An Error-Correction Model of U.S. M2 Demand

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Much applied research in monetary economics has been devoted to the specification of the money demand function. Money demand specification has important policy implications. A poorly specified money demand function could yield, for example, spurious inferences on the underlying stability of money demand—a consideration of central importance in the formulation of monetary policy.

This paper is concerned with one aspect of money demand specification, namely, the choice of the form in which variables enter the money demand function. It is common to specify the money demand function either in log-level form or in log-difference form. The log-level form, popularized by Goldfeld's (1974) work, has often been criticized on the ground that the levels of many economic variables included in money demand functions are nonstationary. Therefore, the regression equations that relate such variables could be subject to "the spurious regression phenomenon" first described in Granger and Newbold (1974). This phenomenon, later formalized in Phillips (1986), refers to the possibility that ordinary least-squares parameter estimates in such regressions do not converge to constants and that the usual t- and F-ratio test statistics do not have even the limiting distributions. Their use in that case generates spurious inferences. In view of these considerations, many analysts now routinely specify the money demand functions in first-difference form.

Quite recently, the appropriateness of even the first-difference specification has been questioned. In particular, if the levels of the nonstationary variables included in money demand functions are cointegrated as discussed in Engle and Granger (1987),[1] then such regressions should not be estimated in first-difference form. This is because level regressions which relate the cointegrated variables can be consistently estimated by ordinary least-squares without being subject to the spurious regression phenomenon described above.[2] One implication of this work is that money demand functions estimated in first-difference form may be misspecified because such regressions ignore relationships that exist among the levels of the variables.

Since there are potential problems with money demand functions specified either in level or in first-difference form, some analysts have recently begun to integrate these two specifications using the theories of error-correction and cointegration. In this approach, a long-run equilibrium money demand model (cointegrating regression) is first fit to the levels of the variables, and the calculated residuals from that model are used in an error-correction model which specifies the system's short-run dynamics.[3] Such an approach permits both the levels and first-differences of the nonstationary variables to enter the money demand function. This approach also makes it easier to distinguish between the short- and long-run money demand functions. Thus, some variables that are included in the short-run part of the model might not be included in the long-run part and vice versa, thereby permitting considerable flexibility in the specification of the money demand function.

This paper illustrates the use of the above approach by presenting and estimating an error-correction model of U.S. demand for money (M2) in the postwar period. The money demand function presented here exhibits parameter stability. Money growth forecasts generated by this function are

[1] Let X_{1t}, X_{2t}, and X_{3t} be three time series. Assume that the levels of these time series are nonstationary but first differences are not. Then these series are said to be cointegrated if there exists a vector of constants (α_1, α_2, α_3) such that $Z_t = \alpha_1 X_{1t} + \alpha_2 X_{2t} + \alpha_3 X_{3t}$ is stationary. The intuition behind this definition is that even if each time series is nonstationary, there might exist linear combinations of such time series that are stationary. In that case, multiple time series are said to be cointegrated and share some common stochastic trends. We can interpret the presence of cointegration to imply that long-run movements in these multiple time series are related to each other.

[2] The usual t- and F-ratio statistics can be used provided some other conditions are satisfied and other adjustments are made. See Phillips (1986) and West (1988).

[3] This approach, popularized by Hendry and Richard (1982) and Hendry, Pagan and Sargan (1983), has been applied to study U.K. money demand behavior by Hendry and Ericsson (1990) and U.S. money demand behavior by Small and Porter (1989) and Baum and Furno (1990).

consistent with the actual behavior of M2 growth during the last two decades or so. A key feature of the results presented here is that consumer spending is found to be a better short-run scale variable than real GNP, even though it is the latter that enters the long-run part of the model.[4]

The plan of this paper is as follows. Section 1 presents the error-correction model and discusses the issues that arise in the estimation of such models. Section 2 presents the empirical results. The summary observations are stated in Section 3.

I. An Error-Correction Money Demand Model: Specification and Estimation

Specification of an M2 Demand Model

The error-correction money demand model has two parts. The first is a long-run equilibrium money demand function

$$rM2_t = a_0 + a_1\, rY_t - a_2\, (R\text{-}RM2)_t + U_t \quad (1)$$

where all variables are expressed in their natural logarithms and where rM2 is real M2 balances; rY, real GNP; R, a short-term nominal rate of interest; RM2, the own rate of return on M2; and U, the long-run random disturbance term. Equation 1 says that the public's demand for real M2 balances depends upon a scale variable measured by real GNP and an opportunity cost variable measured as the differential between the nominal rate of interest and the own rate of return on M2. The parameters a_1 and a_2 measure respectively the long-run income and opportunity cost elasticities. A key aspect of the specification used here is that the own rate of return on M2 is relevant in determining M2 demand (Small and Porter, 1989, and Hetzel and Mehra, 1989). The conventional specification usually omits this variable (see, for example, Baum and Furno, 1990).

