Working Paper 84-7

INADEQUATE TESTS OF THE RATIONALITY OF EXPECTATIONS

by

Roy H. Webb

October 1984

An earlier version of this paper was presented at the Western Economic Association, Las Vegas, June 24-28, 1984. The author is indebted to Bill Lupoletti for his comments and research assistance, and to Don Graling at Chase Econometrics and Rob Wescott at Wharton Econometric Forecasting Associates for their help in assembling time series of forecasts. Views expressed in this paper are those of the author and should not be attributed to the individuals named above nor to the Federal Reserve Bank of Richmond or the Federal Reserve System.

ABSTRACT: INADEQUATE TESTS OF THE RATIONALITY OF EXPECTATIONS

In several recent articles, authors have regressed actual values of macroeconomic aggregates on predicted values and claimed that they were testing the rationality of expectations. This paper interprets those regressions as testing a joint hypothesis of imperfect information and rational expectations. An empirical method is proposed to separate the components of the joint hypothesis. Predictions from two major forecasting services are examined, and results are found that are consistent with rational expectations but inconsistent with the joint hypothesis. It is therefore argued that many purported tests of rational expectations are inadequate.

Inadequate Tests of the Rationality of Expectations

It has become almost commonplace for economists to regress actual values of macroeconomic aggregates on predicted values, and then to interpret the results as tests of the rationality of expectations. Prominent examples include McNees (1978), Friedman (1980), Figlewski and Wachtel (1981), Brown and Maital (1981), Gramlich (1983) and Urich and Wachtel (1984). Many different sources of expectations have been studied, including forecasts from leading consulting services, the Livingston survey of economists, the Commerce Department survey of businesses, the Michigan survey of consumers, and surveys of financial market participants. Variables studied have included prices, interest rates, GNP and its components, and the monetary aggregates.

Such studies have often found results that were interpreted as being inconsistent with the rational expectations hypothesis. It is argued below, however, that what the authors actually tested was the joint hypothesis that (1) expectations were rational and (2) individuals employed a correct economic model. An empirical test is proposed to distinguish the two hypotheses. Based on data from major forecasting services, results are first shown that are inconsistent with the joint hypothesis of rational expectations and a correct model. Further results are shown to be consistent with rational expectations; therefore the assumption that forecasters used a correct model is suspect. Consequently, the results in this paper illustrate the inadequacy of many purported tests of the rational expectations hypothesis.

An Inadequate Test of Rational Expectations Consider the equation

$$A_t = \alpha + \beta P_{t-k} + \varepsilon_t \qquad (1)$$

where A_t is the actual value of a variable at time t, P_{t-k} is the predicted value of A made at time t-k, α and β are coefficients, and ε_t is an error term that is conventionally assumed to be white noise if k=1. For $k > 1$, it can be shown that ε_t would follow an MA(k-1) process if the one-period errors are white noise. The authors mentioned above have asserted that if a particular series of forecasts were found to be biased--that is, if α were found to be significantly different from 0, and/or β were significantly different from 1--then the forecasts would not be consistent with rational expectations. The reasoning is that given a correct model of the process generating a particular variable, the rational expectations hypothesis has been defined as

$$P_{t-k} = E_{t-k}(A_t) \qquad (2)$$

where E_{t-k} is the mathematical expectations operator for expectations formed in period t-k. Taking expectations of both sides in (1),

$$E_{t-k}(A_t) = \alpha + \beta P_{t-k} \qquad (3)$$

Thus $\hat{\alpha} \neq 0$ or $\hat{\beta} \neq 1$ are not consistent with equation (2).

Suppose, however, that the correct model of the economy is not possessed by an individual. Then there is no requirement that equation (2) should hold. The question thus arises whether the joint assumption of rational expectations and knowledge of the correct model is appropriate. According to Robert Lucas and Thomas Sargent, "[I]t has been only a matter of analytical convenience and not of necessity that equilibrium models have used . . . the assumption that agents have already learned the probability distributions they face. [It] can be abandoned, albeit at a cost in terms of the simplicity of the model." (1979, p. 13)

In other words, when <u>constructing</u> an economic model with rational expectations, it is natural to impose the additional hypothesis that individuals know the proposed model. Otherwise, rational expectations would be coupled with an arbitrary assumption that individuals possess a particular misspecified model. In addition, it would be necessary to specify a learning mechanism for individuals to acquire knowledge of the correct model.

