Staggered Price Setting and the Zero Bound on Nominal Interest Rates

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The nominal interest rate cannot be less than zero: no one would choose to hold assets bearing a guaranteed negative nominal return when they could instead hold money, which bears a guaranteed zero nominal return. Does the zero bound have normative implications for monetary policy? The nominal interest rate tends to be low when expected inflation is low, so the lower expected inflation is, the more likely it is that zero nominal interest rates would be encountered. Some have argued that the zero bound's proximity at low inflation constitutes an argument against policy that results in low inflation or deflation.[1] Here we compare moderately deflationary and moderately inflationary regimes using a macroeconomic model to evaluate whether the zero bound introduces distortions that make low inflation undesirable.

The model and the method distinguish our analysis from other recent research on the same topic.[2] The model has optimizing behavior by individuals and firms, with the qualification that firms' price setting is staggered. Other analyses of the zero bound have also used sticky-price models; the zero bound is more likely to be important if nominal disturbances have real effects, as they do with sticky prices. Individuals in the model choose to hold money because it decreases the time they must spend shopping. Other analyses have not modeled money demand. The method we employ involves solving the entire model nonlinearly, which means directly imposing the zero bound on nominal

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[1] Notable examples are Vickrey (1954), Okun (1981), and Summers (1991).

[2] Notable examples include Fuhrer and Madigan (1997), Rotemberg and Woodford (1997), and Orphanides and Wieland (1998). Section 1 contains a discussion of these articles.

interest rates. We then compare the two inflation regimes in several ways, one of which involves using an explicit welfare metric, the representative agent's expected utility.

In the model, a deflationary regime where nominal interest rates are occasionally zero generates *higher* welfare than a moderate inflation regime where nominal interest rates are always positive. This striking result—which conflicts with the spirit if not the letter of previous work—can be attributed to two factors mentioned above. First, the fact that money demand is explicitly modeled means that there is a distortion associated with *positive* nominal interest rates: individuals waste resources economizing on real money balances. Second, while the two-period staggered price-setting requirement makes prices sticky, it does not make *inflation* sticky. When inflation is sticky, as in the models used by Fuhrer and Madigan (1997) and Orphanides and Wieland (1998), for example, the zero bound on nominal interest rates effectively means that real interest rates are constrained in low-inflation regimes. In contrast, in the sticky-*price* model used here, real interest rates are not constrained at low inflation. The monetary authority can create temporary expected inflation when nominal rates are zero, thereby pushing real rates down, as described by Mishkin (1996).

1. BACKGROUND AND RELATED WORK

Nominal interest rates are interest rates on marketable securities or loans denominated in an economy's unit of account. In contrast, real interest rates apply to assets denominated in a market basket of goods and services. Irving Fisher, who used the terms "money interest" and "real interest," is traditionally credited with being the first to distinguish between nominal and real interest rates. Fisher himself acknowledged, however, that he had many predecessors who understood the distinction between nominal and real interest rates to some degree.[3]

In Fisher's original analysis, the relationship between the nominal (money) interest rate and the real interest rate is but a special case of the relationships between interest rates denominated in any two standards of value. The celebrated Fisher equation first appears in "Appreciation and Interest," in an example where the two standards are gold and wheat. But, when Fisher introduces that analysis, he poses the general question, "If a debt is contracted in either of two standards and one of them is expected to change with reference to the other, will the rate of interest be the same in both? Most certainly not" (Fisher 1896, p. 6). The Fisher equation follows three pages later: $1 + j = (1 + a)(1 + i)$,

[3] Fisher's important works on this subject are "Appreciation and Interest," *The Rate of Interest*, and *The Theory of Interest*. See Humphrey ([1983] 1986), and Laidler (1991) for a discussion of Fisher's predecessors.

where j and i are the rates of interest in wheat and gold, respectively, and a is the (certain) expected rate of appreciation of gold in terms of wheat. This generality on Fisher's part is important, because it provides him with a principle for understanding why the money interest rate is bounded by zero. Fisher states that the interest rate cannot be negative in any standard that can be hoarded without loss. The argument is straightforward: individuals would choose to hoard the standard itself rather than hold securities or loans denominated in that standard and yielding negative interest. For perishable standards, however, the situation is different: "One can imagine a loan based on strawberries or peaches contracted in summer and payable in winter with negative interest" (Fisher 1896, p. 32). Since fiat money is storable at near zero cost, it follows that the nominal interest rate in a modern, fiat-money economy is approximately bounded by zero.

The zero bound is clearly a constraint on monetary policy, but is it an important constraint? In order to answer this question, one needs a macroeconomic model and a criterion for measuring importance. To understand the contribution made by this article, one should first know something about the models and criteria used in recent analyses by Fuhrer and Madigan (1997), Rotemberg and Woodford (1997), and Orphanides and Wieland (1998).[4]

Fuhrer and Madigan (1997) and Orphanides and Wieland (1998) use similar models, so we will consider them together. As with our analysis below, they assess the zero bound's importance by comparing their models' performance at a moderate inflation target to that at an inflation target low enough to make the nominal interest rate occasionally zero. Fuhrer and Madigan use a small model that contains (i) a backward-looking IS curve, (ii) an overlapping price-contracting specification, and (iii) a monetary policy reaction function.[5] Orphanides and Wieland's model shares the same contracting specification but disaggregates the IS curve into separate spending equations for consumption, fixed investment, inventory investment, net exports, and government spending. Neither model includes money. Monetary policy operates by changing the short-term nominal interest rate. Long-term real interest rates enter the spending equations, but because the contracting specification makes inflation sticky, persistent changes in the short-term nominal rate generate changes in the long-term real rate. Thus, monetary policy can affect real spending and hence output. In both models, the equations representing private sector behavior are posited rather than derived from explicit optimization problems.

Fuhrer and Madigan evaluate the zero bound's importance by comparing their model's responses to IS curve shocks at inflation targets of zero and 4

[4] The "liquidity trap" literature associated with Keynes ([1936] 1964) and Patinkin (1965) concerned the possibility of a positive lower bound on nominal interest rates. Relating that literature to recent work would be an article by itself.

