Working Paper 81-2

DISCOUNT WINDOW BORROWING, MONETARY POLICY, AND THE POST-OCTOBER 6, 1979 FEDERAL RESERVE OPERATING PROCEDURE*

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Revised September 1, 1981

*The author is Research Officer and Economist at the Federal Reserve Bank of Richmond. Discussions with Tim Cook have been particularly valuable. Thanks are also due to John Boschen, Bob King, Rich Lang, Ben McCallum, John McDermott, Tom Mead, and Jerome Stein. The views here are solely those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

Introduction

This paper is intended to be an analysis of discount window borrowing as it relates to more general issues of monetary control. The topic deserves a new look because of the central role of discount window borrowing under the post-October 6, 1979 "reserve targeting" operating strategy.

The analytical core of the paper is the derivation of a demand for borrowing function based on profit-maximizing bank behavior. It is shown that a basic feature of the nonprice rationing mechanism at the discount window causes the banks to solve a dynamic optimization problem in deciding on optimal current discount window borrowing. The solution of this problem for the structure of the borrowing demand function has implications for the conduct of monetary policy. These are brought out in the latter sections of the paper.

Nonprice Rationing at the Discount Window

If there were no nonprice rationing at the discount window, the Federal funds rate would never rise above the discount rate, because a bank would never pay more for reserves than it would have to pay at the discount window. Since 1965, the Federal funds rate has, on numerous occasions, risen above the discount rate. On two occasions it has remained above the discount rate for roughly two years running. This indicates that an effective form of nonprice rationing is being administered at the discount window.

The basis for this nonprice rationing is spelled out in Regulation A. The condition under which a bank is entitled to "adjustment credit" at the discount window is stated in Regulation A as follows:

> Federal Reserve credit is available on a short-term basis to a depository institution under such rules as may be prescribed to assist the institution, to the extent appropriate, in meeting temporary requirements for funds, or to cushion more persistent outflows of funds pending an orderly adjustment of the institution's assets and liabilities.[1]

The sense of this statement of privilege is that appropriate borrowing should be temporary. The intention is clearly that discount officers and committees should use duration as a fair objective measure of appropriateness, with appropriateness negatively related to duration. This intention is also clearly expressed in the Report of the System Committee on the Discount and Discount Rate Mechanism (1954), where it is suggested that "the duration of borrowing [is] to be used to establish a rebuttable presumption that borrowing [is] for an inappropriate purpose."[2]

Reserve Banks have set up rules for administering their discount windows based on duration as a measure of appropriateness. A common feature of these rules is a set of restrictions on the number of weeks a bank can be "in the window" during a specified period. Such "frequency" restrictions exist for 13-, 26-, and 52-week periods.[3] In general, the rules seem to be designed to apply progressively heavier pressure to banks the more lengthy a given "stay in the window."

From the point of view of modeling borrowing behavior, there are many unsatisfactory features of the nonprice rationing mechanism in force at the Reserve Banks. The nonprice costs imposed on banks are difficult to identify. The frequency guidelines are difficult to incorporate in an operational empirical model of the demand for borrowing. And the lack of uniformity in discount window administration across Reserve Banks contributes to the difficulty in modeling aggregate borrowing. However, these problems are ignored in this paper in order to fully concentrate on the effect of "progressive pressure" in influencing the structure of the bank borrowing function.

A Model of the Bank Borrowing Decision[4]

Major aspects of the bank borrowing decision are described in this section. Banks are assumed to behave rationally and to maximize profits. Because of the mechanism of nonprice rationing at the discount window, banks turn out to care about the past and future in deciding how much to currently borrow. In other words, they face a "dynamic optimization problem." In the following two sections, a simple formal solution to this optimization problem is derived and some characteristics of the bank borrowing function are discussed.

Even a simple version of this dynamic optimization problem is fairly complex. Consequently, a simple form of discount window nonprice rationing mechanism is assumed for this discussion. First, the marginal perceived effective cost of borrowing is assumed to rise with borrowing in the current period. Second, given the current level of borrowing, the marginal perceived effective cost of borrowing is assumed to be positively related to the level of borrowing last period.

