Economic Fundamentals and Bank Runs

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Recently there has been a renewed discussion in the literature about the determinants of bank runs. Two alternative theoretical explanations are usually provided. According to the first theory, bank runs are exclusively driven by changes in economic fundamentals, such as a deterioration in the return on investment. The second theory views bank runs as a consequence of the existence of multiple equilibria. In the latter case, which equilibrium obtains depends on the realization of an extrinsic random variable, often called "sunspots." *Extrinsic uncertainty* is uncertainty in economic outcomes that does not originate directly in changes of economic fundamentals (see Shell and Smith [1992]). The word "sunspots" is intended to convey the idea that these random variables do not directly influence the economic fundamentals of the economy.[1] However, sunspots can still influence economic outcomes to the extent that people believe they do. In this sense, sunspots can be viewed as coordination devices for agents' expectations in decentralized market economies. This is the view adopted in the bank-run literature and in this paper.

Some scholars have recently argued that the multiple-equilibria-plus-sunspots explanation of bank runs is inconsistent with available evidence

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[1] Shell and Smith (1992, 602) write: "The 'sunspot' terminology is a bit of a spoof on the work of Jevons (1884) and his followers, who related the business cycle to the cycle of actual sunspots. To the extent that actual sunspots do affect economic fundamentals this is intrinsic uncertainty, but the overall effects of actual sunspots on economic fundamentals are probably not major. Then, if actual sunspot activity does have substantial impacts on the economy, it must be that it serves a role beyond its effects on fundamentals. Cass-Shell (1983) sunspots are highly stylized; by definition, they represent purely extrinsic uncertainty."

showing that bank runs have historically been strongly correlated with deteriorating economic fundamentals (see Gorton [1988]; Allen and Gale [1998]; and Schumacher [2000]). In this paper I will argue that such a conclusion is not well justified. More specifically, I will show that the multiple-equilibria model of bank runs, combined with a reasonable (and well-accepted) equilibrium selection concept, can provide theoretical justification for the correlation observed in the data. In other words, the presence of an empirical correlation between bank runs and poor economic fundamentals cannot be used to discriminate between the two competing theories. Furthermore, the equilibrium selection story presented here strongly accords with the long-standing belief that some bank runs can be characterized as events resulting from exogenous waves of pessimism and that those mood shifts are more likely when economic conditions are bad or deteriorating.

The empirical evidence that links bank runs to economic conditions has been well documented. Gorton (1988) discusses what he calls the "recession hypothesis," according to which bank panics are closely associated with the business cycle. In a related paper, Miron (1986) presents evidence in favor of the "seasonal hypothesis," which is that bank runs tend to be correlated with seasonal fluctuations in the liquidity needs of depositors. Saunders and Wilson (1996) and Schumacher (2000) discuss evidence on the selectivity of depositors: not all banks are equally likely to experience a run during a panic, and in particular a questionable solvency position prior to the run tends to increase the probability of depositors running on a particular bank.[2]

Gorton (1988) studies bank panics during the National Banking Era (1865–1914). Using data for national banks, Gorton investigates whether the model and variables that explain the behavior of depositors during no-panic situations also explain their behavior during panics. In this sense, panics would not be purely random events; rather, they would be directly correlated with the arrival of new information that determines depositors' desire to withdraw funds from the bank. Gorton finds no evidence for something special happening during panics that cannot be explained by the model that describes the behavior of depositors in no-panic situations. Instead, the evidence seems to suggest that panic events are just the consequence of extreme realizations of the circumstances that explain behavior during normal times. It is important to note, however, that Gorton finds examples in which shocks of equal magnitude to those usually associated with runs did not cause a panic (for example, the November 1887 spike in the liability of failed businesses did not induce a

[2] Calomiris and Mason (1997) find evidence of depositors' confusion during the June 1932 bank panic, but they also find that solvent banks were able to support each other to avoid failure.

