Private Money and Counterfeiting

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P erhaps the most fundamental question in monetary economics pertains to the role of the government in providing money. A widely held view among economists is that the supply of media of exchange is an activity that should not be left to the private sector. Indeed, even Milton Friedman, who in most respects has viewed the economic role of the government quite narrowly, argues in Friedman (1960) that the provision of money is fraught with peculiar market failures and that the government should have a monopoly in the supply and control of the stock of circulating currency.

Monetary systems that include the private provision of circulating media of exchange were not uncommon in the past. In the United States, most of the stock of currency in circulation prior to the Civil War consisted of notes issued by state-chartered banks. The U.S. pre–Civil War monetary system has been judged by some, but not all, as chaotic (Rolnick et al.1997; Rolnick and Weber 1983, 1984), since it included thousands of note-issuing banks and the quality of these notes was difficult to distinguish. Counterfeiting was a problem, and there was sometimes poor information on a particular bank's chances of defaulting. However, the Suffolk Banking System in pre–Civil War New England is thought to have functioned quite efficiently (see Smith and Weber [1998]). In addition, the monetary system in place in Canada prior to 1935 featured private note issue by a small number of chartered banks, and this system also appears to have worked quite well (see Williamson [1999] and Champ, Smith, and Williamson [1996]).

Private money systems are not just of historical interest. In the United States, the government monopoly on the issue of circulating media of exchange

resulted from the federal taxation after the Civil War of the notes issued by state-chartered banks, and from the elimination of the supply of government bonds qualifying as backing for notes issued by national banks. As argued by Schuler (2001), all serious federal impediments to private bank note issue in the United States were removed in 1976 and 1994 (also see Lacker [1996]). Thus, it would seem that private banks in the United States are currently free to issue circulating pieces of paper, though how U.S. regulators would respond to private note issue is uncertain. New transactions technologies also give financial institutions the capability to issue private electronic monies, such as stored-value cards, and several banks have conducted market trials of such products.

The purpose of this article is to study some of the benefits and costs of private money issue. The key benefit of privately issued money is that these private liabilities can intermediate productive assets, much as the deposit liabilities of private banks do. Economic efficiency is enhanced if private money is permitted, as this private money is backed by productive investment, which ultimately enhances production and welfare. If private money is banned, then circulating currency takes the form of barren, unbacked fiat money. However, one cost of having circulating private money is that it can be more easy to counterfeit than fiat money. If counterfeiting is not very difficult, then it can have negative ramifications for social welfare.

I explore these issues here using a search model of money. Early versions of these monetary search models were developed by Kiyotaki and Wright (1989, 1993), with later developments by Trejos and Wright (1995) and Shi (1995). Some of the ideas in this article are closely related to those in Williamson (1999) and Temzelides and Williamson (2001b). In a monetary search model, economic agents typically find it difficult to get together to trade and make transactions, and there are limits on the flows of information. These are frictions which the use of money can help to overcome in the model—and in reality.

My first step will be to investigate the properties of the model when counterfeiting is not possible. I will show that government-supplied fiat money in this context is always detrimental to social welfare. Fiat money displaces private money, resulting in less investment and production and in lower welfare. If the counterfeiting of private money is possible, then with the cost of counterfeiting sufficiently low, monetary exchange may be supported only if private money is prohibited.

In Section 1 I construct the basic model, and first assume that there is no opportunity for the issue of counterfeits. I then study the nature of equilibrium in this model and show that, in the absence of counterfeiting, it is inefficient for fiat money to circulate. In the second section, I permit the costly issue of counterfeit private money and show that the potential for counterfeiting can lead to a classic type of market failure, much as in the "lemons" model of Akerlof (1971).

1. A MONETARY SEARCH MODEL WITH PRIVATE MONEY AND FIAT MONEY

The first step will be to study how an economy works when it is possible for banks to issue private monies that can circulate as media of exchange, and where economic agents can also use fiat money in exchange. In this basic model, there are no private information frictions, and there is no potential for counterfeiting.

In basic search models of money, three key assumptions are usually made: (i) people have difficulty meeting to carry on economic exchange, (ii) there is randomness involved in how people make contact, and (iii) people have limited information about what others are doing. These three assumptions capture important elements of real-world economic activity and economic exchange that help explain the existence and use of money in developed economies.

Assumption (i) is realistic, as it is clearly costly in terms of time and resources for people to get together to trade goods and assets. Shopping takes time, and it is impossible to be in two places at once. While internet shopping has reduced shopping costs dramatically, physical goods are costly to ship and cannot be delivered immediately. Assets appear to be less costly to trade than goods, as trade in assets typically involves only a change in an electronic record of ownership. For example, to trade shares on the New York Stock Exchange, one needs only to communicate with a broker. However, asset exchanges are still costly, and people do not have access to the communication technology required to make some kinds of asset trades at all times and places.

Assumption (ii) is probably the least realistic of the above three assumptions, for most individuals exercise much thought and planning in determining when they will trade goods and assets. For example, an individual's food purchases might involve no randomness at all. He or she plans to visit the supermarket regularly on a particular day of the week, draws up a shopping list, fills it at the supermarket, and returns home. However, people often find themselves in circumstances where they need to make unplanned purchases or sales of goods and assets. A car might break down, requiring one to hire a tow truck and rent a car; an unexpected illness might require that a person sell some stocks and bonds from his or her asset portfolio; a person might find that the supermarket has stocked some unusual food that he or she has a strong preference for and make an unanticipated purchase. Assumption (iii) is clearly realistic, for it is impossible to know all the intricate details of the economic interactions of all the people living in one's own city or town, let alone of all the people living in the world.