The situation is totally different when evaluating real-time forecasts, however. While even the appropriate steady-state model for some variables may be subject to dispute among contemporary economists, it is certainly heroic to assume that any individual knows the quarter-by-quarter dynamic pattern of adjustment between steady states. This observation does not have any implications for the validity of the hypothesis of rational expectations by itself--it simply implies that it may not be appropriate to impute knowledge of a correct model to real-time forecasters.

That is not to say that the rational expectations hypothesis can only be applied to model building. On the contrary, there is an obvious implication of rational expectations for real-time forecasters. At its most general level, the rational expectations hypothesis states that individuals will attempt to capture large, obvious rents, and in the process erode the source of such rents.[1] Applied to forecasts, this principle implies that individuals would not continually make the same costly mistake if it could be easily avoided. Therefore, any in-sample bias that is observed in a particular series of forecasts should not have any value in allowing a forecaster to make better predictions. This implication can be tested, and such tests are implemented below.

A Search for Systematic Bias with Predictive Value Suppose that an investigator were to estimate equation (1) through period T, find that

$\hat{\alpha} \neq 0$ or $\hat{\beta} \neq 1$, and conclude that the forecasts were biased. It would then be possible to use estimated values of α and β to produce a forecast of A_{T+1}, say P^*_{T+1}, which should not contain the bias observed through period T. In symbols,

$$P^*_{T+1} = \hat{\alpha} + \hat{\beta} P_{T+1} \qquad (4)$$

With an additional observation, one could re-estimate equation (1) and compute P^*_{T+2}. Further repetitions could then produce a series of post-sample forecasts from which in-sample bias was removed and which could be compared with the real-time predictions.

If the in-sample bias were systematic, one would expect the simulated unbiased post-sample forecasts to be more accurate than a particular series of real-time predictions. Such a finding would contradict the rational expectations principle as described above, since it would demonstrate that an individual forecaster could have easily improved his forecasts with information available when the forecasts were made, namely the P^* series. On the other hand, if the real-time predictions were more accurate than the simulated forecasts, that would imply that the in-sample bias was not systematic and would be consistent with rational expectations. Thus one would be led to reject the joint hypothesis of rationality plus the correct model but not reject rational expectations itself. That, in turn, would imply that the model employed by an individual forecaster was misspecified at particular times, but was revised in light of past errors.

The hypothetical results stated above are actually observable. Forecasts examined in this section include one-, two-, and four-quarter-ahead forecasts of real GNP and the implicit price deflator from Wharton Econometric Forecasting Associates, and Treasury bill rate forecasts for the same horizons from Chase Econometrics. The Wharton forecasts are from 1969 fourth quarter to 1983 fourth quarter, and the Chase forecasts are from 1970 third quarter to 1983 fourth quarter.[2]

For each variable, equation (1) was estimated[3] through 1976 fourth quarter.[4] The results are shown in the accompanying table. In-sample bias was observed (from t tests on α and β) in all cases except for real GNP, one and two quarters ahead. The next step was to construct the series of post-sample forecasts. For each one-quarter-ahead forecast, coefficients from equation (1) estimated through 1976 fourth quarter were combined with the real-time forecast for 1977 first quarter to give the simulated unbiased forecast for 1977 first quarter, as in equation (4). (Similar procedures were employed for two- and four-quarter-ahead forecasts, with post-sample forecasts beginning in 1977 second quarter and 1977 fourth quarter respectively.) Next, equation (1) was reestimated through 1977 first quarter and the resulting coefficients were used to produce a forecast for 1977 second quarter. This procedure was repeated through 1983 fourth quarter in every case where in-sample bias was found in the pre-1977 data. The regressions were run with a fixed starting date, and also with a moving initial date (in order to allow for possible structural change in the process generating forecast errors). Taking an example of the moving start-date, for the one-quarter-ahead forecast of the GNP deflator the first regression was estimated from 1969 fourth quarter to 1976 fourth quarter, the next regression was estimated from 1970 first quarter to 1977 first quarter, and so forth until the last regression, 1976 fourth quarter to 1983 fourth quarter. Thus both procedures employ only data that would have been available to a forecaster at each particular time.

As indicated in the table, the actual forecasts were more accurate in every case when compared to the revised series constructed from regressions with a fixed starting date, and were more accurate in six of seven cases when compared with the revised series constructed from regressions with a moving starting date. Although the difference in accuracy was fairly small for the one and four quarter forecasts of the deflator, the accuracy differential was larger in several cases and was most dramatic for the two-and four-quarter-ahead interest rate forecasts.

Conclusion The results presented above indicate that observed bias in forecasts from major consulting services did not have predictive value. Those results are consistent with the interpretation that although the forecasters did not possess a complete, correctly specified model of the dynamic evolution of the economy, they did not repeat easily avoidable errors. Thus contrary to a common interpretation, observation of biased forecasts over a particular interval is not sufficient to infer that individuals failed to form expectations rationally.

