

# Bond Price Premiums

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This article provides a detailed introduction to consumption-based bond pricing theory, a special case of the consumption-based asset pricing theory associated with Robert Lucas (1978). To help make the theory more accessible to novices, we organize the article around the two famous interest rate decompositions associated with Irving Fisher. These complementary decompositions relate real or nominal long-term interest rates to expected future short-term interest rates (the expectations theory of the term structure), and relate short- or long-term nominal interest rates to the ex ante real interest rate and the expected inflation rate (the Fisher equation). According to consumption-based theory, the Fisherian relationships hold exactly only under certain restrictive conditions. We show what those conditions are, and we show that generalizations of the Fisherian relationships hold quite broadly in the consumption-based model.

The pure Fisherian relationships are shown to hold only as special cases of the relationship between individual preferences, future economic activity, and the returns on assets. Notable sufficient conditions for the pure expectations hypothesis are that households be neutral to risk and the price level behave like a random walk; the pure Fisher equation requires only risk neutrality.<sup>1</sup> In turn, long-term nominal bond prices may lie above or below the values dictated by the pure expectations hypothesis and the pure Fisher relationship—forward premiums and inflation-risk premiums may be positive or negative.

Interpreting bond prices of various maturities is an important challenge for the Federal Reserve. Nominal bond prices contain information about the public's expectations of inflation and of future short-term rates. And they contain information about the levels of short-term and long-term real interest rates. All these variables can be valuable signals to the Federal Reserve of the

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<sup>1</sup> Risk aversion lies at the heart of much of asset pricing theory. For example, it is what leads us to assume that riskier assets will have a higher average return than safer assets.

appropriateness of its policy.<sup>2</sup> However, extracting these signals requires an understanding of the potential limitations of the pure expectations hypothesis and the pure Fisher relationship.<sup>3</sup>

The article proceeds as follows. In Section 1 we provide a brief historical overview of the two interest rate decompositions. Section 2 lays out a modeling framework for thinking about bond price determination, and derives the basic bond pricing equations from which all else will follow. Section 3 derives the generalized expectations theory of the term structure and Section 4 derives the generalized Fisher equation. Section 5 combines the results of the previous two sections for a general discussion of the yield differential between short- and long-term bonds. Sections 2–5 provide a textbook treatment of bond pricing relationships.<sup>4</sup> Section 6 provides a selective review of applied research based on bond pricing theory. Section 7 concludes the article.

Although the usual statements of the expectations hypothesis and the Fisher equation are made in terms of interest rates, most of our derivations use zero-coupon bond prices. This is for analytical simplicity; working with bond prices is slightly easier, especially when the bonds are zero-coupon bonds. And given an expression for the price of a bond, one can always work out the corresponding interest rate.

## 1. BRIEF HISTORY OF INTEREST RATE DECOMPOSITIONS

The expectations hypothesis of the term structure and the Fisher equation both made early appearances in Irving Fisher's *Appreciation and Interest* (1896).<sup>5</sup> Chapter 2 of that work is devoted to a discussion of the equation, or "effect," that would later bear the author's name. The Fisher equation is typically thought of as relating "real" and "nominal" interest to the expected rate of inflation, but Fisher's analysis in *Appreciation and Interest* is more general. He relates the interest rates between two standards (for example real vs. nominal, or dollars vs. yen) to the relative rate of appreciation of the standards, as

$$1 + j = (1 + a)(1 + i), \quad (1)$$

<sup>2</sup> See Bernanke and Woodford (1997), however, on the risks involved in the Federal Reserve basing its policy actions solely on such data.

<sup>3</sup> The empirical limitations of the pure expectations hypothesis have been well documented, for example, by Campbell and Shiller (1991). There has been less emphasis on violations of the Fisher relationship. Sarte (1998) presents evidence that the violations are small using a standard form of preferences. Kim and Wright's (2005) results suggest larger violations, using a different approach outlined in Section 6.

<sup>4</sup> Textbook treatments are available, see for example, Sargent (1987) and Cochrane (2001). However, they tend to provide fewer details, concentrating instead on the method by which one can price any asset in the consumption-based framework.

<sup>5</sup> See Humphrey (1983) for the intellectual history of the Fisher equation before Fisher. Humphrey shows that the relationship was well understood before Fisher.

where  $i$  is the rate of interest in the appreciating standard,  $a$  is the rate of appreciation, and  $j$  is the rate of interest in the depreciating standard. In Fisher's words,

The rate of interest in the (relatively) depreciating standard is equal to the sum of three terms, viz., the rate of interest in the appreciating standard, the rate of appreciation itself, and the product of these two elements. (p. 9)

In our context,  $j$  is the nominal rate,  $i$  is the real rate, and  $a$  is the expected inflation rate.

In Chapter 5 and to some extent in Chapters 3 and 4, one can find the essence of the expectations hypothesis of the term structure. Most notably perhaps, on pages 28 and 29, Fisher writes,

A government bond, for instance, is a promise to pay a specific series of future sums, the price of the bond is the present value of this series and the "interest realized by the investor" as computed by actuaries is nothing more or less than the "average" rate of interest in the sense above defined.

By " 'average' rate of interest in the sense above defined," Fisher means what we now understand to be the expected future path of short-term rates.

