

# Why Is There Debt?

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The striking feature of debt contracts is that over a wide range of circumstances the payment is fixed and invariant, although occasionally, as in a default, less than the full payment is made. In this article I offer a simple explanation for why such arrangements are widely observed. The explanation relies on recent advances in the theory of financial arrangements under imperfect information. I will argue that the opportunity for borrowers to hide their future resources sharply constrains the degree to which loan repayment can be made contingent on the borrower's future resources.

From one point of view it is not obvious that the ubiquity of debt contracts is a puzzle. A borrower acquires a sum of money today that will be repaid in the future, along with an additional payment, called interest. The interest rate is the price for the temporary use of resources. It seems perfectly natural that this amount is predetermined.

Modern economic theory has taught us to view matters differently. When a loan is made, the lender acquires a **contingent claim**, a promise by the borrower to pay an amount that can depend in any arbitrary, prespecified way on future events.<sup>1</sup> Many familiar contracts actually do involve future payments that are contingent in significant ways. Insurance contracts are promises to make a payment contingent on some particular future loss. Partnership agreements and profit-sharing arrangements make future payments contingent on the uncertain profits of the firm. Traded securities such as stocks, bonds, options, and related derivative products have returns that are highly sensitive to future events. But in a debt contract, the payment is generally *noncontingent* in that the amount does not vary with future circumstances, such as the borrower's wealth. Of course a debt contract *is* contingent to the extent that the

lender does not receive full repayment if the borrower defaults. But although default is an important feature of the arrangement, it occurs relatively rarely.

Finding plausible models in which people agree to debt contracts, although they are allowed to agree to any possible contingent repayment schedule, has proven surprisingly difficult. In fact, in many models people are much better off with a contingent contract than they are with a debt contract. In Section I, I present a simple, two-agent model that shows why standard economic theory predicts that contracts generally should be contingent. The model also serves as a useful starting point for further analysis.

In Section II, I present a model in which the borrower and lender agree to a loan repayment that is noncontingent because the borrower can conceal future resources. Section III points out that this model is deficient because nothing resembling default ever occurs, and then argues that collateral, broadly defined, is an important omitted feature of the model. Next, in Section IV, I present a model in which an implicitly collateralized debt contract, with occasional default, is the chosen arrangement. Three brief sections conclude the paper: Section V surveys literature that has addressed the same question; Section VI briefly discusses some policy implications; and Section VII summarizes the explanation offered here for the ubiquity of debt contracts and notes two remaining unsolved puzzles.

## I. A SIMPLE MODEL OF CONTINGENT CLAIMS

To begin, consider an economy with only two people: a borrower and a lender. The economy lasts for just two time periods; call them periods 1 and 2. Imagine that the two people are farmers, and that the two periods represent the spring and fall of a given year. In the spring the lender harvests a crop: wheat, say. The lender's land produces no crop in the fall. The borrower's land produces no crop in the spring, but will produce a crop in the fall. Both agents would like to consume wheat in both the spring and

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<sup>1</sup> Flood (1991) provides an accessible introduction to models of contingent claims.

the fall. To do so, the borrower must obtain a loan of wheat in the spring, to be repaid from the proceeds of the fall harvest. For simplicity, I ignore the use of wheat in planting, and assume that the crops have already been planted. I also ignore the possibility of storing wheat from the spring to the fall; allowing storage would not affect the results. No other goods are available to these two agents.

To make the contingent nature of the contract of interest, some random event has to occur between spring and fall. I assume that in the spring the amount of the borrower's fall wheat harvest is uncertain. In the fall the harvest is realized, and both agents learn the exact value of the harvest. The payment contract is contingent if it depends on the amount of the borrower's crop. Other sources of uncertainty could have been considered—shocks to the preferences of the two agents for example—but in many ways, uncertainty concerning the borrower's ex post resources is the archetypal setting for financial contracting. If the borrower is a wage earner, for example, future income or employment might be uncertain. If the borrower is an individual entrepreneur, future returns from the venture might be uncertain. If the borrower is an incorporated firm, future liquid resources of the firm might be uncertain.

To proceed, the borrower's harvest in the fall is  $\theta$ , and can take on one of  $N$  values:  $\theta_1, \theta_2, \dots, \theta_N$ , where these are ordered so that  $0 < \theta_1 < \theta_2 < \dots < \theta_N$ .<sup>2</sup> In the spring, both people believe that the probability that  $\theta$  takes on the value  $\theta_n$  is  $\pi_n$ , where  $\pi_n > 0$  for  $n = 1, 2, \dots, N$ , and  $\sum_{n=1}^N \pi_n = 1$ . The lender has a harvest of  $e_1^L$  in the spring. The lender makes a loan advance of  $q$  in the spring, and receives a payment of  $y_n$  in the fall if the harvest is  $\theta_n$ . In the spring the lender's consumption is  $e_1^L - q$ , while the borrower's spring consumption is  $q$ . When the borrower's harvest is  $\theta_n$ , the lender's fall consumption is  $y_n$  and the borrower's fall consumption  $\theta_n - y_n$ . A contract is a set of payments  $\{q, y_1, y_2, \dots, y_N\}$ , and these completely determine the consumptions of the two agents.

I assume that the borrower evaluates the contract  $\{q, y_1, y_2, \dots, y_N\}$  according to the expected utility function

$$u_B(q) + \beta \sum_{n=1}^N u_B(\theta_n - y_n) \pi_n, \quad (1)$$

<sup>2</sup> For there to be a meaningful distinction between a debt contract with occasional default and a more general contingent contract, I need at least three states. The notational burden of  $N$  states is not significantly greater.

where  $\beta$  is a discount factor satisfying  $0 < \beta < 1$ . This is the ex ante expected utility of the borrower in the spring. Similarly, the lender evaluates the contract according to the expected utility function

$$u_L(e_1^L - q) + \beta \sum_{n=1}^N u_L(y_n) \pi_n. \quad (2)$$

The within-period utility functions  $u_B$  and  $u_L$  are assumed to be strictly increasing, continuous, concave and smoothly differentiable.

Contracts cannot require payments that exceed the available resources. Stated formally, contracts must satisfy the following resource feasibility constraints:

$$q \geq 0, \quad (3)$$

$$e_1^L \geq q, \quad (4)$$

$$y_n \geq 0, \quad n = 1, 2, \dots, N, \quad (5)$$

$$\theta_n \geq y_n, \quad n = 1, 2, \dots, N. \quad (6)$$

### Optimal Contracts

To obtain predictions in this simple environment about the arrangements that the two agents will choose, I restrict attention to optimal contracts. A contract is **optimal** if it is feasible and no other feasible contract exists that makes one agent better off, in terms of ex ante expected utility, without making the other agent worse off. Because of the simple nature of the environment, an easy way of finding optimal contracts is by maximizing the weighted average of the two agents' utility functions, subject to the resource feasibility constraints. The weights, sometimes called "Pareto weights," are arbitrary positive numbers, and varying their relative size traces out a range of contracts that gives more utility to one agent and less to the other. If a contract is optimal in this environment, then it is the solution to the constrained maximization problem for some Pareto weights, and vice versa.

The programming problem that finds optimal contracts, then, is the following.

#### Problem 1:

Maximize, by choice of  $q, y_1, y_2, \dots, y_N$ ,

$$\lambda_B \left[ u_B(q) + \beta \sum_{n=1}^N u_B(\theta_n - y_n) \pi_n \right] + \lambda_L \left[ u_L(e_1^L - q) + \beta \sum_{n=1}^N u_L(y_n) \pi_n \right]$$

subject to the resource feasibility constraints (3)-(6).

The Pareto weights  $\lambda_B$  and  $\lambda_L$  are arbitrary positive constants.

To show the properties of optimal contracts, I examine the set of first-order conditions that are necessary and sufficient for a contract to be a solution to Problem 1. If both the borrower and the lender are enjoying positive consumption in the fall for a given state  $\theta_n$ , so that  $0 < y_n < \theta_n$ , then the first-order condition for  $y_n$  is

$$\lambda_L u'_L(y_n) = \lambda_B u'_B(\theta_n - y_n). \quad (7)$$

Condition (7) requires that the marginal utility of the lender's fall consumption, scaled by  $\lambda_L$ , must equal the marginal utility of the borrower's fall consumption, scaled by  $\lambda_B$ . This condition determines  $y_n$  in a manner illustrated in Figure 1. The width of the box in Figure 1 is  $\theta_n$ , the realized harvest outcome to be divided between the two agents. The payment  $y_n$  is measured horizontally from left to right, and the lender's marginal utility, measured vertically, falls as  $y_n$  rises. Similarly, the consumption of the borrower is measured horizontally from right to left, and the borrower's marginal utility rises as  $y_n$  rises. The optimality condition (7) dictates that the payment is determined by the intersection of the two weighted marginal utilities. Identical conditions apply for every other possible harvest outcome; the horizontal dimensions of the box vary with  $\theta_n$ , but otherwise the analysis is the same.

