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Fisherian and Wicksellian Price-Stabilization Models in the History of Monetary Thought

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If money became scarce, as shown by a tendency of the price level to fall, more could be supplied instantly; and if superabundant, some could be withdrawn with equal promptness. . . . The money management would thus consist . . . of buying [securities on the open market] whenever the price level threatened to fall below the stipulated par and selling whenever it threatened to rise above that par.

-Irving Fisher [3, p. 97]

That interest on borrowed money is for one reason or another either below or above the level which would normally be governed by the real rate ruling at the time [is] a circumstance which, so long as it lasts, must cause a progressive rise or fall in prices. . . . [Thus] there should be a common policy, a raising or lowering of bank rates . . . from time to time in order to depress the commodity price level when it showed a tendency to rise and to raise it when it showed a tendency to fall.

-Knut Wicksell [17, pp. 215, 223]

Introduction

Central bankers charged with the responsibility for stabilizing the general level of prices need to know at least two things. First, what causes prices to deviate from their desired fixed target level? Secondly, what policy rule or response most effectively corrects those deviations and restores prices to target?

Historically, proponents of price stability developed two basic reduced-form models to answer these questions. One model, associated with Irving Fisher, attributes price movements to shocks operating through excess money supply and demand. It calls for money-stock adjustments to keep prices at their target level. The other model, associated with Knut Wicksell, ascribes price movements to discrepancies between market and natural (equilibrium) rates of interest. It prescribes interest-rate adjustments to restore prices to target. Although both models are fairly well known, their historical significance has not always been fully appreciated. Until the Keynesian revolution of the 1930s and 1940s they constituted the dominant policy models in nineteenth and twentieth century central banking tradition. In fact, many celebrated economists before Fisher and Wicksell contributed to their development.

Given the importance of price stability as a policy goal, it is useful to reexamine these historical models. As simple, stripped-down prototypes of the more elaborate macroeconomic models employed today, they reveal in sharp focus much about the mechanics of price-level stabilization. In particular, they provide information on the relative price-stabilizing powers of alternative policy feedback rules—e.g., money stock rules versus interest rate rules. Accordingly, the threefold purpose of this article is (1) to describe the structure and logic of the two reduced-form models, (2) to sketch their evolution in the history of monetary thought, and (3) to analyze each to see if they yield dynamic stability such that prices return to target equilibrium following economic shocks. The central message is that both models, if properly formulated, still provide reliable guides to policy.

The Models Outlined

Before tracing the historical development of the models, it is necessary to sketch their essential features so as to identify what particular contributors had to say about each. As presented here, both reduced-form models consist of (1) a price-change equation relating price movements to the variables that cause them and (2) a policy-response function specifying the feedback rule the central bank follows to keep prices on target.

Fisherian Model

The Fisherian model says that prices rise or fall when the existing quantity of money exceeds or falls short of the amount people wish to hold at prevailing prices and real incomes. It also says that policymakers can correct deviations of prices from target by expanding or contracting the money stock (or at least its high-powered base component) as prices are below or above their target level. In symbols:

$$(1) \quad dP/dt = \alpha(M - kPy)$$

$$(2) \quad dM/dt = \beta(P_T - P)$$

where dP/dt denotes price change, P actual prices, P_T their fixed target level, M the money stock, dM/dt its change, k the inverse of money's turnover velocity or the fraction of nominal income people wish to hold in money, y real income, and α and β positive constants.

Thus suppose a money-control error or decrease in money demand produces an excess supply of money. The resulting attempts by cashholders to get rid of the excess cash through spending puts upward pressure on prices according to equation 1. As prices begin to rise above target, the central bank responds by contracting the money stock according to the feedback policy rule represented by equation 2. In this way the central bank eventually contracts the money stock sufficiently to restore prices to target. Such is the underlying logic of the Fisherian model.

Wicksellian Model

The alternative Wicksellian model attributes price movements to the differential between the natural (equilibrium) and market rates of interest. Prices rise when the market rate is below the unobservable natural rate, fall when the market rate exceeds the natural rate, and remain unchanged at a stationary level when the two rates coincide. When prices start to rise or fall the central bank acts to restore them to target by raising or lowering the market rate in proportion to prices' deviation from target. Stated mathematically:

$$(3) \quad dP/dt = \alpha(r - i)$$

$$(4) \quad di/dt = \beta(P - P_T)$$

where r denotes the natural rate, i the market rate, di/dt its adjustment, and the other symbols are as defined above.

