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THE EFFECTS OF OPEN MARKET OPERATIONS IN A MODEL OF INTERMEDIATION AND GROWTH*

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Abstract

We examine a standard model of capital accumulation in which spatial separation and limited communication create a role for money and shocks to portfolio needs create a role for banks. In this context we examine the existence, multiplicity, and dynamical properties of monetary equilibria with positive nominal interest rates. Moderate levels of risk aversion can lead to the existence of multiple monetary steady states, all of which can be approached from a given set of inital conditions. In addition, even if there is a unique monetary steady state, monetary equilibria can be indeterminate, and oscillatory equilibrium paths can be observed. Thus financial market frictions are a potential source of both indeterminacies and enhanced economic volatility.

We also consider the consequences of monetary policy actions that rearrange the composition of government liabilities. Contractionary monetary policy activities can have complicated consequences, depending especially on the nature of the steady state equilibrium that obtains when there are multiple steady states. Under plausible conditions, however, contractionary monetary policy activity raises both the nominal rate of interest <u>and</u> the rate of inflation. Output can either rise or fall.

That financial market conditions "matter," both for the level of real activity and its rate of growth, is now a well-established proposition.[1] A large number of theoretical studies detail how the various functions of financial market institutions affect production and capital accumulation. Yet for many economies the most prominent financial market institution is their central bank, and the most common exogenous source of day-to-day changes in financial market conditions is monetary policy.

How do common monetary policy actions affect capital accumulation, real and nominal rates of interest, and the rate of inflation? Answering this question requires analyzing an economy with at least three kinds of assets: money, government bonds, and capital. Thus, an economy with a fairly rich set of asset markets is needed to examine the consequences of monetary policy actions--such as open market operations--for real activity and asset returns.

While a large theoretical literature exists on the integration of financial markets into standard growth models, little of this literature can accommodate government liabilities in an interesting way.[2] This paper presents a model of capital accumulation with outside liabilities and with banks that operate to insure agents against random liquidity needs.[3] In this context

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[1]For theoretical treatments of this topic see Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Levine (1991), Cooley and Smith (1992), Bencivenga, Smith, and Starr (1993 a,b,c), Khan (1993), Parente (1993), and Greenwood and Smith (1993). Anecdotal evidence supporting the proposition in the text appears in Patrick (1966), Cameron (1967), McKinnon (1973), and Shaw (1973). More formal empirical evidence is presented by Goldsmith (1969), Antje and Jovanovic (1993), and King and Levine (1993 a,b).

[2]Two exceptions to this statement are Azariadis and Smith (1993) and Schreft and Smith (1994). Monetary growth models, like those of Diamond (1965), Tirole (1985), Sidrauski (1967 a,b), and Brock (1974, 1975) do allow for the presence of government liabilities and capital. However, those models do not give an explicit allocative role to financial market institutions, and they deliver a variety of implications about the consequences of monetary policy activity that are refuted by the data. For a discussion of the latter issue see Azariadis and Smith (1993) or Schreft and Smith (1994).

[3]In doing so, it generalizes the model presented in Schreft and Smith (1994).

approaching the low-capital-stock steady state or follow any of the continuum of paths that approach the locally stable high-capital-stock steady state. Economies suffering the former fate will have permanently high nominal interest rates, which is a signal of their financial systems' inefficiency. Economies approaching the high-capital-stock steady state will have relatively low nominal and real rates of interest. In any event, two economies with similar, or even identical, initial capital stocks can end up with different long-run output levels.

The possibility that a locally stable monetary steady state exists, even if the steady state is unique, also indicates that dynamical equilibria of our economy may be indeterminate, even when steady state equilibria are not. Moreover, it can easily transpire that paths approaching the steady state display damped oscillations as they do so. Both of these are new phenomena because virtually all existing monetary growth models deliver unique nonstationary equilibria that converge monotonically to a steady state.[5] Thus the integration of more interesting financial market institutions into models of money and growth is a potential source of both indeterminacy and enhanced economic volatility.

The effects of permanent changes in open market activity depend, of course, on the number of steady state equilibria. When there is a unique monetary steady state, "contractionary" monetary policy actions (that is, increases in the ratio of bonds to money) necessarily raise the nominal rate of interest. If bank portfolio allocations are not too sensitive to changes in the nominal interest rate, contractionary monetary policies also lead to a reduction in steady state output. This occurs for an obvious reason: an increase in government bonds outstanding simply crowds out capital in private portfolios. However, if the demand for reserves by banks is sufficiently sensitive to changes in the nominal interest

[5]The exceptions to this statement are Azariadis and Smith (1993) and Schreft and Smith (1994).

government liabilities.

The scheme of the paper is as follows. Section I describes the environment, while section II considers the nature of trade and the role for banks. Steady state equilibria are analyzed in section III, while section IV addresses dynamical equilibria. Section V comments on the possibility of development trap phenomena; section VI concludes.

I. Environment

We consider an economy consisting of an infinite sequence of two-period-lived overlapping generations, plus an initial old generation. In addition, in each period agents are assigned to one of two locations; we assume that at each date the locations are completely symmetric and that, at the beginning of each period, each location contains a continuum of ex ante identical young agents with unit mass.

We let $t = 0, 1, \ldots$ index time. At each date t there is a single final commodity that is produced using a constant returns to scale technology with capital and labor as inputs. This technology is commonly available to all agents. Any agent using K_t units of capital and L_t units of labor can produce $F(K_t, L_t)$ units of the final good at t. We let $f(k_t) \equiv F(k, 1)$ denote the intensive production function, where $k_t \equiv K_t / L_t$ is the time t capital-labor ratio employed. We assume that f satisfies the following assumptions: (a) $f(0) = 0$, (b) $f'(k) > 0 > f''(k)$, $\forall k \geq 0$, and (c) the standard Inada conditions.

At each date the final good can be either consumed or set aside as an investment to be converted into capital. One unit of the final good set aside at t becomes one unit of capital at $t + 1$ with probability $q \in (0,1]$ and becomes worthless with probability $1 - q$. Investment returns are assumed to be iid on each unit invested. Finally capital, once produced, is used in fraction $\pi \in (0,1)$ of all young agents in each location is randomly selected to be transferred to the other location. Although π is constant and known at the beginning of each period, the identities of the specific agents who are to be relocated are discovered only after savings decisions have been made.

We assume that neither capital investments, nor the consumption good, nor government bonds can be transported between locations. Thus money is the only asset that can be carried between locations, which is the source of its liquidity advantage.[6] In addition we assume that spatial separation and limited communication preclude agents from exchanging privately issued claims originated in "the other" location; hence only currency is useful in interlocation exchange.[7]

For this reason, agents who discover that they are to be relocated will wish to convert their other asset holdings into currency. Thus random relocations play the same role here that "liquidity preference shocks" play in the Diamond-Dybvig (1983) model. As in that context, agents will wish to be insured against the event of premature asset liquidation. The efficient way for this insurance to be provided [see Greenwood and Smith (1993)] is through a bank that takes deposits, holds the primary assets in the model directly, and structures the returns paid to agents in a way that depends on whether they are relocated (in effect, on their date of withdrawal). As in Diamond-Dybvig, all savings will be intermediated through banks of this type. Thus agents who are to be relocated simply make withdrawals from their banks and are

[6]In effect relocated agents face a cash-in-advance constraint on consumption purchases, while agents who are not relocated do not. The inability to use bonds in interlocation exchange can be motivated by the realistic assumption that bonds are issued only in large denominations.