John Hicks (1939) and F. A. Lutz (1940) elaborated on Fisher's version of the expectations hypothesis in the 1930s and 1940s. Their versions of these interest rate decompositions continued to be based on reasoning regarding how returns among different assets should be related. Later, the development of consumption-based asset pricing theory (Lucas 1978) gave a formal foundation to Fisher's reasoning, while making clear that restrictive assumptions were needed for the Fisherian relationships to hold exactly. The discipline provided by consumption-based theory and the rise of rational expectations and dynamic equilibrium modeling in macroeconomics also led economists and finance theorists to de-emphasize certain elements of Fisher's theories regarding interest rates. For example, with respect to the expectations hypothesis, early versions were "usually understood to imply . . . that interest rates on long-term securities will move less, on the average, than rates on short-term securities" (Wood 1964). It is now well understood that whether this will be true depends on the behavior of monetary policy and on the real shocks hitting the economy (Watson 1999). And with respect to the Fisher equation, prior to the consumption-based theory, researchers often emphasized not just the decomposition into real rates and expected inflation, but also the extent to which the real rate was invariant to changes in expected inflation (Mundell 1963). It is now understood that one cannot make general statements about this

invariance, even though a version of the Fisher equation holds under general conditions.

## 2. MODELING FRAMEWORK

We use the modern theory of consumption-based asset pricing, first developed by Robert Lucas (1978), to study bond prices. For our purposes, the crucial elements in this theory are as follows. There is a representative consumer who has an infinite planning horizon, has a standard utility function (exhibiting risk aversion) over consumption each period, and discounts the utility from future consumption at a constant rate.<sup>6</sup> The consumer has a budget constraint which states that the sum of income from sales of real and financial assets (including income from maturing bonds), and income from other sources must not be exceeded by the sum of spending on current consumption, on purchases of real and financial assets, and on any other uses. With this framework, it is possible to price any asset. To do this, we use conditions describing individuals' optimal behavior.

### Preferences and Budget Constraint

The consumer's preferences in period  $t$  are given by

$$v_t = E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}), \quad (2)$$

where  $u(c_{t+j})$  is a utility function that is increasing and strictly concave in consumption ( $c$ ), and the discount factor  $\beta \in (0, 1)$ . Before specifying the budget constraint, it will be helpful to provide more detail about the set of financial assets that play a role in our analysis. They are

1.  $n$ -period real discount bonds; if issued in period  $t$ , they pay off one unit of consumption with certainty in period  $t + n$ . Their price in period  $t$  in terms of goods is  $q_t^{(n)}$ , and the quantity that the consumer purchases in period  $t$  is  $b_t^{(n)}$ .
2.  $n$ -period nominal discount bonds; if issued in period  $t$ , they pay off a dollar with certainty in period  $t + n$ . Their dollar price in period  $t$  is denoted  $Q_t^{(n)}$ , and the quantity that the consumer purchases in period  $t$  is denoted  $B_t^{(n)}$ . Notice that if the dollar-denominated price of consumption at date  $t + n$  is very high, a nominal bond provides little

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<sup>6</sup>The representative consumer idea can be taken literally or can be viewed as a shortcut for the assumption that whatever individual-level heterogeneity does exist has been insured away by the existence of a complete set of Arrow-Debreu securities (state-contingent claims).

consumption. Therefore, an asset that yields a dollar with certainty is still a “risky” asset.

3. One-period nominal forward contracts,  $s$  periods ahead; these contracts represent a commitment in period  $t$  to purchase a one-period nominal discount bond in period  $t + s$  at the pre-specified dollar price  $Q_{t,s}^f$ . The quantity that the consumer commits in period  $t$  to purchase in period  $t + s$  is  $B_{t,s}^f$ .
4. One-period real forward contracts,  $s$  periods ahead; these contracts represent a commitment in period  $t$  to purchase a one-period real discount bond in period  $t + s$  at the pre-specified price in terms of goods  $q_{t,s}^f$ . The quantity that the consumer agrees in period  $t$  to purchase in period  $t + s$  is  $b_{t,s}^f$ .

With this set of assets, the consumer’s flow budget constraint in period  $t$  is

$$\begin{aligned} & \sum_{n=1}^{\infty} Q_t^{(n-1)} B_{t-1}^{(n)} + \sum_{s=1}^{t-1} B_{t-s-1,s}^f + \\ & P_t \sum_{n=1}^{\infty} q_t^{(n-1)} b_{t-1}^{(n)} + P_t \sum_{s=1}^{t-1} b_{t-s-1,s}^f + W_t \\ & = P_t c_t + \sum_{n=1}^{\infty} Q_t^{(n)} B_t^{(n)} + \sum_{s=1}^t Q_{t-s,s}^f B_{t-s,s}^f + \\ & P_t \sum_{n=1}^{\infty} q_t^{(n)} b_t^{(n)} + P_t \sum_{s=1}^t q_{t-s,s}^f b_{t-s,s}^f + Z_t, \end{aligned} \tag{3}$$

where  $P_t$  is the price level—the dollar price of the consumption good,  $Z_t$  is purchases of other assets, and  $W_t$  is labor income and income from all other sources. The left-hand side of (3) represents income and the right-hand side represents spending.

There are several things to note with regard to the budget constraint. First, in period  $t$ , for any  $j > k > 0$ , a  $j$ -period bond issued in period  $(t - k)$  is identical to a  $(j - k)$ -period bond issued in period  $t$ , because they have the same maturity and the same payoff at maturity.<sup>7</sup> Thus, in the budget constraint we include only the latter on the right-hand side. Second, a discount bond that matures in period  $t$  can be thought of as having a price of one dollar (for a nominal bond) or one good (for a real bond) in period  $t$ . Thus, on the left-hand side of the budget constraint we have imposed  $Q_t^{(0)} = q_t^{(0)} = 1$ . Third, the prices of nominal bonds are written in terms of dollars and the prices of real bonds are written in terms of goods; with the budget constraint written in nominal terms, this means that prices of real bonds must be multiplied by  $P_t$ . Finally, it is important to be clear about the forward contracts that appear in the budget constraint. On the left-hand side, the terms  $\sum_{s=1}^{t-1} B_{t-s-1,s}^f$  and

<sup>7</sup> For example, set  $j = 10$ ,  $k = 3$  and  $t = 20$ . In period 20, a ten-period bond issued in period 17 is identical to a seven-period bond issued in period 20.

