FACTORS DETERMINING EXCHANGE RATES: A SIMPLE MODEL AND EMPIRICAL TESTS

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This article constructs and tests a simple static equilibrium model of exchange rate determination.[1] The model assumes a regime of freely floating currencies and posits that the exchange rate, by definition the relative price of two national moneys, is determined by the basic factors underlying the demands for and supplies of those national money stocks. Besides the money supply itself, these factors include real income and interest rates—the latter reflecting expectational influences that enter into exchange rate determination.

The article proceeds as follows. First, it discusses the logic and economic content of the individual equations that constitute the major building blocks of the model. Second, it condenses the model to one reduced-form equation that expresses a functional relationship between the exchange rate and its ultimate determinants. Third, it fits the foregoing equation to the statistical data on several foreign exchange rates, assesses the accuracy of the fit, and discusses some problems involved in testing the model.

The Model and Its Elements The model itself consists of two hypothetical national economies represented by a set of equations containing the following variables. Let M be the nominal money stock (assumed to be exogenously determined by the central bank) and m the demand-adjusted rate of growth of that stock, i.e., the difference between the respective growth rates of the nominal money supply and real money demand, this difference by definition being equal to the rate of price inflation. Furthermore, let D be the real demand for money, i.e., the stock of real (price-deflated) cash balances that the public desires to hold, Y the exogenously-determined level of real income, and i and r the nominal and real rates of interest, respectively. Also let X be the exchange rate (defined as the domestic currency price of a unit of foreign currency), P be the price level, and E be the expected future rate of price inflation. Asterisks are used to distinguish foreign-country variables from home-country variables, and the subscript w denotes the entire world economy. The foregoing elements are linked together via the relationships described below.

Monetary Equilibrium Equations The first part of the model consists of monetary equilibrium equations, one for each country

(1) $P = M/D$ and $P^* = M^*/D^*$.

These equations, which can also be written in the form $M/P = D$, state that the price level in each country adjusts to bring the real (price-deflated) value of the nominal money stock into equality with the real demand for it, thereby clearing the market for real cash balances. This market-clearing price-adjustment process relies chiefly on equilibrating changes in aggregate expenditure induced by discrepancies between actual and desired real balances. For example, if actual balances exceed desired, cash-holders will attempt to get rid of the excess via spending for goods. Given the exogenously-determined level of real output, however, the increased spending will exert upward pressure on prices thereby reducing the real (price-deflated) value of the nominal money stock. Prices will continue to rise until actual real balances are brought down to the desired level. Conversely, a shortfall between actual and desired real balances will induce a cut in expenditure leading to a fall in prices and a corresponding rise in the real value of the money stock. This process will continue until actual real balances are brought into equality with desired balances. To summarize, disequilibrium between actual and desired real balances generates the changes in spending that cause prices to alter sufficiently to eliminate the disequilibrium.

Note that the equations also imply that, given the real demand for money, the price level is determined by and varies equiproportionally with the nominal money supply. This latter result, of course, is the

[1] Variants of the model have been employed by a number of analysts to explain recent exchange rate movements. See in particular the papers by Bilson [1, 2], Frenkel [4], and Putnam and Woodbury [7] cited in the list of references at the end of the article. Much of the relevant empirical work on the model is summarized in the surveys by Isard [5] and Magee [6].

essence of the quantity theory of money. For that reason, the equations could also be called quantity theory equations.

Real Cash Balance Equations National demand for money functions constitute the second part of the model. Written as follows

(2) $D = KYi^{-a}$ and $D^* = K^*Y^*i^{*-a}$

these equations express the public's demand for real cash balances as the product of a constant K and two variables, namely real income and the nominal interest rate. The income variable is a proxy for the volume of real transactions effected with the aid of money and thus represents the transaction demand for money. By contrast, the interest rate variable measures the opportunity cost of holding money instead of earning assets. The parameter —a, which appears as the exponent of the interest rate variable, is the interest elasticity of demand for money. It measures the sensitivity or responsiveness of money demand to changes in the interest rate and is assumed to be a negative number indicating that the quantity of real balances demanded varies inversely with the cost of holding them. For simplicity the numerical magnitude of the interest elasticity parameter is assumed to be the same for both countries. For the same reason the income elasticity of demand for money, as represented by the exponential power to which the income variable is raised, is assumed to possess a numerical value of unity.