Table 1 Financial Panics, 1890–1908 (Miron, 1986)

Major Panics	September 1890 May 1893 December 1899 May 1901 March 1903 October 1907
Minor Panics	February 1893 September 1895 June 1896 December 1896 March 1898 September 1899 July 1901 September 1901 September 1902 December 1904 April 1905 April 1906 December 1906 March 1907 September 1908

panic, while the smaller increase in June 1884 did). Finally, in Tables 1 and 2 we can see that there is some disagreement as to what constitutes a panic. For example, Gorton does not consider the episodes of May 1901 and March 1903 as panics. Furthermore, and more germane to this paper, Tables 1 and 2 suggest that there were several bank panics in periods with no economic recession. Of course, seasonality may be part of the answer in those cases (as discussed by Miron [1986]).[3]

These are interesting findings, but they are not enough to rule out the possibility that, in some cases, banking panics are associated with the existence of multiple equilibrium outcomes (that is, situations where both the panic and the no-panic outcomes are possible). These stylized facts refute only the simplest way of modeling multiple equilibria and even then only under fairly specific conditions. Showing that reasonable theories of multiple-equilibria bank runs are not refuted by the available evidence is important since policy prescriptions depend on the assessment of the economic conditions that generate those bank runs. It would be helpful for policymakers to be able to conclude that multiple-equilibria bank runs are not the norm. However, as I will show here, the evidence discussed above does not allow us to reach that conclusion.

[3] Gorton (1988) finds no evidence of seasonal effects as causes for panics using his definition.

Table 2 Business Cycle and Bank Panics (Gorton, 1988)

NBER Cycle (Peak-Trough)	Panic Date
October 1873 - March 1879	September 1873
March 1882 - May 1885	June 1884
March 1887 - April 1888	No panic
July 1890 - May 1891	November 1890
January 1893 - June 1894	May 1893
December 1895 - June 1897	October 1896
June 1899 - December 1900	No panic
September 1902 - August 1904	No panic
May 1907 - June 1908	October 1907
January 1910 - January 1912	No panic
January 1913 - December 1914	August 1914

The paper is organized as follows. In the next section I discuss a simple model of bank runs that is now standard in the economic literature. I then study the conditions under which multiple equilibria arise, and I review different theories of how an equilibrium is selected in those cases. I show that some of the more appealing equilibrium selection mechanisms are indeed compatible with the available evidence. Finally, in the conclusion I discuss some policy implications.

1. MODELING BANK RUNS

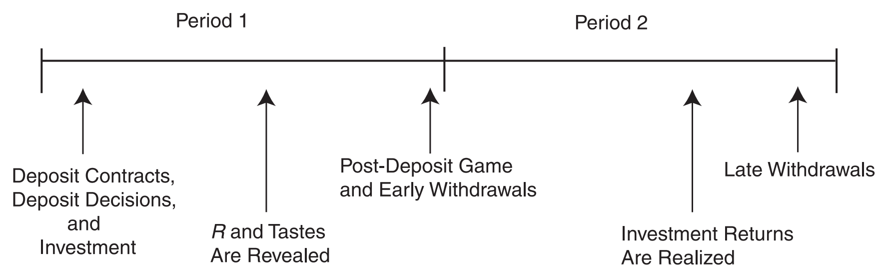
The Environment

The environment is similar to that in Diamond and Dybvig (1983), except that the return on investment is stochastic. There are two time periods, $t = 1, 2$, and a large number of ex ante identical agents (a continuum of agents with unit mass). Each agent is endowed with a consumption good at the beginning of date 1 and none after that. Agents are uncertain about their preferences: some will be impatient and will need to consume at the end of period 1; the rest will be patient and can wait to consume in period 2. At the beginning of period 1 agents do not know whether they will be patient or impatient, but they know that the probability of being impatient at the end of the period is u. Preferences are represented by the following utility function:

$$v(c_1, c_2) = \begin{cases} \frac{1}{\gamma}(c_1)^\gamma & \text{with probability} \quad u \\ \frac{1}{\gamma}(c_1 + c_2)^\gamma & \text{with probability} \quad 1 - u \end{cases},$$

where c_1 is consumption at the end of period 1, c_2 is consumption at period 2, and $\gamma < 1$. The realization of preference types is independent across agents, implying that u will also be the fraction of the population that becomes impatient. Agents' types are not observable and hence patient agents can

Figure 1 Timing



always pretend to be impatient if they wish to do so (impatient agents could pretend to be patient, but this is never the case for the contracts studied below).

There are two saving technologies available: storage and investment. One unit of consumption placed in storage yields one unit of consumption at any future time. For the investment technology, one unit of consumption placed in investment at the beginning of period 1 yields R units in period 2. The return on investment R is a random variable taking values greater than unity and with a probability density function given by $f(R)$. Note that the expected value of R is necessarily greater than one and hence investment is a better technology than storage to save consumption for the second period (that is, for funds that are needed with certainty in the second period). If investment is liquidated early (at the end of period 1), then it yields $x < 1$ units of consumption per unit invested. Hence, investment is an illiquid asset that yields a higher return than storage if held to maturity, but a lower return if liquidated early.