These reduced-form equations are derived from a larger model that explains how the interest rate differential affects (1) real investment and saving, (2) loan supply and demand, (3) money supply and demand, and (4) aggregate supply and demand. Through these factors the rate differential moves the price level.

Thus when the loan rate lies below the natural rate (the rate that equilibrates saving and investment) investors demand more funds from banks than savers deposit there. Assuming banks accommodate these extra loan demands by issuing notes and creating checking deposits, a monetary expansion occurs. Since neither real income nor prices have changed in cashholders' money demand functions, the additional money constitutes an excess supply of cash that spills over into the product market in the form of an excess demand for goods. This excess demand puts upward pressure on prices which continue to rise until the rate differential vanishes. Since the model in its pure credit or inside money version contains no automatic self-equilibrating market mechanism to eliminate the rate differential, the central bank must do the job. To arrest and reverse the price rise the bank must raise the market rate until prices return to target.

Of course if the central bank knew the level of the natural rate it could always keep the market rate there and no price movements would occur. But the essence of the Wicksellian model is that the natural rate is an unobservable variable that moves around under the impact of productivity shocks, technological progress, factor endowment changes, and other real disturbances that cause it to deviate from the market rate. In such circumstances the central bank does not know what the natural rate is. It knows only that the resulting price level movements indicate that the market rate is not at its natural level and must be changed. That is, the bank must adjust the market rate in the same direction that prices are deviating from target, ceasing only when they finally stabilize there.

Historical Evolution of the Models

Having outlined the essential features of the two price-stabilization models, one can readily trace their evolution in the history of monetary thought. At least four classical and neoclassical economists contributed to the development of the Fisherian model: David Hume (1711-1776), David Ricardo (1772-1823), Irving Fisher (1867-1947), and Lloyd Mints (1888-1989). Likewise at least four monetary economists helped advance the Wicksellian model: Henry Thornton (1760-1815), Thomas Joplin (c.1790-1847), Knut Wicksell (1851-1926), and Gustav Cassel (1866-1945).

David Hume

The Fisherian model is much older than Irving Fisher. The origins of the model date back at least to David Hume's 1752 essay "Of the Balance of Trade." There Hume stated the gist of the model's equations, albeit in words rather than algebraic symbols (see Waterman [15, pp. 86-7]). True, as noted below, he substituted the world gold price of goods P_W for target prices P_T in the model's feedback policy rule or money adjustment equation. He also assumed that corrective money stock adjustments were achieved through international specie flows rather than through central bank action. But these are superficial differences only. Basically his equations were those of the Fisherian model.

Hume applied the model to a small open economy operating under a metallic (gold standard) regime with fixed exchange rates and a currency convertible into gold at a fixed price on demand. He showed how inflows and outflows of gold through the balance of payments would operate to correct monetary disequilibria and bring domestic prices in line with given world prices. In his famous exposition of the international price-specie-flow mechanism he assumed a sudden contraction of the domestic money stock and argued that three results would ensue.

First, the money stock contraction would, by reducing the existing quantity of money below the amount people desired to hold, produce domestic price deflation. Prices would fall in proportion to the monetary shortage or excess demand for cash:

$$(5)\quad dP/dt = \alpha(M - kPy).$$

Second, the fall in domestic prices P relative to given foreign (world) prices P_W would generate a trade balance surplus B as cheaper domestic goods outsold dearer foreign ones at home and abroad:

$$(6)\quad B = \beta(P_W - P).$$

Third, the trade surplus would be paid for by a compensating inflow of monetary gold from abroad:

$$(7)\quad B = dM/dt.$$

Substituting equation 7 into equation 6 yields

$$(8)\quad dM/dt = \beta(P_W - P)$$

which implies that the domestic money stock adjusts through specie flows until domestic prices stabilize at the fixed level of world prices as required for balance-of-payments and monetary equilibria. Here is the Fisherian model with (1) world prices replacing target prices and (2) the balance of payments replacing the central bank as adjuster of the money stock.

David Ricardo

Hume applied the model to a metallic or convertible currency regime. Ricardo, writing almost sixty years later, extended Hume's model to an inconvertible paper currency regime with floating exchange rates and a variable price of gold.

