin{aligned}
\widetilde{\alpha} &:= \frac{\alpha\gamma}{1-(1-\gamma)\alpha} \\
\widetilde{z}_{ijt} &:= [1-(1-\gamma)\alpha]\left(\frac{(1-\gamma)\alpha}{R_t}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} z_{ijt}^{\frac{1}{1-(1-\gamma)\alpha}} \\
\widetilde{Z} &:= \overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}}
\end{aligned}
$$

- Note that $(1-\gamma)\alpha$ is capital's share of income

Back - Firm problem

Computation

A firm's wage-bill share is defined by their relative wage:

$$s_{ij}^{wn} = \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{1+\eta}$$

Within a market, an equilibrium can be solved by iterating through the following conditions given a guess of $\mathbf{s}_j^{wn} = \left(s_{1j}^{wn}, \ldots, s_{M_j j}^{wn}\right)$

$$\begin{aligned}
\varepsilon_{ij} &= \begin{cases} s_{ij}^{wn}\theta + \left(1 - s_{ij}^{wn}\right)\eta & \text{Bertrand} \\ \left[s_{ij}^{wn}\frac{1}{\theta} + \left(1 - s_{ij}^{wn}\right)\frac{1}{\eta}\right]^{-1} & \text{Cournot} \end{cases} \\
\mu_{ij} &= \frac{\varepsilon_{ij}}{\varepsilon_{ij}+1} \\
w_{ij} &= \mu_{ij} MRPL_{ij} \\
\mathbf{w}_j &= \left[\int_0^1 w_{ij}^{1+\eta}\, dj\right]^{\frac{1}{1+\eta}} \\
s_{ij}^{wn(NEW)} &= \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{1+\eta}
\end{aligned}$$

We guess equal shares, and then iterate until $\mathbf{s}_j^{wn(NEW)} = \mathbf{s}_j^{wn}$. Back

DRS Computation

Sub in inverse supply curve for n_{ij}:

$$MRPL_{ij} = \omega \mathbf{W}^{(1-\widetilde{\alpha})(\theta-\varphi)} \hat{z}_{ij} \left\{ w_{ij}^{-\eta} \mathbf{w}_j^{\eta-\theta} \right\}^{1-\widetilde{\alpha}}$$

Write the wage in terms of the marginal revenue product of labor:

$$\begin{aligned} w_{ij} &= \mu_{ij} MRPL_{ij} \\ &= \mu_{ij} \omega \mathbf{W}^{(1-\widetilde{\alpha})(\theta-\varphi)} \hat{z}_{ij} \left\{ w_{ij}^{-\eta} \mathbf{w}_j^{\eta-\theta} \right\}^{1-\widetilde{\alpha}} \end{aligned}$$

Use $\mathbf{w}_j = w_{ij} s_{ij}^{-\frac{1}{\eta+1}}$: $w_{ij} = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \mu_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \hat{z}_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} s_{ij}^{-\frac{(1-\widetilde{\alpha})(\eta-\theta)}{\eta+1} \frac{1}{1+(1-\widetilde{\alpha})\theta}}$

We will solve for an equilibrium in 'hatted' variables, and then rescale:

$$\begin{aligned} \hat{w}_{ij} &:= \mu_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \hat{z}_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} s_{ij}^{-\frac{(1-\widetilde{\alpha})(\eta-\theta)}{\eta+1} \frac{1}{1+(1-\widetilde{\alpha})\theta}} \\ \hat{\mathbf{w}}_j &:= \left[\sum_{i \in j} \hat{w}_{ij}^{\eta+1} \right]^{\frac{1}{\eta+1}} \\ \hat{\mathbf{W}} &:= \left[\int \hat{\mathbf{w}}_j^{\theta+1} dj \right]^{\frac{1}{\theta+1}} \\ \hat{n}_{ij} &:= \left(\frac{\hat{w}_{ij}}{\hat{\mathbf{w}}_j} \right)^{\eta} \left(\frac{\hat{\mathbf{w}}_j}{\hat{\mathbf{W}}} \right)^{\theta} \left(\frac{\hat{\mathbf{W}}}{1} \right)^{\varphi} \end{aligned}$$

Back

DRS Computation

These definitions imply that

$$
\begin{aligned}
w_{ij} &= \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \hat{w}_{ij} \\
\mathbf{w}_j &= \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \hat{\mathbf{w}}_j \\
\mathbf{W} &= \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \hat{\mathbf{W}}
\end{aligned}
$$

These definitions allow us to compute the equilibrium market shares in terms of 'hatted' variables:

$$
s_j^{wn} = \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{\eta+1} = \left(\frac{\hat{w}_{ij}}{\hat{\mathbf{w}}_j}\right)^{\eta+1} \tag{3}
$$

Back

DRS Computation

For a given set of values for parameters $\{\overline{\varphi}, \widetilde{Z}, \widetilde{\alpha}, \beta, \delta\}$, we can solve for the non-constant returns to scale equilibrium as follows:

1. Guess $\mathbf{s}_j^{wn} = (s_{1j}^{wn}, \ldots, s_{M_j j}^{wn})$
2. Compute $\{\epsilon_{ij}\}$ and $\{\mu_{ij}\}$ using the industry eq formulas.
3. Construct the 'hatted' equilibrium values as follows:

$$\hat{w}_{ij} = \mu_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \hat{z}_{ij}^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} s_{ij}^{-\frac{(1-\widetilde{\alpha})(\eta-\theta)}{\eta+1}\frac{1}{1+(1-\widetilde{\alpha})\theta}}$$

$$\hat{\mathbf{w}}_j = \left[\sum_{i\in j} \hat{w}_{ij}^{\eta+1}\right]^{\frac{1}{\eta+1}}$$

$$\hat{\mathbf{W}} = \left[\int \hat{\mathbf{w}}_j^{\theta+1} dj\right]^{\frac{1}{\theta+1}}$$

$$\hat{n}_{ij} = \left(\frac{\hat{w}_{ij}}{\hat{\mathbf{w}}_j}\right)^{\eta} \left(\frac{\hat{\mathbf{w}}_j}{\hat{\mathbf{W}}}\right)^{\theta} \left(\frac{\hat{\mathbf{W}}}{1}\right)^{\varphi}$$

4. Update the wage-bill share vector using previous expression (prior slide).
5. Iterate until convergence of wage-bill shares. Back

DRS Computation

Recovering true equilibrium values from 'hatted' equilibrium: Once the 'hatted' equilibrium is solved, we can construct the true equilibrium values by rescaling as follows:

$$\omega = \frac{\widetilde{Z}}{\overline{\varphi}^{1-\widetilde{\alpha}}} \tag{4a}$$

$$\mathbf{W} = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\varphi}} \hat{\mathbf{W}}^{\frac{1+(1-\widetilde{\alpha})\theta}{1+(1-\widetilde{\alpha})\varphi}} \tag{4b}$$

$$w_{ij} = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \hat{w}_{ij} \tag{4c}$$

$$\mathbf{w}_j = \omega^{\frac{1}{1+(1-\widetilde{\alpha})\theta}} \mathbf{W}^{\frac{(1-\widetilde{\alpha})(\theta-\varphi)}{1+(1-\widetilde{\alpha})\theta}} \hat{\mathbf{w}}_j \tag{4d}$$

$$n_{ij} = \overline{\varphi} \left(\frac{w_{ij}}{\mathbf{w}_j}\right)^{\eta} \left(\frac{\mathbf{w}_j}{\mathbf{W}}\right)^{\theta} \left(\frac{\mathbf{W}}{1}\right)^{\varphi} \tag{4e}$$