$P_t \sum_{s=1}^{t-1} b_{t-s-1,s}^f$  represent income from maturing one-period bonds that were purchased under forward contracts entered into in periods earlier than  $t - 1$ . On the right-hand side, the terms  $\sum_{s=1}^t Q_{t-s,s}^f B_{t-s,s}^f$  and  $P_t \sum_{s=1}^t q_{t-s,s}^f b_{t-s,s}^f$  represent purchases of one-period bonds under forward contracts entered into in periods earlier than  $t$ . For example, in period  $t$ , the consumer purchases a quantity  $B_{t-2,2}^f$  of one-period bonds at price  $Q_{t-2,2}^f$  in accordance with a forward contract entered into in period  $t - 2$ . Similarly, the consumer purchases a quantity  $B_{t-3,3}^f$  of one-period bonds at a price  $Q_{t-3,3}^f$  in accordance with a forward contract entered into in period  $t - 3$ . Forward contracts entered into in period  $t$  do not appear in the period  $t$  budget constraint because they do not affect income or spending in period  $t$ ; they do show up in future budget constraints.

As mentioned earlier, by limiting our attention to zero-coupon bonds, it is natural to focus on bond prices rather than interest rates. However, one can easily recover interest rates from bond prices. Let  $R_t^{(n)}$  denote the gross nominal yield on a bond that sells in period  $t$  for price  $Q_t^{(n)}$  and pays one dollar in period  $t + n$ . On a standardized per-period basis, the yield satisfies

$$R_t^{(n)} = \left(1/Q_t^{(n)}\right)^{1/n}; \quad (4)$$

$R_t^{(n)}$  is the constant per-period interest rate that is implied by the price  $Q_t^{(n)}$ . Likewise, for an  $n$ -period real bond we have

$$r_t^{(n)} = \left(1/q_t^{(n)}\right)^{1/n}, \quad (5)$$

and for one-period nominal and real forward contracts entered into in period  $t - s$  for execution in period  $t$ , we have

$$R_{t-s,s}^f = 1/Q_{t-s,s}^f, \quad (6)$$

and

$$r_{t-s,s}^f = 1/q_{t-s,s}^f. \quad (7)$$

### Individual Optimality Conditions

The consumer chooses consumption and holdings of each asset to maximize expected utility subject to the sequence of flow budget constraints. One way to carry out this maximization is to form a Lagrangian from the utility function and the sequence of budget constraints, and then use first-order conditions for consumption and each asset. The Lagrangian is

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j [u(c_{t+j}) + \quad (8)$$

$$\begin{aligned} & \Lambda_{t+j} \cdot \left\{ \sum_{n=1}^{\infty} Q_{t+j}^{(n-1)} B_{t+j-1}^{(n)} + \sum_{s=1}^{t+1} B_{t+j-s-1,s}^f + \right. \\ & P_{t+j} \sum_{n=1}^{\infty} q_{t+j}^{(n-1)} b_{t+j-1}^{(n)} + P_{t+j} \sum_{s=1}^{t+1} b_{t+j-s-1,s}^f + W_{t+j} \left. \right\} \\ & - \Lambda_{t+j} \cdot \left\{ P_{t+j} c_{t+j} + \sum_{n=1}^{\infty} Q_{t+j}^{(n)} B_{t+j}^{(n)} + \sum_{s=1}^t Q_{t+j-s,s}^f B_{t+j-s,s}^f + \right. \\ & P_{t+j} \sum_{n=1}^{\infty} q_{t+j}^{(n)} b_{t+j}^{(n)} + P_{t+j} \sum_{s=1}^t q_{t+j-s,s}^f b_{t+j-s,s}^f + Z_{t+j} \left. \right\} \Big]. \end{aligned}$$

The first-order condition for consumption in period  $t$  is

$$u'(c_t) = P_t \Lambda_t. \tag{9}$$

The multiplier  $\Lambda_t$  is the marginal utility of nominal income.

**Nominal and Real Bonds**

The first-order conditions for  $n$ -period nominal bonds are

$$\Lambda_t Q_t^{(n)} = \beta E_t \left[ \Lambda_{t+1} Q_{t+1}^{(n-1)} \right], \quad n = 1, 2, \dots \tag{10}$$

where we have used the fact that an  $n$ -period bond in period  $t$  becomes an  $n - 1$  period bond in period  $t + 1$ . This expression implies that the price in period  $t$  of an  $n$ -period discount bond is the ratio of the present value of expected marginal utility in period  $t + n$  to marginal utility in period  $t$  :

$$Q_t^{(n)} = \beta^n E_t \left[ \Lambda_{t+n} / \Lambda_t \right], \quad n = 1, 2, \dots \tag{11}$$

To show this, first write (10) for period  $t + 1$  :

$$\Lambda_{t+1} Q_{t+1}^{(n)} = \beta E_{t+1} \left[ \Lambda_{t+2} Q_{t+2}^{(n-1)} \right], \quad n = 1, 2, \dots \tag{12}$$

and substitute the result into (10), dividing both sides by  $\Lambda_t$  and using the law of iterated expectations:

$$Q_t^{(n)} = \beta^2 E_t \left[ (\Lambda_{t+2} / \Lambda_t) Q_{t+2}^{(n-1)} \right], \quad n = 1, 2, \dots \tag{13}$$

If  $n = 1$  then we have (11), because  $Q_{t+2}^{(0)} = 1$ . If  $n > 1$  then repeat the process, substituting for  $\Lambda_{t+2} Q_{t+2}^{(n-1)}$  using (10), etc. Intuitively,  $\Lambda_t Q_t^{(n)}$  is the utility cost of a bond in period  $t$ , and  $\beta^n E_t \left[ \Lambda_{t+n} \right]$  is the expected utility benefit from the payoff at maturity, discounted back to the present. When the agent has optimized over bond holdings these two values are identical.<sup>8</sup> Holding

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<sup>8</sup>The object on the right-hand side of (11) is referred to as the intertemporal marginal rate of substitution.

constant the current marginal utility of a dollar ( $\Lambda_t$ ), a higher bond price  $Q_t^{(n)}$  will correspond to a higher payoff in utility terms, that is a higher  $u'(c_{t+n})$  or a lower  $P_{t+n}$ .