A simple cost of borrowing function that embodies the two essential features of the nonprice rationing mechanism described above may be written

(1) $$C_t = (c_1 B_{t-1} + 1)\frac{c_0}{2}[(B_t + 1)^2 - 1] + d_t B_t$$

where d_t = the period t discount rate

$$c_0, c_1 > 0, \; B_t, B_{t-1} \geq 0$$

This cost function is graphed in Figure 1 for a given current discount rate and lagged level of borrowing. The functional form has a number of reasonable characteristics. First, the cost is zero when current borrowing, B_t, is zero, i.e., the function passes through the origin. Second, the marginal cost of current borrowing is positive and rises with the level of current borrowing, i.e., the function is convex downward. Third, at any level of current borrowing, B_t, the marginal cost of borrowing is positively related to the level of lagged borrowing, B_{t-1}, i.e., roughly speaking the function rotates counterclockwise with a rise in B_{t-1}. Fourth, the marginal cost of current borrowing rises and falls one-for-one with the current discount rate, i.e., again roughly speaking the curve rotates counterclockwise with a rise in the current discount rate.

A bank borrows in the current period (period t) until the marginal cost of an additional dollar of current borrowing just equals the marginal benefit. The first component of the current cost of an additional dollar of discount window borrowing is found by differentiating the cost function with respect to B_t, yielding

$$(2) \qquad (c_1 B_{t-1} + 1) c_0 (B_t + 1) + d_t$$

This component of the current marginal cost rises with B_t, d_t, and B_{t-1}.

In rationally assessing the cost of additional current borrowing, a bank must also consider that current borrowing raises the marginal cost of borrowing in the future through the nonprice rationing mechanism. In particular, the bank must include in its marginal cost of current borrowing the present discounted value of next period's increased marginal cost of borrowing due to an extra dollar of current borrowing. This second component of the current cost of an additional dollar of discount window borrowing is found by updating the B elements in the cost function one period and differentiating with respect to B_t, yielding

$$(3) \qquad b \frac{c_1 c_0}{2} [(B_{t+1} + 1)^2 - 1]$$

where $b \equiv$ a constant rate of time discount[5]

Note that this component of the current marginal cost is zero if next period's borrowing, B_{t+1}, turns out to be zero. But current (B_t) borrowing raises the marginal cost of borrowing next period for any positive, B_{t+1}, borrowing level next period.

The inclusive marginal cost of B_t borrowing is the sum of (2) and (3)

$$(4) \qquad (c_1B_{t-1}) + 1)c_0(B_t + 1) + d_t + b\frac{c_1c_0}{2}[(B_{t+1} + 1)^2 - 1]$$

The current marginal benefit of an extra unit of discount window borrowing is the opportunity cost of obtaining the funds in the Federal funds market, i.e., the current Federal funds rate, f_t.

A bank maximizes profits (the net benefit from borrowing at the discount window) by raising B_t to the point where the inclusive marginal cost of B_t borrowing just equals the marginal opportunity cost.[6] In other words, profit maximizing behavior leads a bank to set B_t so that expression (4) equals the Federal funds rate. This condition, known as the Euler equation, is necessary for B_t to be an optimum. The Euler equation for the bank borrowing problem is

$$(5) \qquad (c_1B_{t-1} + 1)c_0(B_t + 1) + d_t + b\frac{c_1c_0}{2}[(B_{t+1} + 1)^2 - 1] = f_t$$

where $f_t \equiv$ the period t Federal funds rate[7]

As is seen above, the Euler equation is nonlinear in the B's. Since the Euler equation is extremely difficult to solve in its nonlinear form, we shall work with a linearized approximation to the Euler equation.[8] The linearized Euler equation is

(6) $$B_{t+1} + \phi B_t + \frac{1}{b} B_{t-1} = a + hS_t$$

where $S_t \equiv f_t - d_t$

$$\phi \equiv \frac{1}{bc_1}$$

$$a \equiv - \left[\frac{c_0 + B^L(c_0 c_1 + c_0 + bc_1 c_0)}{bc_1 c_0} \right]$$

$$h \equiv 1/bc_1 c_0$$

$B^L \equiv$ long run "normal" borrowing

In technical terms, the linearized Euler equation is a second order difference equation in borrowing, B, that is forced by the spread between the Federal funds rate and the discount rate, S.