Back

DRS Computation

We set the scale parameters $\overline{\varphi}$ and $\widetilde{Z}$ in order to match average firm size observed in the data ($AveFirmSize^{Data} = 27.96$ from Table 6), and average earnings per worker in the data ($AveEarnings^{Data} = \$65,773$ from Table 6):

$$\widehat{AveFirmSize}^{Data} = \frac{\int \left\{\sum_{i\in j} n_{ij}\right\} dj}{\int \left\{M_j\right\} dj} \tag{5a}$$

$$\widehat{AveEarnings}^{Data} = \frac{\int \left\{\sum_{i\in j} w_{ij} n_{ij}\right\} dj}{\int \left\{\sum_{i\in j} n_{ij}\right\} dj} \tag{5b}$$

▸ Back

DRS Computation

To compute the values of $\overline{\varphi}$ and $\widetilde{Z}$ that allow us to match $AveFirmSize^{Data}$ and $AveEarnings^{Data}$, we substitute the model's values for n_{ij}, w_{ij}, and M_j into $AveFirmSize^{Data}$ and $AveEarnings^{Data}$. We repetitively substitute equations (4a) through (4e) into (5a) and (5b). We then solve for $\overline{\varphi}$ and $\widetilde{Z}$:

$$\overline{\varphi} = \frac{\frac{AveFirmSize^{Data}}{\widehat{AveFirmSize}^{Model}}}{\left(\frac{AveEarnings^{Data}}{\widehat{AveEarnings}^{Model}}\right)^{\varphi}} \tag{6}$$

$$\widetilde{Z} = \overline{\varphi}^{1-\widetilde{\alpha}} \left(\frac{AveEarnings^{Data}}{\widehat{AveEarnings}^{Model}}\right)^{1+(1-\widetilde{\alpha})\varphi} \times \hat{\boldsymbol{W}}^{-(1-\widetilde{\alpha})(\theta-\varphi)} \tag{7}$$

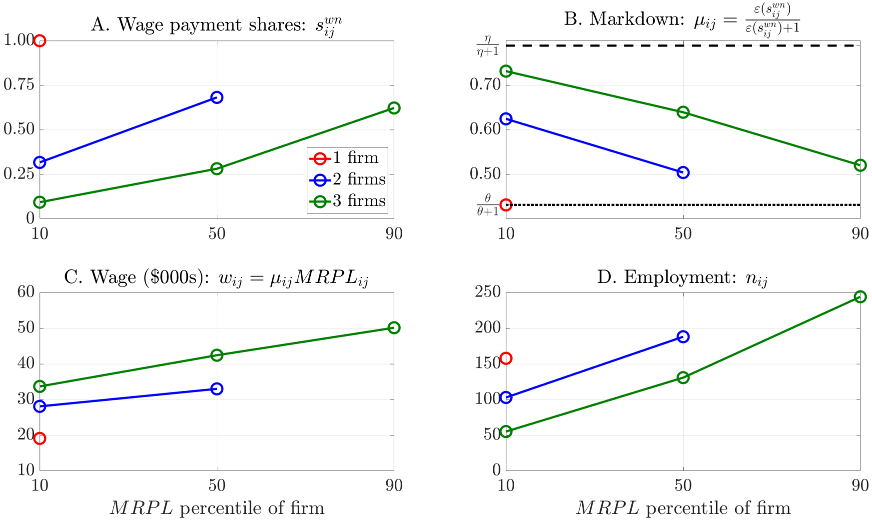
where

$$\widehat{AveFirmSize}^{Model} = \frac{\int \left\{\sum_{i\in j} \hat{n}_{ij}\right\} dj}{\int \left\{M_j\right\} dj}$$

$$\widehat{AveEarnings}^{Model} = \frac{\int \left\{\sum_{i\in j} \hat{w}_{ij}\hat{n}_{ij}\right\} dj}{\int \left\{\sum_{i\in j} \hat{n}_{ij}\right\} dj}$$

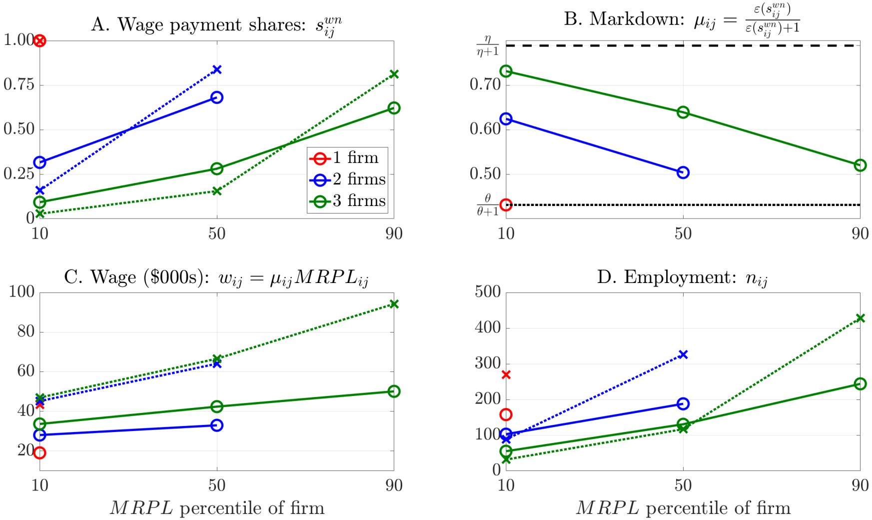
▸ Back

Firms - Local labor market equilibrium



Back to simple graph

Firms - Local labor market equilibrium - Competitive



Back to simple graph

Aggregation – Labor share and concentration

$$ls_{ij} = \frac{w_{ij} n_{ij}}{\widetilde{z}_{ij} \widetilde{Z} n_{ij}^{\widetilde{\alpha}}}$$

$$ls_{ij} = \widetilde{\alpha} \frac{w_{ij}}{\widetilde{\alpha} \widetilde{z}_{ij} \widetilde{Z} n_{ij}^{\widetilde{\alpha}-1}}$$

$$ls_{ij} = \widetilde{\alpha} \frac{w_{ij}}{MRPL_{ij}}$$

$$ls_{ij} = \widetilde{\alpha} \mu_{ij}$$

Let $y_{ij} = \widetilde{z}_{ij} \widetilde{Z} n_{ij}^{\widetilde{\alpha}}$. At the market level, the labor share in market j, LS_j, is given by the following expression:

$$\begin{aligned} LS_j &= \left[\frac{\sum_i y_{ij}}{\sum_i w_{ij} n_{ij}} \right]^{-1} \\ &= \left[\sum_i \left(\frac{w_{ij} n_{ij}}{\sum_i w_{ij} n_{ij}} \right) \frac{y_{ij}}{w_{ij} n_{ij}} \right]^{-1} \end{aligned}$$

