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Michael Dotsey
Federal Reserve Bank of Richmond

Peter Ireland
Federal Reserve Bank of Richmond

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THE WELFARE COST OF INFLATION IN GENERAL EQUILIBRIUM

Michael Dotsey*

and

Peter Ireland*

Research Department
Federal Reserve Bank of Richmond
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Abstract

This paper presents a general equilibrium monetary model in which inflation distorts a variety of marginal decisions. Although individually none of the distortions is very large, they combine to yield substantial welfare cost estimates. A sustained 4% inflation like that experienced in the U.S. since 1983 costs the economy the equivalent of 0.41% of output per year when currency is identified as the relevant definition of money and over 1% of output per year when M1 is defined as money. The results illustrate how the traditional, partial equilibrium approach can seriously underestimate the true cost of inflation.

I. Introduction

A sound judgment regarding the desirability of price stability as the principal goal of monetary policy requires an accurate assessment of the consequences of sustained price inflation. Thus, monetary economists have devoted considerable effort to measuring the welfare cost of inflation. The traditional approach, developed by Bailey (1956) and Friedman (1969), treats real money balances as a consumption good and inflation as a tax on real balances. This approach measures the welfare cost by computing the appropriate area under the money demand curve.

Applications of the Bailey-Friedman analysis, most notably those of Fischer (1981) and Lucas (1981), find the cost of inflation to be surprisingly low. Fischer computes the deadweight loss generated by an increase in inflation from zero to 10% as just 0.3% of GNP using the monetary base as the definition of money. Lucas places the cost of a 10% inflation at 0.45% of GNP using M1 as the measure of money. Since these estimates appear small relative to the potential cost of a disinflationary recession, they provide little support for the idea that price stability is an essential goal for monetary policy.

The inflation tax, however, may distort economic decisions along margins that the partial equilibrium approach of Bailey and Friedman ignores. This paper, therefore, takes a general equilibrium approach to assessing the welfare cost of inflation. A unique feature of the model developed here is an explicit transactions technology that gives rise to a money demand function resembling those estimated with data from the US economy. Thus, the analysis begins by accounting for Bailey-Friedman costs of inflation of the magnitude estimated by Fischer and Lucas.

The Bailey-Friedman approach, however, turns out to capture only a fraction of the total cost of inflation in this model. By explicitly focusing of the role of money in facilitating transactions, the model identifies several other distortions associated with the inflation tax. First, as in Cooley and Hansen (1989, 1991), inflation causes agents to inefficiently substitute out of market activity and into leisure. Second, as suggested by Karni (1974), inflation causes agents to devote productive time to activities that enable them to economize on their cash balances. When adapted to a general equilibrium setting, Karni's specification implies that inflation draws a fraction of the labor force out of goods production and into a distinct financial sector.[1] Finally, the model takes its specification for goods-producing technologies from Romer (1986), so that the allocative effects of inflation can potentially influence the growth rate, as well as the level, of aggregate output. Although none of the additional distortions is very large, they combine to yield estimates of the welfare cost of inflation that are more than three times the size of the Fischer-Lucas estimates.

Black *et al.* (1993), Coleman (1993), De Gregorio (1993), Gomme (1993), Jones and Manuelli (1993), Marquis and Reffett (1993), and Wang and Yip (1993) also examine the effects of inflation in endogenous growth settings. Thus, this paper extends previous work by adding a novel transactions technology to a familiar monetary growth model. Unlike more conventional cash-in-advance specifications, the transactions technology used here can be parameterized to generate a money demand function that is as interest-elastic as those estimated with US data.[2] Consequently, the model is ideally suited for comparing the partial equilibrium Bailey-Friedman cost to the full general equilibrium cost of inflationary policy. Indeed, the results show how the traditional, partial equilibrium approach can seriously underestimate the true cost of inflation and thereby understate the case for price stability.[3]

II. A General Equilibrium Model of the Inflation Tax

A. The Economic Environment

The economy consists of a continuum of markets, indexed by $i\in[0,1)$, arranged on the boundary of a circle with unit circumference. In each market, a distinct, perishable consumption good is produced and traded in each period $t=0,1,2,\ldots$. Hence, the economy's consumption goods are also indexed by $i\in[0,1)$, where good i is sold in market i.

Large numbers of identical households, financial intermediaries, and goods-producing firms inhabit each market i. Enough symmetry is imposed on these agents's preferences, endowments, and technologies that the analysis considers without loss of generality the behavior of a single representative household, a single representative intermediary, and a single representative firm. The representative agents all live at market 0, so that the index i measures the distance of market i from their home.

The government, which otherwise plays no role in the economy, provides households with noninterest-bearing fiat money. It supplies each household with m_0^s units of money at the beginning of period t=0 and augments this supply by making identical lump-sum transfers h_t to all households at the beginning of dates $t=0,1,2,\ldots$. Hence, the per-household money supply m^s_{t+1} at the end of date t satisfies

(1) $$m^s_{t+1} = (1+g_t)m^s_t,$$

where the rate of money growth g_t is given by

(2) $$g_t = h_t/m^s_t.$$

The government announces the complete sequence $\{g_t\}^{\infty}_{t=0}$ of money growth rates at the beginning of period $t=0$. There is no uncertainty, and all agents have perfect foresight.

