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WORKING PAPER 95-1

OPTIMAL DISINFLATIONARY PATHS

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January 1995

\*I would like to thank John Cochrane, Thomas Cooley, Jeffrey Lacker, Kevin Reffett, Stacey Schreft, Alan Stockman, John Weinberg, and seminar participants at Florida State, Iowa, and the Minneapolis Fed for helpful comments and suggestions. The opinions expressed herein are my own and do not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

## **Abstract**

This paper characterizes optimal monetary policy in the context of a general equilibrium model with optimizing agents and staggered price setting. Starting from a steady state with positive inflation, a rapid disinflation is desirable when announcements of future monetary policy are fully credible. Disinflationary policy yields substantial losses in output and employment when the monetary authority lacks credibility; nevertheless, the benefits of disinflation still exceed the costs. Disinflation often fails to be welfare-improving, however, when lost seignorage revenues must be replaced using other distortionary taxes.

## 1. Introduction

The inflation rate fell sharply in the United States after reaching a peak of over 13 percent in 1980. Since 1983, in fact, inflation has stabilized at an average annual rate of 4 percent.<sup>1</sup> The Federal Reserve must reduce the average inflation rate still further, however, if it is to achieve its long-run goal of price stability.<sup>2</sup> This paper uses a general equilibrium monetary model to suggest how the final stage of disinflation should proceed.

Previous work with general equilibrium monetary models provides a simple answer to the question of what the optimal disinflationary path looks like. In the cash-in-advance models of Greenwood and Huffman (1987) and Cooley and Hansen (1989), for instance, monetary policy affects real variables only to the extent that inflation acts as a tax, distorting agents' decisions as they engage in efforts to economize on their cash balances. Optimal monetary policy eliminates the inflation tax by following the Friedman (1969) rule, contracting the money supply to make the nominal interest rate zero. Moreover, these models assume that nominal wages and prices are perfectly flexible, so they imply that the Phillips curve is either vertical or positively-sloped; that is, they identify no short-run costs of disinflation. Hence, the models indicate that the Friedman rule should be adopted immediately.

When prices or wages are not perfectly flexible, however, there may be short-run costs of an immediate disinflation that partially or completely offset the gains from removing the inflation tax. Phelps (1979) and Taylor (1983) analyze the problem of disinflation with models featuring

overlapping labor contracts that fix nominal wages in advance for several periods. Both Phelps and Taylor demonstrate the existence of a disinflationary path along which the initial levels of output and employment are maintained; both show that this path involves a reduction in the rate of money growth that is only gradual. Ball (1994) uses a model of staggered price setting to derive the striking result that a quick disinflation can actually increase output, provided that the path for the money supply is chosen appropriately. Nevertheless, in Ball's model as in Phelps and Taylor's, the instantaneous disinflation called for by the flexible-price cash-in-advance models decreases output and employment in the short run.

Phelps, Taylor, and Ball consider the effects of disinflationary policies on output and employment rather than on welfare. Their results suggest, however, that the optimal disinflationary path when prices or wages are not perfectly flexible differs significantly from the optimal path in the flexible-price case. Indeed, Danziger (1988) shows that an immediate disinflation may fail to be welfare-improving in a model of staggered price setting that is similar to Ball's.

This paper adds to the line of research initiated by Phelps and Taylor and continued by Danziger and Ball by considering the consequences of disinflation in a model where the effects of past inflation are built into current goods prices. The major difference between the model of staggered price setting developed here and those used in previous work, however, is that none of the earlier models provides agents with a motive for economizing on their cash balances in the face of a positive inflation tax. Here, the presence of a cash-in-advance constraint implies that the

inflation tax has the traditional distortionary effects on money demand. The model can therefore be used to compare the short-run costs of disinflation due to nominal rigidity with the long-run benefits from removing the inflation tax. In addition, the model presented here differs from conventional cash-in-advance models only in the extent to which it permits prices to adjust to monetary disturbances.<sup>3</sup> Thus, the implications of staggered price setting are isolated by comparing the optimal monetary policy found here with the immediate disinflation called for by the flexible-price models.

The next section outlines the model of staggered price setting. Section 3 then defines and characterizes the model economy's equilibrium. Section 4 derives the optimal monetary policy and compares it to two alternatives: an immediate switch to zero money growth, consistent with the Federal Reserve's goal of price stability, and the immediate switch to the Friedman rule prescribed by flexible-price cash-in-advance models.

The paper goes on to consider two modifications to the basic model. Sargent (1986) and Goodfriend (1993) emphasize the role of the monetary authority's credibility in determining the outcome of disinflationary policies. Thus, section 5 investigates the effects of disinflation when private agents' expectations respond only gradually to an announced change in policy. Cooley and Hansen (1991) use a flexible-price cash-in-advance model to show that an immediate disinflation fails to be welfare-improving when lost seignorage revenues must be replaced using other distortionary taxes. Section 6 asks whether their result carries over to the environment with staggered price setting. Finally, section 7 concludes.

## 2. A Model of Staggered Price Setting

As indicated above, this section develops a general equilibrium monetary model that borrows most of its structure from the flexible-price cash-in-advance models of Greenwood and Huffman (1987) and Cooley and Hansen (1989). The model departs from more conventional ones, however, by introducing an element of nominal price rigidity. Here, as in Blanchard (1983), a staggered nominal price setting structure is simply imposed and is not derived from more basic, underlying frictions. The framework serves as a tractable way of capturing two ideas: first, that firms cannot continuously adjust their prices and second, that when firms do change their prices, they do not all do so simultaneously. Ohanian and Stockman (1994) provide a list of empirical studies that document rigidity in nominal goods prices.