▸ Back - Labor share and aggregation

Aggregation – Labor share and concentration

Using the definition of the wage-bill share,

$$LS_j^{-1} = \sum_i s_{ij}^{wn} \widetilde{\alpha}^{-1} \mu_{ij}^{-1}$$

$$LS_j^{-1} = \widetilde{\alpha}^{-1} \sum_i s_{ij}^{wn} \left[\frac{\eta+1}{\eta} + s_{ij}^{wn} \left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta} \right) \right]$$

$$LS_j^{-1} = \widetilde{\alpha}^{-1} \frac{\eta+1}{\eta} + \widetilde{\alpha}^{-1} \left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta} \right) HHI_j^{wn}$$

Define the inverse Herfindahl at the market level as $IHI_j^{wn} = (HHI_j^{wn})^{-1}$. Aggregating across markets yields the economy-wide labor share:

$$LS^{-1} = \frac{\int \sum y_{ij}}{\int \sum w_{ij} n_{ij}} = \int \frac{\sum w_{ij} n_{ij}}{\int \sum w_{ij} n_{ij}} \frac{\sum y_{ij}}{\sum w_{ij} n_{ij}}$$

$$= \int s_j^{wn} LS_j^{-1}$$

$$LS^{-1} = \frac{1}{\widetilde{\alpha}} \left(\frac{\eta+1}{\eta} + \left(\frac{\theta+1}{\theta} - \frac{\eta+1}{\eta} \right) \int s_j^{wn} \left(IHI_j^{wn} \right)^{-1} dj \right)$$

▸ Back - Labor share and aggregation

Aggregation – Labor share and concentration

$$\textbf{Wage Bill Herfindahl:} \quad HHI_j^{wn} \equiv \sum_i (s_{ij}^{wn})^2 \quad , \quad s_{ij}^{wn} = \frac{w_{ij} n_{ij}}{\sum_i w_{ij} n_{ij}}$$

$$\textbf{Employment Herfindahl:} \quad HHI_j^{n} \equiv \sum_i (s_{ij}^{n})^2 \quad , \quad s_{ij}^{n} = \frac{n_{ij}}{\sum_i n_{ij}}$$

Note:

$$HHI_j^{wn} = \sum_i \left(\frac{w_{ij}}{\sum_i s_{ij}^n w_{ij}} \right) \left(s_{ij}^n \right)^2$$

1. Employment Herfindahl yields less concentration:

   Since $cov\left(s_{ij}^n, w_{ij}\right) > 0$, then $HHI_j^{wn} > HHI_j^n$

2. $cov\left(s_{ij}^n, w_{ij}\right)$ is endogenous and depends on concentration

Back - Labor share and aggregation

Table: Summary Statistics, Longitudinal Employer Database 1976 and 2014

	(A) Firm-market-level averages	
	1976	**2014**
Total firm pay (000s)	470.90	1839.00
Total firm employment	37.09	27.96
Pay per employee	$ 12,696	$ 65,773
Firm-level observations	660,000	810,000
	(B) Market-level averages	
	1976	**2014**
Wage-bill Herfindahl (Unweighted)	0.45	0.45
Employment Herfindahl (Unweighted)	0.43	0.42
Wage-bill Herfindahl (Weighted by market's share of total employment)	0.19	0.14
Employment Herfindahl (Weighted by market's share of total employment)	0.18	0.12
Firms per market	42.56	51.60
Percent of markets with 1 firm	14.6%	14.7%
National employment share of markets with 1 firm	0.63%	0.36%
Market-level observations	15,000	16,000
	(C) Market-level correlations	
	1976	**2014**
Correlation of Wage-bill Herfindahl and number of firms	-0.22	-0.21
Correlation of Wage-bill Herfindahl and Std. Dev. Of Relative Wages	-0.49	-0.51
Correlation of Wage-bill Herfindahl and Employment Herfindahl	0.98	0.98
Correlation of Wage-bill Herfindahl and Market Employment	-0.20	-0.21
Market-level observations	15,000	16,000

Notes: Tradeable NAICS2 codes (11,21,31,32,33,55).

Back to bar chart Back to calib

	(A) Firm-market-level averages	
	1976	**2014**
Total firm pay (000s)	209.40	1102.00
Total firm employment	19.43	23.21
Pay per employee	$ 10,777	$ 47,480
Firm-Market level observations	3,746,000	5,854,000

	(B) Market-level averages	
	1976	**2014**
Wage-bill Herfindahl (Unweighted)	0.36	0.34
Employment Herfindahl (Unweighted)	0.33	0.32
Wage-bill Herfindahl (Weighted by market's share of total wage-bill)	0.17	0.11
Employment Herfindahl (Weighted by market's share of total wage-bill)	0.15	0.09
Firms per market	75.70	113.20
Percent of markets with 1 firm	10.4%	9.4%
Market level observations	49,000	52,000

	(C) Market-level correlations	
	1976	**2014**
Correlation of Wage-bill Herfindahl and number of firms	-0.20	-0.17
Correlation of Wage-bill Herfindahl and Employment Herfindahl	0.97	0.97
Correlation of Wage-bill Herfindahl and Market Employment	-0.15	-0.16
Market-level observations	49,000	52,000

Notes: **All NAICS.** Back to bar chart Back to calib

Corporate taxes, labor and wages

	$\log n_{ijt+1}$ (1)	$\log n_{ijt+1}$ (2)	$\log w_{ijt+1}$ (3)	$\log w_{ijt+1}$ (4)
$\tau_{s(j)t}$	-0.00164***	-0.00321***	-0.00203***	-0.000913
	(0.000627)	(0.000740)	(0.000647)	(0.000902)
s_{ijt}^{wn}		1.931***		0.147***
		(0.0460)		(0.00835)
$\tau_{s(j)t} \times s_{ijt}^{wn}$		0.0172***		0.00373***
		(0.00490)		(0.000982)
Year FE	Y	Y	Y	Y
Commuting Zone FE	Y	Y	Y	Y
Industry FE	N	Y	N	Y
Firm×State FE	N	Y	N	Y
R-squared	0.034	0.877	0.096	0.797
Round N	4,425,000	4,425,000	4,425,000	4,425,000

Notes: *** $p<0.01$, ** $p<0.05$, * $p<0.1$ Standard errors clustered at State ×year level. Tradeable C-Corps from 2002 to 2014.

▸ Back - Regression specification

Data Appendix

Data:

- Isolate all plants (lbdnums) with non missing firmids, with strictly positive pay, strictly positive employment, non-missing county codes for the continental US (we exclude Alaska, Hawaii, and Puerto Rico)
- isolate all lbdnums with non-missing 2 digit NAICS codes equal to 11,21,31,32,33, or 55.
- We use the consistent 2007 NAICS codes provided by *Fort & Klimek* throughout the paper.
- Define a firm to be the sum of all establishments in a commuting zone with a common firmid and NAICS3 classification.

1. **Summary Statistics Sample:** Our summary statistics include all observations that satisfy the above criteria in 1976 and 2014.
2. **Corporate Tax Sample:** The corporate tax analysis includes all observations that satisfy the above criteria between 2002 and 2014 with an LFO of ‘C’. Firms must operate in at least two markets within a state.

Data Appendix

Table: Sample NAICS3 Codes.

