Sticky Prices, Marginal Cost, and the Behavior of Inflation

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A principal goal of economic modeling is to improve the formulation of economic policy. Macroeconomic models with imperfect competition and sticky prices set in a dynamic optimizing framework have gained wide popularity in recent years for examining issues involving monetary policy. For example, Rotemberg and Woodford (1999b) and McCallum and Nelson (1999) examine the behavior of model economies under a variety of monetary policy rules; Ireland (1995) examines the optimal way to disinflate; and Benhabib, Schmitt-Grohe, and Uribe (forthcoming) and Wolman (1998) study the monetary policy implications of the zero bound on nominal interest rates.[1] Nevertheless, serious questions remain as to whether these models accurately describe the U.S. economy, and therefore as to how one should interpret the results of this research.

One criticism of optimizing sticky-price models is that the relationship between output and inflation they generate is inconsistent with the behavior of these variables in the United States.[2] However, recent research by Sbordone (1998) and Galí and Gertler (1999) has breathed new life into these models by shifting attention away from the relationship between output and inflation and toward one between marginal cost and inflation—the latter being a more fundamental relationship in the models. If firms have some market power, as under imperfect competition, the behavior of their marginal cost of production is an important determinant of how they set prices. In turn, the overall price

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[1] These are but a few of the many papers using such models. For a survey, see Taylor (1999).

[2] See Fuhrer and Moore (1995).

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level and inflation rate are determined by aggregating individual firms' pricing decisions. There is then a clear relationship between the behavior of individual firms' marginal cost and the behavior of inflation. Sbordone (1998) and Galí and Gertler (1999) use this relationship to estimate and evaluate optimizing sticky-price models.[3] They find that such models can accurately replicate the observed behavior of inflation.

In this article, we work through the details of a sticky-price model, making explicit the relationship between marginal cost and inflation just described. We then offer a criticism of the specific form of price stickiness used by Sbordone and Galí and Gertler; essentially, they let prices be implausibly sticky. Plausible forms of price stickiness generate fundamentally different inflation dynamics and hence will be more difficult to reconcile with the behavior of marginal cost and inflation in the United States. However, the methodology introduced by Sbordone (1998) and Galí and Gertler (1999) remains a promising approach for evaluating sticky-price models. We suggest two ways in which this research agenda can continue progressing.

We concentrate on partial equilibrium analysis. The analysis takes as given the average inflation rate and the behavior of demand and real marginal cost. A complete general equilibrium version of our sticky-price framework would include descriptions of factor markets, consumer behavior, and monetary policy. In a general equilibrium, marginal cost and inflation would be endogenous; conditional on private behavior, policy would determine the behavior of inflation. Nonetheless, even in a general equilibrium, one would observe the relationship between marginal cost and inflation that is the focus of this article.

1. FROM INDIVIDUAL FIRMS' PRICING TO AGGREGATE INFLATION

Two central components comprise most of the recent optimizing sticky-price models: (1) monopolistic competition among a large number of firms producing differentiated products and (2) limited opportunities for price adjustment by individual firms. Monopolistic competition makes it feasible for some firms not to adjust their price in a given period; under perfect competition, only firms that charged the lowest price would sell anything. Limited price adjustment means that real and nominal variables interact; output and real marginal cost—both real variables—affect individual firms' pricing decisions, which in turn affect the price level and inflation.

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[3] There is a separate literature dating back at least to the 1960s and continuing today that relates the behavior of inflation to marginal cost in reduced-form econometric models. See, for example, Eckstein and Wyss (1972).

Monopolistic Competition

The first component is monopolistic competition. The monopolistic competition framework most common in recent models is that of Dixit and Stiglitz (1977). The large number of firms mentioned above is represented mathematically by a continuum, and the firms are indexed by $z \in (0, 1)$. Assume that these firms' differentiated products can be aggregated into a single good, interpreted as final output. If $y_t(z)$ is the amount produced by firm z, final output is

$$y_t = \left(\int_0^1 y_t(z)^{(\varepsilon-1)/\varepsilon} dz \right)^{\varepsilon/(\varepsilon-1)}. \tag{1}$$

With this aggregator function and market structure, demand for the good produced by firm z is given by

$$y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} y_t, \tag{2}$$

where $P_t(z)$ is the nominal price of good z, and P_t and the price of one unit of y_t. According to (2), demand for good z has a constant elasticity of $-\varepsilon$ with respect to the relative price of good z, and given the relative price, demand is proportional to the index of final output (y_t). The Appendix contains a detailed derivation of the demand function (2) and shows that the price index (P_t) is

$$P_t = \left(\int_0^1 P_t(z)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{3}$$

The price index has the property that an increase in the price of one of the goods has a positive but not necessarily one-for-one effect on the index. If that good's nominal price is lower (higher) than the price index, an increase in its price raises the price index more (less) than one-for-one, because the good has a relatively high (low) expenditure share.

Limited Price Adjustment

Limited opportunities for price adjustment constitute the second important component of our representative model. We assume that any firm $z \in (0, 1)$ faces an exogenous probability of adjusting its price in period t and that the probability may depend on when the firm last adjusted its price. The probability of adjusting is non-decreasing in the number of periods since the last adjustment, and we denote by J the maximum number of periods a firm's price can be fixed.[4] The key notation describing limited price adjustment will be a vector $\underset{\sim}{\alpha}$; the jth element of $\underset{\sim}{\alpha}$, called α_j, is the probability that a firm adjusts its price in period t, conditional on its previous adjustment having occurred in period $t - j$.

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[4] Looking ahead, one of the specifications we will focus on has $J = \infty$.