I will now describe the model environment, which will consist of a description of the population of economic agents, the preferences of these agents, the available technology, the endowments that are available to produce goods satisfying agents' preferences, and how economic agents can interact. There is a continuum of economic agents, having unit mass, and each agent has

preferences given by

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t [\theta_t u(c_t) - x_t],$$

where E_0 is the expectation operator, conditional on information available to the agent at time 0, r is the subjective discount rate with $r > 0$, θ_t is an independent and identically distributed preference shock with $\Pr[\theta_t = 1] = \Pr[\theta_t = 0] = \frac{1}{2}$, c_t is consumption in period t, $u(\cdot)$ is a utility function that is strictly concave and strictly increasing with $u'(0) = \infty$, and $u(0) = 0$. For convenience I will assume that there exists some $\hat{q}$ such that $u(\hat{q}) - \hat{q} = 0$. Also, x_t denotes production of goods, so that an agent suffers disutility from producing. Thus, a given economic agent will wish to consume in some periods and not in others, with the desire to consume determined at random. An economic agent can in any period produce goods, at a cost in terms of disutility. These goods can be consumed by someone else or can be used as an input to an investment technology. These goods are otherwise perishable, and an economic agent cannot consume his or her own output.

There are two sectors in the economy, the *search sector*, where economic agents are randomly matched and can trade with each other, and the *banking sector*, where an economic agent engages in banking transactions. An agent has no choice about which sector to visit during a given period, in that with probability π he or she visits the search sector, and with probability $1 - \pi$ he or she visits the banking sector. We have $0 < \pi < 1$.

If an agent is in the banking sector at the beginning of the period, that agent has the opportunity to fund an investment project. That is, an investment project is indivisible and requires γ units of goods to initiate, where $\gamma > 0$. We will call the agent's claim to the investment project a bank note, or private money. This claim is indivisible and portable at no cost, but a given agent can carry at most one bank note in inventory. At any time in the future, the bank note can be redeemed. An agent who returns to the banking sector with the bank note can interrupt the investment project and receive a return of R units of consumption goods, which must then be consumed. The payoff R is independent of when the project is interrupted, and once the project is interrupted it will yield no more payoffs. Assume that $u(R) - \gamma > 0$. One can think of the "bank" here as a machine that yields an invisible bank note if γ units of goods are inserted in it. At any period in the future, the bank note can be inserted in the bank machine, in which case the bank machine will yield R units of consumption goods.

Bank notes in the model are intended to capture some important features of the bank liabilities that circulated in the United States before the Civil War or in Canada prior to 1935. In these historical monetary regimes, bank notes circulated hand-to-hand, and they were ultimately redeemable (typically in gold or silver) at the bank of issue. The redemption value of circulating bank

notes did not depend on the length of time between the issue of the note and its redemption. To keep things simple, in the model some of the features of historical bank note issue are assumed as part of the technology, but this assumption is not important for my argument.

In period 0, a fraction M of the population is endowed with one unit each of fiat money. Fiat money is assumed to be an intrinsically worthless and indivisible object. Agents can hold at most one unit of some object, so in equilibrium a given agent will be holding either one bank note, one unit of fiat money, or nothing. The assumption that assets are indivisible is common in search models of money, and this assumption is made for tractability. If assets were divisible and it were feasible for a given agent to hold any nonnegative quantity of a particular asset, then we would have to track the entire distribution of assets across the population over time. In general, this would make the model difficult to work with.[1] Given the assumption of indivisibility of assets and the constraint that any given agent can hold only one unit of some asset, we need only keep track of the fraction of agents in the population holding each asset at each point in time.

If an agent is in the search sector at the beginning of the period, he or she is matched with one other agent for the period. These matches are random except that, for analytical convenience, every match is between an agent who wishes to consume ($\theta_t = 1$) and one who does not wish to consume ($\theta_t = 0$). Thus, I have ruled out matches where there is a double coincidence of wants and both agents in the match wish to consume during the period, and where neither agent wishes to consume. In order for exchange to take place in any of the single-coincidence-of-wants matches, it must be the case that the agent who wishes to consume has an asset (either a bank note or money) and the agent who does not wish to consume has no asset. Clearly, if the agent who wishes to consume has no asset, he or she has nothing to offer in exchange for the other agent's output, and if the agent who does not wish to consume already has an asset, he or she will not accept more assets as he or she is not able to carry them into the future.

One important assumption is that an agent cannot make contact with other agents visiting the banking sector at the same time. That is, suppose that agents arrive at the banking machine sequentially during the period. This prevents agents from making trades while in the banking sector, which simplifies the model. A second important assumption is that agents meeting in the search sector know nothing about others' trading histories. With knowledge of trading histories, it would be possible to support certain types of credit arrangements, which we can think of as being similar to centralized credit card networks. Such credit arrangements are studied in Aiyagari and Williamson (1999, 2000),

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[1] See, however, Green and Zhou (1998), Lagos and Wright (2000), and Shi (1997), where search models with divisibile money are constructed.

Williamson (1999), Temzelides and Williamson (2001a), and Kocherlakota and Wallace (1998). Thus, the model environment rules out credit, which makes it simpler to focus attention on the monetary arrangements of interest here.