The Purchasing Power Parity Equation The third equation of the model is the purchasing power parity relationship

(3) $P = XP^*$

showing how national price levels are linked together via the exchange rate. As indicated by the equation, prices in both countries are identical when converted into a common currency unit at the equilibrium rate of exchange. This means that the exchange rate equalizes such common-currency price levels and, by implication, the buying power of both moneys expressed in terms of a common unit. In other words, exchange-rate adjustment insures that a unit of a given currency commands the same quantity of goods and services abroad when converted into the other currency as it commands at home. This condition of equalized purchasing power is of course necessary if the two national money stocks are to be willingly held and monetary equilibrium is to prevail in both countries. For if the purchasing powers were unequal, people would demand more of the high- and less of the low-purchasing power currency on the market for foreign exchange. The resulting excess demand for the former and the corresponding excess supply of the latter would cause the exchange rate between the two currencies to adjust until purchasing power was equalized and both money stocks were willingly held. Note also that the purchasing power parity equation can be rearranged to read $X = P/P^*$, thus corresponding to the economic interpretation of the exchange rate as the relative price of the two currencies, i.e., as the ratio of the foreign currency's internal value in terms of goods to the domestic currency's internal value in terms of goods. Since the internal value of a unit of currency in terms of a composite market basket of commodities is the inverse of the general price level $1/P$, it follows that the relative price of the two moneys is simply the ratio of the national price levels as indicated by the equation.

Nominal Interest Rate Equations The fourth group of relationships in the model are the nominal interest rate equations, one for each country. Written as follows

(4) $i = r + E$ and $i^* = r^* + E^*$

they define the nominal interest rate as the sum of the real rate of interest and the expected future rate of inflation, the latter variable being the premium added to real yields to prevent their erosion by inflation.

Real Interest Rate Parity Condition The fifth equation expresses the interest-parity condition

(5) $r = r^* = r_w$

according to which the real rate of return on capital assets tends to be everywhere the same and independent of the currency denomination of the asset. This equation reflects the model's assumption of a highly-integrated efficient world capital market. In such a world, capital is mobile internationally, i.e., foreigners can purchase domestic securities and domestic citizens can purchase foreign securities. Given these conditions it follows that real yield equalization is necessary if all asset stocks are to be willingly held. Accordingly, the equation states that real interest rates in both countries are the same and are equal to a given constant world rate r_w. Note that equations 4 and 5 taken together imply that international nominal interest rate differentials reflect differences in expected future national rates of inflation. For example, if the market expects the future rate of inflation to be 12 percent in the UK and 5 percent in the

US, then the UK nominal interest rate will be 7 percentage points above the corresponding US interest rate.

Price Expectations Equations Completing the model are price expectations equations that describe how the public forms its anticipations of the future rate of inflation. These inflationary expectations constitute the anticipated future rates of depreciation of money holdings. As such, they enter the foreign and domestic demand for money functions via the nominal interest rate variables and thereby play an important role in exchange rate determination. Written as follows

(6) $E = m$ and $E^* = m^*$

the price expectations equations state that the expected rate of inflation E is equal to the demand-adjusted rate of monetary expansion m, i.e., the difference between the respective growth rates of the nominal money supply and real money demand.

As written, these equations embody the so-called *rational expectations hypothesis* according to which the public correctly bases its price forecasts on the variable that the model contends actually determines the rate of inflation. This feature insures that the model is internally consistent, i.e., that the equations describing the formation of inflationary expectations are consistent with equations describing how inflation is actually generated. Such consistency is characteristic of the forecasting behavior of rational agents who use knowledge about the actual inflation-generating process in forming expectations of future inflation. Since the model asserts that the actual rate of price inflation is determined by the demand-adjusted growth rate of money (see equation 1), it follows that the expected rate of inflation is determined by that same variable as shown in equation 6.

Linkages and Causation Taken together, the foregoing relationships constitute a simple six-equation model of exchange rate determination. For convenience the model is summarized below.

(1) $P = M/D$ and $P^* = M^*/D^*$.

(2) $D = KYi^{-a}$ and $D^* = K^*Y^*i^{*-a}$.

(3) $P = XP^*$.

(4) $i = r + E$ and $i^* = r^* + E^*$.

(5) $r = r^* = r_w$.

(6) $E = m$ and $E^* = m^*$.

