Firms, Assignments, and Earnings

Edward Simpson Prescott

The U.S. distribution of labor earnings is highly skewed to the right. Roughly, the lowest 50 percent of U.S. households, as measured by individual labor earnings, make 10 percent of total labor earnings. The next lowest 30 percent earn approximately 30 percent and highest 10 percent make 40 percent.[1]

Earnings are also related to a person's position within a firm and employment at a particular firm. Within a firm earnings tend to be associated with rank. The higher is an individual's authority and control, the higher is his compensation. The most extreme manifestation of this is the enormous pay of the top executives of large firms. In 1996 the median pay of chief executive officers of companies in the S&P 500 index was nearly 2.5 million dollars (Murphy 1999).

Across firms earnings tend to increase with firm size. This is particularly true for executives. The elasticity of executive pay with respect to firm size is in the range of 0.20 to 0.35 (Rosen 1992). Earnings for workers also increase with firm size. This is the well-documented wage-size premium (Brown and Medoff [1989] and Oi and Idson [1999]).

The standard neo-classical production function, where output equals a function of aggregate labor and aggregate capital, cannot simultaneously account for these facts. It can generate an unequal distribution of earnings, if some people's labor is more efficient than others. But it has only one economy-wide firm so it is necessarily silent on any relationship between earnings and firm assignments. And even with respect to the distribution of earnings, the

The author would like to thank Andreas Hornstein, Tom Humphrey, Pierre Sarte, and John Weinberg for helpful comments. The views expressed in this article do not necessarily represent the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

[1] These are 1998 numbers taken from the Survey of Consumer Finances as reported by Rodriguez, Diaz-Gimenez, Quadrini, and Rios-Rull (2002). They define labor earnings as wages and salaries plus 85.7 percent of business and farm income.

inequality of labor efficiency that would imply such a distribution seems so unequal as to defy credulity.

For a theory to explain these facts, it needs to solve the problem of *jointly* assigning workers and managers to firms. This paper sketches such a theory that is based on the firm-size model of Lucas (1978) and on the hierarchy models of Rosen (1982, 1992). For simplicity, most of the analysis focuses on firms with only two types of jobs, executives and workers. This is enough to illustrate the connection between pay and rank within and across firms; it also has the advantage of allowing us to discuss the well-documented patterns in executive pay.[2]

In a firm the role of a manager is more important than that of any single worker, just as the role of the chief executive officer is more important than that of any subordinate, manager or worker. Firms are structured as hierarchies in which decisions made by a high-level manager affect the productivity of individuals in lower levels who report directly, or indirectly, to the manager. Decisions made at each successively higher level in a firm affect proportionately more people. Ultimately, the top executive's decisions affect the productivity of everyone within the firm. Figure 1 illustrates.

For this reason it matters a lot who is assigned to the top positions within a firm. For a firm, a small difference in managerial talent at the highest level leads to a big difference in output. As a consequence, within a firm it is best to place the most talented individual at the top, while across firms it is best to place the most talented individual at the largest firm. For both of these reasons, scarce managerial talent can be incredibly valuable. Within firms it leads to earnings inequality over rank. Across firms it lead to earnings inequality over firm size.