[5] It is the same model used in Fuhrer and Moore (1995).

percent. In contrast, Orphanides and Wieland simulate their model using estimated shock processes and compare the variance of output at different inflation targets. The general conclusion of these papers is that at a zero inflation target, monetary policy is significantly constrained by the zero bound, in the sense that the zero bound is encountered regularly, and output is consequently more variable than at a moderate inflation target. The easiest explanation for this result comes from the first example in Fuhrer and Madigan, a permanent shock to the IS curve. The monetary authority responds to this shock by lowering short-term nominal interest rates. When the inflation target is zero, the monetary authority cannot lower the nominal rate by as much as it would choose if the inflation target were 4 percent. With sticky inflation, the decline in the real interest rate is also smaller, and therefore—because of the interest rate effect on spending—there is a larger fall in output at the zero inflation target. This fall in output is presumed to be bad, although that presumption is not implied by the model.

The principal virtue of the analysis conducted by Fuhrer and Madigan (1997) and Orphanides and Wieland (1998) is that it is performed using models that fit a particular sample of data quite well. However, their low inflation experiments are conducted in an economic environment quite different from the data sample. Therefore, the fact that the models' equations are not derived from explicit objective functions makes it doubtful that those equations would be stable in the face of the contemplated policy experiments. Although the model we use has not been shown to fit recent data well, it is valuable because it is set up with explicit objective functions for individuals and firms. This means that the model can legitimately be used for policy and welfare analysis.

Rotemberg and Woodford (1997) come to a slightly different conclusion about the importance of the zero bound as a constraint on monetary policy, using a different model and approach from those of Fuhrer and Madigan and Orphanides and Wieland. As we will also, Rotemberg and Woodford use a sticky-price model whose equations are derived from explicit optimization problems, and they use the utility function of agents in the model to measure the welfare associated with different monetary policy rules. However, Rotemberg and Woodford linearize their model to simplify the analysis, and this precludes them from directly imposing the zero bound. They account for the zero bound indirectly by assuming that the variability of the monetary authority's interest rate instrument is constrained by the average level of interest rates, that is, by the inflation target. Specifically, they assume that the ratio of the standard deviation of the nominal interest rate to the average level of the nominal interest rate can be no greater than the ratio that describes their 1980–1995 U.S. sample. Thus, policy rules that generate high variability of nominal rates are incompatible with low inflation targets. Since a generic implication of models such as theirs is that stable inflation requires volatile nominal interest rates, their assumption implies a sharp tradeoff between the level of inflation and its variability. While this assumption has the effect of increasing the optimal inflation target from zero in their model, the optimum does not move far from zero.

All three papers discussed above exclude money from the models. Rotemberg and Woodford correctly state that the behavior of their model would be unchanged if they used a money-in-the-utility function specification where money was additively separable in the period utility function. However, ignoring money demand also means ignoring the welfare costs of positive nominal interest rates. That is, while the behavior of real and nominal variables may be invariant to incorporating money in an additively separable way, the welfare implications of different monetary policies are not invariant to this modification. Since concern about the zero bound on nominal interest rates boils down to concern about the welfare level associated with very low inflation targets, leaving money out of the model may be an important omission.

2. A MODEL WITH STAGGERED PRICE SETTING

Our analysis of the zero bound's importance for monetary policy is based on an explicit optimizing sticky-price model similar to, but simpler than, the one in Rotemberg and Woodford (1997). As in Fuhrer and Madigan (1997) and Orphanides and Wieland (1998), we impose the zero bound directly, rather than measuring its importance indirectly.[6] However, we take our analysis two steps further. First, we explicitly model money demand (using a shopping-time technology), so there is a force working in *favor* of zero nominal interest rates. Second, no linear approximations are employed to solve the model, which is fundamentally nonlinear.

The model follows the tradition of Taylor (1980), in that price setting is staggered: each firm sets its price for two periods, with one half of the firms adjusting each period.[7] As in Taylor's model, monetary policy is nonneutral in the model because stickiness in individual prices gives rise to stickiness in the price level. There are a continuum of firms, and they produce differentiated consumption goods using labor provided by consumers at a competitive wage as the sole input. Consumers are infinitely lived and use income from their labor, which is supplied elastically, to purchase consumption goods. Consumers hold money in order to economize on transactions time, as in McCallum and Goodfriend ([1987] 1988).

[6] Fuhrer and Madigan use three different approaches, one of which involves directly imposing the zero bound.

[7] The remainder of this section is loosely based on Section 2 in King and Wolman (forthcoming 1999). The model analyzed here differs in that it explicitly motivates money demand with a shopping-time technology.

Consumers

Consumers have preferences over a consumption aggregate (c_t) and leisure (l_t) given by

$$E_t \sum_{t=0}^{\infty} \beta^t \cdot [\ln(c_t) + \chi_t \cdot l_t]\,. \tag{1}$$

The discount factor β is set to 0.985, and the variable χ_t is a random preference shock.[8] The consumer's budget constraint is

$$c_t + \frac{M_t}{P_t} + \frac{B_t/P_t}{1+R_t} = \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + w_t n_t + d_t + \frac{S_t}{P_t},$$

and the time constraint is

$$n_t + l_t + h[M_t/(P_t c_t)] = E, \tag{2}$$

where P_t is the price level, M_t is nominal money balances chosen in period t to carry over to $t+1$, B_t is holdings of one-period nominal zero-coupon bonds maturing at $t+1$, R_t is the interest rate on nominal bonds, w_t is the real wage, n_t is time spent working, d_t is real dividend payments from firms, S_t is a lump-sum transfer of money from the monetary authority, $h[M_t/(P_t c_t)]$ is time spent transacting, and E is the time endowment. Defining real balances to be $m_t \equiv M_t/P_t$, the function $h(\cdot)$ is parameterized as in Wolman (1997):

$$h(m_t/c_t) = \phi \cdot (m_t/c_t) - \frac{\nu}{1+\nu} A^{-1/\nu} (m_t/c_t)^{\frac{1+\nu}{\nu}} + \Omega, \text{ for } m_t/c_t < A \cdot \phi^{\nu},$$

$$h(m_t/c_t) = \Omega, \text{ for } m_t/c_t \geq A \cdot \phi^{\nu}, \tag{3}$$

with $\phi = 1.4 \times 10^{-3}$, $A = 1.7 \times 10^{-2}$, and $\nu = -0.7695$. Transactions time is thus decreasing in real balances and increasing in consumption, up to a satiation level of the ratio of real balances to consumption.