Equilibrium

I will confine my attention here to steady state equilibria, where prices and the distribution of assets across the population are constant over time. There may exist other equilibria in this model, such as deterministic equilibria with cycles and stochastic sunspot equilibria. However, these other equilibria are more difficult to analyze, and our points can be made in a more straightforward way by studying only steady states. The distribution of assets across agents in a steady state is described by (ρ_0, ρ_p, ρ_m), where ρ_0 denotes the fraction of agents in the population holding no asset, ρ_p is the fraction of agents holding bank notes, and ρ_m is the fraction holding fiat money. We have $\rho_0 + \rho_p + \rho_m = 1$. Next, q_p is the price of a bank note in terms of consumption goods. That is, q_p is the quantity of consumption goods that an agent gives up in equilibrium for a bank note. Similarly, q_m denotes the price of fiat money. The other variables we will need to determine are V_0, the expected utility at the end of the period associated with holding no asset, or the *value* to holding nothing, and V_p and V_m, the values to holding a bank note and fiat money, respectively.

Dynamic optimization by the economic agents in the model implies a set of *Bellman equations*. First, V_0 is determined by

$$V_0 = \left(\frac{1}{1+r} \right) \left(\pi \left\{ \begin{array}{c} \left(\rho_0 + \frac{\rho_p}{2} + \frac{\rho_m}{2} \right) V_0 + \frac{\rho_p}{2} \max \left[V_p - q_p, V_0 \right] \\ + \frac{\rho_m}{2} \max \left[V_m - q_m, V_0 \right] \\ + (1 - \pi) \max \left[V_0, V_p - \gamma \right] \end{array} \right\} \right)$$

(1)

In equation (1), the value of holding no asset at the end of the current period is determined by the opportunities this represents for trading in the following period. These opportunities need to be discounted to the present using the discount rate r. In the next period, with probability π the agent will be in the search sector, and will meet an agent with nothing, with a bank note, or with fiat money, with probabilities ρ_0, ρ_p, and ρ_m, respectively. If the agent is in the search sector and meets another agent with nothing, clearly they cannot trade and the agent's value will then be V_0 at the end of the next period. However, if the agent meets someone with a bank note, trade can only take place if the other agent wishes to consume, which occurs with probability $1/2$. If the other agent wishes to consume, then the agent decides whether to trade or not based on what gives him or her the greatest utility. If he or she trades, then q_p goods must be produced at a utility cost of q_p, and the agent receives the bank note, with an associated value V_p and a net expected utility gain of

$V_p - q_p$. However, should the agent not trade, value will remain the same at V_0. Similarly, if the agent meets someone with money, trade will occur only if the other agent wishes to consume, and the agent will trade if $V_m - q_m > V_0$, and will not trade if $V_m - q_m < V_0$. Now, if the agent is in the banking sector, which occurs with probability $1 - \pi$, then he or she can choose to do nothing, which yields a value at the end of the next period of V_0, or a bank note could be purchased, yielding expected utility $V_p - \gamma$.

It is somewhat simpler to rewrite the Bellman equation (1) by multiplying both sides by $1 + r$, and then subtracting V_0 from the left and right sides to obtain

$$r V_0 \quad = \quad \frac{\pi \rho_p}{2} \max \left[V_p - q_p - V_0, 0 \right] + \frac{\pi \rho_m}{2} \max \left[V_m - q_m - V_0, 0 \right]$$
$$+ (1 - \pi) \max \left[0, V_p - \gamma - V_0 \right]. \tag{2}$$

In equation (2), the right-hand side is the net expected flow return that can be obtained when no asset is held at the beginning of the period. In a manner similar to equation (2), for agents holding bank notes and fiat money, respectively, we have

$$r V_p \quad = \quad \frac{\pi \rho_0}{2} \max \left[u(q_p) + V_0 - V_p, 0 \right]$$
$$+ \frac{(1 - \pi)}{2} [u(R) + \max \left(-\gamma, V_0 - V_p \right)], \tag{3}$$

and

$$r V_m = \frac{\pi \rho_0}{2} \max \left[u(q_m) + V_0 - V_m, 0 \right]. \tag{4}$$

Note in equation (3) that in the second term on the right-hand side, the holder of the bank note redeems it in the banking sector only in the case where he or she wishes to consume. Otherwise, it is preferable to continue to hold the note so that it can be traded away or redeemed in the future. In equation (4), the holder of fiat money obtains a return only in the search sector when he or she meets an agent who holds no asset and wishes to consume.

Next, we need to describe how prices are determined in trades between asset holders and those not holding assets. In general, two agents who can potentially trade have a bargaining problem to solve, and the literature has approached bargaining problems of this nature in a variety of ways including using a Nash bargaining solution or a Rubinstein bargaining game (see Trejos and Wright [1995]). Here, I will follow the simplest possible approach, which is to assume that the asset holder has all of the bargaining power and makes a take-it-or-leave-it offer to the agent who holds no asset. That is, the asset holder sets the price for the exchange in such a way that the other agent is just indifferent between accepting the offer and declining. This gives

$$V_p - q_p - V_0 = 0, \tag{5}$$

and

$$V_m - q_m - V_0 = 0. \tag{6}$$

Equilibrium Where Bank Notes and Fiat Money Circulate

We will first examine an equilibrium where bank notes and fiat money are exchanged for goods in the search sector. Since some agents hold bank notes and some agents hold no assets in such a steady state equilibrium, then when the holder of a bank note redeems that note in the banking sector, he or she must be indifferent between acquiring another bank note and holding no asset. If this were not the case, then either the steady state supply of bank notes would be zero, or there would be no agents in the search sector with no assets, so bank notes could not be used in exchange. We then have

$$V_p - \gamma = V_0. \tag{7}$$

Equations (2), (5), (6), and (7), then, imply that $V_0 = 0$. That is, the value of holding no asset is zero, since an agent with no asset then receives no surplus from trading in the search sector with asset holders, and his or her value will not change when visiting the banking sector. Given this, equation (7) implies that $V_p = \gamma$, and (5) and (6) imply, respectively, that $V_p = q_p = \gamma$ and $V_m = q_m$.

