Notes on "Profits and Inflation in the Time of COVID"*

Andreas Hornstein
Federal Reserve Bank of Richmond

November 2, 2023

Contents

Abstract

This note provides background for the Economic Brief "Profits and Inflation in the Time of COVID."

*The views expressed herein are my own and do not necessarily represent the views of the Federal Reserve Bank of Richmond or the Federal Reserve System. I would like to thank David Ramachandran for research assistance. Andreas Hornstein: andreas.hornstein@rich.frb.org

1 Income accounts for the non-financial corporate business sector

Table 1 displays the income side composition of gross value added (GVA) in the non-financial corporate business (NFCB) sector, NIPA Table 1.14. Table 2 displays the role of depreciation for the distinction between gross and net value added.

GVA	Gross value added	T^{net}	Taxes on production and imports less subsidies
		W	Compensation of employees, paid
		D	Consumption of fixed capital, i.e., depreciation
		I^{net}	Net interest and miscellaneous payments
		Tra	Business current transfer payment, net
		T^{inc}	Taxes on corporate income
		P	Profits after tax with inventory valuation adjustment (IVA) and capital consumption adjustment (CCAdj)

Table 1: Income Components of GVA in NFCB Sector

GVA	$=T^{net} + W +$ GOS	Gross value added
GOS	=NOS + D	Gross operating surplus
NVA	=GVA - D	Net value added
NVA	$=T^{net} + W +$ NOS	
NOS	$= I^{net} +$ Tra + Before Tax Profit	Net operating surplus
P^{gross}	$= T^{inc} + P$	Profits with IVA & CCAdj

Table 2: Gross and Net Value Added in NFCB

Most of the work accounting for changes in the NFCB sector's GVA deflator starts with the income side and uses the identity, GVA = T^{net} + W + GOS, and calls the GOS "profits." For our purposes, redefine this expression as

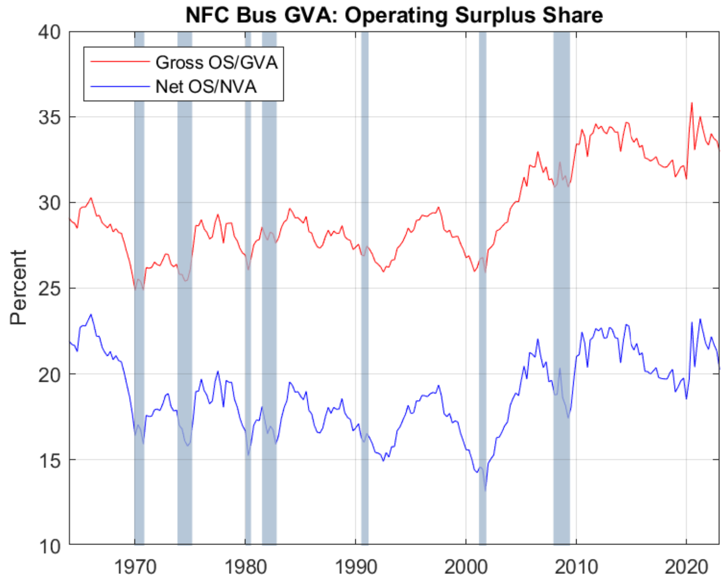
$$Y = T + W + \Pi \tag{1}$$

where Y is GVA, Π denotes GOS, and all variables are in nominal terms.

Figure 1 [Figure 1 in EB] plots the operating surplus share in value-added on a gross respectively net basis. For the period 1965 to 2022, over the business cycle surplus shares tend to decline prior to recessions and tend to increase in the early expansion phases of the cycle. Prior to 2000, the average surplus shares were relatively stable, more so for GOS than for NOS, but average surplus shares have increased since. Furthermore, the surplus shares spiked in 2020 during the Covid pandemic, but have since declined.