Ricardo wrote during the Bank Restriction period (1797-1821) of the Napoleonic Wars when the Bank of England had suspended the convertibility of the pound into gold at a fixed price upon demand. The suspension of specie payments and the resulting move to inconvertible paper was followed by a rise in the paper pound price of commodities, gold bullion, and foreign currencies. A debate then arose over the question: Was there inflation in England and if so what was its cause?

Ricardo's answer was definitive. In various newspaper articles and pamphlets, most notably his 1810 *High Price of Bullion, A Proof of the Depreciation of Bank Notes*, he argued that inflation did exist, that overissue of banknotes by the Bank of England was the cause, and that the premium on gold (the difference between the market and official mint price of gold in terms of paper money) together with the pound's depreciation on the foreign exchanges constituted the proof. He reproached the Bank's directors for having taken advantage of the suspension of convertibility to overissue the currency. And he admonished them to contract the note issue until the preexisting noninflationary price situation was restored. Here is the model's core postulate: that rising prices spell a redundancy of money requiring immediate corrective contraction.

In employing the model, Ricardo dropped Hume's assumption of an observable general level of prices since few reliable general price indexes existed at the time. He argued that given inconvertibility, gold's price and the exchange rate constituted good proxies for the unobservable general price level whose movements they matched almost one-for-one. This tight linkage derived from the notion that the pound price of goods was by definition equal to the pound price of gold times the world (and English) gold price of goods. Likewise it derived from the corresponding idea that the pound price of goods equalled the pound price of foreign currency times the foreign currency price of goods. With the price of goods in terms of gold and foreign currency given and normalized at unity, it followed that the paper pound price of goods moved one-for-one with the pound price of gold and foreign exchange.

Accordingly, in the model's equations he made three small changes. He substituted gold's price and the exchange rate for general prices P. He likewise used gold's premium over the official mint price and the depreciation of the exchange rate to represent price rises dP/dt. Finally, he used gold's mint price and the pre-existing exchange rate to stand for target prices P_T.

He then condensed the equations into his famous *Ricardian definition of excess* according to which if gold commands a premium and the exchange rate is depreciated then the currency is by definition excessive and must be contracted. His definition states that rising prices, or rather their empirical proxies, the gold premium and depreciated exchanges, signify an excess supply of money according to the expression $dP/dt = \alpha(M-kPy)$. His definition also directs the central bank to reduce the money supply when gold's price exceeds its old mint price and when the exchange rate is depreciated relative to its preexisting level. As these two differentials represent the corresponding gap between actual and target prices, one obtains the expression $dM/dt = \beta(P_T-P)$. Hence the Ricardian definition of monetary excess embodies both equations of the model.

Irving Fisher

The two main twentieth century proponents of the monetary model were the American quantity theorists and price stabilizationists Irving Fisher and Lloyd Mints. Fisher employed the model in developing his famous "compensated dollar" rule for stabilizing the purchasing power of the dollar. His rule called for adjusting the gold content of the dollar or its inverse, the official buying and selling price of gold, equiproportionally with changes in the preceding month's general price index. In essence his proposal was based on the relationship: dollar price of goods equals dollar price of gold times gold price of goods. It required adjusting the dollar price of gold to offset movements in the gold price of goods (as proxied by last month's general price index) so as to stabilize the dollar price of goods.

Thus if excess supplies of monetary gold were elevating the price of goods (both in terms of gold and dollars) in the equation $dP/dt = \alpha(M-kPy)$ the monetary authorities would respond with compensating reductions in the dollar price of gold. The fall in gold's price would have a twofold stabilizing effect. It would neutralize the inflationary impact of the rise in the gold price of goods such that dollar prices would remain unchanged. It would also, by rendering gold cheaper to industry and the arts, divert existing stocks from monetary to nonmonetary uses. The result would be to reduce the excess supply of monetary gold that put upward pressure on prices. Money (and prices) would move in the direction dictated by the expression $dM/dt = \beta(P_T-P)$.

Fisher also used the monetary model in developing his alternative proposal to stabilize prices through open market operations. He stated the essentials of the model most clearly in his 1935 book *100% Money*. There he argued (1) that price level movements stem from excess money supplies and demands, (2) that prices can be restored to target via corrective adjustments in the money stock, and (3) that such corrective adjustments can be achieved through open market operations. As he put it:

> If money became scarce, as shown by a tendency of the price level to fall, more could be supplied instantly; and if superabundant, some could be withdrawn with equal promptness. . . . The money management would thus consist, ordinarily, of buying [securities] whenever the price level threatened to fall below the stipulated par and selling whenever it threatened to rise about that par. (p. 97).

