

Exchange Rate Volatility in a Simple Model of Firm Entry and FDI

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In recent years, the field of international trade has experienced a renaissance in theory and measurement, much of which is rooted in the seminal contribution by Melitz (2003). Melitz's theory of heterogeneous firms and entry has changed not only how the field understands trade flows, but also how it views multinational production. It enables more realistic modeling of multinational firm behavior by capturing the fact that only the largest and most efficient manufacturing firms invest abroad and, most importantly, that they earn positive profits.

In this article, we present and analyze a simple model of firm exit and entry in a Melitz-type environment. We apply the notion of endogenous variation in the entry margin to location decisions by domestic and foreign firms. If a firm wants to supply markets abroad, it has to locate production facilities in the foreign country. We interpret the outcome of this decision as foreign direct investment (FDI). Modeling this location decision thus links the theory of FDI with models of multinational enterprises (MNEs). Moreover, this has implications for the determination of international prices and quantities and related macroeconomic issues.

We want to accomplish two things with this article. First, we derive and explain a full set of analytical solutions for all variables of interest in our theoretical model. This comes at the price of some arguably restrictive assumptions. However, by doing so we can cleanly isolate the entry

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mechanism that is at the core of the model and carries over to the variables in the model. Our contribution thus lies in making this mechanism more transparent compared to richer modeling environments that have to rely on numerical solutions. We therefore see this article as an introductory guide to the mechanics of Melitz-style models of multinational firms.

Second, we use the model to take a look at a perennial issue in international finance, namely the determinants of exchange rate volatility and the apparent disconnect with economic fundamentals. Recent discussions of exchange rate determination have increasingly emphasized the possible role of payments earned on FDI and other assets held abroad. Yet, there are few existing models of MNEs and endogenous exchange rates. This article demonstrates that the entry decisions of MNEs influence the volatility of the real exchange rate in countries where there are significant costs involved in maintaining production facilities, even when prices are perfectly flexible. We show that for plausible parameterizations, MNE activity can make the exchange rate more volatile than relative consumption.

The key element of this framework is that a firm's technology depends both on aggregate and idiosyncratic labor productivity. Given fixed costs of entry, this determines a firm-specific threshold productivity level, below which firms do not operate. This threshold moves around with aggregate economic conditions. Moreover, the model implies an endogenous distribution of firm-level productivities that has strong empirical support (e.g., Helpman, Melitz, and Yeaple 2004). In our model, the threshold or entry margin influences the relative volatility of exchange rates, the aggregate price level, and consumption arising in response to productivity shocks. A positive country-specific productivity shock allows both native and foreign-owned firms with lower firm-specific levels of productivity to become profitable. Lower idiosyncratic labor productivity in these new entrants dampens the impact of the country-specific shock on total aggregate productivity. Thus, a positive productivity shock can impact the real exchange rate at the same time entry by progressively less productive firms dampens the effect of the productivity shock on the aggregate price level and consumption.

In order to highlight the entry channel for exchange rate determination and to derive closed-form solutions, we make two simplifying assumptions. First, we segment markets by allowing no cross-border transfers of wealth via portfolio investment and we shut down any real trade linkages, except for those involving the production and remittance activities of multinational firms. These assumptions leave the nominal exchange rate completely determined by flows of currency used for paying local costs of production incurred by overseas branches of MNEs and for repatriating their profits earned abroad. We show that FDI, even in this model without sunk costs of physical capital, can act as the key driver of real and nominal exchange rate movements.

The second assumption is standard in the Melitz-type literature, namely that participation in any market is a period-by-period decision. This simplifies the model's solution considerably since it eliminates the presence of endogenous state variables in the solution. In addition, it yields testable empirical implications linking a country's industrial structure to the volatility of its exchange rate. We find that the behavior of multinationals is most likely to generate excess volatility when FDI is plentiful in sectors with higher industry concentration, higher value-added, and higher barriers to foreign participation relative to domestic production, so that foreign firms tend to be big relative to domestic firms.

The rest of the article considers the role that MNEs can play in explaining the determinants of exchange rates. We begin by placing our analysis within the broader context of the recent literature. We then introduce a simple, stylized model of multinational production. We emphasize the role of entry in determining the aggregate productivity level and the number of different goods available in the economy. Section 3 contains the main analysis of the model. We discuss intuitively the role that market entry plays in the response to shocks to technology for both nominal and real exchange rates, as well as for consumption and other real quantities. We show analytically how this can be decomposed into direct and indirect effects. We then discuss the implication of our model for the exchange rate disconnect puzzle and the volatility puzzle. The last section concludes.

1. RELATION TO THE LITERATURE

It is well known that the volatility of the exchange rate is much higher than that of other macroeconomic variables, such as the aggregate price level and consumption. This produces a fundamental challenge for optimization-based open economy models that link marginal rates of substitution to international goods prices. For instance, Baxter and Stockman (1989) and Flood and Rose (1995) point out that nominal and real exchange rate volatility is typically 10 times higher than the volatility of relative prices and several times greater than the volatility of output or consumption. As demonstrated by Backus, Kehoe, and Kydland (1992), standard open economy business cycle models have difficulty replicating these stylized facts unless implausible substitution elasticities are assumed. The reason is the tight link between marginal rates of substitution and international relative prices that are at the heart of optimization-based frameworks.

This exchange rate volatility puzzle is related to, in the nomenclature of Rogoff (1996), the exchange rate disconnect puzzle. It stipulates that, empirically, exchange rates appear to behave virtually independently of underlying economic fundamentals. Consequently, the ability of modern open economy macroeconomics to explain exchange rate movements has not been

an unqualified success.¹ In this article, we approach this issue not from the goods side, but rather from a perspective of financial flows generated by the operations of MNEs. This removes the burden of having relative quantities match the volatility of relative prices.² In order to capture in the model the disconnect between relative consumption and international prices, we turn to the literature on MNEs and FDI, which de-emphasizes the role of final consumption in favor of production decisions.

Our model draws its motivation from this growing body of work that stresses the potential role of MNEs as one factor driving exchange rate fluctuations. We add the additional consideration that entry by heterogeneous firms affects fluctuations in prices and consumption, and thus exchange rate volatility. Quantitatively, there are several studies that highlight a causal relationship between FDI and the exchange rate. Kosteletou and Liargovas (2000) provide empirical evidence that inflows of FDI Granger-cause fluctuations in the real exchange rate for some European countries. Whether FDI generates appreciating or depreciating tendencies varies by country, a disparity that the authors explain as emerging from each country's use of the inflows to finance either consumption or capital accumulation. Shrikhande (2002) builds a theoretical model that allows for cross-border acquisitions of physical capital. He is able to replicate the observed persistence and time-varying volatility in the real exchange rate using fixed investment costs, similar to the fixed cost of entry in our model. Gourinchas and Rey (2007) find empirical evidence of a recursive relationship between exchange rates and the return on net foreign asset holdings, including FDI, such as we model here.

Whereas the reduced-form correlation between FDI and exchange rate volatility is well established, the direction of causality is widely debated. Specifically, the literature seeking to measure the effect of exchange rate volatility on FDI is vast and conflicted, which further supports the analysis in this article linking them both as endogenous variables. Phillips and Ahmadi-Esfahani (2008) provide an exhaustive survey of these varied empirical and theoretical results. Several articles have recently analyzed entry and production behavior of heterogeneous multinational firms. Russ (2007, 2011) shows that accounting for the source of exchange rate volatility can determine whether the relationship between volatility and FDI is positive or negative. Fillat and Garetto (2010) find evidence that increased uncertainty of any type in the host country can increase the likelihood that firms will export rather than invest abroad. Ramondo, Rappoport, and Ruhl (2010) obtain the result

¹The seminal article in this literature is Meese and Rogoff (1983). Different perspectives on this issue are given by Clarida and Gali (1994) in a value-at-risk framework, and Lubik and Schorfheide (2005) in an estimated dynamic stochastic general equilibrium model.