The quantity index for GVA in the NFCB sector is constructed from the production side. That is, it is based on the goods produced by the NFCB sector net of the intermediate goods purchased. Let y denote

Figure 1: NFCB: Value Added Share of Operating Surplus



this quantity index, that is, real GVA, and the price index is $p = Y/y$ or

$$p = \frac{T}{y} + \frac{W}{y} + \frac{\Pi}{y} = \tau + w + \pi \tag{2}$$

where lower case letters denote the unit cost components of the GVA price index. We can write the change in the price index as an income-share-weighted average of the changes in the unit cost components

$$\hat{p} = \frac{\tau}{p}\hat{\tau} + \frac{w}{p}\hat{w} + \frac{\pi}{p}\hat{\pi} \tag{3}$$

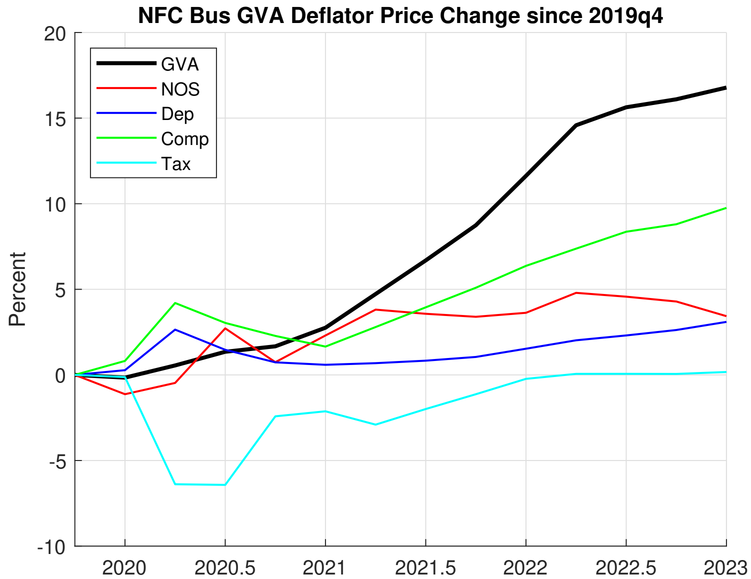
$$= \frac{T}{Y}\hat{\tau} + \frac{W}{Y}\hat{w} + \frac{\Pi}{Y}\hat{\pi} \tag{4}$$

where $\hat{x} = \Delta x/x$ denotes the rate of change for a generic variable x.

Figure 2 [Figure 3 in EB] plots the cumulative percentage changes of the unit cost components of the GVA price index starting in the first quarter of 2020. We can see that changes in labor compensation and GOS, the sum of NOS and Depreciation, contributed about equally to the increase in the GVA price index during the pandemic. In fact, during the first half of 2021 when inflation started to pick up, an increasing NOS made a substantial contribution to inflation. But after the first half of 2021, changes in the NOS no longer contributed to inflation.

It is true that the contribution of GOS (NOS) changes to inflation during the pandemic is large, relative to the pre-pandemic averages. The average contribution of changes in the GOS (NOS) to changes in NFCB inflation from 1965-2019 is about 16% (2%), see Table 3, much less than during the pandemic when it is

Figure 2: Cumulative Change of NFCB Price Index Since 2019q4
Contributions from NOS, Labor Compensation, Depreciation, and Net Production Taxes



close to half (a quarter), see Figure 2.

Income component	Contribution
Business Tax net of Subsidies	3%
Labor Compensation	81%
Depreciation	14%
NOS	2%

Table 3: Average contribution to GVA YoY inflation in NFCB, 1965-2019

2 Measuring markups

This section derives estimates for short-run variations in markups based on Nekarda and Ramey (2020).

Assume that a firm produces an output, y, using a vector of inputs, x, to a constant-returns-to-scale (CRS) production function, $y = f(x; z)$, where z denotes an exogenous productivity shifter. Also, assume that the firm is competitive in its input markets and takes the vector of input prices p_x as given. The cost minimization problem of the firm is

$$C(p_x, y; z) \equiv \min_x p_x x \text{ s.t. } f(x; z) \geq y \tag{5}$$

The first order conditions for the i-th input the solution to the cost minimization problem are

$$p_{x,i} = \lambda \partial f / \partial x_i \tag{6}$$

where λ is the Lagrange multiplier on the constraint. From the envelope theorem, it follows that λ is also the marginal cost of output, $c = \partial C / \partial y$. Furthermore, since production is CRS, the cost function is linear in output

$$C(p_x, y; z) = c(p_x; z) y \tag{7}$$

Note also that we can rewrite equation (6) as

$$c = \frac{p_{x,i}}{\partial f / \partial x_i} \tag{8}$$

and the marginal cost of output is equated across all inputs.

