Krugman's US–Europe Paradox: Understanding its welfare implications

Chen Yeh

Abstract

In this note, we work out Krugman's "very wonkish" model on the apparent paradox between the US and Europe. The goal is to construct the simplest possible framework in which the US grows substantially more than Europe but living standards between the two places stay constant over time. This is consistent with empirical findings from this debate. First, the ratio of the largest European countries' GDP relative to the US has stayed constant at current PPP. Second, growth in real GDP (calculated through national GDP deflators) has been substantially higher for the US. We dissect the assumptions required to achieve Krugman's result and whether it stays robust under a set of alternative, but realistic, assumptions.

1 Starting point: Krugman's toy model

Environment

Suppose there are two places (or "countries"), US and EU, which each have a fixed labor force L. Both countries consume two goods, tech (T) and nontech (N), both costlessly tradable, so each has a single world price. Labor is the only factor of production and mobile across countries. Preferences, which are *identical* across countries, are given by a Cobb-Douglas aggregator:

$$U(T,N) = T^{\alpha} N^{1-\alpha} \tag{1}$$

where $\alpha < \frac{1}{2}$. Nontech is produced using a linear technology in labor only, i.e. $Q_N = L_N$. Furthermore, nontech goods serve as the numéraire, i.e. $p_N = 1$. There is no technological progress for nontech. **Tech** is produced *only in the US* with a linear technology $Q_T = A_T L_T$, where A_T grows at the constant rate ρ.

Equilibrium

Nontech is linear, tradable, and produced in both countries, so wages are equalized and produced at marginal cost. Therefore, we have $p_N = w = 1$ in in both countries. The US tech producer prices at marginal cost:

$$p_T A_T = w = 1 \qquad \Longrightarrow \qquad \frac{\dot{p}_T}{p_T} = -\frac{\dot{A}_T}{A_T}, \tag{2}$$

Thus, the tech price p_T falls at rate ρ. Due to competition and linear production technologies, there are no profits. Total income, equal to GDP, are given by labor income only in both countries, i.e. $Y_{US} = Y_{EU} = wL = L$, and world income is $2L$. Total spending and labor market clearing for tech are characterized by:

$$p_T Q_T = \alpha\left(Y_{US} + Y_{EU}\right) = 2\alpha L, \qquad \text{with} \qquad Q_T = A_T L_T, \tag{3}$$

Then, we have equilibrium output for tech is given by $Q_T = 2\alpha L \cdot A_T$. In addition, US tech labor equals $L_T = 2\alpha L$ (interior since $\alpha < \frac{1}{2}$); the remaining $(1-2\alpha)L$ is US nontech labor.

Then, it becomes clear why this framework generates what Krugman originally intended:

1. US-EU GDP ratio at current prices is one.

$$Y_{US} = wL = L = Y_{EU} \qquad \Longrightarrow \qquad \boxed{\frac{Y_{US}}{Y_{EU}} = 1}$$

As A_T grows, the tech price p_T falls one-for-one. Thus, tech's *value* share is fixed and the ratio never moves. Cobb-Douglas preferences do the work here.

2. US growth diverges from EU growth. Similar to the data, we calculate a country's growth as a weighted average of goods output growth. Then, we get:

$$\begin{aligned} g_{US}^{\text{real}} &\equiv s_T \cdot \frac{\dot{Q}_T}{Q_T} + s_N \cdot \frac{\dot{Q}_N}{Q_N} \\ &= 2\alpha \cdot \rho + (1 - 2\alpha) \cdot 0 \\ &= 2\alpha\rho \end{aligned} \tag{4}$$

where these weights are defined through *output* (not consumption). By construction, there is no growth in EU. Thus, we have:

$$\boxed{g_{US}^{\text{real}} = 2\alpha\rho \quad \text{and} \quad g_{EU}^{\text{real}} = 0}$$

At constant national prices, the US pulls away from EU.

3. Despite diverging growth, relative welfare is constant. Prices are global and preferences are symmetric, so each country faces the same cost-of-living index:

$$\begin{aligned} P &\equiv p_T^{\alpha} p_N^{1-\alpha} \\ &= p_T^{\alpha} \;= A_T^{-\alpha} \end{aligned} \tag{5}$$

In this simple framework, welfare is equivalent to real income per capita which is income deflated with the cost-of-living index. Thus, we have $\mathcal{W}_i = (Y_i/L)/P = p_T^{-\alpha} = A_T^{\alpha}$ in *both* countries, so we must have:

$$\boxed{\frac{d}{dt} \ln\left(\frac{\mathcal{W}_{US}}{\mathcal{W}_{EU}}\right) = 0}$$

That is, welfare across the two countries stays constant over time despite a divergence in real GDP growth. Note that US real GDP grows at $2\alpha\rho$, because the US *produces* the cheapening good for both countries; each country's welfare grows at only $\alpha\rho$, because each country *consumes* only a share of α of it. The extra $\alpha\rho$ that enters the US books is tech it exports to the EU — counted in US output but enjoyed as EU consumption. Thus, output diverges but welfare does not. The current price comparison (which values that export at its falling world price) sees no gap.