Goods Market Structure

As in Blanchard and Kiyotaki (1987), we assume that every producer faces a downward-sloping demand curve with constant elasticity ε. [9] The consumption aggregate is an integral of the differentiated products $c_t = [\int c(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega]^{\frac{\varepsilon}{\varepsilon-1}}$, as in Dixit and Stiglitz (1977).

[8] This value of β implies a steady-state real interest rate of 6.5 percent per annum and hence a steady-state nominal interest rate of about 11.5 percent when there is 5 percent annual inflation. While the number assigned to β has quantitative implications for the results reported below, it does not have qualitative implications.

[9] We assume $\varepsilon = 10$.

Since all producers that adjust their prices in a given period choose the same price, it is easier to write the consumption aggregate as

$$c_t = c(c_{0,t}, c_{1,t}) = \left(\frac{1}{2} \cdot c_{0,t}^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} \cdot c_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where $c_{j,t}$ is the quantity consumed in period t of a good whose price was set in period $t-j$. The constant elasticity demands for each of the goods take the form

$$c_{j,t} = \left(\frac{P^*_{t-j}}{P_t} \right)^{-\varepsilon} \cdot c_t, \tag{5}$$

where P^*_{t-j} is the nominal price at time t of any good whose price was set j periods ago, and P_t is the price index at time t, given by

$$P_t = \left[\frac{1}{2} \cdot \left(P^*_t\right)^{1-\varepsilon} + \frac{1}{2} \cdot \left(P^*_{t-1}\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{6}$$

Optimization

If we attach Lagrange multipliers λ_t and μ_t to the budget and time constraints, respectively, so that λ_t is the marginal value of real wealth and μ_t is the marginal value of time, the first-order conditions for the individual's maximization problem, with respect to c_t, l_t, n_t, B_t, and M_t, are

$$\frac{1}{c_t} = \lambda_t - \mu_t \cdot h'(\cdot)(\frac{m_t}{c_t^2}), \tag{7}$$

$$\chi_t = \mu_t, \tag{8}$$

$$\mu_t = w_t \cdot \lambda_t, \tag{9}$$

$$\frac{\lambda_t}{P_t} = \beta \cdot (1 + R_t) \cdot E_t \frac{\lambda_{t+1}}{P_{t+1}}, \tag{10}$$

and

$$\frac{\lambda_t}{P_t} + \frac{\mu_t}{P_t} \cdot h'(\cdot)(\frac{1}{c_t}) = \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}. \tag{11}$$

In choosing consumption optimally (as in [7]), the individual weighs the benefit of consuming a marginal unit, which is the left-hand side of (7), against the cost, which consists of both forfeited real wealth (the first term on the right-hand side) and time spent transacting (the second term on the right-hand side). In choosing leisure and labor supply optimally (as in [8] and [9]), the individual weighs the marginal value of time against both the marginal utility of leisure and the wage earnings that the time would yield. The choice of bond holdings (equation [10]) equates the marginal value of nominal wealth today to $(1 + R_t)$ times the marginal value of nominal wealth tomorrow. And finally, optimal money holdings (equation [11]) imply that the individual equates the transactions-facilitating benefit to the foregone interest cost of holding money.[10]

Firms

Each firm produces with an identical technology:

$$c_{j,t} = n_{j,t},\ j = 0, 1, \tag{12}$$

where $n_{j,t}$ is the labor input employed in period t by a firm whose price was set in period $t-j$. Given the price a firm charges, it hires enough labor to meet the demand for its product at that price. Firms that do not adjust their price in a given period can thus be thought of as passive, whereas firms that adjust their price do so optimally, that is, in order to maximize the present discounted value of their profits. Given that it has set a relative price $\frac{P^*_{t-j}}{P_t}$, real profits for a firm of type j are

$$\frac{P^*_{t-j}}{P_t} \cdot c_{jt} - w_t \cdot n_{jt}, \tag{13}$$

that is, revenue minus cost.

Optimal Price Setting

Maximization of present value implies that a firm chooses its current relative price, taking into account the effect on current and expected future profits. Substituting into (13) the demand curve (5) and the technology (12), the present discounted value of expected profits is given by

$$c_t \cdot \left[\left(\frac{P^*_t}{P_t} \right)^{1-\varepsilon} - w_t \cdot \left(\frac{P^*_t}{P_t} \right)^{-\varepsilon} \right] +$$

$$\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \cdot c_{t+1} \cdot \left[\left(\frac{P^*_t}{P_{t+1}} \right)^{1-\varepsilon} - w_{t+1} \cdot \left(\frac{P^*_t}{P_{t+1}} \right)^{-\varepsilon} \right] \tag{14}$$

for the two periods over which a price will be in effect. Differentiating (14) with respect to P^*_t and setting the resulting expression equal to zero, one sees that the optimal relative price satisfies

$$\frac{P^*_t}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{\sum_{j=0}^{1} \beta^j E_t[\lambda_{t+j} \cdot w_{t+j} \cdot (P_{t+j}/P_t)^{\varepsilon} \cdot c_{t+j}]}{\sum_{j=0}^{J-1} \beta^j E_t[\lambda_{t+j} \cdot (P_{t+j}/P_t)^{\varepsilon-1} \cdot c_{t+j}]}. \tag{15}$$

[10] The transactions-facilitating benefit is given by $\frac{\mu_t}{P_t} \cdot h'(\cdot)(\frac{1}{c_t})$, and the foregone interest cost is $\frac{\lambda_t}{P_t} - \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}$ (see [10]). A conventional money demand equation can be derived by combining (9)–(11): $m_t/c_t = A \cdot \{[R_t/(1 + R_t)] \cdot (c_t/w_t) + \phi\}^{\nu}$.

Essentially, the optimal relative price equates discounted marginal revenue with discounted marginal cost; the numerator of (15) represents marginal cost and the denominator marginal revenue.[11] In a noninflationary steady state, the firm would choose a markup over marginal cost of $\frac{\varepsilon}{\varepsilon-1}$. In an inflationary or deflationary steady state, the markup would differ from $\frac{\varepsilon}{\varepsilon-1}$, as adjusting firms would take into account the future erosion (or inflation) of their relative price (see King and Wolman [forthcoming 1999] for details). With uncertainty, the markup becomes time varying: it depends on the current and expected future marginal utility of wealth, price level, aggregate demand, and real wage.