NAICS3	Description	NAICS3	Description
111	Crop Production	322	Paper Manufacturing
112	Animal Production and Aquaculture	323	Printing and Related Support Activities
113	Forestry and Logging	324	Petroleum and Coal Products Manufacturing
114	Fishing, Hunting and Trapping	325	Chemical Manufacturing
115	Support Activities for Agriculture and Forestry	326	Plastics and Rubber Products Manufacturing
211	Oil and Gas Extraction	327	Nonmetallic Mineral Product Manufacturing
212	Mining (except Oil and Gas)	331	Primary Metal Manufacturing
213	Support Activities for Mining	332	Fabricated Metal Product Manufacturing
311	Food Manufacturing	333	Machinery Manufacturing
312	Beverage and Tobacco Product Manufacturing	334	Computer and Electronic Product Manuf.
313	Textile Mills	335	Electrical Equipment, Appliance, and Component Manuf.
314	Textile Product Mills	336	Transportation Equipment Manufacturing
315	Apparel Manufacturing	337	Furniture and Related Product Manufacturing
316	Leather and Allied Product Manufacturing	339	Miscellaneous Manufacturing
321	Wood Product Manufacturing	551	Management of Companies and Enterprises

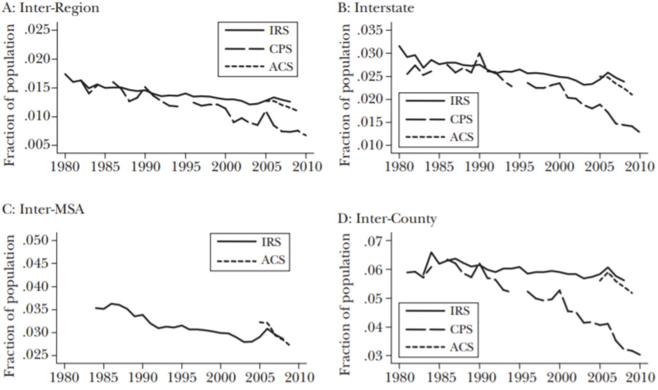
Data Appendix

Table: Commuting Zone Examples

CZ ID, 2000	County Name	Metropolitan Area, 2003	County Pop. 2000	CZ Pop. 2000
58	Cook County	Chicago-Naperville-Joliet, IL Metropolitan Division	5,376,741	8,704,935
58	DeKalb County	Chicago-Naperville-Joliet, IL Metropolitan Division	88,969	8,704,935
58	DuPage County	Chicago-Naperville-Joliet, IL Metropolitan Division	904,161	8,704,935
58	Grundy County	Chicago-Naperville-Joliet, IL Metropolitan Division	37,535	8,704,935
58	Kane County	Chicago-Naperville-Joliet, IL Metropolitan Division	404,119	8,704,935
58	Kendall County	Chicago-Naperville-Joliet, IL Metropolitan Division	54,544	8,704,935
58	Lake County	Lake County-Kenosha County, IL-WI Metropolitan Division	644,356	8,704,935
58	McHenry County	Chicago-Naperville-Joliet, IL Metropolitan Division	260,077	8,704,935
58	Will County	Chicago-Naperville-Joliet, IL Metropolitan Division	502,266	8,704,935
58	Kenosha County	Lake County-Kenosha County, IL-WI Metropolitan Division	149,577	8,704,935
58	Racine County	Racine, WI Metropolitan Statistical Area	188,831	8,704,935
58	Walworth County	Whitewater, WI Micropolitan Statistical Area	93,759	8,704,935
47	Anoka County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	298,084	2,904,389
47	Carver County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	70,205	2,904,389
47	Chisago County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	41,101	2,904,389
47	Dakota County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	355,904	2,904,389
47	Hennepin County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	1,116,200	2,904,389
47	Isanti County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	31,287	2,904,389
47	Ramsey County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	511,035	2,904,389
47	Scott County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	89,498	2,904,389
47	Washington County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	201,130	2,904,389
47	Wright County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	89,986	2,904,389
47	Pierce County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	36,804	2,904,389
47	St. Croix County	Minneapolis-St. Paul-Bloomington, MN-WI Metropolitan Statistical Area	63,155	2,904,389

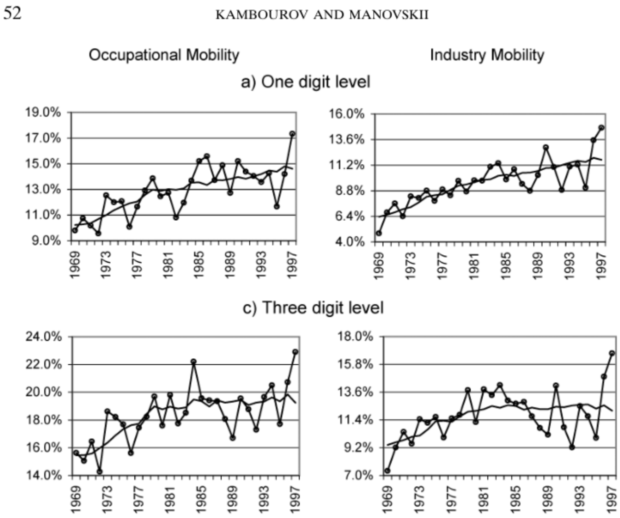
Migration Rates

Figure 2
Annual Internal Migration Rates



Source: Author's calculations based on Internal Revenue Service (IRS), Current Population Survey (CPS), and American Community Survey (ACS) data.
Notes: Current Population Survey and American Community Survey statistics are authors' calculations from microdata excluding residents of group quarters and imputed values of migration. IRS statistics are authors' calculations based on state-level and county-level flows. "MSA" is Metropolitan Statistical Area.

Inter-industry mobility



Summary Statistics

Table: Summary Statistics, C-Corp Sample

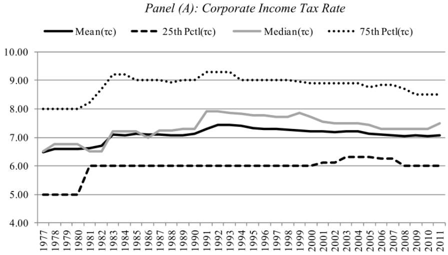
Variable	Mean	Std. Dev.
Corporate Tax Rate in Percent ($\tau_{s(j)t}$)	7.14	3.19
Change in Corporate Tax Rate	0.05	0.78
Total Pay At Firm (Thousands)	2148	19010
Total Employment At Firm	37.99	215.2
Wage Bill Share (s^{wn}_{ijt})	0.03	0.12
HHI - Wage Bill	0.10	0.16
Log Number of Firms per Market [exp(5.56)=259.8]	5.56	2.01
Log Total Employment ($\log n_{ijt}$) [exp(2.39)=10.9]	2.39	1.32
Log Wage ($\log w_{ijt}$) [exp(3.58)=$35k]	3.58	0.71
Observations		4,425,000

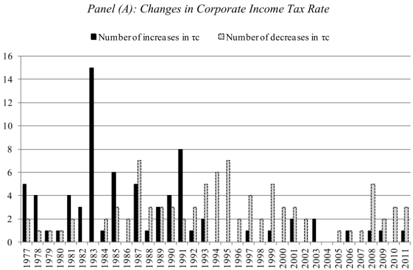
Notes: Tradeable C-Corps from 2002 to 2012.