The second part of the model is a dynamic error-correction equation of the form

$$\begin{aligned}\Delta rM2_t = {} & b_0 + \sum_{s=1}^{n1} b_{1s}\, \Delta rM2_{t-s} \\ & + \sum_{s=0}^{n2} b_{2s}\, \Delta rY_{t-s} \\ & - \sum_{s=0}^{n3} b_{3s}\, \Delta(R - RM2)_{t-s} \\ & + \lambda\, U_{t-1} + \epsilon_t \end{aligned} \quad (2)$$

where all variables are as defined above and where ϵ_t is the short-run random disturbance term; Δ, the first difference operator; $n_i (i = 1,2,3)$, the number of lags; and U_{t-1}, the lagged value of the long-run random disturbance term. Equation 2 gives the short-run determinants of M2 demand, which include, among others, current and past changes in the scale and opportunity cost variables and the lagged value of the residual from the long-run money demand function. The parameter λ that appears on U_{t-1} in (2) is the error-correction coefficient. At a more intuitive level, the presence of U_{t-1} in (2) reflects the presumption that actual M2 balances do not always equal what the public wishes to hold on the basis of the long-run factors specified in (1). Therefore, in the short run, the public adjusts its money balances to correct any disequilibrium in its long-run money holdings. The parameter λ in (2) measures the role such disequilibria plays in explaining the short-run movements in money balances.[5]

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[4] The results presented here are in line with those given in Small and Porter (1989) but differ from those given in Baum and Furno (1990). The error-correction model of M2 demand reported in Baum and Furno does not exhibit parameter stability. One possible reason for this is the use of inappropriate scale and/or opportunity cost variables. The money demand function reported in Baum and Furno measures the opportunity cost variable by a short-term market rate of interest, thereby implicitly assuming that the own rate of return on M2 is zero. Furthermore, real GNP is used in the long-run as well as in the short-run part of the model.

[5] It should, however, be pointed out that the size of the coefficient on the error correction variable in (2) is influenced in part by the nature of serial correlation in the random disturbance term of the long-run money demand model and is not necessarily indicative of the speed of adjustment of money demand to its long-run level. To explain it further, for illustrative purposes assume the restricted simple money demand model of the form

$$m^*_t = a_0 + a_1\, y_t + U_t \quad (a)$$

where changes in money balances follow the partial-adjustment model

$$m_t - m_{t-1} = \delta\, (m^*_t - m_{t-1}),\ 0 < \delta \leq 1 \quad (b)$$

The parameter δ measures the speed of adjustment. m^* is the long-run desired level of real money balances, and other variables are as defined before. Assume now that the random disturbance term U_t in (a) is stationary and follows a simple AR(1) process of the form

$$U_t = \rho\, U_{t-1} + \epsilon_t,\ 0 \leq \rho < 1 \quad (c)$$

The parameter ρ is determined by the nature of shocks to money demand.

Note that the empirical work in the text relies on a long-run demand specification like (a), but allows for more general dynamics than embedded in (b). Equations (a), (b) and (c) imply the following reduced form equation for changes in money balances

$$m_t - m_{t-1} = \delta a_1\, \Delta y_t - \delta(1-\rho)\, U_{t-1} + \delta\, \epsilon_t. \quad (d)$$

Equation (d) resembles the error-correction model of the form (2) given in the text. As can be seen, the size of the coefficient on the lagged level of U_t depends upon two parameters δ and ρ. If ρ is close to unity, then the error-correction parameter will be small even if δ is large.

An important assumption implicit in the above discussion is that the random disturbance term U_t is stationary. Intuitively, this assumption means that actual M2 balances do not permanently drift away from what is determined by long-run factors specified in (1). If this assumption is incorrect so that U_t is in fact nonstationary, then the regression equation (1) if estimated is subject to the spurious regression phenomenon. Furthermore, the coefficient λ in (2) is likely to be zero. To see this, first-difference the equation (1) as in (3)

$$\Delta rM2_t = a_1 \Delta rY_t - a_2 \Delta(R - RM2)_t + U_t - U_{t-1} \quad (3)$$

Assume now that U_t follows a first-order autoregressive process of the form

$$U_t = \rho U_{t-1} + \epsilon_t$$

where ϵ_t is a pure white noise process. Then we can rewrite (3) as in (4)

$$\Delta rM2_t = a_1 \Delta rY_t - a_2 \Delta(R - RM2)_t + (\rho - 1) U_{t-1} + \epsilon_t \quad (4)$$

Equation (4) is similar in spirit to equation (2). If ρ is less than unity so that U_t is stationary, then $\rho - 1$ [which equals λ in (2)] is different from zero. If $\rho = 1$ so that U_t is nonstationary, then $\rho - 1$ [and λ in (2)] is zero. Hence, the dynamic error-correction specification (2) exists if U_t is a stationary variable.

