

TREASURY BILL VERSUS PRIVATE MONEY MARKET YIELD CURVES

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The relationship between time to maturity and yield on securities is of widespread interest to financial market participants and observers. The relationship, known as the term structure of interest rates, provides information about which maturities offer the highest expected returns to investors and which provide the lowest expected costs to borrowers. Plots of the term structure-called yield curves-are shown in Chart 1 for three money market instruments as of the first trading days of December 1984 and December 1985.

Many researchers have studied the term structure of Treasury bill (T-bill) yields and found that investors could expect higher returns, on average, from investing in longer term T-bills. The finding is inconsistent with the pure expectations theory of the term structure, according to which the expected rate of return should be the same at all maturities. In this paper we examine whether this conclusion also applies to the yield curves for private money market instruments by testing the pure expectations theory using yields on three such instruments. We cannot reject the theory for the private money market yield curves. The results suggest that the pure expectations theory may be consistent with the behavior of *money* market participants in general and that the Treasury bill market differs in some way from the private money markets. We demonstrate that the term structure of T-bill rates may differ from those of private money market rates because of a unique characteristic of the T-bill market: only the Treasury can borrow at the T-bill rate.

IMPLIED FORWARD RATES AND THE PURE EXPECTATIONS THEORY

In order to discuss the pure expectations theory of the term structure it is useful to introduce the concept of implied forward rates. The term structure of interest rates at any point in time implies a set of forward interest rates-that is, interest rates on bonds

in the future. Suppose R_1 is the current yield on a one-year discount bond and R_2 is the current annualized yield on a two-year discount bond. The implied forward rate on a one-year bond commencing in one year (F_1) is the rate that equates the two-year return from investing one dollar in the current two-year bond $([1 + R_2]^2)$ to the return from investing one dollar in the current one-year bond and reinvesting the proceeds at the end of one year in a new one-year bond:

$$(1 + R_2)^2 = (1 + R_1)(1 + F_1).$$

This expression can be rearranged to give an expression for the implied forward rate:

$$F_1 = [(1 + R_2)^2 / (1 + R_1)] - 1,$$

which may be represented by the usual linear approximation:

$$F_1 = 2 R_2 - R_1.^1$$

For example, if the rate on one-year bonds is 9 percent and the rate on two-year bonds is 10 percent, the implied forward rate on one-year bonds one year from now is $(2 \times 10) - 9 = 11$ percent.

The pure expectations theory of the term structure states that implied forward rates always equal expected future rates because bonds of different maturities can be considered perfect substitutes.² Although some market participants may have preferences for

¹The linear approximation is used throughout the text to simplify the discussion. The general formula for calculating the implied forward rate, used in the empirical work, is

$$F_m = [(1 + R_{n+m})^{n+m} / (1 + R_n)^n]^{1/m} - 1,$$

where

F_m = implied forward rate on m-year bonds commencing n-years from now,

R_{n+m} = spot rate on n+m-year discount bonds,

R_n = spot rate on n-year discount bonds.

²The discussion here is brief. See Van Horne (1984, pp. 104-12) for a more extensive discussion of the pure expectations theory.

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particular maturities, participants indifferent to maturity are assumed to be sufficiently active in the market to determine the term structure of interest rates. As a result, the expected rate of return is the same for all maturities.

According to the theory, any significant difference between implied forward and expected future rates will be quickly eliminated because it offers market participants profit opportunities. If the implied forward rate were higher than the expected future rate, participants who could borrow at the one-year rate and lend at the two-year rate could lock in a forward one-year investment at a rate higher than the expected future one-year rate. For example; if one-year bonds are trading at 9 percent and two-year bonds are trading at 10 percent, the implied forward rate on one-year bonds commencing in one year is 11 percent. If a market participant believes that in one year the one-year bond rate will be less than 11 percent, say 10.5 percent, he can issue a one-year bond at 9 percent and invest the proceeds in a two-year bond at 10 percent. If the participant's expectations are correct, he can issue a second one-year bond a year later at a rate of 10.5 percent. The participant profits because he earns 10 percent on the two-year bond he invested in and only pays 9.75 percent to borrow with the two one-year bonds.