Back to CCorp Layout

Summary Statistics

Reproduced from Giroud and Rauh (2011):





Back to CCorp Layout

Recovering the elasticities of substitution, η, and θ

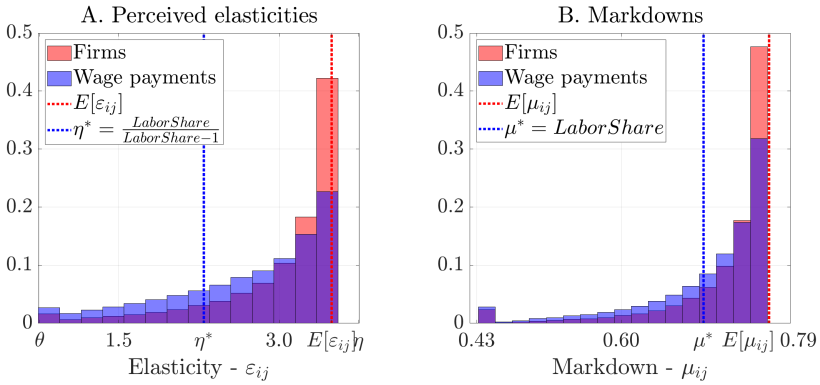
Table: Non-linear regression estimates of substitutability

	(1) Year t	(2) Year t+1
Within market substitutability, η	2.09	3.74
Across market substitutability, θ	0.31	0.76

Notes: We use an evenly spaced grid of labor shares on $[\underline{s}, \overline{s}] = [0.0025, 0.14]$ (within 1 standard deviation of the mean wage-bill share), in conjunction with the OLS regression equation for ϵ_{ij} to generate 56 tuples of labor supply elasticities and wage-bill shares, $\{\epsilon(s_{ijkt}), s_{ijkt}\}$ (one for every grid point). We then use these predicted values as data for $\{\epsilon_{ij}, s_{ij}\}$ to provide non-linear regression estimates of η and θ using the equation for $epsilon_{ij}$.

Back

Distribution of labor supply elasticities and markdowns

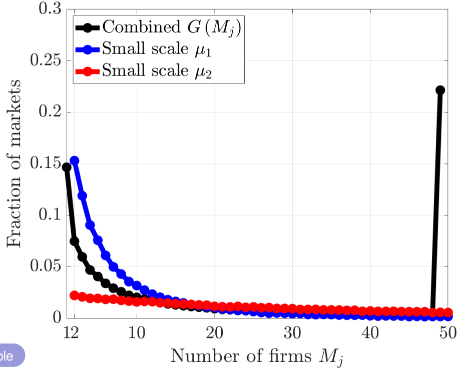


▸ Back - Share dependent labor supply elasticity

Calibration - Number of firms M_j

- 15% of markets have one firm $(M_j = 1)$
- Rest drawn from two Paretos, same shape γ, different scales μ_1, μ_2

Distribution of number of firms M_j	**Mean**	**Std.Dev.**	**Skewness**
Data (LBD, 2014)	51.6	264.9	29.9
Model	51.6	264.9	28.7



▸ Back - Calibration table

Calibration

Table: Estimated parameters

Par.	Description	Value	Targeted Moment	Model	Data
$\widetilde{\alpha}$	DRS parameter	0.984	Labor share	0.57	0.57
$\sigma_{\widetilde{z}}$	Log Normal Standard Deviation	0.391	$E(HHI_j^{wn})$ Payroll wtd.	0.14	0.14
$\widetilde{Z}$	Productivity shifter	23,570	Avg. wage per worker	\$ 65,773	\$ 65,773
$\bar{\varphi}$	Aggregate labor disutility shifter	6.904	Avg firm size	27.96	27.96

Labor share:

- To recover labor-share, we must take a stance on capital's share of income
- Assume $KS = .18$ as in Barkai (2018)

$$\widetilde{\alpha} = \frac{\alpha\gamma}{1-(1-\gamma)\alpha}$$

$$\widetilde{\alpha} = \frac{\alpha\gamma}{1-KS}$$

Back

1. Non-targeted concentration measures

Moment	Model	Data
A. Unweighted		
Wage-bill Herfindahl (unweighted)	0.35	0.45
Std. Dev. of Wage-bill Herfindahl (unweighted)	0.33	0.33
Skewness of Wage-bill Herfindahl (unweighted)	1.07	0.48
B. Weighted		
Wage-bill Herfindahl (weighted by market's share of total payroll)	0.14	0.14
Std. Dev. of Wage-bill Herfindahl (weighted by market's share of total payroll)	0.03	0.20
Skewness of Wage-bill Herfindahl (weighted by market's share of total payroll)	3.01	2.20
C. Correlations of Wage-bill Herfindahl		
Number of firms	-0.52	-0.21
Std. Dev. Of Relative Wages	-0.31	-0.51
Employment Herfindahl	1.00	0.98
Market Employment	-0.75	-0.21

- Model generates $2x$ difference between wtd. and unwtd. HHI^{wn}

Back

Pass-through - Details

Replicate patent experiment in Kline et al (2018):

- Same sample properties (firm size) & same average VA increase
- Model point estimate generated by randomly sampling 1% of firms in the benchmark oligopsonistic economy with size greater than 10 employees (delivers median size of 25.9 in the sample vs 25.2 in Kline et al (2018))
- Then increasing productivity by 30% (delivers 26% increase in y_{ij} / n_{ij} versus $\approx$ 20% in Kline et al (2018))
- Repeat this exercise 100 times.
- Average point estimate over 100 repetitions is reported.

Back - Pass-through

3. Size wage premium

- Replicate size-wage premium regressions in Bloom et al (2018)

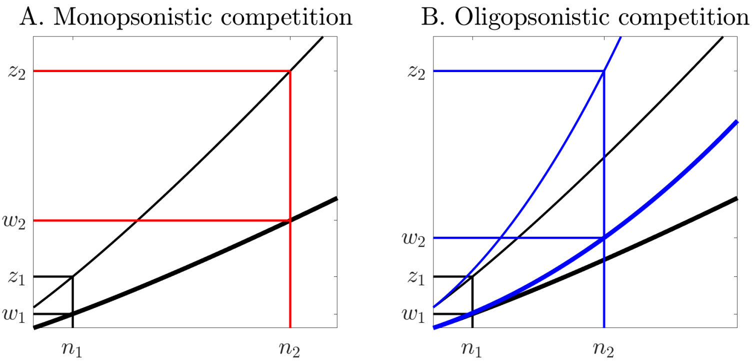
$$\log w_{ij} = \beta_0 + \beta_1 \log n_{ij} + \epsilon_{ij}$$

	Model (1)	**Bloom et al (2018), 1980** (2)	**Bloom et al (2018), 2013** (3)
Elasticity of wage WRT size	0.18	0.11	0.03
Dependent variable	$log(w_{ij})$	Log annual earnings	Log annual earnings
Independent variable	$log(n_{ij})$	Log firm employees	Log firm employees