Following most of the recent literature, including the contributions by Danziger (1988) and Ball (1994), the focus here is on nominal rigidity in the goods market rather than in the labor market. This emphasis reflects the fact that nominal wage contracting models are widely thought to have counterfactual implications for the cyclical behavior of real wages (see, for instance, Mankiw 1987 and King 1990). On the other hand, Cho and Cooley (1992) find that nominal wage setting models do a better job than nominal price setting models at matching a variety of correlations that appear in the data. While their work indicates that it would be useful to consider the nature of optimal monetary policy in economies with wage rigidities as well as price rigidities, this is left as a task for future research.

## 2.1. The Economic Environment

The economy consists of infinitely-lived firms, households, and a government (or monetary authority). The firms are of four types, indexed by  $i \in \{1, 2, 3, 4\}$ ; there are  $N$  identical firms of each type. Firms of different types produce different perishable consumption goods. Hence, there are four types of goods in the economy, also indexed by  $i$ , where good  $i$  is produced by firms of type  $i$ . The  $N$  households are all identical. Each consists of three members: a worker, a shopper, and a bond trader. Time is discrete and indexed by  $t=0, 1, 2, \dots$ .

At the beginning of period  $t=0$ , the monetary authority announces the sequence  $\{H_t\}_{t=0}^{\infty}$  of lump-sum monetary transfers that it plans to make to each household's bond trader at each date  $t=0, 1, 2, \dots$ . The nominal money supply then evolves according to

$$M_{t+1}^s = M_t^s + H_t, \quad t=0, 1, 2, \dots,$$

where  $M_{t+1}^s$  denotes the per-household money supply at the end of period  $t$  and the beginning of period  $t+1$ . By choice of nominal units,  $M_0^s=1$ . For now, the government's activity is limited to making these monetary transfers.

Firms and households perceive the government's time  $t=0$  announcement of its monetary policy  $\{M_{t+1}^s\}_{t=0}^{\infty}$  as fully credible. Hence, firms and households act as if they have perfect foresight from  $t=0$  forward. The behavior of each, given the announced policy  $\{M_{t+1}^s\}_{t=0}^{\infty}$ , is described next.

## 2.2. Firm Behavior

Each firm of type  $i$  has access to a constant returns to scale technology for producing good  $i$ . If the representative type  $i$  firm hires



$n_{it}$  units of labor, then it can produce  $y_{it} = n_{it}$  units of output at time  $t$ .

Firms are constrained in their production and price setting decisions. Specifically, each is required to set a nominal price for its output that is fixed over a four-period interval. During this four-period interval, the firm must supply output on demand at its fixed price. Price setting is staggered so that in each period, all firms of one type are setting new prices while all firms of the other three types are constrained to sell at prices set in a previous period. Type 1 firms set new prices at dates  $t \in T^1 = \{1, 5, 9, \dots\}$ , type 2 firms set new prices at dates  $t \in T^2 = \{2, 6, 10, \dots\}$ , type 3 firms set new prices at dates  $t \in T^3 = \{3, 7, 11, \dots\}$ , and type 4 firms set new prices at dates  $t \in T^4 = \{4, 8, 12, \dots\}$ . Goods prices at  $t=0$  are taken as initial conditions; each firm is constrained to sell at its initial price until the first date at which it is permitted to set a new price.

An individual type  $i$  firm sets a new price at time  $t \in T^i$  taking the other firms' prices as given. If the individual type  $i$  firm chooses to set price  $p$  and all other type  $i$  firms choose to set price  $p_t$ , then the individual firm must produce  $c_{it+j}(p, p_t)$  units of output at each date  $t+j$ ,  $j \in \{0, 1, 2, 3\}$ , where

$$c_{it+j}(p, p_t) = \begin{cases} Nc_{it+j}(p) & \text{if } p < p_t \\ c_{it+j}(p) & \text{if } p = p_t \\ 0 & \text{if } p > p_t \end{cases} \quad (1)$$

and  $c_{it+j}(p)$  is the representative household's demand for good  $i$  at price  $p$  and time  $t+j$ .<sup>4</sup> That is, if the individual firm sets its price below the other firms' price, it must satisfy the demand of all  $N$  households. If it sets the same price as the other firms, it must satisfy the same demand as

the other firms. If it sets its price above the other firms' price, it attracts no demand.

For  $t=0,1,2,\dots$ , let  $w_t$  denote the nominal wage at time  $t$  and  $R_t$  denote the gross nominal interest rate between periods  $t$  and  $t+1$ . The individual type  $i$  firm's discounted profits over the interval during which its price is fixed at  $p$  and the other type  $i$  firms' price is fixed at  $p_t$  are

$$\begin{aligned} \pi(p, p_t) = & (p-w_t)c_{it}(p, p_t) + (p-w_{t+1})c_{it+1}(p, p_t)/R_t \\ & + (p-w_{t+2})c_{it+2}(p, p_t)/(R_t R_{t+1}) + (p-w_{t+3})c_{it+3}(p, p_t)/(R_t R_{t+1} R_{t+2}), \end{aligned} \quad (2)$$

since the individual firm must hire  $c_{it+j}(p, p_t)$  units of labor to meet demand at time  $t+j$ . At each date  $t \in T^i$ , the representative type  $i$  firm chooses  $p$  to maximize  $\pi(p, p_t)$ , taking demands  $c_{it+j}(p, p_t)$ , wages  $w_{t+j}$ , interest rates  $R_{t+j}$ , the other type  $i$  firms' price  $p_t$ , and the price setting rules as given.

