Working Paper 89-2

Cointegration and a Test of the Quantity Theory of Money

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April 1989

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Abstract

The main implication of the Quantity Theory of Money is that long-run movements in the price level are determined primarily by long-run movements in the excess of money over real output. This implication is related to the concept of cointegration discussed in Granger (1986), which states cointegrated multiple time series share common long-run movements. It is shown that the general price level is cointegrated with money, real output, and the nominal rate of interest. These economic variables enter a price equation based on the Equation of Exchange. Furthermore, the appearance of this cointegration in the data seems consistent with the presence of Granger-causality from money and real output to the price level. It is also shown that an inflation equation that incorporates the abovestated implication of the Quantity Theory of Money predicts quite well the actual behavior of inflation during the past decade or so. These results however hold for M2, not M1, measure of money.

One of the most influential economic doctrines is the quantity theory of money (hereafter denoted QTM), which in its simplest form states that long-run movements in the general price level are determined primarily by long-term movements in the excess of money over real output.[1] The theory allows the price level to deviate in the short run from this long-run relationship. However, it is postulated that such deviations would trigger forces which would cause the actual price level to move towards the long-run value implied by the QTM. Such short-run price deviations, according to this theory, are transitory.

This article draws on recent developments in the theory of cointegrated processes to test whether the QTM holds as a long-run equilibrium relation. A price equation summarizing the main determinants of the price level suggested by the QTM is derived. If long-run movements in the price level are related to long-run movements in the variables included in the price equation, then the price level is cointegrated with such variables.[2] If such cointegration exists, then short-term deviations in the price level from its long-run value are stationary, and actual changes in the price level could satisfy an error correction mechanism described in Engle and Granger (1987). In that case, cointegration implies Granger-causation. This article shows that for the M2 measure of money the abovestated implications of the QTM are consistent with the data.[3] In particular, long-run movements in the price

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[1]The theory allows in addition the influence of velocity on the price level. However, it is generally assumed that velocity in the long run is determined by institutional factors and thus does not change or changes slowly over time. For an excellent review of the Quantity theory of money, see Humphrey (1984).

[2]Interpreting cointegration as a long run equilibrium relation is proposed by Engle and Granger (1987).

[3]The results do not hold if M1 measure of money is used.

level appear to be determined primarily by long-run movements in the excess of money over real output.

This article is organized as follows. Section 1 presents a regression equation consistent with the QTM. It also discusses how recent developments in the theory of cointegrated processes could be used to test the main implication of the QTM. Section 2 contains the empirical results and Section 3 the concluding remarks.

I

A Price Equation Consistent with the Quantity Theory of Money

A price equation that summarizes the main determinants of the price level suggested by the QTM can be written as

$$\ln p_t = \beta_1 + \beta_2 R_t + \ln M_t - \ln y_t + n_t \tag{1}$$

where p is the general price level; R, a market rate of interest; M, the quantity of money in circulation; y_t, real output; and n_t, a random error term. ln is the natural logarithm. One could derive this equation from the equation of exchange in which the level of velocity depends upon the opportunity cost variable measured by a market rate of interest.[4] The price equation (1) implies the price level is proportional to the quantity of money, given levels of real output and the market rate of interest.

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[4]This can be seen as follows. The usual equation of exchange expressed in log form is $\ln p = \ln M - \ln y + \ln V$. Moore, Porter and Small (1988) and Hetzel and Mehra (1989), among others, have shown that the level of M2 velocity depends upon the opportunity cost variable measured as the differential between the market rate of interest (R_t) and the own rate of return on money. Since several deposit components of M2 now pay market-determined yields, the own rate of return on money in the long run is likely to move with the market rate. Hence, the long-run value of velocity is likely to depend on the market rate of interest.

The variables in (1) are the long term determinants of the price level. In the short run the actual price level could differ from the value suggested by such determinants. This is implied by the presence of the error term n_t in (1). However, if equation (1) is true, then n_t is a stationary zero mean process,[5] though it could be serially correlated, heteroscedastic, and even correlated with some of the righthand side explanatory variables.

<u>Testing the Quantity Theory of Money: The Issue of Cointegration</u>

If levels of the variables included in the price equation (1) above have stochastic trends, the proposition that this price equation describes the long-run relationship among the variables can be interpreted to mean that the stochastic trend in the price level is related to stochastic trends in money, real output and the nominal rate of interest. This implication is related to the concept of cointegration discussed in Granger (1986), which states cointegrated multiple time series share common stochastic trends.[6] Hence, the long-run implication of the QTM can be examined using the test of cointegration proposed in Engle and Granger (1987). If the QTM is true, then the price level should be cointegrated with money, real output and the market rate of interest.