A competitive firm without market power would take the price of its output, p, as given and would operate with marginal cost equal to price, $p = c$. If the firm has market power, it sets the price for its product as a markup, $\mu \geq 1$, over the marginal cost of production,

$$p = \mu\, c. \tag{9}$$

Substituting for marginal cost from equation (8), we get

$$p = \mu \frac{p_{x,i}}{\partial f / \partial x_i} = \mu \frac{p_{x,i} x_i / y}{f_{x_i} x_i / f} \tag{10}$$

and we can solve the expression for the markup

$$\mu = \frac{\eta_i}{s_i} \tag{11}$$

where $\eta_i = f_i x_i / f$ is the output elasticity and $s_i = p_{x,i} x_i / py$ is the revenue share of the i-th input. Note that this relation holds for any variable input to production. We can write the change in the markup as

$$\hat{\mu} = \hat{\eta}_i - \hat{s}_i \tag{12}$$

where $\hat{x} = \Delta x / x$ denotes the rate of change for a generic variable x.

In the following, we will apply our expression for changes in the markup to GVA in the NFCB sector. Even though we study a subsector of the economy, we use concepts related to aggregate production functions. In particular we assume that real GVA, y, is produced using two inputs, labor ℓ, and capital, k, and productivity z is assumed to be labor-augmenting

$$y = f(k, z\ell) \tag{13}$$

For most of the analysis, labor is assumed to be a variable input, whereas changes to capital may be subject to adjustment costs. Since labor is assumed to be a variable input, we can use expression (12) for labor to construct changes in markups.

Constant output elasticity We can observe the revenue share of labor, but we cannot directly observe the output elasticity. But for Cobb-Douglas production functions

$$y = \prod_{i=1}^{n} x_i^{\alpha_i} \text{ with } \sum_{i=1}^{n} \alpha_i = 1. \tag{14}$$

the output elasticity of each input α_i is constant. Furthermore, if a firm with a CD production firm behaves competitively, that is, its markup is one, then from equation (11) the output elasticity equals the revenue share. Assuming that the production function is CD, the rate of change in markups depends only on the observable rate of change in the labor revenue share

$$\hat{\mu} = -\hat{s}_\ell \tag{15}$$

Overhead labor We may be concerned that not all of employment represents a variable input to production. For example, a fixed amount of overhead labor, $\bar{\ell}$, may be necessary to operate the firm, and only labor in excess of overhead labor, $\tilde{\ell} = \ell - \bar{\ell}$, generates positive output. Assuming that all labor gets paid the same wage, we rewrite the optimal price setting equation (10) as

$$p = \mu \frac{w\tilde{\ell}/y}{\eta_{\tilde{\ell}}} = \mu \frac{(w\ell/y)(\tilde{\ell}/\ell)}{\eta_{\tilde{\ell}}} \Rightarrow \mu = \frac{\eta_{\tilde{\ell}}}{s_\ell \psi} \tag{16}$$

where $\psi = \tilde{\ell}/\ell$ denotes the share of variable labor in total labor input. Again, assuming a constant output elasticity for the variable labor input, we get

$$\hat{\mu} = -\hat{s}_\ell - \hat{\psi} \tag{17}$$

CES production functions Maybe the assumption of a constant output elasticity for labor is too strong. CES production functions represent a standard deviation from CD production functions. CES stands for constant elasticity of substitution between any pairwise input to the production function. A CES production function is of the form

$$y = \left[\sum_{i=1}^{n} \alpha_i x_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \text{ with } \alpha_i, \sigma > 0 \tag{18}$$

where σ is the substitution elasticity between any two inputs. Consider a cost-minimizing firm that uses such a CES production function and takes input prices $p_{x,i}$ as given. Then the FOCs for the optimal input choices are

$$p_{x,i} = \lambda \alpha_i \left(\frac{x_i}{y} \right)^{-1/\sigma} \tag{19}$$

where λ is the Lagrange multiplier on the production constraint, which in turn equals marginal cost. Taking the ratio of the FOCs for two different inputs, we get

$$\frac{\alpha_i}{\alpha_j} \left(\frac{x_i}{x_j} \right)^{-1/\sigma} = \frac{p_{x,i}}{p_{x,j}} \tag{20}$$

Then the rate of change at which a firm varies the relative use of the inputs when their relative price changes is

$$\widehat{\left[\frac{x_i}{x_j} \right]} = -\sigma \widehat{\left[\frac{p_{x,i}}{p_{x,j}} \right]} \tag{21}$$

and, as noted before, σ denotes the substitution elasticity between the two inputs. The limiting case for $\sigma = 1$ is the CD production function where the percentage decline of the relative input use exactly compensates for the relative price increase and the relative shares remain constant.