2 Moving welfare over time: an even "wonkier" model

Krugman's setup generates the patterns of the data qualitatively and implies that, despite a US-EU growth gap, there is no divergence in welfare between the countries. This is exactly the premise he defends in his line of articles. However, as always, this conclusion rests on a set of important assumptions, namely (1) wages are equalized across countries due to mobile labor, and equal technologies for the production of nontech goods, (2) complete US specialization in tech, and (3) symmetric Cobb-Douglas preferences, i.e. expenditure shares in tech and nontech are constant, and equal across countries.

Cobb-Douglas preferences do a lot of work since current price GDP will then be independent of tech prices p_T. However, this is the only equilibrium object that moves over time due to technological progress in tech at rate ρ. Recall that, due to the equalization of wages, welfare in country i is equal to $\mathcal{W}_i = 1/p_T^{\alpha_i}$. Thus, a trivial extension would be to assume asymmetric preferences across countries such that $\alpha_{EU} < \alpha_{US} < \frac{1}{2}$. Then, we would have the following:

$$\boxed{\frac{Y_{US}}{Y_{EU}} = 1} \quad \boxed{g_{US}^{\text{real}} = (\alpha_{US} + \alpha_{EU})\rho \quad \text{and} \quad g_{EU}^{\text{real}} = 0} \quad \boxed{\frac{d}{dt} \ln\left(\frac{\mathcal{W}_{US}}{\mathcal{W}_{EU}}\right) = (\alpha_{US} - \alpha_{EU}) \cdot \rho > 0}$$

That is, we still see consistency with the data (constant ratios of GDP at current prices and diverging growth), but welfare *does* move over time in favor of the US. Under this scenario, the EU would be falling behind the US. Note that the assumption of $\alpha_{US} > \alpha_{EU}$ is not unlikely. According to data from the OECD, total ICT investment as a share of GDP is higher for the US (3.33%) when compared to other large European countries (Germany: 1.33%; France: 2.32%; Italy: 2.14%) in 2014. For any other year since 2001, we also have that ICT investment as a share of GDP is much higher for the US compared to these European countries. In the remainder, we will adjust the model such that the current price GDP ratio between the US and EU is larger than unity and stays constant over time (as consistent with the data). To do so, we will draw inspiration from ABG and assume that the US retrieves profits from tech production.

Everything is the same as before, but we make only two adjustments. First, preferences are asymmetric such that $\alpha_{EU} < \alpha_{US} < \frac{1}{2}$. Second and last, the tech good is still produced only in the US but with a *decreasing returns to scale* technology such that $Q_T = A_T \cdot L_T^{\eta}$ where $\eta \in (0, 1)$.

Equilibrium: update

The mechanics of the equilibrium are almost identical to before. Nontech is produced with a linear technology, tradable, and produced in both countries. Then, under competition, we have: $p_N = w = 1$. Recall that the nontech good is the numéraire. Then, optimality for the US tech producer implies:

$$p_T \, A_T \, \eta \, L_T^{\eta-1} = w = 1 \tag{6}$$

Revenues from tech satisfy $p_T Q_T = L_T/\eta$. By construction, the wage bill consists of a fraction η of these revenues, i.e. $\eta \cdot p_T Q_T = wL_T = L_T$. Then, profits (expressed as a function of tech labor) satisfies:

$$\pi = p_T Q_T - wL_T = \frac{L_T}{\eta} - L_T = \frac{1-\eta}{\eta} L_T \tag{7}$$

US national income is wages plus rents, i.e. $Y_{US} = L + \pi$. For the EU, income consists of wages only, i.e. $Y_{EU} = L$. Market clearing for the tech good implies that total revenues from the tech good consists US and EU spending:

$$\frac{L_T}{\eta} \; = \; \alpha_{US} \, (L + \pi) \; + \; \alpha_{EU} \, L \tag{8}$$