Since all type  $i$  firms are identical, all will choose the same price  $p_t$  in equilibrium and all will earn profits of  $\pi_t = \pi(p_t, p_t)$ . Positive profits are competed away in equilibrium, so that  $\pi_t = 0$  for all  $t=0,1,2,\dots$ . This zero-profit condition, along with equations (1) and (2), is sufficient to determine the equilibrium price  $p_t$  as the solution to

$$\begin{aligned} 0 = & (p_t - w_t)c_{it}(p_t) + (p_t - w_{t+1})c_{it+1}(p_t)/R_t \\ & + (p_t - w_{t+2})c_{it+2}(p_t)/(R_t R_{t+1}) + (p_t - w_{t+3})c_{it+3}(p_t)/(R_t R_{t+1} R_{t+2}). \end{aligned} \quad (3)$$

With  $p_t$  given by the solution to (3) and the demand functions  $c_{it+j}(p)$  consistent with the optimizing behavior of households described below, the individual type  $i$  firm maximizes  $\pi(p, p_t)$  by choosing  $p=p_t$ . Thus, the nominal price  $p_{it}$  of good  $i$  at time  $t$  is given by

$$p_{it} = \begin{cases} p_t & \text{for } t \in T^i \\ p_{it-1} & \text{for } t \notin T^i \end{cases} \quad (4)$$

for  $t=1,2,3,\dots$  and by initial conditions for  $t=0$ .

Although competition eliminates each firm's discounted profits over the course of each interval during which its price is fixed, an individual firm may make a nonzero profit during any single period. That is, while  $\pi_t = \pi(p_t, p_t)$  must be zero for all  $t=0,1,2,\dots$ , it may be that  $\pi_{it}$ , given by

$$\pi_{it} = (p_{it} - w_t) c_{it}(p_{it}),$$

is nonzero for any  $t=0,1,2,\dots$  and  $i \in \{1,2,3,4\}$ . The households own the firms, so the representative type  $i$  firm delivers its current-period profits  $\pi_{it}$  to the representative household's bond trader at the end of each period  $t=0,1,2,\dots$ . If  $\pi_{it} < 0$ , so that the type  $i$  firm incurs a loss in period  $t$ , then the representative bond trader is responsible for covering this loss.

### 2.3. Household Behavior

The representative household's preferences are described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left\{ \sum_{i=1}^4 \ln(c_{it}) + \alpha x_t \right\}, \quad \beta \in (0,1), \alpha > 0, \quad (5)$$

where  $c_{it}$  is the household's consumption of good  $i$  and  $x_t$  is its leisure at time  $t$ . During each period  $t=0,1,2,\dots$ , the three members of the household work, shop, and trade bonds in spatially distinct markets.

The representative household's worker decides how much labor  $n_{it}$  to supply to firms of each type  $i \in \{1,2,3,4\}$  during each period  $t=0,1,2,\dots$

subject to the constraints

$$1 \geq x_t + \sum_{i=1}^4 n_{it}, \quad t=0,1,2,\dots \quad (6)$$

The time endowment in equation (6) is normalized to unity.

The representative household's shopper, meanwhile, purchases consumption goods from firms of each type during each period  $t=0,1,2,\dots$ . Since he shops in different markets from those in which his household's worker is employed, he is required to pay for all of his purchases out of his beginning-of-period money holdings  $M_t$ . That is, he faces the cash-in-advance constraints

$$M_t \geq \sum_{i=1}^4 p_{it} c_{it}, \quad t=0,1,2,\dots \quad (7)$$

At the end of period  $t=0,1,2,\dots$ , the representative household's bond trader receives the wage earnings from the worker, any unspent cash from the shopper, and current-period profits from the firms, and the transfer from the government. The bond trader uses these sources of funds to accumulate the cash  $M_{t+1}$  that his household will carry into the following period. The representative bond trader also borrows from or lends to traders from other households by issuing or purchasing one-period discount bonds. Bonds paying  $B_{t+1}$  dollars at the end of time  $t+1$  trade for  $B_{t+1}/R_t$  dollars at the end of time  $t$ . Thus, the bond trader faces the constraints

$$B_t + M_t + H_t + w_t \sum_{i=1}^4 n_{it} + \sum_{i=1}^4 \pi_{it} \geq \sum_{i=1}^4 p_{it} c_{it} + M_{t+1} + B_{t+1}/R_t, \quad (8)$$

$t=0,1,2,\dots$

The representative household chooses sequences for  $c_{it}$ ,  $n_{it}$ ,  $x_t$ ,  $M_{t+1}$ , and  $B_{t+1}$  to maximize the utility function (5) subject to the

constraints (6)-(8), taking the sequences for  $H_t$ ,  $w_t$ ,  $R_t$ ,  $p_{it}$ , and  $\pi_{it}$  and the initial conditions  $M_0=1$  and  $B_0=0$  as given.

