THE TRADE THEORIST'S SACRED DIAGRAM: ITS ORIGIN AND EARLY DEVELOPMENT

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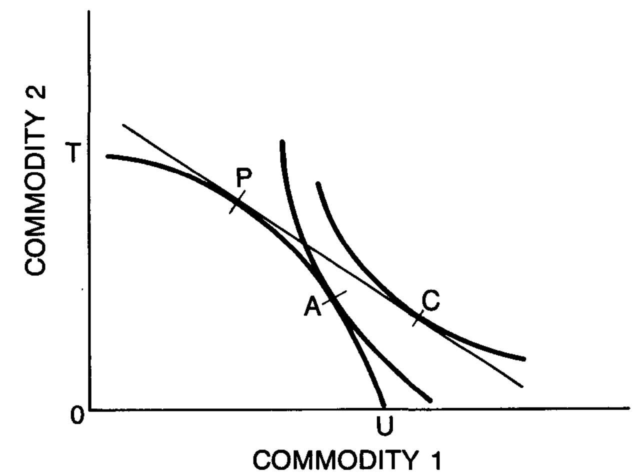
Introduction

In his celebrated 1945 essay on international trade under variable returns in a simple model[1] the noted Dutch economist Jan Tinbergen presented his version of what Robert Baldwin calls "the sacred diagram of the international trade economist" [1, p. 142]. Tinbergen used the diagram, which consists of a transformation or production possibility curve, taste indifference curves, and relative price or terms-of-trade lines, to show how a country gains from the opportunity to trade at a world price ratio different from the closed-economy one (see Figure 1). Given that opportunity, the country does two things. First, it produces the output mix that maximizes its national product valued at world prices. That is, it produces at the point of tangency of the production possibility curve and world price line. Then it trades along that line, exporting products in which it has a comparative cost advantage in exchange for imports of products in which it has a comparative disadvantage, until it reaches its point of maximum satisfaction on its highest attainable indifference curve. There it enjoys a bundle of goods that it could not produce or consume in isolation. Here is the economist's case for free trade captured in a single diagram.

That a simple geometrical diagram would become an icon is hardly surprising. Other economic diagrams have enjoyed that same distinction—the Keynesian cross, Marshallian sissors, Hicksian IS-LM, Edgeworth-Bowley box, Phillips curve, and Knightian circular flow are cases in point. What is surprising is how little has been written on the trade diagram's history. Few systematic surveys of that history exist; textbooks say little about it. Tinbergen himself said nothing about earlier versions of the diagram even though it was 38 years old at the time he presented it. Who invented the diagram? How was it initially received? Who exerted the greatest influence in getting it accepted into trade theory?

Figure 1

TINBERGEN'S DIAGRAM



Before trade the economy produces and consumes at A, the common point of tangency of transformation curve and indifference curve. Given the opportunity to trade at the world price ratio shown by the slope of line PC, it produces commodity bundle P which it then trades for bundle C to reach its point of maximum satisfaction C on its highest attainable indifference curve.

Source: Tinbergen [16, p. 129].

Today these issues still remain unresolved and one finds such writers as Samuelson, Baldwin, Maneschi and Thweatt disagreeing over whether Viner, Lerner, Haberler, or Barone contributed most to the diagram's development.[2] In an effort to rectify this situation and to provide some needed historical perspective, this article traces the evolution of the trade diagram from its 1907 origins to its presentation by Tinbergen in 1945 by which time it had already become the standard geometrical tool of the trade theorist. A word of explanation is in order, however. Today analysts put the diagram to many

[1] Tinbergen's essay, originally entitled "Professor Graham's Case for Protection," was reprinted in 1965 in a slightly abbreviated version as "International Trade Under Variable Returns in a Very Simple Model." See [16].

[2] See Maneschi and Thweatt [12, pp. 375-78] for a review of the controversy.

uses—to depict the effects of protection, of non-economic objectives of tariffs, of domestic market distortions, and of growth on trade, to name just a few. Historically, however, economists chiefly employed it to illustrate trade equilibrium and the gains from trade in a fully competitive economy in which the balance of payments for simplicity consists of the balance of trade. Given this article's historical focus, it too concentrates on those traditional concerns.

Historical Evolution

Historically the diagram evolved through at least eight stages. Each stage saw a different innovator contribute to the diagram's development. Irving Fisher (1907) invented the diagram to illustrate a problem in capital theory, Enrico Barone (1908) extended it to international trade, and Allyn Young (1928) applied it to a hypothetical closed economy operating under constant, decreasing, and increasing returns. Gottfried Haberler (1930) introduced the strictly concave production frontier version into foreign trade theory. Jacob Viner (1931) added community indifference curves to Haberler's diagram and criticized the entire apparatus. Abba Lerner (1932) extended the diagram to the level of the aggregate world economy, Wassily Leontief (1933) applied it to two countries simultaneously, and Jan Tinbergen (1945) elegantly consolidated their results. Except for Young, each analyst used the diagram to emphasize the gains from trade. Of these analysts, it was Haberler and Leontief who had the greatest influence. It was they who convinced trade theorists to add the diagram to their analytical tool kit. What follows describes in chronological order the specific contributions of each of these pioneers.

Irving Fisher

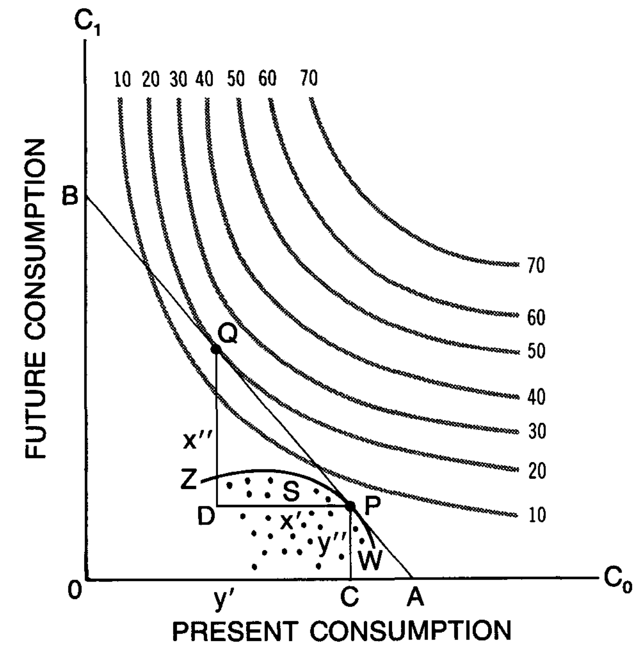
Francis Y. Edgeworth invented indifference curves in his *Mathematical Psychics* in 1881. Similarly, Vilfredo Pareto pioneered the use of transformation curves in his *Manuale di economia politica* in 1906. But Irving Fisher in his 1907 classic *The Rate of Interest* was the first to combine indifference and transformation curves together with market price lines in a single diagram and to use it to illustrate the gains from exchange (see Figure 2).