Now, we will assume that $u(\gamma) - \gamma > 0$, or $\gamma < \hat{q}$, which implies from (3) that the holder of a bank note is willing to trade with an agent holding no asset. Since the equilibrium price of a bank note is γ, the constraint $u(\gamma) - \gamma > 0$ states simply that there is a positive surplus associated with the exchange of a bank note. Then, (3) implies that

$$r\gamma = \frac{\pi \rho_0}{2}[u(\gamma) - \gamma] + \frac{(1 - \pi)}{2}[u(R) - \gamma], \tag{8}$$

and (8) is then an equation that solves for ρ_0, that is,

$$\rho_0 = \frac{2r\gamma - (1 - \pi)[u(R) - \gamma]}{\pi[u(\gamma) - \gamma]}. \tag{9}$$

Since $V_m = q_m$ in equilibrium, we can substitute for V_m in equation (4), and for now we can conjecture that it will always be in the interest of a holder of fiat money to trade for goods at the price q_m. From (4), these steps then give us

$$rq_m = \frac{\pi \rho_0}{2}[u(q_m) - q_m]. \tag{10}$$

Equation (10) then solves for q_m given the solution for ρ_0 from (9). There are two solutions to (10), one where $q_m = 0$, and one where $q_m > 0$. The equilibrium where $q_m = 0$ is uninteresting since the value of holding fiat money is zero, and nothing can ever be purchased with fiat money. However, an agent holding no asset is willing to accept fiat money as that would not make

him or her any worse off. We will confine our attention to the equilibrium where $q_m > 0$. Given this condition, from (10) we have $u(q_m) - q_m > 0$, and our conjecture that the holder of fiat money is always willing to trade in equilibrium is correct. An important result is that, from (8) and (10),

$$q_m < q_p = \gamma, \tag{11}$$

that is, private bank notes exchange for goods in the search sector at a premium over fiat money. This result follows because bank notes have a redemption value in the banking sector, while fiat money does not. Therefore, agents are willing to pay more for the possibility of this higher future payoff.

Now, in the equilibrium we are examining where $q_m = V_m > 0$, when a holder of fiat money goes to the banking sector, he or she will not want to acquire a bank note. Holding fiat money has strictly positive value, while acquiring a bank note implies net expected utility $V_p - \gamma = \gamma - \gamma = 0$. Thus, in a steady state, no one would want to dispose of fiat money balances. This need not necessarily imply that it would never be in anyone's interest to dispose of money along the path the economy takes from the first date to the steady state. However, suppose that there is no fiat money in existence, that the economy converges to a steady state, and that fiat money enters the economy when the central bank chooses holders of bank notes at random in the search sector and replaces each of their bank notes with one unit of fiat money. This action will have no effect on the equilibrium, other than to replace bank notes with fiat money one-for-one, and from the date when money was injected, no one would dispose of fiat money. Therefore, in this sense we can take $\rho_m = M$ in equilibrium, so that the fraction of money holders in the steady state is equal to the quantity of money injected by the central bank. Since $\rho_0 + \rho_p + \rho_m = 1$ in equilibrium, from our solution for ρ_0 in equation (9) we require that $0 < \rho_0 < 1 - M$, which implies

$$0 < 2r\gamma - (1 - \pi)[u(R) - \gamma] < (1 - M)\pi[u(\gamma) - \gamma] \tag{12}$$

From (12), to support an equilibrium where private bank notes circulate, the return on investment, R, cannot be too large or too small. If R is too small, then investment will not be worthwhile, and no one would be willing to hold bank notes. However, if R is too large, then the redemption value of a bank note will be sufficiently attractive that no one will want to trade away a bank note for goods in the search sector.

Note that if $M = 1$, then (12) does not hold for any values of γ, r, and π, since the upper and lower bounds on $2r\gamma - (1 - \pi)[u(R) - \gamma]$ in (12) are then identical. Therefore, there is always some value for M that is sufficiently large that (12) is not satisfied (note that the upper bound decreases with M), and an equilibrium with circulating bank notes and valued fiat money does not exist.

Equilibrium Where Only Fiat Money Circulates

If fiat money circulates and there are no bank notes, then we have $\rho_m = M$, $\rho_0 = 1 - M$, and $\rho_p = 0$. From (6) and (2) we have $V_0 = 0$ and $V_m = q_m$, so equation (4) then implies that q_m is the solution to

$$rq_m = \frac{\pi(1 - M)}{2}[u(q_m) - q_m]. \tag{13}$$

Just as in the previous subsection, I will ignore the equilibrium where $q_m = 0$ and focus on the solution to (13) where $q_m > 0$. Then, from equation (13), $u(q_m) - q_m > 0$, so it will always be in the interest of an agent with fiat money to trade it for goods, as I have implicitly conjectured. In an equilibrium where only fiat money circulates, it cannot be in the interest of any agent to hold a bank note. Were an agent to have a bank note, we would have $V_p = q_p$, from (5), and q_p would be determined, from (3), by

$$rq_p = \frac{\pi(1 - M)}{2}[u(q_p) - q_p] + \frac{(1 - \pi)}{2}[u(R) - q_p]. \tag{14}$$

Then, for it not to be in the interest of an agent to acquire a bank note, it must be the case that $V_p - \gamma \leq 0$, so that an agent prefers to hold no asset rather than acquiring a bank note in the banking sector. This inequality then implies that $q_p \leq \gamma$ or, from (14),

$$2r\gamma - (1 - \pi)[u(R) - \gamma] \geq (1 - M)\pi[u(\gamma) - \gamma] \tag{15}$$