The foregoing equations imply two unidirectional channels of influence—one direct, the other indirect—running from money and income (both exogenous variables) to prices to the exchange rate. Regarding the former channel, the model implies that both exogenous variables affect prices and the exchange rate directly through the monetary equilibrium and purchasing power parity equations. As for the indirect channel, the model implies that the rates of growth of the exogenous variables influence prices and the exchange rate indirectly via the price expectations component of the nominal interest rate variable that enters the demand for money function. More specifically, the model postulates the following causal chain:

1. The demand-adjusted money-stock growth rate determines the expected rate of inflation.
2. Given the real rate of interest, the expected rate of inflation determines the nominal rate of interest.
3. The latter variable, together with the given level of real income, determines the demand for money.
4. Given the demand for money, the nominal money stock determines the price level.
5. Finally, the two price levels, foreign and domestic, together determine the exchange rate.

In brief, when the demand-adjusted money growth rate rises, price expectations also rise and so too does the nominal interest rate (the cost of holding money). This reduces the quantity of real cash balances that people desire to hold, i.e., cashholders will want to get out of money and into goods. The resulting increased spending for goods puts upward pressure on the price level and, via the purchasing power parity nexus, also on the exchange rate. Clearly the linkages run from money stocks and real incomes to prices to the exchange rate.[2] Moreover, all variables affecting the exchange rate do so through monetary channels, i.e., through the demand for and supply of money. In this sense, money demand and supply may be said to constitute the *proximate* determinants of the exchange rate. The *ultimate* determinants, however, are the variables that underlie and determine the monetary factors themselves.

Determinants of the Exchange Rate To show the relationship between the exchange rate and its

[2] Note that reverse causality is effectively ruled out by the assumed exogeneity of the money stock and income variables. Therefore, while these variables can affect the exchange rate, the exchange rate cannot influence them—at least not within the context of the model.

ultimate determinants, simply substitute equations 1 - 2 and 4 - 6 into equation 3 and solve for the exchange rate. The resulting "reduced form" expression is

$$(7) \quad X = [K^*/K][M/M^*][Y^*/Y][i/i^*]^a$$

or, since the nominal interest rate i is the sum of the real interest rate r and the expected rate of inflation E—the latter variable itself being equal to the growth rate of money per unit of money demand m—the equation can be alternatively expressed as

$$(7') \quad X = \left[\frac{K^*}{K}\right]\left[\frac{M}{M^*}\right]\left[\frac{Y^*}{Y}\right]\left[\frac{r+m}{r^*+m^*}\right]^a.$$

Disregarding the fixed constants (the K's), equation 7 (or 7') collects the determinants of the exchange rate into three groups, namely relative money supplies, relative real incomes, and relative nominal interest rates comprised of a fixed real rate component and a variable price expectations component. Of these three groups, the first captures purely monetary influences on the exchange rate while the second and third capture real and expectational influences, respectively.

Regarding monetary and real influences, the equation predicts that a country's exchange rate will depreciate (i.e., rise) if its demand-adjusted money stock is growing faster than in the other country. Conversely, a nation will find its currency appreciating on the foreign exchanges when its money stock grows slower and its real income faster than in the other country. Note that the model's conclusion that rapid real growth results in currency appreciation contradicts the conventional balance of payments view of exchange rate determination. According to this latter approach, income growth tends to depreciate a country's currency by inducing a rise in imports and a consequent trade balance deficit. By contrast, the present model depicts real growth as stimulating not imports but rather the demand for money. Given the nominal money stock, this increased real money demand necessitates a fall in the price level to clear the market for money balances. With foreign prices given, the fall in domestic prices requires an equivalent appreciation of the exchange rate to maintain purchasing power parity. In short, the model predicts that growth-induced rises in the real demand for money will raise the internal and therefore also the external value of a currency.

As for expectational influences, the equation predicts that a rise in the expected rate of inflation in one country (as reflected in its interest rate) relative to the other will cause the former's currency to depreciate on the foreign exchanges. The reason, of course, is that when interest rates rise, desired real cash balances fall. Cashholders attempt to get rid of unwanted balances via expenditure for goods thereby putting upward pressure on prices. According to the model, the rise in prices will be relatively greater in the country experiencing the larger rise in interest rates. In this way increasing relative interest rates cause corresponding increases in relative national price levels that must be offset by exchange rate depreciation to preserve purchasing power parity. Note again that the model's prediction of a direct relation between interest rate movements and exchange rate movements runs counter to the conventional balance of payments view. According to this latter approach, a rising interest rate should lower the exchange rate either by attracting capital from abroad (thereby improving the capital account of the balance of payments) or by reducing domestic expenditure for imports and potential exports (thereby improving the trade balance). This cannot happen in the present model where, instead of strengthening the balance of payments, a rising interest rate induces a shift from cash to goods resulting in domestic inflation and exchange rate depreciation. In short, equation 7 predicts that a country will experience currency depreciation when its relative money stock rises, its relative real income falls, and its relative inflationary expectations rise.

