Pitfalls in Interpreting Tests of Backward-Looking Pricing in New Keynesian Models

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Recently macroeconomists have shown renewed interest in economic models that contain some form of nominal rigidity. These models are referred to generically as New Keynesian models. A particularly important feature of these models is sluggishness in price adjustment. However, there is substantial debate over whether this sluggishness arises from backward-looking adaptive behavior or from forward-looking behavior in the presence of costs in adjusting prices. It is also possible that the economy comprises two types of firms, one type that adjusts the price of its product based on some backward-looking policy and another type that sets its price based on current and anticipated market conditions. Because the nature of price setting is one of the key aspects of New Keynesian models, developing empirical tests that will inform theorists of the correct specification of pricing behavior is essential.

Also, from a policy perspective, understanding how firms set prices is of crucial importance because it determines what the effects of monetary policy will be. For example, as discussed in Ball (1994) and Roberts (1998), credible disinflations are relatively costless in New Keynesian models, but are quite costly from the perspective of traditional backward-looking Keynesian models.

In an attempt to shed empirical light on this question, economists have started investigating the behavior of inflation based on the null hypothesis that

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firms are indeed forward looking. The goal of this work is to test if forward-looking price behavior is consistent with the actual behavior of prices and inflation. This strategy is attractive as a starting point because it is compatible with firms' optimizing behavior. If inhibitions to perfect price flexibility exist, such as adjustment costs or maintaining long-term customer relationships, then it is optimal for a firm to take account of how a chosen price will affect its future profit stream. That is, the firm's pricing decision will be forward looking in much the same way that current investment decisions are based on expectations of future economic conditions. Seminal work in this area has been carried out by Gali and Gertler (1999) and Sbordone (1998).

Many tests used to assess whether forward-looking pricing behavior adequately captures the behavior of inflation also investigate whether the addition of some backward-looking variables appreciably helps explain inflation. A finding that lags of inflation have marginal predictive content is interpreted to mean that a significant fraction of firms are backward looking. Further, this fraction can be estimated. The empirical debate has largely centered on what relevant variables, such as output gaps or marginal cost, should be included in the specification, how to properly measure the variables in question, and the estimation strategy itself. As of yet, there is no general consensus regarding how important forward-looking behavior is in a firm's pricing decisions.[1]

This article takes a different tack. To believe in forward-looking pricing is one thing; it is an entirely different matter to agree on what form that pricing behavior takes. Is it time or state dependent? If time dependent, which of the leading models best describes pricing behavior? Can it be represented by a Calvo-style or quadratic adjustment-cost model? Or is it more amenable to a staggered contracting model in the spirit of Taylor (1980)? As Kiley (1998) and Wolman (1999) have shown, these various models with forward-looking pricing have different implications for how shocks affect the economy and therefore are likely to give rise to different empirical interpretations of pricing behavior. As is also indicated in Guerrieri (2001), the models lead to very different estimable equations. I show that if data are actually generated by a forward-looking model of the Taylor pricing variety, and one estimates a pricing relationship based on Calvo-type behavior, then the conclusion that a significant fraction of firms are backward looking must follow. Thus, the interpretation of various coefficients in existing tests is open to question. Ascertaining the extent of backward-looking pricing behavior may be a more difficult exercise than is currently acknowledged.

[1] Important articles in this literature include Sbordone (1998, 2001), Gali and Gertler (1999), Gali, Gertler, and Lopez-Salido (2001), Rudd and Whelan (2001), Fuhrer (1997), and Roberts (2001).

1. PRICING MODELS

I will begin by outlining two basic pricing models and their implied empirical tests. The first model is the workhorse model of Calvo (1983), which serves as the basis for an important strand of the empirical literature.[2] The second model is a generalization of the more reasonable specification of staggered pricing behavior as postulated by Taylor (1980). The generalization of this model assumes that a fraction of firms change their price in any given period and that at some point every firm will change its price with probability one. The Taylor model can, therefore, be viewed as a truncated version of the Calvo model. I will also exposit the hybrid model of Gali and Gertler (1999), where a fraction of firms follow a backward-looking rule of thumb convention in setting their price.