[7]This formulation follows Townsend (1987), Mitsui and Watanabe (1989), Hornstein and Krusell (1993), and Champ, Smith, and Williamson (1992). The last of these papers contains a detailed discussion of these assumptions and some defense of their realism for the United States and Canada around the turn of the century.

A. Factor Markets

We assume that any agent can run the production process. Producers hire capital and labor in competitive factor markets that operate in each location. Hence capital and labor are each paid their marginal products, so the standard factor pricing relationships obtain:

(3) $r_t = f'(k_t);\quad t \geq 0,$

(4) $w_t = w(k_t) \equiv f(k_t) - k_t f'(k_t);\quad t \geq 0,$

where r_t is the time t rental rate on capital and w_t is the real wage rate at t. Notice that $w' > 0$ holds. In addition, it will be convenient to make one additional technological assumption. Define $\Omega(k) \equiv k/qw(k)$. Then assume that, for all k,

(a.1) $\Omega'(k) > 0.$

Assumption (a.1) is equivalent to

(a.1') $kw'(k)/w(k) \in (0,1);\quad k \geq 0.$

Assumption (a.1) [or (a.1')] holds if, for instance, f is any CES production function with an elasticity of substitution no less than one.

B. Banks

Banks take deposits, hold the model's primary assets directly, and announce deposit return schedules that depend on the depositor's relocation status (or date of withdrawal). In addition, there is free entry into banking. Thus competition ensures that, in equilibrium, The gross real return on money, p_t / p_{t+1}, appears in (6) because agents who are relocated at t are given real balances at that date, which they then carry into $t + 1$. The real return on these money holdings is, of course, p_t / p_{t+1}. The promised real return to depositors, d_t^m, incorporates this consideration.

For the fraction $1- \pi$ of its depositors who are not relocated at t, the bank has promised a gross return of d_t^n per unit deposited. Thus the bank owes $(1- \pi)d_t^n w_t$ to nonmovers, which it pays at date $t + 1$. If $I_t > 1$, which we assume throughout, money is dominated in rate of return. Hence the bank will not carry real balances between periods; or, in other words, it holds money only as a reserve to pay agents who are relocated at t. Thus payments to nonmovers will be financed solely out of the bank's holdings of bonds and capital, so that

$$(7) \quad (1-\pi)d_t^n w_t \leq R_t b_t + q r_{t+1} i_t; \quad t \geq 0$$

must hold.

It will be convenient to represent the bank's choices in terms of the weights attached to different assets in its portfolio. To this end, let $\gamma_t \equiv m_t / w_t$ denote the bank's ratio of reserves to deposits, and let $\lambda_t \equiv i_t / w_t$ denote the ratio of its capital investments to its deposits. Then equation (6) can be rewritten as

$$(6') \quad d_t^m \leq \gamma_t (p_t / p_{t+1}) / \pi; \quad t \geq 0,$$

while (7) takes the form

$$(7') \quad d_t^n \leq \left[q r_{t+1} \lambda_t + R_t (1 - \gamma_t - \lambda_t) \right] / (1-\pi); \quad t \geq 0.$$

Thus $\gamma'(I)$ has the same sign as $(\rho - 1)$.

C. Equilibrium

In equilibrium the factor pricing relationships (3) and (4) must be satisfied, as must the no-arbitrage condition (8). In view of (3), (8) can be rewritten as

(11) $$R_t = qf'(k_{t+1}); \quad t \geq 0.$$

In addition, money supply must equal money demand at each date. Since all demand for money here derives from banks' demands for reserves, in equilibrium

(12) $$m_t = \gamma(I_t)w(k_t); \quad t \geq 0$$

must hold. Finally, $k_{t+1} = qi_t$ must hold. From (5), this condition is equivalent to

(13) $$k_{t+1} = q\left[w(k_t) - b_t - m_t\right]; \quad t \geq 0.$$

These conditions, plus the government budget constraint (1), constitute the full set of equilibrium conditions of the model. For future reference, we note that (1) can be rewritten as

(14) $$R_{t-1}b_{t-1} = m_t - m_{t-1}(p_{t-1}/p_t) + b_t$$

for all $t \geq 1$.

(21) $\quad qf'(k) \geq 1.$

In addition (20) delivers nonnegative values for I iff

(22) $\quad (1+\beta)/\beta \geq qf'(k)$

holds. Finally, equation (20) defines a locus in figure 2 with a slope given by

(23) $\quad dI/dk|_{20} = (1+\beta)qf''(k)/\{1-\beta[qf'(k)-1]\}^2 < 0.$

Thus the locus defined by (20) has the configuration depicted in figure 2.

To obtain the remaining steady state equilibrium condition, substitute (2) and (15) into (17) to obtain

(24) $\quad k = qw(k)\left[1-\gamma(I)(1+\beta)\right].$

Recalling the definition $\Omega(k) \equiv k / qw(k)$, we may rewrite (24) as

(25) $\quad \gamma(I) = \left[1-\Omega(k)\right]/(1+\beta).$

This gives the second steady state relationship that must obtain between I and k.

Equation (25) defines a locus in figure 2 with slope

(26) $\quad dI/dk|_{25} = -\Omega'(k)/(1+\beta)\gamma'(I).$

Thus the slope of the locus defined by (25) is opposite in sign to $\gamma'(I)$. There are now three cases to consider.

the vertical axis at the point $[1-\Omega(0)] / (1+\beta)$. There are therefore three possibilities regarding the existence of steady state equilibria with $I > 1$.

(a) Suppose, as in figure 2.c.i., that $1 < qf'(\underline{k}) < (1+\beta)/\beta$. Then, if any steady state equilibrium with $I > 1$ exists at all, there are necessarily at least two such equilibria. These are represented by points A and B in the figure.

(b) It is possible that $1 < qf'(\underline{k}) < (1+\beta)/\beta$ holds, but that there are no steady state equilibria with $I > 1$. This situation is depicted in figure 2.c.ii.

(c) Figure 2.c.iii depicts an economy with $qf'(\underline{k}) < 1$. This economy necessarily has at least one steady state equilibrium with $I > 1$, and it is possible that this equilibrium is unique.

B. Comparative Statics

We now consider the comparative static consequences (that is, the consequences for steady state equilibria) of open market operations. Specifically, an increase in β constitutes an increase in the bond-money ratio and hence corresponds to an open market sale, while a decrease in β corresponds to an open market purchase. Thus an increase in β represents a contractionary change in monetary policy, as conventionally conceived.

As before, it is necessary to distinguish between three cases in examining the consequences of a change in β.