From the vector $\underset{\sim}{\alpha}$ we derive the fractions of firms in period t charging prices set in periods $t - j$, which we denote by ω_j. To do this, note that

$$\omega_j = (1 - \alpha_j)\omega_{j-1}, \text{ for } j = 1, 2, ..., J - 1,$$

$$\text{and} \tag{4}$$

$$\omega_0 = 1 - \sum_{k=1}^{J-1} \omega_k.$$

This system of linear equations can be solved for ω_j as a function of $\underset{\sim}{\alpha}$. The most common pricing specifications in the literature are those first described by Taylor (1980) and Calvo (1983). Taylor's specification is one of uniformly staggered price setting: every firm sets its price for J periods, and at any point in time a fraction $1/J$ of firms charge a price set j periods ago. The $(J-1)$-element vector of adjustment probabilities for the Taylor model is $\underset{\sim}{\alpha} = [0, ..., 0]$, and the J-element vector of fractions of firms is $\underset{\sim}{\omega} = [1/J, 1/J, ..., 1/J]$. In contrast, Calvo's specification involves uncertainty about when firms can adjust their price. No matter when a firm last adjusted its price, it faces a probability α of adjusting. Thus, the infinite vector of adjustment probabilities is $\underset{\sim}{\alpha} = [\alpha, \alpha, ...]$, and the infinite vector of fractions of firms is $\omega_j = \alpha(1 - \alpha)^j, j = 0, 1, ...$ For the specification we will advocate in Section 3, contrary to Taylor and Calvo, the adjustment fractions are strictly increasing in j and J is finite.

For any pattern of price adjustment, as defined by the α_j or ω_j, the price index (3) can be simplified to reflect the fact that all firms that set their price in the same period will choose the same price.[5] Let $P_{0,t}$ denote the price chosen by adjusting firms in period t. Then the price index can be written as

$$P_t = \left(\sum_{j=0}^{J-1} \omega_j \cdot (P_{0,t-j})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{5}$$

The next step is to show how $P_{0,t}$ is determined.

Optimal Pricing Decisions

In those periods when a firm is able to adjust its price, the price that it chooses will be affected by the pattern of future adjustment opportunities it expects, that is, by $\underset{\sim}{\alpha}$. To determine the optimal price for an adjusting firm, we must first state the firm's profit-maximization problem. If $\pi_{j,t}$ denotes the nominal profits in period t of a firm that charges a price set in period $t - j$, then the

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[5] If there are firm-specific state variables other than price, then all adjusting firms will generally not choose the same price. This would be the case, for example, if firms faced costs of adjusting their labor input. Such a model would be more difficult to analyze, as the number of different types of firms one would need to track would grow without bound over time.

expected present discounted value of profits that the firm is concerned with when it adjusts its price is

$$\Pi_t = E_t \sum_{j=0}^{J-1} \Delta_{j,t+j} \left(\omega_j/\omega_0 \right) \pi_{j,t+j}, \tag{6}$$

where E_t denotes expectation that is conditional on information available when the period t pricing decision is made, and $\Delta_{j,t+j}$ is the discount factor appropriate for discounting nominal profits from period $t+j$ back to period t.[6] The factor (ω_j/ω_0) is the probability that a firm that adjusts its price in period t will still be charging that price in period $t+j$.[7] Although the summation stops with period $t+J-1$, the firm of course cares about its profits further in the future than period $t+J-1$. However, its choice of a price in period t has no bearing on profits beyond period $t+J-1$, because by then a new price will be chosen.[8] From (6), the firm's optimal price sets expected discounted marginal profits to zero:

$$E_t \sum_{j=0}^{J-1} \Delta_{j,t+j}(\omega_j/\omega_0)\frac{\partial \pi_{j,t+j}}{\partial P_{0,t}} = 0. \tag{7}$$

If the firm could adjust its price every period, then ω_j would be zero for all j greater than zero; the optimal price would make marginal profits zero within every period. Price stickiness means that marginal profits are generally nonzero within a period, but the discounted sum of marginal profits is zero.

Profits in a given period are the difference between revenue and costs. For a firm in period $t+j$ that charges a price it set in period t, we will denote the demand it faces and its costs of production by $y_{j,t+j}$ and $TC_{j,t+j}$, respectively. Its profits can then be expressed as

$$\pi_{j,t+j} = P_{0,t} y_{j,t+j} - TC_{j,t+j}. \tag{8}$$

Substituting from the demand function (2) yields

$$\pi_{j,t+j} = P_{0,t} \left(\frac{P_{0,t}}{P_{t+j}} \right)^{-\varepsilon} y_{t+j} - TC_{j,t+j}. \tag{9}$$

Total revenue—the first term in (9)—is simply the product of the price the firm charges and the demand it faces. Total costs will generally depend on

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[6] Below, we will assume that the discount factor is given by the product of nominal interest rates:

$$\Delta_{j,t+j} = \left((1+R_t)^{-1}(1+R_{t+1})^{-1} \cdots (1+R_{t+j-1})^{-1} \right)$$

for $j > 0$, and $\Delta_{0,t} = 1$.

[7] This factor can also be written $\prod_{k=0}^{j}(1-\alpha_k)$, where $\alpha_0 \equiv 0$.

