

A Quantitative Study of the Role of Wealth Inequality on Asset Prices

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There is an extensive body of work devoted to understanding the determinants of asset prices. The cornerstone formula behind most of these studies can be summarized in equation (1). The asset pricing equation states in recursive formulation that the current price of an asset equals the present discounted value of future payments delivered by the asset. Namely,

$$p(s_t) = E[m(s_t, s_{t+1})(x(s_{t+1}) + p(s_{t+1})) | s_t], \quad (1)$$

where $p(s)$ denotes the current price of an asset in state s ; $x(s)$ denotes the payments delivered by the asset in state s ; and $m(s, s')$ denotes the stochastic discount factor from state s today to state s' tomorrow, that is, the function that determines the equivalence between current period dollars in state s and next period dollars in state s' . It is apparent from equation (1) that the stochastic discount factor m plays a key role in explaining asset prices.

One strand of the literature estimates m using time series of asset prices, as well as other financial and macroeconomic variables. The estimation procedure is based on some arbitrary functional form linking the discount factor to the explanatory variables. Even though this strategy allows for a high degree of flexibility in order to find the stochastic discount factor that best fits the data, it does not provide a deep understanding of the forces that drive asset prices. In particular, this approach cannot explain what determines the shape of the estimated discount factor. This limitation becomes important once we want to understand how structural changes, like a modification in the tax code, may affect asset prices. The answer to this type of question requires that the stochastic discount factor is derived from the primitives of a model.

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This is the strategy undertaken in the second strand of the literature.¹ The extra discipline imposed by this line of research has the additional benefit that it allows one to integrate the analysis of asset prices into the framework used for modern macroeconomic analysis.² On the other hand, the extra discipline imposes a cost: it limits the empirical performance of the model. The most notable discrepancy between the asset pricing model and the data was pointed out by Mehra and Prescott (1985). They calibrate a stylized version of the consumption-based asset pricing model to the U.S. economy and find that it is incapable of replicating the differential returns of stocks and bonds. The average yearly return on the Standard & Poor's 500 Index was 6.98 percent between 1889 and 1978, while the average return on 90-day government Treasury bills was 0.80 percent. Mehra and Prescott (1985) could explain an equity premium of, at most, 0.35 percent. The discrepancy, known as the equity premium puzzle, has motivated an extensive literature trying to understand why agents demand such a high premium for holding stocks.³ The answer to this question has important implications in other areas. For example, most macroeconomic models conclude that the costs of business cycles are relatively low (see Lucas 2003), which suggests that agents do not care much about the risk of recessions. On the other hand, a high equity premium implies the opposite, which suggests that a macro model that delivers asset pricing behavior more aligned with the data may offer a different answer about the costs of business cycles.

The present article is placed in the second strand of the literature mentioned above. The objective here is to explore how robust the implications of the standard consumption-based asset pricing model are once we allow for preferences that do not aggregate individual behavior into a representative agent setup.

Mehra and Prescott (1985) consider an environment with complete markets and preferences that display a linear coefficient of absolute risk tolerance (ART) or hyperbolic absolute risk aversion (HARA).⁴ This justifies the use of a representative-agent model. Several authors have explored how the presence of heterogeneous agents could enrich the asset pricing implications of the standard model and, therefore, help explain the anomalies observed in the data. Constantinides and Duffie (1996), Heaton and Lucas (1996), and Krusell and

¹ Lucas (1978) represents the basic reference of the consumption-based asset pricing model. He studies an endowment economy with homogeneous agents and shows how the prices of assets are linked to agents' consumption.

² See Jermann (1998) for an example of a study of asset prices in a real business cycle model.

³ McGrattan and Prescott (2003) argue that the actual equity premium is lower than 6 percent after allowing for diversification costs, taxes, and the liquidity premium of the short-term government bonds.

⁴ The coefficient of absolute risk tolerance is defined as $-\frac{u'(c)}{u''(c)}$.

Smith (1997) are prominent examples of this literature. These articles maintain the HARA assumption, but abandon the complete markets setup. The lack of complete markets introduces a role for the wealth distribution in the determination of asset prices.

An alternative departure from the basic model that also introduces a role for the wealth distribution is to abandon the assumption of a linear ART. This is the avenue taken in Gollier (2001). He studies explicitly the role that the curvature of the ART plays in a model with wealth inequality. He shows in a two-period setup that when the ART is concave, the equity premium in an unequal economy is larger than the equity premium obtained in an egalitarian economy. The aim of the present article is to quantify the analytical results provided in Gollier's article. Preferences with habit formation constitute another example of preferences with a nonlinear ART. Constantinides (1990) and Campbell and Cochrane (1999) are prominent examples of asset pricing models with habit formation. As in Gollier (2001), these preferences also introduce a role for the wealth distribution, but this channel is shut down in these articles by assuming homogeneous agents.

The present article considers a canonical Lucas tree model with complete markets. There is a single risky asset in the economy, namely a tree. This asset pays either high or low dividends. The probability distribution governing the dividend process is commonly known. Agents also trade a risk-free bond. Each agent receives in every period an exogenous endowment of goods, which can be interpreted as labor income. The endowment varies across agents. For simplicity, it is assumed that a fraction of the population receives a higher endowment in every period, that is, there is income inequality. Agents are also initially endowed with claims to the tree, which are unevenly distributed across agents. The last two features imply that wealth is unequally distributed. Agents share a utility function with a piecewise linear ART.

The exercise conducted in this article compares the equilibrium asset prices in an economy that features an unequal distribution of wealth with an egalitarian economy, that is, an economy that displays the same aggregate resources as the unequal economy, but in which there is no wealth heterogeneity. For a concave specification of the ART, this article finds evidence suggesting that the role played by the distribution of wealth on asset prices may be non-negligible. The unequal economy displays an equity premium between 24 and 47 basis points larger than the egalitarian economy. This is still far below the premium of 489 basis points observed in the data.⁵ The

⁵ This number is 129 basis points smaller than the premium documented in Mehra and Prescott (1985). There are two reasons for this. First, the sample period used in the present article is 1871 to 2004, while Mehra and Prescott (1985) use data from 1889 to 1978. Second, the present article uses one-year Treasury bills as a proxy for the risk-free rate, while Mehra and Prescott (1985) use 90-day Treasury bills.

risk-free rate in the unequal economy is between 11 and 20 basis points lower than in the egalitarian economy.

The rest of the article is organized as follows. Section 1 discusses the assumption of a concave ART. Section 2 introduces the model. Section 3 outlines how the model is calibrated. Section 4 presents the results, defining the equilibrium concept and describing how the model is solved. Finally, Section 5 presents the conclusions.