1. Case 1: $\rho = 1$

In this case--which is one where agents have logarithmic utility--$\gamma(I) \equiv \pi$ holds. Thus (25) reduces to nominal interest rate. The effect on the steady state equilibrium value of the capital stock, however, is ambiguous. Whether the capital stock and output rise or fall in this case depends on the elasticity of banks' demands for reserves with respect to the nominal rate of interest and possibly on the magnitude of β. We now state

Proposition 1. Suppose that $\gamma'(I) \leq 0$. (a) Then $dk/d\beta < 0$ if $\rho/(1-\rho) \geq I$ holds. (b) Suppose that $\pi \leq 0.5$, $\rho \leq 1/3$, and $\beta \leq 1$ hold. Then $dk/d\beta > 0$.

Proposition 1 is proved in the appendix.

The intuition of this case is obviously more complex than for case 1. As in case 1, an increase in β--ceteris paribus--introduces more bonds, which tend to crowd out capital. However, the increase in the nominal interest rate also reduces the demand for reserves, which tends to reduce the ratio of total government liabilities to deposits--$(1 + \beta)\gamma(I)$--held in the portfolios of banks. If the latter effect is not too large, the steady state capital stock will fall; if it is large enough, the steady state capital stock will rise. In the second case, a "contractionary" change in monetary policy will not have a contractionary effect on steady state output.

3. Case 3: $\rho > 1$

In this case $\gamma'(I) > 0$ holds, and the determination of a steady state equilibrium is depicted in figures 2.c. For simplicity, we focus here only on the case where $qf'(\underline{k}) > 1$ and in which steady state equilibria exist. Hence there are at least two such equilibria.

As before, an increase in β shifts the locus defined by (20) up and to the right unpleasant monetarist arithmetic result.

In case 1, $dk/d\beta < 0$ necessarily holds; in cases 2 and 3, it might hold. Moreover, in case 2 an increase in β raises the nominal interest rate. If it also raises k, $R = qf'(k)$ falls, so that the inflation rate must increase in the steady state. Thus unpleasant monetarist arithmetic *must* be observed in cases 1 and 2, and it can *always* be observed in case 3. In other words, the possibility that contractionary monetary policy ultimately leads to higher rates of inflation is ubiquitous here.

IV. Dynamics

To study dynamical equilibria, it will be convenient to study the special case of Cobb-Douglas production. Therefore, we henceforth assume that $f(k) = Ak^{\alpha}$, with $\alpha \in (0,1)$.

Since $b_t = \beta m_t$ holds for all $t \geq 0$, equations (12) and (13) imply that the evolution of the capital stock is described by

$$(30) \qquad k_{t+1} = qw(k_t)\left[1-(1+\beta)\gamma(I_t)\right]; \quad t \geq 0.$$

Similarly, the date $t + 1$ version of the government budget constraint [equation (14)] can be written as

$$(31) \qquad m_{t+1}(1+\beta) = m_t(p_t/p_{t+1})\left[1+\beta R_t(p_{t+1}/p_t)\right] \equiv m_t(p_t/p_{t+1})(1+\beta I_t); \quad t \geq 0.$$

Using $R_t \equiv I_t\,(p_t/p_{t+1}) = qf'(k_{t+1})$ to obtain

$$(32) \qquad p_t/p_{t+1} = qf'(k_{t+1})/I_t; \quad t \geq 0,$$

holds, then equation (35) determines a monotone sequence $\{k_t\}$. This sequence converges to a unique steady state equilibrium capital stock, as depicted in figure 4. In addition, (34) reduces to

(36) $$I_t = \alpha / \{(1-\alpha)(1+\beta)[1-\pi(1+\beta)] - \alpha\beta\}; \quad t \geq 0.$$

Thus the nominal interest rate is unchanging over time.[10] Moreover, equations (35) and (36) imply that there is a unique dynamical equilibrium, given the initial capital stock k_0. This dynamical equilibrium converges monotonically to the steady state.

B. Case 2: $\rho \in (0,1)$

To study this case (and the case $\rho > 1$ as well), we define the function $\Psi(I)$ by

(37) $$\Psi(I) \equiv \alpha(1+\beta I)\gamma(I) / (1-\alpha)(1+\beta)I[1-(1+\beta)\gamma(I)].$$

Then we can rewrite equation (34) compactly as

(38) $$\gamma(I_{t+1}) = \Psi(I_t); \quad t \geq 0.$$

Moreover, whenever $\rho \neq 1$, γ^{-1} exists, so that (38) can be expressed in the form

(38′) $$I_{t+1} = \gamma^{-1}[\Psi(I_t)]; \quad t \geq 0.$$

Equations (30) and (38′) are the equilibrium laws of motion for k_t and I_t whenever $\rho \neq 1$.

[10]The value of I_t given by (36) exceeds unity iff $\alpha/(1-\alpha) > 1-\pi(1+\beta) > [\alpha/(1-\alpha)][\beta/(1+\beta)]$ holds. For the case of Cobb-Douglas production, this condition is identical to equation (27).

$I_{t+1} = I_t$ holds whenever (41) is satisfied as an equality--that is, whenever $I_t = I^*$, where I^* is implicitly defined by

(42) $\eta(I^*) \equiv (1-\alpha)(1+\beta) / \alpha.$

A value $I^* > 1$ satisfying (42) exists iff

(a.3) $\eta(1) = (1+\beta)/[1-\pi(1+\beta)] > (1-\alpha)(1+\beta)/\alpha > \lim_{I\to\infty}\eta(I) = \beta$

holds.[11] Since $\rho \in (0,1)$ implies that $\eta'(I) < 0$, if I^* exists it is unique.

When $I_t > I^*$ holds, the fact that $\eta'(I) < 0$ implies that (41) is satisfied and hence that $I_{t+1} > I_t$. Similarily, when $I_t < I^*$, $I_{t+1} < I_t$ holds. Thus the behavior of I_t over time is as depicted in figure 5.

Equation (30) implies that

(43) $k_{t+1} - k_t = qw(k_t)[1-(1+\beta)\gamma(I_t)] - k_t; \quad t \geq 0.$

It follows that $k_{t+1} \geq k_t$ obtains whenever

(44) $[1-\Omega(k_t)]/(1+\beta) \geq \gamma(I_t).$

Since $\rho \in (0,1)$ implies that $\gamma'(I) < 0$, (44) at equality defines an upward-sloping locus along which the per-capita capital stock is constant. Combinations (I_t, k_t) to the right of this locus satisfy (44) as a strict inequality and hence have the capital-labor ratio increasing over time. Thus figure 5 depicts the full dynamical behavior of this economy.

[11]For the case of Cobb-Douglas production, this condition is identical to $(1 + \beta) / \beta > qf'(\tilde{k})$ and $qf'(\underline{k}) > 1$, as described in section III.A.2.

Since $\rho \in (0,1)$ implies that $\gamma'(I) < 0$ and $\eta'(I) < 0$ both hold, it follows that $\partial I_{t+1}/\partial I_t > 1$.

Let λ_1 and λ_2 denote the eigenvalues of J. Clearly $\lambda_1 = \alpha \in (0,1)$ and $\lambda_2 = \partial I_{t+1}/\partial I_t > 1$ both hold, confirming that the unique steady state depicted in figure 5 is a saddle.