Driving Process

The only exogenous variable in the model is the preference shock χ_t, and it is assumed to follow a two-state Markov process:

$$\Pr\left(\chi_t = \overline{\chi} \mid \chi_{t-1} = \overline{\chi}\right) = 0.8, \text{ and}$$

$$\Pr\left(\chi_t = \underline{\chi} \mid \chi_{t-1} = \underline{\chi}\right) = 0.8, \; \underline{\chi} < \overline{\chi}. \tag{16}$$

Thus, χ_t varies between high and low values, and on average each value persists for five periods before switching. This process is not meant to replicate actual features of the U.S. economy. Rather, it is chosen to make the economy alternate between periods of high and low output in a way that makes the real interest rate vary over time. It is by no means the only process that would yield such behavior. The equilibrium behavior of the real interest rate will be affected by monetary policy as well as by the shock process.

Monetary Policy

As described below, we assume that policy is characterized by a feedback rule for the nominal interest rate. One component of the feedback rule is a "target" inflation rate, an inflation rate that the rule would deliver in the absence of shocks. In general, the feedback rule makes the nominal rate a differentiable function of observable variables. In certain states of the world, however, that differentiable function would make the nominal rate negative. In those states of the world, we assume that the policy rule sets $R_t = 0$. Given the nominal interest rate implied by the policy rule, the monetary transfer (S_t) is determined by money demand. Note that money demand is an integral part of the model. It is sometimes asserted that when the monetary authority follows an interest rate rule, money demand can be left out of the model, as it only serves to determine the value of the money supply. Here that is not the case, because the quantity

[11] Note that in this sentence, marginal revenue and cost are with respect to price, not quantity.

of money enters other equations of the model in addition to the money demand equation (specifically [7] and [2]).

The nominal interest rate is the rate on one-period bonds, which are assumed to be in zero net supply. This is somewhat problematic from the standpoint of justifying the zero bound. That is, the zero bound is a necessary characteristic of nominal bonds that are willingly held, but nominal bonds are not actually held in the model (they are priced). This inconsistency can be rectified by assuming that there is a fixed real quantity of outstanding government bonds, and the government pays the interest on those bonds by levying lump-sum taxes as necessary.

Solving the Model

The standard method used for solving dynamic stochastic models such as this one is to calculate the steady state for a given inflation rate, and then linearize the model's equations around that steady state. Linearization would be inappropriate here, because it would rule out imposing the zero bound on nominal interest rates. Instead of linearizing, then, we solve the model using a crude version of the finite element method (see McGrattan [1996]). This method involves picking a grid of points for the model's state variable, P^*_{t-1}, and then finding values of the "control" variables numerically for each grid point and for each value of the preference shock such that the model's equations are approximately satisfied. The solution consists of mappings from the state variable to each of the other variables. Those mappings can be used in conjunction with the stochastic process for the preference shock to simulate the model. Because this solution method involves a finite number of grid points, it necessarily yields only an approximate solution. However, to the extent that the true mappings from the state variables to the other variables are smooth functions, the grid method can yield an extremely accurate solution. Furthermore, the extent that the mappings appear nonlinear gives an indication of the error that would be associated with linearization methods.

3. IMPLICATIONS OF THE ZERO BOUND IN THE MODEL

Using the model described above, one can determine whether the zero bound means that a very low inflation target (here it will be deflation) significantly modifies economic performance relative to a moderate inflation target. For a particular specification of monetary policy, we will simulate the model at moderate inflation and then at moderate deflation, and compare the results along three dimensions. The first involves simulating the model for 30 periods with the same shocks at high and low inflation, and informally comparing the results. The second involves the variances of inflation and output, which has been the conventional metric in the literature on monetary policy rules (see the papers in

Taylor [forthcoming 1999]). Given that the model yields an obvious choice for a welfare function (the representative agent's expected utility), we also compare the two regimes in terms of welfare.

Model Simulations

Recent research on monetary policy has emphasized "Taylor rules," that is, specifications of policy where the monetary authority sets a short-term interest rate as a linear combination of deviations of inflation from a target and deviations of output from some trend or potential level. These rules, popularized by John Taylor (1993), have been shown to be parsimonious approximations of the behavior of actual central banks and to have reasonable properties in certain theoretical models. The rule used below is similar to a Taylor rule, except that instead of inflation on the right-hand side it uses the price level. Concretely,[12]

$$R_t = \max \left\{ \begin{array}{c} R^* + 1.5 \cdot [\ln(P_t) - \ln(\overline{P}_t)] + 1.0 \cdot [\ln(c_t) - \ln(\overline{c})], \\ 0 \end{array} \right\}, \tag{17}$$

where R^* is the steady-state nominal interest rate consistent with the chosen inflation target, $\overline{P}_t$ is a target price-level path that grows at the targeted inflation rate, and $\overline{c}$ is the steady-state level of consumption associated with the inflation target.[13] This rule implies that the price level will always be expected to return to the same trend path. In contrast, the standard Taylor rules imply that inflation will always be expected to eventually return to target, but the price level will be expected to drift away from any previous trend path.[14]

Introduction to the Functions Describing General Equilibrium

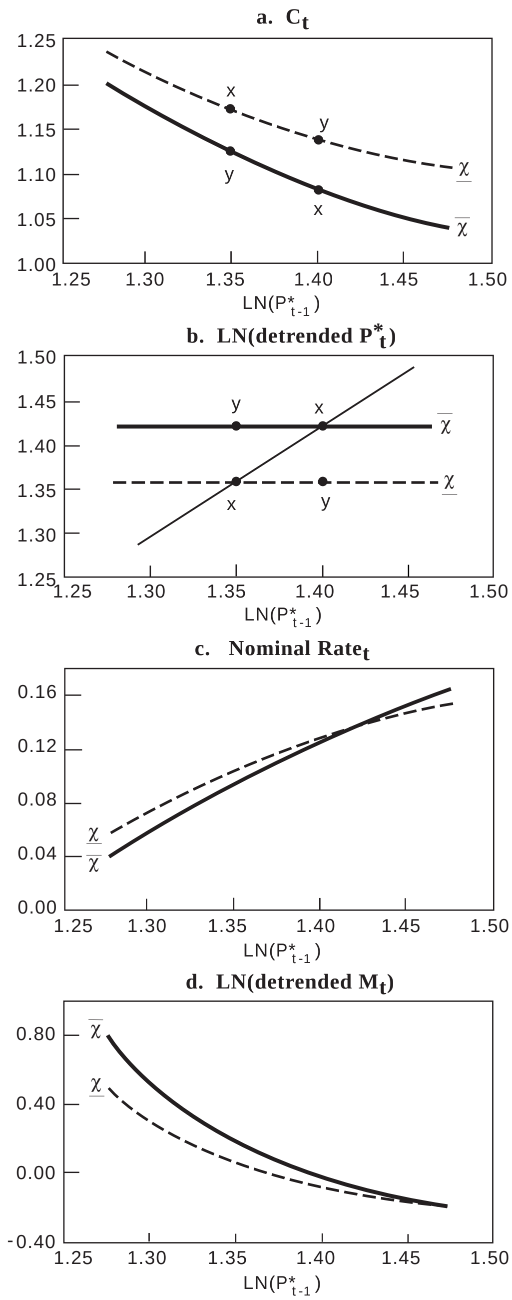
As background to the simulation results, Figure 1 displays the relationships between key endogenous variables and the state variable, which is the price set last period by adjusting firms. Figure 1 is generated with an inflation target of 5 percent. The solid lines show the relationship between P^*_{t-1} (detrended by the targeted inflation rate) and each endogenous variable when the preference shock takes on a high value, and the dashed lines show the relationships when the preference shock takes on a low value. Using panel b, and with knowledge of P^*_0, one can trace out a path for P^*_t by drawing values of χ_t from the stochastic