B. Households and Their Trading Opportunities

The representative household at market $i=0$ has preferences over leisure and the entire continuum of consumption goods as described by the utility function

(3) $$\sum_{t=0}^{\infty}\beta^t\left\{\int_0^1 \ln[c_t(i)]di + BJ_t\right\}, \qquad \beta\in(0,1),\ B>0.$$

Thus, $c_t(i)$ denotes the household's consumption of good i and J_t its leisure at time t.

Following Lucas and Stokey (1983), the representative household is imagined to consist of two members: a worker and a shopper. During each period t, the representative worker rents out his household's capital stock k_t at the real rate r_t and supplies l^g_t units of labor at the real wage w_t to goods-producing firms. He also supplies l^f_t units of labor to financial intermediaries. The worker makes his labor-supply decisions subject to the time constraint

(4) $$1 \geq J_t + l^g_t + l^f_t$$

at each date t.

The representative shopper, meanwhile, travels around the circle in order to acquire goods for his household's consumption. As in Prescott (1987), Schreft (1992), and Gillman (1993), the shopper chooses between two alternative means of making purchases in each market i. His first alternative is to use government-issued money. Since competition equates the nominal price p_t of consumption goods across markets, the shopper may acquire $c_t(i)$ units of good i in exchange for $p_tc_t(i)$ units of money at time t.

The shopper's second alternative for purchasing good i is to enlist the services of a financial intermediary. At a cost of $\gamma(i)$ units of labor, an intermediary verifies the shopper's identity and guarantees his ability to pay, so that a firm in market i is willing to sell its output on credit at time t. The communications and record-keeping costs of facilitating a credit transaction do not depend on the size of the purchase but increase as the shopper travels farther from home. Hence, γ is a strictly increasing function of i. Under the additional assumption that $\lim_{i\to 1} \gamma(i)=\infty$, some goods will always be purchased with cash, and there is a well-defined demand for money.

In exchange for its services at time t, the intermediary in market i charges the representative household the real price $q_t(i)$. Since the intermediary's cost $\gamma(i)$ is independent of the size of the transaction but depends nontrivially on i, competition ensures that the function $q_t(i)$ satisfies these same properties. Thus, the representative shopper may acquire $c_t(i)$ units of good i on credit at time t at a total nominal cost of $p_t[c_t(i)+q_t(i)]$: $p_tc_t(i)$ to pay for the goods themselves and $p_tq_t(i)$ to compensate the intermediary.

Let the indicator function $\xi_t(i)=0$ if the representative shopper purchases good i with money at time t, and let $\xi_t(i)=1$ if he uses an intermediary instead. Let m_t denote the nominal cash balances carried by the shopper into time t; these are augmented at the beginning of the period by the government transfer h_t. Since the shopper must use money whenever he chooses not to hire an intermediary, he faces the cash-in-advance constraint

$$\frac{m_t + h_t}{p_t} \geq \int_0^1 [1-\xi_t(i)]c_t(i)di \tag{5}$$

in each period t.

After consuming its purchases at the end of time t, the household participates in a centralized asset market, where it receives its rental payments r_tk_t and wages $w_t(l_t^g+l_t^f)$ and pays for the goods that it bought on credit earlier in time t. The household uses any excess funds to accumulate the cash balances m_{t+1} that it will carry into period t+1 and to purchase unsold output from the representative firm, which it combines with its depreciated capital stock $(1-\delta)k_t$ in order to carry k_{t+1} units of capital into period t+1.

The representative household is also permitted to borrow from and lend to other households in the end-of-period asset market by issuing or purchasing one-period, nominally-denominated discount bonds. Bonds paying off b_{t+1} units of money in the time t+1 asset market sell for b_{t+1}/R_t units of money in the time t asset market, where R_t is the gross nominal interest rate between t and t+1. Since these bonds are available in zero net supply, $b_{t+1}=0$ must hold as an equilibrium condition in each period t.

As sources of funds in period t, the representative household has its initial money and bond holdings, its beginning-of-period government transfer, its rental and wage receipts, and its capital stock after depreciation. As uses of funds it has its purchases of consumption goods, its payments to intermediaries, and the capital, money, and bonds that it will carry into period t+1. It therefore faces the budget constraint

$$\frac{m_t + b_t + h_t}{p_t} + r_t k_t + w_t(l_t^g + l_t^f) + (1-\delta)k_t \tag{6}$$

$$\geq \int_0^1 c_t(i)di + \int_0^1 \xi_t(i)q_t(i)di + k_{t+1} + \frac{m_{t+1}}{p_t} + \frac{b_{t+1}}{p_t R_t}$$

in each period t. The representative household chooses sequences for $c_t(i)$, $\xi_t(i)$, J_t, l_t^g, l_t^f, k_{t+1}, m_{t+1}, and b_{t+1} to maximize the utility function (3) subject to the time constraint (4), the cash-in-advance constraint (5), and the budget constraint (6), taking the sequences for h_t, r_t, w_t, p_t, $q_t(i)$, and R_t as given. It also takes its initial holdings of capital $k_0>0$, money $m_0=m_0^s$, and bonds $b_0=0$ as given.