The pure expectations theory states that investors who are willing and able to take advantage of such a profit opportunity will continue to borrow at the short-term rate and lend at the long-term rate until the implied forward rate equals the expected future rate. As a result, the shape of the yield curve is determined solely by expectations of future interest rates. If interest rates are not expected to change, the yield curve will be flat. If short-term rates are expected to rise, the yield curve will be upward-sloping : long-term rates will exceed short-term rates by just enough to equate the return from investing in a long-term security to the expected return from investing in a short-term security and rolling it over at the expected higher future short-term rate. Conversely, if short-term rates are expected to fall, the yield curve will be downward-sloping.

Alternatively, the term structure may be affected by factors in addition to expectations of future rates. For example, expected returns may be higher on longer term securities in order to compensate the investor for investing for longer periods. If such is the case, the yield curve will be more upward-sloping than predicted by the pure expectations theory, and implied forward rates will be higher than expected

future rates. Any difference between the implied forward rate (F) and the expected future rate (R^e) is referred to as a term premium (P) :

$$P = F - R^e.$$

TESTING THE PURE EXPECTATIONS THEORY WITH MONEY MARKET YIELDS

Since the pure expectations theory states that implied forward rates equal expected future rates, one can test the theory by determining whether the term premium is zero. Unfortunately, expected future rates are not observable, making it impossible to calculate the term premium on an instrument at a specific time. One can, however, estimate the average term premium over a long period by assuming that market participants form expectations rationally. Under the rational expectations hypothesis, realized future rates equal expected future rates plus a serially uncorrelated forecast error with mean zero. In other words, there is no systematic bias in the market's forecasts. Any systematic difference between implied forward and realized future interest rates can therefore be attributed to term premiums.

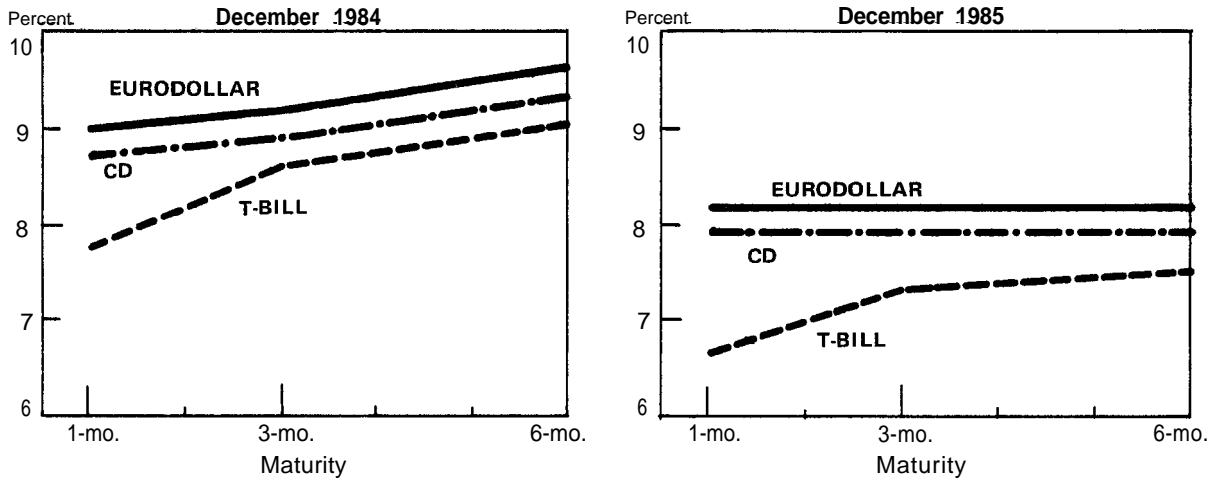
Most studies of the term structure of money market rates have used T-bill yields because T-bills have several qualities that make it easier to isolate the effect of maturity on yield: T-bills are essentially free of default risk, they are identical in all respects except maturity, and they are traded in a highly liquid market. These studies have rejected the joint hypothesis of rational expectations and the pure expectations theory because they have found that implied forward T-bill rates have been significantly higher, on average, than realized future rates.³ Since it is unlikely that the market would systematically overpredict future rates, the difference between implied forward and realized future rates has generally been attributed to term premiums.

