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Characterizing the Unusual Path of U.S. Output During and After the Great Recession

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The growth of the U.S. economy coming out of the 2007–09 Great Recession has been relatively muted when compared to other economic recoveries over the postwar period. Four and a half years into the current recovery, the unemployment rate remains elevated at 6.6 percent, while per capita gross domestic product (GDP) growth has consistently fallen short of its historical average. One interpretation of current economic conditions is that the U.S. economy continues to operate below potential, and that one may soon expect a return to normal conditions driven by increases in cyclical forces like productivity and employment. Another view is that the tepid recovery following the Great Recession has been driven by slower moving forces, and that a notable pick-up in economic activity hinges on variables that tend to change more slowly over time. This article investigates these two perspectives empirically and finds evidence for the latter interpretation.

The focus of the article will be on U.S. per capita GDP, where population is measured as the civilian non-institutional population (i.e., non-military, non-inmates at institutions, 16 years of age and over). As others have noted, the fall in per capita GDP that began in the

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fourth quarter of 2007 was unprecedented in U.S. postwar history.[1] In addition, the higher-than-trend growth rates that typically characterize U.S. economic recoveries were notably absent following the Great Recession: In fact, this was the only recession of the postwar period for which, 16 quarters after its end, per capita GDP had yet to reach its pre-recession peak.

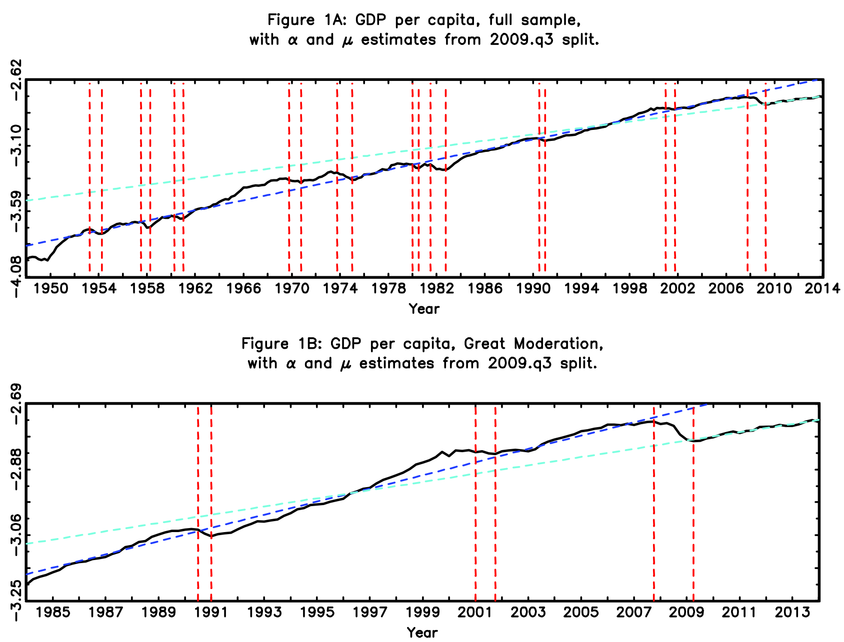
To examine these observations objectively, we first perform some statistical analysis on the per capita GDP time series. Using a range of structural break tests and univariate representations of the process governing U.S. GDP, we present evidence that the Great Recession may have left a scar on the U.S. economy in the form of a long-lasting decline in the level of GDP. Moreover, while we cannot conclusively establish that U.S. per capita GDP growth has shifted to a lower trend, we provide calculations that estimate the likelihood of realized growth rates since the end of the Great Recession to be only 21 percent. To the extent that the Great Recession was driven in part by financial factors, these findings are consistent with work by Reinhart and Rogoff (2014) that highlights the long-lasting effects of financially driven recessions. Finally, we show that unlike every other recession in the postwar period, the fall in and subsequent slow recovery of output during and after the Great Recession cannot easily be explained by shocks typical of the history up to that recession. In this respect, the Great Recession is statistically unique among postwar recessions.

The next part of our analysis focuses on a decomposition of per capita GDP. Since the definition of population used in this article represents the potential workforce of the U.S. economy, our per capita GDP series may be decomposed into the following labor market components: labor productivity, the ratio of employment to the labor force, and the labor force participation rate.[2] The time series behavior of these components can then be further decomposed into different frequencies, highlighting how their contributions to per capita GDP evolve more or less slowly over time. These decompositions lead us to several observations. First, labor productivity and the employment rate tend to move with the business cycle, and although they experienced unusually large negative shocks during the Great Recession, their behavior during and after this recession was not qualitatively different from other postwar recessions in that they soon began to recover. In contrast, the labor force participation rate moves considerably slower over time, and its

[1] For a detailed account that disentangles the various channels underlying the 2007–09 recession, see Stock and Watson (2012).

[2] At times, for convenience given our decomposition, we refer to the ratio of employment to the labor force as the employment rate, although this differs from the more conventional use of the term to denote the ratio of employment to population.

Figure 1 U.S. Per Capita GDP, Logged



behavior during and after the 2007–09 recession differs markedly from that in previous recessions. In this sense, consistent with Stock and Watson (2012), these simple decompositions show that nearly all of the slow recovery in output coming out of the Great Recession stems from a secular decline in the labor force participation rate. Remarkably, in terms of deviations from slow-moving trends, the behavior of per capita GDP and its components in the 2007–09 recession were not unlike that of the other postwar recessions.

This article is organized as follows. Section 1 examines several different univariate characterizations of per capita GDP over the postwar period and conducts a series of exercises that help put the 2007–09 recession and subsequent recovery in the context of previous business cycles. Section 2 decomposes per capita GDP into subcomponents in order to further explore key drivers of its behavior over time. Section 3 concludes.

Table 1 1948:Q1–2013:Q4

	Split Date			
	2008:Q1	2008:Q4	2009:Q2	**2009:Q3**
α Before Split	−3.853	−3.852	−3.850	**−3.849**
α On and After Split	−3.118	-3.455	−3.527	**−3.513**
μ Before Split †	1.904	1.899	1.892	**1.888**
μ On and After Split †	0.558	1.085	1.198	**1.175**
$\chi^2(2)$	*137.32	*180.87	*123.46	***111.04**

Notes: † in annualized growth rates; Critical $\chi^2(2)$ value: 1% 9.21*.

1. UNIVARIATE CHARACTERIZATIONS OF PER CAPITA GDP

Figure 1A illustrates the behavior of the natural logarithm of per capita GDP over the postwar period, from 1948:Q1 to 2014:Q1, where recessions are highlighted by vertical bars. Figure 1B zooms in on the Great Moderation period, 1984:Q1 to 2014:Q1, which we will consider separately since the nature of business cycles appears to be different during this period.[3] The most recent recession clearly stands out as unique in postwar data, both because of the size of the fall in the level of GDP during the recession and because of the tepid growth rate that characterizes the subsequent recovery. We will begin our analysis by using two simple statistical characterizations of the process driving per capita GDP growth to examine the extent to which the recent behavior of per capita output appears unusual in the context of recessions in the postwar era.

Deterministic Trend Model

From looking at Figure 1, a simple linear trend model appears to provide a reasonable first-pass description of the process generating per capita GDP prior to the beginning of the Great Recession in 2007:Q4,

$$y_t = \alpha + \mu t + \varepsilon_t, \tag{1}$$

where y_t denotes the natural logarithm of per capita GDP and ε_t is a mean-zero error term. In Figure 1A, the logarithm of per capita GDP indeed generally appears to have fluctuated around a constant slope over the postwar period. In (1), μ then represents the growth rate of

[3] Aside from changes in volatility of key macroeconomic aggregates, see Gordon (2010) on shifts in various properties of U.S. business cycles over the Great Moderation period.

Table 2 1984:Q1–2013:Q4

	Split Date			
	2008:Q1	2008:Q4	2009:Q2	**2009:Q3**
α Before Split	−3.188	−3.185	−3.182	**−3.180**
α On and After Split	−2.917	−3.066	−3.098	**−3.092**
μ Before Split †	1.977	1.946	1.905	**1.882**
μ On and After Split †	0.558	1.085	1.198	**1.175**
$\chi^2(2)$	*218.21	*219.58	*78.33	***54.84**

Notes: † in annualized growth rates; Critical $\chi^2(2)$ value: 1% 9.21*.

per capita GDP while α captures its log level at some initial date, in this case 1948:Q1.

The dashed lines in Figures 1A and 1B are the best-fit trend lines given by the ordinary least squares (OLS) estimates of α and μ both before and after the end of the Great Recession (2009:Q3). Tables 1 and 2 present findings from standard Chow tests that consider the hypothesis that the Great Recession may have been associated with joint changes in α and μ. Structural break tests for changes in α and μ separately were also carried out. The results (not shown) were similar to those we report in Tables 1 and 2. Table 1 considers the full sample while Table 2 considers only the Great Moderation period. In each table, the Chow tests are carried out using different break dates, from the beginning to the end of the recession as defined by the National Bureau of Economic Research (NBER). The tests allow for autocorrelation and heteroskedasticity in the residuals ε_t and are reported as χ^2 statistics. Regardless of the assumed break date, and over both sample periods, the tests unambiguously reject the null hypothesis of no change in α and μ. Observe that up to a given split date, the growth rate in per capita GDP, μ, averages around 1.9 percent (annualized) but falls considerably lower, to well under 1.2 percent, after the assumed break date.

