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A Business Cycle Analysis of Debt and Equity Financing

Marios Karabarbounis, Patrick Macnamara, and Roisin McCord

The recent turmoil in financial markets has highlighted the need to better understand the link between the real and the financial sectors. For example, a widespread view holds that real shocks can propagate themselves by adversely affecting credit markets (financial accelerator). An informative way to establish such linkages is to look at the co-movement between financial flows and macroeconomic conditions. The magnitude and direction of this relationship can guide our thinking regarding how strong these linkages are and the particular way in which they manifest themselves.

This article takes a modest step in this direction. In particular, we provide an introductory, yet comprehensive, business cycle analysis of firm financing. We first document empirically the cyclical properties of debt and equity issuance. We then build a simple two-period model to analyze the optimal capital structure as well as the response of firm financing to exogenous shocks such as a productivity shock. Finally, we examine how well a fully dynamic, reasonably calibrated, heterogeneous-firm model replicates the business cycle properties of debt and equity issuance.

We document empirical patterns of firm financing based on Compustat for the period 1980–2013. We find that firms issue more debt during expansions. In contrast, the cyclical properties of equity issuance depend on the exact definition of equity. If we define equity issuance using the sale of stock net of equity repurchases (following Jermann and Quadrini [2012]), we find a countercyclical equity issuance (or a

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procyclical equity payout). If we follow Covas and Den Haan (2011) and define equity issuance based on the change in the book value of equity, we find equity issuance to be weakly procyclical. Equity financing through mergers explains much of the discrepancy between the two measures. Stock compensation also explains the discrepancy but to a smaller degree. Moreover, regardless of the measure used, the countercyclicality of net equity issuance is driven by a strongly procyclical dividend payout and not countercyclical gross equity issuance. The data also reveal a substantial degree of heterogeneity in firms' financial decisions. Compared to large firms, the debt issuance of small firms tends to be less procyclical while equity issuance tends to be more procyclical.

To build intuition, we analyze the firm's optimal capital structure within a simple two-period model. Each period, firms receive an idiosyncratic productivity shock. The firm chooses how much to invest and how it will finance this decision. Financing can take the form of a one-period bond (debt) and external equity. The firm chooses debt issuance to balance the tax benefits of debt with the expected bankruptcy costs of default. External equity is also assumed to be costly. We show how the policy functions for investment, debt, and equity vary with internal equity, the costs of issuing equity, and idiosyncratic productivity.

Our fully dynamic model incorporates many of the elements outlined in the two-period model. Firms experience both aggregate and idiosyncratic productivity shocks. Nevertheless, we keep the analysis simple and assume a partial equilibrium framework. The model is calibrated to match several cross-sectional moments as calculated from Compustat. We then examine how well our model can explain the cyclical properties of debt and equity issuance. As in the data, firms issue more debt in response to a positive productivity shock. Higher productivity implies that firms desire to invest more, which makes default more costly and, hence, borrowing easier. Moreover, equity issuance is countercyclical. This is driven by large firms issuing more dividends during expansions. The model also captures the firm-size relationship in firm financing. Specifically, the model is able to match the empirical observation that net equity issuance of small firms is procyclical, while debt issuance is less procyclical than for larger firms.

This article contributes to the literature on firm financing in two ways. First, we highlight how equity financing through mergers and stock compensation can account for the different measures of net equity issuance used in the literature. In particular, we show that if one excludes mergers and stock compensation, the measures used by Covas and Den Haan (2011) and Jermann and Quadrini (2012) (change in book value of equity and net sale of stock, respectively) lead to the same conclusion. Moreover, we show that a countercyclical net equity issuance in the data is driven by dividend payouts falling during recessions, not gross equity issuance increasing during recessions. Although such a distinction is crucial for understanding how firm financing varies over the cycle, it is not stressed in the literature. Second, we test these predictions within a quantitative model of firm financing with heterogeneous firms. Although this is certainly not the first quantitative article of firm financing, our article makes several novel contributions. For example, we build intuition regarding the determinants of firm financing using a simple two-period model. Moreover, using our heterogeneous-firm model we can test if the model captures the empirical firm-size relationship and especially the decomposition of equity financing into gross equity issuance and payout components.

1. RELATED LITERATURE

Our analysis borrows many elements from the work of Covas and Den Haan (2011), who look at disaggregated data from Compustat and document the cyclical properties of firm finance for different firm sizes. Their finding is that debt and (net) equity issuance is procyclical as long as the very large firms are excluded. Hence, Covas and Den Haan (2011) stress the importance of incorporating heterogeneity in quantitative models of firm financing.[1] Jermann and Quadrini (2012) document the cyclical properties of financial flows using aggregate data from the flow of funds accounts. The authors find a procyclical debt issuance but a countercyclical net equity issuance. Their article also examines the macroeconomic effects of financial shocks by constructing a shock series for the financial shock and then feeding the shock into a real business cycle model. Beganau and Salomao (2014) also document financial flows from Compustat. Following the equity definition of Jermann and Quadrini (2012), Beganau and Salomao (2014) also find net equity issuance is countercyclical.

Although the focus on the cyclicality of financial flows has been relatively new, there is ample work on the cross-sectional determinants of capital structure and firm dynamics. Rajan and Zingales (1995) investigate the relationship between leverage and firms' characteristics for a set of countries. They report that most of the empirical regularities found in the United States (such as the positive relationship between firm size and leverage) are also true for other countries. Cooley and

[1] In a related article, Covas and Den Haan (2012) build a quantitative model of debt and equity finance. Our model in Section 4 uses many of their modeling assumptions.

Quadrini (2001) introduce financial frictions in a model of industry dynamics and study the relationship between firm financing and firm size and age. Hennessey and Whited (2007) build a structural model of firm financing and estimate the magnitude of external financing costs. Recently, Katagiri (2014) builds a general equilibrium model of firm financing to study the distribution of leverage.

2. EMPIRICAL ANALYSIS

In this section, we describe several empirical patterns regarding firm financing. We first explain how we construct the variables used in the analysis. We next present aggregate statistics both in the cross-section of firms and along the business cycle. The main findings emerging from the analysis are the following. First, debt issuance is strongly procyclical. Second, the cyclicality of equity issuance depends on the specific measure used. However, smaller firms seem to issue more equity in expansions relative to larger firms, independent of the measure. Third, there is widespread heterogeneity in firm financing decisions.

Data Construction

To construct our variables we use annual data from Compustat. Compustat contains financial information on publicly held companies. Following the literature on firm financing, we focus on the period between 1980 and 2013. Jermann and Quadrini (2012) document that during this period there was a break in macroeconomic volatility as well as significant changes in U.S. financial markets. We exclude financial firms and utilities as these industries are more heavily regulated.[2] One important concern is whether we include firms affected by a merger or an acquisition. For this purpose, we separately report results for two cases. In the first case, we follow Covas and Den Haan (2011) and drop all firm-year observations that are affected by a "major" merger or acquisition. By "major" we mean that the merger or acquisition causes the firm's sales to increase by more than 50 percent. In the second case, we drop all observations affected by any kind of merger. After imposing these restrictions and dropping all observations affected by a major merger, we are left with an unbalanced panel of 19,101 firms and a total of 168,295 firm-year observations. When we also drop observations affected by any merger, we are left with 18,486 firms and 141,379 observations.

[2] For more details on the construction of our data, see Appendix A.

Variable Definitions

The literature uses two different methods to measure equity issuance. Fama and French (2005) and Covas and Den Haan (2011) use changes in the book value of equity (reported on the firm's balance sheet) to measure equity issuance. Jermann and Quadrini (2012) use the "net sale of stock" (from the statement of cash flows) in the construction of equity issuance. To clarify the difference between these two measures, it is useful to define a company's accounting identity:

$$A_{i,t} = SE_{i,t} + RE_{i,t} + L_{i,t}.$$

For firm i at date t, assets $A_{i,t}$ must equal equity plus liabilities $L_{i,t}$ (all variables are book values). Total equity includes retained earnings $RE_{i,t}$, which is the portion of the company's net income it has retained rather than distributed to shareholders as dividends. Therefore, $SE_{i,t}$ is the company's total equity net of retained earnings. This part of the firm's balance sheet reflects equity that the firm has obtained from "external" sources such as sale of common stock.

Under the first definition, equity issuance is the annual change in $SE_{i,t}$ minus cash dividends distributed to shareholders $d_{i,t}$. We subtract cash dividends from our definition because, effectively, they represent one of two ways firms can distribute funds to shareholders: They can buy back stock, which would decrease $SE_{i,t}$, or they can issue dividends, which would decrease $RE_{i,t}$ instead. Therefore, in our first definition, the equity issuance of firm i at date t is

$$\Delta E_{i,t}(1) \equiv \Delta SE_{i,t} - d_{i,t}, \tag{1}$$

where $\Delta SE_{i,t} \equiv SE_{i,t} - SE_{i,t-1}$ is the annual change in $SE_{i,t}$. This corresponds to one of the primary definitions of equity issuance in Covas and Den Haan (2011). Our second definition of equity issuance is defined as follows:

$$\Delta E_{i,t}(2) \equiv \Delta SS_{i,t} - d_{i,t}; \tag{2}$$

$\Delta SS_{i,t}$ is the net sale of stock, which is defined as the gross revenue from the sale of stocks minus stock repurchases. This corresponds to the definition of equity issuance utilized by Jermann and Quadrini (2012).