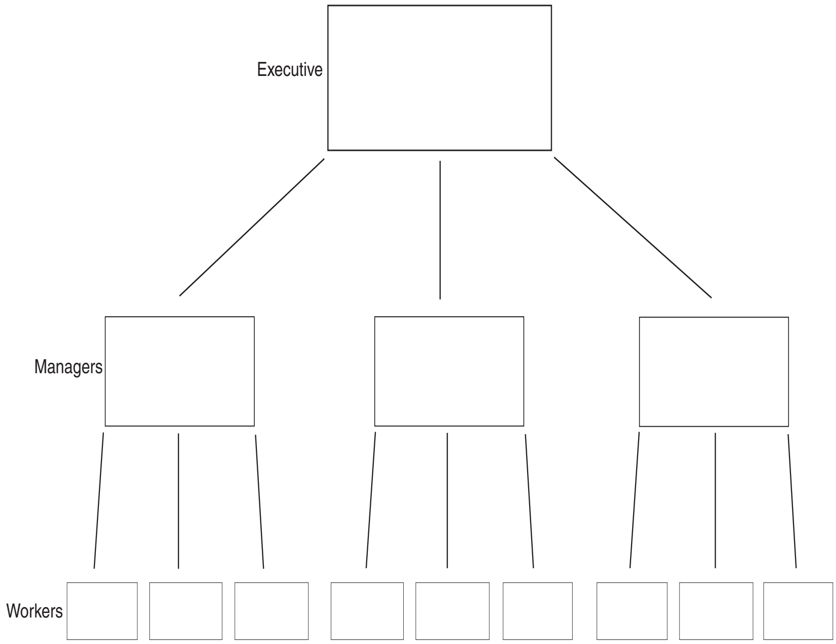
Section 1 studies a problem where people are assigned to be either workers or managers. All firms are a hierarchy with one level of management. This model is a simplification of Lucas (1978). Short discussions of executive pay, the wage-size premium, and marginal product pricing in assignment models are included. Section 2 studies a simple extension of the Lucas model to incorporate multi-level hierarchies as in Rosen (1982). Section 3 provides a concluding discussion.

1. TWO-LEVEL FIRMS

All production in this economy is done by firms. Each firm consists of a manager and a number of workers. A firm's production depends on the talent of

[2] Much of the recent literature on executive compensation has focused on the important question of *how* to pay executives in order to motivate them to act in the best interests of the corporation. That issue is not discussed in this paper. Surveys can be found in Rosen (1992) and Murphy (1999).

Figure 1 Organization of Production within a Firm



Notes: A three-level firm in which decisions made by a manager affect the productivity of all individuals who directly and indirectly report to him. The single individual at the top is the executive, the three individuals at the second level are lower-level managers, and the remaining nine individuals are workers.

the manager and the amount of labor supplied by the workers. The production function is $tf(l)$, where t is the talent of the manager and l is the number of workers working for him. The function $f(l)$ is concave so given a manager there is decreasing returns to scale in the number of workers who work for him. Decreasing returns at the firm level will lead to the existence of multiple firms rather than just one large firm with everyone working under the most talented manager. The number of workers working for a manager is often called a manager's *span of control*. The more talented a manager is the more workers who work for him, and the larger is his span of control.

People differ in their managerial talent. Talent is distributed by $h(t)$ across the population. $H(t)$ is the cumulative distribution function. There is an indivisibility in an individual's job. A person can either be a manager or a worker, but not both at the same time. The problem in this economy is to determine who will be a manager and then how many workers will be assigned to each manager.

Each person must decide whether to be a worker or a manager. If he chooses to be a worker then he receives the labor wage of w, which is independent of his talent. If he chooses to be a manager, he must decide how much labor to hire. He does this by solving

$$\max_{l \geq 0} tf(l) - wl.$$

The first-order condition is

$$tf'(l) - w = 0. \tag{1}$$

A manager's earnings π is equal to $tf(l) - wl$. Naturally, a manager must be paid at least as much as the wage or he would choose to be a worker. Since managerial earnings are increasing in talent there is a unique cutoff level of talent z for which all people with $t \geq z$ are managers and the rest are workers. Let $l(t)$ be the labor hired by a manager of talent t. Then,

$$zf(l(z)) - wl(z) = w. \tag{2}$$

This condition just states that a marginal manager's profit, $zf(l(z)) - wl(z)$, equals his opportunity cost of working, w.

There is also a resource constraint on the supply of labor. It is

$$\int_z^\infty l(t)dH(t) \leq H(z). \tag{3}$$

The left-hand side is labor hired while the right-hand side is labor supplied.

A *competitive equilibrium* is a cutoff level of talent z, an assignment of labor to managers $l(t)$ for $t \geq z$, and a wage w that satisfies the managers' first-order conditions, that is, (1) for all $t \geq z$, indifference for the marginal manager (2), and the resource constraint (3).