For  $n$ -period real bonds the derivations are analogous, though it will be useful to define the marginal utility of consumption as  $\lambda_t \equiv P_t \Lambda_t$ . The price in period  $t$  of an  $n$ -period real discount bond is the ratio of the present value of expected marginal utility in period  $t + n$  to marginal utility in period  $t$ :

$$q_t^{(n)} = \beta^n E_t [\lambda_{t+n}/\lambda_t], \quad n = 1, 2, \dots \quad (14)$$

In contrast to the price of a nominal bond, which depends on the joint properties of the marginal utility of consumption ( $\lambda_t = u'(c_t)$ ) and the price level, the price of a real bond depends only on the marginal utility of consumption.

### *Forward Contracts*

As for forward contracts, the first-order condition for  $s$ -period-ahead one-period nominal forward contracts, committed to in period  $t$  is

$$E_t \Lambda_{t+s} Q_{t,s}^f = \beta E_t \Lambda_{t+s+1}.$$

However, because the forward price is known in period  $t$ , we can bring the price outside the expectation operator:

$$Q_{t,s}^f E_t \Lambda_{t+s} = \beta E_t \Lambda_{t+s+1}. \quad (15)$$

Likewise for real forward contracts, we have

$$q_{t,s}^f E_t \lambda_{t+s} = \beta E_t \lambda_{t+s+1}. \quad (16)$$

From (11) and (15), the price of an  $n$ -period nominal bond is identical to the product of the prices of a sequence of one-period forward contracts,

$$Q_t^{(n)} = Q_t^{(1)} Q_{t,1}^f Q_{t,2}^f \cdots Q_{t,n-1}^f. \quad (17)$$

An  $n$ -period bond and a sequence of forward contracts can each be used to provide a certain return  $n$ -periods ahead. To get a dollar in  $t + n$  using the  $n$ -period bond, one needs to spend  $Q_t^{(n)}$  today, whereas to get a dollar in  $t + n$  using the sequence of forward contracts, one needs to spend  $Q_t^{(1)} Q_{t,1}^f Q_{t,2}^f \cdots Q_{t,n-1}^f$  today. This is easily illustrated in the two-period case: to get a dollar in  $t + 2$  using forward contracts, one needs to have  $Q_{t,1}^f$  in period  $t + 1$ —for this is the forward price of a bond which will deliver a dollar in period  $t + 2$ . In turn, receiving  $Q_{t,1}^f$  in period  $t + 1$  means spending  $Q_t^{(1)} Q_{t,1}^f$  in period  $t$  on one-period bonds—the price of a bond that delivers a dollar in  $t + 1$  is  $Q_t^{(1)}$ , and one needs to purchase  $Q_{t,1}^f$  of these bonds. True arbitrage would be possible if  $Q_t^{(1)} Q_{t,1}^f$  were not equal to  $Q_t^{(2)}$ . The same reasoning holds for a long-term real bond and a sequence of real forward contracts, so we have

$$q_t^{(n)} = q_t^{(1)} q_{t,1}^f q_{t,2}^f \cdots q_{t,n-1}^f. \quad (18)$$



Note that from (17) or (18), the ratio in period  $t$  of the price of an  $n$ -period bond to the price of an  $n - 1$  period bond is equal to the forward price of one-period bond in period  $t + n - 1$ , as of period  $t$ .

The optimality conditions (11) – (16) and the relationships between prices of long bonds and forward contracts (17) and (18) serve as the basis for the generalized Fisher relationship and generalized expectations theory.

### 3. EXPECTATIONS THEORY OF THE TERM STRUCTURE

The standard version of the expectations theory of the term structure states that long-term interest rates are equal to an average of expected future short-term interest rates. We will derive a generalization of this theory, focusing on bond prices instead of rates, and we will see that only under certain conditions does the pure expectations theory hold. Our derivation exploits the fact that a long bond is equivalent to a sequence of forward contracts.

From (17), the price of an  $n$ -period bond is the product of the prices of  $n$  short-term forward contracts. Under the pure expectations hypothesis, the price of an  $n$ -period bond is also equal to the product of the expected prices of future short-term bonds, which we will denote by  $PEH^{(n)}$ , for pure expectations hypothesis:

$$PEH_t^{(n)} = Q_t^{(1)} E_t \left( Q_{t+1}^{(1)} \right) E_t \left( Q_{t+2}^{(1)} \right) \cdots E_t \left( Q_{t+n-1}^{(1)} \right). \quad (19)$$

In a way that we will make more precise shortly, the pure expectations hypothesis holds if covariances involving future bond prices and future marginal utility are zero. It follows from the previous equation and (17) that the deviation of the price of an  $n$ -period nominal bond from  $PEH^{(n)}$  is the product of ratios of forward prices to expected future spot prices:

$$\frac{Q_t^{(n)}}{PEH_t^{(n)}} = \frac{Q_{t,1}^f}{E_t \left( Q_{t+1}^{(1)} \right)} \frac{Q_{t,2}^f}{E_t \left( Q_{t+2}^{(1)} \right)} \cdots \frac{Q_{t,n-1}^f}{E_t \left( Q_{t+n-1}^{(1)} \right)}, \quad (20)$$

or, in shorthand,

$$\frac{Q_t^{(n)}}{PEH_t^{(n)}} = F_{t,1}^{(1)} \cdots F_{t,n-1}^{(1)}, \quad (21)$$

where we call  $F_{t,j}^{(1)}$  the  $j$ -period-ahead forward premium,

$$F_{t,j}^{(1)} \equiv \frac{Q_{t,j}^f}{E_t \left( Q_{t+j}^{(1)} \right)}. \quad (22)$$