Ideally these two measures would be equivalent, as the net sale of stock $\Delta SS_{i,t}$ affects $SE_{i,t}$. Nevertheless, the two definitions lead to different conclusions about the cyclicality of equity issuance. This discrepancy has to do with the way firms choose to issue equity. Apart from equity offerings to the public, equity issuance can take place through mergers, warrants, employee options, grants, and benefit plans among others. Hence, as Fama and French (2005) note, the net sale of stock measure captures only a few of the ways in which firms can raise outside equity. Take, for example, a merger or an acquisition. Suppose a firm acquires another firm by issuing equity to the shareholders of the target firm. This transaction will change the book value of equity. However, it will not alter the sale of stock measure because no actual revenue is raised by the transaction. Moreover, suppose a firm were to compensate its employees with a stock. Again, if equity is measured using the book value of equity, equity issuance will increase. This is because employee compensation will decrease retained earnings and thus increase $SE_{i,t}$, the company's equity net of retained earnings. Meanwhile, as before, the sale of stock measure will not record the equity issuance because no actual revenue is raised.

In the data, a situation in which no firms issue equity (on net) will look the same as a situation in which some firms issue equity while others reduce equity. To uncover such heterogeneity, we break up our first definition of equity issuance into a "gross equity issuance" and "gross equity payouts" component.[3] In particular, we define gross equity issuance $E^I_{i,t}$ to be

$$E^I_{i,t} \equiv \begin{cases} \Delta SE_{i,t} & \text{if } \Delta SE_{i,t} > 0 \\ 0 & \text{if } \Delta SE_{i,t} \leq 0 \end{cases} . \tag{3}$$

Similarly, we define gross equity payouts $E^P_{i,t}$ to be

$$E^P_{i,t} \equiv \begin{cases} d_{i,t} & \text{if } \Delta SE_{i,t} > 0 \\ -\Delta SE_{i,t} + d_{i,t} & \text{if } \Delta SE_{i,t} \leq 0 \end{cases} . \tag{4}$$

Note that $\Delta E_{i,t}(1) = E^I_{i,t} - E^P_{i,t}$ by construction. By looking at gross flows, we can separately identify firms that raise equity and firms that reduce equity.

Moreover, we also consider several other variables of interest. In particular, w_S will denote employee stock compensation; $\Delta RE_{i,t} \equiv RE_{i,t} - RE_{i,t-1}$ is the change in retained earnings. A firm's net debt issuance $\Delta D_{i,t} \equiv D_{i,t} - D_{i,t-1}$ is defined to be the change in the firm's book value of debt between period $t-1$ and t. A firm's net change in sales $\Delta S_{i,t} \equiv S_{i,t} - S_{i,t-1}$ is defined to be the change in the firm's nominal sales between t and $t-1$. Finally, $I_{i,t}$ is the firm's investment while $K_{i,t}$ is the firm's capital stock.

Construction of Group Aggregates

To uncover any underlying heterogeneity in the financing decisions of firms, we sort firms by size. At each date t, we sort firms into four possible groups based on their size (more on the construction of these

[3] We can similarly break up the second definition of equity. However, as discussed earlier, the second definition of equity tends to understate equity issuance.

groups later). Then, for every date t, we aggregate each firm-level variable across all the firms in each bin. To be precise, let $X_{i,t}$ be a variable of interest for firm i at date t. For example, this might be $\Delta D_{i,t}$, the net debt issuance of a particular firm. Let $G_{j,t}$ denote the set of firms in group j at date t. Then, we can construct the group aggregate $X_{j,t}$ as follows:

$$X_{j,t} = \frac{\sum_{i \in G_{j,t}} X_{i,t}}{\sum_{i \in G_{j,t}} K_{i,t}}. \tag{5}$$

The numerator is the sum of $X_{i,t}$ across all firms in group j at date t. Therefore, if $X_{i,t}$ is $\Delta D_{i,t}$, then the numerator of (5) is the net amount of debt issued by all firms in group j at date t. Meanwhile, the denominator of (5) is the total amount of capital in group j at date t. The denominator is used to normalize the resulting aggregate variable and capital is chosen because it is acyclical. Following this procedure, we obtain a time series for the aggregate variable X for each group. Note, however, that the composition of firms in each group varies over time. Not only may a firm transition between groups over time, but the groups may include newly listed firms.

To construct the firm groups, we sort firms based on the previous period's book value of their assets. At each date, we sort firms into four groups. The first group consists of firms with assets below the median ([0, 50]). The second group consists of firms between the 50th and 75th percentile ([50, 75]), and the third group consists of firms between the 75th and 99th percentile ([75, 99]). And finally, the last group consists of firms in the top 1 percent ([99, 100]). As the book value of assets tends to grow over time, we have to be careful in how we determine the asset boundaries for these size groups. Define $A_{50,t}$, $A_{75,t}$, and $A_{99,t}$ to be the asset boundaries between the four size bins. In other words, a firm with assets $A_{i,t} < A_{50,t}$ will be in the [0, 50] group at date $t+1$. Following Covas and Den Haan (2011), we construct $A_{50,t}$, $A_{75,t}$, and $A_{99,t}$ by fitting a (log) linear trend through the asset values that correspond to the 50th, 75th, and 99th percentiles at each time t.

Cross-Sectional Analysis

We begin our analysis by looking at group aggregates for the whole period between 1980 and 2013 (Table 1). Each variable is expressed as a percentage of the group capital stock. In the top panel we exclude major mergers from the sample while in the lower panel we exclude all mergers. Looking at the top panel, we see that relative to their size, small firms tend to issue more debt and equity than large firms. Debt issuance decreases monotonically from 14.1 percent of the group's

Table 1 Summary Statistics

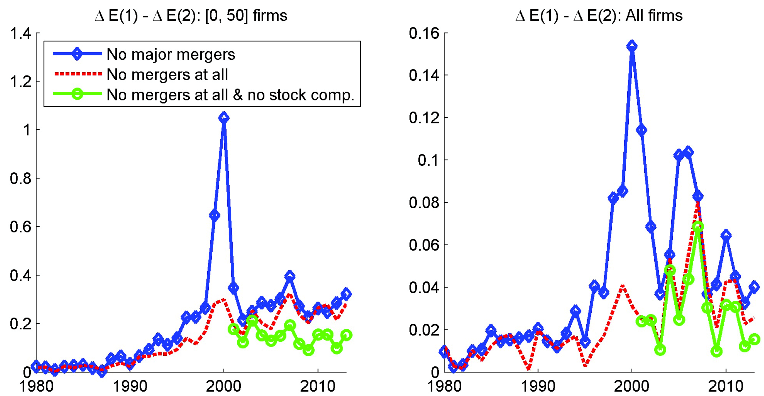
	Size Class (Percent)				
No Major Mergers	**[0, 50]**	**[50, 75]**	**[75, 99]**	**[99, 100]**	**[0, 100]**
ΔD	10.1	8.6	5.4	2.7	4.7
$\Delta E(1)$	69.0	8.6	−3.0	−5.3	−3.0
$\Delta E(2)$	40.6	−1.5	−9.9	−10.0	−9.4
$\Delta E(1) - \Delta E(2)$	28.4	10.0	6.9	4.7	6.3
w_S	7.9	2.6	0.9	0.3	0.8
E^I	79.8	18.0	6.9	4.7	6.9
E^P	10.8	9.4	10.0	10.0	10.0
ΔRE	−35.7	3.6	4.5	3.4	4.0
I	4.6	1.9	0.7	1.1	0.9
ΔS	42.1	28.3	14.5	14.3	15.0
	Size Class (Percent)				
No Mergers At All	**[0, 50]**	**[50, 75]**	**[75, 99]**	**[99, 100]**	**[0, 100]**
ΔD	4.0	0.7	1.7	0.9	1.5
$\Delta E(1)$	76.8	3.6	−5.8	−7.4	−5.6
$\Delta E(2)$	53.3	−1.7	−9.5	−10.7	−9.3
$\Delta E(1) - \Delta E(2)$	23.4	5.3	3.7	3.3	3.7
w_S	8.7	2.6	0.8	0.4	0.8
E^I	87.8	14.1	3.6	2.4	4.0
E^P	11.0	10.5	9.4	9.9	9.6
ΔRE	−49.9	4.1	4.5	6.1	4.8
I	4.8	1.5	0.3	0.5	0.5
ΔS	30.7	15.2	9.4	10.6	9.9

Notes: This table reports the average of various group aggregates between 2001 and 2013. Each variable is expressed as a percentage of the group capital stock. ΔD is net debt issuance. $\Delta E(1)$ is the first measure of net equity issuance and is defined in (1). Similarly, $\Delta E(2)$ is the second measure of equity issuance and is defined in (2). w_S is stock compensation. E^I is gross equity issuance and is defined in (3). E^P is gross equity payouts and is defined in (4). ΔRE is the net change in retained earnings. I is investment. ΔS is the net change in sales. Note that we only have data on stock compensation between 2001 and 2013.

capital stock in the [0, 50] bin to 2.9 percent for firms in the top 1 percent. Equity issuance $\Delta E(1)$ decreases from 64.9 percent of capital for firms in the [0, 50] bin to −3.9 percent for firms in the top 1 percent. For our second measure, $\Delta E(2)$, these numbers are 44.4 percent and −6.4 percent, respectively. Nevertheless, the two measures of equity do differ in a significant way. $\Delta E(2)$, which is based on the net sale of stock, underestimates the amount of equity that firms raise. While the measures differ across all size groups, they are significantly different for smaller firms.