[8] We are implicitly assuming there are no other linkages between profits in the current period and the firms' decisions in prior periods. This assumption may not be innocuous. For example, it rules out dependence of the firm's costs in period t on its production in a prior period.

factor prices, factor utilization, and the level of technology. For now we leave unspecified the determinants of costs. Below we will describe assumptions that imply marginal cost can be easily measured. Differentiating (9) yields an expression for marginal profits:

$$\frac{\partial \pi_{j,t+j}}{\partial P_{0,t}} = (1 - \varepsilon) \left(\frac{P_{0,t}}{P_{t+j}} \right)^{-\varepsilon} \cdot y_{t+j} - \frac{\partial TC_{j,t+j}}{\partial P_{0,t}}. \tag{10}$$

The first term in (10) is marginal revenue with respect to price. Because there is a constant elasticity of demand greater than unity, marginal revenue *with respect to price* is always negative; *lowering* its price will always increase a firm's revenue. The second term is marginal cost with respect to price. It is convenient to express the firm's marginal cost with respect to quantity produced rather than with respect to price; therefore, use the fact that $y_{j,t+j} = \left(\frac{P_{0,t}}{P_{t+j}} \right)^{-\varepsilon} \cdot y_{t+j}$ to write (10) as

$$\frac{\partial \pi_{j,t+j}}{\partial P_{0,t}} = (1 - \varepsilon) \left(\frac{P_{0,t}}{P_{t+j}} \right)^{-\varepsilon} \cdot y_{t+j}$$

$$+ \varepsilon \left(\frac{P_{0,t}}{P_{t+j}} \right)^{-1-\varepsilon} \cdot y_{t+j} \left\{ \frac{\partial TC_{j,t+j}}{\partial y_{j,t+j}} \cdot \frac{1}{P_{t+j}} \right\}. \tag{11}$$

The object in brackets will be referred to as *real marginal cost;* it is the firm's marginal production cost denominated in the final good. The other factors in the second term represent the effect of a change in the price charged on the quantity of goods demanded from the firm. Since real marginal cost plays a major role in what follows, we denote that variable by the shorthand expression

$$\psi_{j,t+j} \equiv \frac{\partial TC_{j,t+j}}{\partial y_{j,t+j}} \cdot \frac{1}{P_{t+j}}. \tag{12}$$

Following up on the above discussion of total costs, real marginal cost will generally depend on variables such as the real wage. Measuring marginal cost directly is generally not a simple matter.

To derive an explicit expression for an adjusting firm's optimal price, first substitute the derivation of marginal profits (11) into the first-order condition (7):

$$E_t \sum_{j=0}^{J-1} \Delta_{j,t+j} (\omega_j / \omega_0)$$

$$\left[(1 - \varepsilon) \left(\frac{P_{0,t}}{P_{t+j}} \right)^{-\varepsilon} y_{t+j} + \varepsilon \left(\frac{P_{0,t}}{P_{t+j}} \right)^{-1-\varepsilon} y_{t+j} \psi_{j,t+j} \right] = 0. \tag{13}$$

Next, multiply (13) by $P_{0,t}^{1+\varepsilon}P_t^{-\varepsilon}$ and rearrange to get

$$P_{0,t} = P_t\left(\frac{\varepsilon}{\varepsilon - 1}\right)\frac{E_t\sum_{j=0}^{J-1}\Delta_{j,t+j}(\omega_j/\omega_0)\left(\frac{P_t}{P_{t+j}}\right)^{-1-\varepsilon}y_{t+j}\psi_{j,t+j}}{E_t\sum_{j=0}^{J-1}\Delta_{j,t+j}(\omega_j/\omega_0)\left(\frac{P_t}{P_{t+j}}\right)^{-\varepsilon}y_{t+j}}. \qquad (14)$$

If the price level and marginal cost are constant, then (14) yields the constant markup that is familiar from static monopolistic competition models: $P_{0,t} = P_t\left(\frac{\varepsilon}{\varepsilon-1}\right)\psi$. This is the markup (or relative price) that maximizes one-period profits. If the price level or marginal cost are not constant, then neither is the relative price that maximizes one-period profits. Therefore a firm whose nominal price may be fixed for more than one period chooses a nominal price that it expects will sacrifice the fewest discounted profits over the life of the price.

Inflation

If aggregate demand (y_t), real marginal cost (ψ_t), and nominal interest rates (equivalently the discount factors $\Delta_{j,t+j}$) are taken as given, then the pair of equations (5) and (14) jointly describe the behavior of the aggregate price level and the price chosen by individual firms. Thus, if we knew the processes governing aggregate demand, real marginal cost, and nominal interest rates, then we could use (5) and (14) to determine the behavior of the price level and hence inflation. In general it is tedious to obtain an explicit expression for inflation. However, it is easy to compute the behavior of inflation. A simple pricing specification will suffice to illustrate the method by which one can compute the behavior of inflation. In analyzing this special case, we will linearize the equations for the price index and for optimal pricing around a steady state with constant inflation rate μ. Linear approximations are also used in the empirical work by Sbordone (1998) and Galí and Gertler (1999).

The special case is a model where no firm sets its price for more than two periods, so that $\underset{\sim}{\alpha} = [\alpha_1]$ and $\underset{\sim}{\omega} = [1/(2-\alpha_1),(1-\alpha_1)/(2-\alpha_1)]$. In this case the price index is

$$P_t = \left(\frac{1}{2-\alpha_1}\cdot P_{0,t}^{1-\varepsilon} + \frac{1-\alpha_1}{2-\alpha_1}\cdot P_{0,t-1}^{1-\varepsilon}\right)^{1/(1-\varepsilon)},$$

and the optimal pricing equation is

$$P_{0,t} = P_t\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{y_t\psi_{0,t} + (1-\alpha_1)E_t[\Delta_{1,t+1}(P_t/P_{t+1})^{-1-\varepsilon}y_{t+1}\psi_{1,t+1}]}{y_t + (1-\alpha_1)E_t[\Delta_{1,t+1}(P_t/P_{t+1})^{-\varepsilon}y_{t+1}]}.$$