C. The Representative Intermediary's Problem

An intermediary in market i hires $\gamma(i)$ units of labor and charges $q_t(i)$ if the representative shopper purchases good i on credit at time t. Thus, the representative intermediary chooses total labor input n_t^f to maximize profits

$$\pi_t^f = \int_0^1 \xi_t(i)q_t(i)di - w_t n_t^f \tag{7}$$

subject to the technological constraint

$$(8) \qquad n_t^f \geq \int_0^1 \xi_t(i)\gamma(i)di$$

at each date t, taking w_t, $\xi_t(i)$, and $q_t(i)$ as given.

D. The Representative Goods-Producing Firm's Problem

The representative goods-producing firm in market i=0 hires k_t units of capital and n_t^g units of labor from households in each period t in order to produce output of consumption good i=0. Its profits in period t are

$$(9) \qquad \pi_t^g = A(k_t)^{\alpha}(n_t^g)^{1-\alpha}(K_t)^{\eta} - r_t k_t - w_t n_t^g, \qquad \alpha\in(0,1),\ \eta>0.$$

The production function in equation (9) contains K_t, the aggregate capital stock per household at time t. Following Romer (1986), capital is interpreted broadly here to include stocks of human capital and disembodied knowledge in addition to physical capital. While goods production features constant returns to scale at the firm level, spillover effects associated with the accumulation of human capital generate increasing returns at the aggregate level. Increasing returns make the economy's growth rate endogenous and possibly dependent on the inflation rate. The representative firm takes the aggregate capital stock K_t as well as the factor prices r_t and w_t as given when maximizing (9).

E. Equilibrium Conditions

A competitive equilibrium in this economy consists of sequences for prices and quantities that are consistent with the solutions to the optimization problems for households, intermediaries, and firms outlined above. Given the initial conditions $k_0=K_0>0$, $m_0=m_0^s$, and $b_0=0$, equilibrium prices and quantities must also satisfy the zero profit conditions

(10) $$\pi_t^g = \pi_t^f = 0,$$

the consistency condition

(11) $$k_{t+1} = K_{t+1},$$

and the market-clearing conditions for goods, labor, money, and bonds

(12) $$A(k_t)^{\alpha+\eta}(n_t^g)^{1-\alpha} + (1-\delta)k_t = k_{t+1} + \int_0^1 c_t(i)di,$$

(13) $$n_t^g = l_t^g,$$

(14) $$n_t^f = l_t^f,$$

(15) $$m_{t+1} = m_{t+1}^s,$$

and

(16) $$b_{t+1} = 0$$

in each period t.

III. The General Equilibrium Effects of the Inflation Tax

Part A of the appendix demonstrates that in equilibrium, there exists a borderline index s_t for each date t such that the representative household purchases all goods with indices $i \le s_t$ on credit and all goods $i > s_t$ with cash. This borderline index is determined by the solution to

(17) $$\gamma(s_t) = [\ln(\lambda_t+\mu_t)-\ln(\lambda_t)]/w_t\lambda_t,$$

where λ_t is the nonnegative multiplier on the budget constraint (6) and μ_t is the nonnegative multiplier on the cash-in-advance constraint (5) from the household's optimization problem. As in Schreft (1992) and Gillman (1993), the shopper uses credit close to home and cash far from home, since intermediation costs increase with distance.

The representative household's optimal $c_t(i)$ is a step function at each date t:

$$(18) \qquad c_t(i) = \begin{cases} c_t^1 = 1/\lambda_t & \text{for } i \le s_t \\ c_t^0 = 1/(\lambda_t + \mu_t) & \text{for } i > s_t \end{cases}.$$

Since $\mu_t \ge 0$, $c_t^1 \ge c_t^0$. As in Prescott (1987), the shopper makes larger purchases on credit and smaller purchases with cash, since the intermediation costs are independent of the size of the transaction.

Equation (18) and the cash-in-advance constraint (5) determine equilibrium money demand as

$$(19) \qquad (m_t + h_t)/p_t = (1 - s_t)c_t^0.$$

The technological constraint (8) and the market-clearing condition (14) determine employment in the financial sector as

$$(20) \qquad l_t^f = \int_0^{s_t} \gamma(i)di.$$

The inflation tax causes the household's cash-in-advance constraint to bind, so that higher rates of inflation tend to be associated with larger values of the multiplier μ_t. Since γ is increasing as a function of i, equation (17) suggests that higher inflation rates are also associated with higher values of s_t. That is, under higher rates of inflation the representative household purchases a wider range of goods with the help of intermediaries.

Equation (18) then indicates that the inflation tax distorts consumption and production decisions in two ways. First, since $c_t^1 > c_t^0$, the representative household purchases different consumption goods in different quantities; its marginal rate of substitution between cash and credit goods deviates from the corresponding marginal rate of transformation. Second, since c_t^0 is decreasing as a function of μ_t, the representative household purchases cash goods in smaller quantities so that overall, market activity is reduced. These are the marginal effects of the inflation tax studied by Cooley and Hansen (1989, 1991). Here, however, the production technology described in equation (9) allows these allocative effects of inflation to change the growth rate, as well as the level, of aggregate output.