True, he applied his diagram to a problem in capital theory rather than to the theory of international trade. That is, he used it to depict an individual's optimum investment decision rather than a country's foreign trade equilibrium. But this difference is only superficial. Like trade theorists after him, Fisher used the diagram to demonstrate the gains from trade (albeit intertemporal rather than international). And like trade theorists, he showed the individual moving along the production possibility frontier to the highest attainable price line and then trading along that line to reach the point of maximum satisfaction. In terms of abstract economic logic, his demonstration matches that of the trade theorists. To Fisher, then, must go the credit for inventing the trade diagram.

His diagram appears on page 409 of *The Rate of Interest*.[3] The transformation or production possibility or (as Fisher called it) opportunity curve ZPW shows an individual's opportunity to transform present consumption (measured on the horizontal axis) into future consumption (measured on the

Figure 2

FISHER'S DIAGRAM



Given the interest rate implicit in the slope of line AB, an investor produces the two-period consumption bundle P having the highest present value. Then he trades that bundle for bundle Q by lending PD units of present consumption for DQ units of future consumption to reach his point of maximum satisfaction Q.

Source: Fisher [4, p. 409].

[3] Fisher also used the diagram in his *The Theory of Interest* (1930). On Fisher's diagram see Hirshleifer [9, pp. 330-32] and Samuelson [15, pp. 29-33].

vertical axis) by investing in real capital projects. The concave shape of the curve represents diminishing returns to investment as the sacrifice of more and more units of consumption today yields smaller and smaller increments to consumption tomorrow.

The set of convex iso-desirability curves (as Fisher called them) labeled 10, 20, 30, etc. constitute the individual's indifference map. Each curve shows alternative combinations of present and future consumption that yield equal satisfaction. Higher curves represent higher levels of satisfaction. Finally, the interest line AB shows the opportunity to convert P dollars of present consumption into Q dollars of future consumption by lending at the market rate of interest shown by the slope of the line. In other words, one can lend as well as invest.

Fisher explains that the individual, if deprived of the opportunity to lend on the money market, would choose the two-period consumption combination shown by the common point of tangency of indifference curve and production possibility curve (point S).[4] This is analogous to the trade diagram's closed-economy equilibrium production and consumption point.

Given the opportunity to lend at the going rate of interest, however, the individual equates that rate with the marginal rate of return on real investment by moving along the production frontier to point P on the highest attainable interest line AB. That is, he chooses the two-period consumption bundle having the highest present value calculated at the market interest rate shown by the slope of AB. Then he trades along that line, lending PD ($= x'$) dollars of current consumption in exchange for DQ ($= x''$) dollars of future consumption, to reach a point of maximum satisfaction Q. In short, given the opportunity to trade at a market price, the individual produces the bundle of goods having the highest market value and then trades it for a preferred bundle lying beyond the production frontier. But this is exactly what a fully competitive open national economy does when given the opportunity to trade at world prices.

Modern users of the trade diagram note that international equilibrium requires the world price ratio be such as to balance trade across nations. In other words, the desired exports of one nation must at the equilibrium price ratio equal the desired imports of another and vice versa. Fisher argued the same about the equilibrium rate of interest. That rate, he said, equates the desired lending of one individual with the desired borrowing of another—that is, it ensures that the legs of the trade triangle PDQ are equal in length but opposite in sign across lenders and borrowers. Thus Fisher did more than specify trade equilibrium conditions for a single individual facing a given market rate. He also specified the market equilibrium conditions that determine that rate. True, he did not show such conditions in his diagram. That is, he did not extend it to the two-person case. But he stated how it could be done. His work presaged later uses of the diagram to depict world trade equilibrium in the two-country case.

Enrico Barone

If Fisher was the first to use the diagram to show the gains from *intertemporal* trade, then Enrico Barone, the Italian mathematical economist and author of the famous article on "The Ministry of Production in the Collectivist State," was the first to use it to depict the gains from *international* trade.[5] In a long footnote in the 1908 edition of his *Principi di economia politica*, he presented a diagram showing pre- and post-trade equilibrium positions for a single national economy that produces and consumes two goods A and B (see Figure 3). His diagram, like Fisher's, consists of three types of curves.

His "production indifference" or transformation curve AB shows the maximum alternative combinations of the two goods the economy can produce from available resources. Its nonlinear curved shape indicates that production takes place under conditions of nonconstant costs. The slope of the curve at any point M represents what Barone called "comparative cost," or the ratio of the marginal costs of production.