It can now be easily seen that if U_t is nonstationary, then the money demand regression estimated in first-difference form is appropriate [as λ in (2) is in fact zero]. On the other hand, if U_t is stationary, then the first-difference regression is misspecified because it omits the relevant variable U_{t-1} [as λ in (2) is in fact nonzero].

Estimation of the Error-Correction Model

If the random disturbance term Ut is stationary, then the money demand regression (2) can be estimated in two alternative ways. The first is a two-step procedure. In the first step, the long-run equilibrium M2 demand model (1) is estimated using a consistent estimation procedure, and the residuals are calculated. In the second step, the short-run money demand regression (2) is estimated with U_{t-1} replaced by residuals estimated in step one (see, for example, Hendry and Ericsson, 1991, and Baum and Furno, 1990). The money demand regression estimated in the first step of this procedure generates estimates[6] of the long-term income and opportunity cost elasticities (a_1 and a_2). The short-run money demand parameter estimates are generated in the second step.

The alternative procedure is to replace U_{t-1} in (2) by the lagged levels of the variables and estimate the short-run and long-run parameters jointly. To explain it further, substitute (1) into (2) to obtain a combined equation

$$\begin{aligned}\Delta rM2_t = d_0 &+ \sum_{s=1}^{n1} b_{1s} \Delta rM2_{t-s} \\ &+ \sum_{s=0}^{n2} b_{2s} \Delta rY_{t-s} \\ &- \sum_{s=0}^{n3} b_{3s} \Delta(R - RM2)_{t-s} \\ &+ d_1 rM2_{t-1} + d_2 rY_{t-1} \\ &+ d_3 (R - RM2)_{t-1} + \epsilon_t \quad (5)\end{aligned}$$

where
$$d_0 = (b_0 - a_0 \lambda)$$
$$d_1 = \lambda$$
$$d_2 = -\lambda a_1$$
$$d_3 = \lambda a_2$$

Equation 5 can be estimated using a consistent estimation procedure and all parameters of (1) and (2) can be recovered from those of (5). For example, the error-correction coefficient λ is d_1; the long-term income elasticity (a_1) is d_2 divided by d_1; and the long-term opportunity cost elasticity (a_2) is d_3 divided by d_1 (see, for example, Small and Porter, 1989).

If one wants to test hypotheses about the long-run parameters of the money demand function (1), it is easier to do so under the second framework than

[6] It should be pointed out that if all of the variables included in (1) are nonstationary, then ordinary least squares estimates of (1) are consistent. However, the usual t- and F-ratio statistics have nonstandard limiting distributions because U_t in (1) is generally serially correlated and/or heteroscedastic. This means one can not carry out tests of hypotheses about the long-run parameters in the standard fashion. Furthermore, if even a single variable in (1) is stationary, then ordinary least squares estimates are inconsistent. West (1988) in that case suggests using an instrumental variables procedure.

under the two-step procedure.[7] The reason is that the residuals in the equilibrium model estimated in step one of the first procedure are likely to be serially correlated and possibly heteroscedastic. Hence, the usual t- and F-ratio test statistics are invalid unless further adjustments are made. In contrast, the residuals in the money demand regression (5) are likely to be well behaved, validating the use of the standard test statistics in conducting inference. In view of these considerations, the error-correction money demand model is estimated using the second procedure, i.e., the money demand function (5).

As noted above, the long-term income elasticity can be recovered from the long-run part of the model (5), i.e., a_1 is d_2 divided by d_1. It may however be noted that the short-run part of the model (5) may yield another estimate of the long-term scale elasticity, i.e., $\sum_{s=0}^{n2} b_{2s}/(1 - \sum_{s=1}^{n1} b_{1s})$. If the same scale variable appears in the long- and short-run parts of the model, then a "convergence condition" might be imposed to ensure that one gets the same point-estimate of the long-term scale elasticity. To explain further, assume that real income appears in the long- and short-run parts of the model and that the long-term income elasticity is unity, i.e., $a_1 = 1$ in (1). This restriction implies that coefficients that appear on rY_{t-1} and $rM2_{t-1}$ in (5) sum to zero. This restriction pertains to the long-run part of the model and is expressed as in (6.1)

$$d_1 + d_2 = 0 \tag{6.1}$$

Furthermore, if the long-term income elasticity computed from the short-run part of the model is unity, then it also implies the following

$$\sum_{s=0}^{n2} b_{2s}/(1 - \sum_{s=1}^{n1} b_{1s}) = 1. \tag{6.2}$$

Equivalently, (6.2) can be expressed as

$$\sum_{s=0}^{n2} b_{2s} + \sum_{s=1}^{n1} b_{1s} = 1.$$

[7] It should be pointed out that these remarks apply to the case in which the equilibrium model (1) is estimated by ordinary least squares, as suggested by Engle and Granger (1987). However, if the equilibrium money demand model is estimated using the procedure given in Johansen and Juselius (1989), then one can conduct various tests of hypotheses of the long-run parameters. The approach advanced in Johansen and Juselius is, however, quite complicated.