### 3. Equilibrium Defined and Characterized

An equilibrium is defined as a collection of sequences for prices and quantities such that: (i) firms behave optimally; (ii) each firm earns zero discounted profits over each interval during which its price is fixed; (iii) households behave optimally; and (iv) the money, bond, goods, and labor markets clear. The money market clears when

$$M_{t+1}^s = M_{t+1}, \quad t=0,1,2,\dots$$

Since bonds are available in zero net supply, the bond market clears when

$$B_{t+1} = 0, \quad t=0,1,2,\dots$$

The goods and labor markets clear when

$$n_{it} = c_{it}, \quad t=0,1,2,\dots, i \in \{1,2,3,4\}.$$

The representative household's objective function is concave, and its constraints are linear, in its choice variables. Thus, the first order conditions for  $c_{it}$ ,  $n_{it}$ ,  $M_{t+1}$ , and  $B_{t+1}$  characterize the solution to the household's problem. These first order conditions imply that in equilibrium,

$$n_{it} = c_{it} = M_t^s / (4p_{it}), \quad t=0,1,2,\dots, i \in \{1,2,3,4\}, \quad (9)$$

$$w_t = \alpha M_{t+1}^s / (4\beta), \quad t=0,1,2,\dots, \quad (10)$$

and

$$R_t = M_{t+2}^s / (\beta M_{t+1}^s), \quad t=0,1,2,\dots \quad (11)$$

Equations (3) and (4) summarize the implications of firm optimization

and the zero-profit condition. In light of equation (9), the demand function in equation (3) is given by  $c_{it+j}(p) = M_{t+j}^s / (4p)$ . Thus, equations (9)-(11) imply that equation (3) can be rewritten as

$$p_t = \frac{\alpha}{4\beta} \left\{ \frac{M_t^s + \beta M_{t+1}^s + \beta^2 M_{t+2}^s + \beta^3 M_{t+3}^s}{M_t^s / M_{t+1}^s + \beta M_{t+1}^s / M_{t+2}^s + \beta^2 M_{t+2}^s / M_{t+3}^s + \beta^3 M_{t+3}^s / M_{t+4}^s} \right\}, \quad (12)$$

t=1,2,3,....

Equations (4) and (9)-(12) express equilibrium prices and quantities in terms of the initial conditions  $M_0^s=1$  and  $\{p_{10}, p_{20}, p_{30}, p_{40}\}$  and the money supply sequence  $\{M_{t+1}^s\}_{t=0}^{\infty}$ . Thus, they show how equilibrium outcomes change as the government varies its monetary policy. Two key effects of monetary policy on the economy's equilibrium are best illustrated by assuming for the moment that money growth  $\gamma_t = M_{t+1}^s / M_t^s$  is constant, with  $\gamma_t = \gamma$  for all  $t=0,1,2,....$

Equation (11) indicates that under the constant money growth rate  $\gamma$ , the nominal interest rate is also constant, with  $R_t = R = \gamma/\beta$  for all  $t=0,1,2,....$  In this model as in the flexible-price cash-in-advance models of Greenwood and Huffman (1987) and Cooley and Hansen (1989), a strictly positive nominal interest rate provides households with an incentive to inefficiently economize on their cash balances by enjoying more leisure and less consumption. The government can remove this source of inefficiency by following the Friedman (1969) rule, with  $\gamma=\beta$ .

Equation (10) shows that under the constant money growth rate  $\gamma$ , the nominal wage  $w_t$  also grows at the constant rate  $\gamma$ . When  $\gamma > 1$ , the representative type i firm sets its price above the nominal wage at date  $t \in T^1$ . During the next three periods, the wage rises while the price

remains fixed; eventually the wage exceeds the fixed price. The firm earns zero discounted profits over the four-period interval by making positive profits early on and incurring losses later. Conversely, when  $\gamma < 1$ , the firm sets its price below the nominal wage at date  $t \in T^1$ , then sees the wage fall during the next three periods until it is smaller than the fixed price. Unless  $\gamma = 1$ , the firm's nominal price moves in a saw-tooth manner relative to the nominal wage. Productive efficiency, however, requires that the firm's price  $p_{it}$  equal its marginal cost  $w_t$  in every period  $t = 0, 1, 2, \dots$ . Equation (10) implies that monetary policy is consistent with this efficiency condition only when the money supply is constant.

This model, therefore, features a tension between the monetary authority's objectives. The government can remove the inflation tax by contracting the money supply, but can guarantee productive efficiency only by holding the money supply fixed. The next section, which characterizes the optimal monetary policy, shows how this tension is resolved in a utility-maximizing way.

#### 4. Optimal Disinflation

Let initial goods prices come from an equilibrium with positive inflation:  $p_{40} = gp_{30} = g^2 p_{20} = g^3 p_{10}$ , where  $g > 1$ . Prior to  $t=0$ , agents expect the money supply to continue expanding at the inflation rate  $g > 1$  forever. At  $t=0$ , however, the monetary authority announces that the money supply will follow an alternative path  $\{M_{t+1}^s\}_{t=0}^{\infty}$  instead. Firms and households believe this announcement and act from  $t=0$  forward with perfect foresight,

taking prices set at  $t=0$  and earlier as given; the policy announcement is perfectly credible. Prices and quantities are determined by equations (4) and (9)-(12). The sequence  $\{M_{t+1}^s\}_{t=0}^{\infty}$  that maximizes the representative household's utility subject to these constraints constitutes the economy's optimal disinflationary path.

To compute the optimal disinflationary path, the preference parameter  $\beta$  is set equal to 0.99, so that each period in the model represents one quarter year. Thus, as suggested by Ball (1994), individual goods prices remain fixed for one-year intervals. Equations (9)-(12) indicate that the preference parameter  $\alpha$  affects only the equilibrium levels of consumption, leisure, wages, and prices. Once their levels are determined,  $\alpha$  affects neither the growth rate of these variables nor the growth rate of money under the optimal policy. Thus,  $\alpha$  is set equal to 10, which guarantees that the nonnegativity constraints  $x_t \geq 0$  and  $n_{it} \geq 0$  do not bind without affecting any of the other results.

Figure 1 shows the optimal disinflationary path starting from a 4 percent annual inflation. It traces out the effects of the optimal monetary policy on the the inflation rate, the net nominal interest rate  $R_t - 1$ , and output. Aggregate output is measured as

$$Y_t = \sum_{i=1}^4 c_{it}$$

and is normalized so that  $Y_0 = 1$ . The market clearing condition for goods and labor implies that  $Y_t$  also serves as an index of total employment.