In terms of marginal utilities, using (11) and (15), the forward premium is

$$F_{t,j}^{(1)} = \frac{\frac{E_t \Lambda_{t+j+1}}{E_t \Lambda_{t+j}}}{E_t \left( \frac{\Lambda_{t+j+1}}{\Lambda_{t+j}} \right)}, \quad (23)$$

and it is straightforward to show that the forward premium is pinned down by the autocovariance properties of marginal utility, or equivalently by the covariance between the future short-term bond price and future marginal utility:

$$F_{t,j}^{(1)} = \frac{E_t \left( \frac{\Lambda_{t+j+1}}{\Lambda_{t+j}} \Lambda_{t+j} \right)}{E_t \left( \frac{\Lambda_{t+j+1}}{\Lambda_{t+j}} \right) E_t \Lambda_{t+j}} \quad (24)$$

$$\begin{aligned} &= 1 + \text{cov}_t \left( \frac{(\Lambda_{t+j+1}/\Lambda_{t+j})}{E_t (\Lambda_{t+j+1}/\Lambda_{t+j})}, \frac{\Lambda_{t+j}}{E_t \Lambda_{t+j}} \right) \\ &= 1 + \text{cov}_t \left( \frac{Q_{t+j}^{(1)}}{E_t Q_{t+j}^{(1)}}, \frac{\Lambda_{t+j}}{E_t \Lambda_{t+j}} \right), \end{aligned} \quad (25)$$

with the last equality following from the law of iterated expectations.<sup>9</sup>

The deviation of the long bond price from the pure expectations hypothesis is thus accounted for by the product of the individual forward premiums ( $F_{t,j}^{(1)}$ ),

$$Q_t^{(n)} = PEH_t^{(n)} \times \left( F_{t,1}^{(1)} \cdots F_{t,n-1}^{(1)} \right). \quad (26)$$

If each of the individual covariances that determine the forward premiums are zero, then the pure expectations hypothesis holds. In turn, forward premiums will be zero if the level of future nominal marginal utility is uncorrelated with its subsequent growth rate. This will be the case, for example, if nominal marginal utility is constant, or if it follows a random walk. Note that for nominal bonds, risk neutrality is insufficient to drive all forward premiums to zero; if the future price level is correlated with its subsequent growth rate, there will be a forward premium, even if investors are risk-neutral.<sup>10</sup> For the case of risk aversion, the behavior of the marginal utility of consumption is crucial for determining the forward premium as well as the inflation-risk premium derived below. In standard models along the lines of Lucas (1978), the marginal utility of consumption is a simple function of consumption itself. Alternatively, one can consider more complicated specifications of  $u(c)$ , or be entirely agnostic on the specification of  $u(c)$ . These approaches are discussed in Section 6.

Why do the conditional covariances between future marginal utility and the subsequent growth rate of marginal utility affect the price of a long-term bond relative to the product of expected future short-term bond prices? Focus on one term ( $F_{t,j}^{(1)}$ ), which is the price premium for a  $j$ -period-ahead forward

<sup>9</sup> From the law of iterated expectations we know, for example, that  $E_t(\Lambda_{t+j+1}) = E_t(\Lambda_{t+j} E_{t+j}(\Lambda_{t+j+1}/\Lambda_{t+j})) = E_t(\Lambda_{t+j} Q_{t+j}^{(1)})$ .

<sup>10</sup> Under risk-neutrality, expected real returns are equated across assets (forward and spot, for example). Because the real return is the nominal return *divided* by the gross inflation rate, expected nominal returns are not necessarily equated.

contract relative to the expected  $j$ -period-ahead spot price of a one-period bond. If the growth rate of the marginal utility of a dollar (price of a one-period bond) is expected to covary positively with the level of the marginal utility of a dollar in  $t + j$ , then you pay a premium at  $t + j$  to lock in at  $t$  the contract that gives you a dollar at  $t + j + 1$ . In this situation, you tend to value a dollar highly for consumption in  $t + j$  precisely when buying a bond requires you to forego a lot of consumption. Thus, the expected spot market looks expensive, which means that the forward price must be high as well.

Note that the  $j$ -period-ahead forward premium can be positive or negative, and thus the overall term premium can be positive or negative. It is common to think of long *rates* as incorporating a positive term premium (meaning that  $\mathbb{E}[F_{t,j}^1] < 0$ ), but if future marginal utility of a dollar is positively correlated with the expected growth rate of marginal utility, then forward premiums and the term premium in rates will be negative.

Of course, we can also think about the forward-spot relationship from a no-arbitrage perspective: agents must be indifferent between committing to buy a one-period bond in period  $t + j$  (the forward contract) and expecting to buy a one-period bond in the spot market at  $t + j$ :

$$Q_{t,j}^f E_t \Lambda_{t+j} = E_t \Lambda_{t+j} Q_{t+j}^{(1)}.$$

Expanding the right-hand side,

$$Q_{t,j}^f E_t \Lambda_{t+j} = E_t \Lambda_{t+j} E_t Q_{t+j}^{(1)} + \text{cov}_t \left( \Lambda_{t+j}, Q_{t+j}^{(1)} \right). \quad (27)$$

Dividing both sides by  $E_t \Lambda_{t+j} E_t Q_{t+j}^{(1)}$  replicates the expression for  $F_{t,j}^1$  above.

#### 4. FISHER RELATIONSHIP

The pure Fisher relationship states that nominal interest rates are equal to real rates plus expected inflation. We will derive a generalization of this expression, focusing again on bond prices instead of interest rates. The derivation follows directly from manipulating the pricing equation for a nominal bond.