### The Nonoptimality of Debt Contracts

I can now demonstrate that in this simple environment, *the payment varies positively with the harvest, and a debt contract will be optimal only under special circumstances.* Consider Figure 2, in which the determination of the payments is illustrated, just as in Figure 1, but for two possible harvest outcomes,  $\theta_n$  and  $\theta_m$ , where  $m > n$ . The box for the larger harvest,  $\theta_m$ , is drawn with the same left edge, so that the origin from which the payments are measured does not move. Consequently, the lender's marginal utility schedule is the same for both harvest outcomes. The origin from which the borrower's consumption is measured shifts to the right, because  $\theta_m > \theta_n$ , so the borrower's marginal utility shifts to the right. If both marginal utility schedules slope down, the point of intersection moves down and to the right going from harvest  $\theta_n$  to  $\theta_m$ . Therefore, the payment  $y_m$  for the larger harvest is larger than  $y_n$ , the payment for the smaller harvest.

Figure 1

### THE DETERMINATION OF FALL PAYMENTS FOR A GIVEN HARVEST, $\theta_n$

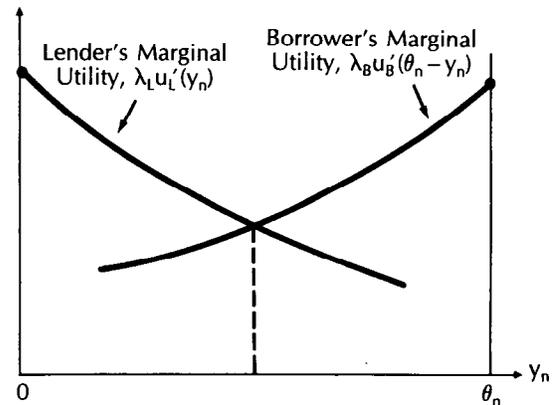
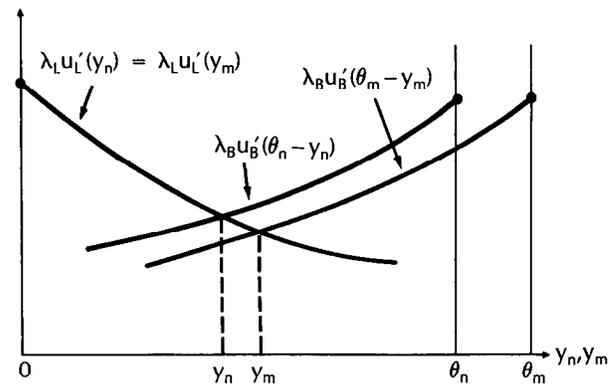


Figure 2

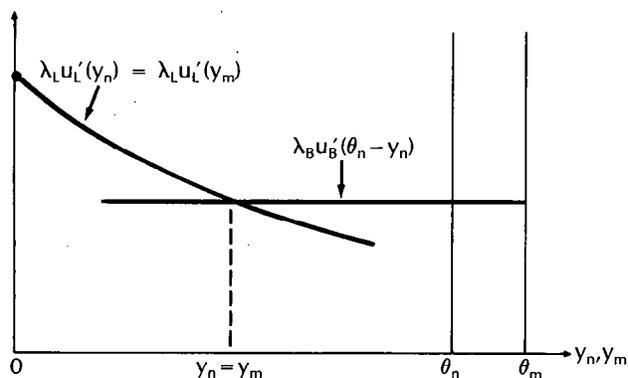
### THE DETERMINATION OF FALL PAYMENTS FOR HARVESTS $\theta_n$ AND $\theta_m$ , WHERE $\theta_m > \theta_n$



Under a debt contract there is a set of harvest outcomes over which the payment made by the borrower in the fall is a constant. It is easy to see what is required for such an arrangement to satisfy the optimality conditions. The lender's optimal consumption must remain the same, and this requires that the marginal utility curve for the borrower be horizontal, as in Figure 3. This in turn requires that the borrower be risk neutral, meaning that the borrower's utility is linear, not strictly concave. In this case a shift to the right in the borrower's marginal utility leaves the point of intersection, and thus the payment to the lender, unchanged. *For a debt contract to be optimal in this environment, the borrower must*

Figure 3

**THE DETERMINATION OF FALL PAYMENTS  
FOR HARVESTS  $\theta_n$  AND  $\theta_m$  WHEN  
THE BORROWER IS RISK NEUTRAL**



be risk neutral, and the lender must be risk averse.<sup>3</sup> This situation is highly implausible.<sup>4</sup>

This simple model highlights the basic principle that if people face contingencies that are observable when they occur, then only under very unusual circumstances would they choose noncontingent arrangements. In general we should expect to see contracts that are contingent on any future event that affects the marginal utilities of the contracting parties, such as shocks to the wealth of either party. For example, under perfect information, as here, we should observe agents insured against idiosyncratic shocks to their own wealth by sharing risk with other agents in the economy. Notably, a wide range of models share this property. The example economy I have described is merely a special case of the classical general equilibrium model of Arrow (1951) and Debreu (1959), extended to allow for uncertainty as in Chapter 7 in Debreu, or Arrow (1964). The principle survives in the most general specifications

<sup>3</sup> The case in which both the borrower and the lender are risk neutral can be ignored. If the two weighted marginal utility schedules do not intersect, then the entire harvest is given to one agent in every state. If the two weighted marginal utility schedules coincide, contractual arrangements are indeterminate.

<sup>4</sup> This seems implausible for at least three reasons. First, risk neutrality seems inconsistent with a wide array of observed arrangements for shedding or dispersing risk. Second, a large class of the population (all debtors) is not likely to have such special preferences. In fact, casual introspection suggests that borrowers are not systematically less risk averse than lenders, and, if anything, they are subject to more idiosyncratic risk than lenders, as in the model presented here. Third, in many economic models the assumption of risk neutrality is made to capture an agent's ability to insure against idiosyncratic risk in perfect capital markets. One would expect, however, that lenders rather than borrowers would have such privileged access.

of the classical model, at least when there are no imperfections in the availability of information. Thus, the ubiquity of debt contracts is puzzling, at least from the viewpoint of classical general equilibrium models.

**II. DEBT CONTRACTS  
IN A MODEL WITH LIMITED INFORMATION**

Apparently, then, to explain debt contracts one must depart from the assumptions of the classical model. In the example above, both the lender and the borrower are fully aware of the realized value of the borrower's harvest; in other words, there is **perfect information**. Suppose instead that the lender is uncertain of the borrower's harvest at the time the payment must be made in the fall. The lender might be forced to rely on the borrower's report about the harvest, especially if there is no independent information available to the lender. In this case the payment might have to be noncontingent, because otherwise the borrower would have reason to make a misleading and self-serving report.

To explore this notion, I now modify the model described above by assuming that the borrower is capable of hiding any amount of the harvest. The hidden crop can be consumed secretly, and hiding is itself costless. The remaining crop, the part not hidden, is displayed to the lender. This is less stringent than assuming that the lender is incapable of observing the harvest at all (pure private information), but still implies that the amount displayed provides only a lower bound on the actual amount of the harvest.<sup>5</sup>

This appears to be a fairly realistic imperfection in information. Often a borrower can divert resources for private benefit that would otherwise be available to repay an obligation. A consumer, for example, can spend freely on current consumption and then default on debts.<sup>6</sup> Similarly, a firm's managers can divert

<sup>5</sup> This assumption is a version of "costly state falsification" (see Lacker and Weinberg, 1989). Costly state falsification is the assumption that one agent can misrepresent the true state at some cost. Perfect information, as in the example in the previous section, is a special case in which all falsification is prohibitively costly. Pure private information is the assumption that the agent can costlessly make any state appear to have been realized. The assumption made here is that the borrower can costlessly make it appear that any lower harvest has occurred, but it is prohibitively costly to make it appear that a larger harvest has occurred.

<sup>6</sup> An individual can spend on consumption just prior to filing for bankruptcy, depleting liquid assets to the detriment of creditors, both in a Chapter 7 liquidation and in a Chapter 13 plan. In addition, the consumer can use funds to purchase "exempt assets" that are then beyond the reach of creditors.

resources in a variety of ways, through direct and indirect managerial compensation, wasteful investment, exploitation of discretion over accounting choices, or favored treatment of particular creditor classes.<sup>7</sup> Often a lender has no direct knowledge of a borrower's total resources and thus must rely on the borrower's own financial statements. At the same time, it often seems as if lenders have or can obtain some information about a borrower. If a borrower claims to have a certain quantity of resources, the lender can ask the borrower for proof of his bank balance or other readily verifiable assets. The borrower is incapable of proving that he does **not** control additional assets; he can display less than his true resources but not more. This informational imperfection is consistent with the observation that parties to financial arrangements are often observed exchanging information at relatively little apparent cost.<sup>8</sup>

### Incentive Constraints

Although the implication of this informational assumption is straightforward, I display the results more formally, since in a more complicated setting examined later the intuition will be less clear and the formalities more important. The borrower now has a choice to make in the fall: if the harvest is  $\theta_n$ , the borrower can display an amount  $\theta_m$ , where  $\theta_m$  can take on the values  $\theta_1, \theta_2, \dots, \theta_n$ . If the borrower displays  $\theta_n$  when the harvest is  $\theta_n$ , nothing is being hidden, while if the borrower displays less than  $\theta_n$ , an amount  $\theta_n - \theta_m$  is being hidden and consumed without the knowledge of the lender.