For our analysis of aggregate markups of GVA, we need only consider two inputs, labor, ℓ, and capital, k. For this purpose, we write the CES production function as

$$y = \left[\alpha_k k^{(\sigma-1)/\sigma} + \alpha_\ell \left(z\ell \right)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{22}$$

where we allow for labor-augmenting technical change z. The output elasticities for the CES function are

$$\eta_\ell = \frac{\partial y}{\partial \ell} \frac{\ell}{y} = \alpha_\ell \left(\frac{z\ell}{y} \right)^{1-1/\sigma} \tag{23}$$

$$\eta_k = \frac{\partial y}{\partial k} \frac{k}{y} = \alpha_k \left(\frac{k}{y} \right)^{1-1/\sigma} \tag{24}$$

Since the CES function is CRS, the output elasticities sum to one

$$1 = \frac{\partial y}{\partial \ell} \frac{\ell}{y} + \frac{\partial y}{\partial k} \frac{k}{y} \tag{25}$$

and we can rewrite the labor elasticity as

$$\eta_\ell^k = \frac{\partial y}{\partial \ell} \frac{\ell}{y} = 1 - \alpha_k \left(\frac{k}{y} \right)^{1-1/\sigma} \tag{26}$$

We can write the change in the labor elasticity as

$$\hat{\eta}_\ell = (1 - 1/\sigma)\left(\hat{z} + \hat{\ell} - \hat{y}\right) \tag{27}$$

for the direct measure, and

$$\hat{\eta}_\ell^k = \frac{-\eta_k}{1-\eta_k}(1 - 1/\sigma)\left(\hat{k} - \hat{y}\right) \tag{28}$$

for the indirect measure. Finally, the change in the markup is

$$\hat{\mu} = \hat{\eta}_\ell - \hat{s}_\ell \text{ respectively } \hat{\mu} = \hat{\eta}_\ell^k - \hat{s}_\ell \tag{29}$$

3 Implementing markup estimates

Baseline markup with(out) overhead adjustment We obtain the baseline markup from the labor revenue share in the NFCB, s_ℓ. For the overhead adjustment, we use the share of production and non-supervisory workers in total payroll employment of non-farm establishments. This sector is broader than NFCB.

In order to calculate variable output elasticities, we need to adjust labor input for productivity changes. But in the presence of markups, productivity changes cannot be measured independently of the level of markups. We consider two options, adjusting TFP measurement for markups estimates from Hall (2018) or for profit margin estimates from Barkai (2020). We choose the latter since the implied markups seems to be more reasonable for the the NFCB sector.

Markups and TFP Solow growth accounting, the standard method to estimate TFP growth, does not require knowledge of the functional form of the production function, but it assumes competitive behavior on input and output markets. Assume that production is $y = f(x_{-\ell}, zx_\ell)$, where $x_{-\ell}$ denotes all inputs but labor, and assume that TFP is labor augmenting. Then the change in output can be written as

$$\hat{y} = \sum_i \eta_i \hat{x}_i + \eta_\ell \hat{z} \tag{30}$$

For a competitive firm that takes input and output prices as given, there is no markup, $\mu = 1$, the output elasticities are equated with the revenue shares, $\eta_i = s_i$, and we can solve for TFP growth in terms of observables

$$\hat{y} = \sum_i s_i \hat{x}_i + s_\ell \hat{z} \tag{31}$$

When firms have market power in their product market, the revenue shares have to be adjusted for the markup, equation (11), and the growth expression becomes

$$\hat{y} = \mu \sum_i s_i \hat{x}_i + \mu s_\ell \hat{z} \tag{32}$$

We can rewrite this expression as

$$\hat{y} = \sum_i s_i \hat{x}_i + \left(1 - \frac{1}{\mu}\right) \hat{y} + s_\ell \hat{z} \tag{33}$$

Hall (2018) estimates the time trend in the Lerner index $\mathcal{L}_t = 1 - 1/\mu_t$ for 60 US industries

$$\mathcal{L}_t = \phi + \psi t \tag{34}$$

and finds that the aggregate Lerner index grew from 0.11 in 1988 to 0.28 in 2015. This corresponds to an increase in markups from 1.12 to 1.39.