Equation (19) suggests that the representative household economizes on its cash balances in the face of a positive inflation tax both by purchasing a wider range of goods without money (i.e., by increasing s_t) and by consuming less of those goods that it purchases with money (i.e., by decreasing c_t^0). Thus, the demand for money is interest-elastic and gives rise to the Bailey-Friedman cost of the inflation tax.

Finally, equation (20) indicates that as the household increases s_t in response to a higher inflation tax, the size of the labor force employed in the financial sector increases. The diversion of labor resources out of productive activity and into finance also contributes to the welfare cost of inflation. Again, the goods-producing technology in (9) provides a channel through which this allocative effect can influence the economy's long-run growth rate.

Thus, the model associates a number of distortions with the inflation tax. It is not possible, however, to assess the magnitude of any of these distortions analytically. Hence, the following sections apply numerical methods to determine the quantitative effects of inflation in general equilibrium.

IV. Model Parameterization

In order to apply numerical methods, specific values must be assigned to the model's parameters. The household's discount rate is set at $\beta=0.99$ and the depreciation rate at $\delta=0.025$ so that each period in the model corresponds to one quarter year. Sustained, balanced growth occurs when the aggregate production function is linear in the capital stock, so $\alpha=0.4$ and $\eta=0.6$. With $A=0.265$, the economy grows at a constant annual rate of 2% (the US average since 1959) under a constant annual inflation rate of 5% (again, the US average since 1959). The representative worker devotes 20% of his time to labor (the figure used by King and Rebelo 1993) under 5% inflation when $B=4.25$.

The magnitude of the Bailey-Friedman cost of inflation hinges on two numbers: the size of the tax base and the interest elasticity of money demand. When the intermediary's cost function is specialized to

$$\gamma(i) = \gamma[i/(1-i)]^{\theta}, \qquad \gamma>0,\ \theta>0, \tag{21}$$

the parameters γ and θ can be chosen so that the size of the tax base and the interest elasticity of money demand in the model match corresponding figures in the US economy.[4] The next section constructs equilibria for two specifications, one in which money is defined as currency and the other in which money is defined as M1. The alternative definitions of money require different sets of values for γ and θ.

Following Cooley and Hansen (1991), the size of the inflation tax base in the US economy is measured by the fraction of all purchases that are made using money. Avery *et al.* (1987) report that in 1984, when inflation was about 4%, US households made 30% of their transactions with currency and 82% of their transactions with M1. These fractions correspond to the value of $1-s_t$ under 4% inflation in the model.

Annual data from 1959-1991 yield estimates of the money demand equations

$$\ln(vc) = 2.88 + 2.73R$$

and

$$\ln(v1) = 1.24 + 5.95R,$$

where vc is the income velocity of currency, v1 is the income velocity of M1, and R is the 6-month commercial paper rate.[5] The OLS coefficients on R in these equations measure the long-run interest semi-elasticity of money demand. An analogous statistic in the model economy is

$$[\ln(v_{10})-\ln(v_0)]/(R_{10}-R_0),$$

where v_{10} and v_0 are the constant annual velocities of money and R_{10} and R_0 are the constant annual nominal interest rates that prevail under constant annual inflation rates of 10% and zero.

To match the tax base and the elasticity figures in the data and model, $\gamma=0.00075$ and $\theta=2.45$ for the currency specification and $\gamma=0.00933$ and $\theta=0.333$ for the M1 specification. With these combinations of γ and θ, the annual velocity of money under 5% inflation in the model economy is 19.9 for currency and 7.6 for M1. Annualized velocity in the model varies inversely with the assumed length of each period; a shorter period length implies a higher annual velocity. Thus, the fact that these figures for velocity are similar in magnitude to the averages of 21.6 for currency and 5.4 for M1 found in US data since 1959, they are also consistent with the identification of one model period as one quarter year.

V. The Quantitative Effects of Inflation in General Equilibrium

This section computes the welfare cost of monetary policies that call for constant rates of money growth. These policies give rise to steady-state equilibria in which all variables grow at constant rates. Part B of the appendix outlines a method for constructing these steady-state equilibria.

Table 1 describes steady-state equilibria under the benchmark policy that yields a constant zero inflation rate.[6] It compares these equilibria to those obtaining under constant 4% (the US average since 1983), and 10% (the alternative policy considered by Fischer 1981 and Lucas 1981) annual rates of inflation. It also reports results from adopting the Friedman (1969) rule, under which the money supply is contracted at the rate of time preference so as to make the nominal interest rate equal to zero. With an annual rate of time preference of about 4% and a 2% annual rate of output growth, the steady state real interest rate in this economy is approximately 6%.[7] Thus, following the Friedman rule generates a 6% annual rate of price deflation.

Under a constant rate of inflation, the representative shopper makes a constant fraction of his purchases with cash. The model is parameterized so that with 4% annual inflation, this constant fraction is about 30% if money is defined as currency and about 80% if money is defined as M1. Table 1 indicates that for either specification, the shopper uses money in a smaller range of transactions when inflation is higher. Thus, the steady-state velocity of money rises with the inflation rate.