As noted earlier, mergers financed through the issuance of stock may also explain part of the difference between the two equity

Figure 1 Effect of Mergers and Stock Compensation on Equity Issuance



Notes: The graph plots the difference between our two measures of equity issuance $\Delta E(1)$ and $\Delta E(2)$ for the period 1980–2013. We plot the difference if (i) no major mergers are included in the sample, (ii) no mergers at all are included in the sample, and (iii) no mergers at all are included in the sample and stock compensation is subtracted from the difference. The left panel shows the differences for firms in the [0, 50] size class. The right panel shows the differences for all firms. In each case, the differences are plotted as a percentage of the group capital stock. Information on stock compensation is available only after 2003.

measures. To investigate how much mergers and acquisitions explain the difference, we repeat our earlier analysis, but we exclude all mergers from the sample. The bottom panel of Table 1 reports the results when all mergers are excluded. While the same results hold as before, firms on average issue less debt than before (1.8 percent versus 5.0 percent). Firms also issue less equity under both definitions (-3.9 percent and -6.2 percent versus -1.7 percent and -6.0 percent, respectively). Overall, the difference between the two equity measures falls by almost half. Moreover, stock compensation (which is not reflected in $\Delta E(2)$) does explain some of the remaining discrepancy between the two measures.[4] In fact, for small firms it is a major explanation for the discrepancy between the two measures. Still, after accounting for mergers and stock compensation, significant differences remain.

[4] However, note that our data for stock compensation only begins in 2001.

Figure 1 shows how our equity measures, $\Delta E(1)$ and $\Delta E(2)$, differ between 1980 and 2013. Similar to Table 1, we plot the difference $\Delta E(1) - \Delta E(2)$ for three different cases: (i) if no major mergers are included, (ii) if no mergers at all are included, and (iii) if no mergers at all are included and we subtract from the difference equity issuance related to stock compensation. The left panel of Figure 1 shows the differences for firms in the [0, 50] size group, while the right panel shows the differences for all firms. This figure highlights how the differences between these two measures have grown since the late 1990s. Moreover, it also demonstrates the importance that mergers and acquisitions have had on equity financing, especially in the late 1990s. $\Delta E(1)$ can capture these effects while $\Delta E(2)$ cannot. However, in the period after 2007, mergers seem to account for only a small part of the discrepancy. Nevertheless, during that period, stock compensation seems to account for a larger fraction of the difference. As seen in Figure 1, this is especially true for firms in the [0, 50] size group.

Finally, from Table 1 (both top and bottom panels) it is readily apparent that small firms grow faster (in terms of sales growth) and invest at a higher rate. Moreover, excluding the top 1 percent, smaller firms have lower growth in retained earnings and ΔRE is even negative for firms in the [0, 50] size group. These results are consistent with the findings of Covas and Den Haan (2011).

Business Cycle Analysis

We next turn to the business cycle analysis of debt and equity issuance. In Table 2, we report the correlation of various group aggregates with real corporate gross domestic product (GDP). To compute these correlations, both GDP and the group aggregates are de-trended with an H-P filter.[5] First consider the top panel of Table 2, which includes results for the case when only major mergers are excluded from the sample. Consistent with Covas and Den Haan (2011) and Jermann and Quadrini (2012), debt issuance is strongly procyclical. The cyclicality is stronger for larger firms. The correlation between debt issuance and corporate GDP increases from 0.536 for the [0, 50] size group to 0.755 for the [75, 99] size group. The correlation falls to 0.547 for firms in the top 1 percent. However, note that there is a relatively small number of firms in this group.[6]

[5] Throughout this article, we use a smoothing parameter of 100 to de-trend annual data.

[6] There are, on average, 31 firms in the top 1 percent every year.

Table 2 Business Cycle Correlations of Debt and Equity Issuance

	Size Class (Percent)				
No Major Mergers	**[0, 50]**	**[50, 75]**	**[75, 99]**	**[99, 100]**	**[0, 100]**
ΔD	**0.536**	**0.611**	**0.755**	**0.547**	**0.785**
	(0.001)	(0.000)	(0.000)	(0.001)	(0.000)
$\Delta E(1)$	**0.345**	0.191	0.016	0.044	0.096
	(0.046)	(0.280)	(0.927)	(0.804)	(0.589)
$\Delta E(2)$	0.243	−0.250	**−0.617**	−0.312	**−0.509**
	(0.166)	(0.155)	(0.000)	(0.072)	(0.002)
w_S	0.010	−0.050	0.066	0.022	0.341
	(0.973)	(0.859)	(0.816)	(0.937)	(0.214)
E^I	**0.353**	0.268	0.306	0.250	**0.363**
	(0.041)	(0.125)	(0.079)	(0.153)	(0.035)
E^P	0.069	0.279	**0.654**	0.314	**0.588**
	(0.697)	(0.110)	(0.000)	(0.071)	(0.000)
	Size Class (Percent)				
No Mergers At All	**[0, 50]**	**[50, 75]**	**[75, 99]**	**[99, 100]**	**[0, 100]**
ΔD	**0.646**	**0.590**	**0.661**	**0.418**	**0.661**
	(0.000)	(0.000)	(0.000)	(0.014)	(0.000)
$\Delta E(1)$	0.264	−0.168	**−0.419**	−0.303	−0.322
	(0.132)	(0.343)	(0.014)	(0.082)	(0.064)
$\Delta E(2)$	0.230	−0.312	**−0.687**	−0.193	**−0.506**
	(0.191)	(0.072)	(0.000)	(0.274)	(0.002)
w_S	0.035	0.027	0.009	−0.005	0.225
	(0.901)	(0.924)	(0.975)	(0.985)	(0.421)
E^I	0.283	−0.040	0.033	−0.113	0.100
	(0.105)	(0.824)	(0.852)	(0.524)	(0.573)
E^P	0.240	0.273	**0.627**	0.314	**0.586**
	(0.172)	(0.119)	(0.000)	(0.071)	(0.000)

Notes: This table reports correlations of group aggregates with real corporate GDP. All variables are de-trended with an H-P filter. The p-values for each correlation are shown in parentheses. Coefficients that are significant at the 5 percent level are shown in bold. ΔD is net debt issuance. $\Delta E(1)$ is the first measure of net equity issuance and is defined in (1). Similarly, $\Delta E(2)$ is the second measure of equity issuance and is defined in (2). w_S is stock compensation. E^I is gross equity issuance and is defined in (3). E^P is gross equity payouts and is defined in (4). Note that we only have data on stock compensation between 2001 and 2013.

Overall, equity issuance, as measured by $\Delta E(1)$, is acyclical. However, according to this measure, equity issuance tends to be procyclical and statistically significant for firms in the [0, 50] size group. The cyclicality of equity issuance monotonically decreases across size groups and becomes essentially uncorrelated with output for the top 1 percent. In particular, the correlation decreases from 0.345 for firms in the [0, 50] size group to 0.044 for firms in the top 1 percent. These results are consistent with Covas and Den Haan (2011). In contrast, if we measure equity issuance using the net sale of stock ($\Delta E(2)$), then equity issuance becomes strongly countercyclical. This is consistent with Jermann and Quadrini (2012). However, even according to this measure, equity issuance of the smallest firms tends to be procyclical (although statistically insignificant) with a correlation of 0.243. Meanwhile, $\Delta E(2)$ is significantly countercyclical for firms in the [75, 99] size group with correlation equal to -0.617. This pattern of financing across firm size is consistent with the net sale of stock measure reported in Covas and Den Haan (2011).

In Table 2, we also report how the cyclicality of $\Delta E(1)$ breaks into a gross equity issuance and gross equity payouts component, both defined in (3) and (4). For smaller firms, gross equity issuance is driving the (pro)cyclicality of net equity issuance. But for all other firm sizes, procyclical gross equity issuance is associated with a more procyclical gross equity payout. Both statistics may explain the weak cyclicality of net equity issuance $\Delta E(1)$. This decomposition can also shed some light on the discrepancy between our net equity measures $\Delta E(1)$ and $\Delta E(2)$. Since $\Delta E(2)$ underestimates gross equity issuance, it is mostly affected by a countercyclical gross equity payout.

Similar to our cross-sectional analysis, we trace the discrepancy in the cyclical behavior of $\Delta E(1)$ and $\Delta E(2)$ to mergers and stock compensation. In the bottom panel of Table 2, we report the business cycle correlations when we exclude all mergers from the sample. In this case, debt issuance is slightly less correlated with GDP but is still strongly procyclical (0.661 versus 0.785). However, according to $\Delta E(1)$, equity issuance for all firms now becomes significantly countercyclical (at the 10 percent level) and significantly countercyclical for the top 25 percent. For example, the correlation for firms in the [75, 99] size group is -0.419 when we exclude all mergers versus 0.016 when we do not. Nevertheless, for the smallest firms, equity issuance according to $\Delta E(1)$ is still procyclical, but it is not significant. Moreover, gross equity issuance is now statistically insignificant for all size groups. For all firms, gross equity payouts is still significantly procyclical. Therefore, the procyclical nature of merger activity[7] appears to play an important role in explaining the differences in the cyclicality between $\Delta E(1)$ and $\Delta E(2)$.