The term premium in T-bill yields, however, may not be representative of the overall money market. Chart 1 shows that the T-bill yield curve has at times shown greater upward slope than the yield curves of other money market instruments, suggesting that the term premium in T-bill yields is bigger than those in the yields on private money market instruments. In fact, the T-bill yield curve has been steeper than the yield curve for negotiable bank certificates of deposit (CDs) on average over the last twenty years. As

³For example Kessel (1965), Roll (1970), McCulloch (1975), and Fama (1984).

Chart 1

MONEY MARKET YIELD CURVES



Note: Secondary market quotes are as of the first trading days of each month. All rates are on an interest-to-follow basis--i.e., as a percentage of actual funds invested. T-bill rates are generally quoted on a bank-discount basis--i.e., as a percentage of par, rather than of actual funds invested. The formula to convert a bank-discount rate (BD) to an interest-to-follow rate (ITF) is $ITF = (360 \cdot BD) / [360 - (N \cdot BD/100)]$, where N is days to maturity.

Source: Salomon Brothers, *An Analytical Record of Yields and Yield Spreads* (New York, 1986).

shown in Table I, the spread between one-month CD yields and one-month T-bill yields has usually exceeded the spread between three-month CD yields and three-month T-bill yields. In turn the latter spread has usually exceeded that between six-month CD yields and six-month T-bill yields. The pattern is persistent: the three-month spread exceeded the six-month spread in eighteen of the twenty years. The T-bill yield curve also has generally been more upward-sloping than the yield curves for commercial paper and Eurodollar deposits.⁴

These observations suggested that one might test the pure expectations theory using yields on private money market instruments and compare the results to those obtained using yields on T-bills. We estimated the average term premiums in the yields on CDs, Eurodollars, commercial paper, and T-bills from 1970 through 1985, the full period for which first day of the month interest rates are available for all the instruments from Salomon Brothers, *An Analytical Record of Yields and Yield Spreads*. For each instrument, we looked at the average difference between the implied forward three-month rate commencing in three months (calculated from the current

⁴The rate on Eurodollar deposits is the London inter-bank offered rate.

three- and six-month interest rates) and the three-month rate realized three months later.⁵ Assuming rational expectations, any significant difference can be attributed to a term premium. We use only one observation per quarter since more frequent observations would overlap, introducing serial correlation into the error terms.

The estimated average term premiums are reported in Table II. For T-bills the average term premium is 61 basis points. Since the estimate is significantly different from zero at the 99 percent confidence level, the pure expectations theory is strongly rejected for the T-bill market. The average term premiums for the private money market instruments, on the other hand, are much smaller and not significantly different from zero in a statistical sense. One cannot reject the pure expectations theory for private money market instruments.

Data are also available as far back as 1964 for T-bills and CDs. Table III shows the average term premiums for T-bills and CDs from 1964 through 1985. The estimated average term premium for

⁵We could not test the theory using one-month interest rates because they imply forward two- and five-month rates when combined with three- and six-month rates and we did not have data on two- and five-month spot rates.

Table 1

CD VS. T-BILL YIELDS: YEARLY AVERAGES

Year	CDs			T-bills			CD less T-bill		
	1-mo.	3-mo.	6-mo.	1-mo.	3-mo.	6-mo.	1-mo.	3-mo.	6-mo.
1966	5.34	5.48	5.63	4.67	4.92	5.19	0.67	0.56	0.44
1967	4.79	5.02	5.22	4.05	4.35	4.73	0.74	0.67	0.49
1968	5.72	5.86	6.00	5.20	5.41	5.62	0.51	0.45	0.38
1969	7.65	7.78	7.91	6.48	6.78	7.11	1.18	1.00	0.80
1970	7.45	7.56	7.65	6.20	6.49	6.73	1.25	1.07	0.93
1971	4.78	4.99	5.21	4.22	4.38	4.62	0.57	0.62	0.59
1972	4.41	4.67	5.02	3.89	4.11	4.59	0.51	0.56	0.43
1973	8.30	8.41	8.31	7.05	7.16	7.47	1.26	1.25	0.84
1974	10.29	10.24	9.98	8.03	7.99	8.26	2.26	2.26	1.71
1975	6.14	6.44	6.89	5.63	5.86	6.28	0.50	0.58	0.61
1976	5.07	5.27	5.62	4.89	5.04	5.39	0.19	0.23	0.23
1977	5.48	5.64	5.92	5.15	5.34	5.68	0.33	0.30	0.24
1978	7.88	8.22	8.61	7.09	7.32	7.88	0.79	0.90	0.73
1979	11.03	11.22	11.44	10.00	10.33	10.59	1.03	0.89	0.85
1980	12.91	13.07	12.99	11.02	11.77	12.05	1.89	1.30	0.94
1981	15.91	15.91	15.77	13.90	14.53	14.82	2.01	1.38	0.95
1982	12.04	12.27	12.57	10.19	10.90	11.71	1.85	1.37	0.86
1983	8.96	9.07	9.27	8.38	8.80	9.13	0.58	0.27	0.14
1984	10.17	10.37	10.68	9.05	9.75	10.26	1.12	0.61	0.42
1985	7.97	8.05	8.25	7.11	7.62	7.96	0.86	0.43	0.29
Average, 1966-85	7.73	7.88	8.04	6.77	7.09	7.43	0.96	0.79	0.61