Empirical Application This article has constructed a simple economic model that states that the bilateral exchange rate between any two national currencies is determined by relative money stocks, relative real incomes, and relative nominal interest rates—the last variable reflecting relative expectations regarding national inflationary prospects. All that remains is to illustrate how the model can be applied in empirical studies of exchange rate determination. With this objective in mind, an attempt is made below to estimate the model's reduced-form exchange rate equation (equation 7) and to use it to explain the behavior of the US/UK and US/Italy exchange rates, respectively, over the post-1972 period of generalized floating. To do this, it is necessary to transform equation 7 into linear form by expressing the variables as logarithms. This step is required because equation 7 is nonlinear, and nonlinear equations are difficult to estimate directly. The resulting log-linear version of equation 7 is written as

$$(8) \quad \ln X = a_0 + a_1(\ln M - \ln M^*) + a_2(\ln Y^* - \ln Y) + a_3(\ln i - \ln i^*)$$

where ln stands for the logarithm of the attached variable and the a's are coefficients to be estimated from the statistical data. Note that according to equation 7 the a priori expected values of the coefficients attached to the money and income variables are unity whereas the coefficient attached to the interest rate variables should lie between zero and unity, consistent with previous empirical estimates of the interest elasticity of demand for money.

Equation 8 was estimated for quarterly US/UK and US/Italy data for the period 1973 I through 1976 II. The money supply variable used for each country was M_1. The income variables used were real gross national product for the US and real gross domestic product for the UK and Italy, respectively. As for the interest rate variables, the treasury bill rate was used for each country in the US/UK equation and the rate on medium-term government bonds was used for each country in the US/Italy equation. All data were taken from the IMF's *International Financial Statistics* with the exception of the figures for UK real gross domestic product, which were taken from the OECD's *Main Economic Indicators*.

The results are shown in Table I below.

Table I

REGRESSION RESULTS FOR US/UK AND US/ITALY EXCHANGE RATES

Quarterly Data: 1973 I - 1976 II

I. Dollar/pound exchange rate

$$\ln X = 5.87 + .49(\ln M_{US} - \ln M_{UK}) + .96(\ln Y_{UK} - \ln Y_{US}) + .24(\ln i_{US} - \ln i_{UK})$$
$$(2.79)^{*} \qquad (2.78)^{*} \qquad (2.34)^{*}$$

$R^2 = .87$ $\quad DW = 1.17$[1]

II. Dollar/lira exchange rate

$$\ln X = -4.44 + .92(\ln M_{US} - \ln M_{IT}) + .70(\ln Y_{IT} - \ln Y_{US}) + .17(\ln i_{US} - \ln i_{IT})$$
$$(3.93)^{*} \qquad (1.32) \qquad (1.62)$$

$R^2 = .87$ $\quad DW = 1.24$[1]

*Indicates statistical significance at the 5 percent level of confidence. t-statistics are given in parentheses beneath the estimated coefficients.

[1]The reported Durbin-Watson statistics are in the inconclusive region in testing for serial correlation. Correcting for serial correlation using the Cochrane-Orcutt method did not significantly alter the results.

In general the empirical results are consistent with the theoretical model. According to the estimated equations, fully 87 percent of the variation of both the dollar/pound and dollar/lira exchange rates are explained by variations in the money stock, real income, and interest rate variables. In both cases the coefficients on the explanatory variables have the expected positive signs. All coefficients are statistically significant at the .05 level except for those on the US/Italy income and interest rate variables. Moreover, the coefficient on the US/Italy money stock variable is close to its expected (theoretical) value of unity, as is the coefficient on the US/UK income variable. The interest rate coefficients in both equations are also consistent with previous empirical estimates of the interest elasticity of demand for money.[3] These results are perhaps better than one might expect considering the extreme simplicity of the model, the degree to which floating rates are managed instead of free, the limited number of observations (14), and the fact that short-run data are used to test a long-run equilibrium model.