To understand how this Euler equation works, recall that a bank's decision on how much to borrow in period t (B_t) depends on historically determined lagged borrowing (B_{t-1}) and on next period's borrowing (B_{t+1}). Now next period's borrowing will be chosen by a bank to satisfy a similar updated Euler equation which embodies the current borrowing choice as a predetermined condition; and each successive period's borrowing will be chosen similarly. In other words, each period's optimal borrowing choice depends on planned borrowing for all future periods. A rational bank must choose a current level of borrowing simultaneously with a desired borrowing path for the entire future. For the path to maximize profits, planned borrowing must satisfy successively updated Euler conditions all along the path. This means that in order to choose a current (B_t) level of borrowing, the bank must solve the Euler equation as a second order difference equation in borrowing (B). Formally, the bank must find a solution to the difference equation

(7) $$B_{t+k+1} + \phi B_{t+k} + \frac{1}{b} B_{t+k-1} = Z_{t+k} \qquad \text{for } k = 0,1,2,3,\ldots$$

where $Z_t \equiv a + hS_t$

The entire optimal borrowing plan chosen in period t can be concisely written

(8) $$B_{t+k} = \lambda_1 B_{t+k-1} - \sum_{i=1}^{\infty}\left(\frac{1}{\lambda_2}\right)^i Z_{t+k-1+i} \qquad k = 0,1,2,3,\ldots$$

where $-1 < \lambda_1 < 0$ and $\lambda_2 < -1$

$Z_t \equiv a + hS_t;\ a < 0,\ h > 0$

$S_t \equiv f_t - d_t$ [9, 10]

<u>Expectations and Discount Window Borrowing</u>

If a bank had perfect foresight about the spread between the funds rate and the discount rate, then equation (8) would in fact be an optimal demand schedule for borrowing from the discount window. It would give current (period t) borrowing as a function of lagged borrowing and current and future spreads. However, a bank does not know what future spreads will be with certainty. A bank must forecast future spreads based on information it has in period t. This point is formalized by rewriting the period t demand for borrowing as

(9) $$B_t = \lambda_1 B_{t-1} - a\left[\frac{1}{\lambda_2} + \sum_{i=2}^{\infty}\left(\frac{1}{\lambda_2}\right)^i\right] - \frac{h}{\lambda_2} S_t - h\sum_{i=2}^{\infty}\left(\frac{1}{\lambda_2}\right)^i \underset{t}{E}[S_{t-1+i}]$$

where $\underset{t}{E}[\ \ ] \equiv$ the mathematical expectation conditional on information available in period t

In (9), the currently observable S_t variable has been separated from the future spreads which must be forecast. The $\underset{t}{E}[S_{t-1+i}]$ term indicates an optimal forecast of future spreads based on information available in period t.

Equation (9) is still not a decision rule since optimal current borrowing B_t is not written as a function of variables that have been observed as of period t. In order to derive a decision rule, it is necessary to specify a process generating the spread. For illustrative purposes, assume the spread follows a first-order autoregressive process. In particular, suppose the S process may be written

(10) $$S_t = \alpha S_{t-1} + \varepsilon_t$$

where $0 < \alpha < 1$

$\varepsilon_t \equiv$ a random disturbance term

This process says that if S is displaced from zero, it will return to zero asymptotically, falling back toward zero by a proportion α each period. This means that we could forecast future movement of S as

(11) $$\underset{t}{E}[S_{t+k}] = \alpha^k S_t$$

In other words, under the assumed simple process determining the spread, the only useful variable in forecasting the spread is the current spread.

Substituting from (11) into (9) yields the following decision rule for period t discount window borrowing

$$(12) \qquad B_t = \lambda_1 B_{t-1} - a\left[\frac{1}{\lambda_2} + \sum_{i=2}^{\infty}\left(\frac{1}{\lambda_2}\right)^i\right] - h\left[\frac{1}{\lambda_2} + \sum_{i=2}^{\infty}\left(\frac{1}{\lambda_2}\right)^i \alpha^{i-1}\right] S_t$$

where $B_t \geq 0$

$-1 < \lambda_1 < 0$

$\lambda_2 < -1$

$0 < \alpha < 1$

$0 < h$

$0 > a$

Equation (12) is a decision rule or an operational demand schedule for discount window borrowing since it shows the optimal level of period t borrowing as a function of variables in a bank's period t information set.