The model is parameterized so that the representative worker devotes approximately 20% of his time endowment to labor. Table 1 shows that as the inflation rate rises, the representative household tends to substitute out of market activity, which requires either money or costly financial services, and into leisure, which can be enjoyed without the use of a means of exchange. In addition to this substitution effect, however, there is a negative wealth effect associated with an increase in the inflation tax. While the substitution effect always dominates in Cooley and Hansen's (1989, 1991) models, Cole and Stockman (1992) find that the wealth effect can easily dominate in their version of the cash-in-advance model in which the use of money can be circumvented at a cost in terms of real resources. The wealth effect can dominate here as well, so that an increase from 4% to 10% inflation actually increases the household's labor supply under the M1 specification.

Higher rates of inflation shift the allocation of the labor force in addition to changing the total labor supply. Table 1 shows that while the fraction of the labor force working for intermediaries is always less than 1.5%, this share rises with the inflation rate. The substitution of labor out of goods production and into leisure and finance tends to reduce the growth rate of output via the spillover effects of aggregate productive activity. But the relationship between inflation and growth is not generally monotonic; the wealth effect that increases the household's labor supply going from 4% to 10% inflation also increases the economy's growth rate under the M1 specification. In general, the effects of inflation on growth are small: 10% inflation reduces the annual growth rate from 2.12% to 2.07% under the currency specification and from 2.03% to 1.97% under the M1 specification.

The welfare cost of inflation is measured, following Cooley and Hansen (1989, 1991), by the permanent percentage increase in the consumption of all goods that makes that representative household as well off under a positive rate of inflation as it is under zero inflation. This figure is converted from a fraction of consumption to a fraction of output by multiplying it by the constant ratio of consumption to output under positive inflation. Table 1 shows that when money is defined as currency, a sustained 4% inflation like that experienced in the US since 1983 has a cost that is equivalent to a permanent 0.41% decrease in output. A 10% inflation costs almost 0.92% of output. When money is defined as M1, a 4% inflation costs 1.08% of output, and a 10% inflation costs 1.73% of output. Table 1 also shows the welfare gain from adopting the Friedman (1969), equivalent to a permanent 0.91% increase in output in the currency specification and a permanent 2.22% increase in output in the M1 specification.

Table 2 reports the Bailey-Friedman cost of inflation in the model economy, computed as the area under the money demand curve that is lost as the steady-state inflation rate increases. Since by construction the model gives rise to a money demand curve that resembles those estimated with US data, the Bailey-Friedman costs are quite similar to those reported by Fischer (1981) and Lucas (1981). With money defined as currency, the Bailey-Friedman analysis puts the cost of a 10% inflation at about 0.06% of output. Both the tax base and the elasticity of demand are larger when money is defined as M1. Hence, the Bailey-Friedman cost of inflation is higher as well: a 10% inflation costs about 0.42% of income.

The Bailey-Friedman approach also indicates that the welfare gain from adopting the Friedman rule is substantial, equal to 0.59% of output, under the currency specification. Recall that for currency, the parameters of the transaction technology (21) are set so that under 4% inflation, the representative household makes only 30% of its purchases with money. The representative household makes all of its purchases with cash under the Friedman rule, since the zero nominal interest rate eliminates the opportunity cost of holding real balances. In order to reduce the fraction of cash transactions from 100% under the Friedman rule to 30% under 4% inflation, the model must give rise to a money demand function that is extremely elastic at low nominal rates of interest. Here, as in Lucas (1993), the high elasticity of money demand at interest rates close to zero implies that there are large welfare gains from moving to the Friedman rule.

Comparing the welfare cost estimates in tables 1 and 2 illustrates that the partial equilibrium analysis of Bailey and Friedman generally captures only a fraction of the total cost of the inflation tax. In addition to its effects on velocity, the inflation tax causes agents to inefficiently allocate productive labor across its various uses. The labor-supply effects may seem small, but they contribute to estimates of the total welfare cost of inflation that are much larger than those obtained using the Bailey-Friedman approach.

In order to isolate the contribution of labor-supply effects to the welfare cost estimates reported in table 1, table 3 considers a version of the model with exogenous growth. This version of the model replaces the production function shown in equation (9) with the more conventional Cobb-Douglas specification

$$(k_t)^{\alpha}(x_t n_t^g)^{1-\alpha}.$$

Instead of the aggregate spillovers emphasized by Romer, sustained growth is now driven by exogenous technological change:

$$x_t = \rho x_{t-1}.$$

As before, $\alpha=0.4$; the economy grows at the annual rate of 2% in steady state when $\rho=(1.02)^{1/4}$.

Table 3 demonstrates that, for the most part, the effects of inflation do not depend on whether growth is endogenous or exogenous. The changes in velocity, total labor supply, and the share of the labor force in finance shown in table 3 are almost identical to those in table 1. The welfare cost estimates, however, are much smaller under exogenous growth. Consider, for example, that in the currency specification with endogenous growth, the annual growth rate falls from 2.12% under zero inflation to 2.07% under 10% inflation. As emphasized by Lucas (1987), policies that induce even small changes in an economy's growth rate have substantial welfare consequences. In fact, comparing tables 1 and 3 reveals that growth effects increase the welfare cost of 10% inflation from 0.20% to 0.91% of output under the currency specification and from 0.92% to 1.73% of output under the M1 specification.