It is important to note that this same method also suggests structural breaks (at the 1 percent level) for both joint and separate changes in α and μ in more than half of the other postwar recessions. However, in the 2007–09 recession, the p-values for all tests are less than 10^{-7}. Only the 1973 recession matches this level of significance, and, in this case, the change in μ is actually positive. In fact, in all other postwar recessions, either the p-values for the results of the Chow test are several orders of magnitude larger than those associated with the 2007–09 recession, or the change in μ is positive rather than negative. Thus, while Chow-type structural breaks were observed in many of the postwar recessions, the downward shift in μ coupled with extremely small p-values make the structural break of the 2007–09 recession somewhat unique.

Stochastic Trend Model

Findings from the simple structural break tests in the previous subsection rely on (1) representing a reasonable data generating process for per capita GDP. The χ^2 statistics shown in Tables 1 and 2 also rely on derivations that hold asymptotically rather than in finite samples. A popular alternative model of per capita GDP instead characterizes the series as having a stochastic trend,

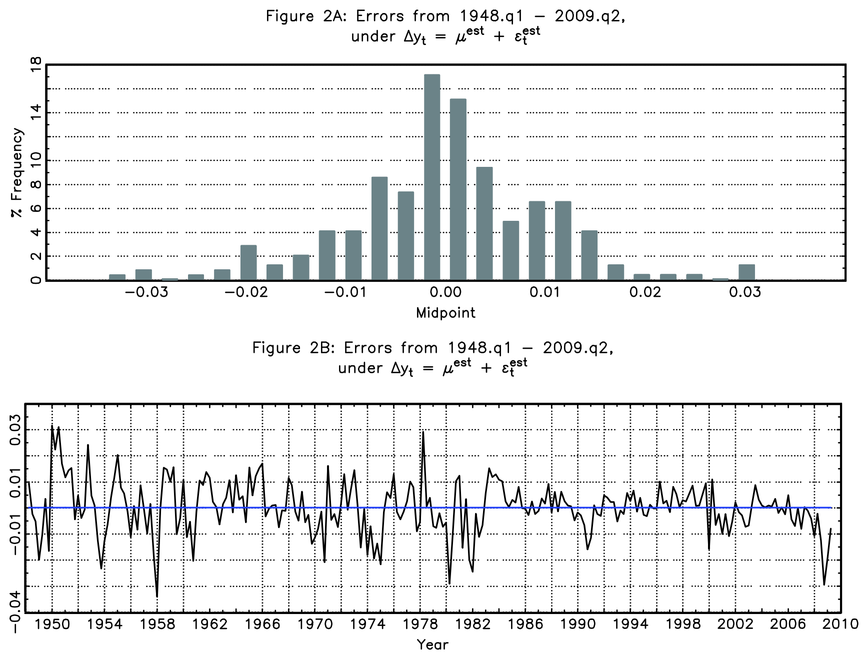
$$y_t = y_{t-1} + \mu + \varepsilon_t. \tag{2}$$

Under this approach, the growth rate of per capita GDP, $y_t - y_{t-1} = \Delta y_t$, is seen as fluctuating around a constant, as described by $\mu + \varepsilon_t$, where ε_t is assumed to be independently and identically (i.i.d.) distributed with mean zero. Importantly, in contrast to equation (1), this stochastic process is such that disturbances, ε_t, have permanent effects on the level of GDP. Nelson and Plosser (1982) argued that many economic series are in fact better described as processes that allow shocks to have permanent effects rather than effects that gradually subside over time. In practice, with finite samples, Stock (1990) and Blough (1992) argue that the question of whether per capita GDP is more accurately characterized as having a deterministic time trend as in (1) or a stochastic trend as in (2) is inherently unanswerable, so that both approaches are worth considering.

Regardless of the assumptions on the data generating process governing per capita GDP, it remains the case that the Great Recession appears unprecedented both in terms of its severity and its slow recovery. To help formalize the notion of the "uniqueness" of the 2007–09 recession, we ask two questions: First, given the set of shocks observed in the postwar period, how likely was the realization of the path characterizing per capita GDP from 2007:Q4 onward? Second, how does this likelihood compare with that of previous recessions in U.S. postwar history? In particular, were recessions preceding the most recent downturn somewhat more plausible considering the history of disturbances incurred up to that recession?

To answer these questions, in contrast to the previous subsection, we explicitly take into account the fact that observations of per capita GDP growth since the 2007–09 recession constitute a finite sample. Thus, let us think of a given date around the start of the Great Recession, denoted date s, from which we are trying to gauge the likely

Figure 2 Residuals from Stochastic Trend Model (2)



path forward for per capita GDP. If date T represents the last date for which we have an observation for per capita GDP, the exercise aims to give us a sense of the likelihood of having observed the realized path $(y_s, y_{s+1}, ..., y_T)$, relative to all other possible paths for per capita GDP, $(y_s^*, y_{s+1}^*, ..., y_T^*)$, given the history of shocks up to date s under the null hypothesis that data is generated by (2). Note that there will be a distribution of paths $(y_s^*, y_{s+1}^*, ..., y_T^*)$, and that the actual observed path $(y_s, y_{s+1}, ..., y_T)$ will generally fall somewhere within that distribution.

To make matters concrete, let s denote 2009:Q3, the start of the recovery. It is then possible to construct estimates of the paths $(y_s^*, y_{s+1}^*, ..., y_T^*)$ by way of bootstrapping, where the observed residuals $(\widehat{\varepsilon}_1, ..., \widehat{\varepsilon}_{s-1})$ from the model (2) are used to represent the unobserved distribution $(\varepsilon_1, ..., \varepsilon_{s-1})$ under the bootstrap procedure. The sample of observed residuals, $\widehat{\varepsilon}_t$, $t = 1, ..., s-1$, is obtained as $\widehat{\varepsilon}_t = \Delta y_t - \widehat{\mu}$, where the OLS estimate $\widehat{\mu}$ is simply the mean of Δy_t. In this case, as indicated in Table 1, $\widehat{\mu}$ is approximately 1.9 percent. Figures 2A and 2B illustrate the properties of the estimated residual, $\widehat{\varepsilon}_t$, from which we are sampling, and which appear close to i.i.d. as assumed. To the extent that some small degree of serial correlation characterizes $\widehat{\varepsilon}_t$, we consider a slightly different variant of (2) later in the article.

The bootstrap algorithm proceeds as follows:

1. Let $(\widehat{\varepsilon}^*_s, \widehat{\varepsilon}^*_{s+1}, ..., \widehat{\varepsilon}^*_T)$ represent a uniformly resampled version of $(\widehat{\varepsilon}_1, ..., \widehat{\varepsilon}_{s-1})$, where $\widehat{\varepsilon}_t = \Delta y_t - \widehat{\mu}$, $t = 1, ..., s-1$, and $\widehat{\mu}$ is treated as true in the bootstrap world.

2. Construct the estimated sample path $(\widehat{y}^*_s, \widehat{y}^*_{s+1}, ..., \widehat{y}^*_T)$ using the stochastic trend model, $\widehat{y}^*_t = \widehat{y}^*_{t-1} + \widehat{\mu} + \widehat{\varepsilon}^*_t$, where the starting value $\widehat{y}^*_{s-1}$ is set to the observed value y_{s-1}.

3. Repeat Steps 1 and 2 many times to obtain a distribution of estimated paths, $(\widehat{y}^*_s, \widehat{y}^*_{s+1}, ..., \widehat{y}^*_T)$.

Figure 3 illustrates examples of four sample paths for $(\widehat{y}^*_s, \widehat{y}^*_{s+1}, .., \widehat{y}^*_T)$, starting in 2009:Q3, generated by drawing disturbances from the period 1948:Q1 to 2009:Q2. Results reported in this section are ultimately based on sample paths calculated from 50,000 Monte Carlo trials. Figure 4A then gives 95 percent confidence intervals for the path of per capita GDP starting in 2009:Q3, given the history of observed shocks and an estimated trend growth rate of roughly 1.9 percent under the null. Two observations are worth noting. First, under the null hypothesis of postwar average trend growth, it is unlikely that today's level of GDP would be back in line with that predicted by the pre-Great Recession trend. This finding holds even when we take into account that, over 50,000 Monte Carlo trials, some sample paths include some of the largest positive shocks to per capita GDP in the postwar period experienced in succession. Second, since 2009:Q3, the observed per capita GDP path has consistently grown below the historical trend growth rate given by the slope of the median (50th percentile) path predicted by the bootstrap simulations.