1. PREFERENCES

It is assumed that agents' preferences with respect to random payoffs satisfy the continuity and independence axioms and, therefore, can be represented by a von Neumann-Morgenstern expected utility formulation. The utility function is denoted by $u(c)$. The utility function is increasing and concave in c . The concavity of $u(c)$ implies that agents dislike risk, that is, agents are willing to pay a premium to eliminate consumption volatility. The two most common measures of the degree of risk aversion are the coefficient of absolute risk aversion and the coefficient of relative risk aversion. The coefficient of absolute risk aversion measures the magnitude of the premium (up to a constant of proportionality) that agents are willing to pay at a given consumption level c , in order to avoid a "small" gamble with zero mean and payoff levels unrelated to c . The coefficient of absolute risk aversion (ARA) is computed as follows:

$$ARA(c) = -\frac{u''(c)}{u'(c)}.$$

The coefficient of relative risk aversion (RRA) also measures the magnitude of the premium (up to a constant of proportionality) that agents are willing to pay at a given consumption level c to avoid a "small" gamble with zero mean, but with payoff levels that are proportional to c . The coefficient of relative risk aversion is computed as follows:

$$RRA(c) = -\frac{cu''(c)}{u'(c)}.$$

The coefficient of ART is the inverse of the coefficient of ARA. The utility function used in this article is reverse engineered to display a piecewise linear ART, namely,

$$ART(c) = -\frac{u'(c)}{u''(c)} = \begin{cases} a_0 + b_0c & \text{if } c \leq \hat{c} \\ a_1 + b_1c & \text{if } c > \hat{c} \end{cases},$$

where $a_1 - a_0 = (b_0 - b_1)\hat{c}$. This equality implies that the ART is continuous. It is assumed that both slope coefficients, b_0 and b_1 , are strictly positive. When $b_1 < b_0$, the ART is concave, and when $b_1 > b_0$, the ART is convex. The standard constant RRA utility function corresponds to the case where $b_1 = b_0$, and $a_1 = a_0 = 0$.

The previous formulation implies that individual preferences can be represented by the following utility function.⁶

$$u(c) = \begin{cases} K_0 (a_0 + b_0 c)^{\left(1 - \frac{1}{b_0}\right)} + K_1 & \text{if } c \leq \hat{c} \\ (a_1 + b_1 c)^{\left(1 - \frac{1}{b_1}\right)} & \text{if } c > \hat{c} \end{cases},$$

where

$$K_0 = - \frac{(b_1 - 1) (a_1 + b_1 \hat{c})^{-\frac{1}{b_1}}}{(b_0 - 1) (a_0 + b_0 \hat{c})^{-\frac{1}{b_0}}},$$

and

$$K_1 = - (a_1 + b_1 \hat{c})^{\left(1 - \frac{1}{b_1}\right)} - K_0 (a_0 + b_0 \hat{c})^{\left(1 - \frac{1}{b_0}\right)}.$$

The present parameterization of the utility function has several advantages. First, it nests the concave and convex ART cases in a simple way. Second, it enables us to introduce a high degree of curvature of the ART. Finally, it helps provide a transparent explanation of the results.

On the Concavity of the Coefficient of Absolute Risk Tolerance

The results in Gollier (2001) suggest that wealth inequality may help in reconciling the model with the equity premium observed in the data as long as agents display a concave ART. This section discusses to what extent this is a palatable assumption.

One possible way to verify the validity of a concave ART is to contrast the testable implications of a concave (or convex) ART in terms of individual savings and portfolio behavior with the data. This is the avenue taken in Gollier (2001). He argues that the evidence is far from conclusive. He documents that even though saving and investment patterns do not seem to favor a concave ART, several studies are able to explain this behavior without relying on a convex ART. More precisely, an increasing and concave ART would imply that the fraction invested in risky assets is increasing with wealth, but at a decreasing rate. This is not observed in the data. However, once the complete information setup is abandoned, one alternative explanation emerges: information does not appear to be evenly distributed across market participants. This is supported by Ivkovich, Sialm, and Weisbenner (forthcoming), who find evidence suggesting that wealthier investors are more likely to enjoy an informational advantage and earn higher returns on their investments, which may feed into their appetite for stocks.

⁶ See Appendix A for a description of how the utility function is recovered from the ART.

In a model without uncertainty, a concave ART would imply an increasing marginal propensity to consume out of wealth. The data contradict this result. But there are various alternative explanations for the increasing propensity to save that do not rely on a convex ART. The presence of liquidity constraints is one of them. The fact that the investment set is not uniform across agents is another one.⁷

Another alternative to test the validity of a concave ART is to use the results from experimental economics. However, Rabin and Thaler (2001) argue that not only is the coefficient of risk aversion an elusive parameter to estimate, but also the entire expected utility framework seems to be at odds with individual behavior. In part, this has motivated the burst of behavioral biases models in the finance literature.⁸ The landscape is different in the macro literature. The expected utility framework is still perceived as a useful tool for understanding aggregate behavior.

The previous arguments suggest that the data do not provide strong evidence in favor of or against a concave ART, which does not invalidate a concave specification of the ART as a possible representation of individual preferences. The rest of the article focuses on this case in order to measure the role of wealth inequality on asset pricing.

2. THE MODEL

This article analyzes a canonical Lucas tree model. The only difference with Lucas (1978) is that our model features heterogeneous agents. We consider a pure exchange economy with complete information. There is a single risky asset in the economy: a tree. There is a unit measure of shares of the tree. The tree pays either high dividends (d_h) or low dividends (d_l). The probability that the tree pays high dividends tomorrow given that it has paid high dividends today is denoted by π_h .⁹ The probability that the tree pays high dividends tomorrow given that it has paid low dividends today is denoted by π_l . There is a measure one of agents in the economy. Agents are initially endowed with shares of the tree and receive exogenous income y in every period. A fraction ϕ of the population is endowed in every period with high income y^r . The remaining agents receive low income y^p .¹⁰ The exogenous income is not subject to uncertainty. This can be viewed as an extreme representation of the fact that labor income is less volatile than capital income. Agents trade in

⁷ See Quadrini (2000).

⁸ See Barberis and Thaler (2003).

⁹ It is assumed that the tree pays high dividends in the first period.

¹⁰ In order to assist the reader, the subscript r stands for “rich,” while the subscript p stands for “poor.”

stocks and one-period risk-free bonds. These two assets are enough to support a complete markets allocation.

The economy is inhabited by a measure 1 of infinitely lived agents. Agents have preferences defined over a stream of consumption goods. Preferences can be represented by a time-separable expected utility formulation, namely,

$$U_0 = E \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] = \sum_{t=0}^{\infty} \sum_{z^t \in Z^t} \beta^t Pr(z^t | z_0) u(c_t(z^t)),$$

where Z^t denotes the set of possible dividend realizations from period 0 up to period t , z^t denotes an element of such a set, $c_t(\cdot)$ denotes a consumption rule that determines the consumption level in period t for a given stream of dividend realizations, and $Pr(z^t | z_0)$ denotes the conditional probability of observing stream of dividend realizations z^t , given that the initial realization is z_0 . Trivially, $z_0 \in \{d_l, d_h\}$.