[12] The interest rate in (17) is a quarterly interest rate, whereas the rates plotted in Figures 1–4 are annual rates.

[13] The inflation target affects steady-state consumption for two reasons. First, the markup chosen by adjusting firms varies with the inflation target in a way that does not exactly offset the inflation erosion of nonadjusting firms' markups. Second, by lowering real balances, higher inflation effectively makes consumption more expensive.

[14] The original motivation for using a price-level target here instead of an inflation target was computational ease. It turns out, however, that analyzing an inflation-targeting policy is no more demanding than analyzing price-level targeting. We are studying inflation-targeting policies in ongoing research.

Figure 1 Functions Mapping State Variable (P^*_{t-1}) to Other Variables at 5 Percent Inflation Target



process governing it. Then, with the path for P_t^* in hand, the relationships in panels a, c, and d can be used to generate paths for the other variables for the given sequence of χ_t. What follows is a discussion of the model's principal mechanisms in light of the relationships shown in Figure 1.

There are essentially two determinants of current-period variables in the model. One is the value of the stochastic preference parameter (χ_t), and the other is the value of the price that adjusting firms set last period. When χ_t takes on a high value, the marginal utility of leisure is high. Agents react by supplying less labor to the market, and this reaction brings with it a decrease in consumption. Thus, in panel a, the level of consumption is low when $\chi_t = \bar{\chi}$. For low values of P_{t-1}^*, the lower level of consumption causes the monetary authority to set a lower value for the nominal interest rate, as in the left-hand part of panel c, and the lower nominal interest rate in turn drives up money demand (panel d). However, when P_{t-1}^* is especially high, the nominal rate is lower in the $\underline{\chi}$ (high-consumption) state. Why is this the case? The feedback rule for monetary policy sets the nominal rate as an increasing function of both consumption and the price level, so it must be that in the high-P_{t-1}^* region the price-level effect dominates in the feedback rule. The policy functions for the price level (not shown) indeed reflect this fact. The price level is higher in the $\bar{\chi}$ state than in the $\underline{\chi}$ state, and the gap between the price levels in the two states is increasing in P_{t-1}^*.

Another perspective on the nominal interest rate functions in panel c comes from thinking about two relationships emphasized by Irving Fisher. We have already seen the "Fisher Equation: I," which states that the nominal interest rate is approximately equal to the sum of the real interest rate and expected inflation.[15] But Fisher also provided the seminal discussion of the relationship between real interest rates and current and future marginal utilities of consumption. Since the real interest rate is the price at which agents can trade current consumption for future consumption, it follows that agents will choose an expected consumption path to equate the real interest rate to the ratio of marginal utilities of current and future consumption. When utility is logarithmic in consumption, as it is here, this "Fisher Equation: II" implies that the real interest rate is approximately equal to expected consumption growth.[16]

From panel a, we know that consumption and the preference parameter move in opposite directions. Further, the stochastic process for the preference parameter is mean reverting, so that when χ_t is low it is expected to increase, and, therefore, consumption is expected to fall. From Fisher's second equation,

[15] For an explanation of why the "Fisher Equation: I" is only approximately correct, see Sarte (1998).

[16] The relationship is only approximate here because the shopping time requirement means that the marginal utility of consumption is greater than the marginal value of a unit of real wealth. To derive this approximate relationship, combine (7) and (10) above.

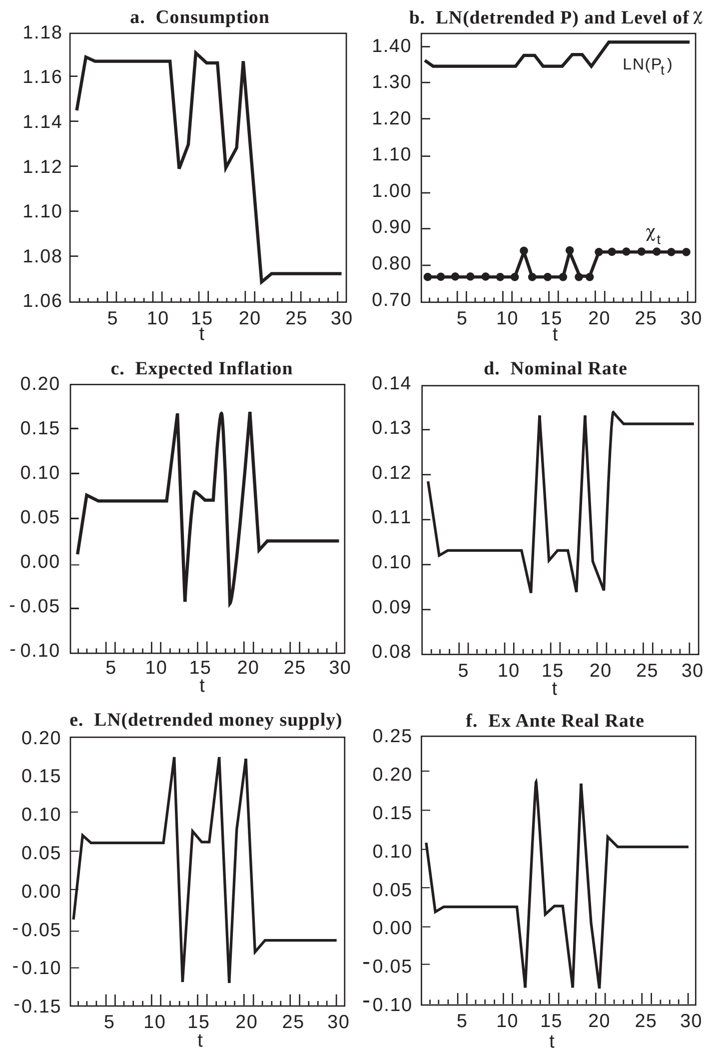
real interest rates are then low when the preference parameter is low. Note, however, that the policy rule typically makes nominal rates high in those cases when we have just argued that real rates are low. From Fisher's first equation, it must then be that high nominal rates correspond to high enough expected inflation to counteract the low real rates. From panel d we can see that monetary policy does in fact deliver high expected inflation when the preference parameter is low. The money supply is low when the preference parameter is low, and mean reversion implies that the money supply is expected to increase in those periods, generating high expected inflation.