Qualitative Features of the Demand Schedule for Discount Window Borrowing

Ignore aggregation problems and consider the aggregate demand for borrowing schedule to have the same form as (12). The demand for borrowing has two main features. First, current borrowing demand is negatively related to lagged borrowing. This is a consequence of a nonprice rationing mechanism at the discount window which discourages continuous borrowing by raising the marginal perceived cost of current borrowing as borrowing in the recent past rises. The particularly simple way in which lagged borrowing enters is due entirely to the assumed simplicity of the cost of borrowing function used here. In practice, the nonprice rationing costs imposed on banks to discourage continuous borrowing are much more complicated and difficult to explicitly identify; so the relationship between current and lagged borrowing is in practice difficult to specify.

Second, current borrowing demand is related to the current spread between the funds rate and the discount rate, S_t. The current spread affects current borrowings through the two channels associated with the two terms in the coefficient on S_t in (12). A higher current spread works through the "$-\frac{h}{\lambda_2}$" term to raise current borrowing demand. A higher spread works through this channel by raising the net marginal benefit to current borrowing.

The "$-h\sum_{i=2}^{\infty}\left(\frac{1}{\lambda_2}\right)^i \alpha^{i-1}$" term captures the effect of the current spread as a predictor of future spreads. "α" plays an important role here. A look at (11) suggests that α can be thought of as measuring a kind of speed of adjustment or steepness of descent of the spread toward zero after a disturbance. A zero α means there is no inertia in the spread; it is expected to move immediately back to zero next period. An α close to one indicates a great deal of inertia in the spread, since it means the spread will return to zero extremely slowly.

The term "$\sum_{i=2}^{\infty}\left(\frac{1}{\lambda_2}\right)^i \alpha^{i-1}$" can be written as $\frac{\alpha}{(\lambda_2 - \alpha)\lambda_2} < 0$. The net coefficient on the spread may be written as $\frac{-h}{\lambda_2 - \alpha} > 0$. The derivative of the net coefficient with respect to α is negative. This means, for example, that a given positive displacement of the spread from zero causes borrowing demand to rise more the faster the expected movement of the spread back toward zero, i.e., the smaller is α.

To see why, compare "corner" cases where α is zero, i.e., the displacement is purely transitory lasting just one period, and α is near one, i.e., the displacement is more persistent. When the high spread is purely transitory, banks expect net benefits to borrowing and actual borrowing to be low tomorrow. On the other hand, if the high spread is more persistent, banks expect the net benefits and actual borrowing to remain high tomorrow. Since the marginal cost component of current borrowing stemming from future planned borrowing is lower in the transitory case, the level of current borrowing at which marginal cost just equals marginal benefit will be reached at a higher level of current borrowing when the spread displacement is transitory.

A particularly simple process for the spread has been assumed to keep this example simple. If the process determining the evolution of the spread were more complicated, the future spread forecast embodied in the borrowing decision rule could be a more complicated function of the current spread. It might also involve lagged spreads and other variables if they play a role in explaining the evolution of the spread.

It bears emphasizing that banks care about lagged borrowing and future spreads in deciding how much to borrow currently because the nonprice rationing mechanism they face at the discount window introduces "progressive pressure" into the cost of discount borrowing, making longer duration borrowing more costly.[11] If appropriate discount window borrowing ought to be temporary, it is reasonable to raise the administrative pressure on borrowing banks with the duration of their borrowing. Duration provides an objective measure of appropriateness and "progressive pressure" provides an automatic inducement for banks to wean themselves from the discount window after an "emergency."

Unfortunately, although "progressive pressure" based on duration is a useful feature of the nonprice rationing mechanism, it introduces a dynamic element into the bank discount window borrowing decision problem. A bank is forced to take account of past borrowing and forecast future spreads to decide how much to currently borrow. Not only does "progressive pressure" make the bank's borrowing decision more difficult, it makes the econometric task of specifying and estimating an aggregate borrowing function for use in monetary control more difficult as well. This last point is discussed in more detail below.

The Borrowing Function and Fed Policy

The Federal Reserve plays a major role in the evolution of the spread between the discount rate and the funds rate. Movements in the spread are heavily influenced by Fed policy. This means that rational bank forecasts of the spread must be based on an understanding of Fed policy toward the spread. Since future expected spread movements influence current borrowing demand, not only the size of coefficients but also the form of the borrowing function depends on Fed policy toward the spread.