Note that in the absence of profits we can define the revenue share of capital as a residual, $s_k = 1 - s_\ell$. But in the presence of profits, we have to assume that this residual contains both profits and capital compensation. Assuming that production is constant returns to scale and that there are no fixed costs, we get the relation

$$1 = \pi + s_k + s_\ell \text{ where } \pi \equiv \frac{(p-c)y}{py} = 1 - \frac{1}{\mu}. \tag{35}$$

Conditional on the markup, we can solve for the capital revenue share

$$s_k = \frac{1}{\mu} - s_\ell. \tag{36}$$

If we calculate TFP paths conditional on an assumed markup, path we have to take into account this correction of capital revenue shares.

Alternatively, Barkai (2020) calculates capital revenue shares directly. For this purpose, he calculates an implicit user cost of capital that serves as the price of capital services. Pure profit rates are then the residual after subtracting the observed labor compensation share and the implicit capital payments share from 1. Barkai (2020) argues that profit shares in GVA of the NFCB sector were increasing from slightly negative in the mid-1980s to about 8% in 2014 with a peak of 12% in 2006-07. One can then use equation (35) to obtain GVA markups between 1.08 and 1.12 for the estimated GVA profit rates. These values are similar to Hall's lower bound estimates for gross output markups.

Markups on gross output and gross value added Hall (2018) estimates markups on gross output, but our exercise involves markups on GVA. Basu (2019) notes that markups on GVA tend to be a multiple of the underlying markups on gross output. In fact, given an observed intermediate input share of about 50%

in most industries, an estimated gross output markup of 1.3 translates to a GVA markup of 1.85. Markups like this have implications for scale elasticities or revenue shares that don't seem credible.

Suppose we have a CRS gross output production function, $y = G(v, m)$, that is separable into a valued added aggregate v and an intermediate input aggregate m, associated cost of value added, c_v, and intermediate price p_m. The firm is assumed to be competitive in its input markets but sets the gross output price as a markup μ_y over marginal cost. The corresponding FOCs for inputs are

$$p_y = \mu_y \frac{p_m}{G_m} = \mu_y \frac{c_v}{G_v} \tag{37}$$

We can rewrite the FOC for intermediate inputs as

$$p_y y = \mu_y \frac{p_m m}{G_m m/y} \Rightarrow \eta_m = \mu_y s_m \text{ and } \eta_v = 1 - \mu_y s_m, \tag{38}$$

since production is CRS. We can rewrite the joint FOCs as

$$\frac{p_m m}{G_m m/y} = \frac{c_v v}{G_v v/y} \Rightarrow \frac{p_m m}{c_v v} = \frac{\eta_m}{\eta_v} \tag{39}$$

By construction, the GVA price is

$$\begin{aligned}
p_v &= \frac{p_y y - p_m m}{v} \\
&= \frac{p_y y - p_m m}{p_m m} \frac{p_m m}{c_v v} c_v \\
&= \left(\frac{1}{s_m} - 1 \right) \frac{\eta_m}{\eta_v} c_v \\
&= \left(\frac{1}{s_m} - 1 \right) \frac{\mu_y s_m}{1 - \mu_y s_m} c_v
\end{aligned}$$