- Model implies 10% larger firm pays 1.8% more

Back - Pass-through

Discussion - Pass-through



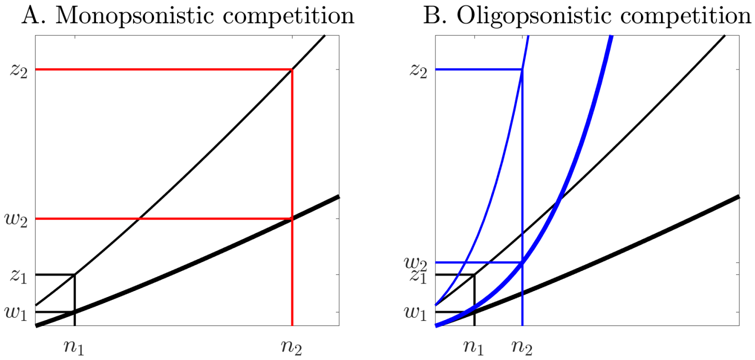
A. Increase in $w_{ij}n_{ij}$, Constant $\bar{\varepsilon}_{ij}$, Constant $\bar{\mu}_{ij} = w_{ij}/z_{ij}$

B. Increase in $w_{ij}n_{ij}$, Lower $\varepsilon(s_{ijt})$, Lower $\mu_{ij} = w_{ij}/z_{ij}$

Oligopolist understands that as wage share grows, labor supply elasticity falls $\varepsilon(s_{ijt}) = \uparrow s_{ijt}\theta + (1 - s_{ijt})\eta$

▸ Back - Discussion: Pass-through

Discussion - Pass-through



A. Increase in $w_{ij} n_{ij}$, Constant $\overline{\varepsilon}_{ij}$, Constant $\overline{\mu}_{ij} = w_{ij} / z_{ij}$

B. Increase in $w_{ij} n_{ij}$, Lower $\varepsilon(s_{ijt})$, Lower $\mu_{ij} = w_{ij} / z_{ij}$

Oligopolist understands that as wage share grows, labor supply elasticity falls $\varepsilon(s_{ijt}) = \uparrow s_{ijt}\theta + (1 - s_{ijt})\eta$

Back - Discussion: Pass-through

Discussion - Wage bill shares and MRPL

Identifying *MRPL*

$$\frac{s^{wn}_{ijt}}{s^{wn}_{ikt}} = \left(\frac{\mu\left(s^{wn}_{ijt}\right)}{\mu\left(s^{wn}_{ikt}\right)}\right)^{1+\eta}\left(\frac{MRPL_{ijt}}{MRPL_{ikt}}\right)^{1+\eta}$$

- Up to a normalization, $\{s^{wn}_{ijt}\}$ can be used to infer $\{MRPL_{ijt}\}$

Back - Equilibrium

Discussion - Wage bill shares and MRPL

Identifying *MRPL*

$$\frac{s^{wn}_{ijt}}{s^{wn}_{ikt}} = \left(\frac{\mu\left(s^{wn}_{ijt}\right)}{\mu\left(s^{wn}_{ikt}\right)}\right)^{1+\eta}\left(\frac{MRPL_{ijt}}{MRPL_{ikt}}\right)^{1+\eta}$$

- Up to a normalization, $\{s^{wn}_{ijt}\}$ can be used to infer $\{MRPL_{ijt}\}$

Implications for measurement in LBD

- Labor markets relatively easy to define
- Wage bill shares observed s^{wn}_{ijt}
- Construct wage bill Herfindahl indices $HHI_{jt} = \sum_i {s^{wn}_{ijt}}^2$
- Contrast with studies of competition in goods markets which do not have local measures of sales shares

  Autor, Dorn, Katz, Patterson, Van Reenen (2018), Phillipon Gutierrez (2018)

Back - Equilibrium

2. Pass-through - Corporate tax effects

After-tax profits with a corporate profit tax

$$
\begin{aligned}
\pi_{ij} &= \pi_{ij}^{Econ.} - \tau_C \pi_{ij}^{Acc.} \\
\pi_{ij}^{Econ.} &= z_{ij} n_{ij}^{\alpha} k_{ij}^{1-\alpha} - w_{ij} n_{ij} - r k_{ij} - \delta k_{ij} \\
\pi_{ij}^{Acc.} &= z_{ij} n_{ij}^{\alpha} k_{ij}^{1-\alpha} - w_{ij} n_{ij} - \lambda r k_{ij} - \delta k_{ij}
\end{aligned}
$$

- Can only write off fraction λ of capital financed by debt

Result

$$
\begin{aligned}
\pi_{ij} &= MRPL\left(z_{ij}, r, \tau_C\right) n_{ij} - w_{ij} n_{ij} \\
MRPL\left(z_{ij}, r, \tau_C\right) &= \frac{1}{1+\tau_C} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \left(\frac{\widetilde{z}_{ij}}{\widetilde{r}}\right)^{\frac{1-\alpha}{\alpha}} \widetilde{z}_{ij} \\
\widetilde{z} &= (1-\tau_C) z_{ij} \\
\widetilde{r} &= (1+\lambda\tau_C) r + (1+\tau_C)\delta
\end{aligned}
$$

▸ Back to corporate tax exercise. ▸ Calibration overview

Perceived vs. uncompensated labor supply elasticity

- Perceived (ε_{ij}):

$$\frac{\partial n_{ij}}{\partial w_{ij}} \frac{w_{ij}}{n_{ij}} = \eta + (\theta - \eta) s_{ij}^{wn}$$

- Uncompensated (Marshallian):

$$\frac{\partial n_{ij}}{\partial w_{ij}} \frac{w_{ij}}{n_{ij}} = \eta + (\theta - \eta) s_{ij}^{wn} + (\psi - \theta) \frac{w_{ij}}{\mathbf{W}} \frac{\partial \mathbf{W}}{\partial w_{ij}}$$

- Perceived $\approx$ uncompensated if $\frac{\partial \mathbf{W}}{\partial w_{ij}} \approx 0$.

- Our perceived elasticity ranges from $\theta = 0.76$ to $\eta = 3.74$. Estimates range from 0.1 to 3, depending on gender, country, and variation used (Evers, Mooij, van Vuuren, 2008).

▸ Back - Labor supply elasticity

Product market discussion

- Labor market power μ_{ijt} identified from product market power in tradeable goods market (the focus of our paper)
- Tradeable goods prices that are set non-competitively by a firm enter the marginal revenue product, $MRPL_{ijt}$
- $MRPL_{ijt}$ is distinct from what we call the labor market markdown
- We recover μ_{ijt} by comparing local labor market responses to corporate tax changes *within* a NAICS3 code
- If tradeable good prices (e.g. furniture prices) do not differ across local labor markets within a state, our estimate of μ_{ijt} only captures labor market power.

▸ Back

Evidence of upward sloping labor supply curves

Generation 1: Exogenous variation in employment demand or wages

- Staiger, Spetz, Phibbs (2010): mandated pay changes in registered nurse market (from national payscale to local)
  - LS elast of .1
- Ashenfelter, Farber, Ransom (2010) provide summary

Generation 2: Vacancy applications and wages

- Banfi and Villena-Roldan (2018): controlling for firm size, job title, and all available observables, higher wage offering attracts more workers
- Belot, M., P. Kircher, and P. Muller (2015): in actual UI sponsored job search office, post fake vacancies with higher wages, those vacancies draw more job searchers

Strong evidence for upward sloping LS curve faced by individual firms, conditional on size

back

Is monopsony power generated by outside options

Theory: Zhu (2011) provides framework with N firms, agents understand outside option is to match with remaining $N-1$ firms

- Wages (asset prices) fall if bargaining breaks down

Empirics: Does outside option affect wages?