In general, if different scale variables appear in the short- and long-run parts of the model, then these restrictions may or may not be imposed on the model.

Tests for Cointegration

An assumption that is necessary to yield reliable estimates of the money demand parameters is that U_t in (1) should be stationary. Since the levels of the variables included (1) are generally nonstationary, the stationarity of U_t requires that these nonstationary variables be cointegrated as discussed in Engle and Granger (1987). Hence, one must first test for the existence of a long-run equilibrium relationship among the levels of the nonstationary variables in (1).

Several tests for cointegration have been proposed in the literature (see, for example, Engle and Granger, 1987, and Johansen and Juselius, 1989). The test for cointegration used here is the one proposed in Engle and Granger (1987) and consists of two steps. The first tests whether each variable in (1) is nonstationary. One does this by performing a unit root test on the variables. The second step tests for the presence of a unit root in the residuals of the levels regressions estimated using the nonstationary variables. If the residuals do not have a unit root, then the nonstationary variables are cointegrated. For the case in hand, if U_t in (1) does not have a unit root, then the nonstationary variables in (1) are said to be cointegrated.

Data and the Definition of Scale Variables

The money demand regression (5) is estimated using the quarterly data that spans the period 1953Q1 to 1990Q4. rM2 is measured as nominal M2 deflated by the implicit GNP price deflator; rY by real GNP; R by the four- to six-month commercial paper rate and; RM2 by the weighted average of the explicit rates paid on the components of M2.

The theoretical analysis presented in McCallum and Goodfriend (1987) implies that the scale variable that appears in a typical household's money demand relationship is real consumption expenditure. Mankiw and Summers (1986) have presented empirical evidence that in aggregate money demand regressions consumer expenditure is a better scale variable than GNP. Their reasoning is based on the observation that some components of GNP, such as business fixed investment and changes in inventories, do not generate as much increase in money balances as does consumer expenditure. The money demand regressions estimated by Mankiw and Summers are in level form and use distributed lags on the scale and interest rate variables. Their empirical work implies that consumer expenditure is a better scale variable than GNP in the short run as well as in the long run. In contrast, Small and Porter (1989) used consumer spending as the short-run scale variable, and GNP as the long-run scale variable. Here I formally test which scale variable is appropriate in the short and long run.[8]

II. EMPIRICAL RESULTS

Unit Root Test Results

The money demand regression (5) includes the levels and first-differences of money, income and opportunity cost variables $rM2_t$, $\Delta rM2_t$, rY_t, ΔrY_t, $(R-RM2)_t$, and $\Delta(R-RM2)_t$. The alternative scale variable considered is real consumer expenditure: rC_t and ΔrC_t. The Augmented Dickey Fuller test[9] is used to test the presence of unit roots in these variables. The test results are reported in Table 1.

These results suggest the presence of a single unit root in $rM2_t$, rY_t and rC_t, implying that the levels of these variables are nonstationary but the first-differences are not. The financial market opportunity cost variable $(R-RM2)_t$ does not have a unit root and is thus stationary.[10]

Cointegration Test Results

The unit root test results presented above imply that except for $rM2_t$ and rY_t all other variables included in the money demand regression (5) are stationary. If $rM2_t$ and rY_t are cointegrated, then (5) can be estimated by ordinary least squares and the resulting parameter estimates are not subject to the spurious regression phenomenon.

The results of testing for cointegration[11] between $rM2_t$ and rY_t are presented in Table 2. As can be seen, the residuals from a regression of $rM2_t$ on rY_t

[8] All the data (with the exception of RM2 and M2) is taken from the Citibank database. M2 for the pre-1959 period and RM2 are constructed as described in Hetzel (1989).

[9] The unit root test procedure used here is described in Mehra (1990).

[10] Schwert (1987) has shown that usual unit root tests may be invalid if time series are generated by moving as well as autoregressive components. In order to check for this potential bias, unit root tests were repeated using longer lags on first-differences of time series. In particular, the parameter n in Table 1 was set at 8 and 12. Those unit root test results (not reported) yielded similar inferences.

[11] For a simple description of this cointegration test see Mehra (1989).

Table 1

Unit Root Test Results, 1953Q1–1990Q4

Z_t	$\rho(t{:}\rho-1=0)$	$\beta(t{:}\beta=0)$	$\Phi_3\ (\rho=1,\ \beta=0)$	n	$\chi^2(1)$	$\chi^2(2)$
	First Unit Root					
$rM2_t$	.97 (−2.2)	.20 (2.0)	2.67	1	.76	1.59
rY_t	.95 (−2.5)	.39 (2.5)	2.50	2	1.50	1.72
rC_t	.94 (−2.5)	.46 (2.5)	3.13	2	.96	1.03
$(R-RM2)_t$	.80 (−4.2)*	.57 (1.2)	9.07*	4	.37	.42
	Second Unit Root					
$\Delta rM2_t$	.59 (−5.3)*			1	.28	.39
ΔrY_t	.31 (−6.5)*			2	.62	1.28
ΔrC_t	.29 (−5.3)*			4	.45	.55
$\Delta(R-RM2)_t$	.09 (−7.0)*			2	.10	.68