To illustrate the connection between firms and pay, we study the case where $f(l) = l^\alpha$ with $0 < \alpha < 1$. The number of employees assigned to a firm, $l(t)$, can be determined from (1). It satisfies

$$l(t) = (w/\alpha)^{1/(\alpha-1)} t^{1/(1-\alpha)}.$$

Because α is between zero and one, the number of employees grows more than proportionately with the manager's talent. Another measure of firm size is firm revenue or output $q(t)$. Its relationship with talent is nearly identical to that of $l(t)$. It is

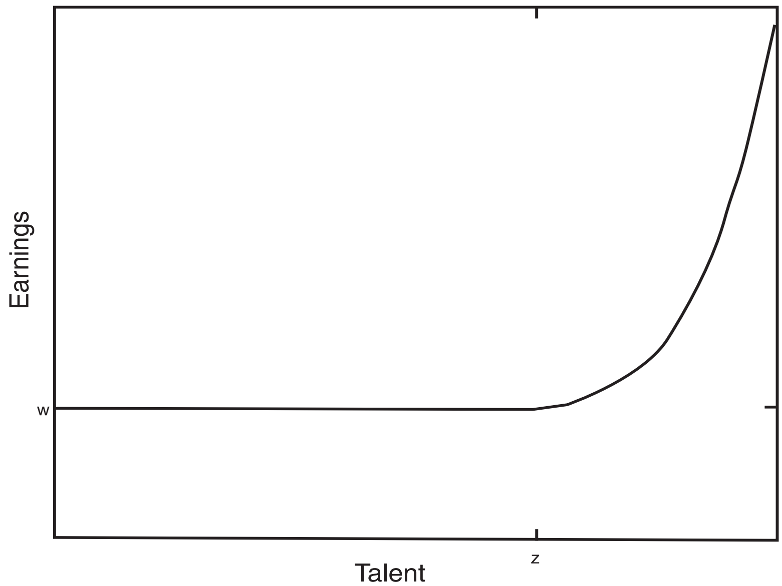
$$q(t) = tl(t)^\alpha = (w/\alpha)^{\alpha/(\alpha-1)} t^{1/(1-\alpha)}. \tag{4}$$

A similar relationship holds for managerial pay. Then,

$$\pi(t) = tl(t)^\alpha - wl(t) = \left((w/\alpha)^{\alpha/(\alpha-1)} - w(w/\alpha)^{1/(\alpha-1)}\right) t^{1/(1-\alpha)}. \tag{5}$$

Managerial pay grows more than proportionately with talent. Small differences in talent at the managerial level lead to large differences in pay (and firm size). The result is appealing because it implies that even with a symmetric

Figure 2 Earnings and Talent



Notes: Earnings as a function of talent. All individuals with talent $t < z$ are assigned to be workers and earn wages w. Individuals with more managerial talent are managers and their pay is an increasing convex function of their talent.

distribution of talent, which has some natural appeal, earnings and firm size will be skewed to the right, as is observed in the data.[3] Figure 2 illustrates the relationship between talent and earnings in this example.

While the relationship of firm size and executive pay to talent is of interest, the applicability of these theoretical results is limited. The talent distribution is not observed and there is little hope of directly observing it. However, the theory does predict a relationship between firm size and executive pay that, for this example, is independent of the talent distribution or talent level. The relationship follows directly from (1). Notice that

$$\frac{q(t)}{l(t)} = tl(t)^{\alpha-1} = \frac{w}{\alpha}. \tag{6}$$

[3] Like the earnings distribution, firm size is highly skewed to the right. This is true for a variety of firm size measures like assets, employment, sales, and others. See Simon and Bonini (1958).

Managerial pay with respect to firm size, as measured by $q(t)$, is

$$\pi(t) = q(t) - wl(t) = (1 - \alpha)q(t). \tag{7}$$

In this example, managerial pay is linear in firm size. (It is also linear in firm size if firm size is defined as number of employees.)