In sum, the equations reported above provide at least modest empirical support for the theoretical model developed earlier in the article. One should not make too much of these results, however. Just as one swallow does not make a summer, two regression equations do not prove a theory. In particular, equation 8 may not fit the data well for other countries and other time periods. In fact, an attempt was made to test the equation against recent data for Canada, Japan, and Germany, as well as for data pertaining to the UK during the early 1920's when that country was off the gold standard. For the first three countries, the equation performed poorly. For the UK from 1920-1924, however, it was at least partially successful. As shown in Table II, the equation performed adequately except for the coefficient on the income variable, which bears the wrong sign. This of course may be due to the unreliability of UK income data for that period rather than to shortcomings inherent in the model.[4] Nevertheless, the fact that the equation does not work well for all countries is reason to interpret the results reported here with caution.

Table II

REGRESSION RESULTS FOR US/UK EXCHANGE RATES

Quarterly Data: 1920 I - 1924 IV

Dollar/pound exchange rate

$$\ln X = -.17 + .55(\ln M_{US} - \ln M_{UK}) - .16(\ln Y_{UK} - \ln Y_{US}) + .10(\ln i_{US} - \ln i_{UK})$$
$$(4.48)^{*} \qquad (-1.55) \qquad (2.77)^{*}$$

$R^2 = .76$ $\quad DW = 1.31$

*Indicates statistical significance at the 5 percent level of confidence. t-statistics are given in parentheses beneath the estimated coefficients.

Problems in Testing the Model In closing this article, it may be appropriate to consider why the data did not exactly fit the model like a

[3] Boorman [3] reports that recent empirical studies of the demand for money suggest an interest elasticity of about −0.2 for short-term rates, quite close to the estimates appearing in Table I.

[4] Since quarterly national income figures are not available for this period, the Federal Reserve's Index of Industrial Production was used as a proxy for US real income. No such official index is available for the UK. Therefore a quarterly industrial production index constructed in 1927 by Rowe [8] was used as a proxy for UK real income. However, the reliability of this index is open to question.

glove. Regarding this question, at least three likely explanations come to mind. First, the model assumes that exchange rates are permitted to float freely while in fact governments still intervene in foreign exchange markets from time to time in order to achieve a managed float. This suggests that there may be some reverse causality running from exchange rates to money, at least in the short run. In brief, the model may not be a completely accurate description of existing exchange rate regimes.

Second, quarterly data may not be suitable for testing what is essentially a model of long-run equilibrium. Quarterly data are short-run data. As such they may be dominated by transitory dynamic adjustment phenomena that are absent in long-run static equilibrium. Annual (or longer) data are more appropriate for testing an equation that is based on assumptions of purchasing power parity, interest rate parity, monetary equilibrium, real income exogeneity, and unidirectional causality between money and exchange rates—all propositions about long-run equilibrium. Unfortunately, the post-Bretton Woods era of floating rates is only four years old, and the number of annual observations is insufficient to test these propositions. Even the number of quarterly observations is distressingly low.

An alternative solution would be to augment the model with additional equations and variables to represent dynamic adjustment processes. While this might permit the specification of short-run influences affecting the exchange rate, it would unduly complicate the model, contrary to the objective of keeping it simple. Note, however, that this latter feature may constitute a third reason for the model's failure to conform exactly to the data, i.e., the model may be far too simple to capture all the influences on the exchange rate. This does not necessarily mean that the model is conceptually unsound. The underlying theory may be correct even though its empirical form is inadequate to fit the facts. Thus the model can be faulted on the grounds that its empirical money demand equations are too simple, that it lacks dynamic adjustment mechanisms, and that it arbitrarily constrains the elasticity coefficients to be the same for each country. These considerations should be kept in mind when interpreting the results of the regression analysis.

Summary This article has developed and estimated a simple model of exchange rate determination. The model states that exchange rate movements are determined by shifts in relative money stocks, relative real incomes, and relative inflationary expectations as manifested in relative interest rate movements. Although the model receives some empirical support from post-1972 data for the dollar/pound and dollar/lira exchange rates, it does not perform well when applied to data for other countries and other time periods. One is therefore advised to take an agnostic attitude regarding the validity of the model until all returns are in. In short, additional experience with floating exchange rates, together with the application of empirical techniques of greater sophistication than those employed here, will be necessary to establish conclusively the validity or invalidity of the model.

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