Notes: Regressions are of the form $Z_t = \alpha + \sum_{s=1}^{n} d_s \Delta Z_{t-s} + \rho Z_{t-1} + \beta T + \epsilon_t$. All variables are in their natural logs; rM2, real balances; rY, real GNP; rC, real consumer spending; R-RM2, the differential between the four- to six-month commercial paper rate (R) and the own rate on M2 (RM2); T, a time trend; and Δ, the first-difference operator. The coefficient reported on trend is to be multiplied by 1000. The parameter n was chosen by the "final prediction error criterion" due to Akaike (1969). The coefficients ρ and β (t statistics in parentheses) are reported. Φ_3 tests the hypothesis $(\rho, \beta) = (1,0)$. $\chi^2(1)$ and $\chi^2(2)$ are Chi square statistics (Godfrey, 1978) that test for the presence of first- and second-order serial correlation in the residuals of the regression.

An "*" indicates significance at the 5 percent level. The 5 percent critical value for t: $\rho-1=0$ is 3.45 (Fuller, 1976, Table 8.5.2) and that for $\Phi_3{:}(\rho=1,\ \beta=0)$ is 6.49 (Dickey and Fuller, 1981, Table VI).

Table 2

Cointegrating Regressions, 1953Q1–1990Q4

$X1_t$	$X2_t$	b	d (t:d=0)	n	$\chi^2(1)$	$\chi^2(2)$
rM2	rY	1.01	−.10 (−3.5)*	1	1.1	1.1
rY	rM2	.98	−.10 (−3.5)*	1	1.1	1.1
rM2	rC	.91	−.05 (−2.3)	1	1.6	2.1
rC	rM2	1.08	−.05 (−2.3)	1	1.6	2.1

Notes: Each row reports coefficients from two regressions. The first regression is the cointegrating regression of the form $X1_t = a + b\,X2_t + U_t$, where U_t is the residual. The second regression tests for a unit root in the residual of the relevant cointegrating regression and is of the form

$$\Delta U_t = d\,U_{t-1} + \sum_{s=1}^{n} f_s\,\Delta U_{t-s}.$$

The coefficient reported from the first regression is b and the coefficient d is from the other regression. n is the number of lags chosen by Akaike's final prediction error criterion. $\chi^2(1)$ and $\chi^2(2)$ are Godfrey (1978) statistics that test for the presence of first- and second-order serial correlation in the residuals of the second regression.

An "*" indicates significance at the 5 percent level. The 5 percent critical value is 3.21 (Engle and Yoo, 1987, Table 3).

(or of rY_t on $rM2_t$) do not possess a unit root, implying that these two variables are cointegrated. Table 2 also presents test results for cointegration between $rM2_t$ and rC_t; those results suggest that $rM2_t$ and rC_t are not cointegrated.

Estimated M2 Demand Regressions

The cointegration test results above imply that the appropriate scale variable that enters the long-run part of the money demand model (5) is real GNP, not real consumer spending.[12] It is, however, still plausible that real consumer spending is a better short-run scale variable than real GNP. In order to examine this issue, (5) is also estimated using ΔrC_t in the short-run part of the model.

The results of estimating (5) are reported in Table 3. The regressions A and B in Table 3 use real GNP and real consumer spending respectively as the short-run scale variable. The long-run part of the model still uses real GNP as the scale variable. The regressions are estimated without imposing the restrictions (6.1) and (6.2). The regressions also included zero-one dummies to control for the transitory effects of credit controls and the introduction of MMDAs and Super-NOWs. As can be seen, both regressions appear to provide reasonable point-estimates of the long-run and short-run parameters. The long-run real GNP elasticity computed from the long-run parts of the models is unity, and the point-estimate of the long-run financial market opportunity cost elasticity ranges between −.10 to −.12. The short-run coefficients that appear on the scale and opportunity cost variables are generally of the correct signs and are statistically significant. The residuals from these regressions do not indicate the presence of any serial correlation (see Chi square and Q statistics reported in Table 3).

The cointegrating regressions between $rM2_t$ and rY_t reported in Table 2 suggest that the estimated long-term real GNP elasticity is not economically different from unity ($\hat{a}_1 = 1.0$ or .98; Table 2). If this hypothesis is true, then it implies that the restriction (6.1) is also true. F1 in Table 3 is the F statistic that tests whether (6.1) is true. F1 is .026 for regression A and .44 for regression B. Both values are small and thus imply that the long-run real GNP elasticity is not different from unity.