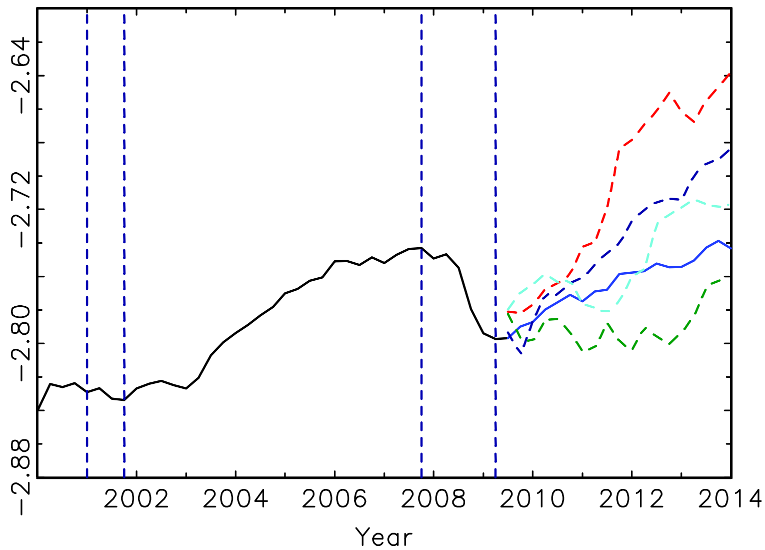
What if we had set s to be 2008:Q1, the first period of decline in the Great Recession? Using (2), we can write per capita GDP at the end of the recession, y_T, as

$$y_T = y_{s-1} + \mu(T - s + 1) + \sum_{j=s}^{T} \varepsilon_j, \qquad (3)$$

so that conditioning on y_{s-1} and μ, y_T is explained by the sequence of shocks $\sum_{j=s}^{T} \varepsilon_j$.

The 95 percent confidence intervals in Figure 4B indicate that the fall in the level of per capita GDP experienced during the Great Recession, together with the subsequent recovery, cannot plausibly be explained by a sequence of bad shocks representative of historical data. As mentioned earlier, recall that the 95 percent confidence intervals illustrated in Figure 4B obtained from a large number of Monte Carlo

Figure 3 Sample Paths Versus Realized GDP Per Capita after 2009:Q3 Split

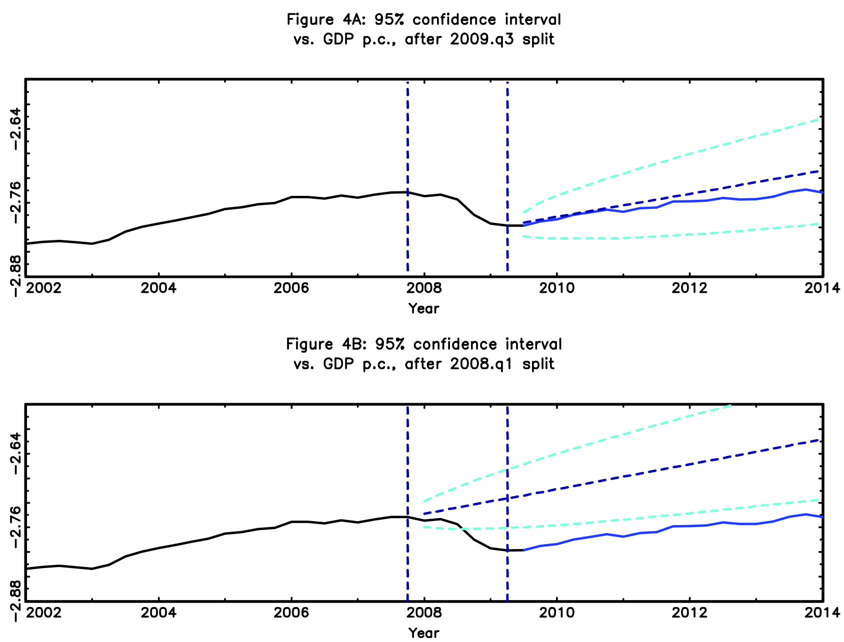


trials contain sample paths that include some of the worst shocks in postwar data experienced in succession.

One way to highlight the sense in which the Great Recession was unique relative to other postwar recessions is to consider previous recessions in the context of the bootstrapping exercise we have just carried out. Thus, Figure 5 illustrates the results obtained from carrying out analogous exercises with respect to the four most recent recessions prior to 2007. On the whole, all previous recessions fall within a 95 percent confidence interval generated by a resampling of shocks up to that recession. Only the 1980–81 recession stands as somewhat of an exception to these findings, but this is only because this recession is followed very soon after by another one, and even in this case, Figure 5 shows that per capita GDP returns to the 95 percent confidence interval as soon as the second recession ends. Statistically, therefore, the Great Recession stands as somewhat unique in the postwar era in that, compared to previous recessions, its severity cannot easily be explained by shocks incurred over the postwar period.

Figure 4 also shows that throughout the recovery period following the 2007–09 recession, per capita GDP has consistently deviated from the median path generated by (2) estimated up to 2007:Q4. Since 2009:Q3, the average per capita GDP growth rate has hovered more

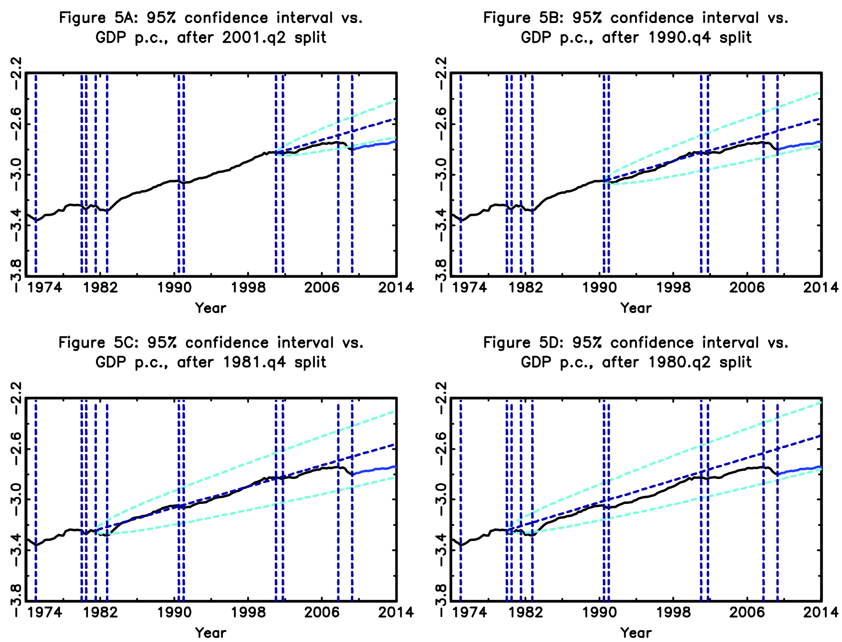
Figure 4 Confidence Intervals from Monte Carlo Trials



than 0.75 percent below the average growth rate prior to the Great Recession. One point of view regarding this is that although GDP continues to evolve below trend, it should be expected to revert back to its historical trajectory at some future date. Another interpretation is that the trend growth rate of GDP has decreased. A test of the latter hypothesis depends on two key considerations: First, the greater the distance between the observed growth rate and the growth rate under the null, the more likely the null will be rejected. In this case, the observed growth rate during the recovery period that started in 2009:Q3 is approximately 1.14 percent while the growth rate under the null was 1.9 percent. Second, the longer the sample period over which the new growth rate is calculated, the more confident we are of its estimate. In the case of the Great Recession, we are roughly 4.75 years into the recovery, or 19 quarters.

As an example, suppose that four quarters have elapsed since the end of the Great Recession, and we now find ourselves in the midst of a weak recovery in 2010:Q3. We want to know whether the observed weakness is enough to reject the null of a growth rate at least as high as 1.9 percent given the stochastic trend model (2) and the history

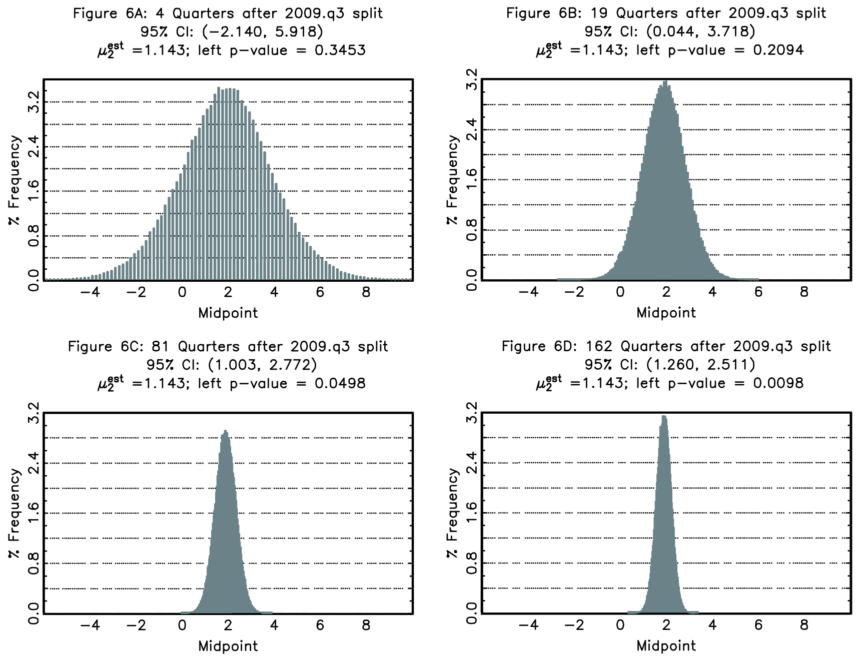
Figure 5 Confidence Intervals for Past Recessions



of observed shocks up to the beginning of the recovery. To address this question, we generate a distribution of estimated growth rates, $\widehat{\mu}^*$, computed from 50,000 Monte Carlo trials of averages over samples of size 4, $(\widehat{y}_s^*, \widehat{y}_{s+1}^*, \widehat{y}_{s+2}^*, \widehat{y}_{s+3}^*)$, generated by the bootstrap algorithm described above with s = 2009:Q3. The resulting distribution is shown in the top left-hand panel of Figure 6. The left p-value associated with a growth rate of 1.14 percent is roughly 35 percent under the null. In other words, our findings indicate a 35 percent probability of experiencing an average growth rate at least as far below the pre-recession growth rate as 1.14 percent over four quarters. Given standard critical values, we cannot reject the null of a growth rate at least as high as 1.9 percent during the recovery. A 95 percent confidence interval in this case ranges from −2.15 percent to 5.91 percent.