Via such operations, the monetary authority could, he claimed, precisely adjust the quantity of money so as to "stabilize the price level at the prescribed point." (p. 90).

Lloyd Mints

Fisher emphasized the efficacy of open market operations. Lloyd Mints's innovation was to note that corrective money stock adjustments could be achieved through government budget deficits and surpluses as well as through open market operations. In his 1946 article "Monetary Policy" and his 1950 book *Monetary Policy for a Competitive Society*, he pointed out that since deficits had to be financed either by new money creation or by expansion of the public debt, one could choose the former route and use those deficits to augment the money stock. Likewise, budget surpluses could be used to contract the money stock rather than to retire the public debt. As to how those deficits and surpluses were to be obtained, he favored variations in tax collections with expenditures held constant. In any case, he argued that the purpose of budget deficits and surpluses is to increase or decrease the money stock M so as to bring prices to target in the equation $dM/dt = \beta(P_T - P)$. Here is his contribution to the Fisherian model.

Historical Development of the Wicksellian Model: Thornton and Joplin

Like the Fisherian model, the alternative Wicksellian interest rate model has its roots in the writings of English classical economists (see Humphrey [6]). Rudiments of the model's price-change equation $dP/dt = \alpha(r-i)$ trace back to Henry Thornton's classic 1802 volume *An Enquiry into the Nature and Effects of the Paper Credit of Great Britain*. There he defined the two interest rates that enter the equation and described the underlying inflationary transmission mechanism through which they operate to raise prices.

He argued that business loan demands depend on a comparison of the loan rate of interest (i) with the expected rate of return (r) on the use of the borrowed funds as proxied by the prevailing rate of profit on mercantile capital. He further argued (1) that a positive profit rate-loan rate differential induces an expansion of loan demands, (2) that banks accommodate these demands by issuing notes and creating checking deposits, and (3) that the resulting monetary expansion, by stimulating aggregate expenditure in an economy already operating close to full employment, puts upward pressure on prices which continue to rise as long as the rate differential persists. Taken together, these arguments imply that rising prices and the money growth that supports them stem from discrepancies between natural (equilibrium) and market (loan) rates of interest as indicated by the expression $dP/dt = \alpha(r-i)$.

Thornton did not state the model's interest-rate adjustment equation $di/dt = \beta(P - P_T)$. But he did note that the Bank of England could have forestalled price rises by setting its loan rate equal to the going rate of profit on capital had statutory usury ceilings not prevented it from doing so. On this point he differed from Wicksell and Cassel both of whom viewed the natural rate as an empirically unobservable variable impossible to target.

Following Thornton, Thomas Joplin in the 1820s and early 1830s added saving and investment schedules to the theoretical inflationary mechanism that leads to the price-change equation $dP/dt = \alpha(r-i)$. He did so in his *Outlines of a System of Political Economy* (1823), *Views on the Currency* (1828), and *An Analysis and History of the Currency Question* (1832). In those works Joplin pointed out that desired investment expenditure constitutes the demand for loanable funds. He noted that saving constitutes part of the supply of such funds. Finally, he stated that an excess of investment over saving caused by a positive natural rate-loan rate differential must be financed by net money creation that puts upward pressure on prices.

Wicksell's Contribution

The pioneering efforts of Thornton and Joplin notwithstanding, economists today chiefly associate the interest rate model with the Swedish economist Knut Wicksell. It was Wicksell who, in the late 1890s and early 1900s, derived the model's reduced-form price-change equation from a full structural model of the inflationary process and who supplied the interest-rate adjustment equation that closed the model. Containing the most complete account of the logic and assumptions underlying the price-change equation, his structural model merits examination in some detail.

Following Wicksell, define the natural rate as the rate that equilibrates saving and investment and that corresponds to the marginal productivity of capital. Likewise define the market rate as the rate banks charge on loans and pay on deposits. Assume that all saving is deposited in banks, that all investment is bank financed, and that banks lend only to finance investment. Let saving and investment be increasing and decreasing functions of the market rate on the grounds that a rise in the rate encourages thrift but discourages capital formation. Assume absolute full employment such that shifts in aggregate demand affect prices and not real output. These definitions and assumptions yield the following equations linking the variables planned real investment I, planned real saving S, market (loan) rate i, natural rate r, loan demand L_D, loan supply L_S, excess money supply X, excess aggregate demand E, money-stock change dM/dt, price-level change dP/dt, and market rate change di/dt.