As before, a contract,  $\{q, y_1, y_2, \dots, y_N\}$ , specifies the spring payment to the borrower,  $q$ , and the fall payment from the borrower,  $y_n$ , contingent on the harvest. Also as before, a contract must satisfy the resource feasibility constraints (3)-(6). Now I impose the further condition, called **incentive feasibility**, that the borrower never has an incentive to hide any of the harvest. If the borrower does not hide any

harvest when the harvest is  $\theta_n$ , his utility is  $u_B(\theta_n - y_n)$ . If the borrower displays  $\theta_m$  when the harvest is  $\theta_n$ , hiding the amount  $\theta_n - \theta_m$ , his utility is  $u_B(\theta_n - y_m)$ . For the borrower to have no positive incentive to hide harvest, it must be true that  $u_B(\theta_n - y_n) \geq u_B(\theta_n - y_m)$ . Therefore, the set of **incentive feasibility constraints** are

$$u_B(\theta_n - y_n) \geq u_B(\theta_n - y_m) \\ \text{for } n=2, \dots, N, \text{ and for } m=1, \dots, n-1. \quad (8)$$

The incentive feasibility constraints stated here can be derived from a deeper formulation that allows contracts that might give the borrower incentive to hide some of the harvest. It can be shown, however, that the results of any arbitrary contract can be replicated by a contract that satisfies the incentive feasibility constraints.<sup>9</sup>

The incentive feasibility constraints can be simplified. To use (8) in finding an optimal contract, the utility of not hiding any harvest must be compared to the utility of displaying *any* amount less than  $\theta_n$ . It turns out that if the constraint for harvest  $\theta_n$  is satisfied for  $m=n-1$ , then the constraints are satisfied for all  $m < n$ . As a result (8) can be reduced to

$$u_B(\theta_n - y_n) \geq u_B(\theta_n - y_{n-1}) \\ \text{for } n=2, \dots, N. \quad (9)$$

In other words, the utility of telling the truth only needs to be compared to the temptation of displaying the next smallest possible harvest  $\theta_{n-1}$ .<sup>10</sup>

An immediate implication of (9) is that a contract is incentive feasible if and only if  $y_n$  is constant or decreasing as  $\theta_n$  increases. For any given harvest outcome, the borrower can make the payment corresponding to that harvest or to any smaller harvest; a given payment is feasible for the corresponding harvest and for any larger harvest outcome. Because

<sup>7</sup> There are many constraints on these abilities, of course. Boards of directors, or other representatives of creditors, monitor some aspects of managerial choice. Discretion over accounting is limited in myriad ways by accounting standards, legal requirements for certification by outside auditors and the like. The fraudulent conveyance provision of the bankruptcy code allows the bankruptcy court to "unwind" distributions to creditors made 90 days or less prior to filing. Nonetheless, managers retain considerable discretion and can often take actions for personal benefit to the detriment of creditors.

<sup>8</sup> In contrast, such readily available observation is difficult to reconcile with pure private information, where it is assumed that the information is completely unavailable to the lender.

<sup>9</sup> The proof is in Appendix A and uses "The Revelation Principle." The terminology is due to Myerson (1979). For an exposition of the Revelation Principle in similar settings see Townsend (1988). A warning is in order here, however; the display of harvest is an "action" and not a "message." In the present setting the distinction is immaterial, but in more general settings in which actions involve real costs, the distinction is important. See Lacker and Weinberg (1989), p. 1350.

<sup>10</sup> To prove this note that (9) implies that for  $n=2, 3, \dots, N$ ,  $y_n \leq y_{n-1}$ , so  $y_n \leq y_m$  for  $m=1, 2, \dots, n-1$ . This in turn implies that (8) is satisfied. The property that only immediately adjacent incentive constraints need to be checked arises in a wide variety of settings.

hiding the harvest is costless, the borrower will make the smallest possible payment and will display the corresponding amount of harvest. Thus the borrower will never have to make a larger payment for a larger harvest than for a smaller harvest.

### Optimality of a Debt Contract

I am now in a position to show that something resembling a debt contract is optimal in this model. As before, a programming problem is solved, but with the addition now of the incentive-feasibility constraints (9). Specifically, I solve

#### Problem 2:

Maximize, by choice of  $q, y_1, \dots, y_N$ ,

$$\lambda_B \left[ u_B(q) + \beta \sum_{n=1}^N u_B(\theta_n - y_n) \pi_n \right] + \lambda_L \left[ u_L(e_1^L - q) + \beta \sum_{n=1}^N u_L(y_n) \pi_n \right]$$

subject to the resource feasibility constraints (3)-(6), and the incentive feasibility constraints (9).

To see why a completely noncontingent contract is optimal, compare the incentive feasibility constraints with the contract that was optimal in Section I. First, recall that incentive-feasible contracts can never be increasing with respect to the harvest  $\theta_n$ , only constant or decreasing, because if  $y_n > y_{n-1}$  and the harvest is  $\theta_n$ , then the borrower would lie in order to make a smaller payment. Second, recall that in Section I, without incentive constraints, risk-sharing alone determined the optimal contract and it had a strictly increasing payment schedule. But such a contract is not incentive feasible, and would always give the borrower an incentive to claim that the smallest possible harvest outcome had occurred. Among the set of contracts that are nonincreasing—and thus incentive feasible—the constant payment schedule is the one that is closest to the optimal contract from Section I in the sense that it has the largest slope. Thus a contract with a constant payment schedule is optimal.<sup>11</sup>

<sup>11</sup> To complete a proof, I need to show that a contract with  $y_n < y_{n-1}$  for some  $n$  cannot be optimal. Suppose that  $y_n < y_{n-1}$  for some particular  $n$ . Then the incentive constraint that relates  $y_n$  and  $y_{n-1}$  is not binding, and  $\phi_n = 0$ , where  $\phi_n$  is the Lagrange multiplier on the  $n^{\text{th}}$  constraint in (9). Therefore, from the first-order conditions we have

$$\beta[\lambda_L u_L'(y_{n-1}) - \lambda_B u_B'(\theta_{n-1} - y_{n-1})] \pi_{n-1} = u_B'(\theta_{n-1} - y_{n-1}) \phi_{n-1} \geq 0$$

A noteworthy feature of this model is that the range of contracts available to the two agents is severely restricted. Because the payment, call it  $R$ , is constant across harvests, *the payment can never be greater than the smallest possible harvest* ( $R \leq \theta_1$ ); otherwise the fixed payment is not feasible for small harvests. This is potentially a quite severe restriction, since the smallest possible harvest could be very different from the expected realization, and could imply a maximum loan repayment that is very small. In this situation, the borrower might be left desiring more credit than he can obtain via any incentive-feasible contract. To see this, one can combine the first-order conditions from Problem 2 to obtain the following equation linking the expected intertemporal marginal rates of substitution of the two agents:

$$\sum_{n=1}^N \frac{\beta u_B'(\theta_n - y_n) \pi_n}{u_B'(q)} = \sum_{n=1}^N \frac{\beta u_L'(y_n) \pi_n}{u_L'(e_1^L - q)} + \frac{\mu_1}{\lambda_B u_B'(q)} \quad (10)$$

The left side of (10) is the borrower's expected intertemporal marginal rate of substitution: the expected value of the ratio of marginal utility in period 2 to marginal utility in period 1. Similarly, the first term on the right side of (10) is the lender's expected intertemporal marginal rate of substitution. The second term contains  $\mu_1$ , the Lagrange multiplier on the constraint,  $y_1 - \theta_1 \leq 0$ . If  $\mu_1 > 0$ , this constraint is binding, and the borrower's expected marginal intertemporal rate of substitution is strictly less than the lender's. This means that the borrower would like to obtain more period 1 consumption in exchange for period 2 consumption, but cannot do so in any feasible contract. In this sense, one might describe such a borrower as constrained or rationed.<sup>12</sup>

and

$$\beta[\lambda_L u_L'(y_n) - \lambda_B u_B'(\theta_n - y_n)] \pi_n = -u_B'(\theta_{n+1} - y_n) \phi_{n+1} \leq 0.$$

Unless  $u_B$  is linear, these two conditions together imply that  $y_n > y_{n-1}$ , but this contradicts the initial supposition that the opposite was true. Thus, a contingent contract cannot satisfy the first-order conditions and thus cannot be optimal.

<sup>12</sup> This feature of the model is an exact restatement of an argument made by Irving Fisher (1930, pp. 210-11). He noted that a borrower's collateral will limit the amount he can borrow, and "[i]n consequence of this limitation upon his borrowing power, the borrower may not succeed in modifying his income stream sufficiently to bring his rate of preference for present over future income down to agreement with the rate or rates of interest ruling in the market" (p. 211).