Note: Rates are secondary market quotes on an interest-to-follow basis (see Chart 1).

Source: Board of Governors of the Federal Reserve System.

T-bills is 53 basis points and the estimate is statistically significant at the 99 percent confidence level. The estimated average term premium for CDs is much smaller and not significantly different from zero. The pure expectations theory is strongly rejected for T-bill yields but not rejected for CD yields.

The above test for term premiums in the yields on private money market instruments is subject to one qualification. The test assumes that the degree of default risk is the same for both maturities. The assumption holds for T-bills since all maturities are essentially free of default risk. In contrast, each of the private money market instruments is subject to some risk of default, and different degrees of expected default loss on three- and six-month private money market instruments could bias the test. In the Appendix, however, we show that under reasonable

assumptions the test is not biased against finding a term premium in the yields on private money market instruments.

EXPLAINING THE DIFFERENCE IN RESULTS

The significant term premium in T-bill yields and the absence of significant term premiums in the private money market yields suggests that the T-bill market differs in some way. The T-bill market does differ in one important respect: whereas there are many issuers in each of the private money markets, only the Treasury can issue T-bills. A key assumption of the pure expectations theory is therefore violated in the T-bill market: market participants, in general, cannot borrow at the T-bill rate. Because the rate at which participants in the T-bill market

Table II
AVERAGE TERM PREMIUMS 1970 Q2 TO 1985 Q4

	<u>T-bills</u>	<u>CDs</u>	<u>Eurodollars</u>	<u>Commercial Paper</u>
Average term premium (in basis points)	61	21	21	14
Standard error	22	26	27	24
t-statistic	2.78	0.79	0.79	0.58
Number of observations	63	63	63	63

Notes:

- (1) The term premium is the difference between the implied forward rate calculated from the three- and six-month spot rates and the realized three-month spot rate three months later.
- (2) The rates are for the first day of the third month of each quarter from Salomon Brothers, An Analytical Record of Yields and Yield Spreads (New York, 1986). These rates are annualized without compounding. Consequently, the formula used to calculate the forward rate is:

$$\left[\left(\frac{1 + R_6/200}{1 + R_3/400} \right) - 1 \right] * 400.$$

can borrow funds is higher than the T-bill rate, they may be unable to profit from the difference between the implied forward and expected future three-month T-bill rates. Of course, the Treasury could reduce the term premium by selling more three- and fewer six-month T-bills, but it has not been willing to do so. Additionally, the term premium in T-bill yields would not exist unless some investors were willing to accept a lower expected yield on three- than on six-month T-bills. This section discusses these points in more detail.

The Treasury's Monopoly Limits Profit Opportunities for Other Investors

The significant term premium in the implied forward three-month T-bill rate indicates that investors have been either unwilling or unable to take full

advantage of the opportunity for expected profit offered by the difference between implied forward and expected future three-month T-bill rates. Investors who were both willing and able would have borrowed at the three-month T-bill rate and invested in six-month T-bills for an expected profit. Such transactions would have tended to push up the three-month rate and push down the six-month rate. If this element of the market was sufficiently large, the implied forward three-month T-bill rate would have been driven down close to the expected future three-month T-bill rate, thereby eliminating the term premium.