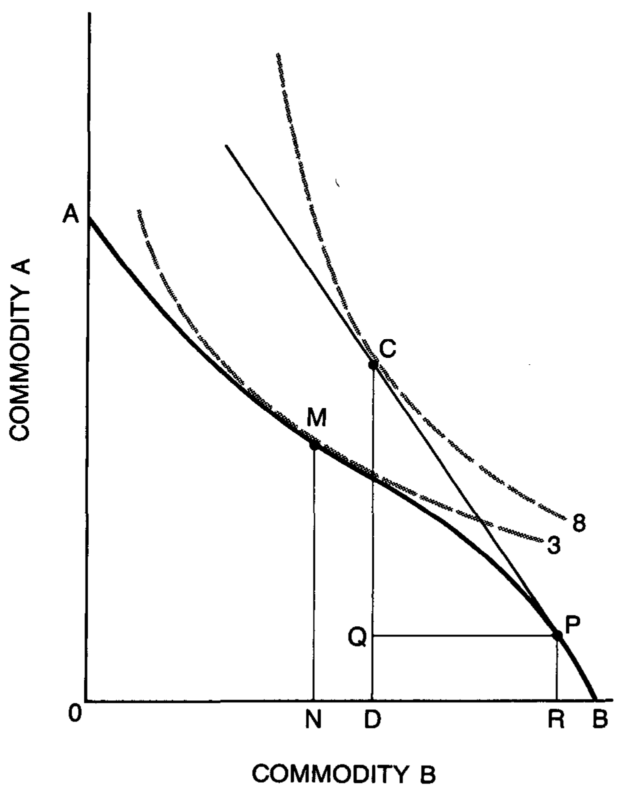
The curves bearing the numbers 3 and 8 are two of a set of community taste indifference curves that represent demand conditions in the economy. Each curve shows alternative commodity bundles yielding equal satisfaction. Higher curves represent higher levels of satisfaction as indicated by the higher numbers they bear. Finally, the curve PC is the world price line whose slope indicates the relative cost of obtaining goods A and B on the world market.

Before trade, the country produces and consumes at the autarky equilibrium point M characterized by the common tangency of production possibility and taste indifference curves. The slope of that tangent represents the domestic pre-trade price ratio and indicates that the country has a comparative cost advantage over the rest of the world in the production of good B.

[4] Fisher omits the relevant indifference curve to avoid cluttering the diagram.

[5] What follows draws heavily on Maneschi and Thweatt [12].

Figure 3
BARONE'S DIAGRAM



Given the opportunity to trade along the world price line PC, the country shifts production from autarky point M to specialization point P. Then it trades commodity bundle P for bundle C by exporting PQ of B for QC of A to reach its point of maximum satisfaction C.

Source: Maneschi and Thweatt [12, p. 381].

When trade opens up at the world price ratio given by the slope of line PC, the country exploits its comparative advantage by moving to production point P where the ratio of domestic marginal costs equals the world price ratio and GNP valued at world prices is maximized. In short, the country produces at the point of tangency of the transformation curve and the (highest attainable) world price line. Then it trades along that line, exporting PQ of good B in exchange for imports of QC of good A, until it reaches the point of maximum satisfaction C. By taking advantage of trade, it separates its production and consumption points and consumes beyond its transformation curve.

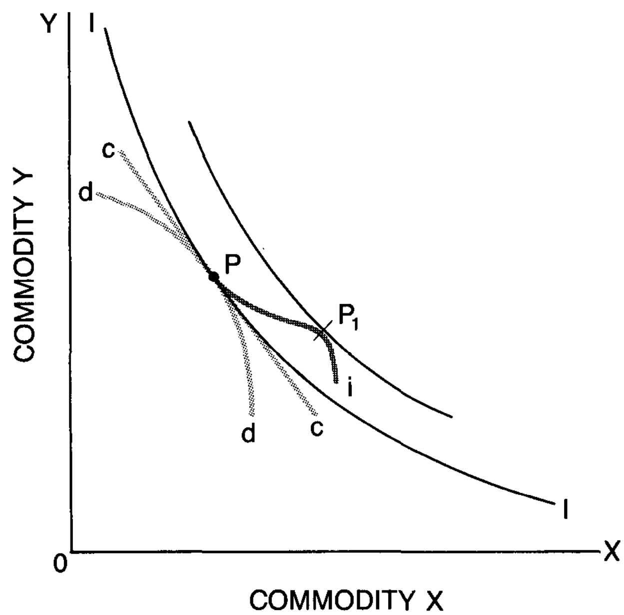
Here are all the elements found in modern versions of the diagram— the three-curve apparatus, the gap between autarky (closed economy) and world prices that makes trade feasible, the movement to the specialization point of maximum-value output, the post-trade separation of production and consumption points, and the trade triangle that reconciles those points. All this was a brilliant performance that should have made Barone the leading figure in the diagram's development. Such, however, was not the case. For all its brilliance, his contribution went largely unnoticed and consequently had no discernible influence on the work of his contemporaries and immediate successors. Barone himself may have been partly responsible for this state of affairs. By burying his diagram in a footnote of the 1908 *Principi* he effectively minimized its importance. That he may have intended to do so is suggested by his failure to include the diagram in his other writings. At any rate it is not to be found in later editions of the *Principi*. When it was finally restored to the 1936 edition it hardly seemed original. By then, other analysts had independently rediscovered the diagram and had developed it beyond Barone. Only in recent years, with the rediscovery of the extremely scarce 1908 edition of the *Principi*, have scholars been able to confirm the originality of Barone's contribution.

Allyn A. Young

After Fisher and Barone, work on the diagram languished. During the next 20 years (1909-1929) only one new version appeared in print and it was inferior to the earlier ones. In apparent ignorance of the contributions of Fisher and Barone, Allyn A. Young in the appendix to his famous 1928 *Economic Journal* article on "Increasing Returns and Economic Progress" presented a closed-economy version of the diagram that according to him derived straight from Pareto (see Figure 4).

Young did not use his diagram to illustrate comparative advantage or the gains from trade. Still he merits recognition for at least three reasons. He anticipated Gottfried Haberler by two years in defining the slope of the production frontier (his "curve of equal costs") as the opportunity cost of producing a unit increase in either good in terms of the amount of the other good sacrificed. Also he explained better than his predecessors that a concave curve reflects increasing opportunity cost, a linear curve constant cost, and a convex curve decreasing cost. Finally, he indicated how increasing returns in one industry might introduce a convex segment into a otherwise concave curve. In this connection

Figure 4
YOUNG'S DIAGRAM



Curves dd and cc are the production frontiers showing increasing costs and constant costs, respectively. Curves dPi and cPi represent cases in which decreasing costs prevail over part of the production frontier. Tangency with indifference curves II, etc., yields equilibrium at P in the first set of cases, P_1 in the second.

Source: Young [18, p. 540].

he discussed stability of closed-economy equilibrium under increasing, constant, and decreasing costs. He correctly noted that stability is ensured in all cases provided collective indifference curves possess greater convexity than the production frontier.