### III. DEFAULT AND COLLATERAL

As a model of debt contracts, there is an obvious deficiency in the optimal contract just described: nothing ever occurs that resembles a **default**, a state in which something less than the fixed payment,  $R$ , is made. The optimal contract is a constant payment, and thus is perfectly risk-free. Many debt contracts are virtually risk-free, but it seems that in most debt arrangements there appears to be at least a remote possibility of default. This possibility is an important feature of the contractual arrangement, even if the probability is small, because a borrower will always be tempted to simulate default. Apparently the environment described above is incompatible with payments that are almost always a fixed amount but occasionally are less.<sup>13</sup> Can economic environments be found that display such contracts?

To guide a search for such environments, let us begin by asking what happens when an individual defaults on an actual debt contract. First, and obviously, the borrower pays less at a given date than was stipulated under the original contract.<sup>14</sup> This is not all that happens, however. If the loan is explicitly collateralized the borrower may be forced to surrender the collateral. Under an "unsecured" obligation the borrower may agree to a restructured payment schedule, promising to make future payments in lieu of the current payment. Sometimes the borrower is forced into legal bankruptcy proceedings, which often involve liquidating assets and using the proceeds to repay claims. For managers of incorporated businesses, bankruptcy involves at least temporary surrender of some control rights associated with the business, because the bankruptcy court or the trustee can assume substantial power over management decisions. The bankrupt that is not liquidated often must agree to a set of restructured claims, as in Chapter 11 reorganizations or Chapter 13 "wage-earner plans" under the U.S. Bankruptcy Code. These outcomes are obviously interrelated, but the salient point is that usually the borrower surrenders something distinct from the originally promised payment: either money at a later date or some other asset or right.

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<sup>13</sup> Indeed, results like those in Section II have been widely known for some time, so perhaps the central problem posed by debt contracts is reconciling the predominantly noncontingent nature of the contract with occasional contingent payments.

<sup>14</sup> I neglect here the phenomenon of "technical default," a common provision in many contemporary debt contracts, in which the borrower has made all requisite payments but has violated some auxiliary covenants. Covenants are important, but I have nothing new to say about them and focus here entirely on default in the sense of payment deficiency.

One is thus led to consider contracts in a multiple-good environment, one in which the borrower has more than one good to sacrifice. In such a setting a contract specifies a payment schedule for each good the borrower will later have available. In principle each payment schedule can be an arbitrary function of future circumstances. A debt contract in this setting is a set of payment schedules with special properties. First, in almost all circumstances fixed noncontingent amounts of a set of goods are paid, fixed sums of money at prespecified dates, for example. Second, in some circumstances less of these goods is paid and positive quantities of some other goods are surrendered, where "other goods" must be interpreted broadly to include legal claims and the like, as described above.

Under what circumstances would such a debt contract be optimal? Let us abstract from multiperiod debt contracts that stipulate a series of payments and focus attention on an obligation to make a single specified payment at a single future date. Consider first the set of states in which the borrower pays the fixed amount of the good—call it money for now. Perhaps the noncontingent nature of the payment schedule over these states can be motivated in exactly the same way as the noncontingent contract of the previous section; if the borrower could hide resources *ex post*, the payment schedule would have to be constant to avoid giving the borrower an incentive to hide.

Now consider the default states in which some other goods are paid. The fixed payment might be larger than the smallest possible amount of money the borrower could have available. When the borrower does not have enough money to make the fixed payment, the actual payment is obviously limited by the amount of money the borrower has. What is to keep the borrower from always feigning these outcomes so as to make the smallest possible payment? With other goods available, the contract could require that if the borrower makes less than the fixed money payment, then some other goods of equal value to the borrower must be transferred to the lender as well. The other goods sacrificed are enough to dissuade the borrower from pretending to be destitute. Thus the transfer of other "collateral" goods ensures that the borrower will not falsely claim to be unable to make the full payment.

Such an arrangement could expand dramatically the set of feasible contracts available to the borrower and lender. In the environment described in Section III, where no other goods were present, the lowest

possible harvest,  $\theta_1$ , placed an upper bound on the size of the fixed payment. With other goods available, a contract can be written with a fixed payment of money that is larger than the smallest amount the borrower might possess *ex post*. The other goods provide a way of relaxing the sharp constraint imposed by the value of the smallest possible harvest outcome in the environment of Section II.

But other puzzles arise in this story. Consider first the set of states in which a noncontingent amount of money is paid. Why, in these circumstances, pay money rather than some other goods, such as those paid in the default-like states? It must be because money, at least in those states, is more valuable to the lender than the other goods, or, equivalently, the other goods are more valuable to the borrower, relative to money, than they are to the lender. This seems like a reasonable condition, one that might be satisfied in many of the circumstances in which debt contracts appear. When a consumer buys a house or a car, say, it is less valuable to the lender, relative to money, than it is to the borrower; the lender would obtain less money by repossessing the collateral and selling it than the borrower would spend to retain it. Consumers quite plausibly could value a good at more than its market price if it is indivisible and consumers only buy one. Similarly, the value to the borrower of all that is forfeited in bankruptcy settlements of various types is usually less than the value of what is received by lenders. Indeed, the difference, regarded as a "deadweight loss," seems to motivate a wide array of arrangements—both in and out of formal bankruptcy proceedings—designed to minimize this loss.

The other goods serve as **collateral**. This is most plain in loans explicitly collateralized by physical goods such as land, structures, chattels, automobiles, or inventories. Often a loan is collateralized by financial instruments such as accounts receivable, warehouse receipts or negotiable securities. Many debts are **implicitly collateralized**, as when income or profits in the more distant future stand behind a promise to make a payment out of income or profits in the near future, or when claims to a portion of the proceeds of liquidation stand behind an unsecured corporate obligation. Even an unsecured creditor can obtain a judgement against a defaulting debtor, allowing the creditor to have the debtor's assets seized to satisfy the claim. While the distinctions between these various means of collateralizing an obligation can be quite important, they are fundamentally similar. Indeed, in almost all instances the nonpayment of a contractual obligation provides the lender

with a legal claim, the content of which is jointly determined by the terms of the original contract and the existing body of contract and bankruptcy law. While the resulting claim can have a wide range of characteristics, it provides the borrower with an incentive to make the stipulated payment whenever possible, to "keep his heart right" in the words of a practitioner.<sup>15</sup> The role of collateral is *not* necessarily to indemnify the lender against potential loss, although it certainly does so to a degree. Rather, collateral is a means of satisfying incentive constraints that ensure voluntary compliance with the terms of the loan agreement.

The main legal distinction between an explicitly collateralized debt and an uncollateralized debt is how the claim stands *vis-à-vis* third parties such as other creditors or a bankruptcy trustee. For example, under the current U.S. law governing secured transactions the difference between secured and unsecured creditors is minor when there is only one creditor.<sup>16</sup> A creditor with a collateralized debt can obtain the collateral to satisfy the claim, rather than see the collateral added to the pool of assets divided among all of the creditors in bankruptcy. This suggests that the essential role of explicit, as opposed to implicit, collateral is related to multilateral financial arrangements, and that uncollateralized lending has much in common with explicitly collateralized lending.<sup>17</sup>

#### IV. COLLATERALIZED DEBT

In this section I describe a two-good economic environment; and I find conditions under which a collateralized debt contract is optimal for the reasons described above. The environment, an extension of the previous example, captures the essential elements of the argument outlined above.<sup>18</sup>

<sup>15</sup> This role was noted by Barro (1976). The quoted practitioner is Chris Carlson, Richmond, VA.

<sup>16</sup> One exception is when the collateral is an "exempt asset" under bankruptcy law, and is thus out of reach of any unsecured creditor but can be recovered under a collateralized loan. Exempt assets include the debtor's "tools of trade," some of the debtor's household goods, and an interest in the debtor's residence. Another exception is when the collateral is an asset that will not pass to the bankruptcy estate, such as the personal assets of the manager of a corporation.

<sup>17</sup> Standard terminology in the theoretical finance literature, unfortunately, is that a debt contract like the one described in Section II is "uncollateralized" while a debt contract like the one described below is "collateralized." The literature treats collateral as if it were exempt assets in a personal bankruptcy, or the personal assets of a manager in a corporate bankruptcy.

<sup>18</sup> The model is a simplified version of my 1991 working paper.

The borrower is assumed to have two goods with which to conceivably repay a loan in the fall. One good is the fall harvest, as before, and the other good can be thought of as **chattels**: durable, portable, personal property such as clothes, furniture or perhaps tools. In a **collateralized debt contract**, when the harvest is sufficient the borrower pays a fixed, non-contingent amount of the harvest (the **payment good**) and none of the chattels (the **collateral good**). When the fall harvest is less than the fixed payment, the entire harvest is given to the lender along with a positive quantity of chattels. For good harvests the borrower does not hide the harvest because some of the chattels would have to be surrendered as well.