That said, it's now been 19 quarters since the end of the recession. Therefore, the top right-hand panel of Figure 6 illustrates the distribution of estimated growth rates analogous to our previous scenario. With more observations over which growth rates are calculated under the null hypothesis, the distribution of $\widehat{\mu}^*$ tightens and the left p-value associated with a 1.14 percent average growth rate falls to 21 percent.

Figure 6 Distributions of Estimated Growth Rates from Monte Carlo Trials



In other words, there is now only a 21 percent chance of observing a growth rate of 1.14 percent or below given historical data. The associated 95 percent confidence interval now shrinks to $(.011, 3.764)$. The bottom two panels in Figure 6 show the distributions, along with the corresponding sample sizes, needed to generate left p-values of 5 percent and 1 percent given a growth rate of 1.14. At the 5 percent critical level, the weak recovery now characterizing the U.S. economy and its disappointing growth rate would have to persist for roughly 20 years before we could unambiguously conclude that we had indeed switched to a new lower trend growth rate.

Initially, it appears that the current weak recovery would have to last for quite a while before we could unambiguously conclude that there has been a change in the trend growth rate. However, the relationship between p-values and sample size is generally convex, which suggests that when the sample size is small, a few more observations can dramatically lower the left p-value of this test. In contrast, the size of the sample under consideration has a relatively small impact when there are many observations. Thus, for example, if the current situation were to extend three and a half more years, there would be only a 15 percent chance of observing such weak circumstances under the null. While not conclusive evidence of a change in trend growth, these calculations nevertheless suggest a relatively low likelihood of having observed the realized path of per capita GDP since 2009:Q3.

So far, we have examined the extreme cases of a pure deterministic trend and a pure stochastic trend model. To the degree that Figure 2B indicates a small degree of serial correlation in the error term of equation (2), a more flexible representation of the data-generating process is given by

$$y_t = y_{t-1} + \rho \Delta y_{t-1} + \mu + \varepsilon_t, \; y_0 \text{ given.} \tag{4}$$

In this case, $\rho \Delta y_{t-1}$ in (4) can be thought of as an error correction term that introduces smoothness in how GDP growth reverts back to trend following a shock, and thus also addresses leftover serial correlation in ε_t in the simpler stochastic trend representation (2). The properties of the estimated errors under this more flexible representation will more closely resemble those of white noise. Repeating the bootstrap exercises described in this section under the more flexible model (4) does not substantively alter our conclusions.

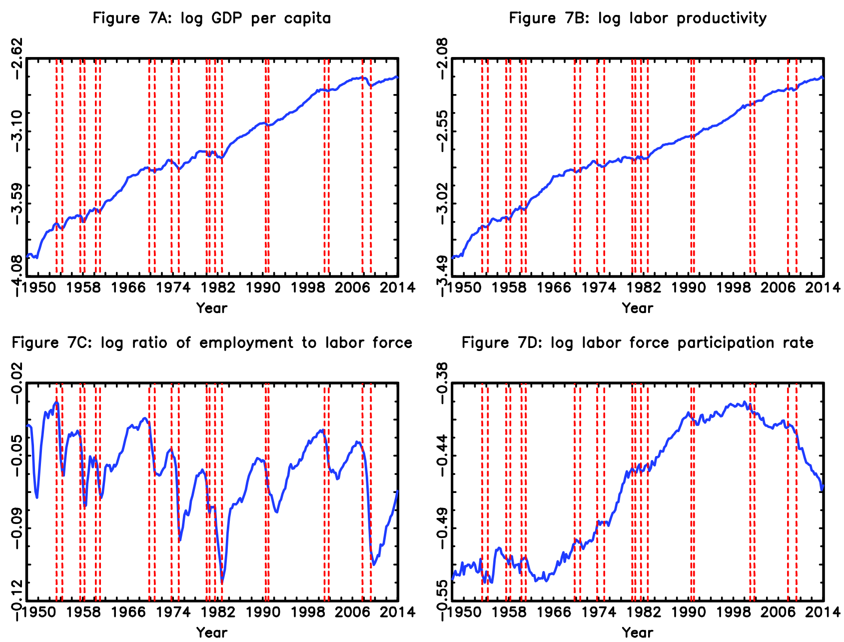
2. DECOMPOSING PER CAPITA GDP

The analysis thus far has provided simple calculations that illustrate how the Great Recession stands as relatively unique in the postwar landscape and suggest that a rapid improvement of the current situation to levels expected from pre-recession trend is questionable. A gradual increase in per capita GDP growth back toward historical trend appears more plausible. However, even in the latter case, every new quarter characterized by below trend growth adds weight to the argument that the U.S. economy has switched to a lower trend growth rate.

To provide further insight into per capita GDP over the postwar period, and in particular its unusual behavior throughout the Great Recession and the subdued recovery that followed, we now decompose per capita GDP into several components and examine the behavior of each of these components individually. Thus, throughout this section, we will work with the following decomposition of per capita GDP:

$$\begin{aligned} \underbrace{y_t - p_t}_{\text{Per Capita GDP}} = & \underbrace{(y_t - e_t)}_{\text{Labor Productivity}} + \underbrace{(e_t - l_t)}_{\text{Ratio of Employment to Labor Force}} \\ & + \underbrace{(l_t - p_t)}_{\text{Labor Force Participation Rate}}, \end{aligned} \tag{5}$$

Figure 7 Decomposition of Per Capita GDP



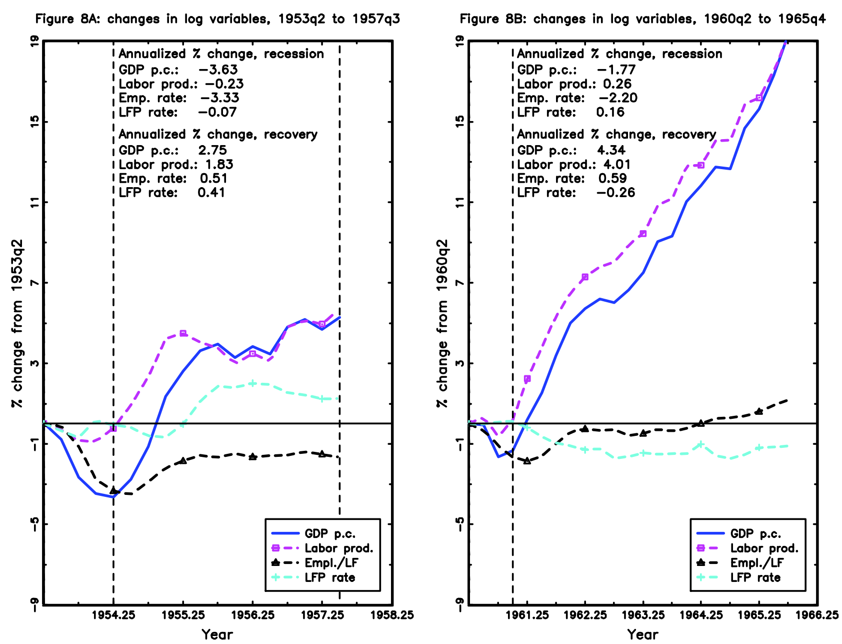
where y_t is real per capita GDP, p_t is the civilian non-institutional population (i.e., non-military, non-inmates at institutions, 16 years of age and over), e_t is employment, and l_t is the labor force, all in logarithm form.[4] We may think of the decomposition in (5) as (roughly) capturing different forces in the economic environment, namely technological considerations that affect primarily labor productivity, demographic and other structural labor market considerations that have a direct bearing on labor force participation, and other labor market factors that affect the unemployment rate.[5] Our objective will be in part to assess how the different components in (5) have contributed to per capita GDP growth during the recessions and recoveries of the postwar period.

In any decomposition of the type in (5), one issue is that the different components making up the series of interest may move at

[4] This decomposition, which lies at the core of our analysis, is a natural one but is by no means the only potentially useful decomposition of GDP. Other non-structural decompositions that can shed insight into the Great Recession might include a breakdown by GDP components in a VAR, a breakdown by regions highlighting the role of housing, or a separation into nominal GDP and inflation.

[5] Note that $e_t - l_t$ is simply one minus the unemployment rate.