First, natural rate-market rate differentials produce corresponding gaps between investment and saving:

$$(9) \quad I - S = a(r - i)$$

where the coefficient a relates the rate differential to the $I-S$ gap.

Second, investment-savings gaps are matched by new money created to finance them:

$$(10) \quad I - S = dM/dt.$$

In other words, since banks create money by lending, monetary expansion occurs when they lend more to investors than they receive in deposits from savers. To see this, denote the investment demand for loans as $L_D = I(i)$. Similarly, denote loan supply as the sum of saving plus new money created by banks in accommodating loan demands; in short $L_S = S(i) + dM/dt$. Equating loan demand and supply ($L_D = L_S$) yields equation 10.

Third, since the demand for money to hold at existing prices and real incomes remains unchanged, the new money created in accommodating loan demands constitutes an excess supply of money X:

$$(11) \quad dM/dt = X.$$

Fourth, cash-holders attempt to get rid of this excess money by spending it. As a result, the excess supply of money spills over into the commodity market in the form of an excess demand for goods as aggregate expenditure at full employment outruns real supply:

$$(12) \quad X = E.$$

Fifth, this excess demand bids up prices, which rise in proportion to the excess demand:

$$(13) \quad dP/dt = kE.$$

Substituting equations (9) through (12) into (13) yields the model's reduced-form price-change equation:

$$(14) \quad dP/dt = \alpha(r - i) \qquad \text{where } \alpha = ka$$

which says that price-level changes stem from the discrepancy between the natural and market rates of interest.

As for the interest-rate adjustment equation that closes the model and brings price movements to an end, Wicksell suggested two. The first:

$$(15) \quad di/dt = b(dP/dt)$$

directs the central bank to adjust market rates in the same direction that prices are moving, stopping only when price movements cease. In Wicksell's own words:

> So long as prices remain unaltered the banks' rate of interest is to remain unaltered. If prices rise, the rate of interest is to be raised; and if prices fall, the rate of interest is to be lowered; and the rate of interest is henceforth to be maintained at its new level until a further movement of prices calls for a further change in one direction or the other. [18, p. 189]

The foregoing rule has one shortcoming: it brings prices to a standstill but leaves them higher or lower than before. Because it fails to restore prices to their pre-existing target level Wicksell replaced it with his second rule which he thought would stabilize prices. That rule:

$$(16) \quad di/dt = \beta(P - P_T)$$

directs the bank to adjust market rates to correct price-level deviations from target.

That Wicksell proposed such a rule to roll back prices to their original level after they had risen or fallen is clearly evident in his writings. It appears in his statement that bank rates should be raised or lowered "to depress the commodity price level when it showed a tendency to rise and to raise it when it showed a tendency to fall." [17, p. 223]. Stronger still is his 1919 proposal to reverse inflation by deflating Swedish prices to their 1914 level.

> In my opinion, we should try to return to the prewar price level. It is difficult to present any valid argument for stopping half way. The means to do this is to maintain a high discount rate . . . in order to reduce the stock of notes to the 1914 level. It is a very painful process, but it is probably better to do it now rather than to wait. [19, p. 27, quoted in 7, p. 465]

He repeated his advice again in 1921 when he argued for

> a withdrawal by the *Riksbank* of the total stock of notes in circulation. Half this stock should be destroyed and the rest returned to the holders of notes . . . , our prices would fall to a level slightly below half the present level of prices. Then it should be the duty of the *Riksbank* to hold *this* level constant. [20, p. 86, quoted in 7, p. 465]

In short, he advocated raising the discount rate so as to contract the money stock and thus lower prices to their pre-existing level. Here is the essence of Wicksell's feedback rule $di/dt = \beta(P - P_T)$. Whether that rule does in fact possess the price-stabilizing powers he sought is discussed below. Before doing so, however, it is necessary to identify Gustav Cassel's contribution to the model.

Cassel's Contribution

Wicksell's policy rule can be criticized as being inferior to the alternative rule of maintaining equality between market and natural rates such that price changes never occur. Gustav Cassel's contribution was to rebut this criticism. In his famous 1928 article "The Rate of Interest, the Bank Rate, and the Stabilization of Prices" he argued that any rule requiring knowledge of the unobservable natural rate was completely non-operational and therefore of little use to central bankers. Policymakers could never know what the natural rate is. But they could observe the price signals generated by departures from the natural rate. And these very signals constitute the arguments of the feedback policy rule $di/dt = \beta(P - P_T)$, thereby rendering that rule operational. On this ground Cassel contended that Wicksell's feedback rule dominated the alternative natural rate rule.