²We should point out, however, that we do not speak to the other part of the disconnect puzzle, namely that exchange rates are essentially unpredictable. This issue is left for a much more empirical treatment than the scope of this article allows.

that real exchange rate volatility can be correlated with lower multinational production relative to arms-length exports when real wages and employment are fixed.

There are important conceptual, empirical, and purely practical reasons for modeling multinational firms characterized by heterogeneous productivity levels. First, it is difficult to explain why some firms, but not all, establish branches abroad, unless there exists some differential in their potential to make a profit, as would occur when firms have differing labor productivity. Second, there are several stylized facts regarding the behavior of MNEs that conflict with the representative firm assumption. Using an extensive data set that joins observations on firm size and employment with intra- and inter-firm trade data, Bernard, Jensen, and Schott (2009) show that multinational firms are larger in size and have greater revenues per worker than firms that do not show evidence of having overseas affiliates. Modeling firm-specific labor productivity as Pareto-distributed generates a pattern of firm sizes that is also Pareto, which conforms to empirical findings by Helpman, Melitz, and Yeaple (2004) and di Giovanni, Levchenko, and Ranci ere (2011), among others. These stylized facts of firm size and distribution are captured by the heterogeneous firm framework.

Finally, introducing heterogeneity in the tradition of Melitz (2003) causes the entire solution of the model to rest only on the lowest productivity level among firms producing in a particular period and a set of exogenous parameters. Pinpointing this threshold productivity level using a zero-profit cutoff condition allows the entire model to be solved numerically without linearization and yields analytical results depicting the influence of shocks to a country's general technological state on the nominal and real exchange rate.

The mechanism we identify, namely that aggregate consumption and prices appear to be much less volatile than the exchange rate because their movement in response to a positive country-specific productivity shock can be dampened by the entry of less productive domestic firms, is akin to a new vein of literature on the exchange rate disconnect puzzle emphasizing the role of transaction costs in trade. Fitzgerald (2008) shows both theoretically and empirically that trade costs based on the geographic distance between countries can explain why relative price levels are much less volatile than the real exchange rate, even when prices are perfectly flexible. Our article abstracts from trade in goods, all local consumption being produced by either domestic firms or resident branches of MNEs. It nonetheless approaches the disconnect puzzle in a similar spirit, asking not why nominal and real exchange rates are so volatile, but why they appear so volatile relative to consumption and relative price levels.

The model closest to ours is Cavallari (2007), which demonstrates that in a framework with heterogeneous firms, exchange rate overshooting may be generated by repatriated profits from multinational firms exploiting a positive

productivity shock overseas. Cavallari relies on sticky prices to drive the result. We show, on the other hand, that entry behavior alone can create exchange rate volatility exceeding that of fundamentals, even with flexible prices. As opposed to the model in Ghironi and Melitz (2005), our framework does not involve the sunk costs or incomplete asset markets that generate, respectively, endogenous persistence in exchange rate behavior and a role for active monetary policy in a study of heterogeneous exporters and exchange rates. However, it is rich enough to demonstrate that production decisions by multinational firms can explain part of the differential in the variance of exchange rates and other macroeconomic variables without nominal rigidities.

2. A SIMPLE MODEL OF ENTRY AND FDI

Our model economy consists of two countries, Home and Foreign, that are identical in every respect. Each country is composed of a representative consumer and a continuum of firms. The consumer enjoys the consumption of goods supplied by both Home and Foreign firms, but derives disutility from supplying labor to firms operating in his home country. Home and Foreign firms are distributed along separate unit intervals. What classifies a firm as Foreign is that it pays a fixed overhead cost denominated in the currency of its host country and repatriates (nominal) profits earned at the end of each production period. This creates a necessity for foreign exchange since the firm's owners can only buy goods using their own home currency.

Furthermore, we assume that there is no trade in goods. Foreign firms can supply the domestic market only by opening production facilities there. Consequently, there is no trade balance, only a capital account in the balance of payments. We also abstract from international borrowing and lending. However, consumers can hold financial wealth in the form of currency that is issued by each country's monetary authority. The relative supply of the two currencies is one of the determinants of the nominal exchange rate.

The Consumer's Problem

The representative consumer in the Home country maximizes lifetime utility

$$\max_{\{C_t, L_t, M_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad (1)$$

subject to the budget constraint

$$P_t C_t + M_{t+1} \leq W_t L_t + M_t + \Pi_t + T_t, \quad (2)$$

and the cash-in-advance constraint

$$P_t C_t \leq M_t. \quad (3)$$

C_t is aggregate consumption, L_t is labor input, M_t is the money stock, W_t is the nominal wage, Π_t are firm profits accruing to the household, and T_t are transfer payments from the government; P_t is the aggregate price index, which we define below. The household discounts future utility streams with $0 < \beta < 1$.

We assume that the period utility function is additively separable, $U(C_t, L_t) = \frac{C_t^{1-\rho}-1}{1-\rho} - \chi L_t$, where $\rho > 0$, $\chi > 0$. Furthermore, we define the consumption aggregator as

$$C_t = \left[\int_0^{n_{h,t}} c_{h,t}(i)^{\frac{\theta-1}{\theta}} di + \int_1^{1+n_{f,t}} c_{f,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (4)$$

with $\theta > 1$. The interval $[0, n_{h,t})$ represents the continuum of all goods $c_{h,t}(i)$ that can possibly be produced by Home-owned firms for the Home market, while the interval $[1, 1 + n_{f,t}]$ represents the continuum of all goods that can be produced by Foreign-owned firms, $c_{f,t}(i)$, for the Home market ($n_{h,t}, n_{f,t} \leq 1$). The specification of the sub-utility function C_t encapsulates a preference for variety in that it is increasing in the number of firms $n_{h,t} + n_{f,t}$ supplying the market. Variations in this extensive margin through entry will therefore be another determinant of the exchange rate.

In solving our model, we assume that the cash-in-advance constraint always binds. This determines aggregate consumption as a function of real money balances, $C_t = \frac{M_t}{P_t}$. From the consumption aggregator, we can derive demand equations for individual goods produced by Home and Foreign firms that are downward sloping in relative prices. Homothetic preferences imply that the demand for each good is a constant proportion of aggregate consumption:

$$\begin{aligned} c_{h,t}(i) &= \left(\frac{p_{h,t}(i)}{P_t} \right)^{-\theta} C_t, \\ c_{f,t}(i) &= \left(\frac{p_{f,t}(i)}{P_t} \right)^{-\theta} C_t. \end{aligned} \quad (5)$$

Finally, the optimality condition for total labor input yields a wage equation:

$$W_t = \chi P_t C_t^\rho. \quad (6)$$

The Firm's Problem

In each country, there is a continuum of firms with plans to put their particular invention into production. We denote firms owned by residents of the Home country with the label h , while firms owned by residents of the Foreign country carry f . The location of production is identified by a “*” for Foreign, and no label for Home. Every firm that decides to enter the market

during period t produces a unique good and operates under a unique, firm-specific productivity level, $\varphi(i)$. We assume that idiosyncratic productivity has a continuous distribution $g(\varphi)$, with support over the interval $(0, \infty)$. Any difference among the pricing rules and production decisions of firms operating in the Home country is due only to differences in φ . Thus, φ is used to index each good and the firm that produces it, instead of the general subscript i .