By linearizing these equations around a steady state with gross inflation equal to μ, we will get a system of expectational difference equations. Before linearizing, rewrite the equations in terms of detrended nominal variables:

$$\tilde{P}_t^{1-\varepsilon} = \left(\left(\frac{1}{2-\alpha_1}\right) \cdot \tilde{P}_{0,t}^{1-\varepsilon} + \left(\frac{1-\alpha_1}{2-\alpha_1}\right) \cdot \mu^{\varepsilon-1}\tilde{P}_{0,t-1}^{1-\varepsilon}\right), \qquad (15)$$

and

$$\tilde{P}_{0,t} = \tilde{P}_t \left(\frac{\varepsilon}{\varepsilon-1}\right)$$

$$\cdot \frac{y_t\psi_{0,t} + (1-\alpha_1)E_t[\Delta_{1,t+1}\mu^{1+\varepsilon}(\tilde{P}_t/\tilde{P}_{t+1})^{-1-\varepsilon}y_{t+1}\psi_{1,t+1}]}{y_t + (1-\alpha_1)E_t[\Delta_{1,t+1}\mu^\varepsilon(\tilde{P}_t/\tilde{P}_{t+1})^{-\varepsilon}y_{t+1}]}. \qquad (16)$$

In (15), (16), and henceforth, the variables $\tilde{P}_{0,t}$ and $\tilde{P}_t$ should be interpreted as deviations from a trend that is growing at rate μ; that is, $\tilde{P}_{0,t} = P_{0,t}/\mu^t$ and $\tilde{P}_t = P_t/\mu^t$.

Linearizing the price index (15) yields

$$\left(\frac{P_0}{P}\right)^{\varepsilon-1}\hat{P}_t = \left(\frac{1}{2-\alpha_1}\right) \cdot \hat{P}_{0,t} + \left(\frac{1-\alpha_1}{2-\alpha_1}\right) \cdot \mu^{\varepsilon-1}\hat{P}_{0,t-1}. \qquad (17)$$

Here (P_0/P) denotes the ratio of the price set by an adjusting firm to the aggregate price level in a steady state where the price level is growing at rate μ, and $\hat{P}_t$ and $\hat{P}_{0,t}$ are logarithmic deviations from the steady-state values of $\tilde{P}_t$ and $\tilde{P}_{0,t}$, respectively. The steady-state ratio (P_0/P) can easily be determined from (15) as $(P_0/P) = \left(\frac{1+(1-\alpha_1)\mu^{\varepsilon-1}}{2-\alpha_1}\right)^{1/(\varepsilon-1)}$. According to (17), if inflation is high enough or the probability of adjustment is low enough, then a given change in prices set in the previous period has a larger effect on this period's price index than does the same change in prices set this period. The reason is that, with inflation eroding relative prices, the relative prices of goods set in the previous period is low, and hence the quantity of those goods purchased is high. Furthermore, with relatively elastic demand, the share of expenditure on the low-priced goods will be higher than the share of expenditure on the high-priced goods, meaning that goods with a price set in the previous period carry greater weight in the steady-state price index.[9]

Linearizing the equation for optimal price of an adjusting firm (16) yields

$$\left(\frac{P_0}{P}\right)\hat{P}_{0,t} = a_1\hat{P}_t + (a_2 - a_3)(E_t\hat{P}_{t+1} - \hat{P}_t) + b_0\hat{\psi}_{0,t} + b_1E_t\hat{\psi}_{1,t+1} + x_t, \quad (18)$$

where

$$a_1 \equiv \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\psi_0 + (1-\alpha_1)\Delta_1\mu^{1+\varepsilon}\psi_1}{1 + (1-\alpha_1)\Delta_1\mu^\varepsilon}\right),$$

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[9] If there is a high adjustment probability, then this expenditure share is low, and the opposite result holds.

$$a_2 \equiv \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{(1 - \alpha_1)\Delta_1\mu^{1+\varepsilon}\psi_1}{1 + (1 - \alpha_1)\Delta_1\mu^\varepsilon}\right)(1 + \varepsilon),$$

$$a_3 \equiv \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\psi_0 + (1 - \alpha_1)\Delta_1\mu^{1+\varepsilon}\psi_1}{1 + (1 - \alpha_1)\Delta_1\mu^\varepsilon}\right)\left(\frac{(1 - \alpha_1)\Delta_1\mu^\varepsilon}{1 + (1 - \alpha_1)\Delta_1\mu^\varepsilon}\right)\varepsilon,$$

$$b_0 \equiv \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\psi_0}{1 + (1 - \alpha_1)\Delta_1\mu^\varepsilon}\right),$$

$$b_1 \equiv \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{(1 - \alpha_1)\Delta_1\mu^{1+\varepsilon}\psi_1}{1 + (1 - \alpha_1)\Delta_1\mu^\varepsilon}\right),$$

and

$$x_t \equiv \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{1}{1 + (1 - \alpha_1)\Delta_1\mu^\varepsilon}\right)\{\psi_0\hat{y}_t + (1 - \alpha_1)\Delta_1\mu^{1+\varepsilon}\psi_1$$

$$\cdot E_t(\hat{\Delta}_{1,t+1} + \hat{y}_{t+1}) - \left(\frac{\psi_0 + (1 - \alpha_1)\Delta_1\mu^{1+\varepsilon}\psi_1}{1 + (1 - \alpha_1)\Delta_1\mu^\varepsilon}\right)[\hat{y}_t + (1 - \alpha_1)\Delta_1\mu^\varepsilon$$

$$\cdot E_t(\hat{\Delta}_{1,t+1} + \hat{y}_{t+1})]\}.$$

If α_1 is low enough and μ is high enough, then $(a_2 - a_3)$ will be positive, in which case (18) says that the price set by an adjusting firm is increasing in the price level and increasing in next period's expected inflation. A firm raises its price as the price level rises because it has an optimal level for its *relative* price (note that [16] can be written with $P_{0,t}/P_t$ on the left-hand side). Expected inflation next period raises a firm's desired price because it means that any price set in the current period will erode in relative terms; firms compensate for the erosion by setting a higher price when they can adjust. The coefficients b_0 and b_1 are positive, which means that the price chosen by adjusting firms responds positively to marginal cost in the current period and to expected future marginal cost. Finally, the variable x_t represents the effects on a firm's optimal price of current and future aggregate demand and the nominal discount factor. We have lumped these factors into the variable x_t in order to focus attention on the fact that (17) and (18) jointly determine the behavior of the price level and adjusting firms' optimal prices, *conditional* on the behavior of real marginal cost and the variables in x_t.