Investors may not be able to profit from the positive term premium in T-bill yields, however, because the rate at which they can borrow three-month money is higher than the three-month T-bill rate. In contrast, many participants in each of the markets for the private money market instruments can profit from any difference between implied forward and expected future rates since they are able to both borrow and lend at approximately equal rates. For example, suppose a bank believes that the future three-month Eurodollar rate will be lower than the forward Eurodollar rate implied by the yield curve. The bank can issue a three-month Eurodollar deposit and place the proceeds in a six-month Eurodollar deposit. The bank will profit if the implied forward three-month Eurodollar rate exceeds the realized future three-month Eurodollar rate. Now consider a trader who believes that the future three-month T-bill rate will

Table III

AVERAGE TERM PREMIUMS 1964 Q2 TO 1985 Q4

	<u>T-bills</u>	<u>CDs</u>
Average term premium (in basis points)	53	17
Standard error	16	19
t-statistic	3.24	0.89
Number of observations	87	87

Note: See notes in Table II.

be lower than the forward three-month T-bill rate implied by the yield curve. Since only the Treasury can issue T-bills, the trader cannot raise three-month funds at the three-month T-bill rate. Rather, if he wishes to fund his purchase of a six-month T-bill by borrowing for three months, his lowest cost source of funds probably will be to enter into a repurchase agreement (RP) with another party.⁶ Under a repurchase agreement, funds are acquired through the sale of a security coupled with a simultaneous agreement to repurchase the security on a specified date at an agreed upon price (and thus an agreed upon rate of interest). For example, by buying a six-month T-bill yielding 10 percent and entering into a three-month repurchase agreement at 9 percent, the trader can secure an investment in a three-month T-bill commencing in three months yielding 11 percent. If in three months the three-month T-bill rate is less than 11 percent, he can sell the T-bill for a profit.

Since the three-month RP rate is invariably higher than the three-month T-bill rate, the forward rate attainable by buying a six-month T-bill and financing it with a three-month repurchase agreement is lower than the forward rate implied by the three- and six-month T-bill rates. Traders can expect to profit only if this "attainable forward rate" is different from the expected future T-bill rate.⁷ The implied forward T-bill rate can therefore be higher than the expected future T-bill rate but not offer any profitable trades. For example, if three-month T-bills are trading at 9 percent and six-month T-bills are trading at 10 percent, the implied forward rate on three-month T-bills commencing in three months is 11 percent. Suppose the expected future three-month T-bill rate is 10.75 percent. If the three-month RP rate is 9.25 percent, the attainable forward three-month T-bill rate is also 10.75 percent. Even though the expected future three-month T-bill rate is less than the forward rate implied by the T-bill yield curve, it is not less than the attainable forward rate. Consequently, investors cannot profit from the gap between the implied forward and expected T-bill rates.

⁶Prior to the development of the RP market, the cheapest way for a trader to finance a six-month T-bill for three months was to get a three-month loan from a bank using the six-month T-bill as collateral.

⁷The term "attainable forward rate" was introduced by Gendreau (1983) in a study of the yields on Treasury bill futures contracts.

Testing for Profit Opportunities

Because traders cannot borrow at the T-bill rate, the positive term premium in T-bill yields does not necessarily mean that they are passing up expected profits. The appropriate test of whether traders have passed up profit opportunities is whether the forward rate attainable by purchasing a six-month T-bill and financing it with a three-month repurchase agreement has been significantly different from the realized future three-month T-bill rate. Ideally, to carry out this test the attainable forward rate should be calculated using the rate on RPs with six-month T-bills posted as collateral. Unfortunately, data on the rates on RPs with specific collateral are not available, but a series on the 90-day RP rate on general government securities collateral starting in September 1979 is available through Data Resources, Inc. It is the closest approximation available of the rate at which traders can borrow three-month money using six-month T-bills as collateral.

The average difference between the attainable forward three-month T-bill rate (calculated from the six-month T-bill and three-month RP rates) and the three-month T-bill rate realized three months later from September 1979 through December 1985 is reported in Table IV. Since the average difference between attainable forward rates and realized three-month rates is only 4 basis points, there is no indication that traders passed up profit opportunities. For comparison, the average term premium in the implied forward three-month T-bill rate over the same period (using the same method as in Table II) is 79 basis points.