The consumer's objective is to maximize the present value of future utility flows. Let us assume for the moment that the price of a stock is given by the function $p(\mathbf{s})$, and the price of a risk-free bond is given by the function $q(\mathbf{s})$, where \mathbf{s} denotes the aggregate state. In the present framework, the aggregate state is fully specified by the dividend realization and the distribution of wealth. Given that the price functions are time-invariant, the consumer's optimization problem can be expressed using a recursive formulation.

The timing within each period is as follows: at the beginning of the period the aggregate tree pays off and agents receive dividend income. After that, they cash in the bonds and stocks purchased in the previous period and receive the exogenous endowment (labor income). The sum of these three components define the cash-on-hand wealth available for investment and consumption. Agents trade in two markets: the market of risk-free bonds and the market of claims to the tree. At the end of the period, they consume the resources that were not invested in stocks or bonds.

The following Bellman equation captures the individual optimization problem of agent i :

$$V_i(\omega, \mathbf{s}) = \underset{a', b'}{Max} \left\{ u(c) + \beta \sum_{s' \in S'(\mathbf{s})} Pr(s' | \mathbf{s}) V_i(\omega'(s'), s') \right\}, \quad (2)$$

subject to

$$\begin{aligned} p(\mathbf{s}) a' + q(\mathbf{s}) b' + c &= \omega, \\ \omega'(s') &= a' [d(s') + p(s')] + b' + y^i, \text{ and} \\ c &\geq 0. \end{aligned}$$

The agent's type, i , depends on the exogenous endowment the agent receives. This means that $i \in \{r, p\}$. There are two relevant state variables for any given individual: the cash-on-hand wealth available at the beginning of the

period (denoted by ω) and the aggregate state of the economy. The aggregate state determines the current prices and the probability distribution over future prices. The state of the economy, \mathbf{s} , is represented by the vector (ω^r, ω^p, d) . The first two components characterize the distribution of wealth, while the last component captures the current dividend realization. The amount of stocks purchased in the current period is denoted by a' . The amount of bonds purchased in the current period is denoted by b' . The next-period state realization is denoted by \mathbf{s}' . The set of possible aggregate state realizations in the following period is denoted by \mathbf{S}' . The aggregate state realization in the next period may depend on the current aggregate state, \mathbf{s} . The function $d(\mathbf{s})$ represents the mapping from aggregate states into dividend payoffs.

The first-order conditions of agent i are represented by equations (3) and (4).

$$p(\mathbf{s}) = \sum_{\mathbf{s}'} Pr(\mathbf{s}' | \mathbf{s}) m_i(\mathbf{s}, \mathbf{s}') [d(\mathbf{s}') + p(\mathbf{s}')]. \quad (3)$$

$$q(\mathbf{s}) = \sum_{\mathbf{s}'} Pr(\mathbf{s}' | \mathbf{s}) m_i(\mathbf{s}, \mathbf{s}'). \quad (4)$$

These two equations illustrate how the asset pricing equation (1) presented at the beginning of this article can be derived from a consumer's optimization problem. The stochastic discount factor of agent i is now a well-defined function of observables (wealth and income), namely

$$m_i(\mathbf{s}, \mathbf{s}') = \frac{\beta u'(c_i(\mathbf{s}'))}{u'(c_i(\mathbf{s}))},$$

where $c_i(\mathbf{s})$ denotes the optimal consumption of agent i in state \mathbf{s} . In equilibrium, equations (3) and (4) must be satisfied for all agents, which means that any individual stochastic discount factor can be used to characterize the equilibrium prices of stocks and bonds.

A recursive competitive equilibrium consists of a set of policy functions $g_r^a(\omega, \mathbf{s})$, $g_p^b(\omega, \mathbf{s})$, $g_p^a(\omega, \mathbf{s})$, $g_r^b(\omega, \mathbf{s})$, price functions $p(\mathbf{s})$, $q(\mathbf{s})$, and an aggregate law of motion $S'(\mathbf{s})$, such that:

1. The policy functions $g_i^a(\omega, \mathbf{s})$, $g_i^b(\omega, \mathbf{s})$ solve the consumer's problem (2) for $i = r, p$.
2. Markets clear,

$$\begin{aligned} \phi g_r^a(\omega^r, \mathbf{s}) + (1 - \phi) g_p^a(\omega^p, \mathbf{s}) &= 1, \text{ and} \\ \phi g_r^b(\omega^r, \mathbf{s}) + (1 - \phi) g_p^b(\omega^p, \mathbf{s}) &= 0 \end{aligned}$$

for all possible values of ω^r , ω^p , and \mathbf{s} .

Table 1 Parameter Values

d_h	d_l	π_h	π_l	y^r	y^p	ϕ	$a^r_{\text{initial period}}$	b_0	b_1	\hat{c}
1.18	0.82	0.87	0.18	4.0	1.0	0.33	1.5	0.5	0.2	2.5

3. The aggregate law of motion is consistent with individual behavior, that is, $\forall \mathbf{s}' = (\omega^{r'}(d'), \omega^{p'}(d'), d') \in S'(\mathbf{s})$ it is the case that

$$\omega^{r'}(d') = g_r^a(\omega^h, \mathbf{s}) [p(\omega^{r'}(d'), \omega^{p'}(d'), d') + d'] + g_r^b(\omega^r, \mathbf{s}) + y^r,$$

$$\omega^{p'}(d') = g_p^a(\omega^p, \mathbf{s}) [p(\omega^{r'}(d'), \omega^{p'}(d'), d') + d'] + g_p^b(\omega^p, \mathbf{s}) + y^p.$$

The above implies that $Pr(\mathbf{s}' | \mathbf{s}) = Pr(d(\mathbf{s}') | \mathbf{s}) \forall \mathbf{s}' \in S'(\mathbf{s})$.

Notice that given that markets are complete, marginal rates of substitution are equalized across agents, states, and periods. Given the time separability of preferences, the equalization of marginal rates of substitution implies that the sequence of consumption levels of rich (poor) agents only depends on the aggregate dividend realization and not on the time period. This means that the individual wealth of rich (poor) agents only depends on the aggregate dividend realization and not on the time period. This simplifies the dynamics of the model: the economy fluctuates over time across two aggregate states characterized by different dividend realizations and wealth distributions. The Appendix provides a detailed description of how the model is solved.