To proceed formally, then, let good 1 in each period be the wheat harvest, as in the previous models, and let good 2 be the collateral good, the borrower's chattels. The borrower is endowed with  $k$  units of chattels, and  $k$  is known ahead of time to both the borrower and the lender. In the event that the fall harvest is  $\theta_n$ , the borrower makes payments of  $y_{1n}$  of good 1 and  $y_{2n}$  of good 2, and consumes  $\theta_n - y_{1n}$  of good 1 and  $k - y_{2n}$  of good 2. As before, the spring loan advance is  $q$ , so that the consumption of the borrower is  $q$ , and the consumption of the lender is  $e_1^L - q$ , the lender's spring endowment minus the loan advance. For simplicity, I assume that the borrower derives utility from consumption of chattels only in the fall.

The expected utilities of the two agents are now

$$u_B(q) + \beta \sum_{n=1}^N [u_B(\theta_n - y_{1n}) + v_B(k - y_{2n})] \pi_n \quad (11)$$

and

$$u_L(e_1^L - q) + \beta \sum_{n=1}^N [u_L(y_{1n}) + v_L(y_{2n})] \pi_n. \quad (12)$$

I have assumed here that both agents have additively separable utility in the fall. The functions  $v_B$  and  $v_L$  are the utilities of the borrower and the lender, respectively, with respect to chattels. I assume that both are continuous, concave and smoothly differentiable. A natural assumption to make is that  $v_B$  is strictly increasing, but  $v_L$  need not be increasing. The function  $v_L$  might be decreasing if the collateral good is worthless to the lender and disposing of it is costly, or if  $y_{2n}$  is viewed as a costly punishment, such as debtor's prison.<sup>19</sup>

<sup>19</sup> Diamond (1984) displays a model of optimal debt contracts that depends on nonpecuniary punishment of the borrower in the event of nonpayment. His model is a special case of the model described here.

The resource feasibility constraints extend naturally to this case:

$$q \geq 0, \quad (13)$$

$$e_1^L \geq q, \quad (14)$$

$$y_{1n} \geq 0, \quad n = 1, 2, \dots, N, \quad (15)$$

$$\theta_n \geq y_{1n}, \quad n = 1, 2, \dots, N, \quad (16)$$

$$y_{2n} \geq 0 \quad n = 1, 2, \dots, N, \quad (17)$$

$$k \geq y_{2n}, \quad n = 1, 2, \dots, N. \quad (18)$$

A **contract** is now a loan advance,  $q$ , and a pair of payment schedules,  $\{y_{11}, y_{12}, \dots, y_{1N}\}$  and  $\{y_{21}, y_{22}, \dots, y_{2N}\}$ , that determine the payments of good 1 and good 2, respectively, for each harvest. As before, contracts might in general give the borrower an incentive to hide some of the harvest in some states, but, as before, we can restrict attention to contracts for which the borrower never has an incentive to hide. Contracts that have this property satisfy the following incentive feasibility constraints:

$$\begin{aligned} u_B(\theta_n - y_{1n}) + v_B(k - y_{2n}) \\ \geq u_B(\theta_n - y_{1m}) + v_B(k - y_{2m}) \\ \text{for } n = 2, \dots, N, \quad m = n - 1. \end{aligned} \quad (19)$$

As in the previous model, I have written the constraint only in terms of the temptation of displaying the next smallest possible harvest  $\theta_{n-1}$ .<sup>20</sup>

Optimal contracts can again be found as the solution to a programming problem, parallel to Problem 2. An optimal contract is a set of numbers  $\{q, y_{11}, y_{12}, \dots, y_{1N}, y_{21}, y_{22}, \dots, y_{2N}\}$  that solve

### Problem 3:

Maximize, by choice of  
 $q, y_{11}, y_{12}, \dots, y_{1N}, y_{21}, y_{22}, \dots, y_{2N},$

$$\begin{aligned} \lambda_B u_B(q) + \lambda_B \beta \sum_{n=1}^N [u_B(\theta_n - y_{1n}) + v_B(k - y_{2n})] \pi_n \\ + \lambda_L u_L(e_1^L - q) + \lambda_L \beta \sum_{n=1}^N [u_L(y_{1n}) + v_L(y_{2n})] \pi_n \end{aligned}$$

<sup>20</sup> The simple argument used in Section II to justify restricting attention to adjacent incentive constraints does not apply in the two-good environment here. The approach is valid nonetheless because I merely want to show that a particular candidate contract is optimal. The set of contracts that satisfy global incentive feasibility constraints is contained in the larger set of contracts that satisfy the weaker local constraints in (19). The candidate contract can be shown to satisfy global incentive feasibility, so if it is optimal relative to contracts satisfying (19), then it is optimal relative to the smaller set of contracts that satisfies global incentive feasibility constraints.

subject to the resource feasibility constraints (13)-(18) and the incentive feasibility constraints (19).<sup>21</sup>

### The Collateralized Debt Contract

Under what conditions, then, does a collateralized debt contract solve Problem 3? To answer this question I need to state precisely what constitutes a collateralized debt contract. To start, the payment schedule for good 1 is

$$y_{1n}^* = y_{1n}^*(R) \equiv \text{MIN}[\theta_n, R]. \quad (20)$$

A fixed, noncontingent amount  $R$  is transferred, unless the harvest is too small and  $\theta_n < R$ , in which case the entire crop is transferred. I have in mind contracts in which  $R > \theta_1$ , contracts that were not incentive feasible in the earlier model with just one repayment good. Figure 4(a) portrays a typical payment schedule for good 1. For future reference, define  $r$  as the largest index number for which  $\theta_r \leq R$ .<sup>22</sup>

To complete the description of a typical collateralized debt contract, I need to specify the schedule of transfers of chattels, the collateral good. I apply two guiding principles: first, ensure incentive feasibility of the resulting contract; and second, minimize the consumption of the borrower's chattels by the lender. *Thus, the chattels payment schedule is the minimal schedule that ensures that the borrower does not have an incentive to cheat and hide some of the harvest.* The schedule is constructed recursively starting with the payment  $y_{2N}^*$ , for the largest harvest, and working down to  $y_{21}^*$ , the payment for the smallest harvest, with the payment set at each step to ensure that the incentive feasibility constraint for that harvest outcome is met with equality. First, the payment  $y_{2N}^*$  can be set freely, so to minimize the payment for this harvest outcome set  $y_{2N}^* = 0$ . Now for any arbitrary  $m < N$ , assume that the chattels payment schedule has already been determined for  $n = m + 1, \dots, N$ . The incentive feasibility constraint relating the payments  $y_{2m}^*$  and  $y_{2n}^*$ , for  $n = m + 1$ , is

$$u_B(\theta_n - y_{1n}^*) + v_B(k - y_{2n}^*) \geq u_B(\theta_n - y_{1m}^*) + v_B(k - y_{2m}^*), \quad n = m + 1. \quad (21)$$

<sup>21</sup> Problem 3 is convex under the additional assumption, which I now make, that  $-u_B^B(c_{21}^B)/u_B^B(c_{21}^B)$ , the coefficient of absolute risk aversion of the borrower with respect to good 1, is non-increasing. Under this condition the set of utilities that satisfy feasibility constraints is convex (even though the constraints are not convex in the choice variables).

<sup>22</sup> Therefore,  $R$  is contained in the half-open interval  $[\theta_r, \theta_{r+1})$ .

If  $y_{2m}$ , payment of chattels from the borrower to the lender, is so small that (21) is violated, then when the harvest is  $\theta_n$  the borrower has an incentive to lie to make the payments  $y_{1m}^*$  and  $y_{2m}$  rather than  $y_{1n}^*$  and  $y_{2n}^*$ . If  $y_{2m}$  is so large that (21) is a strict inequality, then the chattels payment could be reduced without violating incentive feasibility. Choose for  $y_{2m}^*$  the smallest value of  $y_{2m}$  that satisfies (21). *For each harvest outcome, the chattels payment is the smallest possible amount that does not give the borrower an incentive to lie.*

The specific shape of a typical chattels payment schedule is shown in Figure 4(b). For  $\theta_n > R$ , the crop payment is the constant,  $R$ , so for  $m > r$ , (21) reduces to  $v_B(k - y_{2n}^*) \geq v_B(k - y_{2m}^*)$ . Because  $y_{2N}^* = 0$ , we can set  $y_{2m}^* = 0$  for all  $m > r$ . In other words, for harvests greater than  $R$ , the chattels payment is zero. For  $m = r$ , (21) as an equality is

$$u_B(\theta_{r+1} - R) + v_B(k) = u_B(\theta_{r+1} - \theta_r) + v_B(k - y_{2r}^*). \quad (22)$$

This equation determines  $y_{2r}^*$ . For harvests  $\theta_m < \theta_r$  (so that  $m < r$ ), (21) as an equality is<sup>23</sup>

$$u_B(0) + v_B(k - y_{2n}^*) = u_B(\theta_n - \theta_m) + v_B(k - y_{2m}^*), \quad \text{where } n = m + 1. \quad (23)$$