Evaluating the Demand Regressions

The money demand regressions reported in Table 3 are further evaluated by examining their

[12] The long-run money demand functions are assumed to be of the form

$$rM2_t = a_0 + a_1\,rY_t - a_2\,(R-RM2)_t. \qquad (1)$$

A key feature of this specification is that the opportunity cost of holding M2 depends upon the differential between a market rate of interest (R_t) and the own rate of return on M2 ($RM2_t$). This specification thus implies that coefficients that appear on R_t and $RM2_t$ in (1) are of opposite signs but equal absolute sizes. The unit root test results presented in the text implies that $(R-RM2)_t$ is stationary, whereas $rM2_t$ and rY_t are not. The cointegration test results presented in the text implies that $rM2_t$ and rY_t are cointegrated. These results together then imply the presence of a single cointegrating vector among the variables postulated in (1). See Goodfriend (1990).

Table 3

The Error-Correction M2 Demand Regressions, 1953Q1–1990Q4

A. *Real GNP in the Short- and Long-Run Parts of the Model*

$$\Delta rM2_t = \underset{(1.5)}{-.19} + \underset{(4.3)}{.33}\,\Delta rM2_{t-1} + \underset{(1.5)}{.11}\,\Delta rM2_{t-2} + \underset{(1.7)}{.09}\,\Delta rY_t + \underset{(2.0)}{.12}\,\Delta rY_{t-1} - \underset{(5.8)}{.01}\,\Delta(R-RM2)_t - \underset{(4.5)}{.01}\,\Delta(R-RM2)_{t-1}$$

$$-\underset{(1.7)}{.04}\,rM2_{t-1} + \underset{(1.6)}{.04}\,rY_{t-1} - \underset{(2.8)}{.005}\,(R-RM2)_{t-1} - \underset{(2.3)}{.014}\,CC1 + \underset{(1.7)}{.010}\,CC2 + \underset{(4.7)}{.026}\,D83Q1$$

SER = .0055 $\chi^2(1) = .24$ $\chi^2(2) = 2.3$ Q(36) = 23.3

$N_{rY} = 1.0$ $N_{R-RM2} = -.12$ F1(1,139) = .026

B. *Real Consumer Spending in the Short-Run Part and Real GNP in the Long-Run Part*

$$\Delta rM2_t = \underset{(1.9)}{-.24} + \underset{(4.4)}{.31}\,\Delta rM2_{t-1} + \underset{(1.6)}{.12}\,\Delta rM2_{t-2} + \underset{(2.3)}{.17}\,\Delta rC_t + \underset{(2.0)}{.15}\,\Delta rC_{t-1} - \underset{(5.5)}{.01}\,\Delta(R-RM2)_t - \underset{(4.3)}{.001}\,\Delta(R-RM2)_{t-1}$$

$$-\underset{(2.0)}{.05}\,rM2_{t-1} + \underset{(2.0)}{.05}\,rY_{t-1} - \underset{(3.1)}{.005}\,(R-RM2)_{t-1} - \underset{(1.7)}{.009}\,CC1 + \underset{(1.5)}{.009}\,CC2 + \underset{(4.5)}{.025}\,D83Q1$$

SER = .0055 $\chi^2(1) = .001$ $\chi^2(2) = 1.4$ Q(36) = 20.7

$N_{ry} = 1.0$ $N_{R-RM2} = -.10$ F1(1,139) = .44

Notes: The regressions are estimated by ordinary least squares. All variables are defined as in Table 1. CC1, CC2, and D83Q1 are, respectively, 1 in 1980Q2, 1980Q3 and 1983Q1 and zero otherwise. SER is the standard error of the regression; $\chi^2(1)$ and $\chi^2(2)$ are Godfrey statistics for the presence of first- and second-order correlation in the residuals, respectively; Q the Ljung-Box Q statistic; N_{ry} the long-term income elasticity; and N_{R-RM2} the long-term financial market opportunity cost elasticity. The long-term income elasticity is given by the estimated coefficient on rY_{t-1} divided by the estimated coefficient on $rM2_{t-1}$ and the long-term opportunity cost elasticity is given by the estimated coefficient on $(R-RM2)_{t-1}$ divided by the estimated coefficient on $rM2_{t-1}$. F1 is the F statistic that tests whether coefficients on $rM2_{t-1}$ and rY_{t-1} sum to zero. F1 is distributed with F(1,139) degrees of freedom.

structural stability and out-of-sample forecast performance.

Table 4 presents results of the Chow test of structural stability over the period 1953Q1 to 1990Q4. The Chow test is implemented using the dummy variable approach and potential breakpoints covering the period 1970Q4 to 1980Q4 are considered. (The start date is near the midpoint of the whole sample period and the end date near the introduction of NOWs in 1981.) The slope dummies are considered for the long-run as well as for the short-run coefficients.[13] F-S in Table 4 is the F statistic that tests whether slope dummies for the short-run coefficients are zero. F-L tests such slope dummies for the long-run coefficients. F-SL tests all of the slope dummies including the one on the constant term. As can be seen in Table 4, these F statistics generally are not statistically significant and thus imply that the regressions reported in Table 3 do not depict the parameter instability.