Inflation is calculated as the growth rate of the price level  $P_t$ , obtained by dividing nominal expenditure by real output:



$$P_t = Y_t^{-1} \sum_{i=1}^4 p_{it} c_{it}.$$

In figure 1, money and prices grow at the rate of 0.985 percent per period, which corresponds to 4 percent per year, and the nominal interest rate and aggregate output are constant when the economy is in its initial steady state prior to  $t=0$ . The optimal policy calls for an increase in the rate of money growth at  $t=0$ , which counteracts the effect on real goods prices of the slower money growth that occurs later. Phelps (1979) finds that maintaining output and employment in his model of disinflation requires a similar jump in money growth. Since households have already determined their money holdings  $M_0$  when the new policy is announced, the initial increase in money growth does not translate into a distortionary tax.

Real output increases by 2 percent between  $t=0$  and  $t=1$ , partly in response to the unexpected burst of money growth at  $t=0$ , but also because a lower inflation tax reduces leisure and increases consumption at  $t=1$ . In addition, the representative household's demand for real balances increases at the end of  $t=0$  in anticipation of slower money growth during the following period. With output and money demand increasing, the inflation rate starts to fall immediately, even though money growth does not begin to decelerate until  $t=1$ .

After  $t=0$ , money growth converges quickly to a new steady state value that lies between  $\gamma=\beta$  and  $\gamma=1$ . Thus, in the long run, the optimal policy balances the benefits of the Friedman rule, which eliminates the inflation tax, and the benefits of zero money growth, which ensures productive efficiency. Disinflation is largely complete after one year. Since the

inflation tax is permanently reduced, output remains permanently higher, exactly as in flexible-price cash-in-advance models.

Figure 2 displays the effects of immediately adopting a zero money growth policy, consistent with the Federal Reserve's price stability objective. As before, the economy begins in its steady state with 4 percent annual inflation. The inflation rate falls sharply without the initial burst of money growth that occurs under the optimal policy. Output falls slightly at first, but increases later as the effects of reducing the inflation tax take over.

Table 1 reports on the welfare consequences of disinflation, measured as the permanent percentage increase in consumption of all four goods that makes the representative household as well off under continuing inflation as it is under the disinflationary policies. It shows that the gain from eliminating a 4 percent annual inflation using the optimal path is 0.0223 percent of total consumption. The gain from optimally eliminating a 10 percent annual inflation equals 0.0874 percent of consumption. These welfare gains are much smaller than those obtained from immediately adopting the Friedman rule in Cooley and Hansen's (1989) flexible-price cash-in-advance model. Here, the short-run adjustment costs of disinflation partially offset the long-run benefits; hence, the net gain is smaller. Table 1 also reveals that while neither the zero money growth policy shown in figure 2 nor the immediate switch to the Friedman rule prescribed by flexible-price models are optimal under staggered price setting, both come close to the optimum in welfare terms.

These results suggest that it is desirable to disinflate quickly when monetary policy announcements are perfectly credible. Sargent (1986) and

Goodfriend (1993), however, emphasize that the money authority's lack of credibility can be a major stumbling block for disinflationary policy in practice. Under the optimal policy shown in figure 1, households and firms respond to promises of lower inflation in the future even as money growth initially accelerates. The welfare consequence of this policy might be considerably different if agents respond to the faster money growth that they observe instead of the slower money growth that they are promised. Thus, the next section investigates how the optimal disinflationary path changes when the monetary authority's announcements are no longer perceived by private agents as fully credible.

## 5. Optimal Disinflation With Partial Credibility

Following Fischer (1986), partial credibility is incorporated into this perfect foresight model by assuming that agents' expectations of future money growth respond only gradually to an announced change in policy. As before, the economy begins in a steady state under a constant money growth rate  $g > 1$ . As before, the monetary authority announces a new policy  $\{M_{t+1}^s\}_{t=0}^{\infty}$  that it plans to implement at the beginning of  $t=0$ . Firms and households then react as follows.

At the beginning of each period  $t \in T^i$ , type  $i$  firms set new prices based on their knowledge of the actual beginning-of-period money supply  $M_t^s$  and their expectation that the money supply will grow at the constant rate  $g_{t-1}$  from time  $t$  forward. Households make their time  $t$  decisions after they have observed the magnitude of the time  $t$  transfer  $H_t$ . Thus, they

make their decisions based on their knowledge of the actual end-of-period money supply  $M_{t+1}^s$  and their expectation that the money supply will grow at the constant rate  $g_t$  from time  $t+1$  forward.

The expected rate of money growth  $g_t$  evolves according to

$$g_t = g_{t-1} + \varphi[(M_{t+1}^s/M_t^s) - g_{t-1}], \quad \varphi \in (0,1), \quad t=0,1,2,\dots, \quad (13)$$

where  $g_{-1}$  is given by the initial inflation rate  $g$ . Thus, the expected rate of money growth adjusts downward in each period by the fraction  $\varphi$  of the difference between the actual and expected rates of money growth during the period. As  $\varphi$  increases, expectations adjust more rapidly to observed changes in money growth, so that policy can be characterized as more credible. If the actual rate of money growth eventually converges to some constant  $\gamma$ , then (13) implies that the expected rate of money growth  $g_t$  will also converge to  $\gamma$ .