Dynamic Stability of Equilibrium

Without exception all the economists discussed above saw their models as offering reliable guides to policy. None questioned the ability of those models to deliver price stability. It never occurred to them that the models might be dynamically unstable such that policy attempts to stabilize prices would destabilize them instead. They simply assumed that the models' feedback policy rules would always be sufficient to restore prices to target.

It is now time to test the validity of that assumption by formal stability analysis. And it is extremely important to do so. For if the models indeed are dynamically unstable such that attempts to stabilize prices destabilize them instead then those models are useless as policy guides and should have been discarded long ago. It turns out that both models are stable provided one adds a price-change variable to the Wicksellian model's policy response function.

Stability of the Fisherian Model

Demonstrating the dynamic stability of the Fisherian model requires expressing its equations in matrix form and then examining the signs—positive, negative, or zero—of the determinant and trace of the coefficient matrix (see Chiang [2, pp. 638-643]). Expressed in matrix form, the model's equations are:

$$(17)\quad \begin{bmatrix} dP/dt \\ dM/dt \end{bmatrix} = \begin{bmatrix} -\alpha ky & \alpha \\ -\beta & 0 \end{bmatrix} \begin{bmatrix} P \\ M \end{bmatrix} + \begin{bmatrix} 0 \\ \beta P_T \end{bmatrix}.$$

Stability is ensured in this second-order case if the determinant $\beta\alpha$ of the coefficient matrix is positive and the trace $-\alpha ky$ is negative. Since both conditions are met, the model is stable. In other words, the roots of the system's characteristic equation are either real and negative, implying monotonic movement to equilibrium, or they are imaginary with negative real parts, implying convergent cycles. In either case the policy authorities, provided they adhere to the rule of adjusting the money stock to counter price-level deviations from target, can always bring prices back to target. Indeed the model's phase diagram displays this result; prices and the money stock invariably return to equilibrium directly or via convergent counterclockwise paths (see Figure 1).

Oscillatory Behavior of the Wicksellian Model

The same techniques of dynamic stability analysis can be applied to the Wicksellian model. One simply expresses the model in matrix form and examines the signs of the determinant and trace of the coefficient matrix. As shown below, the model generates perpetual oscillations of prices and interest rates about equilibrium until a price-change variable is added to the policy response function. Then the model converges to equilibrium.

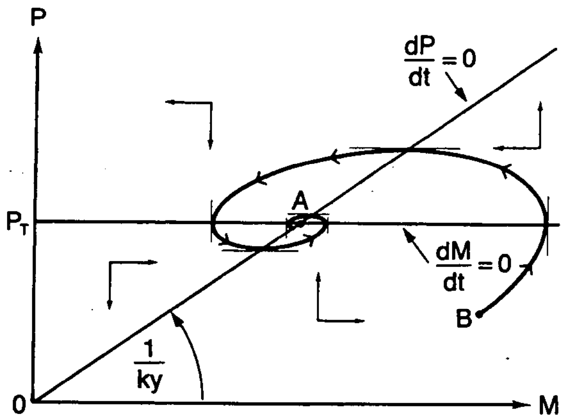
To demonstrate the validity of these assertions write the model $dP/dt = \alpha(r - i)$ and $di/dt = \beta(P - P_T)$ in matrix form:

$$(18)\quad \begin{bmatrix} dP/dt \\ di/dt \end{bmatrix} = \begin{bmatrix} 0 & -\alpha \\ \beta & 0 \end{bmatrix} \begin{bmatrix} P \\ i \end{bmatrix} + \begin{bmatrix} \alpha r \\ -\beta P_T \end{bmatrix}.$$

Examination reveals that the determinant $\alpha\beta$ of the coefficient matrix is positive and the trace is zero. This in turn means that the characteristic roots of the system are imaginary with zero real parts, implying cycles of constant amplitude without convergence or divergence. Thus the best the policymakers can do when adhering to the feedback policy rule of adjusting interest rates to counter price deviations from target is to keep prices cycling forever