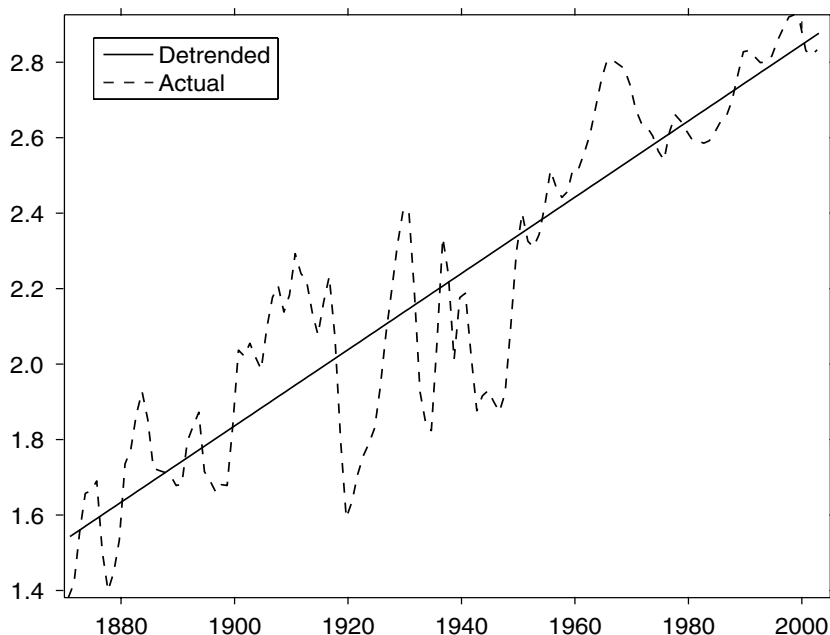
3. CALIBRATION

The baseline parameterization used in this article is described in Table 1. The volatility of dividends is parameterized using the index of real dividends paid by stocks listed in the Standard & Poor's 500 Index.¹¹ First, a linear trend is applied to the logarithm of the series of dividends in order to remove the long-run trend of the series.¹² Second, the exponential function is applied to the filtered series. Figure 1 shows the logarithm of the index of real dividends and its trend. Figure 2 shows the distribution of percentage deviations between the index of real dividends and its trend value. The average deviation over

¹¹ The dividend index can be found in http://www.econ.yale.edu/shiller/data/ie_data.htm. All nominal variables are deflated using the overall Consumer Price Index.

¹² This procedure delivers a smoother trend than what could be found using a Hodrick-Prescott filter with a value of λ equal to 100, which is the value commonly used to detrend annual variables. However, in the present case, a smoothing parameter of 100 implies that a high fraction of the volatility of the detrended series of dividends would be absorbed by movements in the trend, which may underestimate the actual risk perceived by individual investors.

Figure 1 Logarithm of Real Dividends Paid by Stocks in the Standard & Poor's 500 Index

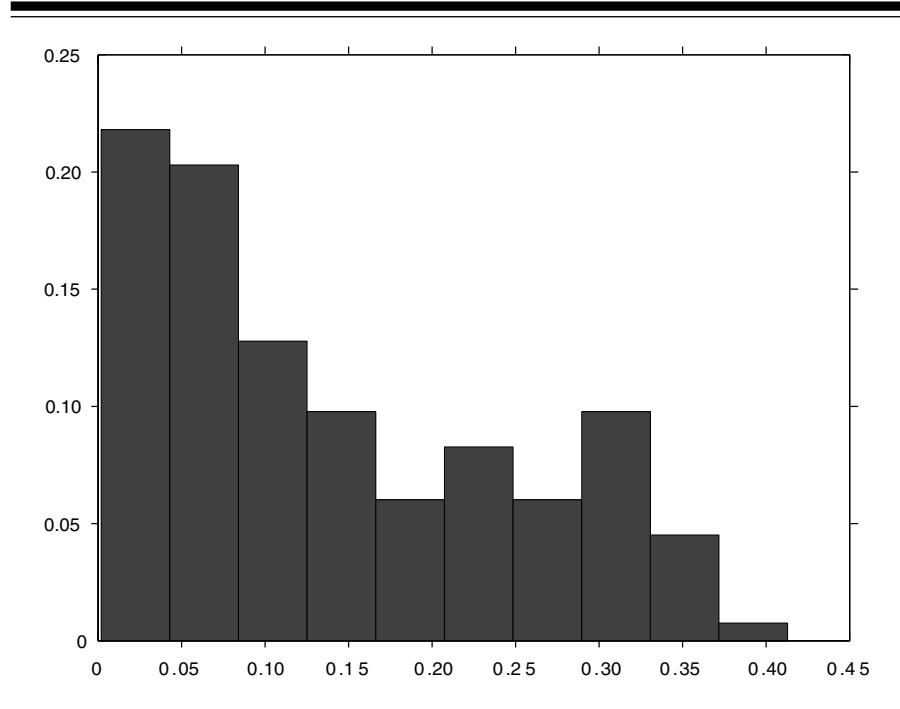


the sample period is 17.6 percent. However, this represents the volatility of a highly diversified portfolio. Several studies document that agents do not diversify as much as standard portfolio theories predict. Thus, the dividend volatility of the stocks actually held by individuals may very well be larger than this measure. The benchmark values of d_h and d_l were chosen to deliver a coefficient of variation of 17.3 percent but we also report results for higher dividend volatility.

In order to estimate the transition probabilities π_l and π_h , the periods with dividends above the trend are denoted as periods of high dividends, and the periods with dividends below the trend are denoted as periods of low dividends. The values of π_h and π_l —the probabilities of observing a period with high dividends following a period with high (low) dividends—were chosen to maximize the likelihood of the stream of high and low dividends observed between 1871 and 2004. A value of $\pi_l = 0.18$ and a value of $\pi_h = 0.87$ are obtained.

Reproducing the degree of inequality is a more difficult job. First, there have been sizable changes in the wealth distribution over the last decades. Second, for the purpose of this article, the relevant measure is the wealth

Figure 2 Histogram of Deviations of Dividends with Respect to Trend Values (in Absolute Value and in Percentage Terms)



inequality among stockholders, which is not readily available. As an approximation, the present calibration focuses only on households that had an income higher than \$50,000 in 1989. Even though this group does not represent the entire population, it represents a large fraction of stockholders.¹³ According to the Survey of Consumer Finances (SCF), 8.6 percent of American families received an annual income higher than \$100,000 in 1989, while the fraction of families receiving an annual income between \$50,000 and \$100,000 in the same year was 22.7 percent.

The first group represents the “rich” agents in the model. The second group represents the “poor” agents in the model. Thus, rich families represent 27 percent of all families with income higher than \$50,000 in 1989. A fraction ϕ equal to 33 percent is used in the article. The exogenous endowment (labor income) received in each period by rich individuals is set equal to 4, while the exogenous endowment of poor individuals is set equal to 1. The initial endowment of stocks of rich individuals is set equal to 1.5, which leaves the

¹³The fact that the characteristics of stockholders may differ from the characteristics of the rest of the population was first pointed out in Mankiw and Zeldes (1991).

poor with an initial endowment of stocks of 0.75. Thus, on average, rich agents receive three times as much income as poor individuals. According to the SCF, the ratio of mean total income between rich and poor was 3.4 in 1989. In addition, the previous parameterization implies a ratio of aggregate “labor income” to capital income (dividends) equal to 2. It is worth stressing that the “poor” in this calibration are not strictly poor. They are intended to represent the set of stockholders who are less affluent. Thus, the previous parameterization yields a distribution of wealth that is less unequal than the overall distribution of wealth.