Implications for Executive Pay

Qualitatively, the theory seems on the mark. Executive pay grows with firm size. Quantitatively, however, some other functional form is needed. In the data the *log* of executive pay is linear in the *log* of firm size, which means that the level of executive pay takes the form

$$pay = b(size)^{\beta}. \tag{8}$$

Numerous studies find that the elasticity, β, is around 0.20-0.35. Elasticities in this range have been found in U.S. data during the 1940s and 1950s (Roberts 1956), U.S. data in the late 1930s (Kostiuk 1989), U.S. data from 1969–1981 (Kostiuk 1989), U.K. data during 1969–1971 (Cosh 1975), and U.S. banking data in the 1980s (Barro and Barro 1990). See Rosen (1992) and Murphy (1999) for more discussion.

One important feature of the data that the model is silent on is the large increase in the ratio of executive pay to worker pay observed over the last 30 years. In 1970 the average executive of an S&P 500 firm made 30 times the average worker wage. In 1996 this ratio was 90 for cash compensation and 210 for realized compensation, which includes the value of exercised stock options (Murphy 1999).

One strategy for addressing this question is to postulate that there was an exogenous change in the technology by changing the production technology to $tAf(l(t))$, where $A > 1$, and f is homogenous of degree α. Interestingly, this has no effect on the economy except to raise everyone's wealth by a factor of A. In particular, set the new wage to Aw and keep the z and the $l(t)$ unchanged from the above model. This allocation satisfies the first-order conditions. Worker pay grows by the factor A and so does managerial pay. Managers still supervise the same number of people and wages and managerial rents increase by the constant factor.

More promising strategies include postulating an exogenous change in the span of control technology, say, from advances in information technology, or by introducing capital. Lucas (1978) includes capital so that the production function is $tf(g(l, k))$, where g is a constant returns-to-scale technology. In his model, as an economy grows wealthier the capital-to-labor ratio in firms increases, there are less firms, and firm size increases. These forces increase executive pay, though the precise effect on the ratio of executive to worker pay is unclear because wages increase as well. Still, the growth in executive pay in the last 30 years seems much greater than can be accounted for by changes in the capital stock so one would guess that other factors are also at work.

Talent as a Worker Input

In the simple model of the previous section there was a constant wage for all workers. A worker was paid the same no matter what firm employed him. In the data, however, there is a premium for working for a larger firm. This well-documented observation has been reported by Brown and Medoff (1989), Idson and Oi (1999), Oi and Idson (1999), Troske (1999), and others. Idson and Oi (1999) report an elasticity of wages with respect to plant size of 0.075 using 1992 data from the Census of Manufactures. This size elasticity implies that an employee who works for a plant that is twice the size of another plant earns 5 percent more than an employee at the smaller plant.

In this section, we modify the production function to allow talent to affect output at the worker level as well as at the managerial level. This will add more earnings inequality. Alone it does not necessarily generate a wage-size premium but it does give some insight into what might generate it.

Let $d(t)$ be the total *talent* of workers assigned to manager t and let the production function now be $tf(d(t))$. The wage w now refers to the payment per unit of hired talent so a worker of talent t is paid wt. A manager's maximization problem is

$$\max_{d \geq 0} tf(d) - wd.$$

The first-order condition is nearly identical to that of the previous problem. It is