The process on the spread assumed in equation (10) may be thought of as policy induced, i.e., a policy rule. In this view, unanticipated changes in the spread might be induced by the Fed as part of its program for monetary control. Thereafter, the spread's return to a normal level might be smoothed. The size of α would be indicative of the degree of smoothing, α close to unity indicating a greater degree of smoothing. The analysis above has shown that the sensitivity of current borrowing to the current spread is smaller the more smoothing or persistence there is in deviations of the spread from a normal long run value. The current borrowing-spread sensitivity depends on Fed policy because the current spread appears in the borrowing function as a predictor of future spreads.

The above analysis has implications for estimation of the borrowing function and its use in implementing monetary policy. First, in order to decide how much to borrow from the discount window, banks must try to understand the Fed policy effect on the spread. Banks must understand Fed policy toward the spread whether it is by direct manipulation of the Federal funds and discount rates or induced through reserve management. This means that if the Fed wants to estimate and utilize a borrowing function, it should make its policy intentions toward the spread as simple and understandable as possible. Any deliberate obfuscation of policy toward the spread makes it more difficult for banks to forecast future spreads. This can only make specification and estimation of the borrowing function more difficult.

Second, if the Fed's implicit policy toward the spread depends on other variables besides the spread such as recent levels of borrowing, the money supply, or the level of interest rates, the policy links between these other variables and the spread should be announced explicitly. This would let banks know what variables would help them forecast the spread and, at the same time, let the Fed know what variables would be in the bank borrowing function. Such a procedure would enable the Fed to more accurately specify the borrowing function and, by implication, to estimate and utilize it more adequately. In short, the Fed, if it wishes to forecast bank borrowing accurately and systematically, must take banks and the public into its confidence.[12]

Third, if the Fed's implicit policy toward the spread changes, the form of the borrowing function will change as well. In particular, the sensitivity of current borrowing to the current spread will depend on the implicit policy toward the spread. This means that when the Fed makes a policy change, altering the time series characteristics of the spread, the borrowing function estimated over historical data from the previous policy regime will not likely remain adequate for use in forecasting bank borrowing under the new policy regime. If the policy change occurs by a sequence of decisions following no clearly discussed or preannounced pattern, it will become known to banks only gradually.[13] This will complicate adaptation of bank borrowing behavior to the new policy environment and further complicate the task of forecasting that behavior.

Discount Window Borrowing Under Post-October 6, 1979 Reserve Targeting

The above analysis is relevant to reserve targeting as it has been carried out by the Federal Reserve since October 6, 1979.[14] The method of discount window administration in force does not allow the Fed to exercise direct control over total reserves. The Fed directly controls nonborrowed reserves only. Under this setup the Fed manipulates nonborrowed reserves to influence the funds rate and other short-term rates of interest and thereby affect the quantity of money demanded. The FOMC chooses an initial intermeeting path for nonborrowed reserves designed to induce the banking system to obtain a target volume of borrowed reserves from the discount window. The target volume of discount window borrowing is chosen, for a given discount rate, to produce the desired level of the funds rate and other short-term rates of interest and ultimately the desired effect on the money supply. This operating procedure depends critically on the link between discount window borrowing and the spread between the discount rate and the funds rate, the link provided by the demand function for discount window borrowing.[15]

The Fed is having difficulty relying on the demand function for discount window borrowing under the new operating procedure. The difficulty is well described in the following account of the means by which the FOMC chooses the initial intermeeting target for borrowed reserves:

> Typically, the Committee [FOMC] has chosen levels [an interim borrowing objective] close to the recently prevailing average--though the level chosen on October 6 was shaded higher to impose some additional initial restraint. Ideally, the assumed initial borrowing level should be such that the resultant mix of borrowed and nonborrowed reserves would tend to encourage bank behavior consistent with the emergence of desired required reserves, and hence of desired monetary growth. In practice, [for a given spread between the discount rate and the funds rate] there seem to be significant short-term variations in the willingness or desire of banks to turn to the discount window. This adds to the difficulty of choosing an appropriate level for path construction purposes, and may necessitate adjustment in a path in response to changes in bank attitudes toward the discount window.[16]

This account indicates that the demand for discount window borrowing appears to the Fed as relatively volatile and difficult to predict.[17] The difficulty the Fed is having in pinning down the borrowing function may be simply due to the complexity and lack of uniformity of the nonprice rationing mechanism in force at the discount window or to the irregular use of the discount rate surcharge. However, the analysis of this paper provides another explanation for the apparent unreliability of the relation between borrowing demand and the spread between the discount rate and the funds rate. First, the analysis shows that as long as "progressive pressure" is employed as a means of nonprice rationing, borrowing demand depends in a potentially complicated way on lagged levels of borrowing and on expected future spreads. Second, unwillingness of the Fed to publicly specify its policy intentions toward the spread makes it difficult for banks to form expectations about future spreads. Both of these factors have likely contributed to the apparent difficulty the Fed has experienced in specifying, estimating, and utilizing a reliable borrowing function in monetary control.