Now, defining

$$\phi \equiv 2r\gamma - (1 - \pi)[u(R) - \gamma], \tag{16}$$

we can conclude from (12) and (15) that an equilibrium exists where bank notes and fiat money circulate if

$$0 < \phi < (1 - M)\pi[u(\gamma) - \gamma], \tag{17}$$

and that an equilibrium where only fiat money circulates as a medium of exchange exists if

$$\phi \geq (1 - M)\pi[u(\gamma) - \gamma]. \tag{18}$$

Inequalities (17) and (18) imply that we are more likely to see bank notes in circulation as the redemption value of a bank note, R, increases (though recall that this redemption value cannot be too large, as we require $\phi > 0$), and that bank notes are less likely to circulate the larger is M, the quantity of fiat money in circulation. With an increase in the redemption value of a bank note, agents are much more willing to acquire notes to be used in exchange. Fiat money displaces private money in circulation, so if the quantity of fiat money is sufficiently large, then private money is driven out of the system.

Note that, if $\phi \leq 0$, then a steady state equilibrium will exist where agents acquire private money, but this private money is not exchanged in the search

sector. Private money is then just held until redemption occurs. As I am primarily interested here in the medium of exchange role of private money, I will assume throughout that $\phi > 0$.

Is It Efficient for Fiat Money to Circulate?

In the equilibrium studied above where private bank notes and fiat money both circulate, an increase in M, the stock of money in circulation, will displace bank notes one-for-one. That is, since ρ_0, the fraction of agents in the population holding no assets, is determined by (9), which does not depend on M, a change in M can only affect the fractions of agents holding fiat money and bank notes. My interest in this section is in determining the welfare effects of changes in M. Is it a good thing for government-supplied fiat money to replace circulating bank notes?

To evaluate changes in welfare for this economy, I will use a welfare criterion of average expected utility across the population in the steady state. Letting W denote aggregate welfare, we have

$$W = \rho_0 V_0 + \rho_p V_p + \rho_m V_m.$$

That is, aggregate welfare is just the weighted average of values (expected utilities) across agents in the steady state. From above, in a steady state equilibrium where bank notes and fiat money circulate, we have $V_0 = 0$, $\rho_p = 1 - \rho_0 - M$, $V_p = \gamma$, $\rho_m = M$, and $V_m = q_m$, where $q_m < \gamma$. These conditions give

$$W = (1 - \rho_0 - M)\gamma + Mq_m.$$

But since $\gamma - q_m > 0$, an increase in M causes W to decrease, so welfare falls as more fiat money is introduced. Ultimately, if M becomes sufficiently large, then bank notes are driven out of the economy altogether. I can show (with some work) that, no matter what M is, welfare cannot be higher when only fiat money circulates than in an equilibrium where bank notes circulate.

The key result here is that fiat money always reduces welfare, which is true because circulating bank notes serve two roles. First, bank notes serve as a medium of exchange and therefore enhance welfare by allowing for production and consumption in the search sector that would otherwise not take place. Second, bank notes support productive investment. The value of bank notes as a medium of exchange encourages agents to hold these assets, and as a result there is productive intermediation activity. Fiat money serves only the medium of exchange function and does nothing to promote private investment. Thus, if fiat money replaces circulating bank notes, then an asset that performs only a medium of exchange function is replacing another asset; this other asset serves as a medium of exchange but also performs an important secondary role in promoting productive investment.

Prior to 1935, the assets used to back the circulating notes that Canadian banks issued were essentially unrestricted. In fact, the issue of circulating notes largely financed bank loans in Canada. This was certainly not true in the United States prior to the Civil War, where notes were issued by state-chartered banks and were typically required to be backed by state bonds. Thus, we can think of private bank notes as financing public investment in the United States and private investment in Canada. In either case, our model captures some elements of the historical role of private money.

The view of private money from the model as I have laid it out thus far is perhaps too sanguine. In practice some private money systems appear to have worked poorly, while others have done quite well. In particular, the monetary system in place prior to the Civil War in the United States certainly appears to have worked poorly, though this is the subject of some debate (Rolnick et al. 1997; Rolnick and Weber 1983, 1984). Indeed, the introduction of the National Banking System in the United States in 1863 and the contemporaneous introduction of a prohibitive tax on state bank notes appears to have been motivated in good part by the view that the existing system of private issue of bank notes by state-chartered banks was inefficient. However, the monetary system in Canada prior to 1935 seems to have been successful in that the notes issued by chartered banks were essentially universally accepted at par and there were only a few unusual circumstances of banks defaulting on their notes (in the United States prior to the Civil War, there was widespread discounting of private bank notes and there were many instances of default on private bank notes).

Three incentive problems are the primary causes of potential inefficiencies in private money systems. First, there might be an "overissue" problem, as discussed by Friedman (1960). That is, if there are many issuers of private money behaving competitively, they will tend to issue notes to the point where they collectively drive the value of private money to zero. Cavalcanti, Erosa, and Temzelides (1999) show, however, that each bank in a private money system (such as the Suffolk Banking System in pre–Civil War New England or the Canadian banking system prior to 1935) could have sufficient incentives to prevent overissue. A key element in these systems was that each private money issuer accepted the notes of other private money issuers for redemption.

The second type of incentive problem arises because private money producers might sell lemons (see Akerlof [1971]). Williamson (1991) shows that if banks differ according to the quality of their asset portfolios, and the holders of bank notes have difficulty distinguishing quality, then the market could be dominated by low-quality private bank notes that bear a low rate of return on redemption. In these circumstances, it is possible that a prohibition on private bank notes, with fiat money circulating as the sole medium of exchange, would be the most efficient monetary arrangement. While lemons problems probably created serious inefficiencies in the United States prior to the Civil

War, these problems appear to have been largely solved in the Suffolk Banking System and in Canada prior to 1935. These two private money systems had key self-regulatory mechanisms that helped prevent lemons problems; furthermore, the Canadian system had an advantageously small number of private note issuers.