Timing

Since agents do not know their preferences until after the opportunity to invest has passed, they pool their endowments in banking coalitions. These banks then allocate some resources into the illiquid investment and provide insurance to their members in case they happen to become impatient at the end of period 1.

Competition in the banking industry drives the banks to offer the best possible available contract to consumers. I restrict the type of contracts that banks can offer to simple deposit contracts that are subject to a sequential service constraint (Wallace [1998]). Under this type of deposit contract, an agent gets the right to either a fixed payment at the end of period 1 (as long as the bank has funds) or a contingent payment in period 2. The sequential service constraint prevents the bank from adjusting the payment to early withdrawers according to the number of agents that decide to withdraw early. The bank must pay a fixed amount until it runs out of funds. This kind of contract is in the tradition of Diamond and Dybvig (1983) and Cooper and Ross (1998). I use it here mainly because of its simplicity and potential descriptive content.[4]

The timing of events is as follows. At the beginning of period 1, the bank, without knowing the value of R, chooses a deposit contract and a portfolio of assets (investment is possible only at this point). This choice can be summarized by the pair (a, η), where a is the payment that the bank will give to depositors if they decide to withdraw early and η is the proportion of total deposits that the bank decides to keep in storage (with $(1-\eta)$ being the proportion that the bank puts in the illiquid investment technology). Also at this time, agents decide whether or not to deposit their funds in the bank. At the end of period 1, the uncertainty about preferences and technology is resolved: agents find out whether or not they are impatient and the value of R is revealed.[5] At this time, then, agents decide whether or not to go to the bank to withdraw their deposits. Impatient agents have no choice but to withdraw early. Patient agents, however, could choose to wait until period 2, which they will do if they are not better off imitating the impatient agents. Whether a patient agent would be better off withdrawing his or her deposits early depends, in general, on what all the other patient agents are doing. Hence, patient agents play a strategic game at the end of period 1. Following Peck and Shell (2003), I shall call it the "post-deposit game." In period 2, the return on the illiquid technology is realized and those agents that did not withdraw their deposits early (at the end of period 1) go to the bank and share the total remaining resources equally.

2. THE POST-DEPOSIT GAME

The source of multiplicity of equilibria in the model lies in the post-deposit game played by patient agents. The expected outcome of this game will determine the bank's investment decisions and the willingness of agents to make deposits in the bank. The details of those problems are presented in Section 4. What is important here is to understand that solving those problems requires knowing what could happen in the post-deposit game. For this reason, I turn next to the study of this game.

[4] See also Ennis and Keister (2003b). In this environment, there are potential gains from making the early payments contingent on the realization of the return on investment R. The contracts studied here do not allow for this possibility. Gale and Vives (2002) and Allen and Gale (1998) do not assume sequential service, but the optimal contract has a structure similar to the deposit contract in the sense that for high values of R the payoff to early withdrawers is not contingent. This is because investment cannot be liquidated (it has zero liquidation value), and for high enough values of R (so that late consumers get more than early consumers), early consumers just divide the available liquid funds among them, resulting in a fixed quantity for each, independent of the value of R. The costly state verification literature provides another justification for the debt contracts (see, for example, Williamson [1986]).

[5] This value of R is common to all investment in the economy. No diversification is possible.

Table 3 Notation

u	Probability of being impatient
γ	Coefficient of relative risk aversion
R	Return on the risky investment
x	Return from early liquidation
$f(R)$	Probability distribution of R
a	Bank payment for early withdrawal
η	Proportion of total deposits held in storage
$\overline{\overline{u}}$	Probability of getting paid in case of run
R^*	Multiple-equilibria threshold for R
$\widehat{R}$	Risk-dominance threshold for R
p_r	Risk factor of the bank-run equilibrium
π	Probability of a bank run

I concentrate only on symmetric pure strategy equilibria.[6] At the end of period 1, the patient agents are faced with the decision of whether to withdraw their deposits early or leave them in the bank until period 2. Let r denote the decision to go to the bank to withdraw (i.e., to run) and n the decision to wait until the next period (i.e., not to run). Let us define as $P_{ij}(R; a, \eta)$ the payoff to a patient agent following action i $(i = r, n)$ given that all other patient agents are following action j $(j = r, n)$. We need only to consider those payoffs because we are looking at symmetric equilibria, where all patient agents act in the same manner. The normal form of the post-deposit game played by patient agents is given by the following matrix:

		Other Patient Agents: Run	Other Patient Agents: No Run
Patient Agent	Run	$P_{rr}(R; a, \eta)$	$P_{rn}(R; a, \eta)$
	No Run	$P_{nr}(R; a, \eta)$	$P_{nn}(R; a, \eta)$

Note that the payoff $P_{ij}(R; a, \eta)$ depends on the return on investment R and on the deposit contract chosen by the bank (a, η). (Note also that deviations by a single player do not change the payoff to the rest of the players because we are assuming that there is a large number of players.)

It is easy to state conditions under which this game has multiple equilibria. In particular, if $P_{rr}(R; a, \eta) > P_{nr}(R; a, \eta)$ and $P_{nn}(R; a, \eta) > P_{rn}(R; a, \eta)$, then running to the bank at the end of period 1 and waiting until period 2 to withdraw are both equilibria of the game. To see this, note that when $P_{rr}(R; a, \eta) > P_{nr}(R; a, \eta)$ holds, if the patient agent thinks that all other patient agents will run to the bank, then it is in her best interest to run as

[6] Symmetry implies that in equilibrium all impatient agents play the same strategy and all patient agents play the same strategy (but perhaps different from the one played by the impatient agents). Pure strategies are those strategies that do not involve randomization over different possible actions (each agent plays a single action with probability one).

well. Therefore, if all patient agents believe that a run will occur, the run does occur and running is a Nash equilibrium of the game. Likewise, when $P_{nn}(R; a, \eta) > P_{rn}(R; a, \eta)$ holds, if the patient agent thinks that no other patient agent will run to the bank, then it is in her best interest not to run. Therefore, if all patient agents believe that there will be no run, there is indeed no run and not running is a Nash equilibrium of the game. In equilibrium, then, all players play the same strategy, and I will denote each equilibrium by the strategy being played in it. Thus, I call the run equilibrium (if it exists) "equilibrium r," and the no-run equilibrium "equilibrium n."

Another important characteristic of this post-deposit game is that the multiple equilibria are usually Pareto-ranked.[7] One equilibrium is better than another equilibrium in the Pareto sense if all players in the former receive a payoff at least as high as in the latter and one or more players receive a strictly higher payoff. In the game studied here, if $P_{nn}(R; a, \eta) > P_{rr}(R; a, \eta)$, then the no-run equilibrium n is Pareto-preferred to the run equilibrium r.

Given the possibility of multiple equilibria, the natural next step is to ask, how does one of the equilibria get selected? I will discuss the answer to this question in the next section.

Before going into the equilibrium selection issue, it is worth noting that we can further characterize the payoff matrix of the post-deposit game. Studying these payoffs will give us a better idea of the conditions that determine the existence of multiple equilibria in the game.

Since the bank chooses the contract before observing the return R, the values of η and a depend only on the probability distribution of R and not on the particular realizations of R. The bank will never choose a contract such that $ua > \eta$ holds. In such a case, the bank will be certain to need to early-liquidate some of the investment in order to pay depositors (even if no patient agent runs). Since early liquidation is costly, this contract is never optimal. I will study the problem of the bank later, but for now let us assume that the distribution of R is such that the bank chooses a contract (a, η) that satisfies $\eta + x(1 - \eta) < a$. This inequality implies that if every agent goes to the bank early, then the bank would run out of resources before being able to pay the promised amount a to each withdrawer. Furthermore, if the inequality does not hold, then there would be no runs in equilibrium. These two inequalities allow us to determine the value of waiting when there is a run, $P_{nr}(R; a, \eta)$, and the value of running when there is no run, $P_{rn}(R; a, \eta)$. First, we have that $P_{nr}(R; a, \eta) = 0$ because if (almost) every agent goes to the bank to withdraw early, then the bank will run out of funds and no payments will be made in the second period. Second, we have that $P_{rn}(R; a, \eta) = P_{rn}(a) = a^{\gamma}/\gamma$ because when only impatient agents withdraw early, total withdrawals are equal to ua

[7] Games with multiple Pareto-ranked equilibria are called "coordination games" in the literature (for a general review, see Cooper [1999]).

and the bank has access to enough liquid funds, $\eta + x(1-\eta)$, to cover that amount.