Finally, the preference parameter b_0 is set equal to 0.5, a_0 is set equal to 0, and b_1 is set equal to 0.2. This implies that a representative agent would display an average coefficient of relative risk aversion of 2.2, which is within the range of values assumed in the literature. The threshold value \hat{c} is set equal to 2.5. This guarantees that the consumption of poor agents always lives in the region with steep ART, and the consumption of rich agents always lives in the region of relatively flat ART.

It should be stressed that the pricing kernel used in the present article is not based on aggregate consumption data. In fact, the consumption process of the two groups considered in the article displays a higher volatility and higher correlation with stock returns than aggregate consumption. The reason for this is twofold. First, there is evidence against perfect risksharing among households.¹⁴ This suggests that using a pricing kernel based on aggregate consumption data can be potentially misleading. Second, as was pointed out by Mankiw and Zeldes (1991), stockholding is not evenly distributed across agents. Guvenen (2006) and Vissing-Jorgensen (2002) provide further evidence that the consumption processes of stockholders and non-stockholders are different. Thus, the pricing kernel of stockholders appear as a more appropriate choice to study asset prices than the pricing kernel implied by the aggregate consumption.

4. RESULTS

The expected return of a tree in state i is denoted by R_i^e , where

$$R_i^e = \pi_i \frac{(p_h + d_h)}{p_i} + (1 - \pi_i) \frac{(p_l + d_l)}{p_i}.$$

The return on a risk-free bond in state i is denoted by R_i^f , where

$$R_i^f = \frac{1}{q_i}.$$

¹⁴ See Cochrane (1991); Attanasio and Davis (1996); Hayashi, Altonji, and Kotlikoff (1996); and Guvenen (2007).

Table 2 Average Returns and Volatility

Variable	Egalitarian Economy	Unequal Economy	Data ¹⁵
Mean Returns on Equity	4.77	4.91	7.86
Mean Risk-Free Rate	3.78	3.67	2.83
Equity Premium	0.96	1.20	4.89
Std. Dev. of Equity Returns	11.43	12.75	14.3
Std. Dev. of Risk-Free Rate	4.23	4.76	5.8

The asset pricing moments are computed using the stationary distribution. In the long run, the probability that the economy is in a state with high dividends is denoted by π , where

$$\pi = \frac{\pi_l}{1 + \pi_l - \pi_h}.$$

The average long-run return on a stock is denoted by R^e . The average long-run return on a bond is denoted by R^f . They are computed as follows:

$$R^e = \pi R_h^e + (1 - \pi) R_l^e, \text{ and}$$

$$R^f = \pi R_h^f + (1 - \pi) R_l^f.$$

Table 2 compares the first two moments of the equilibrium long-run risk-free rate and stock returns in two hypothetical economies. The unequal economy refers to the economy described in Section 2. In the egalitarian economy, however, every agent is initially endowed with the same amount of stocks and receives the same exogenous endowment in every period. The aggregate resources are the same as in the unequal economy.

Table 2 reports that the role of wealth inequality on asset prices is small but non-negligible.¹⁶ The risk-free interest rate in the unequal economy is 11 basis points lower than the risk-free rate in the egalitarian economy. The premium for holding stocks is 24 basis points larger in the unequal economy. As the distribution of wealth becomes more unequal, the gap in the equity premium increases. For example, when each rich agent is initially endowed with 2 stocks, instead of 1.5, the premium for holding stocks is 34 basis points higher in the unequal economy compared to the egalitarian economy.¹⁷

The present model generates a higher equity premium than Mehra and Prescott (1985) for two reasons. First, agents bear more risk by holding

¹⁵ The equity returns correspond to the real returns of the stocks listed in the Standard & Poor's 500 Index. The risk-free interest rate corresponds to one-year Treasury bills. The sample period is 1871–2004.

¹⁶ The actual data reported in Table 2 differ from Mehra and Prescott (1985). See footnote 5.

¹⁷ In this case, the ratio of financial wealth between rich and poor agents is equal to 4. The ratio equals 2 in our benchmark parameterization.

Table 3 Sharpe Ratio and Moments of the Stochastic Discount Factor in the Egalitarian Economy

Aggregate State	Sharpe Ratio	$Corr(m, R^e d_i)$	$E(m d_i)$	$\sigma(m d_i)$
d_h	0.0990	-1	0.099	0.998
d_l	0.0940	-1	0.087	0.919

stocks. The present article features a risky asset that is riskier than the risky asset in Mehra and Prescott (1985). In their model, agents only receive a risky endowment that is calibrated to mimic the behavior of real per capita consumption between 1889 and 1978. In the present setup, the risky endowment mimics the behavior of the dividend process of the stocks contained in the S&P 500 Index, which is more volatile than aggregate consumption. The second reason why the present article delivers a higher equity premium is because stocks provide poor diversification services and, therefore, agents demand a higher premium per unit of risk. This is reflected in a higher Sharpe ratio. The Sharpe ratio—described in equation (5)—measures the excess returns per unit of risk that agents demand for holding stocks. Equation (5) can be obtained from equation (1) after using the property that

$$R_i^f = \frac{1}{E(m | d_i)}.$$

$$\text{Sharpe ratio} = \frac{E(R^e | d_i) - R_i^f}{\sigma(R^e | d_i)} = -\text{Corr}(m, R^e | d_i) \frac{\sigma(m | d_i)}{E(m | d_i)}. \quad (5)$$

Table 3 illustrates the magnitudes of the moments present in equation (5) for the case of the egalitarian economy. The model generates a Sharpe ratio slightly lower than 0.10. This value can be explained by the high negative correlation between the stochastic discount factor and the returns on stocks, and by the relatively high standard deviation of the stochastic discount factor. The perfect negative correlation between the stochastic discount factor and the returns on stocks is due to the assumption of a binary process for dividends.¹⁸

¹⁸ One way to contrast this correlation with the data is to look at the correlation between consumption growth and stock returns. The motivation for this is that when agents display a utility function with a constant coefficient of relative risk aversion, the discount factor has the following form:

$$m(s, s') = \beta \left(\frac{c(s')}{c(s)} \right)^{-\gamma},$$

where γ denotes the coefficient of relative risk aversion. Thus, the stochastic discount factor is inversely proportional to consumption growth. In the present article, the utility function does not display a constant coefficient of relative risk aversion, but there is still a close relationship between consumption growth and the discount factor. In fact, in the present model, the counterpart of a perfect negative correlation between the discount factor and stock returns is a perfect correlation

Table 4 Average Returns and Volatility for the Baseline Parameterization and for a Parameterization with Higher Dispersion of Dividends