Another candidate to explain the discrepancy in the cyclicality of the two measures is stock compensation. Table 2 includes information on the cyclicality of stock compensation by firms. As mentioned

[7] Eisfeldt and Rampini (2006) document that capital reallocation due to acquisitions is procyclical.

earlier, this type of equity issuance is captured by $\Delta E(1)$ but not by $\Delta E(2)$. Therefore, it could help explain the discrepancy between the two measures. However, as we see from Table 2, stock compensation is itself acyclical. Therefore, while it does explain some of the difference in levels between the two measures (especially for small firms), it does not help explain the different cyclicalities.

3. OPTIMAL CAPITAL STRUCTURE: A TWO-PERIOD MODEL

In this section we outline a simple two-period model to explain how the firm chooses its capital structure. Firms are perfectly competitive and produce a single homogeneous good. Capital is the only input in the firm's production function, $zf(k)$, and z is the firm's productivity. Productivity follows an AR(1) process. We denote by $F(z'|z)$ and $f(z'|z)$ the cumulative distribution and probability density functions for next period's productivity z', conditional on the current-period productivity z.

Budget Constraint

The firm enters the first period with an initial level of capital, k, and a required debt payment, b. Given k and b in the first period, the firm (i) produces $zf(k)$, (ii) chooses investment $i = k' - (1 - \delta)k$, (iii) issues dividends d (or raises external equity if $d < 0$), and (iv) issues new debt, $q(z, k', b')b'$. The firm borrows using a defaultable one-period noncontingent bond. It promises to pay b' tomorrow and in return the firm receives $q(z, k', b')b'$ today, where q is the price of the bond. Later in this section we discuss how this price is determined. To facilitate the analysis, we follow Gourio (2013) by assuming that the firm receives a tax subsidy from the government proportional to the amount borrowed. In other words, for every dollar the firm raises in the bond market, the government gives the firm a subsidy of τ.[8]

The firm chooses dividends d, tomorrow's capital k', and debt b' subject to the following budget constraint:

$$d + k' = e(z, k, b) + (1 + \tau)q(z, k', b')b', \tag{6}$$

where $e(z, k, b) \equiv zf(k) + (1 - \delta)k - b$ is defined to be internal equity. Therefore, when choosing tomorrow's capital stock, the firm has access

[8] We are assuming that the tax subsidy takes place at issuance. However, in reality, the implicit tax subsidy takes place when the firm's earnings are taxed, as interest payments can be deducted from corporate taxable income.

to three sources of funding: (i) internal equity e, (ii) debt qb', which is supplemented by the tax subsidy, and (iii) external equity (when $d < 0$).

As discussed in Fazzari, Hubbard, and Petersen (1988), there are many reasons why external equity is costly, including taxes and flotation costs. Thus, we assume that issuing equity is costly and specify the cost $\Lambda(d)$ as follows:

$$\Lambda(d) = \begin{cases} -\lambda_0 d & \text{if } d < 0 \\ 0 & \text{if } d \geq 0 \end{cases} . \tag{7}$$

When $d < 0$, the firm is issuing external equity and the cost is assumed to be proportional to the amount of funds raised. Moreover, note that $\Lambda(d)$ does not appear in (6). Therefore, when $d < 0$, $-d$ is the amount of funds actually received by the firm. However, shareholders actually pay $-d + \Lambda(d) = -(1 + \lambda_0)d$, of which only $-d$ goes to the firm.

Default Decision

We also allow firms to default on their debt obligations. In particular, in period 2, the firm chooses whether it will pay b' or declare bankruptcy. If the firm does not default, it receives

$$V^{ND}(z', k', b') = z' f(k') + (1 - \delta)k' - b'. \tag{8}$$

In this case, the firm's shareholders receive output and the undepreciated capital minus the debt payment. However, if the firm defaults, we assume that the firm can hide and keep a fraction θ of its assets. Therefore, in this case, the firm receives

$$V^D(z', k') = \theta \left[z' f(k') + (1 - \delta)k' \right]. \tag{9}$$

Due to bankruptcy costs, lenders will only recover a fraction $1-\psi$ of the total remaining assets in the case of default. In other words, the lender recovers $(1 - \psi)(1 - \theta) [z' f(k') + (1 - \delta)k']$ when the firm defaults.

Given (8) and (9), the firm will default tomorrow when $V^{ND}(z', k', b'^D(z', k')$. This implicitly defines a productivity threshold $z^*(k', b')$ such that the firm will default if and only if $z'^*(k', b')$. This threshold is defined to be the value of productivity, z^*, such that the firm is indifferent between defaulting and not defaulting: $V^{ND}(z^*, k', b'^D(z^*, k')$. Using (8) and (9), we can then obtain the following functional form for $z^*(k', b')$:

$$z^*(k', b') = \begin{cases} \frac{b'/(1-\theta)-(1-\delta)k'}{f(k')} & \text{if } b' \geq (1 - \theta)(1 - \delta)k' \\ 0 & \text{if } b' < (1 - \theta)(1 - \delta)k' \end{cases} . \tag{10}$$

Consequently, default is only possible when $b' > (1-\theta)(1-\delta)k'$. Moreover, when b' is above this threshold, z^* depends negatively on k' and positively on b'. The more firms invest, the more output and capital the firm will have next period. This will make default more costly. Consequently, the default threshold decreases (i.e., $\partial z^*/\partial k' < 0$). In contrast, the more debt the firm issues, the more attractive default will be next period. In this case, the default threshold will increase (i.e., $\partial z^*/\partial b' > 0$).

Bond Price

We assume there are households willing to lend their savings to firms. The price that lenders charge, $q(z, k', b')$, takes into account the probability that a firm will default, which depends on the firm's choices for k' and b'. Specifically, it is assumed that q is set to guarantee the lender an expected return equal to the risk-free rate r. Hence, q will be given by

$$q(z, k', b') = \frac{1}{1+r}\left[1 - F(z^*(k', b')|z) + \frac{R(z, k', b')}{b'}\right], \qquad (11)$$

where

$$R(z, k', b') \equiv (1-\psi)(1-\theta)\int_0^{z^*(k',b')}\left[z' f(k') + (1-\delta)k'\right] f(z'|z)dz'$$

is the unconditional expected recovery value of the bond in the case of default. Therefore, the price of debt is composed of two terms. With probability $1 - F(z^*|z)$, the firm will not default and the lender receives b'. However, when the firm does default, the lender receives a fraction $(1-\psi)(1-\theta)$ of total assets.

Firm's Problem

We can now write the firm's problem as a dynamic programming problem. Define $V(z, k, b)$ as the value of a firm with productivity z, capital k, and debt b. This value function is given by

$$V(z, k, b) = \max_{d,k',b'}\left\{\begin{array}{c} d - \Lambda(d) + \\ \frac{1}{1+r}\int_0^\infty \max\left\{V^{ND}(z', k', b'), V^D(z', k')\right\} f(z'|z)dz' \end{array}\right\} \qquad (12)$$

subject to the budget constraint in (6), which is repeated here:

$$d + k' = e(z, k, b) + (1+\tau)q(z, k', b')b'.$$

The firm's objective is to choose next period's capital stock k', debt b', and dividends d in order to maximize its lifetime valuation.

Characterizing the Solution

In this subsection we explain what determines the firm's optimal capital structure. To do so, it is useful to first re-write the firm's value function defined in (12). Specifically, using the bond price function defined in (11), the firm's value function can be re-written as[9]

$$V(z,k,b) = \max_{d,k',b'} \Big\{ e(z,k,b) - k' - \Lambda(d) + \tau q(z,k',b')b' - B(z,k',b') + \frac{1}{1+r}\left[E[z'f(k')|z] + (1-\delta)k'\right] \Big\}, \tag{13}$$

subject to the budget constraint in (6). Recall that $e(z,k,b) \equiv zf(k) + (1-\delta)k - b$ is defined to be internal equity. Let $T(z,k',b') = \tau q(z,k',b')b'$ denote the total value of the tax subsidy. This term reflects the tax benefit of debt issuance. Similarly, $B(z,k',b')$ is defined to be the expected bankruptcy costs and is given by

$$B(z,k',b') = \frac{\psi(1-\theta)}{1+r} \int_0^{z^*(k',b')} \left[z'f(k') + (1-\delta)k'\right] f(z'|z)dz'.$$

As before, firms will choose k', b', and d to maximize the firm's lifetime valuation. As is clear from (13), the effect of marginal changes in k' and b' on $T(z,k',b')$ and $B(z,k',b')$ will play a key role in determining the firm's optimal capital structure. To ease the exposition of the firm's problem, we will first consider the case where issuing equity is costless (i.e., $\lambda_0 = 0$) and describe how the optimal policies for k', b', and d are determined. We then allow for costly equity ($\lambda_0 > 0$) and analyze how the firm's optimal choices change.

Costless Equity Issuance

We first assume that $\lambda_0 = 0$, which implies that $\Lambda(d) = 0$ for all d. In this case, the first order conditions for k' and b' become

$$\tau \frac{\partial q}{\partial k'} - \frac{\partial B}{\partial k'} + \frac{E[z'f'(k')|z] + 1 - \delta}{1+r} = 1 \tag{14}$$

$$\tau \left[q + \frac{\partial q}{\partial b'} b'\right] = \frac{\partial B}{\partial b'}. \tag{15}$$

When $\tau = \psi = 0$, the first-order condition for k' in (14) reduces to the familiar expression that the expected marginal product of capital equals interest plus depreciation (i.e., $E[z'f'(k')|z] = r + \delta$). Therefore, the firm invests the first-best amount of k'. Moreover, when $\tau = \psi = 0$,

[9] The readers can find the exact derivation of this expression in Appendix B.

both sides of (15) are always zero. Therefore, the Modigliani-Miller theorem[10] applies and the optimal capital structure is indeterminate. In this case, there is no benefit or cost from issuing debt.