From the fact that the real and nominal multipliers are related by  $\frac{\lambda_t}{P_t} = \Lambda_t$ , we can use (11) to write the price of a nominal bond ( $\beta^n E_t [\Lambda_{t+n}/\Lambda_t]$ ) as

$$Q_t^{(n)} = \beta^n E_t \left[ \frac{\lambda_{t+n}}{\lambda_t} \frac{P_t}{P_{t+n}} \right], \quad (28)$$

or

$$Q_t^{(n)} = \beta^n \left\{ E_t \left[ \frac{\lambda_{t+n}}{\lambda_t} \right] E_t \left[ \frac{P_t}{P_{t+n}} \right] + \text{cov}_t \left[ \frac{\lambda_{t+n}}{\lambda_t}, \frac{P_t}{P_{t+n}} \right] \right\}. \quad (29)$$

This last expression can be used to decompose the price of a long-term nominal bond into the price of a long-term real bond ( $q_t^{(n)}$  from [14]), the expectation of the inverse of inflation ( $E_t (P_t/P_{t+n})$ ), and a term we will call  $\Theta_t^{(n)}$ , which

can be thought of as the inflation-risk premium:

$$\begin{aligned} Q_t^{(n)} &= \left\{ q_t^{(n)} E_t (P_t/P_{t+n}) \right\} \left( 1 + \Theta_t^{(n)} \right) \\ \Theta_t^{(n)} &\equiv \beta^n \text{cov}_t \left[ \frac{\lambda_{t+n}/\lambda_t}{E_t (\lambda_{t+n}/\lambda_t)}, \frac{P_t/P_{t+n}}{E_t (P_t/P_{t+n})} \right]. \end{aligned} \quad (30)$$

Alternatively, converting to interest rates instead of bond prices, we have

$$R_t^{(n)} = r_t^{(n)} \left[ \frac{1}{E_t (P_t/P_{t+n}) \left( 1 + \Theta_t^{(n)} \right)} \right]^{1/n}. \quad (31)$$

If there is zero conditional covariance between the normalized growth rate of marginal utility and the normalized inverse of inflation, then the inflation-risk premium is zero, and we recover the pure Fisher equation. In general, though, the price of a long-term nominal bond exceeds the “Fisher price” when the covariance between the growth of real marginal utility and the inverse of inflation is positive. What is the intuition behind this covariance effect? The bond pays off one dollar. If the covariance is positive, then the consumption value of a dollar is high (low) in those states where the marginal utility of consumption is high (low). In other words, the bond pays off well in terms of consumption when you value consumption highly, so it is worth more than the Fisher price.

Note that like the forward premium, the inflation-risk premium can be positive or negative. We usually think of the inflation-risk premium in rates as being positive, which would correspond to the price premium  $\Theta_t$  being negative. However, this depends entirely on whether the inverse of the future price level is negatively correlated with the future marginal utility of consumption, conditional on time- $t$  information.

## 5. YIELD DIFFERENTIAL AND HOLDING-PERIOD PREMIUM

Above we provided two decompositions of the price of an  $n$ -period nominal bond. The first expressed the bond price in terms of the price of a real bond, the expected inverse of the change in the price level, and an inflation-risk premium. The second decomposition expressed the bond price in terms of the expected product of the prices of future short-term bonds and the product of individual forward premiums at each maturity. We now use these decompositions to study the yield differential between bonds of any two maturities. In addition, in this section we provide an intuitive explanation of the holding-period premium, the expected differential between the return on a long-term bond sold before it matures and the return on a shorter-term bond sold at maturity.

### Yield Differential

Recall that the yield is the inverse of the price, and that we standardize yields so that they are reported on a gross per-period basis:

$$R_t^{(n)} = (Q_t^{(n)})^{-1/n}. \quad (32)$$

We then can express the bond yield using the two decompositions as

$$R_t^{(n)} = \left[ \left\{ q_t^{(n)} E_t (P_t/P_{t+n}) \right\} \left( 1 + \Theta_t^{(n)} \right) \right]^{-1/n} \quad (33)$$

or

$$R_t^{(n)} = \left[ P E H_t^n \times \left( F_{t,1}^{(1)} \cdots F_{t,n-1}^{(1)} \right) \right]^{-1/n}. \quad (34)$$

Since these expressions hold for any  $n$ , the ratio of the yield on an  $n$ -period bond to the yield on a one-period bond can be written using the Fisher decomposition as

$$\frac{R_t^{(n)}}{R_t^{(1)}} = \frac{\left\{ q_t^{(1)} E_t (P_t/P_{t+1}) \right\} \left( 1 + \Theta_t^{(1)} \right)}{\left[ \left\{ q_t^{(n)} E_t (P_t/P_{t+n}) \right\} \left( 1 + \Theta_t^{(n)} \right) \right]^{1/n}} \quad (35)$$

$$= \frac{r_t^{(n)} E_t (P_t/P_{t+1})}{r_t^{(1)} (E_t (P_t/P_{t+n}))^{1/n}} \frac{1 + \Theta_t^{(1)}}{\left( 1 + \Theta_t^{(n)} \right)^{1/n}}. \quad (36)$$

The nominal yield curve slopes upward if some combination of the following is true: (i) long-term real rates exceed short-term real rates, (ii) the value of a dollar is expected to increase at a higher rate in the short term than over the long term, and (iii) the short-term inflation-risk premium in bond prices exceeds the long-term inflation-risk premium in bond prices.

Alternatively, we can use the perspective of the expectations hypothesis to write the yield differential as

$$\frac{R_t^{(n)}}{R_t^{(1)}} = \frac{Q_t^{(1)}}{\left[ P E H_t^{(n)} \times \left( F_{t,1}^{(1)} \cdots F_{t,n-1}^{(1)} \right) \right]^{1/n}}. \quad (37)$$

For  $n = 1$ , note that  $PEH = Q_t^{(1)}$ , and, by convention,  $F_{t,0}^{(1)} = 1$ . Long-term rates exceed short-term rates if short-term bond prices are expected to fall or if forward-price premiums are negative.