Conclusion

A demand schedule for discount window borrowing based on profit maximizing bank behavior has been derived. The form of the borrowing function has been shown to depend on a feature of the nonprice rationing mechanism at the discount window that introduces "progressive pressure" into the cost of discount window borrowing, making longer duration borrowing more costly. "Progressive pressure" based on duration makes lagged borrowing and future spreads between the discount rate and the Federal funds rate relevant to the current borrowing decision. Since movements in the spread are heavily influenced by Fed policy, and since expected spread movements influence borrowing demand, both the size of the coefficients in the borrowing function as well as the form of the function itself depend on Fed policy toward the spread.

This analysis is relevant to reserve targeting as it has been carried out by the Fed since October 6, 1979. Under this operating procedure, the demand function for discount window borrowing has provided the critical link by which nonborrowed reserve control has been made to affect short-term rates of interest and ultimately the money supply. Unfortunately, the relation between discount window borrowing and the spread between the discount rate and the Federal funds rate has appeared to the Fed as volatile and difficult to predict. This analysis suggests that this may be the case because borrowing also depends in a potentially complicated way on lagged borrowing and expected future spreads. In addition, unwillingness of the Fed to publicly specify its policy intentions toward the spread makes it difficult for banks to form expectations about future spreads. Both of these factors have likely contributed to the difficulty the Fed has experienced in specifying, estimating, and utilizing a reliable borrowing function in monetary control.

FOOTNOTES

[1] Federal Reserve Board Rules and Regulations, Regulation A (as adopted effective September 1, 1980), Sec. 201.3, Par. a. Regulation A also entitles depository institutions to get seasonal and other so-called extended credit. Such borrowing is ignored throughout this paper.

A good discussion of discount window administration is found in Board of Governors of the Federal Reserve System, "Operation of the Federal Reserve Discount Window Under the Monetary Control Act of 1980," September 9, 1980.

[2] Board of Governors of the Federal Reserve System, Reappraisal of the Federal Reserve Discount Mechanism, vols. 1-3 (August 1971), p. 41.

[3] See, for example, Federal Reserve Bank of San Francisco, "Guidelines for the Administration of Short-Term Adjustment Credit in the Twelfth Federal Reserve District" (effective April 11, 1977) or Board of Governors of the Federal Reserve System, "Operation of the Federal Reserve Discount Window Under the Monetary Control Act of 1980," September 9, 1980.

[4] The Depository Institutions Deregulation and Monetary Control Act of 1980 requires all depository institutions to maintain reserves with the Federal Reserve. Included in the term depository institution are banks (whether or not they are members of the Federal Reserve System), savings banks, mutual savings banks, savings and loan associations, and credit unions. The act entitles any depository institution in which transaction accounts are held to the same discount and borrowing privileges at the Federal Reserve Banks as Federal Reserve System members. Consequently, depository institutions other than banks have access to the discount window. Since this is the case, this paper should refer to "the depository institution borrowing decision" rather than "the bank borrowing decision." However, in the interest of simplicity the word bank is retained throughout on the understanding that it stands for all depository institutions having access to the discount window.

[5] The rate of time discount may not be constant. But allowing for this greatly complicates the bank's maximization problem and obscures the main implications to be drawn from its solution without contributing any essential new insights.

[6] A formal statement of the bank's maximization problem is given in Appendix A.

[7] With no progressive pressure, c_1 equals zero. In this case, the marginal cost of period t borrowing involves B_t and d_t; neither the past nor the future is relevant to the bank borrowing decision.

[8] The linearization procedure is outlined in Appendix B.