Figure 8 Changes in Log Variables, 1953 and 1960 Recessions



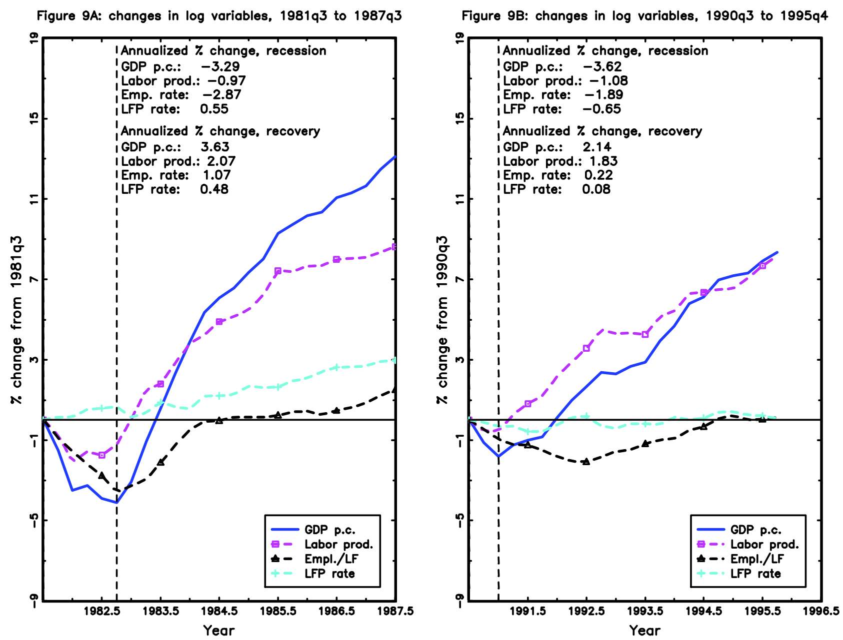
different rates, each potentially having different implications for the series' short- and medium-run forecasts. Thus, let each of the components making up per capita GDP follow a univariate stochastic process, $y_t - e_t = \Theta(L)\varepsilon_{ye,t}$, $e_t - l_t = \Theta(L)\varepsilon_{el,t}$, and $l_t - p_t = \Theta(L)\varepsilon_{lp,t}$, where the ε_ts represent identically and independently distributed disturbances to the individual component series. We then have that

$$y_t - p_t = \Theta(L)\varepsilon_{ye,t} + \Theta(L)\varepsilon_{el,t} + \Theta(L)\varepsilon_{lp,t}, \tag{6}$$

where GDP per capita at any date t reflects the realization of current, and potentially past, disturbances to the individual component series. Suppose now that labor force participation, $\Theta(L)\varepsilon_{lp,t}$, moves relatively slowly over time while the ratio of employment to the labor force, $\Theta(L)\varepsilon_{el,t}$, moves more rapidly. Then a fall in per capita GDP induced by a large shock to labor force participation might imply a relatively slow adjustment back to historical conditions when compared to the case in which the fall in GDP is caused by a shock to the unemployment rate.

Figure 7 illustrates the decomposition of per capita GDP depicted in (5) along with the recession periods indicated by vertical lines. Several observations stand out. First, the slope (or growth rate) of log per

Figure 9 Changes in Log Variables, 1981 and 1990 Recessions

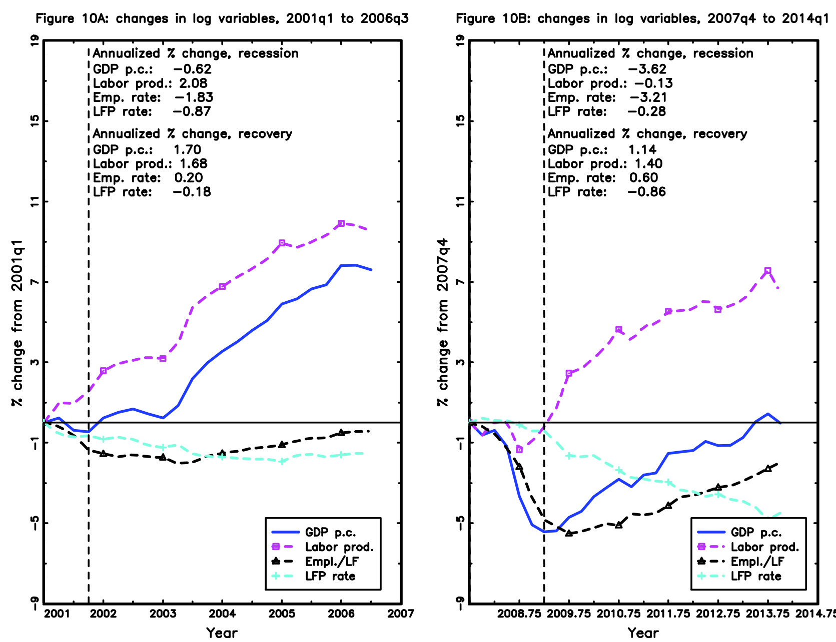


capita GDP generally appears to mimic the slope of log labor productivity. Second, there are nevertheless notable variations in GDP growth over particular periods that are evidently influenced by variations in the unemployment and labor force participation rates. Third, of the latter two variables, the unemployment rate appears to fluctuate with the business cycle, while variations in the labor force participation rate tend to occur more slowly over time.

Taken together, these observations suggest important variations in the way per capita GDP has behaved historically. Thus, in a recent effort to construct long-horizon forecasts of average growth using a univariate framework, Müller and Watson (2013) allow for flexibility in the univariate process governing per capita GDP by allowing the data to be generated by a mix of empirical representations capturing different aspects of its slow moving components. This assumption, in effect, may be thought of as capturing the idea that different components of per capita GDP, which behave noticeably different from each other, play roles of varying importance at different times.

Figures 8 through 10 illustrate the decomposition in (5) during select recessions and recoveries of the U.S. postwar period, using the

Figure 10 Changes in Log Variables, 2001 and 2007 Recessions

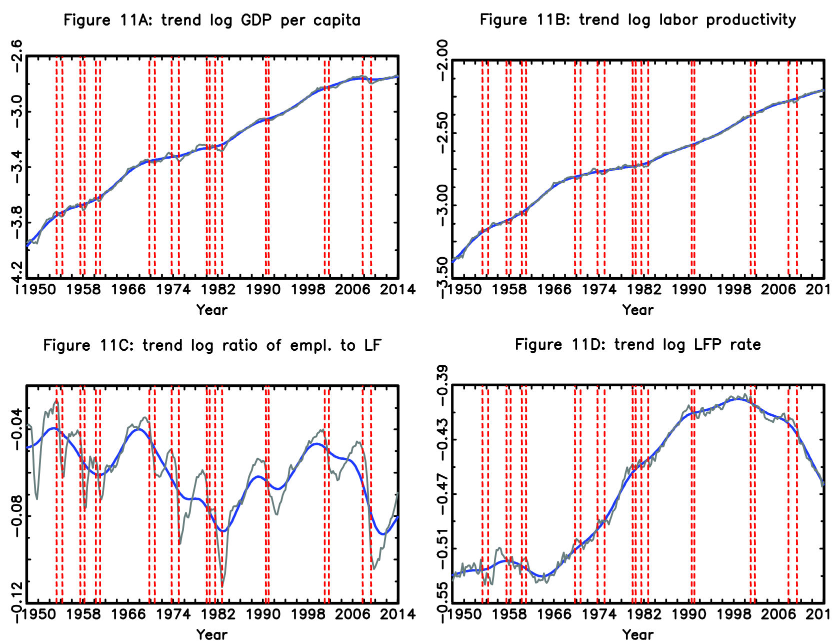


starting quarter of each recession to normalize the component series.[6] On the whole, the fall in per capita GDP during recessions tends to be reflected mostly in a fall in the ratio of employment to the labor force. In contrast, recoveries are generally associated with a pickup in labor productivity. In fact, labor productivity tends not to fall dramatically even during recessions, reflecting the fact that technology is almost always improving. Therefore, the decomposition in (5) reveals that, during most downturns, falling per capita GDP can be accounted for primarily by decreases in $e_t - l_t$ and not the other components.

More recently, however, this pattern has changed. The 2001 and 2007–09 recessions are the only recessions of the postwar period in which the labor force participation rate fell noticeably during both the recessions and subsequent recoveries, dragging down GDP per capita even after the recessions ended. Moreover, the 2007–09 recession and subsequent recovery is the only episode in the postwar period in which, four years after the end of the recession, GDP per capita had yet to

[6] To economize on space, we do not illustrate these decompositions for every postwar recession but the observations we highlight tend to hold across all business cycles.

Figure 11 Trend Components of Per Capita GDP Decomposition



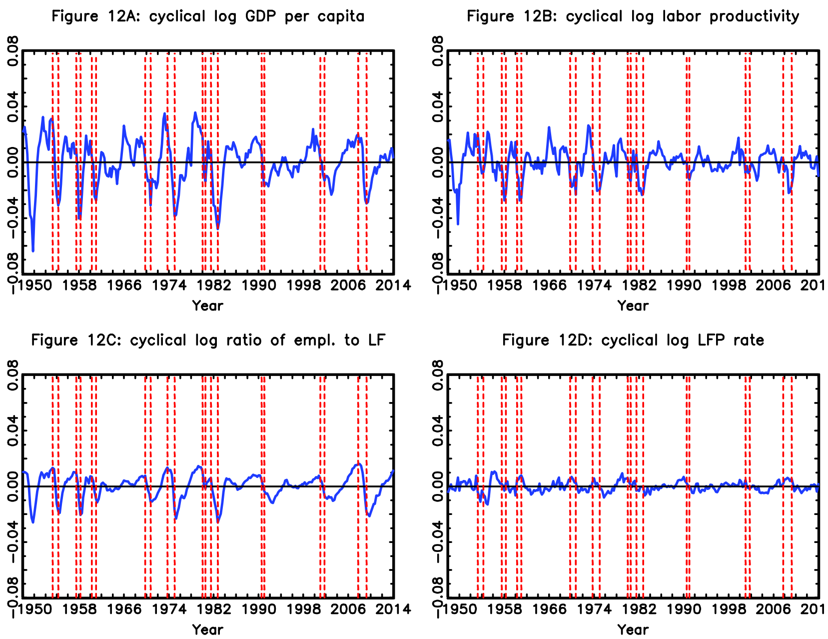
reach its pre-recession peak. However, the behavior of labor productivity in the last two recessions does not differ markedly from the other postwar recessions.