Note that the behavior of real interest rates conflicts with the behavior displayed in the other articles discussed above. There the monetary authority lowers nominal interest rates when output is low, and real rates fall as well. Here, for the most part, the monetary authority also decreases nominal interest rates when output is low. However, real interest rates are to a great extent determined by the shock process in conjunction with Fisher's second equation. For a large class of such processes that includes the one used here, real interest rates are low when output is *high*. More generally, it has proven difficult to produce models where the cyclical behavior of real rates matches the data without resorting to the type of reduced form modeling employed by Fuhrer and Madigan (1997) and Orphanides and Wieland (1998).

Simulated Time Paths

Figure 2 displays the time paths of the variables from Figure 1 other than P_t^*, as well as the price level, the real interest rate, and expected inflation, for a sequence of 30 χ_t drawn from the stochastic process described above. This sequence will be a benchmark for comparison with the low inflation target case below. Focusing first on consumption (panel a), note that there are essentially three regions: low, high, and intermediate. The high-consumption region is attained with any sequence of at least two consecutive low values for χ_t (the realizations of χ_t are plotted in panel b). Likewise, the low-consumption region is attained with any sequence of at least two consecutive high values for χ_t. These regions correspond to the points marked **x** in Figure 1a and b. The intermediate-consumption region corresponds to the transition from one value of the preference shock to the other; these are the points marked **y** in Figure 1a and b. The fact that it takes two periods to transit between the high- and low-consumption regions is an implication of two-period price stickiness. To see this, suppose the economy had been in the low preference parameter/high-consumption state for several periods. If χ_t then took on a high value, in the initial period the state variable (P_{t-1}^*) would be at the level associated with $\underline{\chi}$, so that the economy could not immediately transit to low consumption. If χ_t remained high in the next period, consumption would settle at a lower level, because the state variable had changed; by the period after the shift in χ_t, all

Figure 2 Time Paths from 30-Period Simulation (5 Percent Inflation Target)



firms would have had a chance to adjust their price. If prices were flexible, the transition would be immediate, whereas with prices set for more than two periods the transition would be correspondingly longer.

Note that in some of the periods when consumption takes on an intermediate value, the real rate is negative (Figure 2f). Specifically, this occurs in periods when $\chi_t = \overline{\chi}$ and $\chi_{t-1} = \underline{\chi}$ (periods 12, 17, and 20). Referring back to Figure 1, one can see that in this situation consumption is expected to fall towards the low level associated with $\bar{\chi}$. With consumption expected to fall significantly, the real rate must be negative. Because the inflation target is 5 percent, the zero bound does not inhibit the real rate from going negative. However, one might expect that with a very low inflation target, the real rate would be inhibited from going negative, and thus the zero bound would interfere with the economy's "natural" behavior.

Figures 3 and 4 correspond to Figures 1 and 2, with an inflation target of -5 percent. From Figure 3a–c, we see that for a wide range of values of the state variable, including the region corresponding to high consumption, the nominal rate is zero. This drastically different behavior of the nominal rate, however, does not correspond to significantly different functions for consumption (Figure 3a). The simulation in Figure 4 confirms these results. Whereas we surmised that the nominal rate might hit the zero bound when $\chi_t = \bar{\chi}$ and $\chi_{t-1} = \underline{\chi}$, in fact it hits the bound whenever $\chi_t = \underline{\chi}$. However, consumption behavior is almost indistinguishable from Figure 2, the 5 percent inflation target. From Fisher's second equation, we know that similar consumption behavior must correspond to similar real rate behavior, as confirmed in Figure 4f. How is a zero nominal rate consistent with a negative real rate in periods 12, 17, and 20? From Fisher's first equation, the real rate is the difference between the nominal rate and expected inflation, so in those periods the monetary authority is making expected inflation positive (panel c). The targeted rate of deflation is consistent with periods of high expected inflation, because the policy rule unambiguously makes the expected inflation temporary, and there is no uncertainty about whether the monetary authority will adhere to the policy rule.

Simulations such as those in Figures 2 and 4 are an informal means of evaluating whether the zero bound is important. However, those simulations provide clear evidence—at least in the model used here—that monetary policy can offset the zero bound by generating temporary expected inflation. With real rates thus unconstrained, the existence of the zero bound does not appear to constitute an argument against a low inflation target. Figure 4 illustrates an additional feature of the model that *favors* a very low inflation target. In panels a and f, the series for consumption and real rates from Figure 2, corresponding to a 5 percent inflation target, are reproduced along with the new series corresponding to 5 percent deflation. In panel a, we see that consumption is actually higher in every period with the 5 percent deflation target than it is with the 5 percent inflation target. The lower inflation target corresponds to lower nominal interest rates on average, as is shown clearly in panel d of Figures 2 and 4. Lower nominal interest rates in turn correspond to a smaller money demand distortion, as in Bailey (1956) and Friedman (1969). Individuals hold higher real balances because the opportunity cost of real balances has fallen, and higher real balances effectively make consumption cheaper, because they decrease the time that an individual must spend transacting.