Pursuing now the joint determination of $\hat{P}_{0,t}$ and $\hat{P}_t$, we write (17) and (18) as a system of linear expectational difference equations in the variables $\hat{P}_t$ and $\hat{P}_{0,t-1}$:

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\begin{bmatrix} 0 & 1/(2-\alpha_1) \\ a_3 - a_2 & P_0/P \end{bmatrix} \begin{bmatrix} E_t \hat{P}_{t+1} \\ \hat{P}_{0,t} \end{bmatrix} =
$$

$$
\begin{bmatrix} (P_0/P)^{\varepsilon-1} & -\left(\frac{1-\alpha_1}{2-\alpha_1}\right)\mu^{\varepsilon-1} \\ a_1 - a_2 + a_3 & 0 \end{bmatrix} \begin{bmatrix} \hat{P}_t \\ \hat{P}_{0,t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ x_t + b_0 \hat{\psi}_{0,t} + b_1 E_t \hat{\psi}_{1,t+1} \end{bmatrix}. \tag{19}
$$

The methods described by Blanchard and Kahn (1980) allow one to solve for the behavior of $\hat{P}_t$ given a known process for $\hat{x}_t$ and $\hat{\psi}_{i,t}$.[10] For general specifications of price stickiness—that is, $\underset{\sim}{\alpha}$—the system corresponding to (19) is more complicated. There are additional expected future values of inflation, demand, marginal cost, and discount factors in the analogue to (18), and there are additional past values of optimal prices in the analogue to (17). However, the method for deriving and then solving the system of difference equations is almost identical. For any specification of $\underset{\sim}{\alpha}$ then, the solution to the analogue to (19) describes how the behavior of real marginal cost and x_t translates into the behavior of inflation. This relationship is the basis for the empirical work to be discussed next.

2.   TAKING THE MODEL TO DATA

Our microeconomic-based sticky-price model determines the behavior of the aggregate price level, and hence the inflation rate, in partial equilibrium.[11] From an empirical perspective, this relationship is important because it enables researchers to work with aggregate variables like inflation rather than individual variables like the prices of particular goods. Sbordone (1998) and Galí and Gertler (1999) apply this result in a new and interesting way: they estimate the parameters $\underset{\sim}{\alpha}$, and then test whether the estimated model successfully accounts for actual inflation.

Suppose that all of the models' parameters were known and that data on real marginal cost for different types of firms ($\psi_{j,t}$), aggregate output, and nominal interest rates (the components of x_t), were all available. Then we could use (19) to simulate the behavior of inflation. To simulate, solve the model so that the price level is expressed as a function of the exogenous variables (x_t and

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[10] Note that an initial condition for $P_{0,t-1}$ is also needed; we will assume the steady-state initial condition. The impulse response functions to be presented below are produced using algorithms developed by King and Watson (1998); these algorithms make it easy to solve the more complicated systems that result from more complicated forms of price stickiness.

[11] Recall from above that in a general equilibrium model there would also be a description of monetary policy. While monetary policy would determine the behavior of inflation, inflation behavior would still have to be consistent with individual firms' pricing. And firms would still take the behavior of inflation as given.

$\hat{\psi}_{j,t}$). Then use the observed sequences of exogenous variables to build up a simulated price-level series, from which it is easy to create a simulated inflation series. Of course the parameters are not known, but they can be estimated so that the simulated behavior of inflation is closest to what we observe. Roughly speaking, this is what Sbordone and Galí and Gertler do.[12]

Galí and Gertler make two key assumptions. The first assumption is that price stickiness is given by the Calvo specification, so that only one parameter is related to price stickiness (recall that the Calvo specification is $\alpha_j = \alpha$ for $j = 1, 2, ...\infty$). The second assumption is that all firms produce using identical Cobb-Douglas technologies and the labor market is competitive over the whole economy.[13] We will take up the pricing specification later. Here we explain the importance of the second assumption.

In order for the empirical approach described above to be feasible, the researcher must have access to data on marginal cost. But unlike GDP or inflation, marginal cost is not a data series measured by a government statistical agency. Measurement is lacking for a good reason: the appropriate measure of marginal cost depends on characteristics of the economy which are only imperfectly understood. These characteristics include the competitiveness of factor markets and the extent of adjustment costs firms face in hiring new workers and installing new capital.[14] The assumptions described above surmount this problem, as they imply that the appropriate measure of real marginal cost is labor's share of output—unit labor costs. Estimates of these series are widely available, and hence estimation of α is feasible.

To see how labor's share can reflect real marginal cost, let Y_t be output, let L_t be labor, and let K_t be capital; then with Cobb-Douglas technology, $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. Nominal marginal cost $(\partial TC/\partial Y)$ can be decomposed as $\partial TC/\partial Y = (\partial TC/\partial L) \div (\partial Y/\partial L)$. In a competitive labor market, $\partial TC/\partial L$ is simply the nominal wage (W_t), and $\partial Y/\partial L$ is of course the marginal product of labor—$(1 - \alpha) A_t (K_t/L_t)^{\alpha}$ for the Cobb-Douglas case. Therefore, real marginal cost is $\psi_t = (W_t/P_t) \div \left[(1 - \alpha) A_t (K_t/L_t)^{\alpha} \right]$. If we let w_t denote the real wage $(w_t = W_t/P_t)$, then real marginal cost can be expressed as $\psi_t = w_t L_t \div [(1 - \alpha) Y_t]$. Real marginal cost, then, is proportional to labor's share of output, and variations in labor's share provide a measure of variations in real marginal cost.