C. Case 3: $\rho > 1$

When $\rho > 1$ holds, equations (30) and (38′) continue to describe the equilibrium laws of motion for k_t and I_t. However, $\rho > 1$ implies that $\gamma'(I) > 0$; hence, $I_{t+1} \geq I_t$ holds iff

(50) $\Psi(I_t) \geq \gamma(I_t)$

or, equivalently, iff

(50′) $\eta(I_t) \geq (1-\alpha)(1+\beta)/\alpha.$

As before, $I_{t+1} = I_t$ will be satisfied iff

(51) $\eta(I_t) = (1-\alpha)(1+\beta)/\alpha$

holds. However, here an analysis of equation (51) is more complicated than previously.

It is useful to begin by stating some properties of the function η.

Lemma 2. Suppose that (a.2) and $\rho > 1$ hold. Then

(a) $\eta(1) = (1+\beta)/[1-\pi(1+\beta)] > 0.$

(b) $\eta(I) > 0$ holds for all $I \leq \gamma^{-1}[1/(1+\beta)]$. As $I \uparrow \gamma^{-1}[1/(1+\beta)]$, $\eta(I) \to \infty$.

(c) For all $I \in (1, \gamma^{-1}[1/(1+\beta)])$, $\eta'(I) \leq 0$ holds iff and I_t is constant, as in cases 1 and 2.

When (54) is satisfied, the steady state equilibrium is a sink, implying the indeterminacy of monetary equilibria. In particular, we can choose any initial value I_0 in a neighborhood of I^*, and the economy will asymptotically approach its steady state level. The nominal interest rate will necessarily fluctuate as it does so. Thus (54) is a sufficient condition for financial markets to result in the indeterminacy of an equilibrium and the existence of economic fluctuations.

Example 2. Suppose that $\beta > 0$ and

$$\eta(1) = (1+\beta)/[1-\pi(1+\beta)] > (1-\alpha)(1+\beta)/\alpha$$

hold. As $I \uparrow \gamma^{-1}[1/(1+\beta)]$, $\eta(I) \rightarrow \infty$ and, in addition, (52) must be violated. (In other words $\eta'(I) > 0$ holds for sufficiently large I.) Moreover, suppose that

$$(55) \qquad [(\rho-1)/\rho]\pi(1-\pi)(1+\beta)^2 < 1-\pi(1+\beta).$$

Equation (55) implies that $\eta'(1) < 0$. Then equation (51) is as depicted in figure 7, and if there are any steady state equilibria with $I > 1$, then there are at least two of them. These steady states are labeled A and B in figure 7.

When $I_t = I_A$, $I_{t+1} = I_t$ holds. Moreover, for values of I_t near I_A, $\eta'(I_t) < 0$ holds. Thus $I_t < (>)\ I_A$ locally implies that $I_{t+1} > (<)\ I_t$. Similarly, $I_{t+1} = I_t$ when $I_t = I_B$. However, in a neighborhood of I_B, $\eta'(I_t) > 0$. Hence, $I_t > (<)\ I_B$ locally implies that $I_{t+1} > (<)\ I_t$.

Equation (44) continues to imply $k_{t+1} \geq k_t$; (44) is now is a downward-sloping locus. Thus figure 8 depicts the dynamic behavior of this economy.

D. Numerical Examples

We now provide some numerical examples illustrating many of the possibilities just discussed.

Table 1 displays steady state equilibrium values for an economy with $\rho > 1$ and with $\beta = 0$.[13] The latter feature, of course, guarantees uniqueness of the steady state. As shown in table 1, increases in ρ reduce $|\partial I_{t+1}/\partial I_t|$ and, for $\rho \geq 2.5$, the steady state is a sink. Paths approaching it display damped oscillations as they do so.

Table 2 examines the effects of varying β in an economy that has two steady state equilibria with $I > 1$.[14] As is apparent from the table, increases in β (contractionary monetary policy activities) raise I and reduce k in the high-capital-stock steady state, and hence they necessarily raise the inflation rate (reduce p_t/p_{t+1}). All of these effects are reversed with reference to the low-capital-stock steady state.

Table 2 also illustrates the possibility that the high-capital-stock steady state is a sink. For $\beta < 1.0075$, the parameter values of this example violate both equations (56) and (57). Thus, as the example indicates, the conditions under which the high-capital-stock steady state is a sink are much more general than just those given by the sufficient conditions of proposition 2.

V. Multiple Steady States and "Development Traps"

Models of money and capital accumulation typically deliver a unique monetary steady

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[13]The other parameter values employed in table 1, which are held constant, are $A = 1.1$, $\alpha = 0.5$, $\pi = 0.1$, and $q = 0.5$.

[14]The relevant parameter values other than β are $A = 1.1$, $\alpha = 0.335$, $\pi = 0.25$, and $q = 0.5$.

development trap phenomena here are associated with the possibility that an economy's financial system might operate either relatively efficiently or relatively inefficiently, with the efficiency of the financial system being determined endogenously.

VI. Conclusions

We have examined a model of capital accumulation in which spatial separation and limited communication create a role for money and random shocks to agents' portfolio needs create a role for banks. We have seen that in such a model there is considerable scope for the existence of multiple monetary steady states with positive nominal interest rates, for the indeterminacy of monetary equilibria, and for endogenously arising economic fluctuations.

These possibilities arise because the severity of the financial market frictions that agents face are--at least partly--endogenous in this environment. When nominal interest rates are low, agents perceive the costs of being forced to use currency in interlocation exchange as being correspondingly low, and hence they perceive financial markets as providing, relatively speaking, ample liquidity and as functioning "smoothly." When nominal interest rates are high, on the other hand, agents perceive high costs to being forced to use currency in interlocation exchange, there is a premium on liquidity, and the functioning of financial markets seems less smooth.

When $\gamma' > 0$ holds, low nominal interest rates (well-functioning financial markets) lead banks to invest heavily in capital. Hence the financial system operates with relative efficiency, holds low levels of government liabilities, and finances capital formation. High nominal interest rates, in contrast, are associated with banks holding comparatively large quantities of government liabilities, with the result that there is less financing of capital

Appendix

A. Proof of Proposition 1.

Differentiating equation (20) with respect to β gives

(A.1) $$dI/d\beta = [f''(k)/If'(k)]dk/d\beta - [I^2/qf'(k)][1 - qf'(k) - \beta qf''(k)dk/d\beta].$$

Differentiating equation (25) with respect to β gives

(A.2) $$\gamma(I) + (1+\beta)\gamma'(I)dI/d\beta = -\Omega'(k)dk/d\beta.$$

Substituting (A.1) into (A.2) and rearranging terms yields the expression

(A.3) $$\begin{gathered}\gamma(I) - (1+\beta)I^2\gamma'(I)\{[1 - qf'(k)]/qf'(k)\} = \\ -\{\Omega'(k) + (1+\beta)\gamma'(I)[f''(k)/If'(k)] + [I^2\beta f''(k)/f'(k)]\}dk/d\beta.\end{gathered}$$

Thus, since $\Omega' > 0$ by (a.1) and $\gamma' \leq 0$ by hypothesis, $dk/d\beta$ is opposite in sign to the expression

(*) $$\gamma(I)\{1 - (1+\beta)I[I\gamma'(I)/\gamma(I)]\{[1 - qf'(k)]/qf'(k)\}\}.$$

1. Proof of (a).

Since $1 < qf'(k) < (1+\beta)/\beta$ holds (see figure 3.b),

(A.4) $$(1+\beta)[qf'(k) - 1]/qf'(k) < (1+\beta)\{[(1+\beta)/\beta] - 1\}/[(1+\beta)/\beta] = 1.$$

In addition, equations (9) and (10) imply that holds for all $I \geq 1$. Then (a.2) implies that $\eta(I) > 0$ whenever $I \geq 1$.