Calvo-Style Price Setting

In the Calvo price-setting framework, each firm faces a constant probability, $1 - \theta$, that it will be able to adjust its price in the current period and a corresponding probability of θ that it must charge the same price it charged last period. These features imply two equations governing the behavior of prices. One is a backward-looking price level (p_t) equation that is a weighted average of the nominal prices set by firms in prior periods (p^*_{t-j}). Its log-linear approximation takes a particularly simple form,

$$p_t = (1 - \theta) \sum_{j=0}^{\infty} \theta^j p^*_{t-j} = \theta p_{t-1} + (1 - \theta) p^*_t, \tag{1}$$

where all variables are in logarithms. Equation (1) can also be expressed as a partial adjustment mechanism, $p_t - p_{t-1} = (1 - \theta)[p^*_t - p_{t-1}]$. The partial adjustment interpretation indicates that the price level responds only gradually when p^*_t is raised above p_{t-1}, with the extent of price level adjustment equal to the probability of price adjustment. Equation (2) (also a log-linear approximation) describes forward-looking price setting and reflects the notion that firms understand they may not be able to reset their price in future periods. They appropriately set their price to maximize a discounted expected stream of profits. Thus, current price setting depends on future nominal marginal

[2] The quadratic cost-of-adjustment model developed by Rotemberg (1982) gives rise to a similar pricing equation.

cost,

$$p_t^* = (1 - \beta\theta) \sum_{j=0}^{\infty} (\beta\theta)^j E_t[(\psi_{t+j}/\psi) + p_t]$$
$$= \theta\beta E_t p_{t+1}^* + (1 - \beta\theta)[(\psi_t/\psi) + p_t], \quad (2)$$

where ψ_t is the logarithm of real marginal cost and ψ is the logarithm of the steady state value of real marginal cost and β is the rate at which future utility is discounted. E_t is the conditional expectations operator where expectations are conditioned on all current and past information. Combining equations (1) and (2) yields an equation for inflation of the form

$$\pi_t = \lambda\psi_t + \beta E_t \pi_{t+1}, \quad (3)$$

where $\pi_t = p_t - p_{t-1}$, and $\lambda = (1 - \theta)(1 - \beta\theta)/\theta$.

Using the Calvo model of price adjustment is attractive because of its tractability and parsimony. It is largely because of these two characteristics that the Calvo model has taken center stage in empirical work regarding forward pricing behavior. The model, however, contains a number of unrealistic features. For example, there exists a measurable fraction of firms that have not changed their price for an arbitrarily long time, and these firms produce a significant portion of total output. Accordingly, one would at least expect all firms to change their price after some finite length of time. It is hard to believe that the costs of adjusting prices are so high that it is not beneficial to change prices frequently. Thus, a useful extension of the model would be to set a finite time limit over which a firm's price remains unchanged. Setting such a time limit makes the pricing formulas much more complex and would not be worthwhile if the implications of the added realism were innocuous. Wolman (1999) and Kiley (1998) indicate that this truncated version of the model yields very different behavior than the original Calvo model. I therefore investigate the pricing implications of the truncated model because it may provide a more realistic version of firm behavior.

Generalized Taylor Staggered Price-Setting

In the Taylor framework, as in the Calvo model, a firm that has not changed its price for j periods faces a probability α_j of changing its price, but at some finite horizon J a firm changes its price with probability one. If $\alpha_j = 0$ for all $j < J$, then the model is the basic staggered price-setting model of Taylor (1980) with $1/J$ of firms changing prices each period. Wolman (1999) argues that a more realistic price-setting model would involve monotonically increasing probabilities, $0 \leq \alpha_j \leq \alpha_{j+1} < 1$ for all $j < J$, and $\alpha_J = 1$. His specification implies that a firm that has not changed its price for a number of periods is more likely to change its price than a firm that recently reset its price.