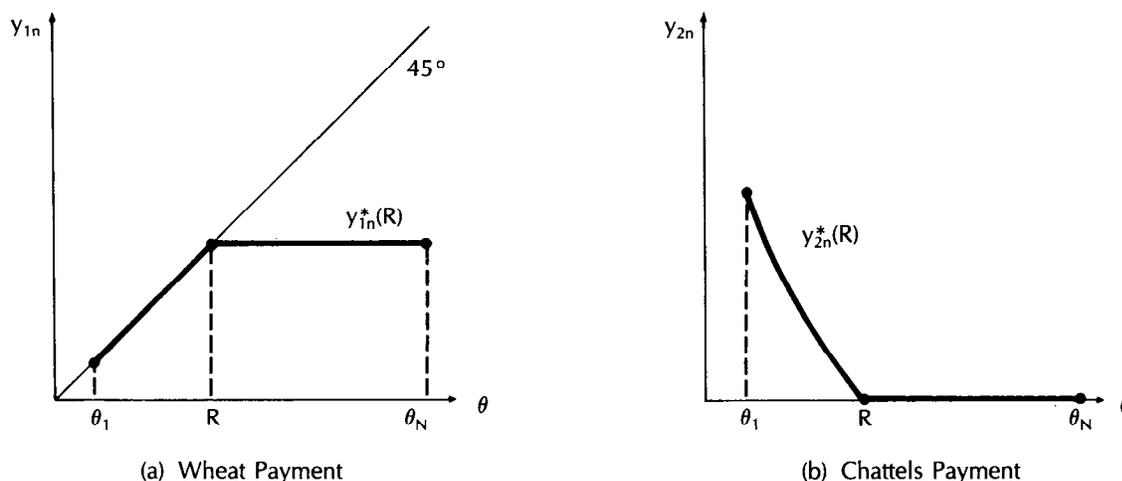
The left side of (23) is the borrower's utility when the harvest is  $\theta_n = \theta_{m+1}$  and he pays  $y_{1n}^* = \theta_n$  and  $y_{2n}^*$ . The right side of (23) is the borrower's utility when the harvest is  $\theta_n = \theta_{m+1}$  and he instead pretends  $\theta_m$  has occurred and pays  $y_{1m}^* = \theta_m$  and  $y_{2m}^*$ . The chattels payment for harvest  $\theta_m$ ,  $y_{2m}^*$ , is set so that the borrower is just indifferent between these two alternatives. Note that the largest transfer of the collateral good is  $y_{21}^*$ , and occurs for the smallest possible harvest,  $\theta_1$ .

To summarize, a collateralized debt contract is described by (20) and (21). For harvests greater than  $R$ , the borrower transfers a fixed amount,  $R$ , of the crop, and none of the chattels. For harvests less than  $R$ , the borrower transfers all of the crop and some amount of the chattels; just enough, for each harvest, to dissuade the borrower from falsely claiming that that harvest has occurred if the harvest is actually larger.

<sup>23</sup> This equation uses the facts that for  $m < r$  and  $n = m + 1$ ,  $\theta_n - y_{1n}^* = \theta_n - \theta_n = 0$ , and  $\theta_n - y_{1m}^* = \theta_n - \theta_m = \theta_{m+1} - \theta_m$ .

Figure 4

A TYPICAL COLLATERALIZED DEBT CONTRACT



The borrower's collateral,  $k$ , can sharply constrain feasible contracts, because a contract cannot require transfer of more collateral than the borrower actually has. The constraint that the largest collateral transfer  $y_{21}^*$  not exceed  $k$  is analogous to the constraint in the model of Section II that the fixed payment not exceed the smallest possible harvest; it places an upper limit on the amount of the fixed payment  $R$ . Although collateral can allow payment schedules which would otherwise be infeasible, feasible payment schedules could still be constrained.<sup>24</sup>

Optimality of a Collateralized Debt Contract

The next task is to examine the first-order necessary and sufficient conditions for Problem 3, and to see whether the collateralized debt contract just described can satisfy those conditions. The objective is to identify conditions on the agent's utility functions, the endowments, and the probability distribution governing the harvest that allow the collateralized debt contract to satisfy the first-order conditions.

One condition that is required for a collateralized debt contract to be optimal is that *the collateral good must be more valuable at the margin to the borrower than to the lender*:

$$\frac{v_B'(k - y_{2n}^*)}{u_B'(\theta_n - y_{1n}^*)} \geq \frac{v_L'(y_{2n}^*)}{u_L'(y_{1n}^*)}, \quad (24)$$

<sup>24</sup> One can easily derive an equation linking the two agents' expected intertemporal marginal rates of substitution in this case, analogous to (10), but with the Lagrange multiplier on the constraint  $y_{21} \leq k$  playing the role of  $\mu_1$ .

or, upon rearranging,

$$\frac{u_L'(y_{1n}^*)}{u_B'(\theta_n - y_{1n}^*)} - \frac{v_L'(y_{2n}^*)}{v_B'(k - y_{2n}^*)} \geq 0. \quad (25)$$

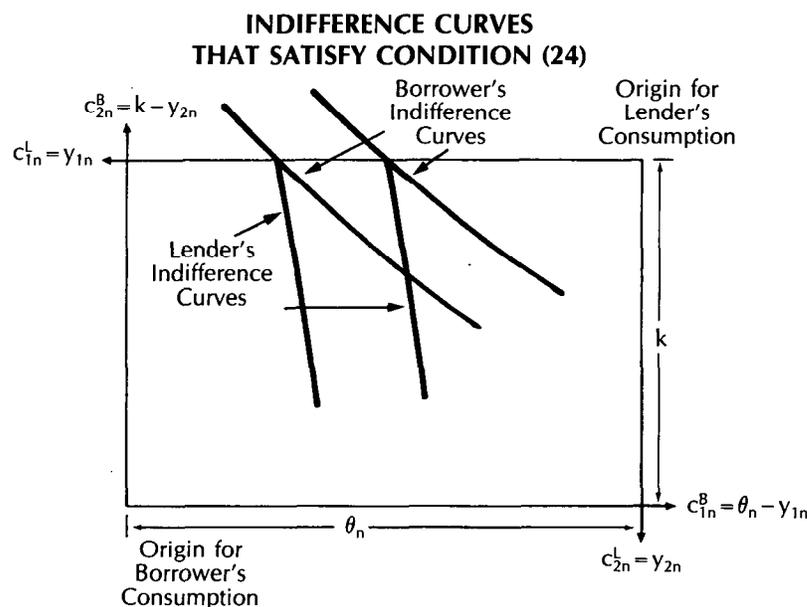
The two ratios in (24) measure the marginal value of the collateral good relative to the harvest good for each agent. The inequality (24) states that the marginal rate of substitution between chattels and wheat is larger for the borrower than for the lender. Indifference curves that satisfy (24) are shown in an Edgeworth Box in Figure 5. Imagine increasing the crop payment,  $y_{1n}$ , and decreasing the chattels payment,  $y_{2n}$ , by infinitesimal amounts in a way that keeps the borrower on the same indifference curve. If (24) holds then such a move along the borrower's indifference curve (to the northwest in Figure 5) *increases* the lender's utility. Thus condition (24) states that, *ceteris paribus*, giving more of the crop to the lender and more of the chattels to the borrower can make one of them better off without making the other worse off.

A second condition for a collateralized debt contract to be optimal is actually a strengthening of the first condition; *the direct benefit of giving more crop to the lender and more chattels to the borrower must be greater than the cost of the second-order effect on incentive constraints*:

$$\left[ \frac{u_L'(y_{1n}^*)}{u_B'(\theta_n - y_{1n}^*)} - \frac{v_L'(y_{2n}^*)}{v_B'(k - y_{2n}^*)} \right] \lambda_L \beta \pi_n - \Delta_n \geq 0, \quad (26)$$

$$\text{where } \Delta_n \equiv \left[ \frac{u_B'(\theta_n - y_{1n}^*) - u_B'(\theta_{n+1} - y_{1n}^*)}{u_B'(\theta_n - y_{1n}^*)} \right] \phi_{n+1}, \quad (27)$$

Figure 5



and where  $\phi_{n+1}$  is the nonnegative Lagrange multiplier on the incentive feasibility constraint for harvest  $\theta_{n+1}$ . The bracketed term in (26) is identical to the left side of (25), and measures the benefit of giving the borrower more of the collateral good and less of the crop when the harvest is  $\theta_n$ . Such a reallocation affects the incentive constraint for harvest  $\theta_{n+1}$ , and the term  $\Delta_n$  is the cost associated with this effect. If the benefit term in (26) is greater than the cost term  $\Delta_n$ , then the debt contract is optimal.