and the GVA markup is a multiple of the gross output markup

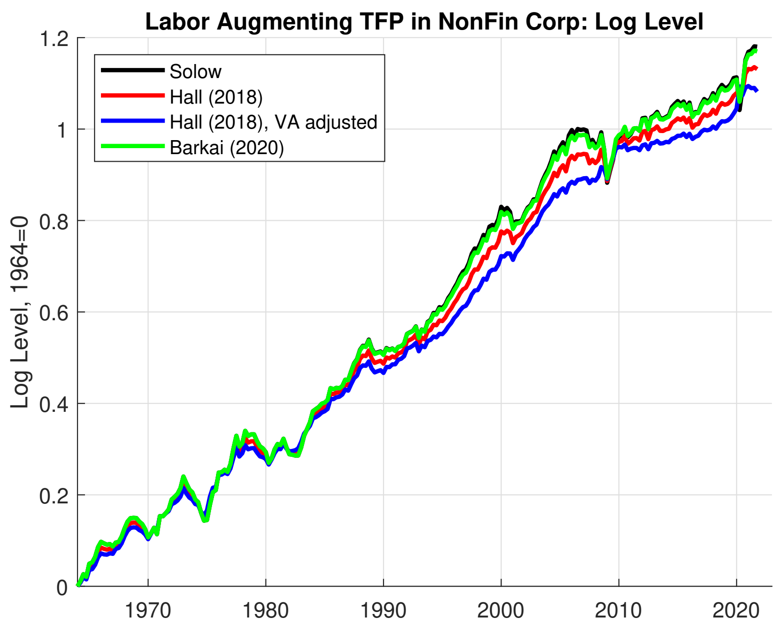
$$p_v = \mu_y \frac{1 - s_m}{1 - \mu_y s_m} c_v = \mu_v c_v \tag{40}$$

At an intermediate input share of $s_m = 0.5$, a gross output markup of 1.1 (1.4) translates to a GVA markup of 1.2 (2.3).

TFP conditional on markups We now calculate measures of labor augmenting technical change for the NFCB conditional on four assumed paths for markups. The first measure is based on the standard Solow residual and uses equation (31) assuming no markups, $\mu = 1$. The second measure is based on the markup augmented Solow residual and uses equation (33) together with Hall's estimate for the markup trend assuming that $\mathcal{L}$ remains unchanged after 2015. This seems conservative since wage shares were actually slightly increasing after 2015. That is, markups were likely to decline. The third measure recognizes that Hall estimates markups for gross output and transforms the estimated gross output markups to GVA markups assuming an intermediate input share of 0.5. The fourth measure uses the markup path implied by the GVA profit rates from Barkai (2020).

Comparing equations (31) and (33), we can see that a positive markup will lower the implied TFP growth rate and the level of TFP relative to the standard Solow case with no markup. We construct the level of TFP by taking the exponential of the cumulative sum of estimated growth rates. We plot the implied log levels of TFP in figure 3. Note that all measures indicate a productivity growth slowdown post-2005 that is more pronounced the higher the assumed markup path is. In the following, we use the markup path implied by Barkai (2020)'s profit rates. It is the one most consistent with the output concept of the NFCB sector.

Figure 3: TFP and Markup Corrections



Labor elasticity for CES production, direct The direct measure of the labor elasticity (23) requires information on real output and labor input, the level of labor augmenting technical change, the CES coefficient on labor, and the substitution elasticity. We follow Nekarda and Ramey (2020) and set the substitution elasticity $\sigma = 0.5$. Rather than calculating the labor elasticity from equation (23) as done in Nekarda and Ramey (2020), we directly proceed to the first order approximation for changes in the labor elasticity, equation (27). Note that either way we require information on labor-augmenting productivity changes. We proceed from equation (30), respectively equation (32), and substitute for the capital revenue share from equation (36),

$$\begin{aligned}\hat{y} &= \eta_k \hat{k} + \eta_\ell \left(\hat{z} + \hat{\ell}\right) \\ &= \mu \left[s_k \hat{k} + s_\ell \left(\hat{z} + \hat{\ell}\right)\right] \\ &= \mu \left[\left(\frac{1}{\mu} - s_\ell\right) \hat{k} + s_\ell \left(\hat{z} + \hat{\ell}\right)\right]\end{aligned}$$

We can solve this expression for effective labor input growth

$$\hat{z} + \hat{\ell} = \frac{\hat{y} - (1 - \mu s_\ell) \hat{k}}{\mu s_\ell}.$$

We now substitute for the effective labor growth in the expression for the change in the labor elasticity (27)

$$\hat{\eta}_\ell = \left(1 - \frac{1}{\sigma}\right) \left(\frac{\hat{y} - (1 - \mu s_\ell) \hat{k}}{\mu s_\ell} - \hat{y}\right)$$

and get

$$\hat{\eta}_\ell = \left(1 - \frac{1}{\sigma}\right) \frac{1 - \mu s_\ell}{\mu s_\ell} \left(\hat{y} - \hat{k}\right) \tag{41}$$

We calculate the change in the labor output elasticity using information on output, the capital stock, the labor revenue share and assumptions on the markup path based on Barkai (2020).