As before, an equilibrium is defined as a collection of sequences for prices and quantities such that firms and households behave optimally and markets clear. The optimal decisions of firms and households now differ from those described in section 3, however. Since type 1 firms set new prices at the beginning of time  $t \in T^1$  based on their knowledge of  $M_t^s$  and their expectation that the money supply will expand forever at rate  $g_{t-1}$ , equation (12) is replaced by

$$p_t = \frac{\alpha g_{t-1} (1 + \beta g_{t-1} + \beta^2 g_{t-1}^2 + \beta^3 g_{t-1}^3) M_t^s}{4\beta(1 + \beta + \beta^2 + \beta^3)}, \quad t=1,2,3,\dots \quad (14)$$

Note that  $p_t$  now depends on firms' expected rate of money growth  $g_{t-1}$ , which may differ from the actual rate of money growth when announcements lack credibility. Individual goods prices are determined by equation (4) for  $t=1,2,3,\dots$  and by initial conditions for  $t=0$ .

Households make their decisions at time  $t$  based on their knowledge of  $M_{t+1}^s$  and their expectation that the money supply will expand forever at rate  $g_t$ . Thus, while equilibrium labor supplies, consumptions, and wages continue to be described by equations (9) and (10), nominal interest rates are now

$$R_t = g_t/\beta, \quad t=0,1,2,\dots \quad (15)$$

Since the current interest rate depends on households' expectations of future money growth, equation (15) indicates that  $R_t$  is determined by  $g_t$ , which may differ from the actual rate of money growth during time  $t$ .

Equations (4), (9), (10), and (13)-(15) express equilibrium prices and quantities in terms of the initial conditions  $M_0^s=1$  and  $\{p_{10}, p_{20}, p_{30}, p_{40}\}$  and the money supply sequence  $\{M_{t+1}^s\}_{t=0}^{\infty}$  when policy announcements are not fully credible. As in the case of full credibility, the optimal disinflationary path is defined as the sequence  $\{M_{t+1}^s\}_{t=0}^{\infty}$  that maximizes the representative household's utility subject to these constraints when the initial prices come from a steady state with positive inflation.

Figure 3 shows the optimal disinflationary path when, as suggested by Fischer (1986),  $\varphi=0.125$ . Instead of the initial burst of money growth that occurs when policy is fully credible, the optimal policy with partial credibility calls for a sharp contraction of the money supply at  $t=0$ . Thus, inflationary expectations begin to ease at once; since  $R_t=g_t/\beta$ , these expectations can be read directly off of the interest rate series. Money growth bounces back, but is still slower than expected, at  $t=1$ ; thereafter, it remains negative. Output falls by almost 1.4 percent in response to the unanticipated monetary contraction and remains below its initial level for

two full years. Eventually, however, output reaches higher levels as the inflation tax recedes. Money growth again converges to a new steady state that lies between the Friedman rule and zero money growth.

While a comparison of figures 1 and 3 reveals that credibility makes a big difference for the effects of disinflation on output and employment, table 2 indicates that the welfare gains under partial credibility resemble those from the full credibility case. Disinflation is easier when policy is more credible so that, for instance, the gain from eliminating a 4 percent annual inflation using the optimal policy equals 0.0184 percent of consumption when  $\phi=0.125$  and 0.0214 percent of consumption when  $\phi=0.750$ . Both of these figures, however, are similar in magnitude to the 0.0223 percent gain under full credibility shown in table 1. Moreover, the alternative policies of immediately switching to zero money growth or the Friedman rule continue to come close to the optimum in welfare terms.

Thus, as suggested by Sargent (1986) and Goodfriend (1993), disinflation yields significant losses in output and employment when the monetary authority lacks credibility. Nevertheless, the results here show that the long-run benefits of a quick disinflation still exceed the short-run costs. Cooley and Hansen (1991) suggest another reason why, in practice, disinflationary policies might fail to be welfare-improving: the government may have to increase other distortionary taxes to replace lost seignorage revenues. Cooley and Hansen conduct their analysis in a flexible-price cash-in-advance model, however; the next section considers whether the presence of other distortionary taxes presents a serious challenge to disinflationary policy in the model with staggered price setting.

## 6. Disinflation With Other Distortionary Taxes

Suppose now that, in addition to controlling the money supply, the government levies a flat-rate tax  $\tau_t$  on workers' labor income, returning the proceeds to households in the form of a lump-sum transfer at the end of each period  $t=0,1,2,\dots$ . The representative household's budget constraints become

$$B_t + M_t + T_t + (1-\tau_t)w_t \sum_{i=1}^4 n_{it} + \sum_{i=1}^4 \pi_{it} \approx \sum_{i=1}^4 P_{it} c_{it} + M_{t+1} + B_{t+1}/R_t, \quad (16)$$

$t=0,1,2,\dots$

In equation (16), the total lump-sum transfer  $T_t$  includes income tax revenues as well as the newly created money  $H_t$ ; that is, while the sequence  $\{T_t\}_{t=0}^{\infty}$  is taken as given by the representative household, it is determined in equilibrium by the government's budget constraints

$$T_t = H_t + \tau_t w_t \sum_{i=1}^4 n_{it}, \quad t=0,1,2,\dots \quad (17)$$

Cooley and Hansen (1991) use a flexible-price cash-in-advance model to establish two results. First, holding the tax rate constant, with  $\tau_t = \tau$  for all  $t=0,1,2,\dots$ , they show that the benefits of disinflation when  $\tau > 0$  exceed those when  $\tau = 0$ . Thus, the presence of another distortionary tax increases the welfare cost of inflation. Second, holding the real value of transfers  $T_t/P_t$  constant, they show that disinflation ceases to be welfare-improving when  $\tau_t$  must adjust to satisfy the government budget constraints (17). They conclude that disinflation is no longer desirable when the

government must use other distortionary taxes to replace lost seignorage revenues.