To understand the cost term  $\Delta_n$ , again imagine increasing the crop payment  $y_{1n}$  and decreasing the chattels payment  $y_{2n}$  by infinitesimal amounts, giving more of the payment good to the lender and more of the collateral good to the borrower, in a way that keeps the borrower on the same indifference curve. In particular, increase  $y_{1n}^*$  to  $y_{1n}^* + \epsilon$ , for some very small  $\epsilon > 0$ , and decrease  $y_{2n}^*$  to  $y_{2n}^* - \delta$ , so as to keep the borrower on a constant indifference curve. This change affects the borrower's incentive to tell the truth when the harvest is  $\theta_{n+1}$ , making it more tempting to display  $\theta_n$  and make the corresponding payments,  $y_{1n}^* + \epsilon$  and  $y_{2n}^* - \delta$ . Specifically,  $u_B(\theta_{n+1} - y_{1n}^* - \epsilon) + v_B(k - y_{2n}^* + \delta) > u_B(\theta_n - y_{1n}^*) + v_B(k - y_{2n}^*)$ , even though  $u_B(\theta_n - y_{1n}^* - \epsilon) + v_B(k - y_{2n}^* + \delta) = u_B(\theta_n - y_{1n}^*) + v_B(k - y_{2n}^*)$  by construction. The change in the right side of the incentive constraint for harvest  $\theta_{n+1}$  [see condition (19)] is approximately

$$[u_B'(\theta_n - y_{1n}^*) - u_B'(\theta_{n+1} - y_{1n}^*)]\epsilon, \quad (28)$$

a nonnegative quantity. The term  $\Delta_n$  is just (28), the amount by which the state  $n + 1$  incentive constraint is tightened, multiplied by  $\phi_{n+1}$ , the Lagrange multiplier, or "shadow value" for that constraint. (The denominator of  $\Delta_n$  rescales  $\phi_{n+1}$  into units of state  $n$  utility.) The term  $\Delta_n$  represents the *cost* of a move toward the northwest boundary of the Edgeworth Box for state  $n$ . Therefore, condition (26) states that the gap between the borrower's and the lender's marginal rate of substitution between chattels and wheat must exceed the cost of an indirect effect on incentive constraints.<sup>25</sup>

There are two intuitive ways to think about condition (26). First, it can be thought of as a lower bound on the gap between the borrower's and the lender's valuation of the collateral good—the bracketed term in (26)—for a given value of the cost term  $\Delta_n$ . If the gap is not large enough, the debt contract is not optimal and the best arrangement involves more frequent transfer of the chattels to the lender. Alternatively, condition (26) can be viewed as an upper bound on the borrower's risk aversion, because the cost term  $\Delta_n$  is approximately proportional to the borrower's coefficient of absolute risk aversion. The

<sup>25</sup> Notice that if  $u_B$  is linear, so that the borrower is risk neutral with respect to good 1, then the derivative  $u_B'$  is a constant,  $\Delta_n$  is zero, and (26) is equivalent to (24). For very small values of  $\theta_{n+1} - \theta_n$ ,  $\Delta_n$  is approximately proportional to  $-u_B''(c_{1n}^{B*})/u_B'(c_{1n}^{B*})$ , the coefficient of absolute risk aversion of the borrower with respect to the payment good. Thus  $\Delta_n$  is larger, ceteris paribus, the more risk averse is the borrower.

incentive constraints prevent the borrower from sharing as much risk as he would like with the lender, so if the borrower is very risk averse the value of relaxing an incentive constraint is large. If the borrower is too risk averse, the cost of indirectly tightening the incentive constraints outweighs the benefit of giving the borrower the chattels, and the debt contract is not optimal.<sup>26</sup>

As mentioned above, collateral is often described as a means of compensating the lender for possible losses in default, but its main role in this model is to secure compliance with the debt agreement. The amount of collateral transferred for a given harvest is just enough to discourage the borrower from pretending a low harvest has occurred when it actually has not. Thus to the borrower, the amount of collateral transferred is equal in value to the shortfall in the crop. The lender is actually worse off when he receives collateral than when the full payment is made, because the collateral is worth less to the lender than to the borrower, relative to the crop. The value of collateral to the lender *does* matter for the arrangement because, the more the lender values the collateral, the lower the interest rate the lender will require.<sup>27</sup> However, the primary function of collateral here is to keep the borrower honest.

## V. RELATED LITERATURE ON DEBT CONTRACTS

Kenneth Arrow (1974), in his 1973 Presidential Address to the American Economics Association, first suggested that private information might be why non-contingent contracts are widely observed. This idea arose in the early economics literature on markets for insurance, particularly medical insurance, in which the absence of insurance arrangements was traced to the nonobservability of some key aspect of future outcomes (see Arrow, 1963, and Spence and Zeckhauser, 1971). This observation has long been taken for granted in the insurance industry itself. For example, an insurance textbook (Angell, 1959) states that one requirement for a hazard to be insurable is

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<sup>26</sup> This reasoning is only heuristic, because independently varying, say, the lender's valuation of the collateral good will affect the cost term as well via the multiplier  $\phi_{n+1}$ . Nonetheless, parametric examples can easily be constructed that match the intuition in the text. Also, one can easily obtain an explicit expression for  $\phi_n$  in terms of the primitive elements of the environment.

<sup>27</sup> The interest rate on a loan is just  $R/q - 1$ . I have in mind a setting in which the lender compares the total return from the loan contract to returns on alternative uses of funds, so the more valuable the collateral the smaller  $R$  has to be.

that "[i]t must be difficult or impossible for the insured to pretend that he has suffered a loss when he has not done so."

Many recent papers have proposed explanations for debt contracts with occasional default. Douglas Diamond (1984) described a model of debt contracts based on private information about the borrower's resources, as here, and based on the idea that a lender can impose "nonpecuniary penalties" on a borrower in the event of default. The amount of the penalty varies with the borrower's reported resources, and is set optimally to ensure that the borrower does not have an incentive to lie. Diamond's model is virtually a special case of the model presented above; the surrender of collateral serves as a penalty in my model, and the collateral good can be interpreted quite broadly as any action that reduces the utility of the borrower. Thus the model presented above unifies the treatment of collateral and penalties in loan contracts, and highlights their essential similarity.

An alternative model of debt contracts was first proposed by Robert Townsend (1979) and is based on the idea that the lender might be able to verify the borrower's report at a cost. If the borrower reports a small harvest, the lender verifies the amount of the harvest and the borrower makes an agreed-upon payment. When the borrower's harvest is sufficient to make the full payment, no verification takes place. The borrower never cheats, because verification would occur and he would be discovered. The debt contract is optimal in such an environment because it minimizes the frequency of costly verification. The logic is closely parallel to that of the model presented in this article. In both models, default involves deadweight loss—the transfer of collateral to the lender in my model and verification in the costly verification model—and the optimal contract seeks to minimize the cost.

Unfortunately, debt contracts are only optimal in the costly verification model in the presence of an ad hoc restriction on contractual arrangements. For each possible report by the borrower, a contract specifies that the lender either verifies or does not. More generally, a contract could specify that for a given report the lender verifies with some probability, not necessarily equal to zero or one. A *deterministic contract* is one in which verification probabilities are all either zero or one, while a *randomized contract* is one in which some verification probabilities are between zero and one. In the costly verification model, debt contracts are optimal only when attention is restricted to deterministic

contracts. Agents in the model can usually improve upon the debt contract with a randomized contract, and when randomized contracts are allowed the optimal contract does not, in general, resemble debt. The reason is that when verification occurs with positive probability, payments can be contingent. Verifying with small probabilities over a wide range of harvest outcomes can provide sufficient incentives and allow improved risk-sharing, while incurring less verification costs on average.<sup>28</sup>

One might think that randomized economic arrangements are unrealistic, and that there must be some as yet undiscovered reason why such arrangements are undesirable, but the possibility of randomization must be taken seriously in this context. Many financial arrangements actually do involve randomized audits, especially when one firm acts as an agent for another and has the opportunity to hide resources. The models presented above do not rely on a restriction to deterministic arrangements.<sup>29</sup>

Michael Jensen and William Meckling (1976) observed that because debt contracts force the borrower to bear all of the risk, he has more incentive than he would under a risk-sharing arrangement to take costly, private, *ex ante* actions that affect his return. This has led some to suggest that perhaps debt is selected over other feasible contingent arrangements because it provides superior incentives to the borrower to take appropriate *ex ante* actions (see Innes, 1990). Unfortunately, if one assumes that the return is freely observable by the lender *ex post*, then the debt contract is optimal only for very special assumptions about preferences and technology, and under strong restrictions on available contracts.<sup>30</sup> If instead one assumes that the return is unobservable, then, as in Section II above, risk-free debt contracts are optimal, independent of the *ex ante* action choice.

Two recent papers, by Oliver Hart and John Moore (1989) and by Charles Kahn and Gur Huberman

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<sup>28</sup> Townsend (1979) recognized this fact, and subsequent research has shown that it is robust. See, for example, Townsend (1988).

<sup>29</sup> No verification is allowed in the models in this paper, as if verification is prohibitively costly, so the issue of randomized verification does not arise. A distinct but related issue concerns randomized payment schedules, which for the same reasons can in some cases improve upon deterministic arrangements. One can easily show, however, that randomized arrangements are never needed in the models above.

<sup>30</sup> The optimality of the debt contract in Innes (1990) requires risk neutrality and restrictions on probability distributions and utilities such that the "monotone likelihood ratio property" holds and effort choice is unique. In addition, only nondecreasing payment schedules are allowed.

(1989), focus on renegotiation in debt contracts. To motivate debt contracts as an optimal arrangement, they assume that the borrower's resources are observed by both the borrower and the lender but are not verifiable by a third party such as a court, and thus "enforceable" contracts cannot be made contingent. One could object by noting that courts often ascertain litigants' wealth, and often enforce highly contingent contracts such as partnership agreements.