Table IV

ATTAINABLE FORWARD VS. REALIZED FUTURE THREE-MONTH T-BILL RATES 1979 Q4 to 1985 Q4

	Attainable Forward Rate Less Realized Future Rate	Implied Forward Rate Less Realized Future Rate
Average difference (in basis points)	4	79
Standard error	50	51
t-statistic	0.09	1.55
Number of observations	25	25

Note: The rates are for the first day of the third month of each quarter. T-bill rates are from Salomon Brothers, *An Analytical Record of Yields and Yield Spreads* (New York, 1986). The RP rates are from Data Resources, Inc.

The Treasury's Behavior

The positive term premium in the implied forward three-month T-bill rate indicates that the Treasury has been willing to issue six-month T-bills at a higher expected interest cost than three-month T-bills. If the Treasury were unwilling to pay a higher expected yield on six-month T-bills, it could issue fewer six-month and more three-month T-bills. Decreasing the supply of six-month T-bills would tend to lower the interest rate on them, and increasing the supply of three-month T-bills would tend to raise their interest rate. These actions would reduce, if not eliminate, the term premium in the T-bill market.

The Treasury's behavior is quite different from the behavior of issuers in the private money markets, who adjust the relative supplies of three- and six-month instruments they issue in response to changes in market rates and in their expectations of future interest rates. The Treasury virtually always sells a roughly equal amount of three- and six-month T-bills at its weekly auction.

Because the rate on six-month T-bills is higher, on average, than the rate on three-month T-bills, it appears that the Treasury could lower its total financing costs by issuing fewer six-month and more three-month T-bills. The potential cost savings from such a change is hard to calculate, however, because it depends on the responsiveness of three- and six-month T-bill rates to changes in supplies—that is, on the interest elasticities of the demands for three- and six-month T-bills. In fact, such a change might not lower the Treasury's financing costs at all. If the Treasury were to issue more three-month T-bills it would have to pay a higher interest rate on all three-month T-bills. If the demand for three-month T-bills were less interest-elastic than the demand for six-month T-bills, then the additional interest cost on three-month T-bills could outweigh the savings from selling fewer of the higher-cost six-month T-bills.

Even if the Treasury could reduce its financing costs by issuing more three-month T-bills, it might not be willing to do so because of other considerations. For example, in recent years the Treasury has been reducing the proportion of debt financed with T-bills in order to increase the average maturity of its debt outstanding. One reason for extending the average maturity is to reduce the year-to-year variation in the interest expense component of the federal budget. Issuing more three- and fewer six-month T-bills would conflict with the policy of debt maturity extension.

The Demand for Short-Term T-Bills

The positive term premium in the implied forward three-month T-bill rate also indicates that some investors have been willing to hold three-month T-bills despite a lower expected return than on six-month T-bills. Further, the absence of a term premium in CD yields implies that investors who held three-month T-bills could have expected higher returns from holding three-month CDs even after adjusting for the possibility of loss due to default on the CDs.⁸ These investors must have had preferences for three-month T-bills over six-month T-bills and over three-month CDs that made them willing to hold three-month T-bills despite a lower expected return.

Broadly speaking there are two possible explanations why some investors are willing to accept a lower expected yield on three-month T-bills. The first is that some investors may be risk averse. They may be willing to accept lower expected returns on three- than on six-month T-bills because six-month T-bills are subject to greater fluctuation in capital value, and they may be willing to accept lower expected returns on three-month T-bills than on CDs because CDs are subject to greater risk of default.

A second possibility is that some investors may be willing to accept lower returns on three-month T-bills because of special characteristics of T-bills. One such characteristic is the role that T-bills play in satisfying numerous institutional and regulatory requirements. For example, Treasury securities are eligible pledging assets against Treasury tax and loan accounts as well as against most state and local government deposits. T-bills are also widely accepted as collateral for selling short various financial securities. T-bills can be used instead of cash to satisfy initial margin requirements against futures market positions. For many of these purposes investors might prefer three- to six-month T-bills because the benefit from holding T-bills is expected to accrue for only a short time. Such might be the case, for example, if T-bills were held as collateral for volatile government deposits or as margin for short-term futures contracts.