- Jager, Schoefer, Young, Zweimüller (2018): outside options not strong determinant of wages
  - Four large reforms of UI in Austria.
  - Wage response less than 1 cent per 1.00 dollar UI increase
  - Nash-bargaining implies 39 cent per 1.00 dollar UI increase in calibrated model
- Hagedorn, Karahan, Manovskii, Mitman (2014): important determinants
  - County border-pair identification strategy

back

Welfare - Output decomposition - Details

- Compute counterfactual output with scale effect only:

$$n_{ij}^{s} = n_{ij} \frac{\int \sum n_{ij}^{c} \, dj}{\int \sum n_{ij} \, dj}$$

$$y_{ij}^{s} = \widetilde{z}_{ij} \widetilde{Z} \left(n_{ij}^{s} \right)^{\widetilde{\alpha}}$$

- We then compute the share of gains due to reallocation:

$$\frac{\frac{\int \sum y_{ij}^{c} \, dj}{\int \sum y_{ij} \, dj} - \frac{\int \sum y_{ij}^{s} \, dj}{\int \sum y_{ij} \, dj}}{\frac{\int \sum y_{ij}^{c} \, dj}{\int \sum y_{ij} \, dj} - 1}$$

- Share output gains due to reallocation: 26%
- Share output gains due to scale: 74%

Back - Welfare - Output decomposition

Minimum wage - Appendix

Household - Additional constraint: Labor supply less than labor demand:

$$n_{ijt} \leq \underline{n}_{ijt}$$

- Define $\lambda_t \nu_{ijt}$ as associated multiplier
- λ_t is the multiplier on the budget constraint
- ν_{ijt} is marginal utility of sending a worker to firm with a binding $w_{ij} = \underline{w}$
- $\widetilde{w}_{ijt} = w_{ijt} - \nu_{ijt}$ is the *perceived wage*

Firm - Problem as before with added constraint:

$$w_{ijt} = \begin{cases} \overline{\varphi}^{-\frac{1}{\varphi}} \mathbf{N}_t^{\frac{1}{\varphi}} \left(\frac{\mathbf{N}_{jt}}{\mathbf{N}_t}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijt}}{\mathbf{N}_{jt}}\right)^{\frac{1}{\eta}} & , \quad \text{if} \quad n_{ijt} > \underline{n}_{ijt} \\ \underline{w} & , \quad \textit{otherwise} \end{cases}$$

Result - Equilibrium can be solved in perceived wages $\widetilde{w}_{ijt}$

Back

Minimum wage - Appendix

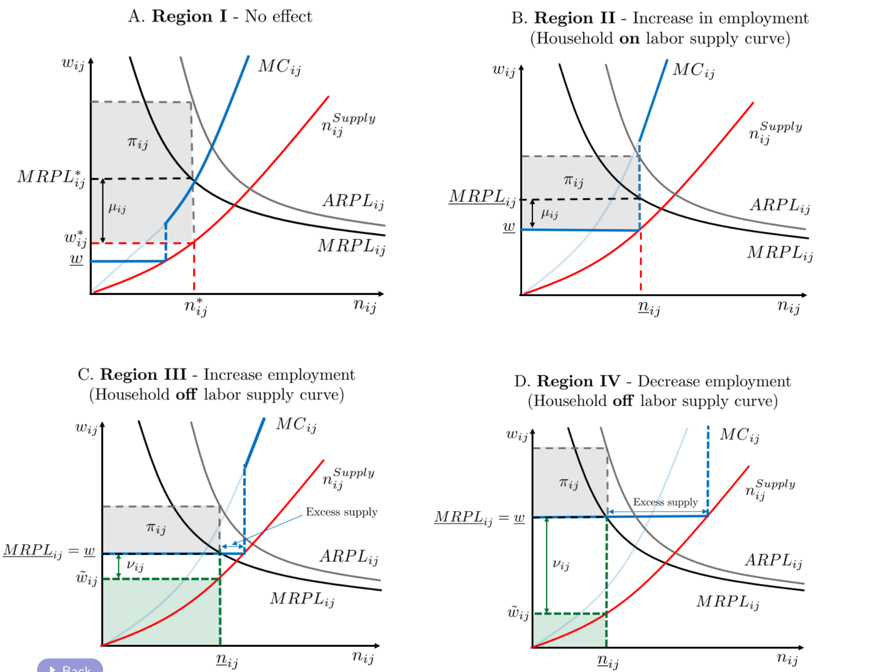
Define the *perceived* wage-bill share:

$$\widetilde{s}_{ijt} = \frac{(w_{ijt} - \nu_{ijt})n_{ijt}}{\sum_{i\in j}(w_{ijt} - \nu_{ijt})n_{ijt}}$$

Define the *perceived* sectoral and aggregate wage indexes:

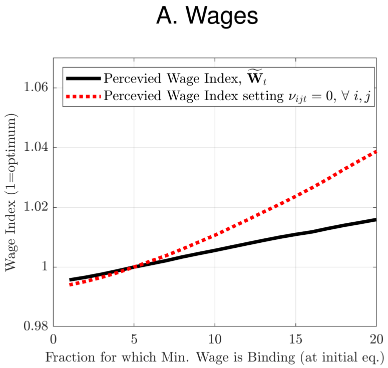
$$\widetilde{\mathbf{W}}_{jt} := \left[\sum_{i\in j}\Big(w_{ijt} - \nu_{ijt}\Big)^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{\mathbf{W}}_{t} := \left[\int \widetilde{\mathbf{W}}_{jt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}} .$$

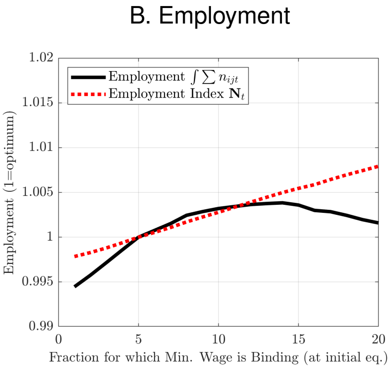
▸ Back


A. Region I - No effect
w_{ij}
MC_{ij}
n_{ij}^{Supply}
π_{ij}
$MRPL^*_{ij}$
μ_{ij}
$ARPL_{ij}$
w^*_{ij}
$\underline{w}$
$MRPL_{ij}$
n^*_{ij}
n_{ij}
B. Region II - Increase in employment
(Household on labor supply curve)
w_{ij}
MC_{ij}
n_{ij}^{Supply}
π_{ij}
$MRPL_{ij}$
μ_{ij}
$\underline{w}$
$ARPL_{ij}$
$MRPL_{ij}$
$\underline{n}_{ij}$
n_{ij}
C. Region III - Increase employment
(Household off labor supply curve)
w_{ij}
MC_{ij}
n_{ij}^{Supply}
Excess supply
π_{ij}
$\underline{MRPL}_{ij} = \underline{w}$
ν_{ij}
$\tilde{w}_{ij}$
$ARPL_{ij}$
$MRPL_{ij}$
$\underline{n}_{ij}$
n_{ij}
D. Region IV - Decrease employment
(Household off labor supply curve)
w_{ij}
MC_{ij}
π_{ij}
n_{ij}^{Supply}
Excess supply
$\underline{MRPL}_{ij} = \underline{w}$
ν_{ij}
$ARPL_{ij}$
$MRPL_{ij}$
$\tilde{w}_{ij}$
$\underline{n}_{ij}$
n_{ij}