$$tf'(d) - w = 0. \tag{9}$$

Marginal managers are indifferent to managing and working. This condition is

$$zf(d(z)) - d(z)w = zw. \tag{10}$$

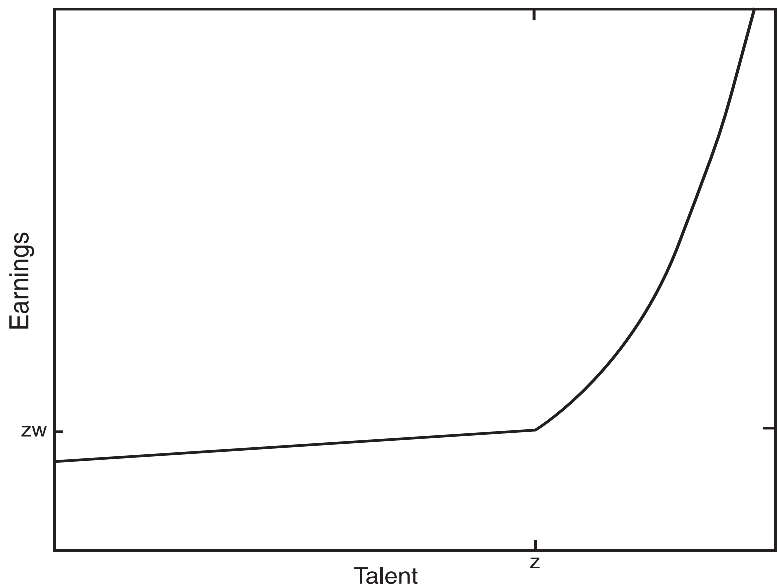
Notice that now the opportunity cost of managing is the marginal manager's talent times the wage.

Finally, the resource constraint on available talent is

$$\int_z^{\infty} d(t)dH(t) \leq \int_0^z t dH(t). \tag{11}$$

The primary advantage of this formulation is that worker pay varies with talent. Figure 3 describes the dependence of pay on talent for the production function $f(d) = d^{\alpha}$. Unlike the previous model, worker pay now varies and is linear with talent. However, the relationship between managerial talent and managerial pay is identical to that in the previous model. The connection between firm size and managerial pay is also the same.

Figure 3 Earnings and Talent



Notes: Earnings as a function of talent when talent is also an input into production. All individuals with talent $t < z$ are assigned to be workers and earn wages wt. Individuals with more managerial talent are managers and their pay is an increasing convex function in their talent.

The model is silent, however, on worker pay and firm size because for any given level of talent supplied to a firm, there are many combinations of differentially talented workers that can provide that total amount of talent. For example, a firm could have a small number of highly talented individuals or a large number of less talented individuals. Still, if there was a reason for the most talented workers to be assigned to the most talented managers and so on down the talent ladder until everyone was assigned, then there would be a wage-size premium. This kind of matching is referred to as *positive assortative matching*. One way to generate such a reason would be to make the production function highly complementary in the talent of the managers and workers. Kremer (1993) studies one such firm-level production function in which several tasks need to be performed simultaneously. If *any* of these tasks are performed unsuccessfully, then no output is produced. Talent improves the probability of success so this form of complementarity generates positive assortative matching and a positive wage-size premium.

Marginal Product

In this model all factors are paid their marginal product. This might not appear to be the case if one was to use the firm-level production function, $tf(l(t))$, to determine marginal product. However, that is not the right production function for determining the margin.

This model is an *assignment* model, and the right margin for determining marginal product is at the level of the production sector, which in this model is the entire economy. What this economy does is take as its inputs the numbers of people at each level of talent and then creates managers and workers, combines them into firms, and produces the output. Firms are really an intermediate good. The production function at the economy level is linear in these inputs so factors are paid their marginal products as in classical distribution theory.

More formally, the production sector solves

$$\max_{z,l(t)} \int_z^\infty tf(l(t))dH(t)$$

subject to

$$\int_z^\infty l(t)dH(t) \leq H(z). \tag{12}$$

The inputs into this production sector are the numbers of each type t. The supply of these inputs is, of course, the distribution $h(t)$. The economy is linear, or constant returns to scale, in the input. If w is the multiplier to (12) then the marginal product of $t \geq z$ is $tf(l(t)) - wl(t)$ and of $t < z$ is w. Managers are paid what is left over. Their pay is a residual, which is called a rent in the Ricardian tradition, but it is still a marginal product with respect to the production sector.