Another special characteristic of T-bills is that the

⁸ Assume that the annualized expected default loss on a three-month CD is no greater than on a six-month CD. Assume also that the expected yield on six-month T-bills is no greater than the expected yield on six-month CDs. Then the fact that the spread between the rates on three-month CDs and three-month T-bills is greater than the spread between the rates on six-month CDs and six-month T-bills implies that the expected yield on three-month T-bills is less than the expected yield on three-month CDs.

interest income on them is not subject to state and local income taxes. Because of peculiarities in the tax laws, most large investors, such as banks and corporations, nevertheless do have to pay taxes on T-bill *interest* income.⁹ Hence, this tax advantage accrues mainly to individual investors. If individuals have a preference for liquidity that is not shared by large investors, they may be willing to accept a lower yield on three- than on six-month T-bills while large investors are not willing to accept a lower expected yield on three- than on six-month private money market instruments.

FURTHER IMPLICATIONS

Time-Varying Term Premiums

Since traders in the T-bill market cannot borrow funds at the T-bill rate, movements in the spread between the rate at which they can borrow and the T-bill rate may cause the term premium to vary over time. Movements in the spread between the RP rate and the T-bill rate change the spread between the implied forward rate and the attainable forward rate. If traders keep the attainable forward rate equal to the expected future rate by maximizing expected profits, such movements also change the spread between the implied forward rate and the expected future rate, i.e., change the term premium.

An example helps demonstrate how movements in the spread between the RP rate and the T-bill rate can affect the term premium. Assume that the three-month T-bill rate is 9.5 percent, the six-month T-bill rate is 10 percent, and the three-month RP rate is 9.75 percent. Assume also that the expected future three-month T-bill rate is 10.25 percent (equal to the attainable forward rate). Since the implied forward rate of 10.50 percent is 25 basis points higher than the expected future rate, the term premium is 25 basis points. Now, if the three-month T-bill rate falls to 9.25 percent and other rates are unchanged, then the implied forward rate rises to 10.75 percent. The implied forward rate is now 50 basis points higher than the expected future rate, but since the attainable forward rate is still equal to the expected future rate there are no profitable trading opportunities. In this case the term premium increased from 25 to 50 basis points simply because of an increase in the spread between the three-month RP rate and the three-month T-bill rate.

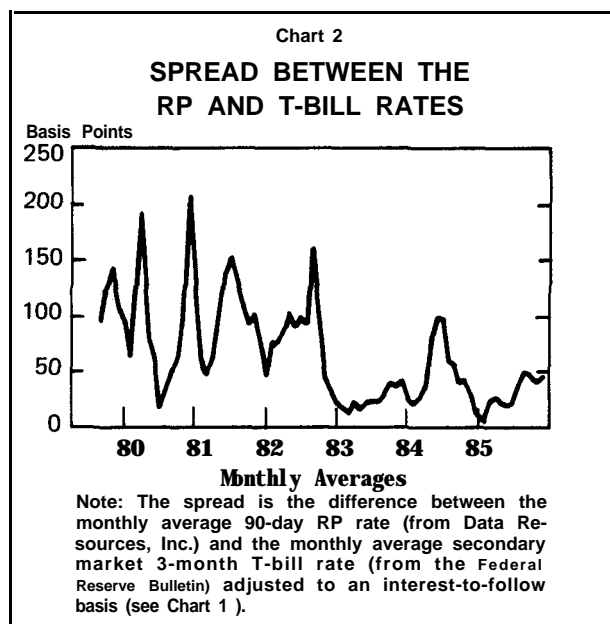
⁹Cook and Lawler (1983) provide details on the taxation of T-bill interest income for different investors.

Movements in the spread between the three-month RP rate and the three-month T-bill rate have been substantial, as shown in Chart 2. These movements may explain why **some** researchers have found evidence of a time-varying term premium in the T-bill market.¹⁰

T-Bill Futures Rates and Implied Forward Rates

The difference between the interest rate at which private investors can borrow and the interest rate on T-bills also helps explain why implied forward T-bill rates have been higher than the rates on T-bill futures contracts. If investors could both borrow and lend at the T-bill rate, any significant difference between implied forward rates and futures rates would offer profitable arbitrage opportunities. Investors could lock in a risk-free profit by borrowing money at the three-month T-bill rate, investing in a six-month T-bill and simultaneously entered into a futures contract to sell a three-month T-bill three months in the future. Private investors, however, cannot carry out this set of transactions because they cannot borrow at the T-bill rate. As pointed out by Gendreau (1985) the relevant rate comparison for arbitrage opportunities is between the forward rate attainable by investors through buying a T-bill and financing it

¹⁰Researchers who have found a time-varying term premium in the T-bill market include Kessel (1965), Friedman (1979), and Jones and Roley (1983).



with a term RP and the rate on the corresponding T-bill futures contract. Gendreau compared these rates and found that the attainable forward three-month T-bill rate was lower, on average, than the futures rate and that the difference was statistically insignificant.