The third type of incentive problem in private money systems is the potential issue of counterfeits. In terms of its function as a medium of exchange, a counterfeit is much like a lemon of extremely poor quality. If sufficient care is put into its production, the counterfeit will pass undetected in many circumstances as a medium of exchange, but in contrast to a genuine bank note it has no redemption value. While we might view Williamson (1991) as applying to counterfeiting as well as to poor-quality banking, no one has analyzed the counterfeiting problem in the context of a monetary search model. Thus, exploring the implications of counterfeiting in our model in the next section will prove useful.

2.   A MODEL WITH COUNTERFEITING

One potential problem with a private money system is that this money may be counterfeited. Indeed, the counterfeiting of private bank notes appears to have been common in the United States prior to the Civil War. Clearly, government-issued fiat currency is also subject to counterfeiting, but there may exist economies of scale in counterfeit-prevention technologies and in the enforcement of counterfeiting laws. Thus, the modifications I make in the model will include the assumptions that private bank notes can be counterfeited at some cost and that (for simplicity) the cost of counterfeiting fiat money is infinite. There will be a tradeoff, then, between the benefits from the circulation of private money—the promotion of productive investment—and the costs of private money—the promotion of inefficient counterfeiting. I will show that there are circumstances in which the possibility of counterfeiting fundamentally changes the nature of the equilibria that can exist. Indeed, a ban on private money may be necessary to support a stationary equilibrium with monetary exchange.

I will assume that a counterfeit bank note can be created when an agent is in the banking sector, at a cost δ in units of goods, where $0 < \delta < \gamma$, so that it is more costly to produce a genuine bank note than a counterfeit. This counterfeit note can potentially be exchanged for goods in the search sector, but there is no investment project backing the note, and so it has no redemption value. In meetings with other agents in the search sector, a counterfeit note can be detected with probability η, if it is offered in exchange, where $0 < \eta < 1$, but otherwise the counterfeit goes undetected and is indistinguishable from a private bank note. If a counterfeit is detected, then it is confiscated and

destroyed. We will let ρ_f denote the fraction of agents holding counterfeit notes in equilibrium, with $\rho_0 + \rho_p + \rho_m + \rho_f = 1$.

We determine the value of holding a counterfeit, V_f, in a manner similar to (2), (3), and (4), as

$$rV_f = \frac{\pi \rho_0 (1-\eta)}{2} \max \left[u(q_u) + V_0 - V_f, 0 \right] + \frac{\pi \rho_0 \eta}{2} (V_0 - V_f). \quad (19)$$

In equation (19), note in the first term on the right-hand side that if the counterfeit goes undetected, it sells at the same price as a private bank note which cannot be identified—that is, at the price q_u, which is the price of a bank note of unidentified quality. Also note, in the second term, that if detection takes place in the search sector, the note is confiscated and the agent will have value V_0 at the end of the period. I assume for convenience that counterfeits can always be recognized in the banking sector. Thus, an agent with a counterfeit bank note arriving in the banking sector will hide the counterfeit and not attempt to redeem it.

We also need to modify equation (2), since agents with no asset can encounter an agent with a counterfeit note with whom they might trade. We have

$$\begin{aligned}
rV_0 &= \frac{\pi(1-\eta)(\rho_p + \rho_f)}{2} \max \left[\frac{\rho_p V_p + \rho_f V_f}{\rho_p + \rho_f} - q_u - V_0, 0 \right] \\
&\quad + \frac{\pi \eta \rho_p}{2} \max \left[V_p - q_p - V_0, 0 \right] \\
&\quad + \frac{\pi \rho_m}{2} \max \left[V_m - q_m - V_0, 0 \right] \\
&\quad + (1-\pi) \max \left[0, V_p - \gamma - V_0, V_f - \delta - V_0 \right]. \quad (20)
\end{aligned}$$

In the first term on the right-hand side of equation (20), the agent sometimes cannot distinguish between a genuine bank note offered in exchange and a counterfeit. In this circumstance, I assume that if the agent accepts the note, he or she learns before the end of the period whether or not it is a counterfeit. Whether the note is accepted depends on the expected value of the note to the agent. In the second term on the right-hand side, the agent has encountered an agent with a bank note and has been able to verify that it is not a counterfeit. The third term on the right-hand side of equation (20) takes account of the agent's opportunity to produce a counterfeit bank note when in the banking sector. The analogue of equation (3) is

$$\begin{aligned}
rV_p &= \frac{\pi \rho_0 \eta}{2} \max \left[u(q_p) + V_0 - V_p, 0 \right] \\
&\quad + \frac{\pi \rho_0 (1-\eta)}{2} \max \left[u(q_u) + V_0 - V_p, 0 \right] \\
&\quad + \frac{(1-\pi)}{2} [u(R) + \max (-\gamma, V_0 - V_p, V_f - \delta - V_p)]. \quad (21)
\end{aligned}$$

Equation (21) takes account of the fact that an agent with a bank note can potentially encounter agents with assets who both recognize and do not recognize his or her bank note as not being a counterfeit; it also accounts for the fact that the agent can create a counterfeit in the banking sector when a bank note is redeemed. Equation (4) remains the same.

In trades where the holder of a bank note or counterfeit makes a take-it-or-leave-it offer to an agent holding no asset who does not recognize the quality of the asset, we obtain

$$\frac{\rho_p V_p + \rho_f V_f}{\rho_p + \rho_f} - q_u - V_0 = 0, \tag{22}$$

while equations (5) and (6) continue to hold.