For ease of comparison with the basic Calvo model, I assume $\alpha_j = \alpha < 1$ for all $j < J$, and for tractability take $J = 3$. As in the Calvo model, price-setting behavior is characterized by two equations (see the appendix), a backward-looking equation describing the price level,

$$p_t = \omega_0 p_t^* + \omega_1 p_{t-1}^* + \omega_2 p_{t-2}^*, \tag{4}$$

and a forward-looking equation depicting optimal price-setting,

$$p_t^* = \rho_0(\psi_t + p_t) + \rho_1 E_t(\psi_{t+1} + p_{t+1}) + \rho_2 E_t(\psi_{t+2} + p_{t+2}). \tag{5}$$

Both of these equations are linearizations around zero inflation of the nonlinear equations that exactly describe model behavior, and the variables in both are expressed as logarithmic deviations from steady state.[3] The parameters ω_j represent the fraction of firms that have not changed their price for j periods and are a function of α. The ρ's arise from the linearization of the optimal price-setting equation and involve the probability α and the time discount factor β that agents use when discounting future utility.

Combining (4) and (5) yields the following difference equation in inflation and marginal cost:

$$\begin{aligned} &\{1 + c_1 L + c_2 L^2 + c_3 L^3\} E_{t-2}\pi_{t+2} = \\ &-\{1 + a_1 L + a_2 L^2 + a_3 L^3 + a_4 L^4\} E_{t-2}\psi_{t+2}, \end{aligned} \tag{6}$$

where E_{t-2} is the expectations operator conditional on information as of $t-2$ and L is the lag operator.[4] As mentioned, the Calvo and Taylor models of price setting result in very different nominal behavior, and these differences carry over to the empirical tests of forward-looking pricing. Equation (6) is the analogue to (3) and contains a number of important differences. First, lagged inflation enters this expression, as does lagged marginal cost. Also, the lead structure in (6) is more complicated, and expectations are conditioned on more distant past information. The different conditioning set will have implications for the admissibility of variables as instruments in the estimation carried out below.[5]

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[3] The price level equation is given by $P_t = \left[\sum_{j=0}^{2} \omega_j P_{t-j}^{*(1-\epsilon)}\right]^{\frac{1}{1-\epsilon}}$, where variables are in levels and ϵ is the elasticity of demand for the firm's product. The optimal price-setting equation is $p_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{j=0}^{2} \beta^j E_t\{(\omega_j/\omega_0)\cdot(\lambda_{t+j}/\lambda_t)\cdot\psi_{t+j}\cdot(P_{t+j}/P_t)^{\varepsilon}\cdot y_{t+j}\}}{\sum_{j=0}^{2} \beta^j E_t\{(\omega_j/\omega_0)\cdot(\lambda_{t+j}/\lambda_t)\cdot(P_{t+j}/P_t)^{\varepsilon-1}\cdot y_{t+j}\}}$, where y is the firm's level of output. For more detail concerning the derivation in the text, see Dotsey, King, and Wolman (1999).

[4] In deriving (6), use was made of the fact that unity is one of the roots of the fourth order polynomial describing the behavior of deviations in the price level around its steady state (see the appendix).

[5] Similar observations are made by Guerrieri (2001).

Further, the different empirical implications generated by (6) apply to more realistic pricing models such as the one used by Wolman (1999), who assumes that the probability a firm will change its price is increasing in the elapsed time since its last price adjustment, and to state-dependent models of the type explored in Dotsey, King, and Wolman (1999). The essential characteristic of these types of models is that they generate higher order difference equations in inflation. They do so as long as firms exist that have not adjusted their price for more than two quarters, a feature needed to match microdata on firm pricing, and as long as all firms adjust their price in some finite time interval.