Substituting (7) into (8) and solve for L_T, we obtain equilibrium labor tech and profits:

$$L_T = \frac{\eta(\alpha_{US} + \alpha_{EU}) \, L}{1 - \alpha_{US}(1-\eta)} \tag{9}$$

$$\pi = \frac{(1-\eta)(\alpha_{US} + \alpha_{EU})L}{1 - \alpha_{US}(1-\eta)} \tag{10}$$

Both objects are *independent of* A_T: they do not move over time. Note that the US also produces nontech whenever $\alpha_{US} + \eta\,\alpha_{EU} < 1$, but this holds automatically whenever $\alpha_{EU} < \alpha_{US} < \frac{1}{2}$. The equilibrium prices for tech can be retrieved from the US tech firm's optimality condition (6). Then, we get:

$$p_T = \frac{L_T^{1-\eta}}{\eta A_T} \tag{11}$$

$$= \frac{1}{\eta}\left(\frac{\eta(\alpha_{US} + \alpha_{EU})\,L}{1 - \alpha_{US}(1-\eta)}\right)^{1-\eta} \cdot A_T^{-1} \tag{12}$$

Thus, as before, we have that $\frac{\dot{p}_T}{p_T} = -\frac{\dot{A}_T}{A_T} = -\rho$. Repeating our exercise from before, we get:

1. US-EU GDP ratio at current prices is larger than one (as consistent with the data) and constant.

$$\boxed{\frac{Y_{US}}{Y_{EU}} = \frac{L+\pi}{L} = 1 + \frac{(1-\eta)(\alpha_{US} + \alpha_{EU})}{1 - \alpha_{US}(1-\eta)} > 1}$$

where $\eta \to 1$ brings us back to the original Krugman framework. The parameter η determines the level of profits for the US tech producer and causes a wedge in the *level* of the two places' GDP.

2. US growth diverges from EU growth. Similar to the data, we calculate a country's growth as a weighted average of goods output growth. Recall that $\frac{\dot{Q}_T}{Q_T} = \frac{\dot{A}_T}{A_T} + \eta \cdot \frac{\dot{L}_T}{L_T} = \rho$. Then, we get:

$$g_{US}^{\text{real}} \equiv s_T \cdot \frac{\dot{Q}_T}{Q_T} + s_N \cdot \frac{\dot{Q}_N}{Q_N}$$

$$= \frac{p_T Q_T}{Y_{US}} \cdot \rho + \frac{p_N Q_N}{Y_{US}} \cdot 0$$

$$= \left(\frac{L_T/\eta}{L+\pi}\right)\rho = \frac{\alpha_{US} + \alpha_{EU}}{1 + (1-\eta)\alpha_{EU}} \cdot \rho \tag{13}$$

where these weights are defined through *output* (not consumption). By construction, there is no growth in EU. Thus, we have:

$$\boxed{g_{US}^{\text{real}} = \frac{\alpha_{US} + \alpha_{EU}}{1 + (1-\eta)\alpha_{EU}} \cdot \rho \quad \text{and} \quad g_{EU}^{\text{real}} = 0}$$

At constant national prices, the US pulls away from EU. Note that we are back to the original Krugman framework when $\alpha_{US} = \alpha_{EU} = \alpha$ and $\eta \to 1$.

3. In the presence of diverging growth, EU welfare does start lagging behind US welfare. Prices are global, and we have *country-specific* Cobb-Douglas preferences:

$$P_i \equiv p_T^{\alpha_i} p_N^{1-\alpha_i}$$

$$= p_T^{\alpha_i} = A_T^{-\alpha_i} \tag{14}$$

Once again, welfare is equivalent to real income per capita which is income per capita deflated with the country-specific cost-of-living index. Thus, we have:

$$\mathcal{W}_{US} = \frac{L+\pi}{P_{US}} = (L+\pi) \cdot A_T^{\alpha_{US}} \tag{15}$$

$$\mathcal{W}_{EU} = \frac{L}{P_{EU}} = L \cdot A_T^{\alpha_{EU}} \tag{16}$$

Then, US welfare drifts away from the EU since $\alpha_{US} > \alpha_{EU}$:

$$\boxed{\frac{d}{dt}\ln\left(\frac{\mathcal{W}_{US}}{\mathcal{W}_{EU}}\right) = (\alpha_{US} - \alpha_{EU}) \cdot \rho > 0}$$

Note that decreasing returns to scale induces profits for the US which determines the *level* of the US-EU GDP ratio at current prices. The asymmetry in preferences $\alpha_{US} > \alpha_{EU}$ causes a divergence in welfare over time. Intuitively, the US consumes more of the good whose prices falls over time (i.e., tech), so its cost-of-living index falls faster as well.