This test for cointegration consists of two steps. The first step tests whether each variable in equation (1) has a stochastic trend. That is

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[5]The stationarity of n_t is required by the QTM, which regards any short term deviations from the price level implied by values of R, M and y as transitory.

[6]Let x_t be a vector of N component time series, each first difference stationary. Then x_t is said to be cointegrated if there exists a vector α such that $Z_t = \alpha x_t$ is stationary. The intuition behind this definition is that even if each element of x_t is nonstationary, there might exist linear combinations of such multiple time series that are stationary. In that case, multiple time series are cointegrated and share some common stochastic trends. We can interpret the presence of cointegration to imply that the stochastic trends (long-run movements) in these multiple time series are related to each other.

investigated by performing the unit root tests on the variables. The second step tests whether stochastic trends in these variables are related to each other. In particular, the question of interest here is whether the stochastic component in the price level is related to stochastic components in money, real output and the nominal rate of interest. This can be examined by estimating the cointegrating regression of the form (2)

$$\ln p_t = \gamma_0 + \gamma_1 R_t + \gamma_2 \ln M_t + \gamma_3 \ln y_t + e_t \qquad (2)$$

and then testing whether the residual e_t in (2) has a unit root or not. If e_t in (2) does not appear to have a unit root while the lefthand and righthand variables had each a unit root, then the variables are said to be cointegrated.[7] In that case, ordinary least squares estimates of the parameters of (2) are consistent.[8] Furthermore, as shown in West (1988), these ordinary least squares estimators have even asympototic normal distributions if, in addition to sharing a common unit root, the unconditional mean of first differences of the nonstationary variables is non-zero. In that case standard inference procedures based on t and F values can proceed in the usual

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[7]The test for cointegration proposed in Engle and Granger (1987) thus consists of finding linear combinations of nonstationary multiple time series that are stationary. This test is particularly suited to test the QTM, because the particular linear combination of interest is the one in which the price level is the dependent variable and money, real output and the nominal interest rates as the righthand side variables. Alternative test procedures for cointegration among multiple time series look for the number of common stochastic trends and are based on transformations of the original variables (see, for example, Stock and Watson (1988), Johansen (1988) and Bossaerts (1988)). These alternative test procedures are useful if the objective is to find the number of common trends.

[8]If levels of the nonstationary variables included in equation (2) are not cointegrated, then ordinary least squares estimators of this equation would not possess any desirable asymptotic properties. As shown in Phillips (1986), ordinary least squares estimates of the regression parameters would not converge to constants, and the usual t and F-ratio test statistics would not possess limiting distributions but diverge as the sample size T goes to infinity.

way.[9] The implication is that estimated coefficients of the cointegrating regression can be used to test hypotheses about the long-run impact of money, real output and the nominal rate of interest on the price level.

<u>Short-Run Dynamics: Cointegration and Granger-Causality</u>

Granger (1988) also points out that if a pair of series are cointegrated, then there must be causation in at least one direction.[10] Suppose one finds that the residual e_t in (2) is stationary. This result implies the presence of cointegration. The question of interest is whether the presence of cointegration implies changes in the price level are jointly Granger-caused by lagged levels of money, real output and the nominal rate of interest, as suggested in Granger (1988).[11] I present below some evidence consistent with the presence of such causality. Furthermore, I also show that an inflation equation incorporating the equilibrium relationship exhibited in

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[9]The standard errors of the estimated coefficients need to be adjusted to allow for the presence of time dependent and heteroscedastic error term e_t in (2).

[10]To illustrate, assume we have two series x and y. Assume that x and y are both integrated of order one (i.e., both series are stationary after first differencing) and that they are cointegrated. Then, as shown in Granger (1988), they satisfy an error-correction model of the form

$$\Delta x_t = \text{lagged } \Delta x_t,\ \Delta y_t + \gamma_1 Z_{t-1}$$

$$\Delta y_t = \text{lagged } \Delta x_t,\ \Delta y_t + \gamma_2 Z_{t-1}$$

where Z_t is the residual from the cointegrating regression and where one of γ_1, $\gamma_2 \neq 0$. Since Z_{t-1} depends upon lagged levels of x and y the model above implies that either Δx or Δy (or both) must be caused by lagged levels of the variables. The intuition behind this result is that for a pair of series to have attainable equilibrium, there must be causation between them to provide the necessary dynamics.