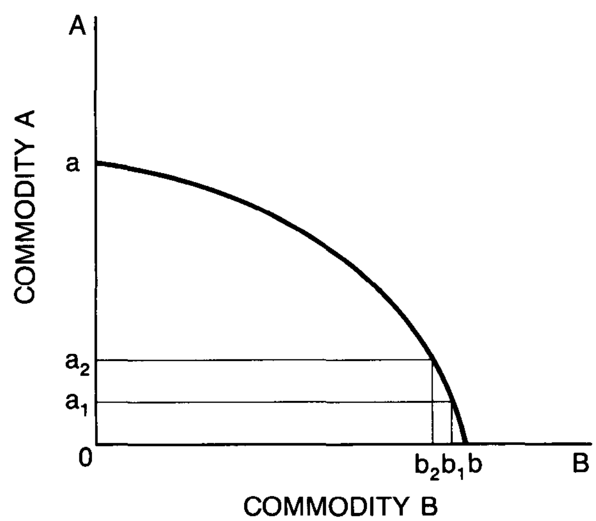
As for collective indifference curves, he noted that their location on the diagram assumes a fixed distribution of income when in fact that distribution and thus the indifference map itself changes with movements along the production frontier. In other words, a reallocation of production from good X to good Y redistributes income from X producers to Y producers and thus shifts the indifference map. For this reason he thought such curves should be treated as an expository device and not as a rigorous conception. His discussion anticipated Lerner and Tinbergen, both of whom analyzed decreasing costs, and Viner, who criticized the concept of community indifference maps.

Gottfried Haberler

We have seen how Fisher in 1907 invented the diagram, how Barone in 1908 extended it to international trade, and how Young in 1928 applied it to the closed economy. In 1930, however, Gottfried Haberler in his seminal paper on comparative cost, did what none of his predecessors had done.[6] He introduced into international trade theory a strictly concave production possibility curve showing diminishing returns and increasing costs in the production of both goods (see Figure 5). Fisher and Young, of course, had worked with such concave transformation curves, but not within the context of international trade theory. Barone, on the other hand, had used transformation curves to analyze foreign trade. But the curves he used were not strictly concave.

[6] See Haberler [6] for an English translation of his 1930 paper from the original German. Haberler's diagram and its underlying analysis also appears in Chapter 10 of his *The Theory of International Trade, with Its Applications to Commercial Policy* (1936).

Figure 5
HABERLER'S STRICTLY CONCAVE PRODUCTION FRONTIER



The concavity of curve ab shows that successive unit increases in one good require progressively larger decreases in the other. The opportunity cost of each good increases as more is produced.

Source: Haberler [6, p. 10].

Nor had Haberler's predecessors adequately explained the reasons for the curve's concave shape. Such concavity they attributed to diminishing returns and increasing costs without specifing the forces causing these phenomena. Haberler, however, explained the causes of the curve's concavity by invoking the notion of specific and nonspecific factors of production. Specific factors he defined as those tied to a particular industry and suitable to the production of no other good. Nonspecific factors on the other hand are those freely transferable between industries and equally suited to the production of both goods.

Using a two-good, three-factor model, he assumed that each good requires for its production one specific factor which it uses exclusively and a nonspecific factor shared in common with the other industry. Combining increasing amounts of the nonspecific factor with fixed amounts of a specific one to produce more of either good yields decreasing increments of output, i.e., diminishing returns. Thus the amount of one good sacrificed to free enough nonspecific resources to produce a unit increase in the other good must rise as output of the latter good increases. The same thing would happen, Haberler noted, if all resources, though mobile, were not equally well-suited for different employments. For example, suppose that of the nation's fixed stock of resources all initially employed in producing A, part is better suited to producing B. One might think of mountainous land better suited to skiing or mining than to wheat production. Transferring such resources to B at first results in a large rise in the output of that good at the cost of little sacrifice of A. Beyond some point, however, continued expansion of B necessitates the transfer of resources less and less suited to B production and more and more suited to A production. At that point the opportunity costs of B in terms of A sacrificed rises. Either case, Haberler said, yields a smooth concave curve with the marginal opportunity cost of transforming one good into the other rising continuously over the whole range of the curve.

Finally, Haberler better than any of his predecessors explained the place of the transformation curve in the theory of comparative advantage. According to him, the curve together with demand conditions (indifference curves) determines an economy's production point and thus relative commodity costs in the absence of trade. On the assumption that prices equal costs, those curves also determine relative commodity prices. Differences in these autarky relative costs and prices across nations reflect comparative advantages that make trade mutually advantageous. When trade takes place at the equilibrium world price ratio each nation tends to specialize in the production of the commodity of its comparative advantage. As it does so, however, it incurs increasing opportunity costs. Specialization continues up to the point at which marginal opportunity costs equal world prices, i.e., up to the point at which the transformation curve just touches the world price line. Each nation then trades along that line, exporting its comparative advantage commodity in exchange for the other commodity, until it reaches its point of maximum satisfaction.

Haberler's analysis had a galvanizing effect on his contemporaries. In quick succession Jacob Viner, Abba Lerner, and Wassily Leontief combined his concave transformation curve with collective indifference curves to obtain the basic diagram of the trade theorist. Each of these writers, however, put the diagram to somewhat different uses described below.

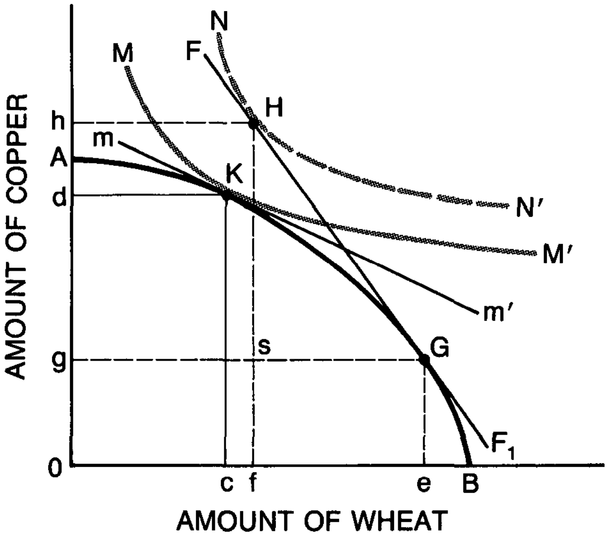
Jacob Viner

Viner's version of the diagram, presented in a lecture at the London School of Economics in January 1931 but not published until the 1937 appearance of his *Studies in the Theory of International Trade*, shows before- and after-trade equilibria for a single country (see Figure 6). Before trade, the country produces and consumes at point K on the highest attainable indifference curve tangent to the production frontier. When presented with the opportunity to trade at a world price ratio different from the autarky one—this difference indicated by the different slopes of the price lines FF_1 and mm '—the country shifts production to point G and then trades along the world price line, exporting Gs units of wheat in exchange for imports of sH units of copper. In so doing, it ends up consuming commodity bundle H lying on a higher indifference curve than the autarky bundle K consumed before trade.