Labor elasticity for CES production, indirect The indirect measure of the labor elasticity (26) requires information on the ratio of real capital input to output, the CES coefficient on capital, and the substitution elasticity. Again, we set the substitution elasticity $\sigma = 0.5$. Again, we proceed directly to the first order approximation of the change in the labor elasticity, equation (28). We substitute for the level of the capital elasticity using the markup expression for capital, equation (11), and substitute for the capital revenue share from equation (36),

$$\hat{\eta}_\ell^k = \left(1 - \frac{1}{\sigma}\right) \frac{1 - \mu s_\ell}{\mu s_\ell} \left(\hat{y} - \hat{k}\right) \tag{42}$$

This is the same equation as (41). So, once we use a consistent adjustment for markups, the first order approximations of changes of the direct and indirect measures of labor elasticity are the same.

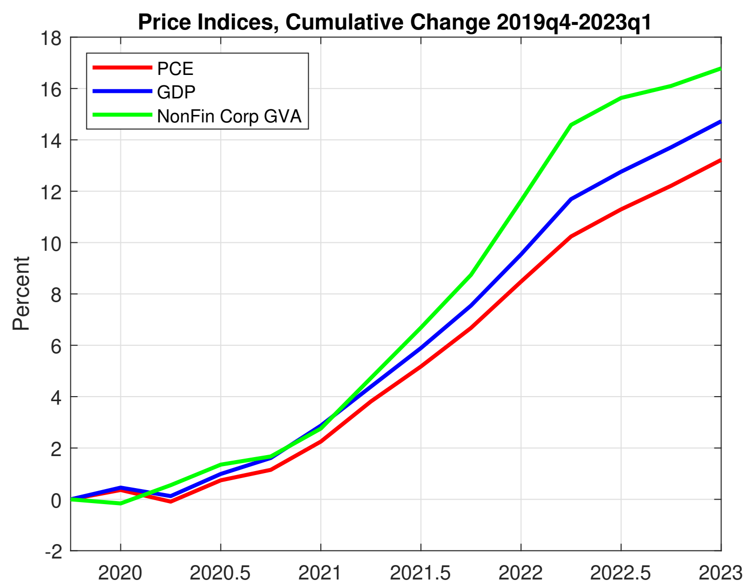
4 Some useful facts

- The overall GDP deflator, the PCE deflator, and the NFCB price deflator increase at similar rates from 1964 to present. In particular, they increased by roughly the same amount from 2019q4 to 2023q1:

15% for GDP, 13% for PCE, and 17% for the NFCB sector. See Figure 4.

- In the 2020s, the NFCB sector represents about 85% of corporate business, 66% of business, and 50% of overall GDP. The share of the NFCB sector has been declining over time. See Figure 5.
- In the nonfarm business sector, production and non-supervisory workers make up about 82.5% of total payroll pre-pandemic and about 81.5% post-pandemic. The share of production workers has fluctuated in a range between 83.5% in the 1960s and 80.5% in the 1980s. See Figure 6.

Figure 4: Price Indices: PCE, GDP, and NFCB



5 Data

Our data cover mainly the NFCB sector. We also use some additional data to place the NFCB sector in the overall economy, and we use some data from other sectors to supplement our markup calculations.

- Y, y: gross value added of the NFCB sector, quarterly SA nominal and chained real from NIPA 1.14
  - $p = Y/y$
- W, T, GOS: income components of GVA of the NFCB sector, quarterly SA nominal from NIPA 1.14
- ℓ: hours worked of the NFCB sector, quarterly SA quantity index 2012=100 from Productivity & Cost tables
- s_ℓ: labor revenue share of the NFCB sector, SA percent from P&C tables