[11]As is quite clear from the illustration given in ft. note 10, the presence of cointegration does not necessarily imply that money, real output and the nominal rate of interest would jointly Granger-cause the rate of inflation.

(1) does reasonably well in predicting the actual behavior of inflation during the last decade or so.

II

This section presents the empirical results. The data used are quarterly and cover the sample period 1952Q1 to 1988Q4. The general price level (p) is measured by the implicit GNP deflator; real output (y), by real GNP; nominal money (M), by M2 measure of money; and opportunity cost (R), by 4-6 month commercial paper rate.[12] I first present unit root test results and show that levels of variables included in the price equation (1) have each a unit root. However, levels of these variables appear to be cointegrated in the sense that the residual from the 'cointegrating regression' of the form (2) is stationary.

Test Results for Cointegration

The test used to detect a unit root in a given time series X_t is the Augmented Dickey-Fuller (ADF) test and is performed estimating the following regression

$$\Delta X_t = a + \sum_{s=1}^{n} b_s \, \Delta X_{t-s} + c\, T + d\, X_{t-1} + \epsilon_t \tag{3}$$

where ϵ_t is the i.i.d. disturbance and n is the number of lagged values of first differences that are included to allow for serially correlated errors. If there is a unit root in X_t, then the estimated coefficient d above should not be different from zero. The results of estimating (3) for price level,

[12]The opportunity cost of holding M2 is measured by the market rate of interest. This reflects the assumption made here that in the long run the own rate of return on M2 depends on the market rate of interest.

real output, M2, and the nominal interest rate data are presented in Table 1. These test results are consistent with the presence of a unit root in each of the relevant variables.[13]

Table 2 presents results of regressing the price level on levels of money, real output and the nominal rate of interest as in equation (2). The estimates of this regression are presented in the upper panel of Table 2. The middle panel of this Table presents the first ten autocorrelations of the residual from the cointegrating regression. These autocorrelations appear to decline quite rapidly, suggesting that the residual is stationary. The lower panel presents results of applying the formal ADF test for detecting a unit root in the residual series. Since there does not appear to be any serial correlation in first differenced residual series, I also present results without including own lags. The estimated coefficient that appears on the lagged level of the residual in the relevant regression range between -.2 (n=0) to -.3 (n=4) and is significantly different from zero at the .05 level.[14] This result implies that the residual e_t is stationary.[15]

[13]I get similar results using the unit root tests proposed in Phillips and Perron (1988). Their procedure accounts for non-independent and non-identically distributed errors using non-parametric adjustment to the standard Dickey-Fuller procedure. The adjusted Dickey-Fuller t values denoted as Z (t_α) in Phillips and Perron (1988) for lnp, ln y, ln M2, and RCP are, respectively, -2.0, -2.1, -2.3 and 3.0. The 5% critical value (Fuller (1976), Table 8.5.2) is -3.45. None of the t values is significant, implying the existence of a unit root each in the data.

[14]The unit root test proposed in Phillips (1987) when applied to the residual e_t yielded the following regression $e_t = .76\ e_{t-1} + \hat{E}_t$. The t statistic for the hypothesis that the estimated coefficient on e_{t-1} in the regression above is unity (denoted as Z_t in Phillips (1987)) is -4.4. The 5% critical value (Engle and Yoo (1987), Table 3) is 4.0. This result indicates that the residual e_t does not have a unit root.

[15]I also examined the stationarity of the residual e_t if the opportunity cost variable in the regression (2) was measured as the differential between the market rate of interest and the own rate of return on money. These test results continue to support the conclusion that the residual e_t is stationary.

The results presented above imply that levels of the variables entering the price equation are nonstationary but cointegrated.[16] The parameter estimates of (2) presented in Table 2 are therefore consistent. The coefficients that are estimated on ln M2 and ln y_t are close to unity, as implied by the quantity theory. Since the residual e_t is serially correlated and possibly heteroscedastic, the least squares standard errors were corrected as suggested in West (1988). The parameter estimates shown in Table 2 still exceed their standard errors by a substantial margin[17].

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[16]The Engle and Granger test as done here suggests the presence of at least one cointeprating vector. I also computed the likelihood ratio test, -2lnQr, proposed in Johansen (1988), for the no of linearly independent cointegrating vectors r, or equivalently the number of common unit roots (4-r) in a VAR (4) for the set of four time series included in equation (3) of the text. The likelihood ratio test values for their being 3, 2, 1, and 0 cointegrating vectors are 1.2, 10.5, 30.3 and 58.5, respectively. The ninety five percent quantiles for r = 3, 2, 1, and 0 are 4.2, 12.0, 23.8 and 38.6, respectively (Johansen (1988), Table 1). This evidence is consistent with the presence of at least two linearly independent cointegrating vectors in the four-variable system.