However, when $\tau > 0$ and $\psi > 0$, the Modigliani-Miller theorem no longer applies. As seen in (14), the tax subsidy and bankruptcy costs now affect the firm's investment decision. By affecting the net tax benefit, $\tau q b' - B$, a marginal change in k' now has an additional benefit or cost. Consequently, whether the optimal k' is above or below the first-best level of k' depends on how a marginal change in k' affects the net tax benefit. Under our benchmark parameterization, $\tau \frac{\partial (qb')}{\partial k'} > \frac{\partial B}{\partial k'}$, we imply that k' can be higher than the first-best level of k'. Moreover, when $\tau > 0$ and $\psi > 0$, debt is beneficial to the firm because it increases the tax subsidy it receives. At the same time, more debt makes default more likely and increases the expected costs of bankruptcy. Consequently, as seen in (15), firms choose b' to equate the marginal tax benefits of debt with marginal bankruptcy costs.

The left panel of Figure 2 provides a visual characterization of the optimal capital structure. Since external equity is costless, internal and external equity are perfect substitutes. Hence, internal equity does not have any effect on the optimal value for k' and b', which are both horizontal lines. In what follows, we denote by k^* and b^* the firm's optimal choice of k' and b' when $\lambda_0 = 0$. Given that $k' = k^*$ and $b' = b^*$ for any value of e, it follows from the firm's budget constraint in (6) that the optimal dividend policy is then just a straight line (with a slope of 1). Firms with low (or even negative) internal equity are able to choose $k' = k^*$ because they can issue equity costlessly. Firms with large amounts of internal equity choose $k' = k^*$ and also choose to issue a positive dividend.

Costly Equity Issuance

Now we assume that external equity is costly (i.e., $\lambda_0 > 0$). In this case, the first-order conditions for b' become

$$(\tau + I_{d<0}\lambda_0(1+\tau)) \left[q + \frac{\partial q}{\partial b'} b' \right] = \frac{\partial B}{\partial b'}. \tag{16}$$

This condition will only hold when $d \neq 0$. In the case of costly external equity, the marginal cost of an additional unit of debt is the same. Nevertheless, there is potentially an additional benefit of debt. In particular, an additional unit of debt allows the firm to substitute away from costly external equity. As seen in (16), a marginal increase in b'

[10] See Modigliani and Miller (1958).

means that the firm is able to raise $(1+\tau)\left[q+\frac{\partial q}{\partial b'}b'\right]$ in extra funds through the debt market (and through an additional tax subsidy). For each unit of extra funds raised, the firm is able to save on the external equity cost λ_0.[11]

Similarly, the first-order condition for k' is now given by

$$\tau\frac{\partial q}{\partial k'}-\frac{\partial B}{\partial k'}+\frac{E[z'f'(k')|z]+1-\delta}{1+r}=1+I_{d<0}\lambda_0\left[1-(1+\tau)\frac{\partial q}{\partial k'}b'\right]. \tag{17}$$

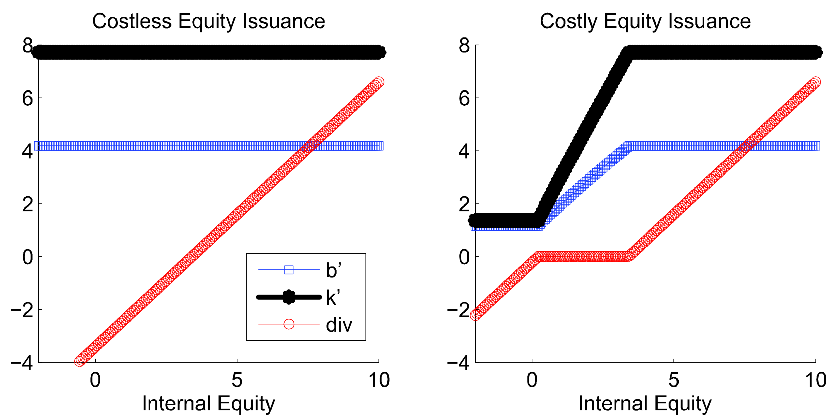
This condition only holds with equality when $d\neq 0$. In the case of costly external equity, the marginal benefit of additional investment is the same. However, there is now potentially an additional cost associated with increasing k'. When the firm is already relying on external equity ($d<0$), the additional unit of k' must be financed with expensive external equity. Since a higher k' tends to lower the price on existing debt, the firm only needs to raise $1-(1+\tau)\frac{\partial q}{\partial k'}b'$ of external equity. For every unit of additional external equity the firm raises, it must pay the cost λ_0.

The right panel of Figure 2 plots the policy functions for k', b', and d as a function of internal equity when external equity is costly. Examination of Figure 2 reveals that firms now behave differently depending on how much internal equity they have (their initial size). There are three regions of interest: (1) firms with low levels of internal equity, (2) firms with medium levels of internal equity, and (3) firms with high levels of internal equity.

First consider firms with low (but not necessarily negative) levels of internal equity. From Figure 2, it can be seen that $k'<k^*$, $b'<b^*$, and $d<0$. Because these firms start out with low levels of internal equity, they need to issue equity to reach even low levels of k'. Consequently, it is still beneficial to issue even a small amount of external equity to increase their investment. However, because of the cost, they do not issue as much as they would when $\lambda_0=0$. Nevertheless, even though they choose $b'<b^*$, it is the case that $b'/k'>b^*/k^*$. Because of the high cost of external equity, they still do substitute toward more debt relative to a lower level of k'. As internal equity increases they substitute external with internal equity while maintaining the same amount of investment and debt issuance.

[11] We should note that in the infinite-horizon version of this model, issuing debt will be associated with one more cost. In particular, the firm might want to issue less debt in case it ends up receiving a bad draw tomorrow and issuing costly equity to avoid default. This is a precautionary savings mechanism for the firm. In our two-period version there are only positive payments to shareholders in the second period.

Figure 2 Policy Functions



Notes: Both panels plot k', b', and d as a function of internal equity. Left panel: External equity is costless ($\lambda_0 = 0$). Right panel: External equity is costly ($\lambda_0 > 0$). The policy functions at the left panel coincide with the policy functions at the right panel for high values of internal equity.

Now consider firms with medium levels of internal equity. These firms choose $k' < k^*$ and $b' < b^*$, but also $d = 0$. Intuitively, the first-order conditions for b' and k' in (16) and (17) do not hold with equality. Because they have more internal equity, they avoid issuing costly external equity. Instead, they rely only on internal funds and debt to finance investment. However, firms in this region will use any additional internal equity to increase their investment (while maintaining $d = 0$). As a result, both k' and b' are increasing with e. Moreover, as firms obtain more internal equity, b'/k' is decreasing toward b^*/k^*.

Finally, consider firms with high levels of internal equity. These firms have so much internal equity that they are able to choose $k' = k^*$ and $b' = b^*$ without having to raise external equity. When external equity was costless, they chose $d > 0$. Costly external equity has no effect on them because they were not raising external equity anyway. Hence, their behavior coincides with the case of costless external equity where investment and debt issuance are constant and the firms are issuing positive dividends.

Cyclicality of Debt and Equity Issuance

Here we use our stylized framework to analyze the effects of productivity changes (z) on investment, debt, and equity issuance (k', b', and

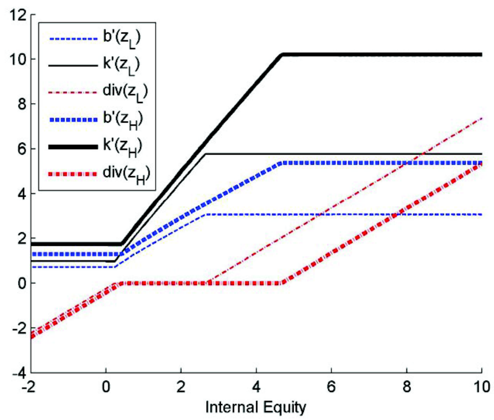
d, respectively). Figure 3 plots the policy functions for k', b', and d as a function of internal equity when external equity is costly. We plot the policy functions when productivity is low ($z = z_L$) and when productivity is high ($z = z_H$). A higher value of productivity will affect the firm's capital structure in two ways. First, internal equity $e(z, k, b) \equiv zf(k) + (1 - \delta)k - b$ will increase. Second, if shocks are autocorrelated (which is true in our simple example), a higher z in the first period will imply a higher expected z' in the next period. Using Figure 3, we can distinguish between the two since we plot how the policy functions change for a given amount of internal equity.

Looking at Figure 3, we see that higher productivity shifts k' upward since the marginal benefit of investing increases (see Equation [14]). This means that a fraction of previously unconstrained firms will find themselves constrained since the same amount of e will not be enough to sustain the larger amount of investment. Debt issuance b' will also increase. As firms invest more, the default threshold decreases for any given $b' > (1 - \theta)(1 - \delta)k'$ (i.e., $\partial z^*/\partial k' < 0$). This increases the borrowing capacity of the firm and lowers the marginal bankruptcy costs for each individual b'. Since the tax benefit of debt is $\tau qb'$, the higher borrowing capacity also increases the marginal benefit of issuing debt. Both effects cause b' to increase for a given level of internal equity. The increase in debt issuance is not uniform across firm sizes though. Smaller firms issue less debt than larger firms.