Figure 1

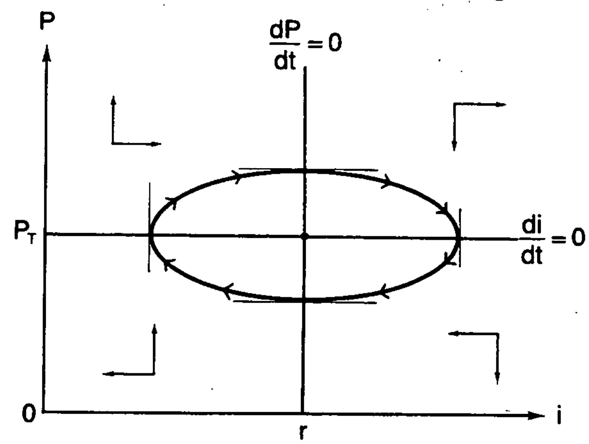
Fisherian Model's Phase Diagram



This diagram depicts the dynamical behavior of the two-equation monetary model $dP/dt = \alpha(M - kPy)$ and $dM/dt = \beta(P_T - P)$. The positively sloped line shows all P-M combinations that yield zero excess money supply such that prices do not change. It is the graph of the expression $P = (1/ky)M$ obtained by setting dP/dt equal to zero in the model's first equation. Points above the line represent situations of excess demand for money putting downward pressure on prices (see vertical arrows). Points below the line represent situations of excess supply of money putting upward pressure on prices (see vertical arrows).The horizontal line graphs the expression $P_T = P$ obtained by setting dM/dt equal to zero in the model's second equation. The line shows that when prices are on target no corrective money stock changes are required. Points above the line represent positive price deviations from target requiring contractions of the money stock (see horizontal arrows). Points below the line represent negative price deviations from target requiring expansions of the money stock (see horizontal arrows).Starting from any disequilibrium point B prices and money will converge to equilibrium A either directly or via the counterclockwise path shown.

Figure 2

Wicksellian Model's Phase Diagram



This diagram depicts the dynamical behavior of the Wicksellian two-equation interest rate model $dP/dt = \alpha(r - i)$ and $di/dt = \beta(P - P_T)$. The vertical line graphs the expression $i = r$ obtained by setting dP/dt equal to zero in the model's first equation. The line shows that when the market rate equals the natural rate no price changes occur. Points to the left of the line represent situations in which the market rate is below the natural rate causing prices to rise (see vertical arrows). Points to the right of the line represent situations in which the market rate is above the natural rate causing prices to fall (see vertical arrows). The horizontal line graphs the expression $P = P_T$ obtained by setting di/dt equal to zero in the model's second equation. The line shows that when prices are on target no corrective interest rate changes are required. Points above the line represent positive price deviations from target requiring corrective rises in the market rate (see horizontal arrows). Points below the line represent negative price deviations from target requiring corrective falls in the market rate (see horizontal arrows). Formal stability analysis reveals that the coefficient matrix of this system has a zero trace and a positive determinant. This means that the characteristic roots are imaginary with real parts zero, implying cycles of constant amplitude without convergence. The system ceaselessly orbits equilibrium without approaching it.

around target. Indeed the model's phase diagram displays this result: the path of prices and interest rates orbits ceaselessly around equilibrium without approaching it (see Figure 2). True, prices conform to target *on average* over the whole cycle. But they also are forever rising and falling. Clearly this is not the sort of absolute price stability Wicksell or Cassel sought. Their model represented by matrix equation 18 cannot deliver such stability.

Response Function Fully Specified

The foregoing result stems from the particular policy response function embedded in the Wicksellian model. That response function derives from Wicksell's advice to the policymakers to adjust interest rates to counter price deviations from target. Consistent with that recommendation response function $di/dt = \beta(P - P_T)$ contains but one argument, namely the gap $P - P_T$ between actual and target prices. As noted above, however, Wicksell also postulated an alternative response function containing price changes dP/dt as the independent variable. Incorporating that variable into equation 4 yields the augmented or fully specified function:

$$\text{(19)} \quad di/dt = \beta(P - P_T) + b(dP/dt)$$

that directs the authorities to adjust the market rate in response to two variables, namely price changes and the gap between actual and target price levels. In other words, the equation's last term b(dP/dt) halts inflation or deflation in its tracks while the first term $\beta(P - P_T)$ seeks to undo the damage already done by bringing prices back to target. This rule seems eminently sensible. Certainly the Federal Reserve, if charged with the duty to stabilize prices, would respond to emerging inflation and deflation as well as to price gaps.