Although a wide range of literature examines the effects of debt contracts or the choice between debt and some other particular contract, the form of the contracts available to agents is generally taken as given. Thus this literature often has little to say about why contracts are limited to particular forms.

## VI. SOME POLICY IMPLICATIONS, BRIEFLY NOTED

Recommended public policies toward credit markets are often predicated on models in which debt contracts play a prominent role, and so a model that explains debt contracts might have novel policy implications. What novel prescriptions for government credit policy might be suggested by the model described here? A complete answer is beyond the scope of the paper and is the subject of continuing research, but some tentative conclusions are possible.

Many policy prescriptions are sensitive to the assumption that capital markets are "perfect," meaning that people can borrow or lend as much as they like on the same terms. For example, the Ricardian Equivalence Theorem revived by Barro (1974), which states that under certain conditions government debt policy is irrelevant, depends critically, as Barro noted, on perfect capital markets. In the model I presented above, the capital market imperfection is derived endogenously from informational constraints, but a blanket endorsement of policy prescriptions that depend on capital market imperfections seems unwarranted. Rather, one needs to assess how the informational imperfection affects the policymaker's ability to improve on private arrangements; in some cases the policymaker may be as sharply constrained as private agents.

One category of potentially useful measures might be termed "collateral enhancement." I showed above how the quantity of collateral available to the borrower could sharply constrain the loan contract. Under current U.S. law, there are limits to the collateral a consumer can offer; one cannot offer to a

prospective lender one's imprisonment for nonpayment of a debt, for example. Moreover, under the "fresh start" provision of the Bankruptcy Act one cannot waive the right to discharge unsatisfied debts in bankruptcy. Consumers presumably could obtain more credit if they could offer to be imprisoned or could waive the right to discharge a debt, because such stiff penalties would make larger repayments credible. Interestingly, debts arising from government guaranteed educational loans are not dischargeable in bankruptcy during the first five years following the date that the first payment becomes due [11 U.S.C. § 523(a)(8)]. The claim in bankruptcy represented by a guaranteed student loan is thus more burdensome than a dischargeable claim, and presumably allows improved loan terms for the borrower or the lender. The analysis of the present paper suggests that allowing borrowers to waive the right to discharge debts in bankruptcy might improve the functioning of credit markets. However, there might be compelling countervailing reasons for the prohibition of waivers of discharge that are not taken into account by the models presented above; see Jackson (1985) for a discussion.

Another possible rationale for government credit policy concerns the valuation of collateral. Suppose the borrower in the model described above faces two possible lenders who differ only in the value they place on the borrower's chattels. The optimal arrangement is for the borrower to obtain a loan from the lender who values the collateral good most highly, since this will provide the borrower with a lower interest rate. If, for some reason, a borrower's collateral has a social value that is higher than its private value to lenders, due to an externality of some type, then direct government lending or government loan guarantees might be warranted. To justify such policies one would have to argue that the public valuation of the collateral is higher than its highest private valuation, and one can legitimately question whether this condition holds for many current loan-guarantee programs.<sup>31</sup>

Beyond these simple observations, little is known as yet about the policy implications of models like

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<sup>31</sup> William Gale (1990) has described credit market models in which borrowers have private information beforehand about the riskiness of their future resources. He shows that in such models government loan guarantees targeted to high-risk borrowers can improve efficiency. In his 1991 paper he applies this model to existing federal credit programs and calculates that policy is likely to be quite inefficient. Debt contracts are assumed in his model, rather than derived endogenously, and it is unclear how the analysis would be affected by the latter.

the one presented above. On one hand, it is difficult to imagine policy interventions that make some people better off without making anyone worse off in this type of model, other than the two just mentioned. In particular, based on this model alone there does not seem to be an efficiency rationale for loan subsidies or more general interest rate manipulations. Such policies could have important consequences for the distribution of welfare, of course, but would have to be evaluated by criteria other than Pareto optimality. On the other hand, the model leaves out some features, such as *ex ante* private information, that some economists claim rationalize credit market intervention.<sup>32</sup> The claims usually pertain to markets that are dominated by the use of debt contracts, and yet the claims are based on models in which debt contracts are imposed, rather than derived as optimal. It is not yet known whether the conclusions of those models would survive if they were modified so that debt contracts arise endogenously, as in the model I have presented here.

## VII. CONCLUDING REMARKS

So why is there debt with occasional default? My answer has two components. First, borrowers can fool lenders about their circumstances, so having the borrower share risk with the lender gives the borrower an irresistible temptation to cheat. Thus payment schedules are noncontingent in such situations. Second, if the borrower is incapable of making the stipulated payment, the lender has recourse either to explicit or to implicit collateral. Such recourse is sufficient to dissuade the borrower from withholding payment.

It is worth pointing out that important puzzles concerning debt contracts remain unsolved. The sole source of uncertainty here is the borrower's future resources, and it seems quite reasonable to assume that borrowers can hide resources from lenders. But much of the uncertainty that faces borrowers and lenders concerns widely observed events about which neither is able to lie. Examples include publicly known prices and published economic data. The theory of Section I predicts that repayment contracts ought to be contingent on many publicly observed events. For example, officially published data on average prices of consumer goods are widely available, and are closely correlated with the real value

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<sup>32</sup> See Stiglitz (1988).

of the monetary payments made by debtors to creditors. Why are so few debt contracts indexed for inflation?

A second puzzle is perhaps related to the first. A vast literature in monetary economics is motivated by the observation that money is widely used in spot exchanges for goods. And yet, almost all debt contracts are repaid in money as well. Perhaps the widespread use of money to settle debts is an

equally important puzzle. The model described above does not have an explicit role for money, but the logic of the model suggests a rudimentary answer. The borrower might have some sort of advantage relative to the lender in selling the crop, and therefore returns money rather than the crop itself to the lender. This answer is rudimentary because it does not explain just why the borrower would have such an advantage. Evidently, much remains to be learned about financial arrangements such as debt contracts.

## APPENDIX A

### A Derivation of the Incentive Feasibility Constraints

In this appendix I show that any pattern of consumptions by the two agents that can be achieved by any arbitrary contract, possibly giving the borrower an incentive to hide the harvest, can be achieved by a contract that satisfies the incentive feasibility constraints and does not give the borrower an incentive to hide any of the harvest. Therefore, a given consumption pattern can be achieved if and only if it results from a contract that satisfies the incentive feasibility constraints. The argument is presented in the model of Section II, but can easily be extended to cover the model of Section IV.

To begin, take as given an arbitrary contract  $\{q, y_1, y_2, \dots, y_N\}$ , that satisfies the resource feasibility constraints (13)-(18), and consider a given harvest  $\theta_n$ , where  $n > 1$ . The borrower can display any harvest  $\theta_m$ , where  $m$  can equal  $1, 2, \dots, n$ , and  $m$  is chosen to maximize  $u_B(\theta_n - y_m)$ . Define  $y_n^*$  as the payment the borrower actually makes after optimally choosing a utility maximizing display. It does not matter if the utility maximizing display is not unique, because the utility maximizing payment is always unique. The payment  $y_n^*$  clearly satisfies  $u_B(\theta_n - y_n^*) \geq u_B(\theta_n - y_m)$  for  $m = 1, 2, \dots, n$ .

Now consider an arbitrary harvest  $\theta_p < \theta_n$ , and define  $y_p^*$  analogously as the utility maximizing pay-

ment for the harvest  $\theta_p$ . Clearly,  $y_p^* = y_m$  for some  $m$  in the set  $\{1, 2, \dots, p\}$ . Since  $p < n$ , it is also true that  $y_p^* = y_m$  for some  $m$  in the set  $\{1, 2, \dots, n\}$ ; in other words, the utility maximizing payment for the harvest  $\theta_p$  is a payment that could have been made for the harvest  $\theta_n$ . As a result, the payment  $y_p^*$  can provide no more utility when the harvest is  $\theta_n$  than the utility maximizing payment  $y_n^*$ . Therefore,  $u_B(\theta_n - y_n^*) \geq u_B(\theta_n - y_p^*)$ . Since both  $n$  and  $p$  are arbitrary, this condition holds for  $n = 2, \dots, N$ , and for  $p = 1, \dots, n - 1$ . These are exactly the incentive feasibility constraints (19).

I have defined a set of payments  $\{y_1^*, y_2^*, \dots, y_N^*\}$ , the utility maximizing payments chosen by the borrower when the contract is  $\{q, y_1, y_2, \dots, y_N\}$ . Now define a new contract  $\{q, y_1^*, y_2^*, \dots, y_N^*\}$ , by substituting the actual payments for the originally stipulated payments. This new contract satisfies the incentive feasibility constraints, and thus does not provide any positive incentive to hide harvest. The new contract results in consumption patterns for both the borrower and the lender that are identical to those resulting from the original contract. Because the original contract is arbitrary, I have shown that any consumption patterns that can be achieved can also be achieved under a contract that provides no incentive to hide the harvest.

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