2. MULTIPLE-LEVEL HIERARCHIES

Extending the basic assignment model to hierarchies with more than two levels is conceptually straightforward, but it can be difficult analytically. In this section, a simple extension is provided.[4] The purpose is to generate a production hierarchy, like that illustrated in Figure 1, to introduce slightly more complicated managerial production functions, and to discuss relative pay levels between levels of management.

Production is limited to three-level hierarchies. As before, workers and managers jointly produce a good according to the production function $tf(l)$. However, this good is no longer final output but an intermediate good that is used by a second-level manager to produce the final output. Let the intermediate good be called m. Final output is $tg(m)$, where t is the talent of the second

[4] See Rosen (1992) for analysis of a problem where managers have a fixed span of control.

level manager. The intermediate goods are used to create a tractable example. The goal is to model firms organized like those illustrated in Figure 1.

The price of a unit of the intermediate good is λ and, as before, the price of labor is w. We need to solve for an assignment of labor to level-one managers, $l(t)$, an assignment of the intermediate good to level-two managers, $m(t)$, and cutoff values z_1 and z_2 that correspond to the cutoff talent levels between workers and level-one managers and between level-one managers and level-two managers, respectively.

The competitive equilibrium is set up so that the level-one manager hires the labor and creates the intermediate good, which he sells to the level-two managers. Despite this separation, we will interpret the level-one managers and workers who create the intermediate good for a level-two manager as being within the same firm. The problem can be formally set up in this way, but it is much more complicated to write down.

A level-two manager's problem is

$$\max_{m \geq 0} tg(m) - \lambda m.$$

The first-order condition is

$$tg'(m) - \lambda = 0. \tag{13}$$

The marginal level-two manager, z_1, must be indifferent to working as a level-one manager, that is,

$$z_2 g(m(z_2)) - \lambda m(z_2) = \lambda z_2 f(l(z_2)) - wl(z_2). \tag{14}$$

A level-one manager's problem is

$$\max_{l \geq 0} \lambda t f(l) - wl.$$

The first-order condition is

$$\lambda t f'(l) - w = 0. \tag{15}$$

The marginal level-one manager, z_2, must be indifferent to working as a worker,

$$\lambda z_1 f(l(z_1)) - wl(z_1) = w. \tag{16}$$

The final conditions for a competitive equilibrium are the resource constraints that the intermediate good used by level-two managers equals the intermediate good produced by combinations of workers and level-one managers

$$\int_{z_2}^{\infty} m(t)dH(t) \leq \int_{z_1}^{z_2} tf(l(t))dH(t), \tag{17}$$

and that the labor used by level-one managers equals the labor supplied

$$\int_{z_1}^{z_2} l(t)dH(t) \leq H(z_1). \tag{18}$$

With these formulas, we can derive similar relationships to those derived earlier. For all managers at the same level in a hierarchy, the relationship of pay with respect to talent look similar to those studied earlier. The curvature of pay with respect to talent may differ between managers assigned to different levels of a hierarchy. This will depend on the properties of the production functions f and g. These functions need not be the same since managing at lower levels within a firm may be very different than managing at higher levels.

A commonly observed feature of managerial pay within a firm is that there is a much larger difference in pay between levels of a hierarchy than within given job classifications (Rosen 1992). In our simple model, such a feature could be obtained if level-one managers were appropriately assigned to level-two managers. But more generally, it would be desirable to formalize the model so that it mattered which level-one manager was assigned to which level-two manager. This brings up the issue of positive assortative matching raised earlier in our discussion of the wage-size premium. These effects would seem to matter for junior executives as well.

3. CONCLUSION

The hierarchy illustrated by Figure 1 captures some features of firms, but it really postulates that each branch within a firm operates separately from the others. Some parts of a firm operate in this way, but there are others, like personnel, maintenance, legal, and audit, that provide services to all parts of a firm. Their outputs are essentially intermediate inputs into the production of the final output by other parts of the firm. The literature rarely considers these features, yet if firms do anything special, it is that they do joint production of activities that are not as efficiently supplied on the market. It would seem desirable to introduce these features into some of the firm production functions that have been studied in the literature.