[17]It is not clear if standard t and F values could be used to test hypotheses based on the estimated price equation (3). This is so because certain conditions stated in West (1988) are not satisfied. For example, even though the unconditional mean of lnM and lny is nonzero, that of R is not. Moreover, the formal test done for the number of common unit roots suggests the presence of more than a single unit root (see footnote 16). However, as shown in West (1988), the price equation (1) can be reformulated as an equation with a single nonstationary righthand explanatory variable as shown below in (B)

$$\ln p_t = \gamma_o + \gamma_1 \ln(M_t/y_t) + \gamma_2 (R_t - RM_t) + V_t \qquad (B)$$

where RM is the own rate of return on money. The opportunity cost variable is measured as the differential between the market rate and the own rate of return on money. The unit root tests suggest that (R-RM) is stationary, but $\ln(M_t/y_t)$ is not. The unconditional mean of $\ln(M_t/y_t)$ is nonzero. The least squares estimation of (B) yielded

$$\ln p_t = 5.04 + .98 \ln(M_t/y_t) + .014 (R_t - RM_t) + \hat{V}_t$$

The coefficient that appears on money in (B) is not different from its theoretical unitary value and is statistically different from zero (t value is 87.3). It should also be pointed out that the coefficient that appears on (R-RM) above is .014, which is not too different from the value .011 found on R in equation (2) of the text. See also footnote 15.

Short-Run Dynamics: Evidence on Causality and Forecasting Performance

I now present some evidence on the extent to which short-run movements in the rate of inflation are influenced by deviations of the price level from the long-run equilibrium relation (2) estimated here. The evidence consists of estimating the following regression

$$\Delta^2 \ln p_t = f_0 + \sum_{s=1}^{n1} f_{1s}\, \Delta^2 \ln p_{t-s} + \sum_{s=1}^{n2} f_{2s}\, \Delta \ln M_{t-s} + \sum_{s=1}^{n3} f_{3s}\, \Delta \ln y_{t-s} + \sum_{s=1}^{n4} f_{4s}\, \Delta R_{t-s} + \lambda\, e_{t-1} + \epsilon_{1t} \qquad (4)$$

and then testing whether λ is statistically different from zero.[18] I also estimate these regressions for a shorter sample period, 1952Q1-1979Q4.[19]

Table 3 presents estimates of the coefficient that appears on the error-correction variable e_{t-1} in regression (4). As can be seen, the estimated coefficient $\hat{\lambda}$ is generally negative as expected and is statistically different from zero.[20] This result implies that lagged levels, as opposed to first differences, of money, real output and the nominal rate of interest are relevant in modeling short-run dynamics of the rate of inflation. The earlier empirical work which tested causality using only first differenced variables might be suspect, because they might have missed detecting causality that

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[18]In equation (4) the price level is in second differences whereas money, real output and the nominal rate of interest in first. This specification is suggested by unit root tests results. In particular, unit root tests indicated the presence of two unit roots in the price level, implying that this regressor needs to be differenced twice.

[19]The residual series e_t used in the shorter-sample period is constructed using parameters of the 'cointegrating regression' (2) estimated over the whole sample period 1952Q1 to 1988Q4. I get similar results if the 'cointegration regression' is estimated over the shorter period and its residual series is used.

[20]The coefficient that is estimated on the error correction variable remains negative and is statistically significant if the price level regressor in (4) is in first differences.

enters through the error-correction variable (Granger (1988)).

How well does the price equation incorporating the error-correction mechanism explain the long-run behavior of inflation? I present below some evidence on this issue by examining the out-of-sample forecasting performance of a price regression of the form (4) over the period 1977 to 1988. In particular, the price regression that underlies this exercise is

$$\Delta^2 \ln p_t = f_0 + \sum_{s=1}^{4} f_{1s}\, \Delta^2 \ln p_{t-s} + f_2\, \Delta R_{t-1} + \lambda\, (\ln p_{t-1} - \ln \overset{*}{p}_{t-1}) + \epsilon_t \tag{5}$$