(b) Immediate from $\gamma(1) = \pi$.

(c) Straightforward differentiation of (40) yields

(A.10) $$\eta'(I)/\eta(I) = -[I(1+\beta I)]^{-1} + (1+\beta)\gamma'(I)/[1-(1+\beta)\gamma(I)].$$

The statement in the text is obtained by rearranging terms in (A.10). $\eta'(I)<0$ follows from the fact that $\rho \in (0,1)$ implies $\gamma'(I) < 0$. □

C. Proof of Lemma 2.

(a) is immediate from $\gamma(1) = \pi$.

(b) From equation (40) it is apparent that $\eta(I) > 0$ iff $1/(1 + \beta) \geq \gamma(I)$.

(c) Equation (A.10) gives an expression for $\eta'(I)/\eta(I)$. Rewrite this as

(A.11) $$I\eta'(I)/\eta(I) = -(1+\beta I)^{-1} + [I\gamma'(I)/\gamma(I)]\{(1+\beta)\gamma(I)/[1-(1+\beta)\gamma(I)]\}.$$

Now use

$$I\gamma'(I)/\gamma(I) = (\rho-1)[1-\gamma(I)]/\rho$$

in (A.11) to obtain

(A.12) $$I\eta'(I)/\eta(I) = -(1+\beta I)^{-1} + [(\rho-1)/\rho][1-\gamma(I)]\{(1+\beta)\gamma(I)/[1-(1+\beta)\gamma(I)]\}.$$

For all I as stated, $\eta(I) > 0$ holds. Hence $\eta'(I) \leq 0$ holds iff the right-hand side of (A.12) is nonpositive. But this is equation (52) in the text. □

But then (56) implies that $\partial I_{t+1}/\partial I_t \geq 0$ when evaluated at $I_t = I_A$. This establishes (a).

(b) $\partial I_{t+1}/\partial I_t < 1$ holds at $I_t = I_A$, as just argued. Thus it remains to show that $\partial I_{t+1}/\partial I_t > -1$ at that same point. The proof is by contradiction.

Suppose to the contrary that--at the point $I_t = I_A$--

(A.19) $\partial I_{t+1}/\partial I_t \leq -1.$

(A.15) and (A.19) then imply that

(A.20) $[2-(1+\beta)\gamma(I_A)]/[1-(1+\beta)\gamma(I_A)] \leq [\rho/(\rho-1)]/[1-\gamma(I_A)](1+\beta I_A).$

Moreover, $\gamma(I_A) < 1/(1+\beta)$ holds, so that $[1-\gamma(I_A)]^{-1} < (1+\beta)/\beta$. But then (A.20) implies that

(A.21) $[\rho/(\rho-1)](1+\beta)/\beta(1+\beta I_A) > [2-(1+\beta)\gamma(I_A)]/[1-(1+\beta)\gamma(I_A)] > 1.$

Since $I_A > 1$ holds, (A.21) implies that $\rho/(\rho-1)\beta > 1$, contradicting (57). This establishes (b) □

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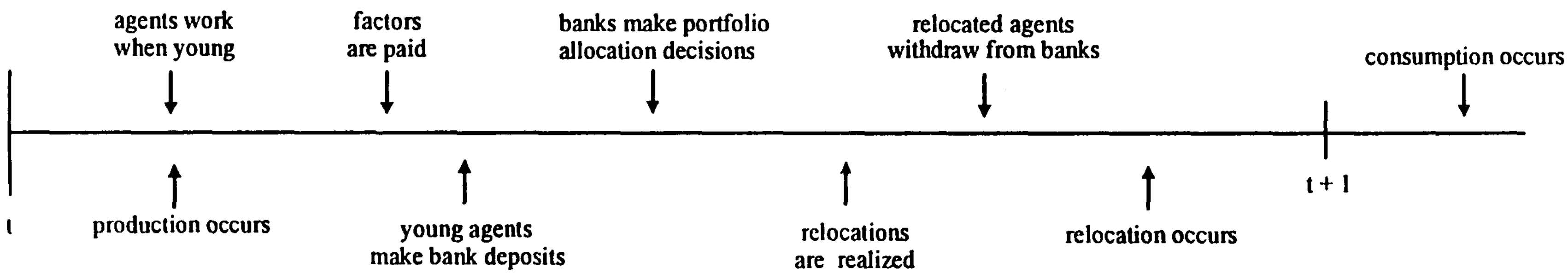
Table 1

ρ	Steady State I	Steady State k	dI_{t+1}/dI_t
1.4	1.1146	.0609	-2.7865
1.7	1.1163	.0607	-1.5947
2.5	1.1189	.0604	-0.7459
3.0	1.1198	.0603	-0.5599
4.0	1.1211	.0601	-0.3737
10.0	1.1234	.0599	-0.1248

Parameter values are:

$\beta = 0$, $A = 1.1$, $\alpha = 0.5$, $\pi = 0.1$, $q = 0.5$

Figure 1
Timing of Events



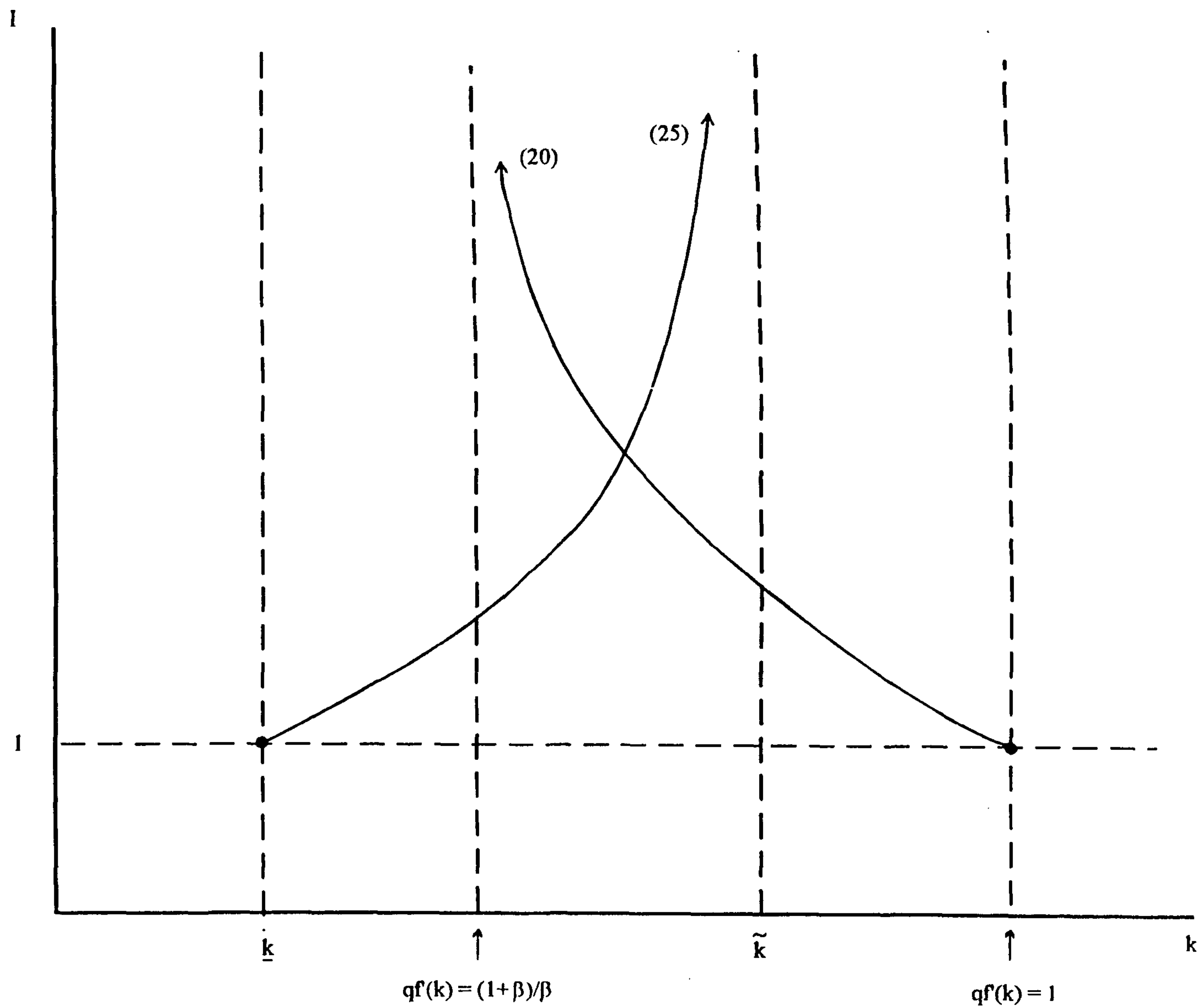


Figure 2b

Steady State Equilibrium

$\rho \in (0,1)$