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[12] These authors each use different estimation methods. However, it is fair to summarize both of those methods as ones that choose the model's parameters in order to best fit observed inflation.

[13] These authors also linearize around a zero inflation steady state, which simplifies things further. Sbordone allows marginal cost to vary according to when firms last adjusted their price, whereas Galí and Gertler do not. As such, Sbordone's analysis is more general than what we describe.

[14] Similar problems are involved in the measurement of output. Arguably, however, the problems are more severe for marginal cost.

Galí and Gertler (1999) find that, with labor's share as a proxy for real marginal cost, a Calvo pricing model explains post–1960 U.S. inflation quite well. Their estimate of α is roughly 0.2, implying that firms keep their prices fixed for about five quarters on average. This result is striking. It runs counter to the claims of Fuhrer and Moore (1995) and others that forward-looking sticky-price models are inconsistent with the behavior of U.S. inflation. Galí and Gertler reconcile these results by emphasizing that previous work explained inflation through the behavior of output. As is clear from (19), however, the key variable for explaining inflation is real marginal cost rather than output.[15] Thus Galí and Gertler argue that the main empirical difficulty is not in explaining inflation behavior with a forward-looking sticky-price model, but in reconciling the behavior of output with the behavior of real marginal cost.

3.   INFLATION DYNAMICS ARE SENSITIVE TO THE PRICING STRUCTURE
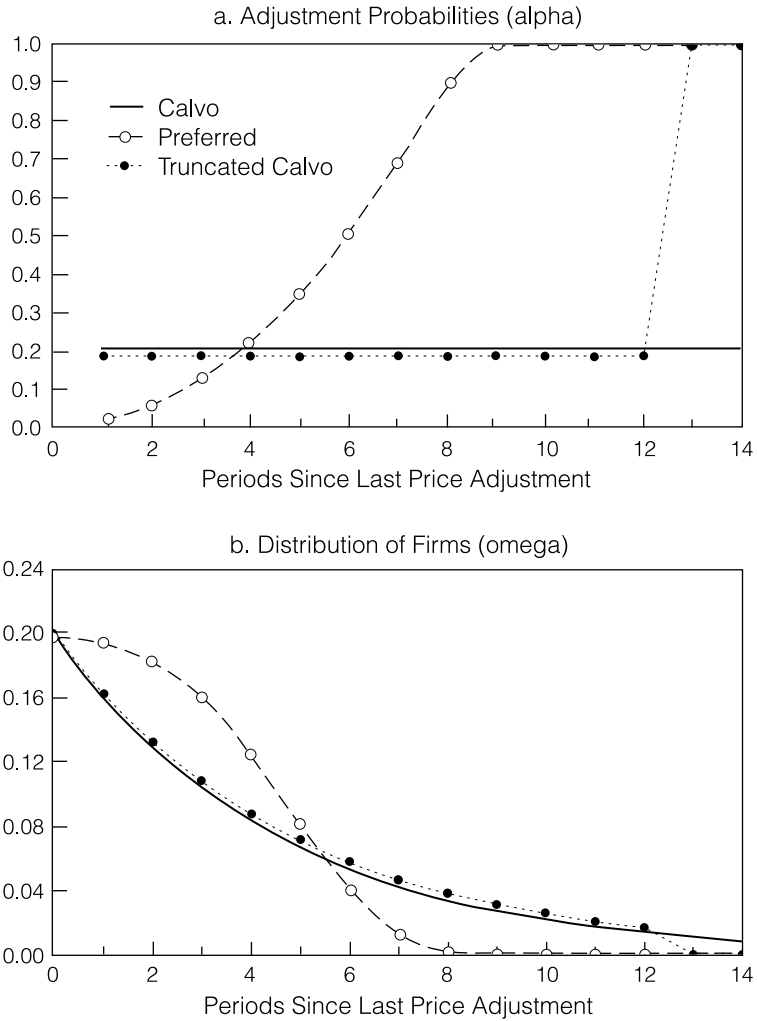
One might think that because the Calvo model fits inflation data well, it must be an appropriate model. However, another aspect of the data is fundamentally at odds with the Calvo model. Unfortunately, it appears difficult to eliminate this discrepancy without changing the implications for inflation dynamics.

Recall that the Calvo specification posits a common price-adjustment probability (α) for all firms. From (4), we see that the distribution of fractions of firms is then given by $\omega_j = \alpha(1 - \alpha)^j$, for $j = 1, 2, ...$ That is, a positive fraction of firms charges a price set arbitrarily many periods in the past. This is clearly a counterfactual implication. However, the Calvo specification allows for a characterization of inflation dynamics even simpler than (16), and it may be worth paying the price of an infinite distribution in order to gain this simplification. Supporting this view is the fact that the fractions of firms become arbitrarily small as the number of periods increases; for example, if $\alpha = 0.2$, less than 0.02 percent of firms charge a price set more than ten years in the past. With numbers that small, it is difficult to believe that the Calvo specification could produce dynamics qualitatively different than those associated with a more plausible specification generating the same average duration of a fixed price. Recent work by Kiley (1998), however, suggests that two such models would produce qualitatively different dynamics. To investigate the implications for inflation of different specifications of price stickiness, we first estimate a univariate autoregression for labor's share (used here to represent real marginal cost). We then compare the sticky-price model's impulse response functions of inflation to a shock to labor's share for the different specifications.

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[15] When average inflation ($\mu - 1$) is nearly zero, the coefficients on demand and nominal interest rates in (19) will be small.