Equilibrium

Given the possibility that counterfeit bank notes will be issued, an equilibrium can be of three types. First, there could be an equilibrium where fiat money and bank notes circulate, but where it is in no one's interest to issue a counterfeit. Second, it could be that only fiat money circulates as a medium of exchange, with no bank notes in circulation, and therefore with no opportunities for circulating counterfeits. Third, fiat money, bank notes, and counterfeits could all circulate in equilibrium. We will consider each of these possibilities in turn.

Bank Notes and Fiat Money Circulate, with No Counterfeiting

This equilibrium is similar in most respects to that considered in the previous section, where bank notes and fiat money circulate but there are no opportunities to issue counterfeits. That is, equations (7), (8), (9), (10), and (12) all hold. Here, given that there are opportunities to counterfeit, it cannot be in the economic interest of anyone to issue a counterfeit in equilibrium.

If a counterfeit were issued, from (19) and (22) its value V_f would be determined by

$$2r V_f = \frac{\phi}{[u(\gamma) - \gamma]}[(1 - \eta)u(\gamma) - V_f], \tag{23}$$

where ϕ is defined as in (16). That is, if a counterfeit were issued, it would be negligible relative to the quantity of notes in circulation, and it would trade at the price $q_u = \gamma$ so long as it went undetected when offered in exchange. For it not to be in the interest of an agent to issue a counterfeit when in the banking sector, we must have $V_f \leq \delta$, which gives, from (23),

$$\phi[(1 - \eta)u(\gamma) - \delta] \leq 2r\delta[u(\gamma) - \gamma]. \tag{24}$$

Thus, since $\phi > 0$, (24) essentially states that the cost of counterfeiting, δ, must be sufficiently large and the probability of detection η must also be sufficiently large for this equilibrium to exist. However, note that even if $\eta = 0$ and no counterfeits can be detected in use, (24) will hold if δ is sufficiently large. It is also true that, for any $\delta > 0$ there is some sufficiently large value for η such that (24) will hold. That is, a sufficiently high detection probability will discourage counterfeits no matter how cheap they are to produce.

Another interpretation of condition (24) is the following. Suppose that the economy is in a steady state equilibrium with circulating private money and fiat money and an infinite cost of producing counterfeits. Then suppose that there was an unanticipated innovation to the counterfeiting technology that reduced δ so that condition (24) did not hold. It would then be in the interest of agents to issue counterfeits, which would upset the steady state equilibrium.

Only Fiat Money Circulates

When counterfeiting is possible, a potential outcome is that private money is not issued in equilibrium, and only fiat money circulates as a medium of exchange. Counterfeits would always be identifiable in such an equilibrium, since there would be no private money in circulation, and it would then be an equilibrium for no one to accept counterfeits. This equilibrium will be identical in all respects to the one considered in the previous section, where counterfeiting was not possible. Thus, for this equilibrium to exist, condition (18) must hold.

Bank Notes, Fiat Money, and Counterfeits Circulate

In the final case I consider, bank notes, fiat money, and counterfeits are all exchanged for goods in the search sector. This is the most complicated of the three cases to analyze.

Here, since an agent who holds no asset never receives any surplus in trading, we will have $V_0 = 0$. Also, when an agent is in the banking sector and is not holding an asset, then he or she must be indifferent among the following choices: continue to hold no asset; acquire a bank note; acquire a counterfeit. That is, since bank notes are continually retired through redemption, and some counterfeits are detected each period and removed from circulation, there must be a flow of new bank notes and counterfeits each period. Further, some agents must wish not to hold assets in the steady state, otherwise there will be no goods offered for sale in the search sector. Thus, it must be the case that agents are indifferent among the above three options in the steady state. Thus, we must have $V_0 = V_p - \gamma = V_f - \delta$. Then, given that $V_0 = 0$, we have $V_p = \gamma$ and $V_f = \delta$. This in turn implies, from (5), (6), and (22), that $q_p = \gamma$, $q_m = V_m$,

and

$$q_u = \frac{\rho_p \gamma + \rho_f \delta}{\rho_p + \rho_f}. \tag{25}$$

From (19) and (21) we obtain, respectively,

$$2r\delta = \pi \rho_0 [(1 - \eta)u(q_u) - \delta], \tag{26}$$

and

$$2r\gamma = \pi \rho_0 [\eta u(\gamma) + (1 - \eta)u(q_u) - \gamma] + (1 - \pi)[u(R) - \gamma]. \tag{27}$$

Then, equations (26) and (27) solve for ρ_0 and q_u, and since $\rho_p + \rho_f + \rho_m = 1 - \rho_0$, and $\rho_m = M$, then given a solution for q_u, we can use (25) to solve for ρ_p and ρ_f. Solving (26) and (27) for ρ_0 and $u(q_u)$, we obtain

$$\rho_0 = \frac{\phi - 2r\delta}{\pi[\eta u(\gamma) - \gamma + \delta]}, \tag{28}$$

$$u(q_u) = \frac{\delta\phi - 2r\delta(1 - \eta)u(\gamma)}{(\phi - 2r\delta)(1 - \eta)}. \tag{29}$$

Now, we require that $0 < \rho_0 < 1 - M$, or, from (28),

$$2r\delta < \phi < \pi(1 - M)[\eta u(\gamma) - \gamma + \delta] + 2r\delta. \tag{30}$$

Also, (25) implies that $u(q_u) < u(\gamma)$ and $u(q_u) > u(\delta)$ so, respectively, from (29), we must have

$$\phi[(1 - \eta)u(\gamma) - \delta] > 0, \tag{31}$$

and

$$\phi < \frac{2r\delta[u(\delta) - u(\gamma)]}{u(\delta) - \delta}. \tag{32}$$

But (30) then implies that $\phi > 0$, since $\delta > 0$, and (32) implies $\phi < 0$, since $\delta < \gamma$ and $u(\gamma) - \gamma > 0$. This resulting contradiction tells us that this type of equilibrium cannot exist.