This idiosyncratic component is distinct from an aggregate time-varying disturbance A_t , which denotes the country-specific state of technology available to all firms operating in the Home country. Technology is thus characterized by

$$y_{h,t}(i) = A_t \varphi(i) l_{h,t}(i), \quad (7)$$

where $l_{h,t}(i)$ is the amount of labor used by Home firm i for production in the Home country. The country-specific productivity parameter for the Home country, A_t , is defined by

$$\log A_t = \phi \log A_{t-1} + \varepsilon_{A_t},$$

where $\varepsilon_{A_t} \sim N(0, \sigma_{\varepsilon_A}^2)$.

Home firms operating in the Home country maximize profits subject to consumer demand. They also bear a fixed overhead cost of production, f_h , denominated in units of aggregate output. The profit maximization problem is thus

$$\begin{aligned} \max_{p_{h,t}(\varphi)} \left\{ \pi_{h,t}(\varphi) = p_{h,t}(\varphi) c_{h,t}(\varphi) - W_t \frac{c_{h,t}(\varphi)}{\varphi A_t} - P_t f_h \right\} \\ \text{s.t. } c_{h,t}(\varphi) \leq \left(\frac{p_{h,t}(\varphi)}{P_t} \right)^{-\theta} C_t, \end{aligned} \quad (8)$$

where we have used the market clearing condition $c_{h,t}(\varphi) = y_{h,t}(\varphi)$, and substituted out labor input $l_{h,t}(\varphi)$ with the production function. Assuming an interior solution, that is, where the firm has already entered and operates in the Home market, the first-order condition for profit maximization is then

$$\frac{\partial \pi_{h,t}(\varphi)}{\partial p_{h,t}(\varphi)} : c_{h,t}(\varphi) + \frac{\partial c_{h,t}(\varphi)}{\partial p_{h,t}(\varphi)} p_{h,t}(\varphi) - \frac{\partial c_{h,t}(\varphi)}{\partial p_{h,t}(\varphi)} \frac{W_t}{\varphi A_t} = 0. \quad (9)$$

We can now derive the optimal price-setting condition by substituting the derivative of the demand equation into the firm's first-order condition:

$$p_{h,t}(\varphi) = \frac{\theta}{\theta - 1} \frac{W_t}{\varphi A_t}. \quad (10)$$

As is typical in models with Dixit-Stiglitz-type preferences for variety, firms set prices as a markup over marginal costs. Moreover, for a given wage, higher productivity firms charge a lower price since they have lower marginal costs.

The same steps can be used to derive the pricing equation for Foreign-owned firms operating in the Home country. The profit maximization problem

is

$$\begin{aligned} \max_{p_{f,t}(\varphi)} \left\{ \pi_{f,t}(\varphi) = \left(\frac{1}{S_t} \right) \left[p_{f,t}(\varphi) c_{f,t}(\varphi) - W_t \frac{c_{f,t}(\varphi)}{\varphi A_t} - P_t f_f \right] \right\} \\ \text{s.t. } c_{f,t}(\varphi) \leq \left(\frac{p_{f,t}(\varphi)}{P_t} \right)^{-\theta} C_t, \end{aligned} \quad (11)$$

where S_t is the nominal exchange rate at time t , measured in units of Home currency per unit of Foreign currency. The term f_f denotes the fixed cost paid by Foreign-owned firms operating in the Home country. The fixed cost is denominated in units of the aggregate output of the host country and paid in units of local currency. It can be thought of as an overhead cost, or, more abstractly, as the cost of capital with 100 percent depreciation. The pricing rule for Foreign goods produced and sold in the Home country turns out to be identical, since firms face the same Home-country wage and are influenced by the same country-specific productivity shocks:

$$\frac{\partial \pi_{f,t}(\varphi)}{\partial p_{f,t}(\varphi)} : \left(\frac{1}{S_t} \right) c_{f,t}(\varphi) + \left(\frac{1}{S_t} \right) \frac{\partial c_{f,t}(\varphi)}{\partial p_{f,t}(\varphi)} p_{f,t}(\varphi) - \left(\frac{1}{S_t} \right) \frac{\partial c_{f,t}(\varphi)}{\partial p_{f,t}(\varphi)} \frac{W_t}{\varphi A_t} = 0, \quad (12)$$

from which it follows immediately that

$$p_{f,t}(\varphi) = \frac{\theta}{\theta - 1} \frac{W_t}{\varphi A_t}. \quad (13)$$

More productive firms, that is, those having a high level of labor productivity φ , will charge lower prices, sell more units, and earn higher revenues and profits.

We now define a few more concepts that will prove useful in solving the model. Let $\eta_{h,t}(\varphi)$ and $\eta_{f,t}(\varphi)$ be the distributions of firm-specific productivity levels observed among active Home- and Foreign-owned firms. The aggregate price level P_t , which is the price index that minimizes expenditure on a given quantity of the aggregate consumption index in equation (4) is then given by³

$$P_t = \left[n_{h,t} \int_0^{\infty} p_{h,t}(\varphi)^{1-\theta} \eta_{h,t}(\varphi) d\varphi + n_{f,t} \int_0^{\infty} p_{f,t}(\varphi)^{1-\theta} \eta_{f,t}(\varphi) d\varphi \right]^{\frac{1}{1-\theta}}. \quad (14)$$

Substituting the pricing rules for individual goods, the expression reduces to

$$P_t = \left(\frac{\theta}{\theta - 1} \right) \frac{W_t}{A_t} \left[n_{h,t} \int_0^{\infty} \varphi^{\theta-1} \eta_{h,t}(\varphi) d\varphi + n_{f,t} \int_0^{\infty} \varphi^{\theta-1} \eta_{f,t}(\varphi) d\varphi \right]^{\frac{1}{1-\theta}}. \quad (15)$$

³ See Melitz (2003) and Russ (2007) for a discussion of the computation of the aggregate price level and average firm-specific level of labor productivity.

We now define the average firm-specific productivity level for firms owned by country j as $\bar{\varphi}_{j,t} = \left[\int_0^\infty \varphi^{\theta-1} \eta_{j,t}(\varphi) d\varphi \right]^{\frac{1}{\theta-1}}$. It follows that the production-weighted average firm-specific level of labor productivity $\bar{\varphi}_t$ can be written as

$$\bar{\varphi}_t = \left[\frac{n_{h,t}}{N_t} \bar{\varphi}_{h,t}^{\theta-1} + \frac{n_{f,t}}{N_t} \bar{\varphi}_{f,t}^{\theta-1} \right]^{\frac{1}{\theta-1}}, \quad (16)$$

where $N_t = n_{h,t} + n_{f,t}$ is the composite continuum of goods available in the Home economy, which equals the number of firms. Using these expressions for average firm productivity together with the wage equation (6) and the cash-in-advance constraint in (15), we can finally express the aggregate price level as

$$P_t = \left(\chi \frac{\theta}{\theta-1} \frac{N_t^{\frac{1}{1-\theta}}}{\bar{\varphi}_t A_t} \right)^{\frac{1}{\rho}} M_t. \quad (17)$$

The price level is decreasing in the number of goods available since consumers have a preference for variety, which makes for a more expensive consumption bundle. It is decreasing in aggregate productivity and the index of average idiosyncratic productivities.

The Zero-Profit Cutoff Condition

The production side of the economy is characterized by a continuum of prospective Home and Foreign entrepreneurs distributed, respectively, over $[0, 1)$ and $[1, 2]$, but only firms that can expect to be sufficiently productive to recoup the overhead cost will choose to produce in a particular period. Any firm may enter, depending on whether its total productivity, φA_t , is high enough to result in revenues sufficient to cover this per-period fixed cost.