Except for the concavity of the production possibility curve, Viner's diagram is virtually the same as Barone's. But Viner did one thing that neither Barone nor anyone else had done up to that time. He pointed to certain logical flaws in the diagram's construction and questioned its usefulness in showing the gains from trade.

In particular, he focused on the shortcomings of community indifference maps and production possibility curves. Community indifference maps were suspect because they embodied the assumption of a fixed distribution of income when in fact trade would change that distribution and thus the indifference map itself. Likewise the production

Figure 6
VINER'S DIAGRAM



Given the opportunity to trade at world prices shown by the slope of line FF_1, the economy shifts production from autarky bundle K to bundle G, which it then trades for preferred bundle H by exporting Gs wheat for sH copper.

Source: Viner [17, p. 521].

possibility curve was flawed because it assumed perfectly inelastic (fixed) factor supplies when in fact those supplies vary with changes in their prices. Trade, by changing factor prices, would change the quantities of factors supplied and thus the production possibility curve itself. Nor was this the only problem. The curve, Viner noted, also embodied the assumption of factor indifference between alternative uses when in reality factors may prefer one employment to another. Assuming factors employed in the industry of their preference are paid the value of their marginal product there, they must receive a premium over that to induce them to work in the other industry. In that case, factor costs to one industry will not equal sacrificed factor product in the other, and the cost of securing a unit increase in either good is not accurately measured by the quantity of the other good given up.[7] Viner's conclusion was straightforward. Job preferences and the resulting compensating pay differentials drive a wedge between commodity relative prices and the ratio of factor marginal products reflected in the slope of the transformation schedule. In other words, prices would not reflect opportunity costs as Haberler supposed.

Viner's trenchant criticisms proved less than devastating. For the production possibility curve was simply too useful a tool to abandon. Despite its restrictive assumptions, it captured the essence of a country's commodity supply conditions. For that reason, trade theorists chose the diagram and its underlying opportunity cost interpretation over Viner's real cost interpretation.

Abba Lerner

Unlike Viner, Lerner accepted the trade diagram uncritically. He used it to depict trade equilibrium for the aggregate world economy in a two-country model.[8] His demonstration, as presented in his celebrated 1932 *Economica* article on "The Diagrammatical Representation of Cost Conditions in International Trade," required three steps.

First, he derived the world transformation curve by optimally adding national production possibilities at equal marginal cost ratios. He did so by sliding one country's production possibility block along the other's with the slopes or marginal opportunity cost ratios always kept equal (see Figure 7). In this way he traced out an efficient world production possibility frontier, something nobody had done before.

Second, he confronted this world production frontier with a global community indifference curve which he implicitly derived by aggregating over the underlying country curves (not shown by him). The resulting common point of tangency of the two curves determines the world production and consumption points as well as the equilibrium terms of trade.

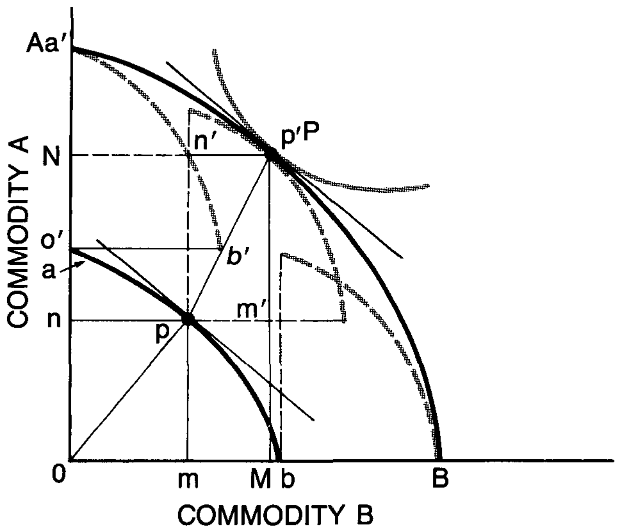
Finally, he located each country's post-trade production point by moving the world terms-of-trade line parallel to itself until it just touched the individual production possibility curves. He did not identify the consumption point or the exports and imports of each

[7] An example will suffice. Industry A pays each unit of labor a real wage w_A equal to its marginal product there. But that same labor unit costs industry B the amount $w_A + d$, where d is the wage differential or pay premium that compensates for the nonpecuniary disadvantages (subjective disutility) of working in B. Thus labor's cost to B exceeds its foregone product in A by the factor d. Similarly, labor's marginal product in B equals its wage rate there, $w_A + d$. But that same unit of labor costs A only w_A. Thus labor's cost to A understates its sacrificed alternative product by the factor d. True costs deviate from opportunity cost.

[8] On Lerner see Mundell [13, pp. 147-48] and Samuelson [15, p. 645].

Figure 7

LERNER'S DERIVATION OF THE WORLD TRANSFORMATION CURVE



Moving one country's production block along the other's traces out the world transformation curve AB. The diagram shows three successive positions of the second country's block o'a'b' as it slides along the first country's production frontier ab. Tangency of transformation curve and indifference curve yields world equilibrium at P with country post-trade production points being p' and p, respectively.

Source: Lerner [11, p. 90].

nation. But he did remark that both nations would benefit from trade even if they possessed identical concave transformation curves provided their indifference maps differed. His remark anticipated Wassily Leontief's geometrical demonstration of this case.

He also showed what the world production possibility curve looks like when at least one of the countries produces under conditions of increasing returns such that its production frontier is convex. Richard E. Caves neatly summarizes his analysis.