External equity issuance will increase (or dividend payout will decrease) in response to an increase in productivity. Firms with low amounts of internal equity e will increase their equity issuance to sustain a larger amount of investment. Since equity issuance is costly, they will change their issuance by only a small amount. Firms with a medium level of e will not issue equity or distribute any dividends. However, the set of (constrained) firms that do not distribute any dividends will increase. Similarly, firms with a high level of e will decrease the amount of dividends that they pay out.

Hence, for a given amount of internal equity our simple model predicts a procyclical debt and equity issuance. Of course, as stated before, e will also increase if z increases. A larger internal equity will represent a movement along the policy functions. This can potentially increase debt issuance but decrease external equity issuance (or increase dividend payout). So while debt issuance is definitely procyclical, equity issuance might be procyclical or countercyclical depending on how strong the opposing effects are. Based on Figure 3 it seems that for smaller firms the equity issuance is more likely to be countercyclical but for larger firms it is more likely to be procyclical.

Figure 3 Policy Functions for Different Levels of Productivity



Notes: We plot k', b', and d as a function of internal equity for a low value of productivity z_L and a high value of productivity z_H. Thin lines are used for z_L and thick lines are used for z_H. External equity is costly in both cases.

4. FULL MODEL

Utilizing the basic ingredients of our stylized two-period model in Section 3, we now build a fully dynamic model with heterogeneous firms and aggregate productivity shocks. Nevertheless, to keep the analysis simple, we assume a partial equilibrium framework.

Entrepreneurs and Firms

The economy is populated by a continuum of entrepreneurs. Each entrepreneur operates a firm. Entrepreneurs, and thus the firms they operate, differ with respect to their idiosyncratic productivity z. Firms are perfectly competitive and produce a single homogeneous good. Capital k and labor l are inputs into the firm's production function, $y = Az(k^\alpha l^{1-\alpha})^\gamma$, where A is aggregate productivity. We assume that $\gamma \in (0, 1)$, implying that there are decreasing returns to scale at the firm level. With the assumption of perfect competition, diminishing returns to scale enable heterogeneity to exist in equilibrium. Assuming a competitive labor market, the firm's profits can be denoted by

$$\pi(A, z, k) = \max_l \{Az(k^\alpha l^{1-\alpha})^\gamma - wl\}, \tag{18}$$

where w is the real wage. Since this is a partial equilibrium analysis, the wage w is normalized to 1.

We assume that both $\ln z$ and $\ln A$ follow an AR(1) process:

$$\ln z' = \rho_z \ln z + \varepsilon^z$$
$$\ln A' = \rho_a \ln A + \varepsilon^A,$$

where $\varepsilon^z \sim N(0, \sigma_\varepsilon^z)$ and $\varepsilon^A \sim N(0, \sigma_\varepsilon^A)$. Since z is an idiosyncratic shock, ε^z is assumed to be independent of ε^A. We denote by $F(z'|z)$ and $f(z'|z)$ the cumulative distribution and probability density functions for next period's productivity z', conditional on the current productivity z. Similarly, let $p(A'|A)$ denote the probability density function for A', conditional on current aggregate productivity A.

Every period firms choose how much capital to invest for next period k'. Investment is subject to a capital adjustment cost $g(k, k')$. We will assume that this function takes the form $g(k, k') = \phi\frac{(k'^2}{k}$. This will guarantee a gradual transition of firms toward their optimal size. Firms issue bonds b', which are priced at $q(A, z, k', b')$. This price will be determined endogenously based on the investment and debt issuance decisions of the firm as well as the idiosyncratic and aggregate shocks. As in Section 3, firms receive a tax subsidy from the government, $\tau q(A, z, k', b')b'$. Firms also have the option of distributing dividends ($d > 0$) or issuing equity ($d < 0$). As in Section 3, we assume that external equity is costly. However, now we specify the cost $\Lambda(A, d)$ as follows:

$$\Lambda(A, d) = \begin{cases} A^{-\lambda_1}\frac{\lambda_0}{2}d^2 & \text{if } d < 0 \\ 0 & \text{if } d \geq 0 \end{cases} .$$

Following Covas and Den Haan (2012), we assume that equity issuance costs are lower during expansions. This assumption will be critical to match the procyclicality of equity issuance in the data.

After the firm chooses k', b', and d, it may exit next period. We assume there are two reasons a firm may exit. First, a constant fraction η will exogenously be forced to exit. In this case, it is assumed that the entire firm value is destroyed. This implies that the firm will default and both the entrepreneur and lender will recover nothing. Second, depending on tomorrow's realization of A' and z', some entrepreneurs will endogenously default on their debt obligations. In this case, we assume that the firm is liquidated but that the entrepreneur lives on to found a new firm (a start-up). We discuss this default decision in the next subsection in more detail.

Default Decision

In deciding whether or not to default, the entrepreneur compares the value of "not defaulting" to the value of "defaulting." We define $V^{ND}(A,z,k,b)$ to be the value of not defaulting for a firm with state (A,z,k,b). Similarly, we define $V^{D}(A,z,k)$ to be the firm's value of default. These value functions will be defined below. Given these value functions, the firm's total value $V(A,z,k,b)$ is defined to be

$$V(A,z,k,b) = \max\left\{V^{ND}(A,z,k,b), V^{D}(A,z,k)\right\}. \tag{19}$$

If $V^{ND}(A,z,k,b) \geq V^{D}(A,z,k)$, the firm pays back its debt b and continues its operations. Otherwise, the firm chooses not to pay back its debt b and defaults.

The value of not defaulting, $V^{ND}(A,z,k,b)$, is then defined to be

$$V^{ND}(A,z,k,b) = \max_{d,k',b'} \left\{d - \Lambda(A,d) + \frac{1-\eta}{1+r} E\left[V(A',z',k',b')|A,z\right]\right\} \tag{20}$$

$$\text{s.t.} \quad d = \pi(A,z,k) + (1-\delta)k - b + (1+\tau)q(A,z,k',b')b' - k' - g(k,k'). \tag{21}$$

If the firm does not default, it chooses how much to invest (k'), how much debt it will issue (b'), and if it will distribute dividends ($d > 0$) or issue equity ($d < 0$). It makes these decisions subject to the budget constraint in (21). As noted earlier, the firm must also pay an equity issuance cost ($\Lambda(A,d) > 0$) if it issues equity ($d < 0$). Next period, with probability η, the entrepreneur receives the exogenous exit shock and receives nothing. With probability $1-\eta$, however, the firm does not exogenously exit. In this case, depending on tomorrow's realization of A' and z', the firm can decide tomorrow whether to default or continue operating.

If the firm defaults, it shuts down its operations and is liquidated. Nevertheless, the entrepreneur can hide a fraction θ of the firm's undepreciated capital. Moreover, the entrepreneur can start a new firm next period. Hence, the owner can transfer his idiosyncratic productivity to a different project while eliminating his debt obligations. Given these assumptions, the value of defaulting, $V^{D}(A,z,k)$ is assumed to be

$$V^{D}(A,z,k) = \left\{\theta(1-\delta)k + \frac{1}{1+r} E\left[V^{s}(A',z')|A,z\right]\right\}, \tag{22}$$

where $V^{s}(A',z')$ is the value of a start-up tomorrow with aggregate productivity A' and idiosyncratic productivity z'. This value function will be defined later.

In general, we can define a threshold $z^{*}(A,k,b)$ such that firms with capital k, debt b, and idiosyncratic productivity lower than $z^{*}(A,k,b)$

will default. This threshold is defined to be the value of idiosyncratic productivity z^* such that the firm is just indifferent between defaulting and not defaulting:

$$V^{ND}(A, z^*, k, b) = V^D(A, z^*, k). \tag{23}$$

Consequently, this default threshold will depend on the aggregate level of productivity (A) as well as the firm's individual levels of capital (k) and debt (b). The default threshold z^* increases if debt b is large and decreases if capital k is large or if the economy is booming (A is high).

Bond Price

The firm issues bonds that are purchased by risk-neutral households. Households lend $q(A, z, k', b')b'$ to firms today, and in return the firm promises to pay b' next period. Given that the default is possible, the price $q(A, z, k', b')$ is set to guarantee the lender an expected return equal to the risk-free rate r. Consequently, the bond price will be given by

$$q(A, z, k', b') = \frac{1-\eta}{1+r}\left[1 - F(z^*(A', k', b')|z) + \frac{R(A, z, k', b')}{b'}\right], \tag{24}$$

where

$$R(A, z, k', b') = (1-\psi)(1-\theta)\int_0^\infty \int_0^{z^*(A',k',b')} (1-\delta)k' f(z'|z)p(A'|A)dz'dA'$$

is the unconditional recovery value of the bond. With probability η, the firm receives an exogenous exit shock and the lender receives nothing. However, with probability $(1 - \eta)$, the firm does not receive an exit shock. In this case, the firm does not default with probability $1 - F(z^*|z)$ and the lender receives b'. However, if the firm defaults, then the lender receives fraction $(1 - \psi)(1 - \theta)$ of its undepreciated capital. The parameter θ controls how much of the capital stock the entrepreneur can hide while ψ reflects the bankruptcy costs.