Figure 1 Three Pricing Specifications

a. Adjustment Probabilities (alpha)

Periods Since Last Price Adjustment

b. Distribution of Firms (omega)

Periods Since Last Price Adjustment

The three pricing specifications we analyze are illustrated in Figure 1. Panel a shows the patterns of adjustment probabilities (α^s), and panel b shows the distributions of fractions of firms (ω^s). The solid lines represent a Calvo specification close to that estimated by Galí and Gertler.[16] The dashed lines are

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[16] Galí and Gertler's specification is slightly different, as they allow for a fraction of firms to be "rule-of-thumb" price setters. However, these firms turn out to be unimportant for their results.

arguably a more reasonable specification: no firms charge a price set more than eight quarters ago, but the average duration of a fixed price is five quarters, just as for the solid-line Calvo specification. The dashed line, which we will refer to as our preferred case, has further appeal in that it is the kind of price-adjustment pattern generated if firms face a distribution of fixed costs of price adjustment, as in Dotsey, King, and Wolman (1999). Finally, the dotted line is an intermediate case: as in the Calvo case, firms face a constant adjustment probability for the first 12 quarters that they are charging a price, but the adjustment probabilities jump to one after the twelfth quarter.

Figure 2 shows the response of inflation to a marginal cost shock (as prox-ied for by labor's share) under these three specifications of price stickiness. In studying these pictures, it is important to keep in mind that the Calvo specifica-tion (solid line) has been shown to be consistent with the behavior of inflation in the United States when labor's share is used to represent marginal cost. For that case, the response of inflation to a marginal cost shock is relatively small, but fairly persistent. In contrast, for the preferred specification, where the adjustment probabilities are smoothly increasing and the distribution of firms does not extend beyond eight quarters, inflation responds much more strongly to the marginal cost shock; the magnitudes of the increase and subsequent decrease in inflation are roughly three times as large as the corresponding magnitudes for the Calvo case. Although the intermediate case gives results closer to Calvo, still the impact effect of the marginal cost shock on inflation is nearly 50 percent greater in the intermediate case than it is for pure Calvo pricing.

Figure 1 can help us to understand the dramatic difference between infla-tion behavior under the Calvo and preferred specifications. Even though in both cases roughly the same fraction of firms adjusts their price in a given period, for the Calvo case a higher fraction of the adjusting firms are themselves recent adjusters (in panel a, α_j is relatively high for low j in the Calvo case). Since recent adjusters have already responded to a recent shock, their effect on the price level is small. By contrast, in the preferred case most of the adjusting firms have last adjusted several periods ago. Thus, in periods immediately following a shock, the model registers significant additional adjustment to that shock. For an adjusting firm, this means that it responds more strongly to a shock in the preferred case; to not do so would mean that its relative price would move too far from the desired level in ensuing periods.

That the impulse response functions differ sharply for the Calvo and our preferred case has a direct bearing on whether Galí and Gertler's empirical results are sensitive to the assumption of Calvo pricing. Recall they found that the dynamics of U.S. inflation could be closely replicated by a model of Calvo pricing, where the main "forcing variable" for inflation was labor's share, which proxies for real marginal cost. The impulse response function of inflation to marginal cost is one way of summarizing the model's dynamics. Because the

**Figure 2   Impulse Response Functions of Inflation to a
Marginal Cost Shock**
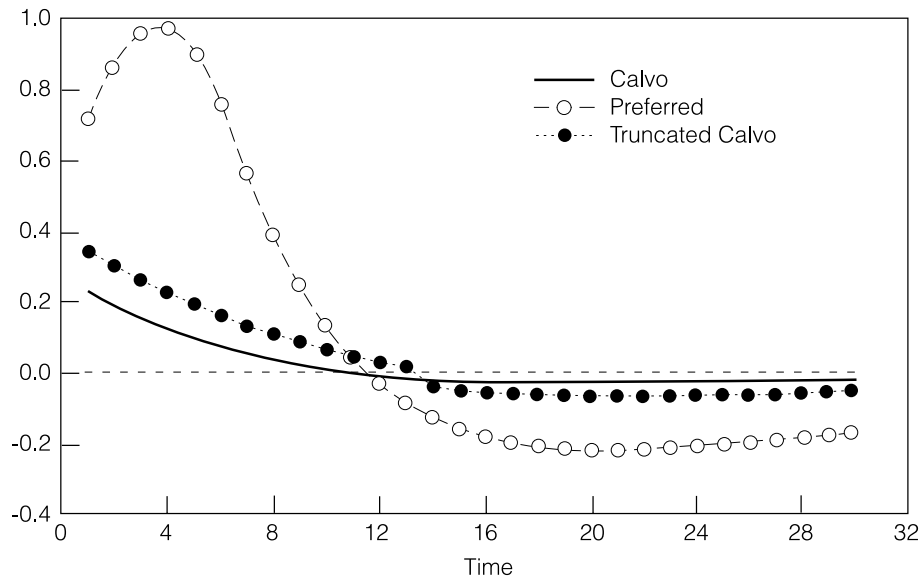
Calvo model matches inflation dynamics, its impulse response function is the "correct one" for matching the behavior of inflation. The preferred case gives such a different impulse response function that it could not also match inflation behavior when driven by the same marginal cost process; inflation would have to be much more volatile than observed in the data, or real marginal cost would have to be much smoother. We conclude, then, in support of Kiley's (1998) finding that the Calvo model is an extreme special case, not just a convenient simplification. Modifying the form of price stickiness so that (1) the probability of price adjustment is a smoothly increasing function of time since last adjustment, and (2) no firm keeps its price fixed more than eight quarters leads to dynamics fundamentally different than those of the Calvo model, even if one holds constant the average length of time a price is held fixed. With such a change, it will no longer be possible to match inflation dynamics with labor's share proxying for real marginal cost.

4.   SHOULD WE GIVE UP ON STICKY-PRICE MODELS?

One interpretation of our critique is that models with imperfect competition and sticky prices are poor descriptions of the data and as such should be abandoned.