> He proved that increasing returns necessitate complete specialization by at least one country. This can occur not only when both countries' transformation curves are convex to the origin, but also if one (national) transformation curve is convex while the other shows a constant rate of transformation, or even concavity to the origin, so long as the convexity of the one exceeds the concavity of the other. There will normally be points on the world transformation curve where more than one pattern of international specialization is efficient. No matter which of the two countries specializies completely, the same commodity totals will be produced. Another trait of such a point is if a change in world tastes is moving the world production combination past one, the optimal pattern of specialization may shift markedly [3, pp. 162-63].

Wassily Leontief

In the year after Lerner's article appeared, Leontief in his paper on "The Use of Indifference Curves in the Analysis of Foreign Trade" completed Lerner's demonstration of world trade equilibrium. He did so by depicting for both countries the post-trade consumption points and trade triangles that connect those points with their corresponding production points, something Lerner had failed to do. Unlike Lerner, however, he did not work with world production possibility and taste indifference curves. Instead, he focused on the curves of each country, combining them together in a single chart. In this way he was able to use the diagram to show how trade affects both countries simultaneously.

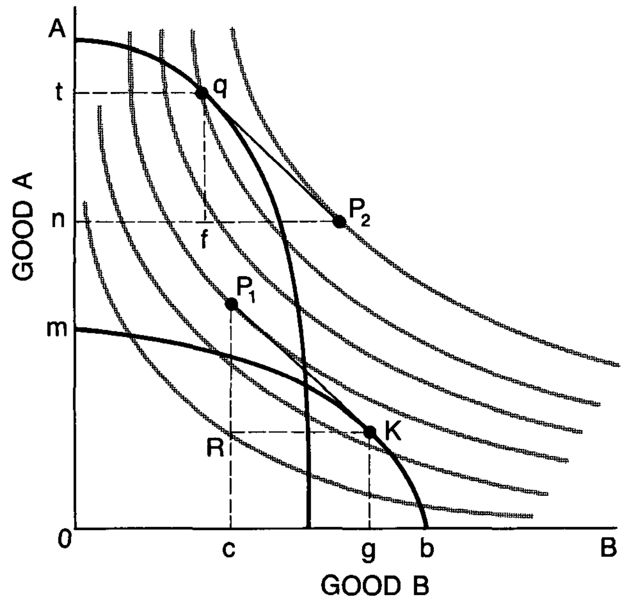
He showed how gains from trade arise when (1) production conditions alone and (2) demand conditions alone differ across countries. In the first case, countries have different production possibility curves but identical indifference maps (see Figure 8A). In the second case (anticipated by Lerner), production possibility curves are the same and only indifference maps differ across countries (see Figure 8B).

Figure 8A depicts the first case. Here the country possessing the vertically elongated transformation curve produces at q where its output valued at world prices is maximized. Then it trades along the relative price line qP_2, exporting qf of good A against imports of fP_2 of good B, and consumes at P_2, a point it could not reach before trade when it was constrained to consume on its production possibility curve. Likewise the other country gains by producing its highest valued output at K, trading along the price line KP_1, and consuming at P_1 beyond its production possibility frontier.

As for equilibrium conditions, Leontief specified that the price lines connecting the production and consumption points must be of the same slope and length for both countries. The first condition ensures that both countries face the same price ratio or terms of trade. The second ensures that exports of one country equal imports of the other. In other words, it ensures that the trade triangles P_1RK and qfP_2 are the same, as required for international equilibrium.

Figure 8a

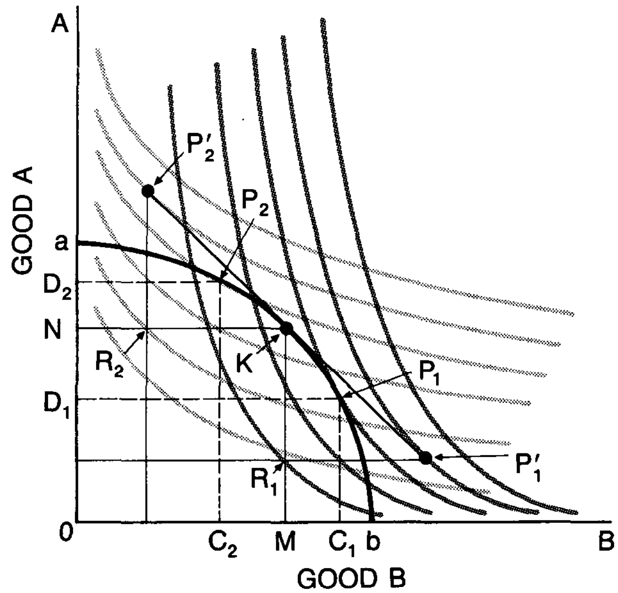
Different Transformation Curves, Identical Indifference Maps



One country produces at q and exports qf of A for fP_2 of B. The other produces at K and exports KR of B for RP_1 of A. The equilibrium world price ratio shown by the common slope of lines qP_2 and P_1K must be such as to make the trade triangles identical.

Figure 8b

Identical Transformation Curves, Different Indifference Maps



Both countries produce at K, one exporting KR_1 of A for $R_1P'_1$ of B, the other exporting KR_2 of B for $R_2P'_2$ of A. The equilibrium world price ratio must be such as to make the trade triangles identical.

Source: Leontief [10, pp. 25, 27].

Trade also enables countries to consume beyond their production possibility curves when only demand conditions (indifference maps) differ. Leontief's second diagram shows why: different demand conditions result in different pre-trade equilibrium points on the production possibility curve. At these different points, comparative costs differ making trade advantageous.

Thus before trade the country with the steeper indifference curves initially consumes and produces at P_1 on its production possibility curve while the other country does the same at P_2. The different slopes of the production possibility curve at those two autarky points show that comparative costs differ across countries making trade profitable. When trade takes place at the equilibrium price ratio given by the slope of line $P'_2P'_1$, each country produces at K and exports the good in which it has a (pre-trade) cost advantage. The first country exports KR_1 of good A for imports of $R_1P'_1$ of good B, reaching consumption point P'_1 in the process. Similarily, the other country exports R_2K of good B in exchange for imports of $R_2P'_2$ of good A, and consumes at P'_2 beyond its production possibility curve. Both countries gain from trade despite having identical production frontiers. Here in Leontief's 1933 diagram is everything and more found in the earlier constructions of his predecessors.