### Holding-Period Premium

There are many ways that one can transport money or goods from period  $t$  to period  $t + j$ . Until now, we have emphasized the comparison between buying a  $j$ -period bond and buying a sequence of one-period bonds. Another option,

however, is to purchase an  $i$ -period bond, where  $i > j$ , and sell the bond in period  $t + j$  for the price  $Q_{t+j}^{(i-j)}$ , which is uncertain as of period  $t$ . Of course, the bond pricing relationships derived previously must imply that the consumer is indifferent between these two strategies. That is,

$$E_t [\Lambda_{t+j}] = \frac{Q_t^{(j)}}{Q_t^{(i)}} E_t [\Lambda_{t+j} Q_{t+j}^{(i-j)}]. \quad (38)$$

The left-hand side is the expected payoff in period  $t + j$  in utility terms to buying one  $j$ -period bond in period  $t$  for price  $Q_t^{(j)}$ ; note that the only uncertainty with respect to this strategy involves the marginal utility of a dollar in period  $t + j$ . The right-hand side is the expected return in period  $t + j$  in utility terms to spending the same amount,  $Q_t^{(j)}$ , on an  $i$ -period bond in period  $t$  and selling the bond in period  $t + j$ . With this strategy, there is uncertainty both about the marginal utility of a dollar in period  $t + j$  and about the price at which one can sell the bond in period  $t + j$ .

An intuitively appealing property similar to the pure expectations hypothesis is that the expected dollar return to these two strategies should be the same. By now, it is probably clear that while the expected utility return must be the same (as reflected in [38]), the expected dollar return will generally be different for the two strategies. The dollar return to buying the  $j$ -period bond and holding it to maturity is certain and given by  $1/Q_t^{(j)}$ . The expected dollar return to buying the  $i$ -period bond and selling it in period  $t + j$  is given by

$$\frac{E_t [Q_{t+j}^{(i-j)}]}{Q_t^{(i)}}. \quad (39)$$

So the expected “premium” for holding an  $i$ -period bond for  $j$  periods is

$$H_t^{(i,j)} = \frac{Q_t^{(j)} E_t [Q_{t+j}^{(i-j)}]}{Q_t^{(i)}}. \quad (40)$$

From (38) we can write this premium as

$$H_t^{(i,j)} = \frac{E_t [\Lambda_{t+j}] E_t [Q_{t+j}^{(i-j)}]}{E_t [\Lambda_{t+j} Q_{t+j}^{(i-j)}]} \quad (41)$$

or

$$H_t^{(i,j)} = \frac{1}{1 + \text{cov}_t \left( \frac{Q_{t+j}^{(i-j)}}{E_t Q_{t+j}^{(i-j)}}, \frac{\Lambda_{t+j}}{E_t \Lambda_{t+j}} \right)} \quad (42)$$

$$= \frac{1}{1 + \text{cov}_t \left( \frac{E_{t+j}(\Lambda_{t+i}/\Lambda_{t+j})}{E_t(\Lambda_{t+i}/\Lambda_{t+j})}, \frac{\Lambda_{t+j}}{E_t \Lambda_{t+j}} \right)}. \quad (43)$$

The holding-period premium is driven by the same uncertainty as the forward premium, except over a possibly longer horizon. If future marginal utility is positively conditionally correlated with the future price of an  $(i - j)$ -period bond, then the  $i$ -period bond will tend to generate capital gains when they are highly valued, so individuals will not require a high expected return. That is, the relative expected return  $H_t^{(i,j)}$  will be low when  $\text{cov}_t \left( \frac{\Lambda_{t+j}}{E_t \Lambda_{t+j}}, \frac{Q_{t+j}^{(i-j)}}{E_t Q_{t+j}^{(i-j)}} \right)$  is positive.

Of course, we can also use our earlier derivations to express the holding-period premium in ways related to the pure expectations hypothesis and the Fisher equation. Using the definition of the pure expectations hypothesis, we have from (26)

$$H_t^{(i,j)} = \frac{E_t \left[ P E H_{t+j}^{(i-j)} \times \left( F_{t+j,1}^{(1)} \cdots F_{t+j,i-j-1}^{(1)} \right) \right]}{E_t \left( Q_{t+j}^{(1)} \right) E_t \left( Q_{t+j+1}^{(1)} \right) \cdots E_t \left( Q_{t+i-1}^{(1)} \right) \times \left( F_{t,j}^{(1)} \cdots F_{t,i-1}^{(1)} \right)}. \quad (44)$$

Very loosely speaking, this expression relates the holding-period premium to the conditional covariance between expected future short prices and expected future forward premiums. Analogously, using the Fisher equation, we have from (30)

$$H_t^{(i,j)} = \frac{E_t \left( P_t / P_{t+j} \right) \left( 1 + \Theta_t^{(j)} \right) E_t \left[ \left\{ q_{t+j}^{(i-j)} \left( P_{t+j} / P_{t+i} \right) \right\} \left( 1 + \Theta_{t+j}^{(i-j)} \right) \right]}{q_{t,j}^f q_{t,j+1}^f \cdots q_{t,i-1}^f E_t \left( P_t / P_{t+i} \right) \left( 1 + \Theta_t^{(i)} \right)}. \quad (45)$$

Again, loosely speaking, this expression relates the holding-period premium to the conditional covariance between the future inflation-risk premium, and the pure Fisher component of the future  $(i - j)$ -period bond price.