Finally, the model considered here is a static model. If taken to a dynamic environment, then the strategy would be to assume that managers and firms are reallocated each period to whichever type of firm the market sees fit to assign them. For some purposes, that abstraction is fine but it misses a very important part of the managerial assignment problem. Managers infrequently move between firms. Indeed, an important activity of a firm is to identify and develop managerial talent for future promotion through its ranks. The career ladder within a firm is a very important device for solving this assignment problem and to an extent it operates separately from the market. Understanding this mechanism might be quite important for understanding the internal distribution of pay between levels of a hierarchy within a firm. For some work along this line see Lazear and Rosen (1981), Meyer (1994), or the papers surveyed in Rosen (1992).

REFERENCES

Barro, Jason R., and Robert J. Barro. 1990. "Pay, Performance and Turnover of Bank CEOs." *Journal of Labor Economics* 8: 448–81.

Brown, Charles, and James Medoff. 1989. "The Employer Size-Wage Effect." *Journal of Political Economy* 97 (October): 1027–59.

Cosh, Andrew. 1975. "The Renumeration of Chief Executives in the United Kingdom." *Economic Journal* 85 (June): 75–94.

Idson, Todd L., and Walter Y. Oi. 1999. "Workers are More Productive in Large Firms." *Papers and Proceedings of the American Economic Association* 89: 104–8.

Kostiuk, Peter F. 1989. "Firm Size and Executive Compensation." *Journal of Human Resources* 25: 90–105.

Kremer, Michael. 1993. "The O-Ring Theory of Economic Development." *Quarterly Journal of Economics* 108: 551–75.

Lazear, Edward P., and Sherwin Rosen. 1981. "Rank Order Tournaments as Optimum Labor Contracts." *Journal of Political Economy* 89: 841–74.

Lucas, Robert E., Jr. 1978. "On the Size Distribution of Business Firms." *Bell Journal of Economics* 9 (Autumn): 508–23.

Meyer, Margaret A. 1994. "The Dynamics of Learning with Team Production: Implications for Task Assignment." *Quarterly Journal of Economics* 109 (November): 1157–84.

Murphy, Kevin J. 1999. "Executive Compensation." In *Handbook of Labor Economics* 3b, edited by Orley Ashenfelter and David Card. Amsterdam: Elsevier Science: 2485–563.

Oi, Walter Y. 1983. "Heterogeneous Firms and the Organization of Production." *Economic Inquiry* 21 (April): 147–71.

______________, and Todd L. Idson. 1999. "Firm Size and Wages." In *Handbook of Labor Economics* 3b, edited by Orley Ashenfelter and David Card. Amsterdam: Elsevier Science: 2165–214.

Roberts, David R. 1956. "A General Theory of Executive Compensation Based on Statistically Tested Propositions." *Quarterly Journal of Economics* 70 (May): 270–94.

Rodriguez, Santiago Budria, Javier Diaz-Gimenez, Vincenzo Quadrini, and Jose-Victor Rios-Rull. 2002. “Updated Facts on the U.S. Distributions of Earnings, Income, and Wealth.” Federal Reserve Bank of Minneapolis *Quarterly Review* 26 (Summer): 2–35.

Rosen, Sherwin. 1982. “Authority, Control, and the Distribution of Earnings.” *Bell Journal of Economics* 13: 311–23.

_______________. 1992. “Contracts and the Market for Executives.” In *Contract Economics*, edited by Lars Wein and Hans Wijkander. Cambridge, Mass. and Oxford: Blackwell: 181–211.

Simon, Herbert A., and Charles P. Bonini. 1958. “The Size Distribution of Business Firms.” *American Economic Review* 48: 607–17.

Troske, Kenneth R. 1999. “Evidence on the Employer Size-Wage Premium from Worker-Establishment Matched Data.” *Review of Economics and Statistics* 81 (February): 15–26.