We now determine the idiosyncratic productivity level that is sufficient for a firm to generate non-negative revenue net of entry costs. We identify the lowest productivity level, $\hat{\varphi}$, that allows a firm to enter into production using the Zero-Profit Cutoff (ZPC) condition. Formally, the ZPCs for Home- and Foreign-owned firms operating in the Home country are given by

$$\pi_{h,t}(\hat{\varphi}_{h,t}) = p_{h,t}(\hat{\varphi}_{h,t})c_{h,t}(\hat{\varphi}_{h,t}) - W_t l_{h,t}(\hat{\varphi}_{h,t}) - P_t f_h \stackrel{!}{=} 0 \quad (18)$$

and

$$\pi_{f,t}(\hat{\varphi}_{f,t}) = \left(\frac{1}{S_t} \right) [p_{f,t}(\hat{\varphi}_{f,t})c_{f,t}(\hat{\varphi}_{f,t}) - W_t l_{f,t}(\hat{\varphi}_{f,t}) - P_t f_f] \stackrel{!}{=} 0, \quad (19)$$

respectively. Analogous expressions apply to entry in the Foreign market.

We substitute the optimal pricing equation, the goods demand function, and the expression for real balances into the respective ZPCs. After straightforward, but tedious algebra, we arrive at the following intermediate expression for the productivity threshold values:

$$\hat{\varphi}_{j,t} = \theta f_j \left(\frac{\theta}{\theta - 1} \frac{\chi}{A_t} \right)^{1/\rho} \left(\bar{\varphi}_t N_t^{\frac{1}{\theta-1}} \right)^{\theta-1-1/\rho}, \quad j = h, f. \quad (20)$$

The threshold values are identical for both Home and Foreign firms except for the differences in the fixed cost of entry. Furthermore, the difference between the thresholds depends only on the ratio of the fixed costs they pay to produce in the Home market:

$$\hat{\varphi}_{f,t} = \left(\frac{f_f}{f_h} \right)^{\frac{1}{(\theta-1)}} \hat{\varphi}_{h,t}. \quad (21)$$

Firms with a higher entry cost need to have higher productivity to stay active. This is a recurring theme in the FDI literature, as there is substantial empirical evidence showing that only the highest-productivity firms engage in foreign direct investment.

We can derive similar expressions from the ZPCs for the Foreign country:

$$\hat{\varphi}_{j,t}^* = \theta f_j^* \left(\frac{\theta}{\theta - 1} \frac{\chi}{A_t^*} \right)^{1/\rho} \left(\bar{\varphi}_t^* N_t^{*\frac{1}{\theta-1}} \right)^{\theta-1-1/\rho}, \quad j = h, f. \quad (22)$$

The structure of the threshold condition is identical to the one for the Home country, but we allow for potentially different entry costs. Moreover, we assume that the fixed cost involved in production abroad is sufficiently large that a firm producing abroad will always produce in its native country as well ($\hat{\varphi}_{f,t}^* \leq \hat{\varphi}_{f,t}$). Thus, our model does not capture issues of geographic preference in firm location.

There are two *ceteris paribus* observations we can make at this stage. First, the threshold is decreasing in aggregate productivity. In a cyclical upswing, firms' idiosyncratic productivity does not have to be as high to generate enough revenue to cover the fixed cost. Second, for large enough values of the substitution elasticity θ , the threshold is increasing in both the average productivity $\bar{\varphi}_t$ and the number of firms operating in the home country N_t . Both effects reflect the influence of competition. The marginal firm operating in the Home country needs to have higher idiosyncratic productivity, both when average firm productivity is higher and when it is competing with a large number of other firms. We should note, however, that these deliberations are partial equilibrium in nature, as an exogenous rise in aggregate productivity presumably increases the number of firms, but lowers average productivity. In order to go much further we now need to make distributional assumptions on the nature of the firm-specific productivity process. Once $g(\varphi)$ is specified, equation (20) is sufficient to pinpoint the minimum level of labor productivity for Home and Foreign firms entering the Home market.

The Number of Firms

As described in Helpman, Melitz, and Yeaple (2004) and Russ (2007), the equilibrium distribution of firm-specific productivity levels for firms owned by country $j \in [h, f]$ is truncated, so that firms with productivity levels too low to earn at least zero profits do not produce in period t . These low-productivity firms are plucked from the formulation of the aggregate price and output levels, leaving a truncated equilibrium distribution:

$$\eta_{j,t}(\varphi) = \left\{ \begin{array}{ll} 0 & \text{for } \varphi < \hat{\varphi}_{j,t} \\ \frac{g(\varphi)}{1-G(\hat{\varphi}_{j,t})} & \text{for } \varphi > \hat{\varphi}_{j,t} \end{array} \right\}. \quad (23)$$

This allows us to determine the number of firms in the economy. Denote $n_{j,t}$ for firms owned by residents of country j who enter the Home market ($j \in [h, f]$). It follows that this is simply the probability that *any* firm holds an idiosyncratic productivity parameter greater than $\hat{\varphi}_{j,t}$. Specifically, $n_{j,t} = 1 - G(\hat{\varphi}_{j,t})$. For instance, as $\hat{\varphi}_{f,t}$ increases, the proportion of Foreign-owned firms entering the Home market falls. Such an increase means that a Foreign firm must have a greater idiosyncratic level of labor productivity to expect to enter without incurring a loss. We can thus write average productivity levels in the Home- and Foreign-owned sector as

$$\bar{\varphi}_{j,t} = \left[\frac{1}{1 - G(\hat{\varphi}_{j,t})} \int_{\hat{\varphi}_{j,t}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi \right]^{\frac{1}{\theta-1}}, \quad j = h, f. \quad (24)$$

Using this expression and the definition of productivity index (16) we find that

$$\bar{\varphi}_t N_t^{\frac{1}{\theta-1}} = \left[\int_{\hat{\varphi}_{h,t}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi + \int_{\hat{\varphi}_{f,t}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi \right]^{\frac{1}{\theta-1}}. \quad (25)$$

We further assume, for purposes of exposition, that idiosyncratic productivity is drawn from a Pareto distribution. The Pareto distribution is used widely in the literature on firm entry and FDI as it describes firm size and rank distribution well. Specifically, the probability and cumulative density functions are given by, respectively, $g(\varphi) = k\varphi^{-(k+1)}$ and $G(\varphi) = 1 - \varphi^{-k}$, with the shape parameter $k > 0$.⁴ This specification now allows us to compute the integrals in the above expression. After several steps, using condition (21), we can solve for the threshold productivity level for Home firms as a function of the exogenous aggregate productivity shock alone:

$$\hat{\varphi}_{h,t} = \psi_0 A_t^{-\frac{\theta-1}{k\rho(\theta-1)+(\theta-1)-k}}, \quad (26)$$

⁴We normalize the location parameter of the distribution to 1, which implies a support of $[1, \infty)$.

where ψ_0 is a constant.⁵ We note that for the underlying Pareto distribution to have bounded variance, we need $k > 2$. Furthermore, for the integral to exist, we have to assume $k > \theta - 1$. We also note that the distribution of firm-specific productivity induces a distribution over the idiosyncratic productivities of active firms in the Home country, which is Pareto itself.

It can be easily verified that the exponent on A_t in equation (26) is positive for all ranges of parameter values for ρ and θ . However, if the Pareto shape parameter k becomes very large relative to the coefficient of relative risk aversion, then firms become less disperse (that is, heterogeneity becomes less important). Moreover, θ should not be too large relative to k since otherwise consumers' love of variety is not strong enough to keep them from buying only the cheapest goods. We thus find that an increase in aggregate productivity leads to a fall in the threshold level of idiosyncratic productivity that firms need to cross in order to cover the fixed costs of operation.