Let us now define $\overline{\overline{u}} \equiv [\eta + x(1-\eta)]/a < 1$ as the probability of being paid when every agent goes to the bank early. This formula is a direct consequence of assuming that agents take random positions in the line formed at the bank's window and that there is a sequential service constraint. Thus, we have that $P_{rr}(R; a, \eta) = P_{rr}(a, \eta) = \overline{\overline{u}}a^{\gamma}/\gamma$. It is important to note that P_{nr}, P_{rn}, and P_{rr} are not functions of the particular realization of R. The only payoff that is a direct function of the realization of R is that for late withdrawals when there is no run, that is

$$P_{nn}(R; a, \eta) = \frac{1}{\gamma}\left(\frac{R(1-\eta) + (\eta - ua)}{1-u}\right)^{\gamma}.$$

Note that $P_{nn}(R; a, \eta)$ is a continuous, increasing, and unbounded function of R. Hence, there exists a threshold value R^* such that if $R > R^*$, we have that $P_{nn}(R; a, \eta) > P_{rn}(a) = a^{\gamma}/\gamma$ and the post-deposit game is a multiple-equilibria coordination game. If $R < R^*$, the post-deposit game has a unique equilibrium in which all agents withdraw their deposits at the end of period 1. In summary, the payoff matrix for the post-deposit game is:

		Other Patient Agents: Run	Other Patient Agents: No Run
Patient Agent	Run	$\frac{1}{\gamma}\overline{\overline{u}}a^{\gamma}$	$\frac{1}{\gamma}a^{\gamma}$
	No Run	0	$\frac{1}{\gamma}\left(\frac{R(1-\eta)+(\eta-ua)}{1-u}\right)^{\gamma}$

3. EQUILIBRIUM SELECTION IN THE POST-DEPOSIT GAME

There is an extensive literature on equilibrium selection in games. This literature has concentrated some attention on 2×2 games with multiple equilibria. The post-deposit game of the previous section can be thought of as just an example of a 2×2 symmetric game with the potential for multiple equilibria (i.e., a 2×2 symmetric coordination game).[8] In this section, I will review some of the basic ideas from this literature and discuss how they apply to the bank-run problem at hand.

It is useful at this point to introduce the concept of *equilibrium selection mechanism* (ESM). An ESM is a probability distribution that assigns, to each equilibrium of the game, a probability indicating how likely it is to be the result of play. For the post-deposit game under consideration, an ESM is a function that for each possible triplet (R, a, η) assigns a probability π to

[8] Usually we refer to a 2×2 game as a game that is played by two individuals who each have two possible pure strategies that they can choose to play. In the post-deposit game, agents play a "game" against the population that is often called a "macroeconomic game." See Cooper (1999) for an extensive discussion on the subject.

the run equilibrium (r) and a probability $(1-\pi)$ to the no-run equilibrium (n). These probabilities must be feasible in the sense that, for given values (R, a, η), if the run equilibrium does not exist, then $\pi = 0$, and if a run is the only equilibrium, then $\pi = 1$. It is important to note that there is a degree of coordination being assumed from the outset: agents know that the only possible outcomes are those where all the rest of the agents play in the same manner (and this coordination is common knowledge). The ESM provides some structure to the coordination problem but does not explain why and how coordination arises. In this sense, the concept of an ESM can be thought of as a generalization of the traditional sunspot approach: there is still in place an exogenous coordination device on which all agents base their actions. The innovation is that the general ESM allows for the probability of each equilibrium to depend on exogenous and endogenous variables in the model.

The next natural question is, where does the function $\pi(R, a, \eta)$ come from? In the traditional sunspot approach, the function π is a constant between zero and one when feasible (i.e., when both equilibria exist). Another commonly used criterion for equilibrium selection is to assume that the best equilibrium (in the Pareto sense) will be selected. In this case, the ESM is such that the probability π is equal to zero as long as the no-run equilibrium exists and switches to unity when only the run equilibrium exists. Yet there are other possible forms that the function π may take and that can be reasonably justified. I review some of these forms next.