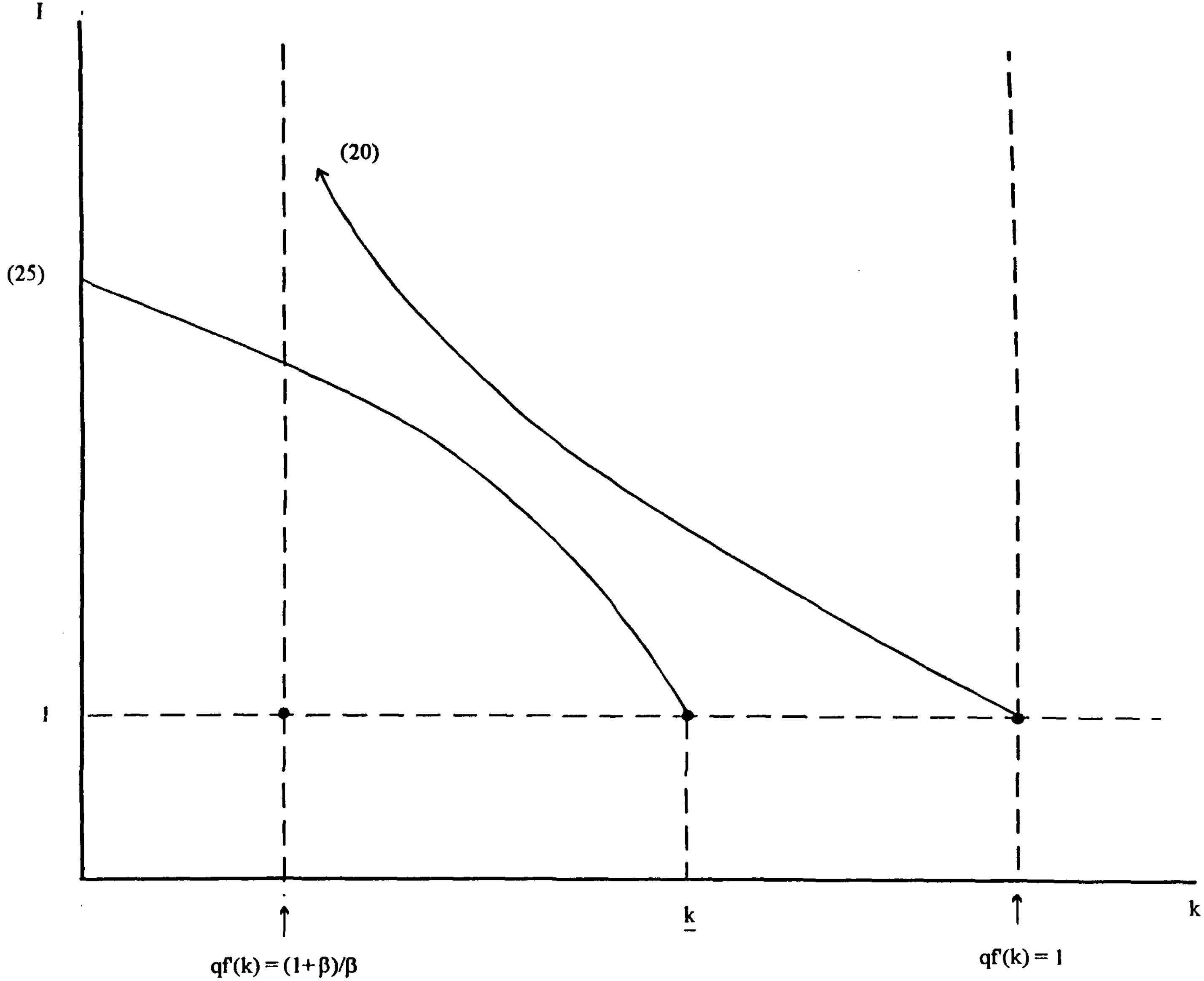


Figure 2.c.ii

Steady State Equilibria

$\rho > 1$

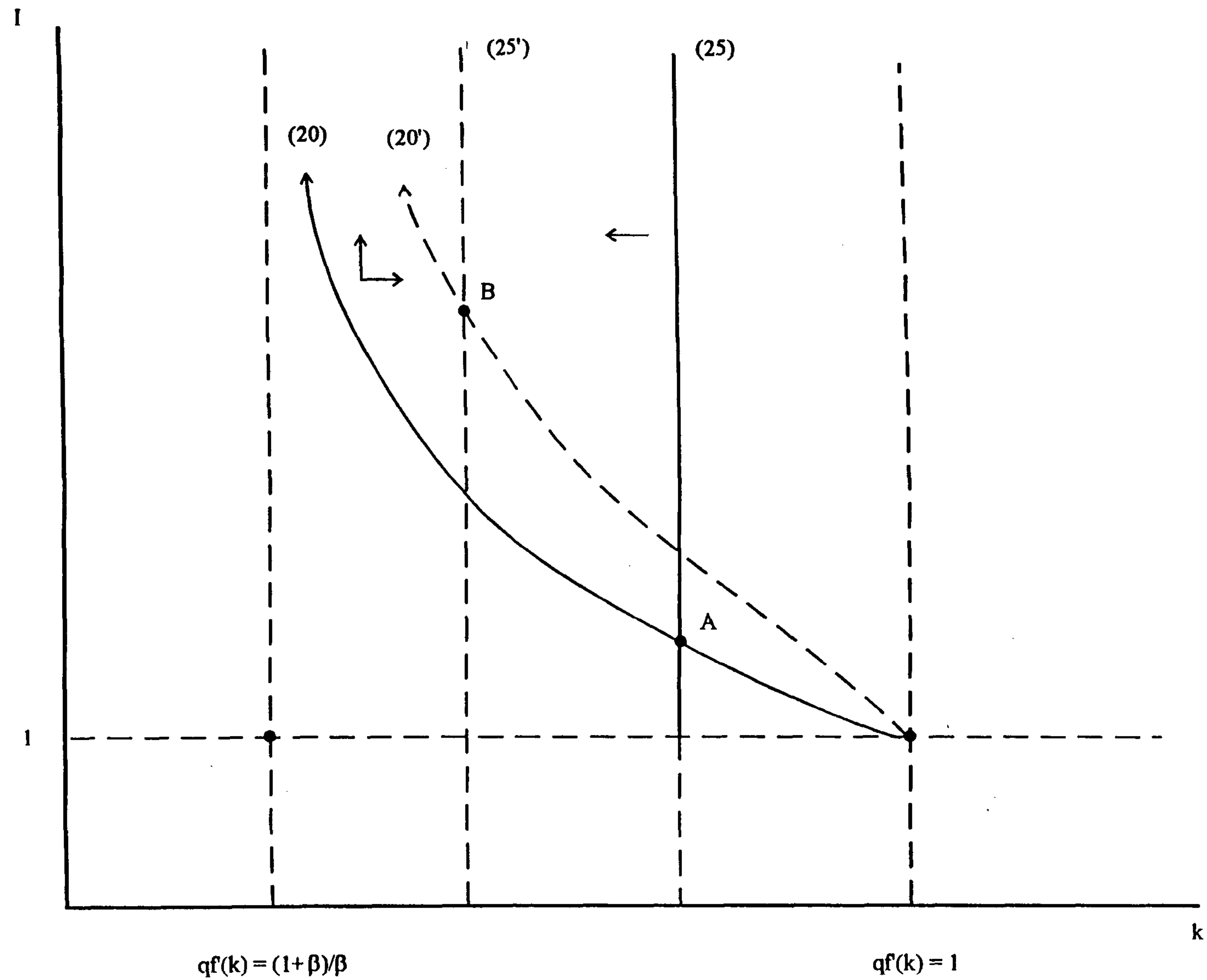


Figure 3.a

Contractionary Monetary Policy

$\rho = 1$

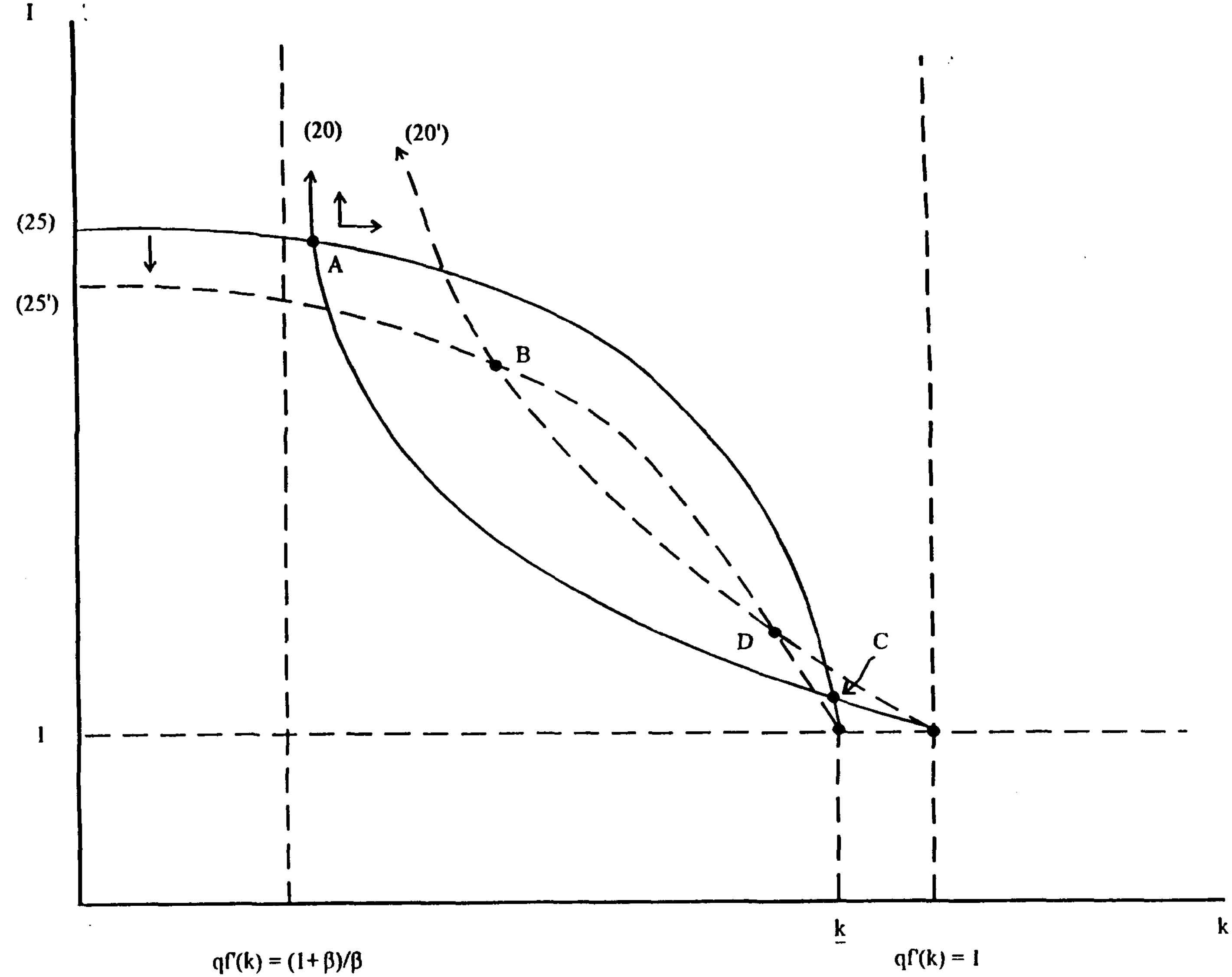