We can now compute the remaining endogenous variables. The total number of varieties equals

$$N_t = n_{h,t} + n_{f,t} = \hat{\varphi}_{h,t}^{-k} + \hat{\varphi}_{f,t}^{-k} = \left[1 + \left(\frac{f_f}{f_h} \right)^{\frac{1}{\theta-1}} \right] \hat{\varphi}_{h,t}^{-k}. \quad (27)$$

An increase in aggregate productivity lowers the threshold for both Home- and Foreign-owned firms, which raises their numbers in the Home economy. It can also be quickly verified that the average firm-specific productivity level is

$$\bar{\varphi}_{j,t} = \left(\frac{k}{k - (\theta - 1)} \right)^{\frac{1}{\theta-1}} \hat{\varphi}_{j,t}, \quad j = h, f.$$

It is proportional and increasing in the threshold level. An increase in $\hat{\varphi}_{j,t}$ reflects the exit of less productive firms, and thus implies that average idiosyncratic productivity rises. Similarly, the measure of aggregate firm productivity,

$$\bar{\varphi}_t = \left[1 + \left(\frac{f_h}{f_f} \right)^{\frac{k-(\theta-1)}{(\theta-1)}} \right]^{\frac{1}{\theta-1}} \left[1 + \left(\frac{f_f}{f_h} \right)^{\frac{1}{\theta-1}} \right]^{\frac{1}{1-\theta}} \left(\frac{k}{k - (\theta - 1)} \right)^{\frac{1}{\theta-1}} \hat{\varphi}_{h,t}, \quad (28)$$

is increasing in the Home- and Foreign-owned average productivities and thus the threshold, $\hat{\varphi}_{h,t}$.

⁵ Specifically,

$$\psi_0 = \left\{ (\theta f_h)^\rho \left(\frac{\theta - 1}{\theta} \right)^\kappa \left[1 + \left(\frac{f_h}{f_f} \right)^{\frac{\kappa - (\theta - 1)}{(\theta - 1)}} \right]^{\frac{\rho(\theta - 1) - 1}{(\theta - 1)}} \left(\frac{k}{k - (\theta - 1)} \right)^{\frac{\rho(\theta - 1) - 1}{(\theta - 1)}} \right\}^{\frac{\theta - 1}{k\rho(\theta - 1) + (\theta - 1) - k}}$$

The Balance of Payments and the Exchange Rate

We now close the model by describing international transactions. There is no international borrowing and lending and domestic agents are restricted to holding only domestic currency. Moreover, there is no international trade in goods. Instead, domestic consumers can satisfy their demand for Foreign products from Foreign firms that have located their production facilities in the Home country. The only exchange across borders is through the repatriation of profits, as we assume that entry costs of Foreign firms have to be paid in terms of the host country's currency.

Equilibrium in the Foreign exchange market requires that the number of units of Home currency being offered for exchange by overseas branches of Foreign multinationals repatriating their profits equal the number of units of Home currency demanded by overseas branches of Home multinationals repatriating their own profits. This condition is the multinational analog to the condition for a world with exporters described in Bacchetta and van Wincoop (2000):⁶

$$S_t n_{h,t}^* \pi_{h,t}^* (\bar{\varphi}_{h,t}^*) = n_{f,t} \pi_{f,t} (\bar{\varphi}_{f,t}). \quad (29)$$

Using the ZPC condition and the solution for the average firm-specific productivity level, we have

$$\begin{aligned} n_{f,t} \pi_{f,t} (\bar{\varphi}_{f,t}) &= n_{f,t} [p_{f,t} (\bar{\varphi}_{f,t}) c_{f,t} (\bar{\varphi}_{f,t}) - W_t l_{f,t} (\bar{\varphi}_{f,t}) - P_t f_f] \\ &= n_{f,t} P_t f_f \left[\left(\frac{k}{k - (\theta - 1)} \right) - 1 \right]. \end{aligned}$$

Applying the same process to the left-hand side of the balance-of-payments equation yields an expression for the nominal exchange rate:⁷

$$S_t = \frac{f_f n_{f,t} P_t}{f_h^* n_{h,t}^* P_t^*}. \quad (30)$$

The exchange rate is determined by three factors: first, the relative size of the entry costs; second, the number of firms operating in the respective foreign markets. This determines the overall volume of capital account transactions. *Ceteris paribus*, if the number of Foreign firms operating in the Home country is relatively large, then their domestic currency denominated profits have to be

⁶ See Russ (2007) for a derivation of the aggregation of profits, which is also described in Melitz (2003). Intuitively, we have to aggregate over all individual Foreign-owned firms operating in the respective host countries. As it turns out, this can be expressed as the product of the profit of the firm with average productivity $\pi_{f,t} (\bar{\varphi}_{f,t})$ and the number of firms $n_{f,t}$. Similar reasoning applies to Home firms operating abroad.

⁷ The expression is considerably simplified by the assumption that both countries are identical except for the exogenous shock processes. We regard this as a clean benchmark and a starting point for further work.

exchanged against a relatively smaller supply of foreign currency denominated profits. Hence, their relative value and thus the price of domestic currency is low (i.e., the exchange rate S_t is high). The third factor are domestic price levels, as in any quantity-theoretic model. What differentiates our framework from a standard exchange rate model is the presence of frictions in the form of entry costs into foreign markets.

We now perform the final steps in deriving an analytical solution for the model. The price level P_t in equation (17) depends on the total number of firms N_t , aggregate firm productivity $\bar{\varphi}_t$, the money supply M_t , and the exogenous shock A_t . We can substitute the reduced-form expressions for the endogenous variables in the price level equation, which yields

$$P_t = \psi_0^{\frac{k-(\theta-1)}{\rho(\theta-1)}} \psi_1 \left(\frac{1}{A_t} \right)^{\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}} M_t, \quad (31)$$

where $\psi_1 = \left(\frac{\kappa\theta}{\theta-1} \right)^{\frac{1}{\rho}} \left[1 + \left(\frac{f_f}{f_h} \right)^{\frac{1}{(\theta-1)}} \right]^{-\frac{1}{\rho(\theta-1)}} \left(\frac{k}{k-(\theta-1)} \right)^{-\frac{1}{\rho(\theta-1)}}$. The nominal price level is increasing in the money stock with unit elasticity, while it is decreasing in the productivity shock. We will make a quantitative assessment of the productivity elasticity below. We also note that the solution for aggregate consumption can be found from this expression, using the cash-in-advance constraint, that is, $C_t = \frac{M_t}{P_t}$.

We assume that both countries are identical with respect to their economic structure, except that they are driven by independent shocks. We can therefore

write the price level ratio $\frac{P_t}{P_t^*} = \left(\frac{A_t^*}{A_t} \right)^{\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}} \frac{M_t}{M_t^*}$. These two expressions reflect the cash-in-advance constraint for money holding, which delivers a quantity-theoretic result, but with a twist. The relative and absolute price level is unit-elastic in money supply, but moves inversely with (relative) productivity. We also find it useful to compute the expression for the relative consumption ratios between the two countries, namely $\frac{C_t}{C_t^*} = \left(\frac{A_t}{A_t^*} \right)^{\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}}$.