Since these results indicate that growth effects play a large role in generating the welfare cost estimates reported in table 1, it is worth noting that empirically, Kormendi and Meguire (1985), Fischer (1991), and De Gregorio (1993) find that differences in inflation do contribute significantly to explaining cross-country differences in growth. Levine and Renelt (1992) argue that results from cross-country studies such as these are not generally robust. However, the results in table 1 also explain why the inflation-growth rate link may be difficult to detect in the data: the changes in growth rates are small and not always monotonic. Finally, note that although the economy's growth rate increases going from 4% to 10% inflation under the M1 specification, welfare still decreases. Policies that promote growth do not always increase welfare.

VI. Summary and Conclusions

In the general equilibrium model developed here, the inflation tax distorts a variety of marginal decisions. Agents inefficiently economize on their holdings of real cash balances. They substitute out of market activity by taking more leisure. They divert productive resources out of goods production and into finance.

The model shows that individually, none of these distortions is very large. By construction, the model's money demand function matches those estimated with US data. Hence, the Bailey-Friedman cost of inflation in the model is similar in magnitude to the figures obtained with US data by Fischer (1981) and Lucas (1981), which are too small to justify the expense of a disinflationary recession. Similarly, the effects of inflation on the total labor supply and its sectoral allocation are small. These labor-supply effects are allowed to influence the economy's long-run growth rate, but a 10% inflation turns out to reduce the growth rate by only 0.05% compared to a regime of price stability.

The various small distortions, however, combine to yield substantial estimates of the total cost of inflation. A 4% inflation like that experienced in the US since 1983 costs the economy 0.41% of output per year when currency is identified as the relevant definition of money and over 1% of output per year when M1 is defined as money. These higher estimates strengthen the case for making price stability the principal objective for monetary policy.

More generally, the findings demonstrate the usefulness of general equilibrium models for policy evaluation. In this case, a partial equilibrium approach to measuring the welfare cost of suboptimal policy grossly underestimates the true welfare effects. Only when all of the policy-induced distortions are allowed to interact can reliable estimates be obtained.

Appendix

This appendix derives the equilibrium conditions presented as equations (17)-(20) in the text. It also outlines methods for numerically constructing competitive equilibria under policies that call for constant money growth rates.

A. Derivation of Equilibrium Conditions

In the representative household's optimization problem, let λ_t be the nonnegative multiplier on the budget constraint (6) and let μ_t be the nonnegative multiplier on the cash-in-advance constraint (5). Let $c_t^0(i)$ be the household's consumption of good i at time t if it purchases this good with money; let $c_t^1(i)$ be the household's consumption of good i at time t if it purchases this good on credit. The first order conditions from the household's problem are

(A.1) $$c_t^0(i) = c_t^0 = 1/(\lambda_t+\mu_t)$$

(A.2) $$c_t^1(i) = c_t^1 = 1/\lambda_t$$

(A.3) $$c_t(i) = [1-\xi_t(i)]c_t^0 + \xi_t(i)c_t^1$$

(A.4) $$\xi_t(i) = \begin{cases} 1 \text{ if } \ln(c_t^1)-\lambda_t[c_t^1+q_t(i)]\geq\ln(c_t^0)-(\lambda_t+\mu_t)c_t^0 \\ 0 \text{ otherwise} \end{cases}$$

(A.5) $$\lambda_t w_t = B$$

(A.6) $$\lambda_t = \beta\lambda_{t+1}[r_{t+1}+(1-\delta)]$$

(A.7) $$\lambda_t/p_t = \beta(\lambda_{t+1}+\mu_{t+1})/p_{t+1}$$

(A.8) $\lambda_t/p_tR_t \;=\; \beta\lambda_{t+1}/p_{t+1}$.

The first order condition from the representative intermediary's problem is

(A.9) $$n_t^f \;=\; \int_0^1 \xi_t(i)\gamma(i)di.$$

The first order conditions from the representative goods-producing firm's problem are

(A.10) $$r_t \;=\; \alpha A(k_t)^{\alpha-1}(n_t^g)^{1-\alpha}(K_t)^\eta$$

(A.11) $$w_t \;=\; (1-\alpha)A(k_t)^\alpha(n_t^g)^{-\alpha}(K_t)^\eta.$$

Equation (A.9) and the zero-profit condition $\pi_t^f=0$ imply that

(A.12) $q_t(i) \;=\; w_t\gamma(i)$.

Equations (A.1)-(A.4) and (A.12) imply the existence of the borderline index s_t satisfying

(A.13) $\gamma(s_t) \;=\; [\ln(\lambda_t+\mu_t)-\ln(\lambda_t)]/w_t\lambda_t$

such that

(A.14a) $\xi_t(i)=1,\; c_t(i)=c_t^1$ for $i\le s_t$

(A.14b) $\xi_t(i)=0,\; c_t(i)=c_t^0$ for $i>s_t$.

Equations (A.13) and (A.14) correspond to (17) and (18) in the text. Equation (A.14) and the cash-in-advance constraint (5) imply equation (19) in the text. In light of (A.14) and the market-clearing condition (14), equation (A.9) can be rewritten as equation (20) in the text.