	$d_h = 1.18$ and $d_l = 0.82$		$d_h = 1.25$ and $d_l = 0.75$	
	Egalitarian	Unequal	Egalitarian	Unequal
Mean Returns on Equity	4.77	4.91	5.34	5.62
Mean Risk-Free Rate	3.78	3.67	3.41	3.21
Equity Premium	0.96	1.20	1.87	2.34
Std. Dev. of Equity Returns	11.43	12.75	16.20	18.16
Std. Dev. of Risk-Free Rate	4.23	4.76	5.87	6.62

As far as the standard deviation of the stochastic discount factor is concerned, it can be approximated by

$$\sigma(m) \approx \gamma \sigma(\Delta \ln c),$$

where γ stands for the coefficient of relative risk aversion and $\sigma(\Delta \ln c)$ represents the standard deviation of the growth rate in consumption (see footnote 18). In the model, the coefficient of relative risk aversion of the representative agent is above 2.2, while the volatility of the growth rate in consumption is slightly below 0.05. This value is higher than the standard deviation of the growth rate of aggregate consumption (below 2 percent in the postwar years), but it does not differ significantly from the estimates of the standard deviation of consumption growth of stockholders. Mankiw and Zeldes (1991) estimate a standard deviation of consumption growth of U.S. stockholders slightly above 3 percent over the period 1970–1984.¹⁹

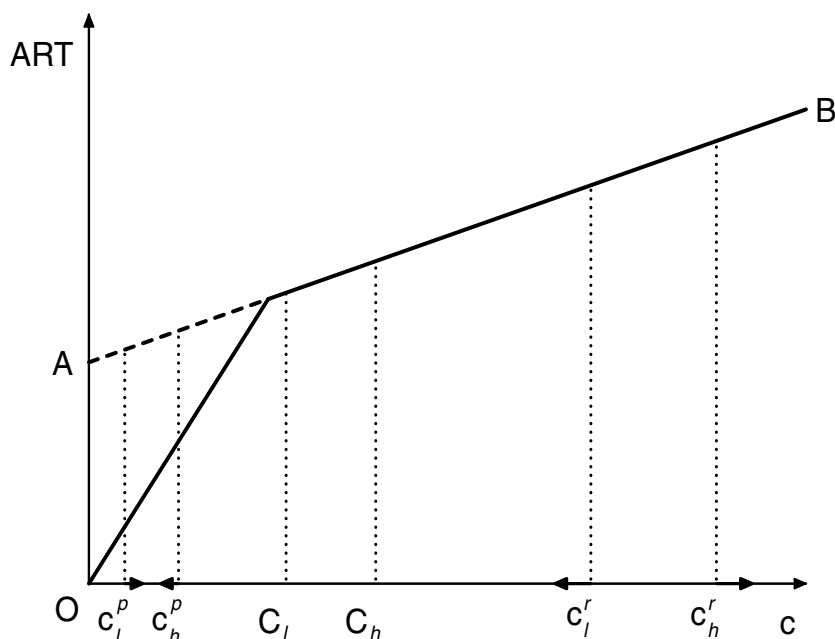
Table 4 shows that as the dispersion of dividends increases to 24 percent, the equity premia in the unequal economy is 47 basis points larger in the unequal economy compared to the egalitarian economy. The risk-free rate is 20 basis points lower in the unequal economy compared to the egalitarian case. A dispersion of dividends of 24 percent is not such a large figure once we internalize the fact that investors do not diversify as much as standard portfolio theories predict.²⁰

between consumption growth and stock returns or excess returns ($R^e - R^f$). Mankiw and Zeldes (1991) find that the correlation between consumption growth and excess returns ranges from 0.26 to 0.4 using aggregate data, and it can be as high as 0.49 when the data refer to the consumption of shareholders.

¹⁹ Attanasio, Banks, and Tanner (2002) find a standard deviation of consumption growth of stockholders ranging from 3.7 to 6.5 percent in the case of the UK.

²⁰ See Ivkovich, Sialm, and Weisbenner (forthcoming).

Figure 3 Effect of a Concave ART on the Marginal Rates of Substitution of Rich and Poor Agents



Notes: C_i denotes average consumption in state i , c_i^r denotes consumption of a rich agent in state i when the ART is represented by AB, and c_i^p denotes consumption of a poor agent in state i when the ART is represented by AB. The arrows illustrate how the consumption of rich and poor agents move when the ART is given by the curve OB, instead of AB.

Interpretation of the Results

Gollier (2001) shows that in an economy with wealth inequality, the ART of the hypothetical representative agent consists of the mean ART of the market participants. Thus, when the ART is concave, Jensen's inequality implies that the ART of a hypothetical representative agent in an economy with wealth inequality is below the ART of the representative agent in an economy with an egalitarian distribution of wealth. In turn, Gollier shows that this implies that the equity premium in an economy with an unequal distribution of wealth is higher than the equity premium in an economy with an egalitarian distribution of wealth. This result holds regardless of whether the ART is increasing or decreasing with consumption. The baseline parameterization used in this article considers the first case, which appears to be in line with the data. It implies that in equilibrium, wealthier agents bear more aggregate risk.

Even though Gollier (2001) relies on a two-period model, the results in this section suggest that his results also hold in an infinite-horizon setup. An intuitive explanation is provided in Figure 3. The graph describes how the consumption of rich and poor agents is affected by the nonlinearity of the ART. The solid line describes the ART. If the ART was linear and represented by the dashed line AB , the economy would behave as if there was a representative agent. In this case, the consumption levels of rich and poor agents in state i would correspond to points like c_i^r and c_i^p , respectively. C_i denotes the average per capita consumption in state i . In equilibrium, the marginal rates of substitution are equalized across agents:

$$\frac{u'(c_i^r)}{u'(c_i^p)} = \frac{u'(C_i)}{u'(C_h)} = \frac{u'(c_i^p)}{u'(c_i^r)}.$$

Poor agents are more risk-averse when the ART is represented by the solid curve OB , instead of AB . This means that at the prices prevailing when the economy behaves as if there was a representative agent, poor individuals are willing to consume less than c_h^p in the high dividend state and more than c_l^p in the low dividend state. Thus, the “new” equilibrium consumption levels of rich and poor agents must move in the direction of the arrows. Notice that the marginal rate of substitution for rich agents ($u'(c_i^r)/u'(c_i^p)$) is higher in the economy with concave ART, compared to the economy with linear ART (curve OB versus curve AB).