Let us start by defining the *risk factor* of equilibrium j, for $j \in \{r, n\}$ as the smallest probability p such that if a player believes that with probability strictly greater than p all the other players are going to play action j, then action j is the unique optimal action to take (see, for example, Young [1998]). Hence, the risk factor of the run equilibrium (r) is given by the solution to the following equation:[9]

$$p_r P_{rr} + (1 - p_r) P_{rn} = p_r P_{nr} + (1 - p_r) P_{nn}.$$

Therefore,

$$p_r = \frac{P_{nn} - P_{rn}}{(P_{rr} - P_{nr}) + (P_{nn} - P_{rn})}$$

is the risk factor of the run equilibrium. When both equilibria exist (run and no-run), the only payoff that depends on R is P_{nn}, and this payoff is increasing in R. Hence, p_r is an increasing function of R. This result is rather intuitive. It says that the higher the return on investment R, the higher the belief probability of a run p must be in order to induce a patient agent to run on the bank.

[9] The payoffs are still a function of the triple (R, a, η), but I choose not to explicitly write this dependence in order to simplify notation.

An equilibrium j is *p-dominant* if the equilibrium action j is the unique best response to any belief of the player that puts probability at least $p \in [0, 1]$ on the other players playing action j (see Morris, Rob, and Shin [1995]). Hence, the run equilibrium is p_r-dominant.

If the risk factor of the run equilibrium p_r is less than or equal to one-half, then the run equilibrium is *risk dominant* (Harsanyi and Selten [1988]). *Risk dominance* has been used as a criterion for equilibrium selection: the risk-dominant equilibrium will be the one selected and played. This criterion has an appealing interpretation. If each player is uncertain about the action of the other players, it is plausible that he or she would assign equal probability to each of the possible outcomes (a flat or diffuse prior). If the risk factor of equilibrium j is less than one-half, that is, if equilibrium j is risk dominant, and if players have flat priors about the actions of the other players, then equilibrium j will be the one played. In the post-deposit game, when each player assigns equal odds to all of his or her opponents playing either action r or n, then the players will choose to play the action of the risk-dominant equilibrium. In terms of the definition of ESM, the risk dominance criterion assigns probability one to the risk-dominant equilibrium.

Another way of motivating an equilibrium selection rule in games with multiple equilibria is to study learning dynamics under repeated iterations of the static (stage) game. See, for example, Kandori, Mailath, and Rob (1993); Young (1998); and Matsui and Matsuyama (1995). These papers concentrate on games with two players and assume that there are frictions limiting the ability of agents to adjust their strategies. Kandori, Mailath, and Rob also assume bounded rationality on the part of the agents playing the dynamic game (in the form of myopic behavior and some propensity to make mistakes). It is interesting to note that the learning dynamics under these assumptions tend to select (as the frictions or the probability of mistakes vanish) the risk-dominant equilibrium as the one most likely to be played. Temzelides (1997) extends this work and applies it to the bank-run model.

Ennis and Keister (2003a) study a learning model that induces a probability distribution over the possible equilibria of a 2×2 macroeconomic coordination game. We show that the probability of equilibrium j induced by this learning process is strictly decreasing in the risk factor of equilibrium j and can take values strictly lower than one even when equilibrium j is risk dominant. In terms of the previous ESM terminology, we have that the function π is a decreasing function of p_r and may take values strictly between zero and one. Since p_r is an increasing function of R (the fundamentals), we have that the probability of a run π is a decreasing function of R. That is, the better the fundamentals (R), the less likely is a bank-run event. In Ennis and Keister (2003b) we apply these ideas to study the effect of bank runs on economic growth.

Let us now go back to the case of equilibrium selection based on the traditional sunspot approach. Assume that the return on investment R takes values only in the interval (R^*, ∞), where R^* is the threshold such that for values of R greater than R^* there are multiple equilibria of the post-deposit game. In other words, assume that the contract is such that the no-run equilibrium exists for every possible value of R. Assume also that a binomial sunspot random variable determines which equilibrium is selected. Because both equilibria exist for every value of R, the probability of a bank run is always given by the constant probability associated with the sunspot realization that coordinates agents to "run" to the bank. This is the sense in which the previous literature on bank runs has dismissed the sunspot explanation for not conforming with the observed correlation of bank runs with economic fundamentals.