where $\ln \overset{*}{p}_{t-1}$ is simply set to $\ln (M_{t-1}/Y_{t-1})$. That is, the long-run value of the price level ($\ln p^*$) is determined primarily by the excess of the log of money over real output.[21] The regression is estimated first over the period 1952Q1 to 1976Q4 and dynamically simulated out-of-sample over the 4-quarter and 8-quarter periods, 1977Q1 to 1977Q4 and 1977Q1 to 1978Q4. The errors that occur predicting the inflation rate are calculated for the 4-quarter and 8-quarter horizons. The end of the estimation period is then advanced by four quarters, and estimation and out-of-sample simulation is repeated using this new period 1952Q1 to 1977Q4. That procedure is repeated until the price equation is reestimated and simulations prepared based on data ending in each fourth quarter through 1987Q4 (in case of 4-quarter horizon) and 1986Q4 (in case of 8-quarter horizon). Table 4 reports the errors from

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[21]Lag lengths of the right hand side variables in (5) were selected by the 'final prediction error' criterion. First differences of money and real output did not enter the regression. This inflation regression appears to pass several tests of model adequacy. In particular, the regression passes the Chow test of parameter stability over the period 1952Q1 to 1988Q4. The Godfrey test for serial correlation and heteroscedascity did not indicate the presence of serial correlation or heteroscedasticity in the residuals of (5). See also Hallman, Porter, and Small (1989) for another price regression that is similar in spirit to (5).

this exercise. As can be seen in the Table, this equation does a reasonable job in predicting the rate of inflation. The bias is small and the root mean squared error (RMSE) is 1.11 percentage points for the 4-quarter horizon and 1.44 percentage points for the 8-quarter horizon. On balance, these results imply that forecasts of the rate of inflation from the price equation estimated under the restriction implied by the QTM are not out of line with the actual behavior of inflation during the last decade or so.[22]

III

Concluding Remarks

The quantity theory of money does appear to hold as a long run equilibrium relation in the sense that short-term deviations in the general price level from the value implied by the excess of money over real output are stationary. The proposition that the rate of inflation in the long run is determined primarily by the rate of growth of money in excess of real output is consistent with the data.

However, these results hold only for the M2 measure of money and reflect perhaps the underlying stability of the M2 demand behavior recently noted in Rasche (1987), Moore, Porter, and Small (1988) and Hetzel and Mehra (1989). Other analysts (see for example, Reichenstein and Elliott (1987) and Mehra (1988)) have also produced evidence consistent with the view that money measured by M2 remains relevant in predicting the long-term behavior of inflation.

[22]This is not to suggest that the price equation (6) is the best forecasting equation but rather it captures quite well the long-term behavior of the price level.

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Table 1

Augmented Dickey-Fuller Test Results

Augmented Dickey-Fuller Regression: $\Delta X_t = \hat{a} + \sum_{s=1}^{4} \hat{b}_s \Delta X_{t-s} + \hat{c}\,T + \hat{d}\,X_{t-1} + \hat{\epsilon}_t$

Time Series X_t	$\hat{d}$ (t value)	F	Q(sl)
$\ln p_t$	-.006 (-1.96)	32.4*	26.7(.87)
$\ln M2_t$	-.013 (-2.3)	19.9*	41.8(.23)
$\ln y_t$	-.047 (-2.1)	6.1*	21.9(.97)
R_t	-.13 (-3.2)	8.0*	38.7(.35)

Notes: The Augmented Dickey-Fuller regression is estimated over the period, 1952Q1-1988Q4. p is the price level; y, real GNP; M2, M2 measure of money; and R, the 4-6 month commercial paper rate. ln is natural logarithm and Δ, the first difference operator. $\hat{d}$ is the estimated coefficient that appears on the lagged level of the variable in question and the associated parenthesis contain the t value; the 5 percent critical value of the t statistic is 3.45 (Fuller (1976), Table 8.5.2). F is a test of the null hypothesis that four lagged values of ΔX do not enter the Dickey-Fuller regression stated above. Q(sl) is the Ljung-Box Q-statistic based on 36 autocorrelations of the residual and sl is the significance level.

* significant at .05 level.