We can now define the real exchange rate $RE R_t = S_t \frac{P_t^*}{P_t} = \frac{f_f}{f_h} \frac{n_{f,t}}{n_{h,t}^*}$. In order to provide a closed-form solution, we need to determine the relative number of Foreign firms operating in their respective host countries, $\frac{n_{f,t}}{n_{h,t}^*}$. We note that $n_{f,t} = \hat{\varphi}_{f,t}$, namely the value of the productivity threshold. We can substitute this into the definition of the real exchange rate:

$$RE R_t = \left(\frac{f_f}{f_h} \right) \left(\frac{A_t}{A_t^*} \right)^{\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}}. \quad (32)$$

The real exchange rate depends only on relative productivity levels. Since $\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k} > 0$, an increase in productivity at home increases the real exchange rate and the relative price of the domestic consumption bundle falls. This is the standard supply effect on the real exchange rate, as *ceteris paribus*

the productivity increase leads to higher output, lower prices, and thus a lower price level, which makes Foreign-produced goods more expensive. The elasticity coefficient is the same as the one we identified before in the price level. This shows that real exchange rate movements are driven by real factors. This conjecture is borne out when we compute the nominal exchange rate:

$$S_t = \frac{f_f n_{f,t} P_t}{f_h^* n_{h,t}^* P_t^*} = \frac{f_f M_t}{f_h M_t^*}. \quad (33)$$

In the absence of any nominal friction, there is no effect of the money supply on real variables.

Closing the Model

We now discuss the remaining general equilibrium and aggregation conditions that close the model. Expressions for all reduced-form solutions are listed in Table 1. We first compute the solution for the labor supply. Noting that $c_{h,t}(i) = y_{h,t}(i)$, we can use the firm-specific demand function in equation (5) and the production function in equation (7) to find labor input for firm i :

$$\begin{aligned} l_{h,t}(i) &= \left(\frac{p_{h,t}(i)}{P_t} \right)^{-\theta} \frac{C_t}{A_t \varphi(i)} = \\ &= \left(\frac{\theta - 1}{\theta} \frac{1}{\kappa} \right)^\theta C_t^{1-\rho\theta} A_t^{\theta-1} \varphi(i)^{\theta-1}. \end{aligned} \quad (34)$$

The second line is derived by using the solution for firm i 's optimal price (10) and the wage (6). This relationship applies to all firms making non-negative profits.

We can thus aggregate over all Home firms that operate domestically:

$$\begin{aligned} L_{h,t} &= \int_0^{n_{h,t}} l_{h,t}(i) di = \left(\frac{\theta - 1}{\theta} \frac{1}{\kappa} \right)^\theta C_t^{1-\rho\theta} A_t^{\theta-1} \int_0^{n_{h,t}} \varphi(i)^{\theta-1} di \\ &= \frac{1}{\theta} \left(\frac{\theta - 1}{\theta} \frac{1}{\kappa} \right)^\theta C_t^{1-\rho\theta} A_t^{\theta-1} \hat{\varphi}_{h,t}^{-k(\theta-1)}, \end{aligned} \quad (35)$$

where the last equality uses $n_{h,t} = \hat{\varphi}_{h,t}^{-k}$. We can derive a virtually identical expression for Foreign firms operating in the Home market, whereby we rely on the assumption that they face the same demand schedules and the same labor market. The only difference is that Foreign firms pay a higher fixed cost for entry, which results in a higher productivity threshold.

Table 1 Closed-Form Solutions**Variables**

$\hat{\varphi}_{h,t} = \psi_0 A_t^{-\frac{\theta-1}{k\rho(\theta-1)+(\theta-1)-k}}$	Productivity Threshold Home Firms
$\hat{\varphi}_{f,t} = \left(\frac{f_f}{f_h}\right)^{\frac{1}{\theta-1}} \hat{\varphi}_{h,t}$	Productivity Threshold Foreign Firms
$n_{h,t} = \hat{\varphi}_{h,t}^{-k}$	Number of Home Firms
$n_{f,t} = \hat{\varphi}_{f,t}^{-k}$	Number of Foreign Firms
$N_t = n_{h,t} + n_{f,t}$	Total Number of Firms at Home
$C_t = \psi_0^{-\frac{k-(\theta-1)}{\rho(\theta-1)}} \psi_1^{-1} A_t^{\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}}$	Aggregate Home Consumption
$L_{h,t} = \frac{1}{\theta} \left(\frac{\theta-1}{\theta} \frac{1}{\kappa}\right)^{\theta} C_t^{1-\rho\theta} A_t^{\theta-1} \hat{\varphi}_{h,t}^{-k(\theta-1)}$	Employment at Home Firms
$L_{f,t} = \frac{1}{\theta} \left(\frac{\theta-1}{\theta} \frac{1}{\kappa}\right)^{\theta} C_t^{1-\rho\theta} A_t^{\theta-1} \hat{\varphi}_{f,t}^{-k(\theta-1)}$	Employment at Foreign Firms
$L_t = L_{h,t} + L_{f,t}$	Aggregate Home Employment
$RER_t = \left(\frac{f_f}{f_h}\right) \left(\frac{A_t}{A_t^*}\right)^{\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}}$	Real Exchange Rate
$S_t = \frac{f_f M_t}{f_h M_t^*}$	Nominal Exchange Rate
$P_t = \psi_0^{-\frac{k-(\theta-1)}{\rho(\theta-1)}} \psi_1 \left(\frac{1}{A_t}\right)^{\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}} M_t$	Price Level

Coefficients

$$\psi_0 = \left\{ \left(\theta f_h \right)^{\rho} \left(\frac{\theta-1}{\theta} \right) \kappa \left[1 + \left(\frac{f_h}{f_f} \right)^{\frac{\kappa-(\theta-1)}{(\theta-1)}} \right]^{\frac{\rho(\theta-1)-1}{(\theta-1)}} \left(\frac{k}{k-(\theta-1)} \right)^{\frac{\rho(\theta-1)-1}{(\theta-1)}} \right\}^{\frac{\theta-1}{k\rho(\theta-1)+(\theta-1)-k}}$$

$$\psi_1 = \left(\frac{\kappa\theta}{\theta-1} \right)^{\frac{1}{\rho}} \left[1 + \left(\frac{f_f}{f_h} \right)^{\frac{1}{(\theta-1)}} \right]^{-\frac{1}{\rho(\theta-1)}} \left(\frac{k}{k-(\theta-1)} \right)^{-\frac{1}{\rho(\theta-1)}}$$

Aggregate labor supply is found by aggregating over the individual labor supplies:

$$\begin{aligned} L_t &= L_{h,t} + L_{f,t} = \int_0^{n_{h,t}} l_{h,t}(i) di + \int_0^{n_{f,t}} l_{f,t}(i) di = \\ &= \frac{1}{\theta} \left(\frac{\theta-1}{\theta} \frac{1}{\kappa} \right)^{\theta} C_t^{1-\rho\theta} A_t^{\theta-1} \left(\hat{\varphi}_{h,t}^{-k(\theta-1)} + \hat{\varphi}_{f,t}^{-k(\theta-1)} \right). \end{aligned} \quad (36)$$

This expression isolates the three effects working on labor input. Since consumption and leisure are substitutes, the wage increases in aggregate consumption. Unless $\rho < \frac{1}{\theta}$, which would imply that households are close to being risk-neutral, increases in aggregate consumption, driven by productivity increases, reduce labor. A countervailing effect is coming from labor demand,

whereby productivity shocks directly raise employment. The third element is the entry effect identified earlier. Productivity improvements lower the thresholds for both Home and Foreign firms, entry occurs, and labor demand rises.