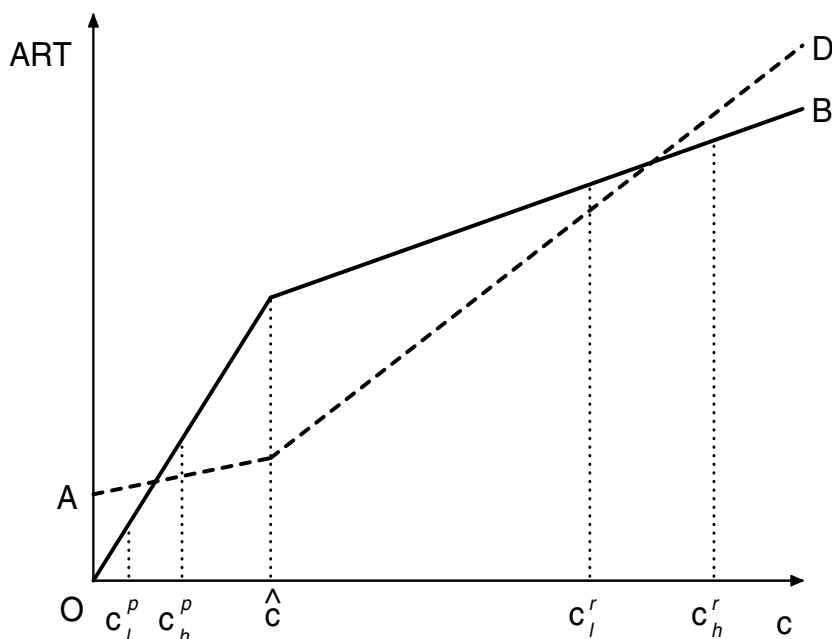
From the perspective of a rich individual, the mean price of stocks must therefore decrease. The reason is that the tree is paying low returns in states that have now become more valuable (low consumption) and high returns in states that have become less valuable (high consumption). Since markets are complete and the marginal rate of substitution are equalized across agents, poor agents agree with their rich counterparts. As a consequence, the average equity premium asked to hold stocks is larger than in the economy with a representative agent.²¹

The Role of the Concavity of the Coefficient of Risk Tolerance

In order to illustrate the role played by the curvature of the ART, this section illustrates how the equity premium changes with alternative parameterizations of the ART. The comparative statics exercise is reduced to alternative parameterizations of b_0 . In order to make the results comparable with the ones

²¹ Note that the ranking of consumption in Figure 3 respects the ranking of consumption given by the baseline parameterization. In particular, the average consumption level is always above the threshold value \hat{c} .

Figure 4 Concave and Convex Specifications of ART with the Same Coefficient of ART for Rich and Poor Agents



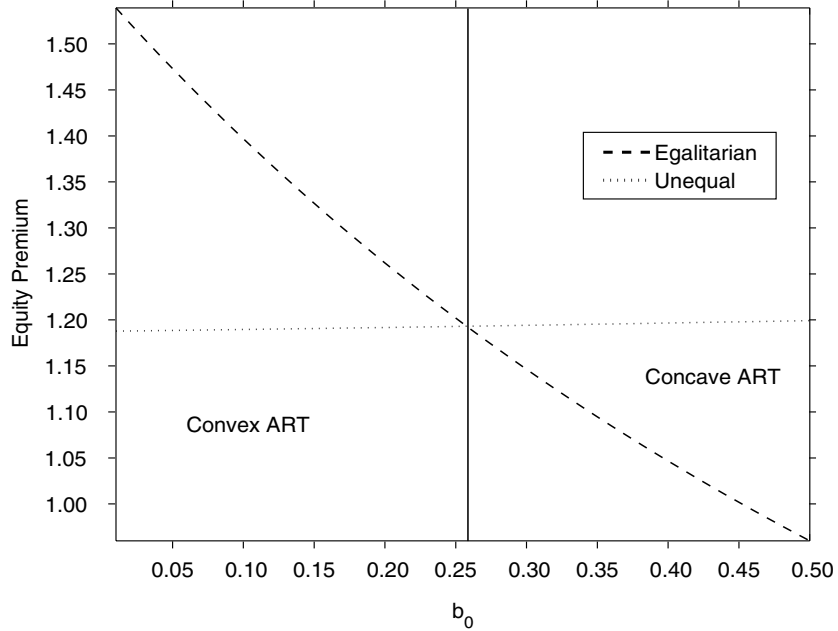
Notes: The solid line OB represents the baseline case with a concave ART. The dashed line AD represents a case with a convex ART. In both cases, the average ART is the same for poor and rich agents.

presented before, this section considers alternative values of b_0 , but restricts the remaining parameters in the utility function change in such a way that, on average, the ART remains constant in the economy with wealth inequality. This is best illustrated in Figure 4. The graph displays two parameterizations of the ART: the solid line OB represents the baseline case with a concave ART. The dashed line AD represents a case with a convex ART. The line AD features a lower slope (lower b_0) on the first segment of the piecewise linear formulation. The remaining coefficients of the line AD are chosen to satisfy the following conditions: average ART is the same for poor and rich agents, and the change in the slope of the ART occurs at \hat{c} .²²

Figure 5 shows that when the ART is convex, the equity premium is larger in an economy with an egalitarian distribution of wealth compared to the

²² Note that the equilibrium allocation of consumption of poor and rich agents in good and bad states does not depend on the shape of the utility function. This is because of the complete markets assumption.

Figure 5 Equity Premium as a Function of b_0



Notes: The ART is concave (convex) for values of b_0 above (below) 0.27. The equity premium is barely affected by b_0 in the economy with wealth inequality due to the fact that the average degree of ART of poor and rich agents is kept constant.

economy with wealth inequality. Conversely, when the ART is concave, the equity premium is lower in the economy with an egalitarian distribution of wealth. These findings are in line with the results of Gollier (2001).

An alternative interpretation of Figure 5 is that if the data are actually generated by the economy with wealth inequality, using a representative agent model would generate biased predictions. A representative agent model—that implicitly assumes that every agent is endowed with the same wealth level—would overestimate the equity premium in the case with convex ART and underestimate the equity premium in the case with concave ART.

5. CONCLUSIONS

The objective of this article is to quantify how robust the asset pricing implications of the standard model are once alternative preference specifications are considered. The exercise is motivated on the grounds that there is no strong evidence in favor of the constant ARA or constant RRA utility representations usually used in the finance and macroeconomic literature. Following Gollier

(2001), the article focuses on a case with a concave ART. In the economy analyzed in this article, the heterogeneity of individual behavior is not washed out in the aggregate. This introduces a role for the wealth distribution in the determination of asset prices. The model is parameterized based on the historic performance of U.S. stocks and approaches the salient features of the wealth and income inequality among stockholders. For the baseline parameterization, the equity premium is 0.24 percent larger in the unequal economy compared to the economy in which the wealth inequality is eliminated. The premium increases if we allow for the fact that agents typically hold portfolios that are more concentrated than the market portfolio. For example, if the stocks display standard deviation of dividends of 25 percent, the increase in the equity premium in the unequal economy increases to slightly less than half a percentage point. This suggests that the role played by the distribution of wealth on asset prices may be non-negligible.