Entry

As noted earlier, there are two reasons firms exit in this model. First, a fraction η of firms will exogenously exit. The entrepreneurs of these firms are assumed to be replaced by "new" entrants. Therefore, while a constant fraction of entrepreneurs exit each period, a constant mass of entrepreneurs are born each period. These new entrepreneurs are assumed to draw their initial idiosyncratic productivity from the invariant distribution for z. Second, some of the remaining firms will endogenously choose to default. The entrepreneurs of these firms, however, are able to continue. In particular, these entrepreneurs can start a new firm (start-up) in the next period.

Therefore, in every period, firms will be destroyed and created at the same time. Because firms are assumed to be born with no capital, a start-up will have zero profits in the first period. Then, a start-up firm will choose how much to invest (k'). This investment can be financed by raising equity ($d < 0$) or by issuing debt (b'). Let $V^s(A, z)$ denote the value of a start-up with aggregate productivity A and idiosyncratic productivity z. This value is defined to be

$$V^s(A,z) = \max_{d,k',b'} \left\{ d - \Lambda(A,d) + \frac{1-\eta}{1+r} E\left[V(A',z',k',b') | A,z \right] \right\} \quad (25)$$

$$\text{s.t.} \qquad d = (1+\tau) q(A,z,k',b') b' - k'.$$

Therefore, the problem of a start-up is very similar to the problem of a continuing firm. However, a start-up begins its life with no debt and no assets. Because the start-up has no initial capital, it is assumed that it does not pay any capital adjustment costs.

Timing

The timing of the economy can be described as follows.

1. All entrepreneurs/firms receive productivity draws A and z.
2. A fraction η of firms are exogenously destroyed.
3. Surviving firms with state $\{z, A, k, b\}$ decide to default if $z < z^*(A, k, b)$. Firms that default exit.
4. Firms that did not default, as well as new start-ups, make investment and firm financing (debt and equity) decisions.

5. QUANTITATIVE ANALYSIS

In this section we quantitatively characterize our model of firm financing. We calibrate our model either using parameters commonly used in the literature or targeting specific moments computed in the data. We compare the model's predictions for the same set of statistics computed from Compustat in Section 2.

Table 3 Parameter Values

Parameter	Description	Value	Target
r	Real interest rate	0.04	Standard
δ	Depreciation rate	0.10	Standard
α	Capital share	0.36	Standard
γ	Returns to scale	0.65	Gomes and Schmid (2010)
η	Exit rate	0.04	Cooley and Quadrini (2001)
ψ	Bankruptcy cost	0.25	Arellano, Bai, and Zhang (2012)
λ_0	Equity issuance cost	0.75	Covas and Den Hann (2012)
λ_1	Equity issuance cost	20	Covas and Den Hann (2012)
ρ_z	Persistence of z	0.55	Clementi and Palazzo (2014)
ρ_a	Persistence of A	0.68	Clementi and Palazzo (2014)
σ_ε^z	Standard deviation of ε^z	0.18	S.D. of sales growth
σ_ε^A	Standard deviation of ε^A	0.016	Clementi and Palazzo (2014)
τ	Tax credit	0.07	Mean leverage
θ	Hidden fraction	0.93	Mean default
ϕ	Capital adjustment cost	0.10	Mean of sales growth

Notes: This table reports the parameter values used in the quantitative model. Each parameter is calibrated either based on the literature or targeting a specific moment.

Calibration

All parameter values are reported in Table 3. The model is computed at an annual frequency. We normalize the wage rate to 1 and set an annual risk-free rate of 4 percent. The depreciation rate is set at 10 percent, a value commonly employed in the literature. The capital share equals $\alpha = 0.36$ and, based on Gomes and Schmid (2010), the decreasing returns to scale parameter is $\gamma = 0.65$. The firms' exit rate η is set to 0.04 based on Cooley and Quadrini (2001). Bankruptcy cost equals $\psi = 0.25$ based on Arellano, Bai, and Zhang (2012). Following Covas and Den Haan (2012), we assume that equity issuance costs are lower during expansions and set $\lambda_0 = 0.75$ and $\lambda_1 = 20$.

The persistence of idiosyncratic productivity $\rho_z = 0.55$ is based on Clementi and Palazzo (2014). Although the authors provide an estimate for σ_ε^z, we choose to use this parameter to match a specific moment (see below). We also borrow their estimates to calibrate the persistence and standard deviation of the aggregate productivity process. In particular, ρ_a is set to 0.68 and σ_ε^A is chosen to be 0.016.

The remaining parameters, $\{\tau, \theta, \phi, \sigma_\varepsilon^z\}$, are chosen to match specific model moments. In particular, a higher tax benefit τ will encourage firms to issue more debt and increase their leverage ratio. Therefore, to match the mean leverage ratio observed in Compustat, τ is set to 0.02. Conditional on the value of ψ, a larger value of θ induces more firms to default since they can hide and keep a larger fraction of their assets. Thus, to match the mean default rate in the economy, θ is set to 0.93. The adjustment cost parameter ϕ affects how fast firms grow. Hence, to match the average cross-sectional growth rate of sales, ϕ is set to 0.10. Finally, a larger dispersion in idiosyncratic productivity will lead to a larger dispersion in the growth of sales. With a value of $\sigma_{\varepsilon}^{z} = 0.18$, the model matches the cross-sectional standard deviation of sales growth.

Steady-State Results

We start by characterizing the steady state of the economy. In the steady state, aggregate productivity is constant in every period ($A = 1$). Based on our policy functions, we simulate a panel of firms and track their behavior over time. We use the stationary distribution to construct several statistics and compare them to the ones computed from Compustat. Table 4 gives a summary of the results.

In Compustat the distribution of leverage across firms is found to be highly skewed to the right. Excluding firms at the top 1 percent of the distribution, the average leverage ratio is 27 percent. Our model economy is able to match this statistic by targeting the tax credit τ. In contrast, leverage ratios are more dispersed in the data than our model. The standard deviation of leverage in Compustat is 0.37, much higher than the model's result of 0.15. A reason for this failure is the relatively low value for the persistence of idiosyncratic productivity ρ_z. If idiosyncratic shocks are not very persistent then even unproductive firms can easily get access to credit. Indeed, we have experimented with higher values of ρ_z and found that the standard deviation of leverage increases. Moreover, the model can perform well with respect to sales growth. The mean of sales growth in the model is 0.12, very close to the value computed in the data (0.11). This statistic was targeted using the adjustment cost parameter ϕ. The model can also capture the dispersion in sales growth rates (0.45 in the model versus 0.51 in the data). To match this moment, we used the dispersion of idiosyncratic productivity shocks σ_{ε}^{z}.

Table 4 Steady-State Results

Statistic	Data-Compustat Small Firms	Data-Compustat Large Firms	Data-Compustat All	Model Small Firms	Model Large Firms	Model All
Mean (Leverage)	0.27	0.28	0.27	0.30	0.23	0.27
S.D. (Leverage)	–	–	0.34	–	–	0.15
Mean (Sales Growth)	0.12	0.10	0.11	0.21	0.04	0.12
S.D. (Sales Growth)	–	–	0.51	–	–	0.45

Notes: This table shows the steady-state results for the mean and standard deviation of leverage and sales growth, respectively. Statistics on leverage and sales growth are calculated from the data (Compustat).

We next compare the behavior of small versus large firms. Using data from Compustat and consistent with Rajan and Zingales (1995) and Cooley and Quadrini (2001), we find a positive relationship between leverage and total assets. However, the differences seem to be minor as firms with assets smaller than the median have a leverage equal to 0.27 while firms with assets larger than the median have a leverage equal to 0.28. In our model, these numbers are 0.30 and 0.23, respectively. In Section 3, we saw that as firms obtain more internal equity, the ratio $\frac{b'}{k'}$ decreases. Smaller firms (with lower internal equity) substitute more toward debt to avoid using costly external equity. Moreover, due to decreasing returns to scale, the model replicates qualitatively the empirical observation that smaller firms grow faster. In the model, sales growth is 0.21 for small firms and 0.04 for large firms. In Compustat, these numbers are 0.12 and 0.10, respectively. In general the model captures the basic features of the data with some success.

Business Cycle Results

We now allow the economy to experience aggregate productivity shocks. To avoid further computational complexity we assume that the prices do not adjust in response to productivity changes. If we allowed for a general equilibrium framework, we would have to keep track of the distribution of firms over debt, capital, and equity, which would greatly increase the state space.