The consumption effect is generally not strong enough to overturn the direct productivity effect outside of sticky price models, hence the overall effect of productivity shocks on employment is positive. But this is reinforced through the entry mechanism, which implies that in our Melitz-type framework, labor input is likely to be more volatile than in standard models. The reduced-form expression for L_t is straightforward to compute, but lengthy. We thus only report the elasticity of L_t with respect to aggregate productivity: $(\theta - 1) + (1 - \rho)\theta \frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}$. It is composed of the direct effect from productivity, $(\theta - 1)$; the second terms amalgamate the indirect effects from consumption-leisure substitutability and entry. In the benchmark case of log-utility, $\rho = 1$ and the indirect effects cancel each other out. When agents are less risk-averse, $0 < \rho < 1$, then the indirect effects amplify labor movements, and have a dampening effect otherwise. We will discuss this insight in more detail below.

The remaining reduced-form solutions are now easy to compute. We forgo discussion of these as they simply reiterate the main themes. The expressions are listed in Table 1. Finally, the model is closed by specifying monetary policy. We assume the money supply evolves according to a simple monetary base rule subject to i.i.d. injections,

$$M_{t+1} = M_t + \varepsilon_{M_t}, \quad (37)$$

where $\log \varepsilon_{M_t} \sim N(0, \sigma_{\varepsilon_M}^2)$. Seigniorage revenue is rebated to the household: $T_t = M_{t+1} - M_t = \varepsilon_{M_t}$. This completes the specification of the model.

3. DISCUSSION

The logic behind the model emerges most clearly by considering the effects of a productivity shock. We first note that the model contains no endogenous propagation mechanism. Any persistent effects thus stem entirely from serial correlation in the exogenous disturbances. In other words, there are no intertemporal tradeoffs to consider. However, this allows us to cleanly isolate the entry mechanism at play, which is something that is not easily discernible in richer environments built around the Melitz-framework (e.g., Ghironi and Melitz 2005).

Model Mechanics

Suppose aggregate productivity A_t unexpectedly increases by 1 percent. Because overall productivity, composed of aggregate and firm-specific

productivity, rises, firms can expect to generate higher revenue out of which the fixed cost of entry can be more easily financed. The idiosyncratic productivity threshold thus falls for both Home and Foreign-owned firms and entry occurs. If $f_h < f_f$, relatively more Home firms enter, but the overall number of firms in the economy, N_t , increases. The elasticity of the number of firms with respect to productivity can be found by combining equations (26) and (27). This yields an elasticity coefficient of $\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k} > 0$. As it turns out, this is a key coefficient for the behavior of the model. We will analyze its determinants in more detail in the next section.

The flip side of more firms operating in the economy is that it has adverse effects on several productivity measures. Since there are now more lower productivity firms after the positive aggregate productivity shock, average idiosyncratic productivity for home and foreign firms, $\bar{\varphi}_{j,t}$, $j = h, f$, and for the overall economy, $\bar{\varphi}_t$, falls. Vice versa, a decline in aggregate productivity raises average productivity since firm entry declines relative to its steady state. The model thus captures a cleansing effect of recessions and the observed increase in average firm productivity over the course of a downturn. In a similar vein, this also illustrates how measured total factor productivity can be a misleading indicator for actual firm productivity due to the composition effect caused by entry and exit.

The effect on other real quantities is quickly established. The solution for consumption comes directly from the cash-in-advance constraint. Its responsiveness to productivity is again given by the previous coefficient. An increase in aggregate productivity lowers the aggregate price level in equation (31) with the same elasticity coefficient and raises the real exchange rate. As we pointed out before, there is no effect on the nominal exchange rate since the real exchange rate freely adjusts to equilibrate the balance of payment flows generated by the increased FDI from the low to the high productivity country. More Foreign firms enter the domestic market and produce output, which increases $n_{f,t}$. However, the domestic price level falls due to the supply effect, which lowers the nominal value in domestic currency terms of the Foreign-operated firms. As the expressions for the nominal exchange rate show, see equations (30) and (33), these two effects exactly offset each other. We also want to point out that the model preserves monetary neutrality. Money supply shocks only affect the nominal exchange rate, see equation (33).

Entry and Exchange Rate Volatility

We now use the analytical solutions derived above to study the relationship between the nominal exchange rate, the real exchange rate, and the underlying fundamentals. The first issue we discuss is the relationship between the exchange rates and the fundamental shocks, namely the money supply and productivity processes. The background to this discussion is the so-called

exchange rate disconnect puzzle, which stipulates that, empirically, exchange rates appear to behave independently of underlying economic fundamentals—that they are virtually autonomous processes best captured by a unit root model (see Meese and Rogoff 1983). A corollary of this puzzle is that the behavior of real quantities is well captured by underlying shocks, whereas exchange rates are not.

We first note that the nominal and real exchange rates are driven by different shock processes, that is, the dichotomy in this framework between the effects of real and nominal shocks is preserved. Movements in the real exchange rate are explained by movements in relative productivity levels, see equation (32), with an elasticity coefficient of $\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}$. The properties of the underlying driving processes thus carry over to the exchange rates. High persistence in the latter would therefore have to be generated by a high degree of persistence in productivities. One problematic issue is that the underlying shock processes are generally not observable. Consequently, the literature thus often uses the alternative metric of relative consumption. As the expression $\frac{C_t}{C_t^*} = \left(\frac{A_t}{A_t^*}\right)^{\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k}}$ shows, this is the same as for the real exchange rate up to a scale factor. Real exchange rates thus move one-to-one with relative consumption ratios. In other words, there is no exchange rate disconnect puzzle in this framework. As in the standard literature with trade in goods, movements in relative consumption are closely tied to the real exchange rate. However, we want to point out again that the only cross-country linkage here is via the capital account in terms of repatriated profits. What proxies for the international risk-sharing condition is thus the balance of payments condition.

We now turn to the other dominant issue in the international macro literature, namely the exchange rate volatility puzzle. There are two aspects to this: one, the relative volatilities of nominal and real exchange rates, and two, the relative volatilities of exchange rates and the underlying shocks. We find it convenient to express the moments in terms of natural logarithms:

$$s_t = \text{const.} + m_t - m_t^*,$$

$$\text{rer}_t = \text{const.} + \frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k} (a_t - a_t^*).$$

Assuming independence amongst the exogenous shock processes, we thus find that the volatility of the exchange rates is given by

$$\sigma_s^2 = \sigma_m^2 + \sigma_{m^*}^2,$$

$$\sigma_{\text{rer}}^2 = \left[\frac{k(\theta-1)}{k\rho(\theta-1)+(\theta-1)-k} \right]^2 (\sigma_a^2 + \sigma_{a^*}^2).$$

As we already pointed out in the discussion above, nominal and real exchange rate movements move independently from each other. It follows that the relative volatilities of the exchange rates are essentially arbitrary in this framework and that the model imposes no restrictions on their co-movement. This is the

outcome of the two arguably extreme assumptions: the lack of international trade in goods and assets (besides profit flows) and the identical nature of both countries. Nevertheless, we regard this result as an interesting benchmark for future literature.