The out-of-sample forecast performance is evaluated by generating the rolling-horizon forecasts of the rate of growth of M2 as in Hallman, Porter and Small (1989).[14] The relative forecast performance of the two competing money demand models is compared over the period 1971 to 1990.[15]

Table 5 reports summary statistics for the errors that occur in predicting M2 growth over one-year-,

[13] The results reported in Table 3 suggest that the restriction $a_1 = 1$ is not inconsistent with the data. This constraint was imposed on the long-run part of the model while implementing the test of stability.

[14] The forecasts and errors were generated as follows. Each money demand model was first estimated over an initial estimation period 1953Q1 to 1970Q4 and then simulated out-of-sample over one to three years in the future. For each of the competing models and each of the forecast horizons, the difference between the actual and predicted growth was computed, thus generating one observation on the forecast error. The end of the initial estimation period was then advanced four quarters and the money demand equations were reestimated, forecasts generated, and errors calculated as above. This procedure was repeated until it used the available data through the end of 1990.

[15] The money demand models that underlie this simulation exercise are from Table 3. The predicted values are, however, generated under the constraint that the long-term scale variable elasticity is unity whether computed from the long-run part or from the short-run part of the model. The out-of-sample prediction errors from the error-correction money demand models estimated with this constraint are generally smaller than those from models estimated without the constraint.

Table 4

Stability Tests, 1953Q1–1990Q4

Break Point	Equation A			Equation B		
	F-S	F-L	F-SL	F-S	F-L	F-SL
1970Q4	.6	1.1	1.0	.3	.8	.5
1971Q4	.6	.5	.7	.8	.6	.7
1972Q4	1.9	.1	1.6	1.7	.3	1.2
1973Q4	1.4	.7	1.2	1.4	2.7	1.4
1974Q4	1.9	1.1	1.6	1.6	1.9	1.4
1975Q4	.9	.4	.7	.8	.0	.7
1976Q4	1.2	.4	1.0	1.0	1.2	.9
1977Q4	2.1*	.6	1.6	1.7	.2	1.1
1978Q4	1.8	1.2	1.3	1.6	.9	1.2
1979Q4	1.7	.6	1.2	1.4	.3	1.0
1980Q4	1.3	1.6	1.2	1.3	1.1	1.1

Notes: The reported values are the F statistics that test whether slope dummies when added to equations A and B (reported in Table 3) are jointly significant. The breakpoint refers to the point at which the sample is split in order to define the dummies. The dummies take values 1 for observations greater than the breakpoint and zero otherwise. F-S tests whether slope dummies for the short-run coefficients are zero and are distributed F(6,131) degrees of freedom. F-L tests whether slope dummies for the long-run coefficients are zero and are distributed F(2,131) degrees of freedom. F-SL tests whether all of slope dummies including the one on the constant term are zero and are distributed F(9,131) degrees of freedom.

An "*" indicates significance at the 5 percent level.

two-year-, and three-year-ahead periods. Statistics for regression A are shown within brackets. The period-by-period errors are reported only for the M2 demand regression with real consumer spending as the short-run scale variable. These results suggest two observations. The first is that the regression with real consumer spending provides more accurate forecasts of M2 than does the regression with real GNP. For all forecast horizons the root mean squared errors from regression B are smaller than those from regression A (see Table 5). The second is that the error-correction model with real consumer spending as a short-run scale variable does reasonably well in predicting the rate of growth of M2. The bias is small and the root mean squared error (RMSE) is 1.0 percentage points for the one-year horizon. Moreover, the prediction error declines as the forecast horizon lengthens.

The out-of-sample M2 forecasts are further evaluated in Table 6, which presents regressions of the form

$$A_{t+s} = a + b\,P_{t+s},\; s = 1, 2, 3, \qquad (7)$$

where A and P are the actual and predicted values of M2 growth. If these forecasts are unbiased, then $a=0$ and $b=1$. F statistics reported in Table 6 test the hypothesis (a,b) = (0,1). As can be seen, these F values are consistent with the hypothesis that the forecasts of M2 growth are unbiased.

III.
Summary Remarks

The money demand equations have typically been estimated either in log-level form or in log-difference form. The recent advances in time series analysis have highlighted potential problems with each of these specifications. As a result, several analysts have begun to integrate these two specifications using the theories of error-correction and cointegration. In this approach, a long-run money demand model is first fit to the levels of the variables, and the calculated residuals from that model are used in an error-correction model which specifies the system's short-run dynamics. Such an approach thus allows both the levels and first-differences of the relevant variables to enter the money demand regression.

Using the above approach, this paper presents an error-correction model of M2 demand in the postwar period. It is shown here that real GNP, not real consumer spending, should enter the long-run part of the model. The point-estimate of the long-run real GNP elasticity is not different from unity. Real consumer spending however appears more appropriate in the short-run part of the model. The error-correction model with real consumer spending as a short-run scale variable provides more accurate out-of-sample forecasts of M2 growth than does the model with real GNP. However, both of these models are stable by the conventional Chow test over the sample period 1953Q1 to 1990Q4.