A Hybrid Calvo Model

To investigate whether backward price-setting behavior is also needed to explain the data, economists have postulated that only a fraction of firms base their price on optimizing behavior and that the remaining firms use a rule of thumb based on past prices and inflation. Within the Calvo framework, Gali and Gertler (1999) describe one such rule that leads to a relatively tractable hybrid Phillips curve. Their pricing rule is depicted by

$$p_t^b = \overline{p}_{t-1}^* + \pi_{t-1}.$$

Backward-looking firms set their price, p_t^b, based on an index reflecting the behavior of all firms who changed their price last period, $\overline{p}_{t-1}^*$, and on a correction term involving lagged inflation, π_{t-1}.

In turn, the current price index reflecting the behavior of all price setters is given by

$$\overline{p}_t^* = (1-\varpi)p_t^* + \varpi p_t^b,$$

where ϖ is the fraction of firms that are backward looking. As long as forward-looking price setters compose a significant fraction of firms, the price index of newly set prices will be dominated by forward-looking firms. In the presence of low rates of inflation, the backward-looking price setter's price will not depart far from $\overline{p}_{t-1}^*$. Taken in conjunction, these two assumptions imply that prices set by backward-looking price setters will not depart very far from an optimizing price.

The hybrid model just described implies an equation describing inflation of the form

$$\pi_t = \lambda\psi_t + \gamma_f E_t\pi_{t+1} + \gamma_b\pi_{t-1}, \tag{7}$$

where λ, γ_f, and γ_b are, respectively, nonlinear functions of the discount rate, the probability of price adjustment, and the fraction of firms that are forward looking. In estimation of equations of the form (7), a significant coefficient on lagged inflation is generally taken to imply some departure from rationality on the part of agents (see Roberts [1998]). In the hybrid model, the departure is represented by a fraction of firms that are backward looking, and, as in Gali and Gertler (1999), that fraction is readily ascertained by uncovering the fundamental parameters of the model.[6] I show below that this interpretation is part of a joint hypothesis, an important component of which is the Calvo model of pricing. If prices are indeed forward looking, but are generated from behavior consistent with the generalized Taylor-style pricing model, then the interpretation may not be correct.

2. INTERPRETING TESTS FOR BACKWARD-LOOKING BEHAVIOR

In this section, data are generated from a generalized Taylor staggered pricing model and then used in tests based on Calvo-style pricing to investigate the estimated presence of backward-looking price setting given the knowledge that all firms in the model are forward looking. Other than the pricing behavior, which is depicted by equations (4), (5), and (6), the particular details of the model are not overly important. What is important is that data on marginal cost and inflation are being generated in a manner that is consistent with the underlying state variables of the model. Such treatment is consistent with the empirical work in this area, where only the pricing behavior is carefully exposited. The full model is that of Dotsey and King (2001) without intermediate inputs, and it is driven by shocks to money growth, technology, money demand, and government spending. Thus, the state variables are the aforementioned shocks, past relative prices, and the capital stock.[7]

Before I test the model, it is worth reiterating an important feature of the pricing equations, namely that real marginal cost is the appropriate variable to be included in the determination of inflation. This point has been strongly emphasized by Gali and Gertler (1999) and Sbordone (1998). Many authors, however, have used the output gap, defined as the deviation of the level of output from its long-run or trend level, as the principal determinant of inflation.[8] If we take the various New Keynesian sticky price models as the null to be tested, we see that this alternative procedure is a mistake.

Output-gap measures produce serious problems of measurement error under the null of a New Keynesian model. Under suitable assumptions about

[6] Another interpretation is that expectations only adapt gradually to their rational value.

[7] The shocks all have a standard deviation of 1 percent and the respective autoregressive parameters are 0.8, 0.6, 0.7, and 0.4 for technology, money supply, money demand, and government spending. Thus, I have made no attempt to accurately calibrate the driving processes.