Figure 3 Functions Mapping State Variable (P^*_{t-1}) to Other Variables at 5 Percent Deflation Target

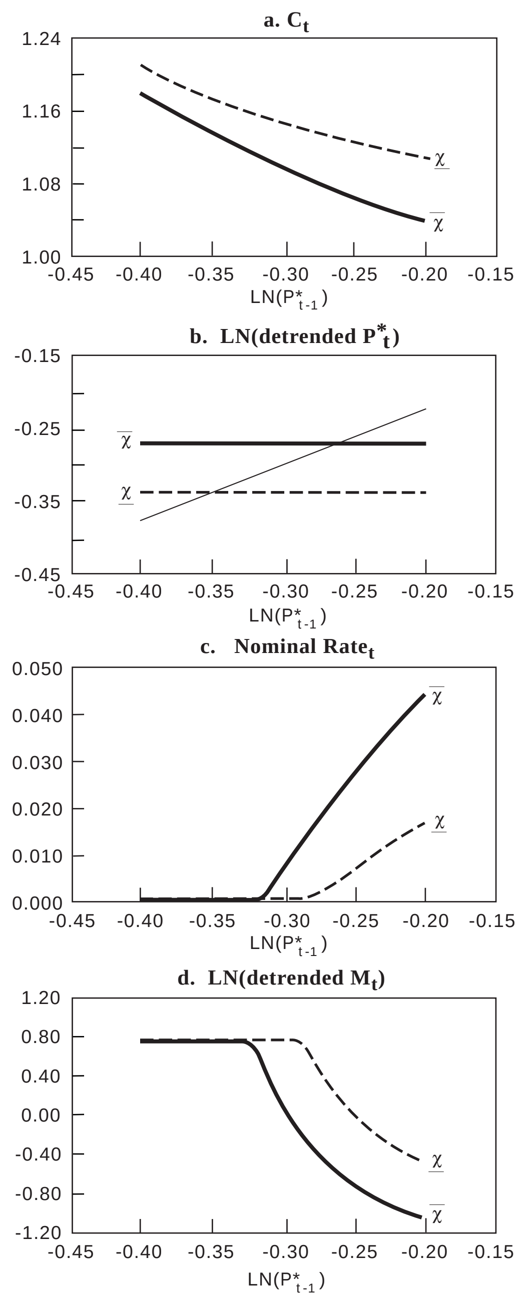
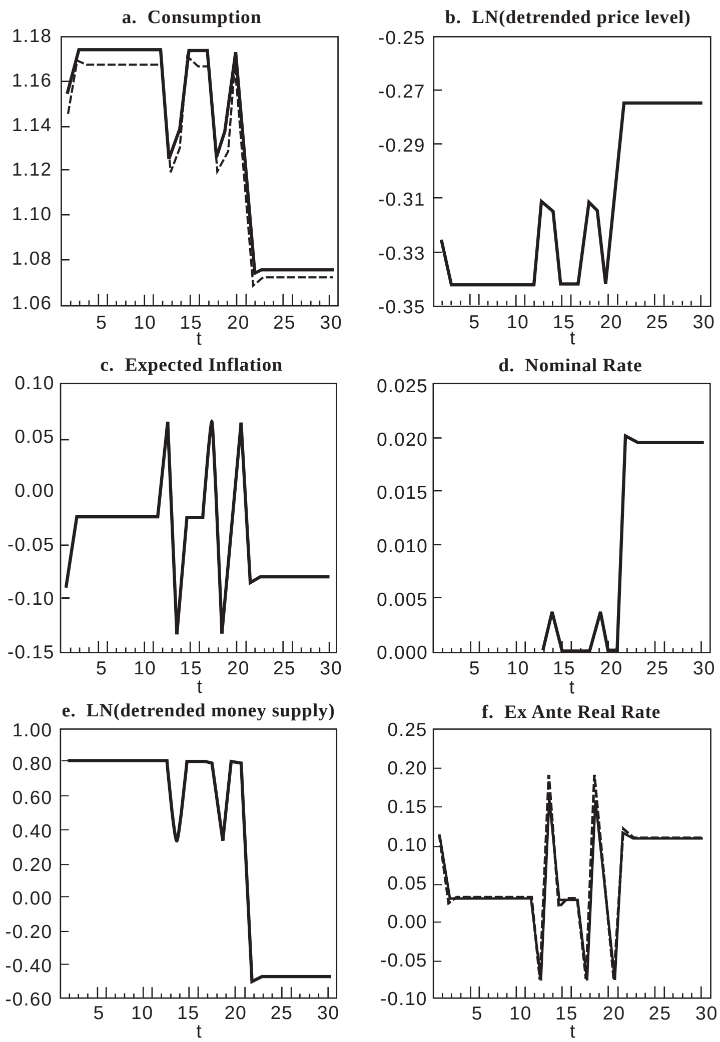


Figure 4 Time Paths from 30-Period Simulation (5 Percent Deflation Target)



Note: Dashed series are from Figure 2 (5 percent inflation target).

Variances

The simulations in Figures 2 and 4 provide strong evidence on the importance of the zero bound, and the welfare results below give the bottom line. To enhance comparability with the articles by Rotemberg and Woodford (1997) and Orphanides and Wieland (1998), we also provide information on variability at high and low inflation targets. Table 1 shows the standard deviations of some of the main variables in the model for both regimes, based on simulations of 5,000 periods. As suggested by Figures 1–4, the variability of consumption is barely affected by the inflation target. On the other hand, the nominal interest rate is much less variable when the inflation target makes zero occasionally binding. There is a tradeoff in the model between the average level of inflation and the minimum feasible variability of inflation, just as described in Rotemberg and Woodford (1997). Also as in that paper, the large difference in nominal interest rate variability in the two regimes translates into only a small difference in inflation variability. A striking feature of Table 1 is the tremendous increase in money supply variability in the deflation regime. This can be traced to the fact that the money demand function exhibits increasing sensitivity to nominal interest rates as the nominal interest rate falls.

Table 1 Standard Deviations in the Two Policy Regimes

	Consumption	Inflation	Nominal rates	Money
5 percent inflation	0.0427	0.0706	0.0145	0.0910
5 percent deflation	0.0435	0.0786	0.0093	0.7562

Welfare

The motivation for this article came from the idea that low inflation targets might be bad because of distortions introduced by the zero bound on nominal interest rates. It is clear from the simulations presented thus far that in fact the real (as opposed to nominal) distortions associated with the zero bound are small. Nevertheless, it is interesting to know whether the inflation or deflation regime is preferred on welfare grounds. When the zero bound is not a factor, a welfare comparison will hinge on the other distortions present in the model. Those other distortions involve the inflation tax and the interaction between sticky prices and monopolistic competition. The inflation tax distortion makes deflation preferable to inflation. Sticky prices and monopolistic competition make the optimal inflation target near zero, so neither 5 percent inflation nor deflation targets would obviously be preferred to the other on that basis. It therefore seems likely that the unambiguous effect of the inflation tax will dictate that the lower inflation regime is preferred. However, to resolve the issue definitively, we must compare the representative individual's expected utility in the inflation and deflation regimes.