We prefer a constructive interpretation, which focuses on the assumptions that guarantee labor's share would be a good approximation to real marginal cost. The first interpretation assumes that labor's share does represent real marginal cost. In this case, if Calvo pricing is the only form of price stickiness consistent with inflation dynamics, but is unacceptable for reasons discussed above, then we should give up on this entire class of sticky-price models. On the other hand, if labor's share does not represent real marginal cost, then a more plausible pricing specification might be consistent with data on inflation and marginal cost, correctly measured.

To justify using labor's share as a stand-in for real marginal cost, we assume that all firms produce using identical Cobb-Douglas technologies and that there is an economywide competitive labor market. These assumptions clearly represent an oversimplification. Possibly by constructing a richer marginal cost structure, one could reconcile a plausible sticky-price model with the behavior of inflation. Sbordone (1998) has already analyzed a simple generalization for marginal cost. She assumes the presence of a competitive labor market but allows factor ratios and hence marginal cost to vary depending on when a firm adjusted its price. Sbordone also maintains the assumption of Calvo-style pricing, so it is unclear whether that particular generalization of the marginal cost structure can generate realistic inflation dynamics when combined with a realistic pricing specification.

Once one is willing to relax the assumptions about factor markets and technology, a wide range of behavior for marginal cost is possible; typically, real marginal cost will not simply correspond to labor's share. Rotemberg and Woodford (1999a) work through several formulations: non-Cobb-Douglas technology, overhead labor, overtime pay, labor adjustment costs, labor hoarding, and variable capital utilization. Incorporating these features means that to explain marginal cost one would need not only labor's share but also such variables as output, labor input, the marginal wage, current and expected future growth of labor input, the fraction of labor input which is idle, and hours per worker. Rotemberg and Woodford cite several papers that have pursued these ideas in an attempt to learn about real marginal cost. Our interpretation of Sbordone's and Galí and Gertler's work suggests that a next step would be to study whether more refined estimates of marginal cost can help reconcile a plausible sticky-price specification with the behavior of inflation.

Another worthwhile endeavor would be to use direct evidence on price stickiness to choose α, and then use the relationship between marginal cost and inflation to estimate the behavior of marginal cost. In other words, instead of using independent evidence on marginal cost to estimate the form of price stickiness, one would be using independent evidence on pricing to estimate the behavior of real marginal cost.

5.   CONCLUSIONS

Current optimizing sticky-price models imply a tight relationship between real marginal cost and inflation. We have worked through the steps in this relationship in detail: expressions for the price index (5) and for a price-setting firm's optimal price (14) imply a linear system (19) that approximates the behavior of the price level (and inflation) primarily as a function of real marginal cost.

In interesting recent empirical work, Sbordone (1998) and Galí and Gertler (1999) use the relationship between marginal cost and inflation to estimate sticky-price models and evaluate how well these models explain actual inflation. Their results are positive in that they find sticky-price models are able to explain U.S. inflation quite well. However, this empirical work relies on the Calvo pricing specification, where firms face a positive probability of having their price fixed for an arbitrarily long time. This pricing specification is clearly implausible. We have shown—building on work by Kiley (1998)—that if labor's share is used to proxy for real marginal cost, a more plausible pricing specification generates inflation dynamics inconsistent with the data.

Continued progress in empirical evaluation of sticky-price models will require intensive study of the factors determining real marginal cost. With more refined estimates of real marginal cost, it may be possible to reconcile a plausible sticky-price specification with data on inflation. Conversely, to the extent that we are confident a particular pricing specification is correct, the link between real marginal cost and inflation should allow us to come up with independent estimates of real marginal cost. Such estimates will help us learn about other aspects of the economy's structure, such as the form of technology, the competitiveness of factor markets, and the extent of adjustment costs in hiring labor and installing capital. Ultimately, this knowledge will facilitate constructing more accurate general equilibrium models, which can then be used for the kind of policy analysis mentioned at the outset.

APPENDIX

Derivation of Demand Function and Price Index

To derive the demand function (2), solve the following problem: minimize the cost of purchasing a given level of final output $\bar{y}$ by choosing appropriate levels of $y(z)$, $z \in (0, 1)$:

$$\mathcal{L} = \left[\int_0^1 P(z) \cdot y(z) dz \right] + P \left[\bar{y} - \left(\int_0^1 y(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \right],$$

where $P(z)$ denotes the nominal price of good z. The Lagrange multiplier on the quantity constraint is the price level P, because the multiplier has the interpretation of the marginal cost of an additional unit of final output, and that is precisely the price index. The first-order conditions for this problem are

$$P(\tilde{z}) = P \left(\int_0^1 y(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{1}{\varepsilon-1}} y(\tilde{z})^{\frac{-1}{\varepsilon}}$$

$$\tilde{z} \in (0, 1).$$

Using the definition of y in (1), these conditions simplify to

$$y(\tilde{z}) = \left(\frac{P(\tilde{z})}{P} \right)^{-\varepsilon} y \tag{20}$$

$$\tilde{z} \in (0, 1); \tag{21}$$

demand for a firm's product is increasing in the level of aggregate demand (y) and decreasing in the relative price the firm charges.

Now we show how the price index is calculated as a function of the prices $P(z)$. Substitute (20) into (1):

$$y = \left\{ \int_0^1 \left[\left(\frac{P(\tilde{z})}{P} \right)^{-\varepsilon} y \right]^{(\varepsilon-1)/\varepsilon} dz \right\}^{\frac{\varepsilon}{\varepsilon-1}},$$

which implies

$$1 = \left\{ \int_0^1 \left(\frac{P(\tilde{z})}{P} \right)^{(1-\varepsilon)} \right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

and thus

$$P = \left\{ \int_0^1 P(\tilde{z})^{1-\varepsilon} d\tilde{z} \right\}^{1/(1-\varepsilon)}. \tag{22}$$

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