Stability of Equilibrium

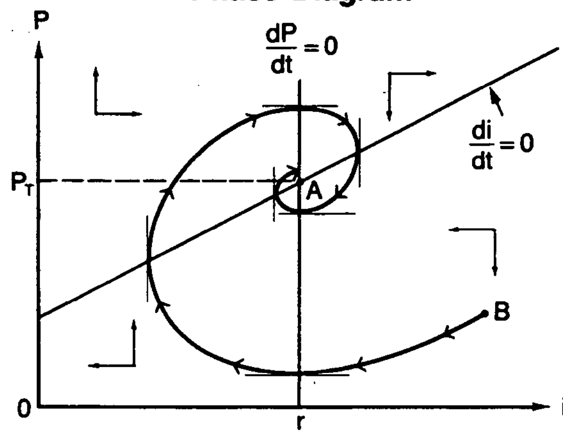
Incorporation of the price-change variable into the policy response function renders the Wicksellian model dynamically stable. To show this, first substitute equation 3 into equation 19 to obtain di/dt $= \beta(P - P_T) + b\alpha(r - i)$. Then express this equation together with equation 3 in matrix form:

$$(20)\quad \begin{bmatrix} dP/dt \\ di/dt \end{bmatrix} = \begin{bmatrix} 0 & -\alpha \\ \beta & -b\alpha \end{bmatrix} \begin{bmatrix} P \\ i \end{bmatrix} + \begin{bmatrix} \alpha r \\ b\alpha r - \beta P_T \end{bmatrix}.$$

Stability requires that the coefficient matrix possess a negative trace and a positive determinant. The model passes both tests. The trace $-b\alpha$ is negative and the determinant $\beta\alpha$ is positive as required. This means one of two things: Either the roots of the system's characteristic equation are real and negative, implying monotonic movement to equilibrium, or they are imaginary with negative real parts, implying convergent cycles. In either case the policy authorities, provided they adhere to the rule of adjusting interest rates to counter price movements and price-level deviations from target, can always bring prices back to target. Indeed, the model's phase diagram displays this result. Instead of orbiting continuously around equilibrium, prices and interest rates invariably return to equilibrium via a convergent clockwise path (see Figure 3). In short, the fully specified Wicksellian model yields dynamic stability after all. It follows that central banks conducting monetary policy through Wicksellian interest-rate adjustment rules have not been seriously misadvised.

Figure 3

Fully Specified Wicksellian Model's Phase Diagram



This diagram depicts the dynamical behavior of the two-equation interest rate model $dP/dt = \alpha(r - i)$ and $di/dt = \beta(P - P_T) + b(dP/dt) = \beta(P - P_T) + b\alpha(r - i)$ obtained by adding a price change variable to the policy response function. The vertical line graphs the expression $r = i$ obtained by setting dP/dt equal to zero in the model's first equation. The line shows that when the market rate equals the natural rate no price changes occur. Points to the left of the line indicate that the market rate is below the natural rate causing prices to rise (see vertical arrows). Points to the right of the line indicate that the market rate is above the natural rate causing prices to fall (see vertical arrows). The upward-sloping line graphs the expression $P = [P_T - (b\alpha/\beta)r] + (b\alpha/\beta)\, i$ obtained by setting di/dt equal to zero in the model's second equation. Points above the line represent situations in which prices are too high requiring corrective rises in the interest rate (see horizontal arrows). Points below the line represent situations in which prices are too low requiring corrective falls in the interest rate (see horizontal arrows). Formal stability analysis reveals that the system is dynamically stable. Starting from any disequilibrium point B prices and interest rates will converge to equilibrium A either directly or via the clockwise path shown.

Conclusion

The main conclusions of this paper can be stated succinctly. Two models—monetary and interest-rate—historically have dominated analytical discussions of the policy problem of price-level stabilization. Of these, the Fisherian monetary model unambigously yields price stability. By contrast, the Wicksellian interest rate model in which policymakers adjust market rates in response to gaps between actual and target prices does not deliver the absolute price stability its authors sought. Instead it yields perpetual oscillations of prices about their target level. Such an outcome can be avoided by adding a price-change variable to the model's policy response function. Doing so renders the model dynamically stable such that the policymakers can always restore prices to target. Policymakers can rest assured that neither the Fisherian model nor the augmented or fully specified version of the Wicksellian model will lead them astray.

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