We calculate expected utility by performing 1,000 simulations of 1,000 periods each, with each simulation beginning from a random value for the state variable. The initial condition is chosen by simulating the model for 50 periods, starting from the steady state, and then setting $P_0 = P_{50}$. Each simulation ($k = 1$ to $1,000$) yields a value for $U_k \equiv \sum_{t=0}^{1,000} \beta^t \cdot [\ln(c_t) + \chi_t \cdot l_t]$, and then expected utility is given by $E(U) = 1,000^{-1} \cdot \sum_{k=1}^{1,000} U_k$. With values for expected utility in both regimes, we compare the regimes by pretending that they were generated in a steady state. We calculate the average per-period utility in the two regimes and then the percentage increase in consumption that would make an agent living in the lower utility regime just as well-off as an agent in the higher utility regime. The results of this exercise are that an agent living in the inflationary regime would be indifferent between receiving a 2.6 percent increase in per-period consumption and switching to the deflationary regime.

To illustrate the importance of the inflation tax in these results, we can repeat the comparison of the two inflation regimes with a slight modification. That modification is to eliminate the money demand distortion; we modify (7) to $\lambda_t = 1/c_t$ and replace (11) with $M_t = P_t \cdot c_t$. With the inflation tax eliminated, the 5 percent inflation target regime is marginally preferred to the 5 percent deflation target regime, although the difference in welfare is minuscule compared to the difference found (with opposite sign) when the inflation tax played a role. The results from eliminating the money demand distortion mean that money demand is crucial in making the deflationary regime welfare-superior to the inflationary regime. However, even without the money demand distortion, the fact that the nominal interest rate is occasionally zero in the deflationary regime does not significantly affect the behavior of real variables. In particular, the policy rule is still able to generate temporarily high expected inflation when real rates need to be negative.

Open Questions

With respect to the specific model used here, at least three modifications would be interesting to analyze. The first modification deals with the specification of price stickiness. Structural models of sticky inflation are ad hoc, but they have been shown to fit recent data well. It should be possible to modify the price block of the current model to make inflation sticky. The resulting specification would not simply repeat the work of Orphanides and Wieland (1998) and Fuhrer and Madigan (1997), because it would incorporate money demand. Solving such a model would be more computationally intensive than solving the model in this article, because it would include additional state variables associated with the pricing specification.

The second modification is related to the first; it involves changing the policy rule from the price-level form to the more common inflation form. Possibly with such a rule and a low inflation target the monetary authority would be less able to generate the temporary expected inflation necessary to drive real rates negative. More generally, it would be interesting to study the properties of a wide range of rules and to find out what the optimal rule is. Experiments with a rule that specifies the money supply instead of the nominal interest rate as the policy instrument yield similar results to those above, in that the deflationary regime is preferred to the inflationary regime. The interest rate rule generates higher welfare than the money rule, but that comparison is limited, focusing on two specific rules as opposed to classes of rules. In terms of optimal rules, King and Wolman (forthcoming 1999) find that it is optimal to stabilize the price level if the money demand distortion is nonexistent. With that distortion present, optimal policy will undoubtedly involve some deflation, but it is not clear exactly what the optimal policy rule is.[17]

The third modification is one that takes more seriously the fiscal aspect of monetary policy. Work by Woodford (1996) and Sims (1994) emphasizes the joint behavior of fiscal and monetary policy. This joint behavior might be especially relevant when interest rates are near zero, because at zero nominal interest rates, fiscal and monetary policy effectively become unified; money and government bonds are perfect substitutes.

Apart from the specifics of the model, the assumption that agents in the model have perfect information about the policy rule is crucial. We found that zero nominal interest rates did not prevent the real rate from falling, because the monetary authority could generate expected inflation when the nominal rate was zero. Agents know that any inflation that ensues will be temporary, and that the monetary authority remains committed to its stated inflation target, so these occasional periods of high expected inflation do not trigger inflation scares. In practice, central banks might have concerns about being able to generate occasional episodes of high expected inflation without endangering the credibility of their low inflation target. In principle it would be possible to analyze this sort of issue in an extension of the current framework.

A fundamental assumption underlying all recent work on the zero bound is that negative ex ante real interest rates are occasionally a natural characteristic of the U.S. economy. It is a trivial matter to look at data on ex post real rates and see that at the short end of the yield curve they have been negative on many occasions. It is less clear that ex ante real rates have been negative. From Irving Fisher, we know that real rates defined by the CPI can be negative only to the extent that the market basket that makes up the CPI is not storable at zero cost. Undoubtedly the inclusion of various services and perishable goods means that in principle the ex ante real rate can be negative. Nonetheless, lack of consensus about how to estimate inflation expectations means that widely accepted series for ex ante real rates do not exist.

[17] The approach taken in this article would suggest defining the optimal policy rule as the rule that generates the highest level of unconditional expected utility. King and Wolman (forthcoming 1999) use a different criterion; they ask what policy rule is implied by assuming that the monetary authority maximizes agents' expected utility given some arbitrary initial conditions.

4. CONCLUSIONS

Two general conclusions are supported by the theoretical analysis in this article. First, the way money demand is modeled is important for how one evaluates the zero bound on nominal interest rates. Existing work presumes that the zero bound makes low inflation bad, because it prevents monetary policy from optimally responding to shocks. But monetary theory supports a strong benefit to zero nominal interest rates, namely, eliminating inefficiencies associated with holding "too little" money. The existence of those inefficiencies contributed to the result in this article that, taking into account the zero bound, a regime with moderate deflation yields higher welfare than a regime with moderate inflation. The second conclusion is that stickiness of inflation is crucial in generating costs of low inflation associated with the zero bound. If prices are sticky but inflation is not, then real rates can fall even if nominal interest rates are very low: the monetary authority simply creates some expected inflation if it wants to drive real rates down.

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