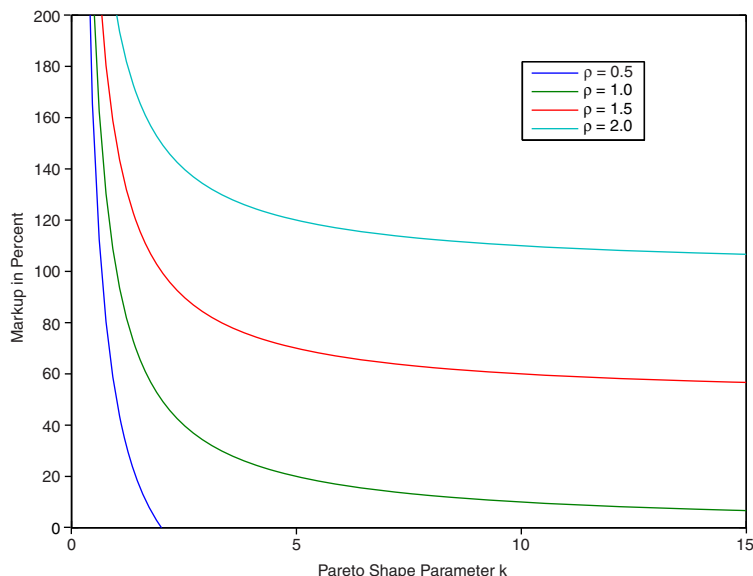
What the model is not silent about, however, is the second aspect of the exchange rate volatility puzzle, namely the degree of amplification of fundamental shocks inherent in the entry mechanism. The key for this is the coefficient:

$$\frac{k(\theta - 1)}{k\rho(\theta - 1) + (\theta - 1) - k} \begin{cases} > 1 & \text{Amplification} \\ = 1 & \text{Equiproportional} \\ < 1 & \text{Dampening} \end{cases} .$$

This translates into the following restriction on the parameters: Shocks are amplified (dampened) through the entry mechanism if $\rho < (>) \frac{\theta}{\theta-1} - \frac{1}{k}$. We thus expect productivity shocks to be amplified (i) when θ is low (and the markup high), (ii) when k is large, and (iii) when the degree of risk aversion is low.

We can assess quantitatively whether these conditions are reasonable. Estimates for the shape parameter k of the Pareto distribution and the substitution elasticity θ run the gamut in the literature. Estimates of the dispersion of firm size from Helpman, Melitz, and Yeaple (2004) suggest a value of $k = 11$. Furthermore, di Giovanni, Levchenko, and Ranci ere (2011) suggest that in any Melitz-model k should be roughly equal to θ . $\theta = 11$ implies a markup of 10 percent, which is an often used value in the macroeconomics literature. On the other hand, Broda and Weinstein (2006) and Feenstra, Obstfeld, and Russ (2011) find values for θ between 2 and 3, which would imply markups between 50 percent and 100 percent.

Figure 1 depicts iso-curves for the equation $\frac{\theta}{\theta-1} = \rho + \frac{1}{k}$, at which there is neither an amplification nor a dampening effect. We report curves for four values of the coefficient of relative risk aversion, $\rho = 0.5, 1.0, 1.5, 2.0$. Areas above and to the right of each curve imply an amplification effect, while below and to the left indicate a dampening effect. It is obvious that an amplification effect generally requires a low degree of risk aversion. This stems from the fact that, with low risk aversion, households willingly substitute into and out of leisure, which implies high labor volatility as we discussed above. At even moderate degrees of risk aversion, for instance, $\rho = 2$, an amplification effect can be ruled out except for implausibly high markups above 100 percent. In a baseline case with log-utility, $\rho = 1$, a value of the shape parameter of $k = 11$, implies a markup of at least 9.1 percent, or $\theta < 12$, for amplification of productivity shocks on real variables; whereas for the alternative case of $\theta = 3$ (and a markup of 50 percent), a value of $k > 2$ would be required. None of these baseline cases appear implausible. In fact, a markup of 10 percent and log-utility is quite standard in the macro literature. However, they are predicated on a narrow range for the risk-aversion parameter. Any

Figure 1 Amplification Iso-Curves

Notes: The graph depicts the iso-curves $\frac{\theta}{\theta-1} = \rho + \frac{1}{k}$ for various values of the coefficient of relative risk aversion ρ . Areas above and to the right of each curve imply an amplification effect, while below and to the left indicate a dampening effect.

amplification that occurs can be sizeable, however. For instance, when $\rho = 1$, $k = 11$, and $\theta = 4$ (implying a markup of 33 percent), the amplification effect is 32 percent. Given the stylized nature of the model, this appears to us as quite large.⁸

Testable Implications

Given the nature of the quantitative exercise above, any potential empirical statements would have to be heavily qualified. However, the analysis yields several interesting testable implications regarding when amplification effects are most likely to be important. First, Figure 1 shows that the lower is the Pareto shape parameter k , the greater is the range of elasticities for which

⁸ We should point out a further caveat to our analysis. The various exchange rate puzzles are typically discussed for high frequency data of a quarter or less. In our framework, the time period is arguably of a much lower frequency since the FDI process of physically locating production abroad takes place on a longer time scale.

amplification effects arise. Thus, we would expect a generally positive causal relationship between multinational firm activity and the relative volatility of the exchange rates for countries with a large degree of multinational activity in industries with a higher dispersion in firm size (that is, a low k) and thus higher industry concentration.

Second, it is apparent that for countries with FDI in manufacturing sectors focused on the production of products with high markups (that is, highly differentiated goods with a low elasticity of substitution), amplification effects are much more likely, even with higher measures of market concentration indicative of low k . Finally, regardless of the size of these parameters, countries and industries with higher fixed costs for multinationals relative to domestic firms (high $\frac{f_f}{f_h}$) will exhibit greater amplification effects. Higher fixed costs may arise due to difficulties connected with obtaining crucial information about the host market, communicating and coordinating with headquarters, or surmounting technological hang-ups. Thus, all else equal, we would expect excess volatility stemming from multinational firm activity to be decreasing in the quality of a country's infrastructure and institutions, and increasing in the level of technological sophistication of its main manufacturing sectors in which FDI plays a key role.⁹

In short, the most promising avenue for the Melitz-type framework we developed to make a contribution to the international trade and macro literature is through an amplification effect of shocks and a variable entry and exit mechanism. The quantitative importance of this mechanism rests on a narrow (though commonly used) set of parameter values within the boundary of what is likely empirically founded. Our quantitative analysis points to three testable implications for researchers seeking to investigate the causal link between multinational activity and excess volatility.

4. CONCLUSION

We build a simple model of market entry with heterogeneous firms and multinational production. We are able to characterize the solutions for all variables analytically, which allows us to identify the key mechanism in the model without having to resort to numerical methods. Fluctuations in the net profits repatriated by multinational firms can generate real and nominal exchange rate volatility. Variability in repatriated profits, since it is entirely dependent upon consumption in our Melitz-type framework with homothetic preferences and constant markups, does not generate a disconnect—variability in the real

⁹We note that when the Pareto shape parameter k is less than 2.5, as is the case in estimates for all industries by di Giovanni, Levchenko, and Ranci ere (2011), the degree of risk aversion is not a prime determinant of whether amplification effects arise due to multinational behavior. Thus, the degree of risk aversion should be of second-order importance in an empirical analysis of the causal effects of FDI on excess volatility.

exchange rate is driven by exactly the same factors and to the same degree as relative consumption. However, there is a potential for disconnect between the real and the nominal exchange rate: the first is driven by productivity shocks and the second by monetary shocks.

In addition, we derive conditions under which the volatility of the real exchange rate can deviate from the volatility of underlying productivity shocks, dampened or amplified by the entry and exit and profit remittances of multinational firms. A reasonable range of parameters can produce either effect. Amplification, that is, excess volatility, emerges under the most commonly used set of parameters, which is remarkable in that it occurs even though prices are fully flexible and markups are constant. In particular, we find that excess volatility in our flexible-price framework is most likely when the distribution of firm size is more fat-tailed, when industries in which FDI is important are highly differentiated with high levels of technical sophistication that generate large coordination costs specific to multinationals, and in countries with low levels of infrastructure and institutional development. In this way, we link a macroeconomic puzzle to the microeconomics of industry structure using the tools from the New Trade Theory.

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