B. Construction of Steady-State Equilibria

Given the initial conditions $k_0=K_0>0$, $m_0=m_0^s$, and $b_0=0$, a competitive equilibrium consists of the sequences $\{h_t,\ g_t,\ m_{t+1}^s,\ n_t^g,\ r_t,\ w_t,\ k_{t+1},\ K_{t+1},\ c_t(i),\ \xi_t(i),\ J_t,\ l_t^g,\ l_t^f,\ m_{t+1},\ b_{t+1},\ p_t,\ q_t(i),\ R_t,\ n_t^f\}_{t=0}^\infty$. When monetary policy calls for a constant rate of money growth, $g_t=g$ for all $t=0,1,2,\ldots$ and equations (1) and (2) in the text determine $\{h_t,\ m^s_{t+1}\}^{\infty}_{t=0}$. In light of the consistency condition (11) and the market-clearing conditions (13)-(16), the task of constructing a competitive equilibrium reduces to finding sequences $\{r_t,\ w_t,\ k_{t+1},\ c_t(i),\ \xi_t(i),\ J_t,\ l^g_t,\ l^f_t,\ p_t,\ q_t(i),\ R_t\}^{\infty}_{t=0}$ that satisfy the remaining equilibrium conditions.

Equation (A.5) can be used to solve for $w_t=B/\lambda_t$. Substituting this solution into (A.13) yields

(A.15) $$\gamma(s_t) = [\ln(\lambda_t+\mu_t)-\ln(\lambda_t)]/B.$$

Equations (A.9) and (A.14) determine $l^f_t=n^f_t$, $\xi_t(i)$, and $c_t(i)$ in terms of s_t, λ_t, and μ_t. Equation (19) in the text can be rewritten as

(A.16) $$p_t = m^s_{t+1}(\lambda_t+\mu_t)/(1-s_t),$$

which determines p_t in terms of s_t, λ_t, μ_t, and the exogenously-given m^s_{t+1}. The household's first order condition (A.8) then determines R_t in terms of s_t, s_{t+1}, λ_t, λ_{t+1}, μ_t, μ_{t+1}, and the constant g, while equation (A.12) determines $q_t(i)=B\gamma(i)/\lambda_t$ in terms of λ_t.

When $\alpha+\eta=1$, equations (A.10) and (A.11) determine $l^g_t=n^g_t$ and r_t in terms of k_t and λ_t:

(A.17) $$l^g_t = [(1-\alpha)A\lambda_t k_t/B]^{1/\alpha}$$

(A.18) $$r_t = \alpha A[(1-\alpha)A\lambda_t k_t/B]^{(1-\alpha)/\alpha}.$$

The solutions for l^g_t and l^f_t and the household's time constraint (4) then determine J_t as a function of k_t, s_t, and λ_t.

Substituting these results into the goods market-clearing condition (12) and the household's first order conditions (A.6) and (A.7) yields

$$(A.19)\quad k_{t+1} = Ak_t[(1-\alpha)A\lambda_t k_t/B]^{(1-\alpha)/\alpha} + (1-\delta)k_t - s_t/\lambda_t - (1-s_t)/(\lambda_t+\mu_t)$$

$$(A.20)\quad \lambda_t = \beta\lambda_{t+1}\{\alpha A[(1-\alpha)A\lambda_t k_t/B]^{(1-\alpha)/\alpha}+(1-\delta)\}$$

$$(A.21)\quad \lambda_t(1-s_t) = [\beta/(1+g)](\lambda_t+\mu_t)(1-s_{t+1}).$$

Equations (A.15) and (A.19)-(A.21) represent a system of four equations in $\{k_{t+1},\ s_t,\ \lambda_t,\ \mu_t\}_{t=0}^{\infty}$.

Define the variables $k_t^*=k_{t+1}/k_t$, $\lambda_t^*=\lambda_t k_t$, and $\mu_t^*=\mu_t k_t$. In equilibria where $k_t^*=k^*$, $s_t=s^*$, $\lambda_t^*=\lambda^*$, and $\mu_t^*=\mu^*$, equations (A.15) and (A.19)-(A.21) become

$$(A.22)\quad \gamma(s^*) = [\ln(\lambda^*+\mu^*)-\ln(\lambda^*)]/B$$

$$(A.23)\quad k^* = A[(1-\alpha)A\lambda^*/B]^{(1-\alpha)/\alpha} + (1-\delta) - s^*/\lambda^* - (1-s^*)/(\lambda^*+\mu^*)$$

$$(A.24)\quad k^* = \beta\{\alpha A[(1-\alpha)A\lambda^*/B]^{(1-\alpha)/\alpha}+(1-\delta)\}$$

$$(A.25)\quad \lambda^* = [\beta/(1+g)](\lambda^*+\mu^*),$$

which can be solved numerically for the constants k^*, s^*, λ^*, and μ^*.