The out-of-sample forecasts presented here suggest that M2 growth in the 1980s is well predicted by the error-correction model that uses real consumer spending as a short-run scale variable. The rate of growth in real consumer spending, which averaged 3.97 percent in the 1983 to 1988 period, decelerated to 1.2 percent in 1989 and .2 percent in 1990. The rate of growth in M2 has also decelerated over the past two years. The money demand model presented here implies that part of the recently observed deceleration in M2 growth reflects deceleration in real consumer spending and is not necessarily indicative of any instability in M2 demand behavior.

Table 5

Rolling-Horizon M2 Growth Forecasts, 1971–1990

Year	1 Year Ahead Actual	1 Year Ahead Predicted	1 Year Ahead Error	2 Years Ahead Actual	2 Years Ahead Predicted	2 Years Ahead Error	3 Years Ahead Actual	3 Years Ahead Predicted	3 Years Ahead Error
1971	12.6	12.4	.2	–	–	–	–	–	–
1972	12.0	10.9	1.1	12.3	11.4	.9	–	–	–
1973	6.9	8.9	–1.9	9.5	9.3	.2	10.5	10.1	.5
1974	5.7	5.9	–.2	6.3	7.7	–1.3	8.2	8.4	–.2
1975	11.4	10.1	1.3	8.6	8.4	.2	8.0	8.5	–.5
1976	12.5	12.4	.1	11.9	11.4	.5	9.9	9.7	.2
1977	10.6	11.1	–.5	11.6	11.8	–.2	11.5	11.5	–.0
1978	7.7	8.6	–.9	9.1	9.6	–.4	10.3	10.5	–.3
1979	7.8	8.7	–.9	7.7	8.5	–.8	8.7	9.1	–.4
1980	8.6	8.7	–.1	8.2	8.9	–.7	8.0	8.6	–.6
1981	8.9	8.4	.5	8.7	8.7	.0	8.4	8.8	–.4
1982	8.7	7.9	.8	8.8	7.9	.8	8.7	8.4	.4
1983	11.5	9.6	1.9	10.1	8.7	1.4	9.7	8.5	1.2
1984	7.7	6.4	1.3	9.5	8.6	.9	9.3	8.4	.9
1985	8.3	8.5	–.2	8.0	7.5	.6	9.2	8.7	.5
1986	8.8	7.4	1.4	8.7	8.1	.6	8.3	7.6	.7
1987	4.2	3.1	1.1	6.5	5.1	1.4	7.2	6.3	.9
1988	5.0	5.8	–.8	4.6	4.1	.5	6.0	5.3	.7
1989	4.5	4.4	.1	4.8	5.2	–.4	4.6	4.4	.2
1990	3.8	5.1	–1.3	4.2	4.7	–.5	4.5	5.1	–.6
Mean Error			.16[–.16][a]			.19[.0][a]			.17[.0][a]
Mean Absolute Error			.85[1.19][a]			.66[.94][a]			.49[.64][a]
Root Mean Squared Error			1.02[1.43][a]			.77[1.14][a]			.57[.77][a]

Notes: Actual and predicted values are annualized rates of growth of M2 over 4Q-to-4Q periods ending in the years shown. The predicted values are generated using the money demand equation B of Table 3. (See footnote 14 in the text for a description of the forecast procedure used.) The predicted values are generated under the constraint that the long-term scale variable elasticity is unity whether computed from the long-run part or from the short-run part of the model.

[a] The values in brackets are the summary error statistics generated using the money demand regression A of Table 3.

Table 6

Error-Correction M2 Demand Models: Out-of-Sample Forecast Performance, 1971–1990

Short-Run Scale Variable	1 Year Ahead a	1 Year Ahead b	1 Year Ahead F3(2,18)	2 Years Ahead a	2 Years Ahead b	2 Years Ahead F3(2,17)	3 Years Ahead a	3 Years Ahead b	3 Years Ahead F3(2,16)
Real Consumer Spending	.0	1.01	.2	.1	1.0	.6	.5	.96	.9
	(.8)	(.09)		(.6)	(.08)		(.6)	(.08)	
Real GNP	.4	.93	.2	.6	.91	.3	.8	.89	.7
	(1.2)	(.13)		(1.1)	(.12)		(.8)	(.09)	

Notes: The table reports coefficients (standard errors in parentheses) from regressions of the form $A_{t+s} = a + b\,P_{t+s}$, where A is actual M2 growth; P predicted M2 growth; and s (= 1,2,3) number of years in the forecast horizon. The values used for A and P are from Table 5. F3 is the F statistic that tests the null hypothesis (a,b) = (0,1), and are distributed F with degrees of freedom given in parentheses following F3.

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