Figure 5: Size of Nonfinancial Corporate Business

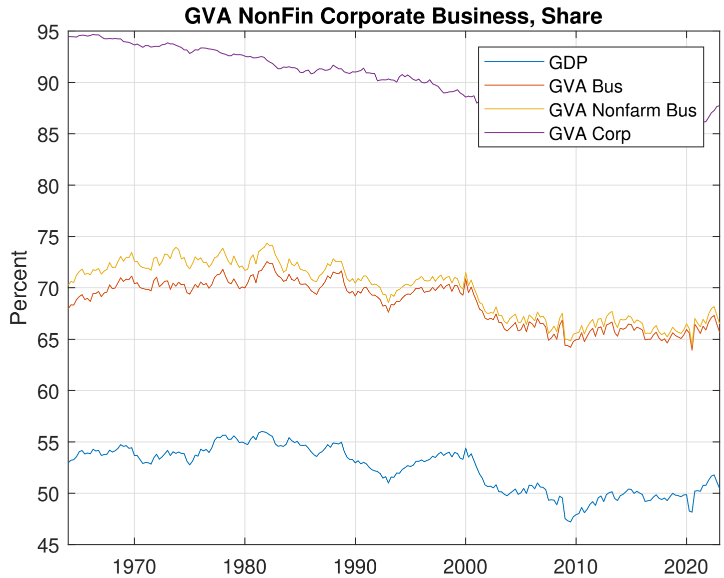
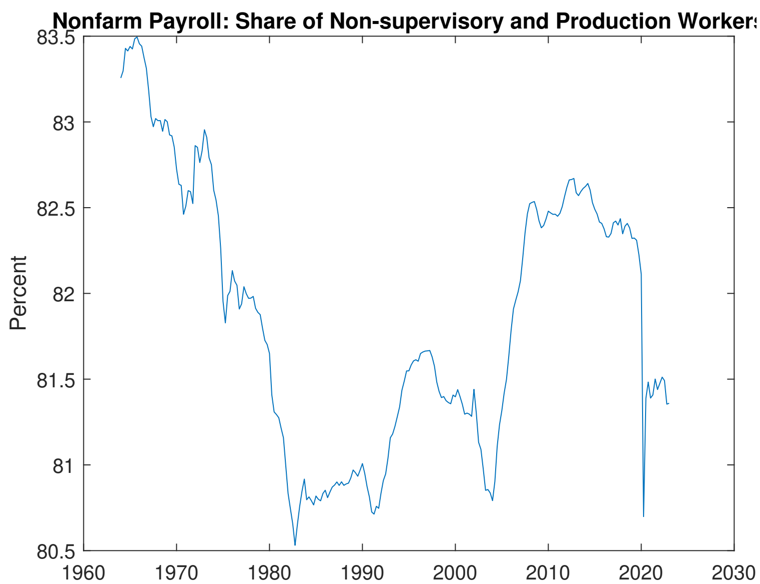


Figure 6: Share of Non-Supervisory Workers in Nonfarm Payroll



- $e, \tilde{e}$: nonfarm payroll employment, total and production & nonsupervisory workers only from Establishment survey, quarterly averages of SA monthly data
  - $\psi = \tilde{e}/e$
- k: non-residential fixed assets of the NFCB sector, quantity index, annual end of year data from Fixed Asset Tables

  - quarterly data from spline interpolation

- u: capital and labor utilization, SA quarterly growth rates from FRB-SF Nonfarm Business sector TFP calculations.

  - Level is exponential of cumulative growth rates, normalized to mean one from 2000-19.

- p^{GDP}, p^{PCE}: Price index for GDP and PCE, quarterly SA from NIPA Expenditure side of GDP tables 1.1.5&6

- Y_i: Nominal GDP, GVA for Business, Nonfarm Business, Nonfinancial Corporate, quarterly from NIPA Domestic Income by Sector 1.3.5

The quarterly NIPA, P&C, and payroll data are available up to 2023q1. The annual capital stock series is available up to 2021.

References

Barkai, S. (2020). Declining labor and capital shares. *Journal of Finance 75*(5), 2421–2463.

Basu, S. (2019). Are price-cost markups rising in the United States? A discussion of the evidence. *Journal of Economic Perspectives 33*(3), 3 – 22.

Hall, R. E. (2018). Using empirical marginal cost to measure market power in the US economy. Technical report.

Nekarda, C. J. and V. A. Ramey (2020). The cyclical behavior of the price-cost markup. *Journal of Money, Credit, and Banking 52*(0Supp), 319 – 353.