The initial condition $k_0>0$ and the definition $k^*=k_{t+1}/k_t$ can then be used to find k_{t+1} for all t=0,1,2,.... The definitions $s^*=s_t$, $\lambda^*=\lambda_t k_t$, and $\mu^*=\mu_t k_t$ determine s_t, λ_t, and μ_t for all t=0,1,2,.... With the sequences $\{k_{t+1},\ s_t,\ \lambda_t,\ \mu_t\}_{t=0}^{\infty}$ in hand, all other equilibrium prices and quantities can be deduced. All of these prices and quantities are either constant or grow at a constant rate.

Notes

[1]This general equilibrium interpretation of Karni's insight is consistent with Yoshino's (1993) finding that inflation and employment in banking have been positively correlated over time in the US and other countries.

[2]In Cooley and Hansen's (1989) single-good cash-in-advance model, for example, the velocity of money is practically constant. Thus, money demand is highly interest-inelastic. Cooley and Hansen (1991) use a multiple-good cash-in-advance model that, in principle at least, allows velocity to vary with the inflation rate. Benabou (1991), however, demonstrates that this alternative cash-in-advance formulation also generates a very inelastic money demand function.

[3]See Imrohoroglu's (1992) work for a very different general equilibrium model that also yields the result that the Bailey-Friedman analysis captures only a fraction of the total cost of inflation.

[4]Cooley and Hansen (1991) choose one parameter to match the size of the inflation tax base in their model with the analogous figure from the US data. Similarly, Lacker and Schreft (1993) choose one parameter to match the interest elasticity of money demand in their model and the US data. Thus, the approach taken here combines the methods of these earlier studies by setting two parameters in order to match both the size of the tax base and the interest elasticity in the model and data.

[5]All data are taken from the *Economic Report of the President* (1993). The series for velocity are constructed using GDP as the measure of income.

[6]Zero inflation serves as a benchmark here since it is also used as a benchmark by Fischer (1981) and Lucas (1981) and since, as noted by Carlstrom and Gavin (1993), price stability is the most widely-cited objective for monetary policy in the US economy.

[7]Although this 6% real interest rate may seem high, it is similar in magnitude to those that typically arise in models of sustained growth. King and Rebelo's (1993) model, for example, yields a 6.5% real rate in steady state.

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Table 1.--The Welfare Cost of Inflation

	Annual Inflation Rate			
	Friedman Rule	0 Percent	4 Percent	10 Percent
Currency Specification				
Annual Rate of Money Growth	-0.0394	0.0212	0.0618	0.1228
Fraction of Transactions Using Cash	1.0000	0.3452	0.3012	0.2646
Annual Velocity of Money	5.7129	16.7262	19.3199	22.2562
Total Labor Supply	0.2016	0.2008	0.2005	0.2002
Fraction of Labor Force in Finance	0.0000	0.0017	0.0028	0.0041
Annual Rate of Output Growth	0.0218	0.0212	0.0210	0.0207
Welfare Cost (Percentage of Output)	-0.9142	0.0000	0.4087	0.9155
M1 Specification				
Annual Rate of Money Growth	-0.0394	0.0203	0.0605	0.1216
Fraction of Transactions Using Cash	1.0000	0.9482	0.8051	0.5173
Annual Velocity of Money	5.7129	6.0247	7.1356	11.3350
Total Labor Supply	0.2016	0.1987	0.1981	0.2003
Fraction of Labor Force in Finance	0.0000	0.0007	0.0042	0.0148
Annual Rate of Output Growth	0.0218	0.0203	0.0197	0.0197
Welfare Cost (Percentage of Output)	-2.2159	0.0000	1.0767	1.7273

Table 2.--The Bailey-Friedman Cost of Inflation

	Annual Inflation Rate			
	Friedman Rule	0 Percent	4 Percent	10 Percent
Currency Specification				
Welfare Cost (Percentage of Output)	-0.5876	0.0000	0.0147	0.0605
M1 Specification				
Welfare Cost (Percentage of Output)	-0.0137	0.0000	0.0573	0.4220

Table 3.--The Welfare Cost of Inflation With Exogenous Growth

	Annual Inflation Rate			
	Friedman Rule	0 Percent	4 Percent	10 Percent
Currency Specification				
Annual Rate of Money Growth	-0.0394	0.0200	0.0608	0.1220
Fraction of Transactions Using Cash	1.0000	0.3470	0.3021	0.2650
Annual Velocity of Money	5.7043	16.6176	19.2472	22.2107
Total Labor Supply	0.2013	0.2006	0.2004	0.2001
Fraction of Labor Force in Finance	0.0000	0.0017	0.0027	0.0041
Annual Rate of Output Growth	0.0200	0.0200	0.0200	0.0200
Welfare Cost (Percentage of Output)	-0.1569	0.0000	0.0875	0.2032
M1 Specification				
Annual Rate of Money Growth	-0.0394	0.0200	0.0608	0.1220
Fraction of Transactions Using Cash	1.0000	0.9488	0.8035	0.5158
Annual Velocity of Money	5.7043	6.0194	7.1520	11.3744
Total Labor Supply	0.2013	0.1986	0.1982	0.2004
Fraction of Labor Force in Finance	0.0000	0.0007	0.0042	0.0149
Annual Rate of Output Growth	0.0200	0.0200	0.0200	0.0200
Welfare Cost (Percentage of Output)	-0.1903	0.0000	0.2739	0.9231