In short, Leontief brought the diagram to its highest stage of development up to the mid-1940s and established it as the standard geometrical tool of the international trade textbooks. It was his version, showing as it does in one Cartesian plane the mutual gains from trade and the international equilibrium conditions for both countries simultaneously, that entered such influential early texts as D.B. Marsh's *World Trade and Investment* (1951) and Charles Kindleberger's *International Economics* (1953). Even today one finds it in such leading texts as Caves' and Jones' *World Trade and Payments* and W. Ethier's *Modern International Economics*.

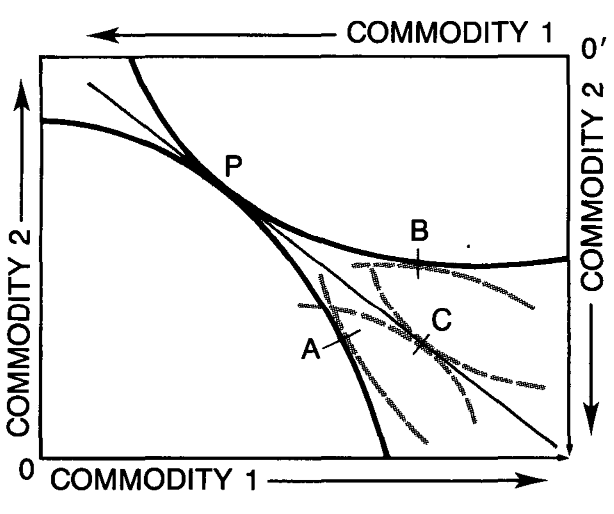
Jan Tinbergen

That Leontief's diagram had by the 1940s already become the standard way to depict international equilbrium under conditions of increasing costs and competitive markets is evident from a glance at Tinbergen's 1945 contribution. His treatment of this case differs in no essential way from Leontief's. Like Leontief he shows the individual open economy producing at the point of tangency of the production frontier and world price line and then trading along that line to reach the consumption point of maximum satisfaction (see Figure 1). And like Leontief he shows that a similar outcome holds for the other country whose exports must also equal the imports of the first and vice versa.

Tinbergen extends Leontief's analysis in two minor respects. He lets production possibility curves *and* indifference maps differ across countries. And he depicts the two-country equilibrium in a box diagram showing the second country's system of coordinates lying diagonally opposite those of the first (see Figure 9). But these are merely trivial differences in mode of presentation. The results he obtains are exactly the same as those shown in Leontief's diagrams.

Figure 9

TINBERGEN'S TWO-COUNTRY DIAGRAM OF WORLD TRADE EQUILIBRIUM



Country A's coordinates are plotted from 0, country B's from 0′. Global equilibrium requires both countries produce and consume at common points of tangency P and C on the world price line PC.

Source: Tinbergen [16, p. 137].

Only when he drops Leontief's assumptions of competitive behavior and increasing costs does he develop some novel results. He considers three cases, two of which yield the perverse outcome that trade may worsen rather than improve a country's welfare. He takes first the case of decreasing costs prevailing in both industries such that the transformation curve becomes convex rather than concave to the origin. He shows here that with trade the only stable equilibria in production are the terminal points on the curve representing complete specialization in one good or the other. Which good the country chooses to produce upon the opening of trade depends on the slope of the world price line and the shape of the indifference curves. Either choice will yield gains from trade.

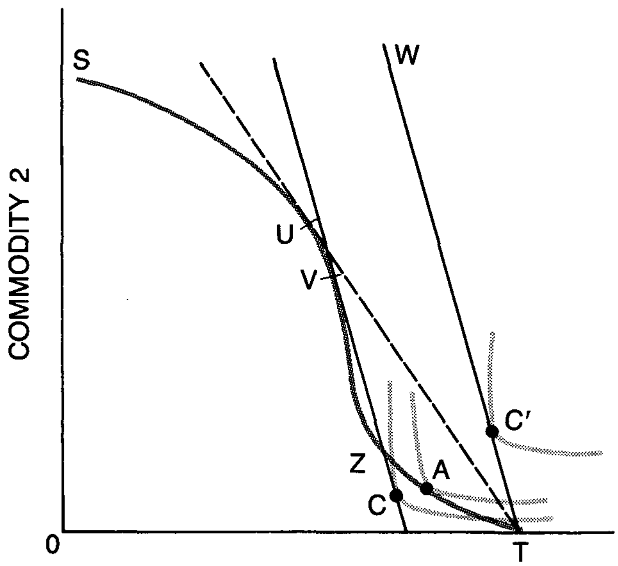
Next he considers the case (anticipated by Young and Lerner) in which decreasing costs prevail in one industry and increasing costs in the other such that the production frontier contains convex and concave segments. He argues that in this case an open trading economy may myopically choose production and consumption points that worsen its welfare compared to the no-trade situation (see Figure 10). That is, given the world price ratio shown by the slope of line VC, the economy chooses production point V and consumption point C which is inferior to autarky point A. But he then notes that this construction assumes that producers and consumers lack perfect knowledge of their opportunities. Otherwise they would produce at T and trade along the price line TW (same slope as VC) to reach consumption point C′ which is superior to the autarky point.

Last he presents a case in which monopoly pricing in the industry possessing a comparative cost advantage distorts relative commodity prices and causes the country to produce and export the wrong good, namely the good in which it has a comparative disadvantage. Depending on the shape of the indifference map, the economy may be better off or worse off than before trade (see Figure 11).

These results of course differ from Leontief's. But Tinbergen reached them with the same geometrical tools. He changed the shape and location of the diagram's curves, to be sure. But he put those curves to their traditional use to depict international equilibrium and the gains (or losses) from trade, albeit for anomalous cases. In this respect, his work continued the tradition stretching from Barone to Leontief.