Following Cooley and Hansen, the first part of table 3 assumes that the income tax rate is constant, with  $\tau_t = \tau^0$  for all  $t=0,1,2,\dots$ . The economy begins in a steady state with positive inflation, and monetary policy announcements are once again fully credible. When  $\tau^0=0$ , there is no distortionary labor tax and the welfare gains from immediately adopting the zero money growth policy and the Friedman rule are the same as in table 1. When  $\tau^0=0.23$ , the value chosen by Cooley and Hansen to match the US data, the welfare benefits of both disinflationary policies increase dramatically. Starting from a 4 percent annual inflation, switching immediately to zero money growth yields a welfare gain of 0.231 percent of consumption. Immediately adopting the Friedman rule yields a gain equal to 0.448 percent of consumption.

In the second part of table 3, the economy begins in a steady state with a constant labor tax rate  $\tau^0$  and a constant inflation rate  $g>1$ . The government switches to zero money growth or the Friedman rule at the beginning of  $t=0$ , but increases the labor income tax  $\tau_t$  as necessary to hold real transfers  $T_t/P_t$  fixed. The table shows that when  $\tau^0=0$ , the zero money growth policy continues to yield a small welfare gain starting from both 4 and 10 percent annual inflations. The Friedman rule, however, ceases to be welfare-improving starting from a 4 percent inflation. Moreover, both disinflationary policies yield sizable losses in welfare when  $\tau^0=0.23$ .

Thus, the results displayed in table 3 confirm Cooley and Hansen's findings by showing that the welfare consequences of disinflation depend



critically on how the government copes with the loss of its seignorage revenues. On the one hand, disinflation becomes much more attractive when the government cuts spending so that the income tax does not have to rise. On the other hand, disinflation often ceases to be welfare-improving when government spending is held fixed, so that all of the lost seignorage must be replaced through higher income taxes.

## 7. Conclusion

Inflation has averaged 4 percent annually in the United States during the past decade. Since the Federal Reserve lists price stability among its principal long-run objectives, a central question for monetary policy is: What is the best way to reduce the average rate of inflation? The flexible-price cash-in-advance models of Greenwood and Huffman (1987) and Cooley and Hansen (1989) provide a simple answer to this question by indicating that the rate of money growth should be reduced immediately to make the nominal interest rate equal to zero, as called for by Friedman (1969). But what if there are nominal rigidities that make disinflation costly in the short run? Previous work by Phelps (1979), Taylor (1983), Danziger (1988), and Ball (1994) suggests that the optimal disinflationary path may then differ considerably from an immediate switch to the Friedman rule.

None of these earlier models with nominal rigidities provides agents with an incentive to economize on their cash balances in the face of a positive inflation tax, however. The model of staggered price setting

developed here shows that once this traditional distortionary effect of inflation on money demand is accounted for, the optimal policy does involve a rapid disinflation. Starting from a 4 percent annual inflation, the optimal disinflationary path eliminates inflation within one year with no losses in output and employment. In addition, immediately switching to zero money growth, consistent with the Federal Reserve's goal of price stability, or adopting the Friedman rule, as called for by flexible-price models, yields a welfare gain that is almost as large as that obtained from following the optimal path.

The model also indicates that disinflation results in significant losses in output and employment when the monetary authority's policy announcements are not fully credible. By isolating credibility as a chief determinant of the effects of disinflation, the results corroborate Sargent's (1986) studies of the European hyperinflations of the 1920's and the British experience under Thatcher as well as Goodfriend's (1993) analysis of disinflation in the US since 1979. In the model, however, the long-run benefits of disinflation still exceed the short-run costs. Even with partial credibility, zero money growth and the Friedman rule continue to be welfare-improving policies.

Finally, the results support Cooley and Hansen's (1991) finding that the presence of other distortionary taxes has major effects on the welfare consequences of reducing inflation. The model shows that disinflation often fails to be welfare-improving when the government must replace lost seignorage revenues by increasing the tax on labor income. Thus, the results indicate that reducing the government's revenue requirements so that tax increases are not necessary is crucial to the success of any

disinflationary policy.

Aiyagari (1990) provides a detailed critique of the Federal Reserve's price stability objective. The model developed here addresses some of Aiyagari's concerns. On the one hand, disinflation reduces the shoe leather costs associated with a binding cash-in-advance constraint. On the other hand, it requires short-run losses in output and employment, particularly when the monetary authority lacks credibility. The model indicates that the benefits of disinflation exceed the costs, provided that income tax increases are not required to make up for lost seignorage revenues. However, the model does not capture several of the other elements discussed by Aiyagari, including the lack of indexation in the US capital income tax, the possible association of greater inflation variability with the level of inflation, and the fact that a large fraction of the US currency supply appears to be held overseas. A full consideration of these issues awaits future research.

## Notes

<sup>1</sup>These figures refer to the annual growth rate of the consumer price index for all urban consumers, CPI-U, as reported in the *Economic Report of the President* (1994, Table B-59).

<sup>2</sup>Black (1990) and Hoskins (1991) list price stability as the Federal Reserve's principal long-run objective.

<sup>3</sup>In this respect, the model is closest in spirit to those developed by Lucas (1988), Cho and Cooley (1992), and Ohanian and Stockman (1994).

<sup>4</sup>In general, the representative household's demand for good 1 will depend on its income and the prices of the other three goods as well as the price of good 1. The notation used here suppresses this dependence in order to focus on the effects of type 1 firms' price setting decisions. The dependence will be accounted for, however, in deriving equilibrium conditions.