Trends and Cycles

As mentioned earlier, the various components in our decomposition of per capita GDP contribute differently to the aggregate series. Labor productivity, for instance, mostly contributes a steady increase over time, or an upward "trend," to GDP per capita. That said, the term "trend" is somewhat charged and can mean very different things in different contexts.

For the purposes of this article, we will mainly take the approach of thinking in terms of particular frequencies of a series of interest. Following the literature on business cycles and NBER practice, the business cycle component of a series will be defined as the component made up of cyclical frequencies corresponding to periods less than eight years. The remaining slower moving components, made up of cycles

Figure 12 Cyclical Components of Per Capita GDP Decomposition



with periods greater than eight years, may be thought of as one definition of "trend." Since the period, p, of a cycle is given by $\frac{2\pi}{\omega}$, where ω is its frequency, and eight years represents 32 quarters, business cycle frequencies are then given by $\omega \in [\pi/16, \pi]$ when using quarterly data. Conversely, "trend" frequencies are given by $\omega \in [0, \pi/16)$.

Definition 1 *The trend of per capita GDP corresponds to its component cycles with frequencies* $\omega \in [0, \pi/16)$.

The motivation underlying this approach is in part that slower moving cycles are thought to be generally determined by forces outside policymaking, such as ongoing technological progress or changes in demographics. From Figure 7, it is likely the case that the bulk of the contributions of labor productivity to per capita GDP occur at frequencies lower than business cycle frequencies. Contributions of labor productivity to the business cycle component of per capita GDP, relative to those of the other two components, however, may nevertheless be significant.

Balanced Growth

In considering the decomposition (5), it is useful to think about balanced growth implications. In particular, we can think of balanced growth theory as providing long-run relationships that should broadly hold between the variables depicted in (5). Thus, suppose that output, Y_t, is produced by way of the technology

$$Y_t = A_t K_t^{\alpha}(Z_t E_t)^{1-\alpha},\ 0 < \alpha < 1,$$

where A_t denotes multifactor productivity, K_t is the capital stock, E_t is labor input, and Z_t represents a composition effect that increases the productivity of labor. Further, let L_t and P_t denote the labor force and population respectively, and let the growth rate of a given variable, x_t, be given by g_x. Then, along a balanced growth path, where ratios of variables are constant, we have that

$$g_Y = g_A + \alpha g_K + (1-\alpha)(g_Z + g_E).$$

But, along a balanced growth path, $g_Y = g_K$, so the above equation simplifies to

$$g_Y = \left(\frac{1}{1-\alpha}\right) g_A + (g_Z + g_E).$$

In the long run, it must also be the case that

$$g_E = g_P = g_L.$$

From (5), we have that

$$\underbrace{\left(\frac{1}{1-\alpha}\right) g_A + (g_Z + g_E) - g_P}_{\text{Per Capita GDP growth}} = \underbrace{\left[\left(\frac{1}{1-\alpha}\right) g_A + (g_Z + g_E) - g_E\right]}_{\text{Labor Productivity growth}}$$

$$+ \underbrace{(g_E - g_L)}_{\text{Employment Rate growth}}$$

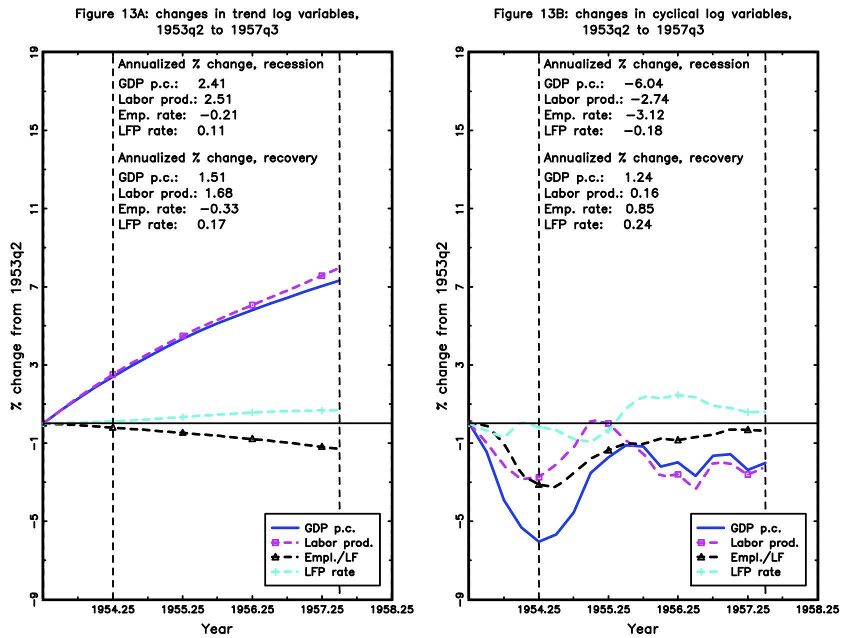
$$+ \underbrace{(g_L - g_P)}_{\text{Labor Force Participation Rate growth}}$$

or, using the balanced growth relationships,

$$\underbrace{\left(\frac{1}{1-\alpha}\right) g_A + g_Z}_{\text{Per Capita GDP growth}} = \underbrace{\left(\frac{1}{1-\alpha}\right) g_A + g_Z}_{\text{Labor Productivity growth}}. \tag{7}$$

Ultimately, therefore, per capita GDP growth follows labor productivity growth, and both are determined by technological parameters.

Figure 13 Changes in Trend and Cyclical Log Variables, 1953 Recession

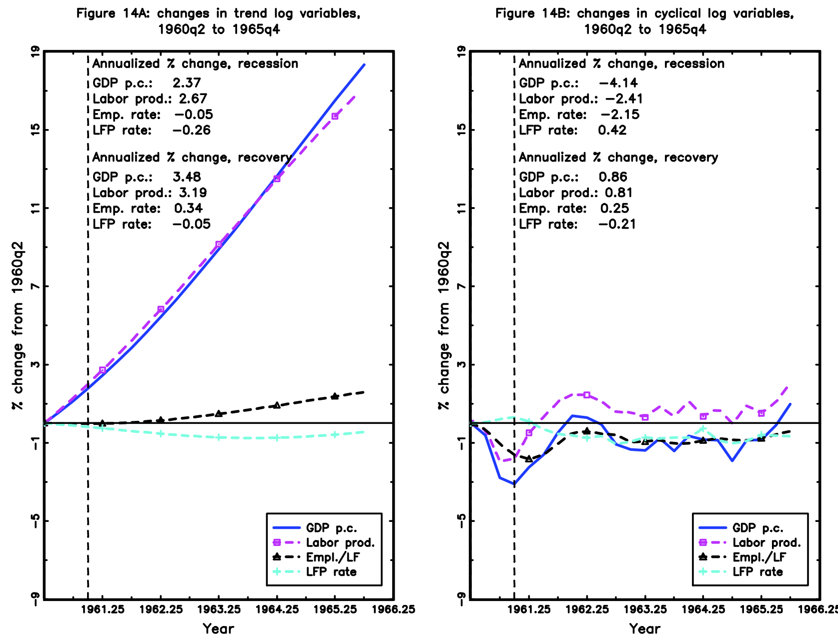


Beyond this observation, it is also important to recognize that balanced growth calculations, where we may think of $\left(\frac{1}{1-\alpha}\right) g_A + g_Z$ as an alternate definition of trend, are only informative in terms of long-run relationships. This represents a single frequency in the frequency domain, frequency zero, among all of the periodic variations that make up per capita GDP. Put another way, the mean growth rate is (in a sense) a single cycle of infinite period among all of the cycles that make up per capita GDP growth.

Definition 2 *The trend of per capita GDP is* $\left(\frac{1}{1-\alpha}\right) g_A + g_Z$.