Table 5 reports the correlation between debt and equity issuance with aggregate output. To facilitate the comparison with the data, we include information from the top panel of Table 2 that excludes only major mergers from the sample. In Section 2, we showed that mergers are an important way that firms raise equity. The model replicates the positive correlation between debt issuance and aggregate output (0.868 in the model versus 0.785 in the data). As explained in Figure 3, a higher productivity increases k', allowing the firm to issue more debt. Table 5 also reports how the cyclicality differs among small and large firms. In Section 2, we documented that the cyclicality is stronger for larger firms (excluding the top 1 percent). Our model replicates this pattern and can match very closely the cyclicality of firms in the [75, 99] bin (0.737 in the model versus 0.755 in the data). In response to an increase in productivity, a small firm may disproportionately increase b' by disproportionately decreasing external equity issuance. In contrast, large firms that issue a small amount of external equity will

Table 5 Business Cycle Results

	Size Class (Percent)				
Data	**[0, 50]**	**[50, 75]**	**[75, 99]**	**[99, 100]**	**[0, 100]**
ΔD	**0.536**	**0.611**	**0.755**	**0.547**	**0.785**
$\Delta E(1)$	**0.345**	0.191	0.016	0.044	0.096
$\Delta E(2)$	0.243	−0.250	**−0.617**	−0.312	**−0.509**
E^I	**0.353**	0.268	0.306	0.250	**0.363**
E^P	0.069	0.279	**0.654**	0.314	**0.588**
	Size Class (Percent)				
Model	**[0, 50]**	**[50, 75]**	**[75, 99]**	**[99, 100]**	**[0, 100]**
ΔD	0.260	0.244	0.737	0.277	0.868
ΔE	0.447	−0.326	−0.714	−0.942	−0.764
E^I	0.528	0.445	0.356	—	0.386
E^P	0.287	0.335	0.715	0.942	0.759

Notes: This table reports the model-generated business cycle properties of debt and equity issuance. This table also reports empirical statistics as calculated in Section 2. For simplicity we report empirical measures that exclude only major mergers from our sample. For the empirical section, we show coefficients that are significant at the 5 percent level in bold.

increase b' in a relatively proportional manner.[12] As a result, we find the correlation between debt issuance and output (productivity) to be much higher in the case of large firms. A similar nonlinearity occurs for the largest firms when they start distributing dividends, which explains why the correlation decreases for that group.

The model also generates a countercyclical equity issuance. In Section 2, we documented that equity issuance can be weakly procyclical or countercyclical depending on the way we measure equity. Moreover, we have shown that much of the procyclicality is due to raising equity through mergers and that the cyclicality becomes negative if we just consider net sale of stock. In the model, net equity issuance ΔE is strongly countercyclical. Similar to the data, we break net equity issuance ΔE into a gross equity issuance E^I and a gross equity payout E^P component, with $\Delta E = E^I - E^P$. Our decomposition reveals that the strong countercyclicality of net equity issuance is driven by a strongly procyclical gross dividend payout. In the model, smaller firms prefer to raise more gross equity than paying out gross dividends during expansions. This leads to a procyclical equity finance for firms in the [0, 50] bin, similar to what we observe in the data. Nevertheless, the

[12] To understand these properties better we refer the reader to Figure 3. Although in our fully dynamic model we include quadratic capital adjustment costs and quadratic equity issuance costs, the basic properties of the policy functions remain intact.

procyclicality of equity issuance for small firms relies on our assumption (among others) of countercyclical equity issuance costs. Overall, the model is consistent with the empirical patterns we see in equity financing.

6. CONCLUSION

This article provides an introductory, yet comprehensive, business cycle analysis of firm financing. We first document several empirical patterns of debt and equity issuance based on data from Compustat. While we find that debt issuance is strongly procyclical, the cyclicality of net equity issuance depends on the exact definition used. If we define equity using the net sale of stock (following Jermann and Quadrini [2012]), we find net equity issuance to be countercyclical. Alternatively, if we define equity issuance using the change in the book value of equity (following Covas and Den Haan [2011]), we find net equity issuance to be weakly procyclical. Nevertheless, we find that equity financing through mergers and, to a lesser extent, stock compensation can explain much of the discrepancy between the two measures. Moreover, regardless of the measure used, the countercyclicality of net equity issuance is driven by a strongly procyclical gross payout to equity and not countercyclical gross equity issuance. Overall, these empirical findings should be useful in evaluating theoretical models, which stress the role of the financial sector in propagating aggregate fluctuations. Of particular interest, perhaps, is the heterogeneous behavior of firm financing and the role of mergers and acquisitions.

To help build intuition, we analyze the firm's optimal capital structure within a simple two-period model. Then, to determine how well our framework can match the cyclical properties of firm financing, we build a fully dynamic quantitative model. The model features heterogeneous firms that endogenously choose their capital structure by balancing the tax benefits against the bankruptcy costs of debt issuance and the expenses associated with equity issuance. The model generates a procyclical debt and countercyclical net equity issuance. Moreover, the model can match the firm-size relationship regarding debt and especially equity issuance. Overall, the model is useful for illustrating the important mechanisms involved. While firms issue more debt to finance more investment, the model highlights that equity issuance provides conflicting motives for the firm. On the one hand, firms would like to issue more equity (which may be costly) to finance more investment. On the other hand, firms would like to pay out more dividends in good times. For most firms the second effect dominates in our model.

However, to generate procyclical net equity issuance for small firms, we assume that equity issuance costs are lower during expansions.

APPENDIX A: DATA SOURCES

We obtain annual data from Compustat between 1980 and 2013. We exclude financial firms (SIC 6000–6999) and utilities (SIC 4900–4999). We drop any firm-year observations if we do not have any information on assets, capital stock, debt, or both equity measures. We drop observations that violate the accounting identity by more than 10 percent. We drop firms affected by 1988 accounting change (GM, GE, Ford, Chrysler).[13] We only include firms reporting in USD. One important concern is whether we include firms affected by a merger or an acquisition. For this purpose, we separately report our results for two cases. In the first case, we follow Covas and Den Haan (2011) and drop all firm-year observations that are affected by a "major" merger or acquisition. By "major" we mean that the merger or acquisition causes the resulting firm's sales to increase by more than 50 percent. In the second case, we drop all observations affected by any kind of merger. To identify whether a firm was involved in a merger, we use the footnote code on sales. Compustat assigns the footnote code AB if the data reflects a major merger or acquisition. Meanwhile, footnote code AA reflects other acquisitions.

SE is defined as the book value of stockholder's equity (data item #216) minus retained earnings (data item #36). $\Delta E(1)$ is defined to be the annual change in SE minus cash dividends (data item #127). The net sale of stock is defined to be the funds received from the issuance of common and preferred stocks (data item #108) minus equity repurchases (data item #115). $\Delta E(2)$ is defined to be the net sale of stock minus cash dividends. RE is the balance sheet item for retained earnings (data item #36). w_S is stock compensation (data item #398). Sales is given by data item #12, which represents gross sales (i.e., the amount of actual billings to the customers). Total assets is the book value of assets (data item #6). We define debt as the sum of debt in current liabilities (data item #34) and long-term debt (data item #9). The capital stock K is (net) property, plant, and equipment (data item

[13] See Bernake, Campbell, and Whited (1990) for details.

#8). Investment I equals capital expenditures on property, plant, and equipment (data item #30).

And finally, we obtain real corporate GDP from the Bureau of Economic Analysis's National Income and Product Accounts. Particularly, we use Table 1.14, which reports the gross value added of domestic nonfinancial corporate business, in billions of chained (2009) dollars.

APPENDIX B: SIMPLIFIED VALUE FUNCTION

In (12), the firm's problem was given by

$$V(z,k,b) = \max_{d,k',b'} \left\{ \begin{array}{c} d - \Lambda(d) + \\ \frac{1}{1+r}\int_0^\infty \max\left\{V^{ND}(z',k',b'), V^D(z',k')\right\} f(z'|z)dz' \end{array} \right\},$$

subject to the budget constraint, which is

$$d + k' = e(z,k,b) + (1+\tau)q(z,k',b')b'.$$

Using the definitions of $V^{ND}(z',k',b')$ and $V^D(z',k')$ in (8) and (9), we can re-write the firm's value function as follows:

$$V(z,k,b) = \max_{d,k',b'} \left\{ d - \Lambda(d) + \frac{1}{1+r}\int_0^\infty [z'f(k') + (1-\delta)k']f(z'|z)dz' \right.$$
$$\left. \begin{array}{c} -\frac{(1-F(z^*(k',b')|z))b'}{1+r} \\ -\frac{1-\theta}{1+r}\int_0^{z^*(k',b')}[z'f(k') + (1-\delta)k']f(z'|z)dz'] \end{array} \right\}.$$

When we substitute for d using the firm's budget constraint, this becomes

$$V(z,k,b) = \max_{d,k',b'} \left\{ \begin{array}{c} e - k' - \Lambda(d) + (1+\tau)q(z,k',b')b' \\ +\frac{1}{1+r}\left[E[z'f(k')|z] + (1-\delta)k'\right] \end{array} \right.$$
$$\left. \begin{array}{c} -\frac{(1-F(z^*(k',b')|z))b'}{1+r} - \\ \frac{1-\theta}{1+r}\int_0^{z^*(k',b')}[z'f(k') + (1-\delta)k']f(z'|z)dz'] \end{array} \right\}.$$

Moreover, in (11), the bond price was defined to be

$$q(z,k',b') = \frac{1}{1+r}\left[1 - F(z^*(k',b')|z) + \frac{R(z,k',b')}{b'}\right],$$

where

$$R(z,k',b') \equiv (1-\psi)(1-\theta)\int_0^{z^*(k',b')}\left[z'f(k') + (1-\delta)k'\right]f(z'|z)dz'.$$

Therefore, using that $qb'(1+r) = [1 - F(z^*)]\, b' + R$, we arrive at (13):

$$V(z,k,b) = \max_{d,k',b'} \left\{ \begin{array}{l} e - k' - \Lambda(d) + \tau q(z,k',b')b' + \\ \frac{1}{1+r}\left[E[z'f(k')|z] + (1-\delta)k'\right] + \end{array} \right.$$

$$\left. - \ \frac{\psi(1-\theta)}{1+r}\int_0^{z^*(k',b')} [z'f(k') + (1-\delta)k']f(z'|z)dz' \right\}.$$

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