[9] The solution procedure is discussed in Sargent [1979], Chapters 9 and 14. Without knowledge of the actual relative sizes of c_1 and b, there is no way of sufficiently restricting the characteristic roots of this difference equation. However, the illustrative purpose of the solution in this paper is well served by assuming that c_1 and b are such as to make $-1 < \lambda_1 < 0$ and $\lambda_2 < -1$. A transversality condition is obtained by assuming that if the spread between the discount rate and the funds rate were expected to be constant for all time, then planned borrowing would eventually converge to a constant optimal level.

[10] The optimal borrowing plan has been derived implicitly only for the case where B and f - d are both positive. Although B can never be negative and current borrowing cannot be nonzero unless current f - d is positive, these constraints have not been formally built into the solution.

[11] See Footnote 7.

[12] This statement is paraphrased from Lucas [1976], p. 42. The points made in this section are based on arguments originally advanced at a more general theoretical level in Lucas' paper.

[13] This statement is also paraphrased from Lucas [1976], p. 40.

[14] Technical descriptions of the post-October 6, 1979 operating procedure may be found in "Monetary Policy and Open Market Operations in 1979," pp. 60-64, "Techniques of Monetary Policy," and "The New Federal Reserve Technical Procedures for Controlling Money." Federal Reserve experience with the new operating procedure is discussed in Board of Governors of the Federal Reserve System, Federal Reserve Staff Study, New Monetary Control Procedures.

[15] The Fed has allowed the funds rate to fall below the discount rate for prolonged periods since October 6, 1979. When this happens there is no incentive for banks to borrow at the discount window, and borrowing volume becomes very small. The borrowing function plays no role in the operating procedure in such situations. Whenever the funds rate has been below the

discount rate, the operating strategy has essentially reverted to the pre-October 6, 1979 policy of direct funds rate control.

[16] "Monetary Policy and Open Market Operations in 1979," Federal Reserve Bank of New York Quarterly Review (Summer 1980): 60.

[17] The following comment from "Monetary Policy and Open Market Operations in 1979," p. 63, provides a specific illustration of the difficulty the Fed has had in utilizing the borrowing function:

> The behavior of the Federal funds rate during November and December was somewhat puzzling, as it often did not follow a usual relationship to the volume of discount window borrowings. The Federal funds rate did decline in early November, when borrowing dropped, but then continued to fall through the rest of the month, while borrowings stabilized around $1.8 billion to $1.9 billion. The average funds rate slipped as low as 12 1/2 percent in the final week of the month, compared with about 13 3/4 percent at the start. However, in December, when borrowings declined further, though irregularly, ranging between $1.2 billion and $1.7 billion after the first week, the funds rate jumped back up to around 13 3/4 to 14 percent through December and into January.
>
> Normally, one would not have anticipated a drop in the Federal funds rate in late November when borrowings were steady. Nor would one have expected the rate to rise and then stay up in December as borrowings resumed their decline.

Levin and Meek [1981] and Keir [1981] contain additional evidence of the difficulty the Fed has had in using the borrowing function in monetary control.

APPENDICES

Appendix A

Maximize (at each point in time t) the discounted present value

$$V_t = \sum_{j=0}^{\infty} b^j \left[f_t B_t - (c_1 B_{t-1} + 1)\frac{c_0}{2}[(B_t + 1)^2 - 1] - d_t B_t \right]$$

by choosing a sequence for $\{B_{t+j}\}_{j=0}^{\infty}$ subject to $B_{t-1} = \overline{B}_{t-1}$, a transversality condition and a known sequence $\{f_{t+j} - d_{t+j}\}_{j=0}^{\infty}$.

Appendix B

A linear approximation to the Euler equation is constructed as follows. Define $\Delta X_t \equiv X_t - X^L$, where X^L is a long run value. Then

$$[c_1(B^L + \Delta B_{t-1}) + 1]c_0(B^L + \Delta B_t + 1)$$
$$+ b\frac{c_1 c_0}{2}[(B^L + \Delta B_{t+1} + 1)^2 - 1] = (f - d)^L + \Delta(f_t - d_t)$$

Assume B^L is small and redefine $B_t \equiv \Delta B_t$, so that the linearized Euler equation may be written

$$B_{t+1} + \frac{1}{bc_1} B_t + \frac{1}{b} B_{t-1} = -\left[\frac{c_0 + B^L(c_0 c_1 + c_0 + bc_1 c_0)}{bc_1 c_0}\right]$$
$$+ \frac{1}{bc_1 c_0}(f_t - d_t)$$

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