In practice, we tend to be concerned with more than the long run, and there may be a range of slow-moving variations in per capita GDP outside frequency zero on which policy may nevertheless have very little effect. Demographic changes underlying changes in labor force participation might be an example of such variations. It is in this sense that the definition of trend in terms of frequencies corresponding to periods longer than eight years is potentially useful. In particular, a "gap"

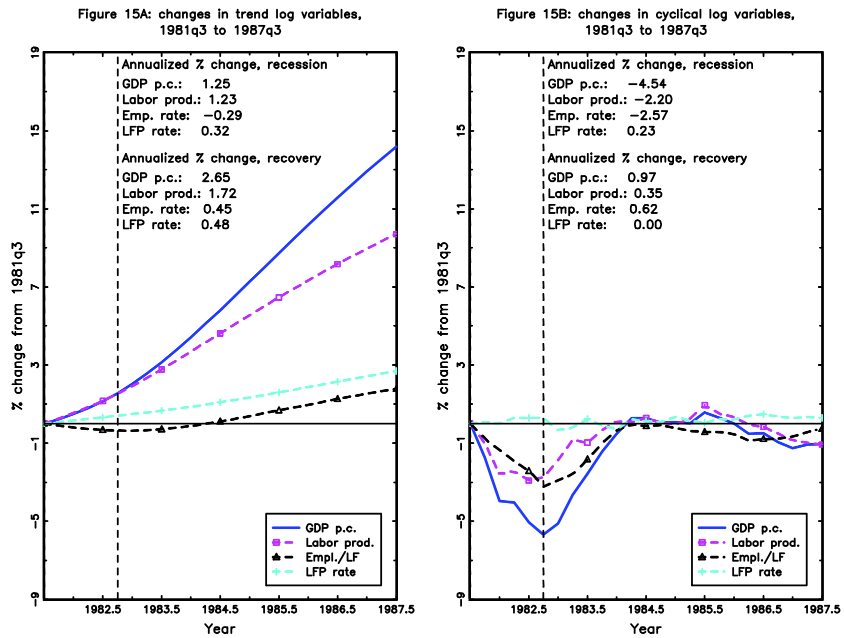
Figure 14 Changes in Trend and Cyclical Log Variables, 1960 Recession



between $y_t - p_t$ and $\alpha + \left[\left(\frac{1}{1-\alpha}\right) g_A + g_Z\right] t$, for some constant α, may be one that is expected to close very slowly or more rapidly depending on the source of the shock and the frequency at which it moves. So, if the labor force participation rate, $l_t - p_t$, experiences a negative shock, we might expect $y_t - p_t$ to fall short of $\alpha + \left[\left(\frac{1}{1-\alpha}\right) g_A + g_Z\right] t$ for a relatively long period, with policy having very little ability to quicken the closing of this gap.

Finally, there is an alternative definition of "gap" that is more model-based, defined as the deviations of sticky price allocations from flexible price allocations in a setting with nominal rigidities. To work with this definition, one must take a stance on the degree of price stickiness and the nature of the shocks affecting the economy at a given time. Comparisons with this more formal notion of trend, while important, are beyond the scope of this article.

Figure 15 Changes in Trend and Cyclical Log Variables, 1981 Recession

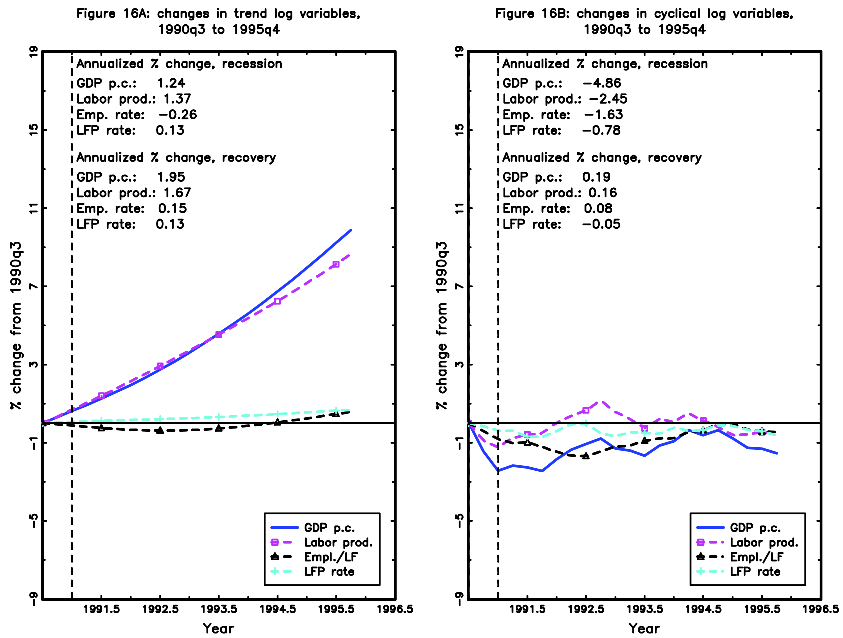


Trends and Cycles in the Decomposition of GDP

Figures 11 and 12 illustrate the trend and cyclical components of the different per capita GDP components in (5). The decomposition into trend and business cycle components is carried out using a Hodrick-Prescott (HP) filter with smoothing parameter of 1,600, given the quarterly data. Note that, because of the linearity of the HP filter, the trends of each of the per capita GDP components add up to trend per capita GDP, and likewise for the cyclical components.[7] The figures suggest that most of the variation in labor productivity and the labor force participation rate is driven by slow-moving cycles (with periods greater than eight years), while variations in the unemployment rate are more frequent. This is particularly evident in Figure 12, where

[7] Because the HP filter is a two-sided filter, estimation of the trend is biased toward the end of the sample. Depending on the nature of the data-generating mechanism, it takes roughly two years for estimation of the trend to settle.

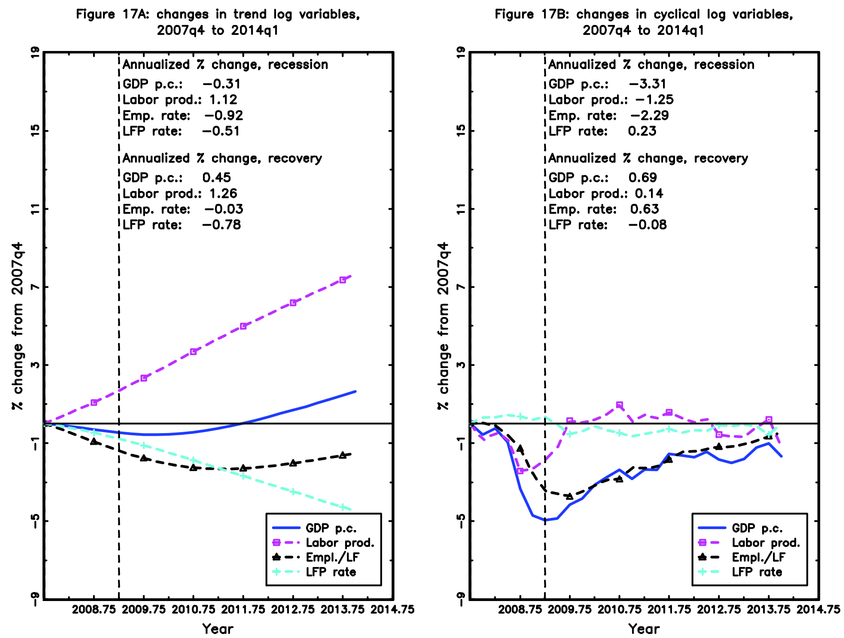
Figure 16 Changes in Trend and Cyclical Log Variables, 1990 Recession



the deviations from trend in the labor force participation rate indeed appear small.

Figures 13 through 17 illustrate the same decomposition as those in Figures 8 through 10, but are presented in terms of cycles and trends. Annualized growth rates for each of the series in Figures 8 through 10 are now broken down into contributions from "cyclical" and "trend" components. Examination of Figure 13, which illustrates the 1953 recession, reveals that the trend behavior of the series shown in the left-hand panel matches well with textbook balanced growth calculations described in the previous subsection. Trend log per capita GDP and log labor productivity have the same slope (i.e., grow at the same rate), while the trend unemployment and labor force participation rates stay relatively constant. This observation also applies to the 1957, 1960, 1980, 1981, 1990, and 2001 recessions. The slopes of labor productivity vary somewhat, ranging from 1.6 percent in the 2001 recession to 2.7 percent in the 1960 recession. However, the recessions of the 1970s, and especially that of 2007 shown in Figure 17, present a different story. In the most recent recession in particular, while labor productivity has

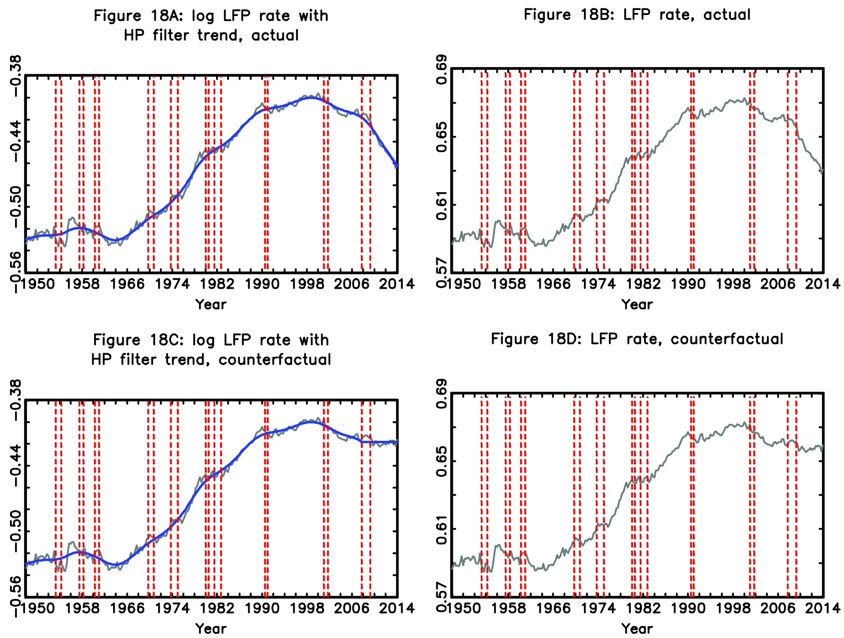
Figure 17 Changes in Trend and Cyclical Log Variables, 2007 Recession



steadily trended upward in a way typical of the postwar period, the labor force participation rate has clearly trended downward, noticeably dragging the growth rate of per capita GDP down from that of labor productivity. Remarkably, the behavior of the series' cyclical components, depicted in the right-hand panel of Figure 17, appears relatively similar to that of other postwar recessions. Put another way, at business cycle frequencies, the Great Recession is not so dissimilar to other postwar recessions. Its "uniqueness" resides almost entirely in slow-moving components of per capita GDP, in this case mostly the labor force participation rate. For the current recovery period, a small negative output gap relative to trend still persists.