[8] For example, see Fuhrer (1997) and Roberts (2001).

technology and factor markets, the relationship between marginal cost and potential output, y_t^* (which is output that would occur if prices were counterfactually flexible), is $\psi_t - \overline{\psi} = \kappa(y_t - y_t^*)$, where $\overline{\psi}$ is steady state marginal cost. The right-hand-side term may be rewritten as the sum of two terms, $(y_t - y_t^{trend}) + (y_t^{trend} - y_t^*)$, where y^{trend} is some measure of trend output. The first term corresponds to the output gap and the last term embodies the measurement error associated with using the output gap. The bias induced by this measurement error will depend on the way trend output is measured and other features of the economy, notably the conduct of policy. For example, if policy kept the price level constant, then there would be no variation in marginal cost in response to a technology shock. There would, however, be variation in the output gap, causing its coefficient in a Phillips curve relationship to be biased downward. In response to other shocks and to other policy rules, the effects of the misspecification on the estimated coefficients could become quite complicated.

Testing the Generalized Taylor Price-Setting Model

The test of forward-looking pricing behavior is implemented by using the equation describing inflation. In our example, this test would be based on equation (6) and could be carried out using the Generalized Method of Moments (GMM). The correct orthogonality condition is

$$\begin{aligned} E_{t-2}\{[\pi_t + (1/c_2)(\pi_{t+2} + c_1\pi_{t+1} + c_3\pi_{t-1} \\ + \psi_{t+2} + a_1\psi_{t+1} + a_2\psi_t + a_3\psi_{t-1} + a_4\psi_{t-2})]s_{t-2}\} = 0, \end{aligned} \tag{8}$$

where in this example the instruments should be the twice-lagged states from the economic model, s_{t-2}. Thus, under the null of a generalized Taylor price-setting model, the equation describing inflation should be tested using a fairly complicated orthogonality condition that includes lags of marginal cost and inflation. Again, in performing the test one should use the actual states as instruments. In practice, a Calvo-type model is tested with instruments that are not the true states. The actual set of state variables is not used in the test because the econometrician does not have access to a time series on the past prices set by adjusting firms or the various economic shocks. Thus, under the null of generalized Taylor-style price setting, the tests commonly used to determine whether forward-looking price setting explains the behavior of inflation are misspecified. Relevant variables and restrictions are omitted, and the instrument set is incorrectly specified.

Testing the Calvo Model

To analyze the potential consequences of model misspecification, I investigate the empirical results when tests that assume the underlying model is of the Calvo variety are conducted on data generated by a generalized Taylor price-setting model. I perform two sets of estimates, one based on a sample of 25,000 observations, referred to as the population estimates, and the other based on 500 simulations involving samples of 200 observations, referred to as the finite sample estimates.

Based on equation (3), the orthogonality condition is

$$E_t\{(\pi_t - \lambda\psi_t - \beta E_t \pi_{t+1})z_t\} = 0,$$

where z_t is an instrument vector containing three lags each of inflation, labor share, and output, and, as described above, λ is a combination of the time preference parameter β and the probability that a firm will not be able to reset its price, θ.[9] The population estimates of these two parameters are 0.58 and 0.35, whereas the average finite sample estimates are 0.56(0.24) and 0.36(0.035), with standard errors in parenthesis. The estimate of β is well below its true value of 0.99 and is also substantially less than that estimated by Gali and Gertler (their estimate is 0.926).[10] The estimate of θ implies a mean lag in the Calvo model of roughly 1.5 quarters, which is smaller than the true mean lag of 2.4 quarters. Gali and Gertler's estimate of θ implies a rather long mean lag of 8.6 quarters and indicates that three-period staggering is insufficient for capturing the underlying price stickiness in the U.S. economy. Restricting the coefficient on β to one only slightly affects the estimate of θ. In population the estimate is 0.37 and in sample it is 0.38(0.038).