However, note that if R *can* be below R^* with positive probability, then for those realizations, regardless of the sunspot variable, the probability of a run will be equal to unity. In such a case, even though sunspots still play an important role in coordinating the agents when there are multiple equilibria, the probability of bank runs will be higher for lower values of R, and indeed the probability of observing a bank run will be the highest (equal to one) when the fundamentals deteriorate sufficiently (that is, when $R < R^*$). In this sense, even the traditional sunspot approach can account for some of the correlation of bank runs with economic fundamentals. Economic fundamentals determine whether multiple equilibria exist, and then probabilities have to adjust to reflect this fact.[10]

Furthermore, the traditional sunspot approach seems too simplistic for this environment, and the risk-dominance-based selection mechanism appears to be a reasonable extension. We can think that the risk dominance ESM is the case where the particular sunspot variable that coordinates patient agents to run to the bank is correlated, in a specific way, with the stochastic variable R determining fundamentals. Risk dominance provides discipline and intuition to this correlation.

In particular, the risk dominance criterion divides the support of the distribution of R into two sets: the set where $R < \widehat{R}$, in which the run equilibrium is risk dominant, and the set where $R > \widehat{R}$, in which the no-run equilibrium is risk dominant. We can think that there is an associated sunspot random

[10] Ironically, the model in the second part of the paper by Allen and Gale (1998) can be used to provide a good example of this situation. For some parameter values their model has multiple equilibria. Their equilibrium analysis delineates three relevant regions for the possible realization of the return on the risky asset R. When R is very low, the equilibrium has a bank run; when R is very high, there are no bank runs in equilibrium; and for intermediate values of R, there are multiple equilibria: both having a bank run and not having a bank run are possible equilibrium outcomes. Therefore, just using a simple sunspot variable to determine which of the two equilibria will be observed in the intermediate region of R would deliver the historical correlation: as fundamentals deteriorate (as R goes from high to low levels), the probability of bank runs first goes from zero to positive (the value associated with the sunspot) and then to unity when fundamentals are so poor that a bank run is unavoidable.

variable s, perfectly correlated with R, such that whenever R takes values in the interval $[1, \widehat{R}]$, the variable s takes the value r, and whenever R takes values in the interval $(\widehat{R}, \infty)$, the sunspot variable s equals n. If agents associate values of $s = r$ with a run situation and values of $s = n$ with a no-run situation, the equilibrium selection process is still driven by sunspots (the variable s), but it generates a correlation of bank runs with the behavior of fundamentals. It is worth noting that for most values of R, both equilibria still exist, even though one of them is risk dominant. What determines which equilibrium will be played is a matter of how agents get coordinated. Coordination is driven by the sunspot variable s. Risk dominance can be thought of as the justification for why the particular sunspot random variable s has been selected as a coordination device over all possible variables that may be available. Note that there is a higher level of coordination among agents in the choice of the relevant sunspot variable. This interpretation of sunspots is in fact associated with another argument that has been used to explain the appearance of such coordination devices: sunspots can be viewed as the limiting case of situations in which agents are overreacting to some small movement in economic fundamentals. Manuelli and Peck (1992) formalize this argument.

Finally, it should be clear at this point that the more general ESM approach (Ennis and Keister [2003a]), in which the probability of a bank run π is a decreasing function of R, is also consistent with both the multiplicity of equilibria and the correlation of bank runs with economic fundamentals. In fact, with this approach the probability π can be strictly between zero and unity and at the same time be dependent on R. This feature seems very appealing, since the historical correlation was never perfect: sometimes bank runs did not occur even though economic fundamentals were as bad as or worse than in periods where a bank run did occur (see Gorton [1988]).

4. THE BANK'S PROBLEM

In Section 2 we assumed that agents would be willing to deposit their funds in the bank and that the bank would choose a contract with some specific properties. This section provides the justification for those assumptions.

Given that the banking system may be subject to runs, agents might choose not to participate in the banking system.[11] In that case, their payoff would be given by the following "autarky" problem

$$V_A \equiv \max_{\eta} \int \left(u \frac{(\eta + x(1-\eta))^{\gamma}}{\gamma} + (1-u) \frac{(\eta + R(1-\eta))^{\gamma}}{\gamma} \right) f(R)\, dR,$$

[11] For the sake of simplicity, I am restricting agents to deposit either *all* their resources in the bank or nothing at all. Ennis and Keister (2003b) consider the case where agents can deposit just part of their initial resources in the bank. This is an important extension in environments where bank runs can happen with positive probability, as is the case in this paper.

subject to $0 \leq \eta \leq 1$. At the beginning of period 1, the agent decides how to split the endowment between storage (η) and investment ($1 - \eta$). At the end of period 1, the agent finds out whether she is patient or impatient. If she is impatient, then she liquidates the investment and consumes (funds are useless for her in the second period). If she is patient, then she stores the liquid funds and consumes in the second period both the liquid funds and the return on investment (recall that we are assuming that $R > 1 > x$).