REGRESSION AND SIMULATION RESULTS

Variable	Forecast Horizon (quarters ahead)	Regression Coefficients (through 1976:4)						Root-Mean Square Forecast Errors (through 1984:1)		
		α	β	ρ_1	ρ_2	θ_1	θ_2	Published	Revised, Fixed Start	Revised, Moving Start
Real GNP	1	0.27 (1.10)	0.75 (0.23)					*		
	2	0.16 (1.49)	0.73 (0.42)			0.64		*		
	4	1.89 (1.84)	0.42 (0.27)	0.76				2.34	2.66	2.84
GNP Deflator	1	2.17 (0.72)	0.76 (0.12)	0.43				1.86	1.91	1.79
	2	5.05 (1.03)	0.19 (0.16)			1.44	0.45	1.36	2.46	1.99
	4	5.56 (1.07)	0.15 (0.11)	1.50	-0.64			1.76	2.06	2.07
Treasury Bill Rate	1	4.32 (1.23)	0.21 (0.16)	0.80				1.80	2.35	2.13
	2	11.92 (2.32)	-0.35 (0.09)	1.12	-0.19			2.58	4.21	3.91
	4	4.87 (1.25)	0.20 (0.15)	1.39	-0.57			3.56	5.71	5.09

Real GNP and the GNP deflator are expressed as percent changes at annual rates, and the interest rate is expressed as a percentage. For multi-step forecasts of GNP and the deflator, the growth rates are average rates over the entire interval. Regression coefficients refer to equation (1), with estimated standard errors in parentheses below the estimated coefficients. ρ_1 and ρ_2 are Cochrane-Orcutt estimates of autogressive parameters; θ_1 and θ_2 are nonlinear least squares estimates of moving average parameters. Forecast errors represent the difference between actual and predicted values. The "published" column is derived from published forecasts and the latest revision of data available on June 1, 1984. Asterisks in that column indicate that no bias was found in the sample period and therefore no post-sample forecasts were calculated.Revised forecasts were calculated by equation (4).

References

Brown, Bryan W. and Maital, Shlomo. "What Do Economists Know? An Empirical Study of Experts' Expectations." Econometrica 49 (March 1981): 491-504.

Figlewski, Stephen and Wachtel, Paul. "The Formation of Inflationary Expectations." Rev. Econ. Statistics 63 (February 1981): 1-10.

Friedman, Benjamin M. "Survey Evidence on the 'Rationality' of Interest Rate Expectations." J. Monetary Econ. 6 (October 1980): 453-465.

Gramlich, Edward M. "Models of Inflation Formation." J. Money, Credit and Banking 15 (May 1983): 155-173.

Lucas, Robert E. Review of "A Model of Macroeconomic Activity" by Ray C. Fair. J. Econ. Literature 13 (September 1975): 889-890.

______ and Sargent, Thomas J. "After Keynsian Macroeconomics." Federal Reserve Bank of Minneapolis Quarterly Rev. 3 (Spring 1979): 1-16.

McNees, Stephen K. "The 'Rationality' of Economic Forecasts." A.E.R. 68 (May 1978): 301-305.

Urich, Thomas and Wachtel, Paul. "The Structure of Expectations of the Weekly Money Supply Announcement." J. Monetary Econ. 13 (March 1984): 183-194.

FOOTNOTES

1. For example, see Lucas (1975).

2. The forecasts for GNP and the deflator were converted from levels to rates of change in order to remove much of the effects of routine data revision. However, the 1980 benchmark revision of the National Income and Product Accounts included definitional changes that had small effects on growth rates as well as levels.

3. More often than not, the errors from equation (1) did not follow the theoretically derived process. This may be an artifact of the small sample. Also, the timing of data receipt and revision is more complicated than is normally assumed when deriving the time series properties of forecast errors. In any event, an AR or MA process that seemed to adequately fit the data through 1976 fourth quarter was used for the estimates shown in columns 3-8 and in the post-sample predictions summarized in columns 10 and 11. It is interesting to note that going from OLS to either a Cochrane-Orcutt or nonlinear estimation procedure invariably made a substantial difference in the estimated standard errors, and often resulted in sizable changes in coefficient estimates. Conversely, results showed much less change when various AR or MA process were assumed. Therefore it does not appear likely that the results in this article depend on the particular estimation procedure employed.

4. The fourth quarter of 1976 is approximately the midpoint of the sample for the various series. No experimentation was conducted for other dates.