APPENDIX A: DERIVATION OF THE UTILITY FUNCTION

Start from a linear formulation of the ART,

$$-\frac{u'(c)}{u''(c)} = a + bc. \quad (\text{A.1})$$

The above inequality implies that the primitive functions of any transformation of both sides of equation (A.1) must be equalized. In particular,

$$\int \frac{u''(c)}{u'(c)} dc = \int -\frac{1}{a + bc} dc. \quad (\text{A.2})$$

Thus,

$$\ln [u'(c)] = -\frac{1}{b} \ln (a + bc) + C_0, \quad (\text{A.3})$$

where C_0 is a real scalar.

Equation (A.4) is obtained after applying the exponential function to both sides of equation (A.3),

$$u'(c) = e^{C_0} (a + bc)^{-\frac{1}{b}}. \quad (\text{A.4})$$

Finally, equation (A.5) is obtained after integrating both sides of equation (A.4),

$$u(c) = e^{C_0} (a + bc)^{1-\frac{1}{b}} \frac{1}{(1-\frac{1}{b})b} + \tilde{C}_1 = \tilde{C}_0 (a + bc)^{1-\frac{1}{b}} + \tilde{C}_1, \quad (\text{A.5})$$

where \tilde{C}_1 is another real scalar. Equation (A.5) implies that the piecewise linear formulation of the ART considered in this article generates four constants that need to be determined: two constants \tilde{C}_0 and \tilde{C}_1 for each of the two combinations of coefficients (a_i, b_i) . In order to pin down the values of these constants, four restrictions are imposed. In the first section of values of the ART—characterized by the parameters a_0 and b_0 —the constants \tilde{C}_0 and \tilde{C}_1 are chosen so that $u(c)$ and $u'(c)$ are continuous. In the second section of values of the ART—characterized by the parameters a_1 and b_1 —the constants \tilde{C}_0 and \tilde{C}_1 are normalized to take values of 1 and 0, respectively. This normalization does not affect any of the results, given that an expected utility function is unique only up to an affine transformation (see proposition 6.B.2 in Mas-Colell, Whinston, and Green [1995]).

APPENDIX B: SOLVING FOR THE EQUILIBRIUM

The present model features complete markets. A well-known result in this setup is that, in equilibrium, marginal rates of substitution across states and periods are equalized across agents. This implies that

$$\frac{u'(c_h^r)}{u'(c_h^p)} = \frac{u'(c_l^r)}{u'(c_l^p)} = \frac{1 - \lambda}{\lambda}, \quad \text{with } \lambda \in (0, 1), \quad (\text{B.1})$$

where c_i^j denotes the consumption of agent j in a state where the tree pays dividends d_i . The value of λ is determined in equilibrium.

The two equalities in equation (B.1), jointly with the aggregate resource constraints

$$\phi c_r^h + (1 - \phi) c_p^h = d_h + \phi y^r + (1 - \phi) y^p, \text{ and}$$

$$\phi c_r^l + (1 - \phi) c_p^l = d_l + \phi y^r + (1 - \phi) y^p,$$

fully determine the allocation of consumption as a function of λ . In turn, the consumption levels $c_i^j(\lambda)$ can be used to retrieve the market prices implied by λ . Market prices must satisfy equations (B.2)–(B.5), which are derived from the first-order conditions of a rich individual.²³

$$p_h(\lambda) = \beta \left[\pi_h (d_h + p_h(\lambda)) + (1 - \pi_h) \frac{u'(c_l^r(\lambda))}{u'(c_h^r(\lambda))} (p_l(\lambda) + d_l) \right], \quad (\text{B.2})$$

²³ Given that marginal rates of substitution are equalized across agents, the same prices are obtained using the first-order condition of poor individuals.

$$p_l(\lambda) = \beta \left[\pi_l \frac{u'(c_h^r(\lambda))}{u'(c_l^r(\lambda))} (d_h + p_h(\lambda)) + (1 - \pi_l) (p_l(\lambda) + d_l) \right], \quad (\text{B.3})$$

$$q_h(\lambda) = \beta \left[\pi_h + (1 - \pi_h) \frac{u'(c_l^r(\lambda))}{u'(c_h^r(\lambda))} \right], \text{ and} \quad (\text{B.4})$$

$$q_l(\lambda) = \beta \left[\pi_l \frac{u'(c_h^r(\lambda))}{u'(c_l^r(\lambda))} + 1 - \pi_l \right], \quad (\text{B.5})$$

where $p_i(\lambda)$ denotes the price of a share of the tree in a period when the tree has paid dividends d_i , and $q_i(\lambda)$ denotes the price of the risk-free bond in a period when the tree has paid dividends d_i .

Notice that only the aggregate resource constraint has been used until this point. In order to pin down values of λ consistent with the equilibrium allocation, an additional market-clearing condition must be considered. We use the market-clearing condition for stocks. An initial condition is also required. For this reason, it is assumed that the tree pays high dividends in the first period. The results are not sensitive to this. Equations (B.6) and (B.7) define the two initial conditions that the demand for bonds and stocks of agent j ($a_h^j(\lambda)$ and $b_h^j(\lambda)$) must meet,

$$\omega_h^j - p_h(\lambda) a_h^j(\lambda) - q_h(\lambda) b_h^j(\lambda) = c_h^j(\lambda), \text{ and} \quad (\text{B.6})$$

$$y^j + a_h^j(\lambda) (p_h(\lambda) + d_h) + b_h^j(\lambda) = \omega_h^j, \quad (\text{B.7})$$

for $j = r, p$, and initial wealth levels ω_h^r , and ω_h^p . Equation (B.6) states that the investment decisions of an agent of type j must leave $c_h^j(\lambda)$ available for consumption in the first period. The second equation states that the cash-on-hand wealth available at the beginning of the second period in a state in which the tree pays high dividends must equal the initial wealth (recall that the tree pays high dividends in the first period). The logic behind the second condition is the following. Given the stationarity of the consumption and price processes, the discounted value of future consumption flows in the first period is identical to the discounted value of future consumption flows in any period in which the tree pays high dividends. This means that the discounted value of claims to future income must also be equalized across periods with high dividend realizations, which implies that equation (B.7) must hold.

Thus, the value of λ consistent with the equilibrium allocation must satisfy

$$\phi a_h^r(\lambda) + (1 - \phi) a_h^p(\lambda) = 1.$$

Finally, the following equation must also hold:

$$y^j + a_l^j(\lambda) [p_h(\lambda) + d_h] + b_l^j(\lambda) = \omega_h^j, \quad (\text{B.8})$$

for $j = r, p$. The above equality states that if the tree has paid low dividends in the current period, the cash-on-hand wealth available at the beginning of the following period in a state where the tree pays high dividends must be equal to the initial wealth of the agent. Equations (B.7) and (B.8) imply that, in equilibrium, the individual portfolio decisions are independent of the current dividend realization, that is,

$$a'_h{}^j = a'_l{}^j \text{ and } b'_h{}^j = b'_l{}^j \text{ for } j = r, p.$$

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