A bank could always choose a contract that eliminates the possibility of experiencing a run. I will call the best contract with such property the "run-proof contract." A contract is run-proof if there is enough liquidity in the bank to pay all agents the amount a at the end of period 1. But because the contract is run-proof, patient agents actually wait until the second period to withdraw. The problem of a bank choosing the run-proof contract is the following:

$$V_{RP} \equiv \max_{a,\eta} \int \left(u\frac{a^{\gamma}}{\gamma} + (1-u)\frac{1}{\gamma}\left(\frac{R(1-\eta)+(\eta-ua)}{1-u}\right)^{\gamma}\right) f(R)\, dR,$$

subject to

$$a \leq \eta + x(1-\eta),\ a \geq 0, \text{ and } 0 \leq \eta \leq 1.$$

The first constraint is the run-proof constraint. It says that even if all agents go to the bank in the first period (i.e., early), the bank will not run out of funds.

Finally, after having studied equilibrium selection in the post-deposit game, we are now in a position to write down the problem faced by the bank at the beginning of period 1. It is important to note that the probability of a run may depend on the contract chosen by the bank and hence the bank will take this effect into account when determining the best possible contract. Formally, the bank's problem is given by

$$\begin{aligned} V \equiv \max_{a,\eta} \int [\pi(R,a,n)P_{rr}(a,\eta) + \\ (1-\pi(R,a,\eta))\left(u\frac{a^{\gamma}}{\gamma} + (1-u)P_{nn}(R,a,\eta)\right)]f(R)\, dR, \end{aligned}$$

subject to $a \geq 0$ and $0 \leq \eta \leq 1$. Note that P_{rn} does not enter the problem directly. It may, however, enter the problem indirectly through the determination of $\pi(R,a,n)$, as in the case of the ESM based in risk dominance or adaptive learning.

When we have $V_A < \max\{V, V_{RP}\}$, the agents will choose to deposit their funds at the bank. When we have $V > V_{RP}$, the bank will choose the contract that allows for the possibility of bank runs according to the ESM that is operating in the economy (that is, according to the given function $\pi(R,a,\eta)$). It is important to note that if there exist values of R such that $R < R^{*}$ and $f(R) > 0$, then for those values of R we must have that $\pi(R,a,\eta) = 1$ because the post-deposit game has a unique (run) equilibrium for those values of R.

Diamond and Dybvig (1983) show that when the return on investment R is not stochastic (and greater than unity) and the probability π is arbitrarily set at zero, the bank chooses a contract (a, η) for which a bank run is a possible equilibrium of the post-deposit game played by the patient agents. Hence, using arguments of continuity, it can be shown that there exist functions $f(R)$ and $\pi(R, a, \eta) > 0$ such that a bank solving the problem V described above will also choose a contract that admits runs (that is, a contract such that $\eta + x(1 - \eta) < a$ holds).

5. CONCLUSION

I have shown that even when bank runs are driven by self-fulfilling expectations in environments with multiple equilibria, the historical correlation of bank runs with poor economic fundamentals can still be accounted for. More evidence would be necessary to reject the case of bank runs originating in situations with multiple equilibria. For now, when we observe a bank run, we cannot in principle confidently discard the possibility that another equilibrium with no bank run was also possible. This conclusion is important from a policy standpoint. In some cases, multiple-equilibria bank runs can be avoided by the design of off-equilibrium policies that are hence never observed. For example, the suspension of convertibility could make the run situation I have presented no longer an equilibrium of the post-deposit game (as proposed by Diamond and Dybvig in their original paper). But because suspension would occur only when there is a run and runs are not equilibrium outcomes anymore, the suspension of payments will not be observed. An important qualification is that, like many other off-equilibrium threats, this policy entails a certain ability of the bank to commit to actually implementing the policy if it becomes necessary.

There is another important policy implication of the ideas presented here. In the multiple-equilibria case, bank runs are usually not optimal and in general the policymaker would like to avoid them (or at least lower their probability). Contrary to this position, Allen and Gale (1998) present the case of bank runs that are not the consequence of a coordination failure and that are in fact part of the optimal arrangement for risk sharing in the economy. The policymaker would not want to avoid the Allen-Gale type of bank runs. Determining which of the two cases is driving a particular episode is an important issue that the policymaker would need to carefully evaluate.

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