## 6. APPLICATIONS OF THE THEORY

The derivations above provide a textbook-like guide to bond price decompositions from the perspective of consumption-based asset pricing theory. As we stated at the outset, these decompositions can be a useful input into the formulation of monetary policy, contributing to an understanding of the term structure of real and nominal interest rates and expected inflation.<sup>11</sup> But any contribution to our understanding of these variables requires taking the theory to the data, and this, in turn, requires making some assumptions about

<sup>11</sup> We emphasize the usefulness for monetary policy, but there are other applications of the theory. Many areas of economics emphasize the behavior of real interest rates, and financial market practitioners use the kind of theories outlined here to aid in the pricing of interest rate derivatives.

the unobservable variables  $\Lambda$  or  $\lambda$ , the marginal utility of nominal or real consumption. According to the pure expectations hypothesis and the pure Fisher equation, marginal utility is extraneous: armed with an estimate of expected inflation, we can directly estimate the real rate from data on nominal rates; likewise, armed with data on the term structure of nominal rates, we can directly calculate the expected path of future short-term rates. Absent risk neutrality, however, marginal utility takes center stage, for it is the covariance of the marginal rate of substitution (marginal utility growth) with the evolution of the price level that pins down the inflation-risk premium (see [30]); and it is the autocovariance properties of marginal utility that determine the forward premium (see [24]).

Researchers applying theory to data on bond prices have gone in two directions concerning the degree of structure they impose on the marginal utility of consumption. The “pure” consumption-based approach takes a stand on the form of  $u(c)$  in (2) and uses data on consumption and inflation to estimate the parameters of  $u(c)$  and the stochastic processes for consumption and inflation. The combination of estimated preference parameters and stochastic processes then comprise a model of bond prices; most importantly, the specification of  $u(c)$  together with data on  $c$  make marginal utility “observable.” Campbell (1986) is a particularly accessible example of this strand of the literature, albeit an example that includes only real bonds; he uses a simple specification of  $u(c)$  and the stochastic process for consumption and derives closed form expressions for interest rates of all maturities. Campbell’s paper is primarily pedagogical.<sup>12</sup>

Two recent papers that use more complicated forms of preferences, include nominal bonds and make a serious attempt to match data are Wachter (2006) and Piazzesi and Schneider (2006).<sup>13</sup> Wachter uses a habit-persistence specification similar to the one Campbell and Cochrane (1999) apply to equity pricing. She argues that the model “accounts for many features of the nominal term structure of interest rates.” Most importantly, from our perspective, the model-based forward premiums that Wachter computes help to account for the empirical disparity between long-term rates and the corresponding average of expected future short-term rates—that is, the violation of the pure expectations hypothesis. However, there is still a noticeable divergence between the actual time series for short-term rates and the path implied by Wachter’s model. Piazzesi and Schneider use the recursive utility preference specification of Epstein and Zin (1989) and Weil (1989). They emphasize the inflation-risk premium in long-term bonds that arises when inflation brings bad news

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<sup>12</sup> See also Ireland (1996) and Sarte (1998).

<sup>13</sup> Both Wachter (2006) and Piazzesi and Schneider (2006) use preference specifications that are not encompassed by (2). However, for our purposes, we can view their approaches as involving complicated specifications of  $u(c)$ .



about future consumption growth. Although their model is broadly consistent with the behavior of the term structure, the short-term rates implied by their model are substantially less volatile than the data. Regarding the approach taken by Wachter (2006) and by Piazzesi and Schneider (2006), Campbell (2006) writes, “The literature on consumption-based bond pricing is surprisingly small, given the vast literature given to consumption-based models of equity markets.” We can thus expect much more work of this sort in the coming years.

Ravenna and Seppälä (2006) is one recent example of studying the term structure of interest rates in a consumption-based model that endogenizes consumption—the papers mentioned in the previous two paragraphs treat consumption (and inflation) as exogenous. Ravenna and Seppälä embed the asset pricing apparatus in a New Keynesian business cycle model. They argue that their model accounts for the cyclical properties of interest rates and the rejections of the pure expectations hypothesis, but they do not provide time series comparisons of data and model-generated interest rates. Given the high degree of structure required, matching the data with this approach is a daunting task.

The second major empirical application of bond pricing theory is known as the “no-arbitrage” or “arbitrage-free” approach. With this approach, one avoids making any parametric assumptions about the form of  $u(c)$ . What the no-arbitrage approach does carry over from the theory laid out previously is the idea that there exists a marginal rate of substitution that prices all bonds; that is, (11) holds for some strictly positive random variable  $m_t \equiv \Lambda_t/\Lambda_{t-1}$ . A good introduction to this approach is Backus, Foresi, and Telmer (1998), and a recent example is Kim and Wright (2005). Those authors and many others assume that the time-series behavior of the yield curve is driven by a small number of latent factors. The arbitrage-free approach has the advantage of being able to fit observed time series on bond prices quite well, thereby opening the door to discussing relatively small changes in the term structure of real or nominal rates. For example, Kim and Wright provide time series plots of the forward rate along with the estimated expected short rate and the estimated term premium. However, this approach has limitations from the perspective of macroeconomics in that it does not provide a framework for studying the joint determination of bond prices and macroeconomic outcomes. Indeed, Duffee (2006) writes, “some readers . . . call this a nihilistic model of term premia.” He views the approach in a positive light, though, as “an intermediate step in the direction of a correctly specified economic model of premia, not an end in itself.”

## 7. CONCLUSION

Nominal and real interest rates are often viewed from the perspectives of the intuitively appealing Fisher relationship and pure expectations hypothesis. Modern asset pricing theory implies that those relationships should not be expected to hold exactly if investors are risk-averse. We have used that theory to describe how the deviations from the Fisher relationship and the pure expectations hypothesis depend on particular covariances. In the process, we have meant to provide an introduction to the consumption-based modeling of bond prices. From the standpoint of macroeconomics and monetary policy, the value of this approach is that it allows researchers to interpret the behavior of the term structure of real and nominal bond prices in ways that relate to macroeconomic activity and monetary policy.

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