Figure 10

TRADE EQUILIBRIUM WITH A MIXED TRANSFORMATION CURVE

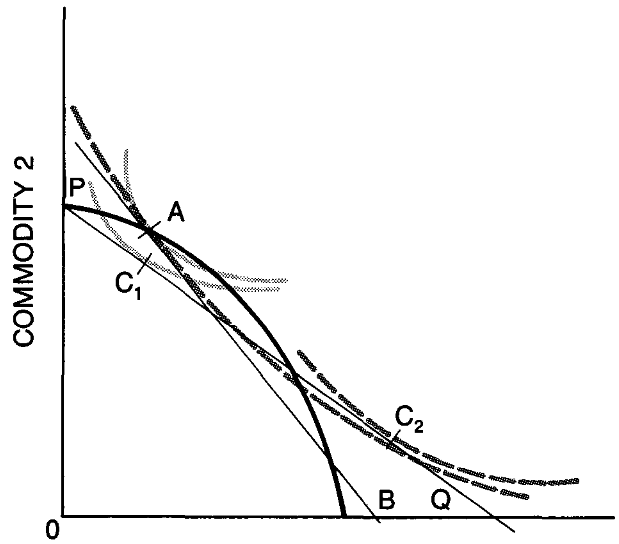


Imperfect knowledge and a mixed (concave-convex) transformation curve can make the country worse off with than without trade. At the world price ratio given by the slope of line VC, the economy produces at V and consumes at point C which lies on a lower indifference curve than the autarky point A. Conversely, with perfect knowledge the economy produces at T and consumes at C′, reaping a clear gain.

Source: Tinbergen [16, p. 133].

Figure 11

MONOPOLY PRICING IN THE COMPARATIVE ADVANTAGE INDUSTRY



Monopoly pricing raises the relative price of good 1 (slope of line AB) above its relative marginal cost (slope of the production frontier at autarky point A) and makes it appear that comparative advantage lies in good 2 when in fact it lies in good 1. Consequently, when trade opens up at the world price ratio given by the slope of line PQ, the economy specializes in the wrong good, producing at P and trading along line PQ to reach point C_1 or C_2 depending on the location of the indifference map. Trade yields losses in the first case, gains in the second.

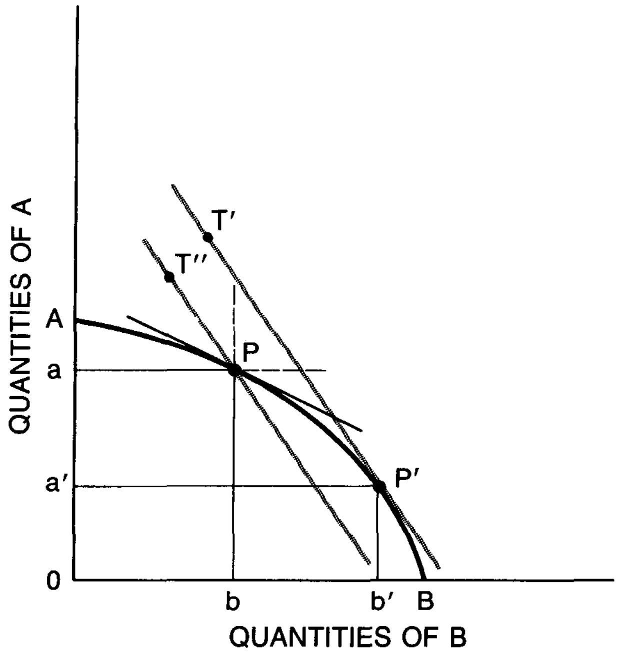
Source: Tinbergen [16, p. 136].

The Diagram Since Tinbergen

After Tinbergen, Haberler in 1950 used the diagram to distinguish between the consumption (exchange) and production (specialization) components of the total gain from trade. The total gain of course is the jump from the autarky consumption point to the preferred point on the (highest attainable) world price line just touching the production possibility curve. Of this total, the consumption gain stems from the opportunity to exchange the pre-trade bundle of goods at world prices. Haberler shows this gain as the movement from P to T″ along a world price line passing through the pre-trade consumption point (see Figure 12). Added to this is the production gain stemming from the opportunity to produce the highest valued bundle of commodities measured at world prices. Haberler shows this gain as the movement from T″ to T′ that results when the economy produces the output mix whose marginal opportunity cost just equals the world terms of trade.

The point of Haberler's demonstration is this: of the two sources of gain, exchange and specialization, the first is fundamental. For, as the diagram shows, exchange yields gains even in the absence of specialization (that is, in the absence of a change of production). The economy simply trades its given autarky bundle for a preferred one at world prices. By contrast, specialization without exchange yields no gains. For it never pays to produce the output mix valued highest at world prices when one cannot trade at those prices: in such cases the autarky mix

Figure 12
HABERLER ON THE GAINS FROM TRADE



Consumption (exchange) gains are shown by the jump from P to T″ as the economy swaps its autarky bundle for a preferred one at world prices. Production (specialization) gains are shown by the further jump to T′ that results when the economy produces and trades the output bundle P′ having the highest value at world prices. Trade yields gains even in the absence of specialization.

Source: Haberler [8, p. 38].

is preferred. On the contrary, specialization without trade yields losses since a closed economy must be self-sufficient (diversified) in all goods. In short, exchange rather than specialization is the necessary and sufficient condition for trade gains.

Haberler's demonstration did not exhaust the diagram's potential: new uses were found for it. Haberler himself employed it again in 1950 to illustrate the infant industry argument for protection. In 1952 James Meade employed it to derive trade indifference curves used in advanced trade geometry. Harry Johnson in 1964 used it to depict noneconomic objectives of tariffs. Jagdish Bhagwati in 1957 used it to show the effects of technological progress on the terms of trade and national welfare. Robert Mundell in 1957 used the diagram to show how international factor mobility negates the protective effects of tariffs. Haberler in 1950, Bhagwati and Ramaswami in 1963, and Johnson in 1965 employed the diagram to analyze domestic market distortions (divergences between private and social marginal costs) arising from external economies or diseconomies and rigid factor prices. The best corrective, they showed, is not a tariff but rather taxes and subsidies in the sector in which the distortions arise.

In all these uses the diagram proved its strength and versatility. So much so that trade theorists will undoubtedly employ it again and again. When they do, they will owe a large debt of gratitude to the pioneers who developed this powerful tool. Even today, if one understands the diagram one understands the logic of comparative advantage and gains from trade.

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