Back

Minimum wage - Scale effects





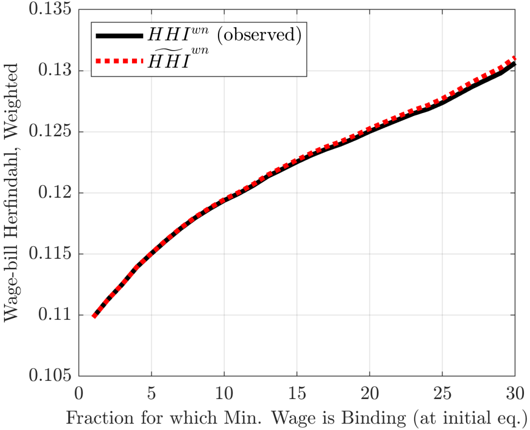
(i) Perceived wages, which determine n_{ij}, do not increase as much

(ii) Small firms shrink, (enter Region IV), employment falls

(iii) HHI monotonically increases, implying falling labor share

▸ Back

Minimum wage - Concentration



- E.g. Would imply decline in labor share of 2 ppt over this range

Back

Minimum wage - Appendix

- Initialize the algorithm by (i) guessing a value for $\widetilde{\mathbf{W}}_t^{(0)}$, (ii) assuming all firms are in *Region I*, which implies guessing $\nu_{ijt}^{(0)} = 0$. These will all be updated in the algorithm.

1. Solve the sectoral equilibrium:
   1.1 Guess perceived shares $\widetilde{s}_{ijt}^{(0)}$.
   1.2 In *Region I*, where minimum wage does not bind, solve for the firm's wage as before, except with the perceived aggregate wage index $\widetilde{\mathbf{W}}_t$ instead of $\mathbf{W}_t$:

$$w_{ijt} = \left[\omega\mu\left(\widetilde{s}_{ijt}\right) \widetilde{\mathbf{W}}_t^{(1-\widetilde{\alpha})(\theta-\varphi)} \widetilde{z}_{ijt} \widetilde{s}_{ijt}^{(l)-\frac{(1-\widetilde{\alpha})(\eta-\theta)}{\eta+1}} \right]^{\frac{1}{1+(1-\widetilde{\alpha})\theta}}$$

   1.3 In all other regions *Region II, III, IV*, set $w_{ijt} = \underline{w}$.
   1.4 Compute perceived wages using the guess $\nu_{ijt}^{(k)}$: $\widetilde{w}_{ijt} = w_{ijt} - \nu_{ijt}^{(k)}$
   1.5 Update shares using $\widetilde{w}_{ijt}$:

$$\widetilde{s}_{ijt}^{(l+1)} = \frac{\widetilde{w}_{ijt}^{1+\eta}}{\sum_{i\in j} \widetilde{w}_{ijt}^{1+\eta}} \qquad \left(:= \frac{\widetilde{w}_{ijt} n_{ijt}}{\sum_{i\in j} \widetilde{w}_{ijt} n_{ijt}} = \frac{\widetilde{w}_{ijt}\overline{\varphi}\left(\frac{\widetilde{w}_{ijt}}{\widetilde{\mathbf{W}}_{jt}}\right)^{\eta}\left(\frac{\widetilde{\mathbf{W}}_{jt}}{\widetilde{\mathbf{W}}_t}\right)^{\theta}\widetilde{\mathbf{W}}_t^{\varphi}}{\sum_{i\in j} \widetilde{w}_{ijt}\overline{\varphi}\left(\frac{\widetilde{w}_{ijt}}{\widetilde{\mathbf{W}}_{jt}}\right)^{\eta}\left(\frac{\widetilde{\mathbf{W}}_{jt}}{\widetilde{\mathbf{W}}_t}\right)^{\theta}\widetilde{\mathbf{W}}_t^{\varphi}} \right)$$

   1.6 Iterate over (b)-(e) until $\widetilde{s}_{ijt}^{(l+1)} = \widetilde{s}_{ijt}^{(l)}$.

1. Recover employment n_{ijt} according to the current guess of firm region. First use $\widetilde{w}_{ijt}$ to compute $\widetilde{\mathbf{W}}_{jt}$, $\widetilde{\mathbf{W}}_t$. Then by region:
   (I) Firm is unconstrained:
$$n_{ijt} = \overline{\varphi}\left(\frac{w_{ijt}}{\widetilde{\mathbf{W}}_{jt}}\right)^{\eta}\left(\frac{\widetilde{\mathbf{W}}_{jt}}{\widetilde{\mathbf{W}}_t}\right)^{\theta}\widetilde{\mathbf{W}}_t^{\varphi}$$
   (II) Firm is constrained and employment is determined by the household labor supply curve at $\underline{w}$:
$$n_{ijt} = \overline{\varphi}\left(\frac{\underline{w}}{\widetilde{\mathbf{W}}_{jt}}\right)^{\eta}\left(\frac{\widetilde{\mathbf{W}}_{jt}}{\widetilde{\mathbf{W}}_t}\right)^{\theta}\widetilde{\mathbf{W}}_t^{\varphi}$$
   (III),(IV) Firm is constrained and employment is determined by firm $MRPL_{ij}$ curve at $\underline{w}$:
$$n_{ijt} = \left(\frac{\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ijt}}{\underline{w}}\right)^{\frac{1}{1-\widetilde{\alpha}}}$$
2. Update $\nu_{ijt}^{(k)}$:
   2.1 Use n_{ijt} to compute $\mathbf{N}_{jt}$, $\mathbf{N}_t$.
   2.2 Update ν_{ijt} from the *household's* first order conditions:
$$\nu_{ijt}^{(k+1)} = w_{ijt} - \overline{\varphi}^{-\frac{1}{\varphi}}\left(\frac{n_{ijt}}{\mathbf{N}_{jt}}\right)^{\frac{1}{\eta}}\left(\frac{\mathbf{N}_{jt}}{\mathbf{N}_t}\right)^{\frac{1}{\theta}}\mathbf{N}_t^{\frac{1}{\varphi}}$$
3. Update $\widetilde{\mathbf{W}}_t^{(k)}$:
   3.1 Compute $\widetilde{w}_{ijt} = w_{ijt} - \nu_{ijt}^{(k+1)}$
   3.2 Use $\widetilde{w}_{ijt}$ to update the aggregate wage index to $\widetilde{\mathbf{W}}_t^{(k+1)}$.
4. Update firm regions:
   4.1 Compute profits for all firms: $\pi_{ijt} = \widetilde{Z}\widetilde{z}_{ijt}n_{ijt}^{\widetilde{\alpha}} - \underline{w}n_{ijt}$.
   4.2 If in sector j there exists a firm with $w_{ijt} < \overline{w}$, then move the firm with the lowest wage into *Region II*.
   4.3 If in sector j there exists a firm that was initially in *Region II* and has negative profits $\pi_{ijt} < 0$, move that firm into *Region III*.[1]
5. Iterate over (1) to (5) until $\nu_{ijt}^{(k+1)} = \nu_{ijt}^{(k)}$ and $\widetilde{\mathbf{W}}_t^{(k+1)} = \widetilde{\mathbf{W}}_t^{(k)}$.

▸ Back