Estimating a Calvo model when the true model involves three-period Taylor-type contracts implies both a misspecification and that the instruments are correlated with the error term. The correlation arises because the true error term includes two lags of marginal cost, as well as expectational errors of future inflation and marginal cost that are based on information up to two periods ago. This correlation is confirmed by the rejection of the overidentifying restrictions. This rejection of the orthogonality of the instruments also occurs when actual data is used. Although Gali and Gertler indicate that their instruments pass the test for overidentification, that result appears to be due to the choice of a number of poor instruments. When I perform the above estimation on their data, using a set of instruments similar to the one used in

[9] The specification in terms of structural parameters is $E_t\{(\theta\pi_t - (1-\theta)(1-\beta\theta)\psi_t - \theta\beta E_t\pi_{t+1})z_t\}$ and corresponds to specification 1 in Gali and Gertler (1999).

[10] If I use Gali and Gertler's method 2, the population estimates for β and θ are 0.72 and 0.47, respectively.

testing model data, I replicate their point estimates almost exactly.[11] However, the model fails the test for instrument orthogonality at 10 percent significance levels.[12] The analysis presented in this article indicates that the failure may be a result of underlying price behavior that conforms in fact more closely with a staggered price-setting model.

Testing the Hybrid Model

I will now test to see if lagged inflation is statistically significant when added to the Calvo specification. From equation (6), which describes the behavior of inflation in the true model, one would expect lagged inflation to be significant in the estimation. However, because the coefficients c_2 and c_3 are both negative, one might expect the coefficient on lagged inflation to be negative. The orthogonality condition in the GMM estimation is

$$E_t\{(\pi_t - \lambda\psi_t - \gamma_f E_t\pi_{t+1} - \gamma_b\pi_{t-1})z_t\} = 0,$$

where $\lambda = (1 - \varpi)(1 - \theta)(1 - \beta\theta)/\phi$, $\gamma_f = \beta\theta/\phi$, $\gamma_b = \varpi/\phi$, and $\phi = \theta + \varpi(1 - \theta(1 - \beta))$. The population estimates of β, θ, and ϖ are 0.60, 0.36, and 0.10, respectively, and the finite sample estimates are 0.60(0.24), 0.37(0.038), and 0.13(0.073). The latter estimates imply a value of $\lambda = 0.94$, $\gamma_f = 0.47$, and $\gamma_b = 0.25$. The value of γ_b is exactly the same as that found by Gali and Gertler on U.S. data.

The positive coefficient on the lagged inflation term occurs because the error term in the regression includes not only expectational errors, but also lagged marginal cost terms and a term involving two period leads of inflation and marginal cost. Further, when the instrument set is insufficiently lagged, the expectational errors will also be correlated with the instruments. Thus, the coefficients in the regression will be biased. The bias involves complicated terms arising from the relationships between the instruments and the explanatory variables as well as from the correlations between the omitted variables that appear in the error term and the variables in the regression. Regarding the latter, the correlation between lagged marginal cost and lagged inflation is 0.75 and between twice-lagged marginal cost and lagged inflation is 0.55. If one estimates the linear relationship implied by the above orthogonality condition, ignoring the relationship between $(\lambda, \gamma_b, \gamma_f)$ and (β, θ, ϖ), then it turns out that γ_f is biased downward and the other two coefficients are biased upward. Thus, the misspecification inherent in the Calvo model implies a downward

[11] Using method 2, my estimate of β is 0.965 compared to their estimate of 0.941, and my estimate of θ is 0.895 while theirs is 0.884.