Figure 3.c

Contractionary Monetary Policy

$\rho > 1$

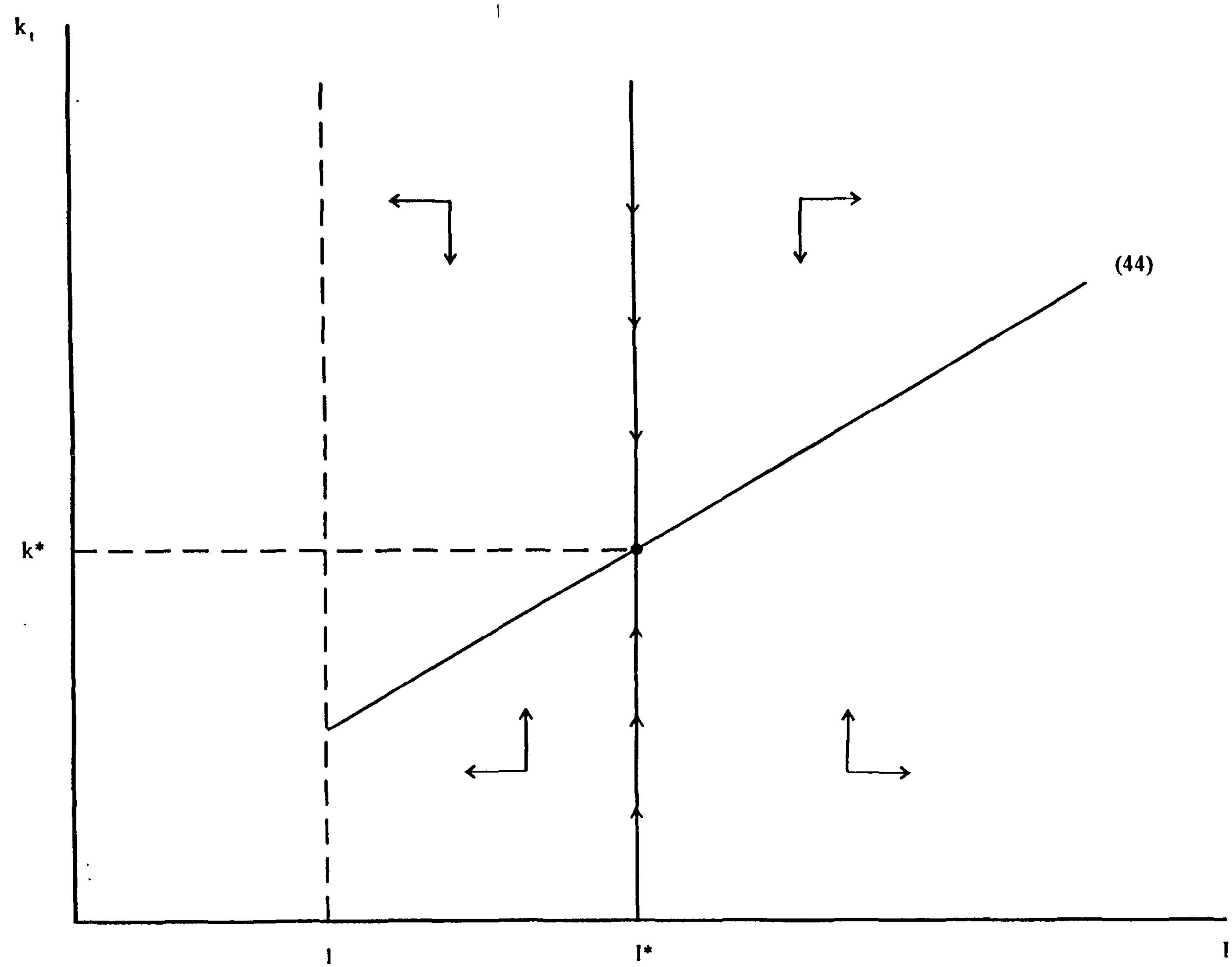


Figure 5

Dynamical Equilibria

$\rho \in (0,1)$

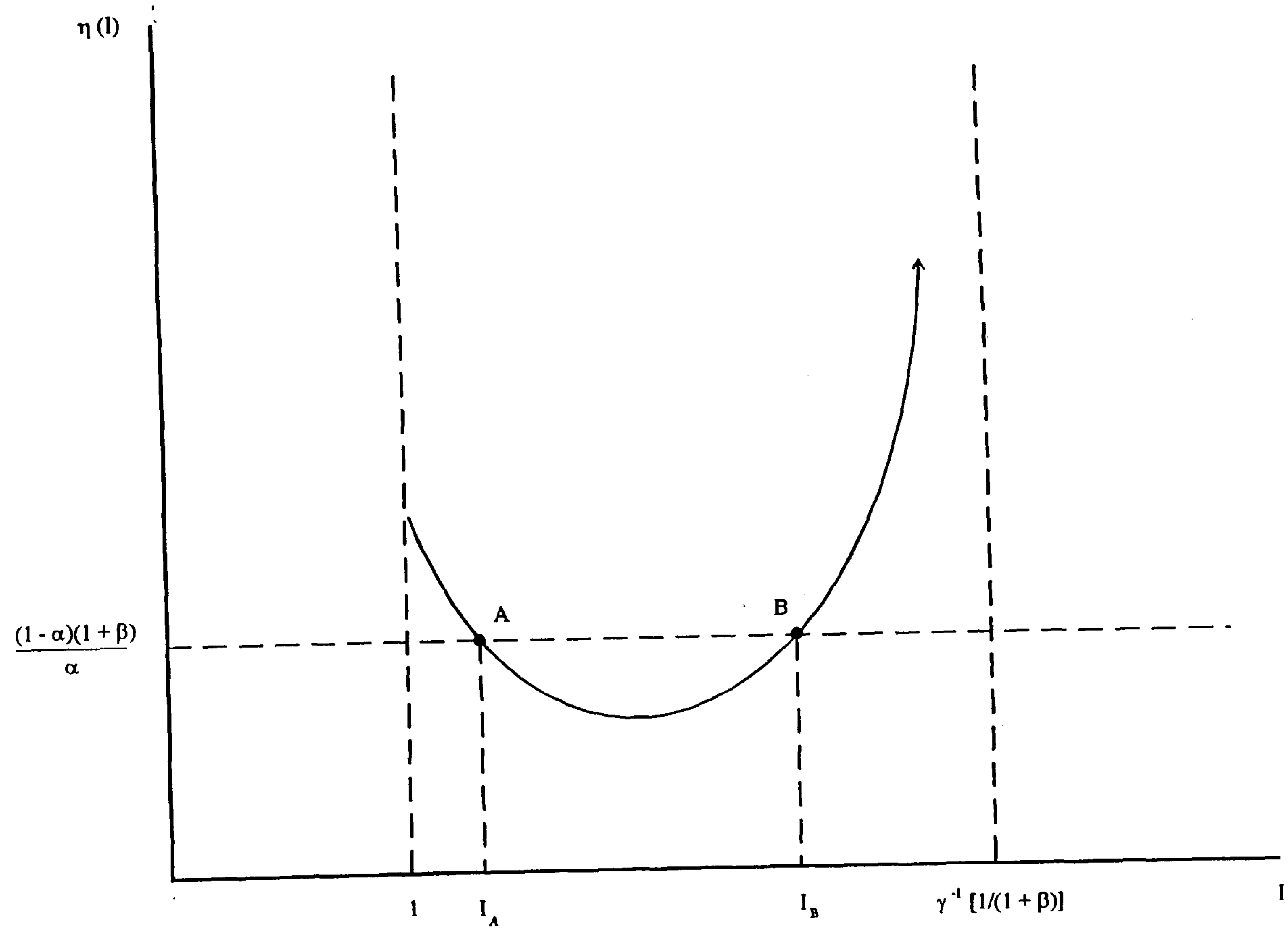


Figure 7