CONCLUSIONS

The evidence presented in this article confirms the conclusions of other studies that the pure expectations theory does not completely explain the term structure of Treasury bill rates. There is strong evidence of a positive average term premium in the implied forward three-month T-bill rate. The behavior of the term structure of T-bill yields, however, appears to be atypical of the money market in general. Based on the evidence presented in this article, one cannot reject the pure expectations theory as an explanation of the term structure of private money market yields. The difference in results suggests that the T-bill market differs in some way from the private

money markets. In fact, a key assumption of the pure expectations theory is violated in the T-bill market because market participants in general cannot borrow at the T-bill rate. They may therefore be unable to profit from the positive term premium in T-bill yields. Only the Treasury can issue T-bills and it has been willing to pay a term premium to issue six-month T-bills.

Thus, conclusions from studies of the term structure of T-bill yields should not be generalized to the yields on private money market instruments. For example, although investors in three-month T-bills can expect higher returns on average from investing in six-month T-bills, investors in three-month CDs cannot necessarily expect higher returns from investing in six-month CDs. Finally, because the term premium in T-bill yields may result from unique characteristics of the T-bill market and the pure expectations theory is consistent with the term structures of private money market yields, the pure expectations theory appears to be consistent with the behavior of money market participants in general.

APPENDIX

This Appendix describes the effect of default-risk on the test for a term premium. It derives the relationship between the *measured* term premium based on promised yields and the *true* term premium based on expected yields, that is, yields that have been adjusted for expected default loss. We assume continuously compounded rates of return, for which the linear approximation of the implied forward rate is exact.

The expected yield on a bond is equal to the promised yield less the expected default loss:

$$(1) ER_t^1 = R_t^1 - EDL_t^1,$$

where

ER_t^1 = annualized expected rate of return on an i-period bond,

R_t^1 = annualized promised rate of return on an i-period bond,

EDL_t^1 = annualized expected default loss on an i-period bond.

Now, the measured implied forward rate on one-period bonds one period in the future observed at

time t ($MIFR_t^1$) is calculated using promised rates of return:

$$(2) MIFR_t^1 = 2R_t^2 - R_t^1;$$

and the measured term premium (MTP_t^1) is the difference between the measured implied forward rate and the expected future promised rate:

$$(3) MTP_t^1 = MIFR_t^1 - E_t(R_{t+1}^1).$$

The true implied forward rate on one-period bonds one period in the future observed at time t ($TIFR_t^1$) is calculated using expected rates of return:

$$(4) TIFR_t^1 = 2ER_t^1 - ER_t^1;$$

and the true term premium (TTP_t^1) is the difference between the true implied forward rate and the expected future expected rate:

$$(5) TTP_t^1 = TIFR_t^1 - E_t(ER_{t+1}^1).$$

Substitute equations 4, 1, 2, and 3 into equation 5 to obtain

$$(6) TTP_t^1 = MTP_t^1 - 2EDL_t^1 + EDL_t^1 + E_t(EDL_{t+1}^1).$$

The test for a term premium is biased against finding a positive term premium if the measured term premium is less than the true term premium: if

$$(7) \text{MTP}_t^1 < \text{TTP}_t^1,$$

or equivalently (using equation 6) if

$$(8) 2\text{EDL}_t^2 < \text{EDL}_t^1 + E_t(\text{EDL}_{t+1}^1).$$

The test is therefore not biased against finding a positive term premium if the annualized expected default loss on two consecutive one-period bonds is less than or equal to two times the annualized ex-

pected default loss on a two-period bond. The only circumstance that would bias the test against finding a term premium in, for example, CD yields would be a probability of default on consecutive three-month CDs that was higher than the probability of default on a six-month CD. This notion seems quite implausible when applied to the high-grade money market instruments used in this study, and we know of no empirical evidence to support it. There is consequently no reason to believe that default risk would bias the test against finding a term premium in the yields on private money market instruments.

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