[12] My instrument set is three lags of inflation, labor share, and output growth.

bias in the estimated importance of forward-looking behavior and an upward bias in the importance of backward-looking behavior.[13]

Fundamental Inflation

I now compute what is termed *fundamental inflation* in order to analyze how well inflation predicted by the estimated model matches inflation generated by the theoretical model. Using the estimates from the regression with once-lagged instruments, I can calculate fundamental inflation (inflation that is generated entirely by the pricing equation of the model) as in Gali and Gertler (1999) by solving difference equation (7). One eigenvalue of this difference equation, δ_1, is less than one while the other, δ_2, lies outside the unit circle. The solution for fundamental inflation is

$$\pi_t = \delta_1 \pi_{t-1} + \frac{\lambda}{\delta_2 \gamma_f} \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2} \right)^k E_t \psi_{t+k}.$$

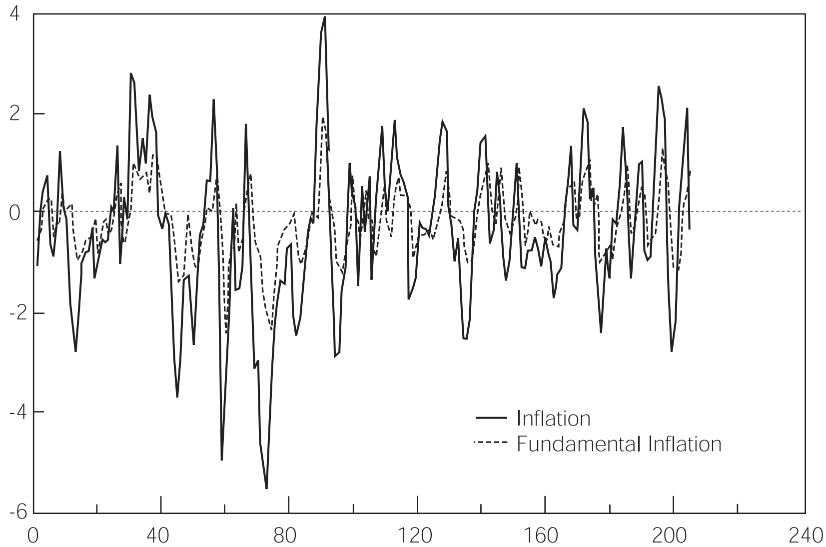
To calculate the summation term, I estimate a Vector Autoregression (VAR) describing inflation and marginal cost on a typical simulation of the model. The simulation in question produced estimates of β, θ, and ϖ of 0.61, 0.36, and 0.095, respectively. The VAR included four lags of each variable, and the forward-looking sum was derived from the estimated equations. Figure 1 depicts the results. Fundamental inflation explains much of the actual movement in inflation, and there is no evidence of systematic bias. The correlation between fundamental and actual inflation is 0.71. In calculating fundamental inflation, it is important to note that even if the coefficient on lagged inflation is small, backward-looking behavior may be important for the dynamics of inflation. The importance arises because the dynamics are governed by the eigenvalues, which are in turn functions of all the underlying parameters.

3. CONCLUSION

This article critically examines the common interpretation of a finding that lagged inflation helps explain the behavior of current inflation. The common interpretation is that some departure from optimality exists in the pricing behavior of firms. A popular explanation of this departure involves the presence of backward rule-of-thumb behavior by some fraction of firms, but irrational forecasting of expected inflation is sometimes also invoked as an explanation. Here, I use a generalized Taylor pricing model as a data-generating mechanism and show that incorrectly basing tests on pricing behavior of the type described

[13] This type of bias may also be present in Fuhrer (1997) because his model fails to account for sufficient lags of the output gap.

Figure 1



by a Calvo model can produce significant coefficients on lagged inflation even though all firms are rational and forward looking. Thus, the interpretation of a significant coefficient on lagged inflation in a pricing equation may be more subtle than is currently realized.