While the trend labor force participation rate has fallen significantly since the start of the last recession, thereby mitigating the strength of the subsequent recovery in per capita GDP, a word of caution is in order. As mentioned earlier, the HP filter-based decomposition of a given series into business cycle and trend components tends to be biased toward the end of the sample, and it typically takes two years or more for the trend decomposition to settle. Because of this, one still

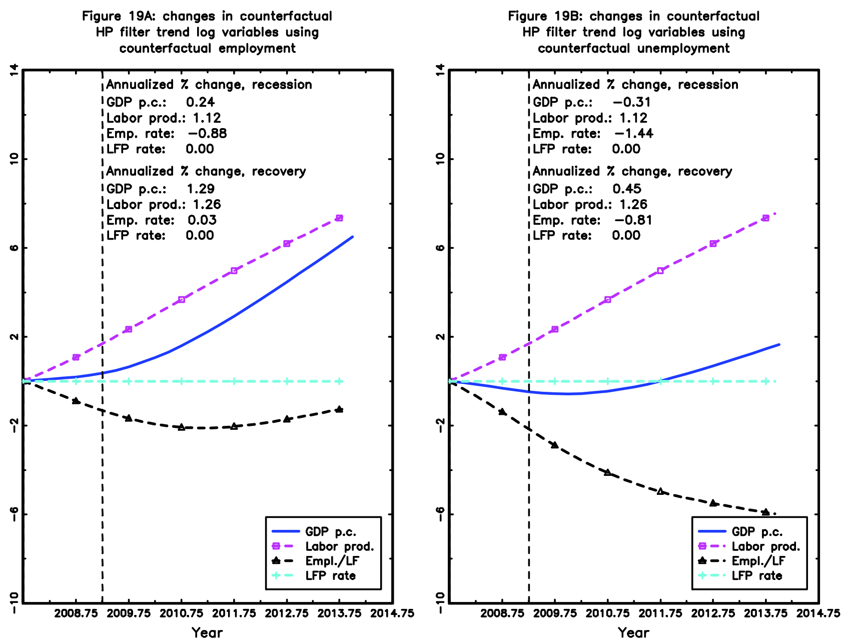
Figure 18 Labor Force Participation Rate, Actual Versus Counterfactual



might suspect that the large decline in the labor force participation rate can, in fact, be explained to a degree by cyclical factors related to the recession. If this were the case, our suggestion that the unusual behavior of output can be explained by secular changes in its components would be tenuous. However, the HP filter-based trends of the labor force participation rate, defined as component cycles with periods greater than eight years, are very similar to those calculated by Kudlyak (2013) using demographic information including age, gender, and cohort effects. In other words, a considerable portion of low frequency variations in the labor force participation rate are essentially explained by demographic factors; for example, one might attribute part of the recent low frequency decline in the labor force participation rate to the slow movement into retirement of the baby boomers.[8] If, as Kudlyak's article indicates, demographic factors are driving the decline in labor force participation, one might expect the recovery of labor force

[8] See Fujita (2014) for a detailed explanation of the causes underlying declines in the labor force participation rate.

Figure 19 A Counterfactual Exercise



participation—and therefore per capita GDP—to be protracted, with little room for improvement from policymakers.[9]

Counterfactual Labor Force Participation Rates

This subsection further investigates the extent to which the recent decline in the trend labor force participation rate has potentially contributed to the tepid recovery of per capita GDP following the Great Recession. Specifically, we carry out a counterfactual exercise in which, similar to Erceg and Levin (2013), the trend labor force participation rate flattens out after 2007:Q4. In this exercise, the counterfactual

[9] The decomposition we study, being an identity, is not necessarily inconsistent with the notion of financial factors having played a key role in the way the Great Recession played out. However, one expects that the productivity subcomponent of this decomposition, among all three subcomponents, might have been most influced by such factors, rather than the labor force participation rate where demographics clearly have a role. Indeed, productivity and employment experienced a more pronounced decline relative to other recessions, but these components appear to have recovered at a pace not too different from that of other recessions.

trend labor force participation rate is defined relative to low frequency variations isolated by the HP filter. A comparison of this counterfactual labor force participation rate series to the actual one is shown in Figure 18.

In any counterfactual calculation of this type, changing the labor force series, LF_t, to reflect a different trend path for the labor force participation rate means that we must also change either the employment series, E_t, the unemployment series, U_t, or both, so that the identity $LF_t = E_t + U_t$ continues to hold under the counterfactual.[10] We consider two polar cases: an "optimistic" case in which all of the additional labor force participation is matched by an increase in employment, and a "pessimistic" case in which the extra labor force participation is reflected by increased unemployment. Thus, the pessimistic case might be interpreted as one in which the distinction between being out of the labor force and being unemployed is not substantive for the counterfactual increase in labor force participation. In contrast, the optimistic case might be interpreted as one in which the counterfactual increased labor force participation assumes away any labor market mismatch issues or other forces that could potentially produce mismatched or discouraged workers who then leave the labor force.

The resulting implications for (HP filter-based) trend GDP per capita are shown in Figure 19.[11] In the pessimistic case, as expected, when the counterfactual increase in the labor force participation series is simply matched by increased unemployment, the path of per capita GDP is unaffected, but the ratio of employment to the labor force falls.

In the optimistic case, a flattening out of the trend labor force participation rate after 2007:Q4 results in a gain of roughly 0.8 percent in per capita GDP growth during the recovery beginning in 2009:Q3. In a sense, this figure represents an upper bound on what a flattening out of the labor force participation rate after the Great Recession might have implied for per capita GDP growth. At the same time, to the degree that the current recovery in per capita GDP has fallen short of historical trend growth by roughly 1 percent, a considerable portion of that difference may be accounted for by the behavior of the trend labor force participation rate. In principle, the implications of a flattening of the trend labor force participation rate lie somewhere between the two cases depicted in Figure 19.

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[10] Here, the behavior of population is taken as given so that a counterfactual labor force series is easily constructed by multiplying the counterfactual labor force participation rate by population.

[11] In these calculations, trend labor productivity is assumed to be unchanged.

3. CONCLUDING REMARKS

A simple decomposition of per capita GDP traces the unusual behavior of output during and after the Great Recession to a large and steady decline in the labor force participation rate. The magnitude and persistence of this decline are unprecedented in U.S. postwar history, much as the fall in per capita GDP that accompanied the Great Recession was unprecedented. Moreover, the fact that the labor force participation rate moves slowly over time, at frequencies much lower than those characterizing business cycles, presaged a muted recovery from the 2007–09 recession relative to other recoveries throughout the postwar period. The persistently slow recovery of per capita GDP might continue to cause concern and potentially warrants further inquiries into the factors—particularly demographic ones—that drive fluctuations in the labor force participation rate. Such inquiries could help determine whether government policy can and should be used to raise the rate of economic growth in the years ahead.

REFERENCES

Blough, Stephen R. 1992. “The Relationship between Power and Level for Generic Unit Root Tests in Finite Samples.” *Journal of Applied Econometrics* 7 (July–September): 295–308.

Erceg, Christopher, and Andrew Levin. 2013. “Labor Force Participation and Monetary Policy in the Wake of the Great Recession.” International Monetary Fund Working Paper WP/13/245 (July).

Fujita, Shigeru. 2014. “On the Causes of Declines in the Labor Force Participation Rate.” Federal Reserve Bank of Philadelphia Research Rap Special Report (February).

Gordon, Robert J. 2010. “Okun’s Law and Productivity Innovations.” *American Economic Review* 100 (May): 11–15.

Kudlyak, Marianna. 2013. “A Cohort Model of Labor Force Participation.” Federal Reserve Bank of Richmond *Economic Quarterly* 99 (First Quarter): 25–43.

Müller, Ulrich, and Mark Watson. 2013. “Measuring Uncertainty about Long-Run Predictions.” Mimeo, Princeton University.

Nelson, Charles, and Charles Plosser. 1982. "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications." *Journal of Monetary Economics* 10: 139–62.

Reinhart, Carmen, and Kenneth Rogoff. 2014. "Recovery from Financial Crises: Evidence from 100 Episodes." *American Economic Review* 104 (May): 50–5.

Stock, James H. 1990. "'Unit Roots in Real GNP: Do We Know and Do We Care?' A Comment." *Carnegie-Rochester Conference Series on Public Policy* 32 (January): 63–82.

Stock, James H., and Mark W. Watson. 2012. "Disentangling the Channels of the 2007–09 Recession." *Brooking Papers on Economic Activity* 44 (Spring): 81–156.