Table 2

Cointegration Test Results

A. Estimates of a Cointegrating Regression; 1952Q1-1988Q4

$$\ln p_t = \hat{\gamma}_0 + \hat{\gamma}_1 \ln M_t + \hat{\gamma}_2 \ln y_t + \hat{\gamma}_3 R_t + \hat{e}_t \quad (A)$$

$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$
1.0	-1.2	.011
(011, .024)	(.028, .063)	(.001, .002)

B. Autocorrelations (from 1 to 10) of the residuals from the Cointegrating Regression (A)

1	2	3	4	5	6	7	8	9	10
.76	.56	.46	.36	.23	.15	.13	.12	.09	.05

C. Augmented Dickey-Fuller Test of Residuals

$$\Delta\hat{e}_{t-s} = \sum_{s=1}^{n} b_s \, \Delta\hat{e}_{t-s} + \hat{d}\, \hat{e}_{t-1} + \hat{\epsilon}_t$$

	$\hat{d}$ (t value)	F	Q(sl)
n=4	-.26(-4.0)*	.90	28.7(.68)
n=0	-.23(4.4)*		35.7(.48)

Notes: See notes in Table 1 for definition of variables. In the top panel above, two values in the parenthesis below the estimated coefficients are the standard errors. The first value is the least squares value, whereas the second value allows for $\hat{e}_t$ to be serially correlated and heteroscedastic and is calculated as in West (1988). F and Q(sl) are defined as in Table 1.

*The 5% and 10% critical values are, respectively, 4.02 and 3.71 (Engle and Yoo (1987), Table 3).

Table 3

Estimates of the Coefficient on the Error Correction Variable in the Inflation Equation

Changes in the Inflation Rate

$$\Delta^2 \ln p_t = f_o + \sum_{s=1}^{n1} f_{1s}\, \Delta^2 \ln p_{t-s} + \sum_{s=1}^{n2} f_{2s}\, \Delta \ln M_{t-s} + \sum_{s=1}^{n3} f_{3s}\, \Delta \ln y_{t-s} + \sum_{s=1}^{n4} f_{4s}\, \Delta R_{t-s} + \lambda\, e_{t-1} + \epsilon_{1t} \quad (A)$$

	Sample Period 1953Q1-1988Q4				Sample Period 1953Q1-1979Q3			
Lag Length (n1, n2, n3, n4)	$\hat{\lambda}$(t value)	F1(df)	F2(df)	Q(sl)	$\hat{\lambda}$ (t value)	F1(df)	F2(df)	Q(sl)
(4,0,0,0)[a]	-.04(-2.0)	16.3*(4,137)		33.3(.45)	-.07(-2.9)	15.9*(4,100)		25.5(.69)
(4,4,4,4)	-.05(-2.4)		1.4(12,125)	34.8(.38)	-.06(-2.8)		1.63(12,88)	30.5(.44)
(4,0,0,1)[a]	-.04(-1.7)			33.8(.42)	-.06(-2.7)			26.8(.63)

Notes: The variable e_{t-1} included in the price equation (A) above is the lagged value of the residual from the cointegrating regression reported in Table 2. F1 tests the null hypothesis that lagged values of the dependent variable do not enter the regression (A). F2 tests the null hypothesis that lagged values of changes in nominal money, real output and the nominal interest rate do not enter the regression (A). df is the degrees of freedom. See also Notes in Table 1.

[a]Lag lengths selected by the final prediction error criterion due to Akaiki (1969).
*Significant at .05 level.

Table 4

Out-of-Sample Forecasting Performance of the Restricted Price Equation; 1977-1988

Estimation Period Ends In	4-quarter Ahead Period			8-quarter Ahead Period		
	Actual	Predicted	Error	Actual	Predicted	Error
1976Q4	6.6	6.7	-.09	7.1	7.1	.01
1977Q4	7.7	7.2	.43	8.1	7.2	.84
1978Q4	8.5	7.7	.74	8.9	7.4	1.53
1979Q4	9.4	7.6	1.79	8.8	7.9	.98
1980Q4	8.3	9.6	-1.29	6.7	8.9	-2.19
1981Q4	5.0	7.1	-2.05	4.3	6.9	-2.56
1982Q4	3.6	5.0	-1.4	3.5	5.0	-1.53
1983Q4	3.3	4.1	-.71	3.1	3.9	-.78
1984Q4	2.9	3.1	-.20	2.8	3.3	-.52
1985Q4	2.7	3.4	-.67	2.9	4.0	-1.13
1986Q4	3.0	4.3	-1.28	3.5	4.5	-.96
1987Q4	4.0	3.5	.51			
Mean Error			.34			-.69
Mean Absolute Error			.93			1.25
Root Mean Squared Error			1.11			1.44

Notes: The error-correction variable in the price equation (see equation (5) in the text) is computed using the restriction that the equilibrium price level (in logs) equals $\ln(M2_t/y_t)$. The numbers in the Table are the annualized rate of growth of the price level (4Q to 4Q) over the out-of-estimation forecast horizons.