A Prohibition on Private Bank Notes

Suppose now that the government can prohibit the issue of private bank notes. That is, assume that the government has the ability to monitor the production of bank notes, but is not able to monitor the production of counterfeits. Then, if there is a public prohibition on the production of private bank notes, everyone knows that a note offered in exchange is a counterfeit. There is then a steady state equilibrium in which counterfeits are never accepted in exchange, but fiat money is.

In this equilibrium, the price obtained for fiat money in exchange, q_m, is determined by equation (13). In contrast to the equilibrium where only fiat

money circulates but private bank note issue is permitted, we do not require that condition (18) holds here for an equilibrium to exist. That is, an equilibrium where fiat money circulates under the prohibition of private money exists for all parameter values.

Existence of a Stationary Equilibrium

I have now determined that, if an equilibrium exists, it must either be one where bank notes and fiat money circulate and counterfeits do not, or where only fiat money circulates. Further, there are restrictive conditions under which fiat money will circulate when private money issue is permitted, and an equilibrium where fiat money circulates always exists when private money issue is prohibited. In this section I want to explore the possibilities for existence of stationary equilibria under counterfeiting and what they mean for the design of monetary systems.

For an equilibrium with the coexistence of fiat money and bank notes, we know from above that conditions (17) and (24) must hold. Alternatively, for an equilibrium with fiat money only, when private money issue is permitted, condition (18) must hold. Now, if

$$\delta \geq \frac{(1 - M)\pi(1 - \eta)u(\gamma)}{2r + (1 - M)\pi}, \tag{33}$$

then if (17) holds, so does (24); under these circumstances the potential issue of counterfeits is irrelevant. That is, if counterfeiting is sufficiently costly, as defined by (33), then so long as $\phi > 0$, a stationary equilibrium with monetary exchange exists where either private money and fiat money circulate (if (17) holds) or where only fiat money circulates (if (18) holds).

Now, if

$$\delta < \frac{(1 - M)\pi(1 - \eta)u(\gamma)}{2r + (1 - M)\pi}, \tag{34}$$

and

$$\phi \in \left(\frac{2r\delta[u(\gamma) - \gamma]}{(1 - \eta)u(\gamma) - \delta}, (1 - M)\pi[u(\gamma) - \gamma] \right), \tag{35}$$

then no equilibrium exists where private money issue is permitted. Under these circumstances, where the cost of counterfeiting is sufficiently small, a stationary equilibrium with monetary exchange can only be supported if there is a prohibition on private money issue.

Thus, the potential for counterfeiting makes a key difference here for the effects of government intervention in the issue of media of exchange. Without the possibility of counterfeiting, the circulation of private money is unambiguously good for economic welfare. If private money were banned under these circumstances, welfare would decrease, and even the introduction of more

government-supplied fiat money into the economy would be detrimental as it inefficiently displaces private money. However, if counterfeiting is possible at a sufficiently low cost, then there can be a classic market failure of the type that can occur in the lemons model of Akerlof (1971). That is, there cannot be an equilibrium where private money and counterfeits coexist, but the issue of private money would induce a flood of counterfeits, so an equilibrium can only exist if there is a prohibition on the issue of private money. Monetary exchange in this case is supported with government supplied fiat money and a ban on private money.

3. CONCLUSION

I have shown, with the aid of a search model of money, some of the benefits and costs of a monetary system where private money can be issued. The issue of private money yields a social benefit in that it leads to productive financial intermediation, which can increase welfare. However, the potential for counterfeiting in this system can also lead to the possibility that monetary exchange can be supported only if private money issue is prohibited.

Though my analysis yields some interesting insights, there are important qualifications to what I have done here. First, I made a very simple assumption in the model: that fiat money could not be counterfeited, while private money could be counterfeited at a cost. While it may be the case that there exist economies of scale in monitoring for counterfeit money, which could imply the optimality of a government monopoly in currency provision, it seems unlikely that fiat money would in general be more difficult to counterfeit than private money. The cost of counterfeiting depends in part on the technology used to produce the money that the counterfeiter is trying to replicate. For example, the new Federal Reserve $20 note is much harder to counterfeit than the old one. In a world with many private money issuers, each private money issuer may invest too little in foiling counterfeiters relative to the social optimum, and it could be that some form of government intervention would correct this market failure. However, to address this issue properly would require a more complicated model with alternative private money production technologies.

Second, a key feature of the monetary search model that lends it tractability is that money is indivisible. Of course, money is certainly indivisible in practice, but the fact that we cannot divide money into denominations smaller than one cent cannot matter much. In our model, agents can carry at most one unit of money, and as a result money is not neutral, which is an undesirable property of the model. Changing the number of units of money in existence will change real variables in the model in the long run, and this was important for some of our results. In particular, we should be skeptical that the result in Section 1 that fiat money displaces private money one-for-one would hold

if money were perfectly divisible. Some authors, in particular Lagos and Wright (2000) and Shi (1997), have studied tractable search models of divisible money. However, it remains to be seen whether these models have much to contribute above and beyond, for example, standard cash-in-advance models.

The model in this article can be extended to examine issues related to the clearing and settlement of private monies, as in Temzelides and Williamson (2001a,b). A more complete model of banking can be embedded in this framework, too, where the banks in the model share some of the features of banks in practice, such as diversification and the transformation of assets (see Williamson [1999]).

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