APPENDIX

I derive the underpinnings of equation (6) for an economy that has zero inflation. Let α_j denote the probability that a firm that last changed its price $j \leq 3$ periods ago changes its price in the current period, and let $\alpha_3 = 1$. Defining $\eta_j = 1 - \alpha_j$, then the fraction of firms that change their price in the current period, $\omega_0 = 1/(1 + \eta_1 + \eta_1\eta_2)$, the fraction that last changed their price one period ago, $\omega_1 = \eta_1/\ (1 + \eta_1 + \eta_1\eta_2)$, and the fraction that last changed their price two periods ago, $\omega_2 = \eta_1\eta_2/\ (1 + \eta_1 + \eta_1\eta_2)$. The price level under generalized Taylor pricing is given by

$$P_t = \left[\sum_{j=0}^{2} \omega_j P_{t-j}^{*(1-\epsilon)} \right]^{\frac{1}{1-\epsilon}},$$

where variables are in levels and ϵ is the elasticity of demand for the firm's product. The optimal price-setting equation is

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{2} \beta^j E_t\{(\omega_j/\omega_0) \cdot (\lambda_{t+j}/\lambda_t) \cdot \psi_{t+j} \cdot (P_{t+j}/P_t)^{\varepsilon} \cdot y_{t+j}\}}{\sum_{j=0}^{2} \beta^j E_t\{(\omega_j/\omega_0) \cdot (\lambda_{t+j}/\lambda_t) \cdot (P_{t+j}/P_t)^{\varepsilon-1} \cdot y_{t+j}\}},$$

where y is the firm's level of output and $\beta^j E_t(\lambda_{t+j}/\lambda_t)$ is the rate at which profits are discounted. For more detail concerning the derivation of these two equations, see Dotsey, King, and Wolman (1999).

Log-linearizing the expression for the price level around zero steady state inflation yields $p_t = \omega_0 p_t^* + \omega_1 p_{t-1}^* + \omega_2 p_{t-2}^*$, which is (4) in the text. Log linearizing the equation for the optimal price yields

$$p_t^* = \rho_0(\psi_t + p_t) + \rho_1 E_t(\psi_{t+1} + p_{t+1}) + \rho_2 E_t(\psi_{t+2} + p_{t+2}),$$

which is (5), where $\rho_0 = 1/\Delta$, $\rho_1 = \beta\eta_1/\Delta$, and $\rho_2 = \beta^2\eta_1\eta_2/\Delta$, and $\Delta = 1 + \beta\eta_1 + \beta^2\eta_1\eta_2$. The linearization turns out to be so compact because at zero inflation many of the terms cancel out (for a general derivation, again see Dotsey, King, and Wolman [1999]).

Combining (4) and (5) for the prices p_t^*, p_{t-1}^*, and p_{t-2}^* yields the following difference equation:

$$\{1 + a_1 L + (a_2 - 1/(\omega_0\rho_2))L^2 + a_3 L^3 + a_4 L^4\}E_{t-2}p_{t+2} =$$
$$\{1 + a_1 L + a_2 L^2 + a_3 L^3 + a_4 L^4\}E_{t-2}\psi_{t+2},$$

where $a_1 = (1 + \beta\eta_1)/(\beta\eta_2)$, $a_2 = (1 + \beta\eta_1^2 + \beta^2\eta_1^2\eta_2^2)/(\beta^2\eta_1\eta_2)$, $a_3 = (1 + \beta\eta_1\eta_2)/(\beta^2\eta_2)$, and $a_4 = 1/\beta^2$. One of the roots of the polynomial on $E_{t-2}p_{t+2}$ is one, and factoring this root yields (6),

$$\{1 + c_1 L + c_2 L^2 + c_3 L^3\}E_{t-2}\pi_{t+2} =$$
$$-\{1 + a_1 L + a_2 L^2 + a_3 L^3 + a_4 L^4\}E_{t-2}\psi_{t+2},$$

where $c_1 = 1 + (1 + \beta\eta_1\eta_2)/(\beta\eta_2)$, $c_2 = -(1 + \eta_2 + \beta\eta_1\eta_2)/(\beta^2\eta_2)$, and $c_3 = -1/\beta^2$.

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