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Table 1.--Welfare Gain from Monetary Policy

Monetary Policy	Initial Annual Inflation Rate	
	4 Percent	10 Percent
Optimal Path	0.0223	0.0874
Zero Money Growth	0.0196	0.0831
Friedman Rule	0.0174	0.0803

*Notes:* Figures refer to the permanent percentage increase in consumption that makes the representative household as well off under continuing inflation at the initial rate as it is under the optimal policy, zero money growth, or the Friedman rule.



Table 2.--Welfare Gain from Monetary Policy With Partial  
Credibility

Monetary Policy	Initial Annual Inflation Rate	
	4 Percent	10 Percent
$\varphi=0.125$		
Optimal Path	0.0184	0.0749
Zero Money Growth	0.0171	0.0739
Friedman Rule	0.0136	0.0694
$\varphi=0.250$		
Optimal Path	0.0202	0.0809
Zero Money Growth	0.0185	0.0793
Friedman Rule	0.0159	0.0759
$\varphi=0.500$		
Optimal Path	0.0211	0.0839
Zero Money Growth	0.0192	0.0820
Friedman Rule	0.0170	0.0791
$\varphi=0.750$		
Optimal Path	0.0214	0.0849
Zero Money Growth	0.0195	0.0828
Friedman Rule	0.0173	0.0800

Notes: See notes to table 1.

Table 3.--Welfare Gain from Monetary Policy With Distortionary Labor Taxation

Labor Tax Rate Constant		
Monetary Policy	Initial Annual Inflation Rate	
	4 Percent	10 Percent
$\tau^0=0.00$		
Zero Money Growth	0.0196	0.0831
Friedman Rule	0.0174	0.0803
$\tau^0=0.23$		
Zero Money Growth	0.231	0.595
Friedman Rule	0.448	0.812
Government Spending Constant		
Monetary Policy	Initial Annual Inflation Rate	
	4 Percent	10 Percent
$\tau^0=0.00$		
Zero Money Growth	0.00520	0.0307
Friedman Rule	-0.00212	0.0220
$\tau^0=0.23$		
Zero Money Growth	-0.112	-0.298
Friedman Rule	-0.243	-0.456

Notes: See notes to table 1.

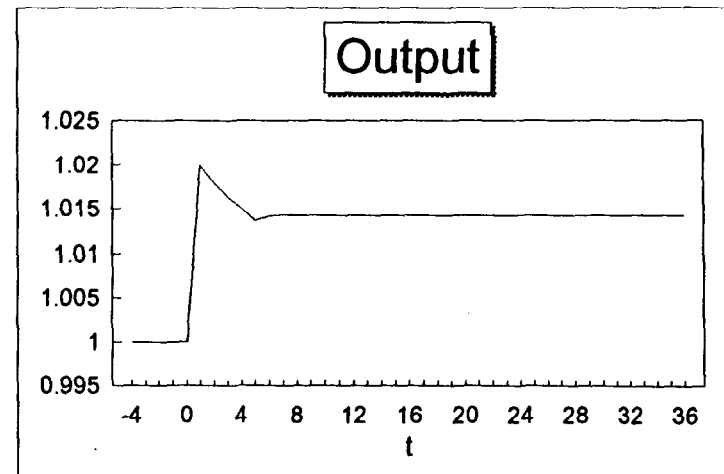
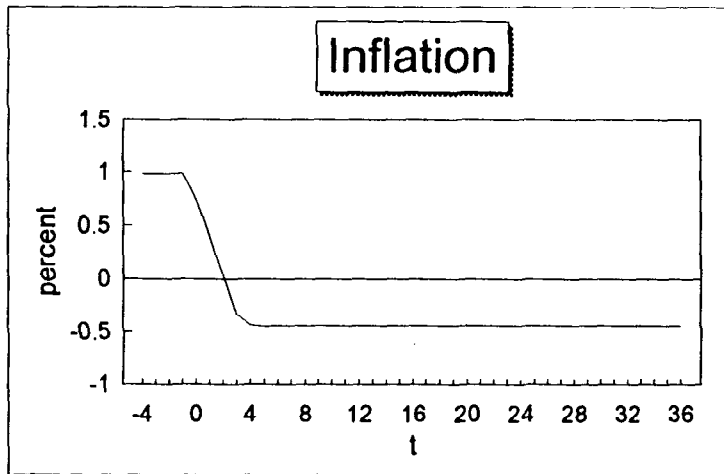
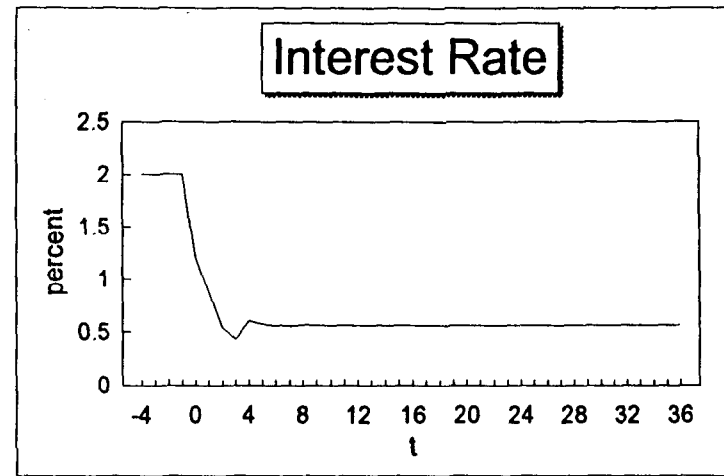
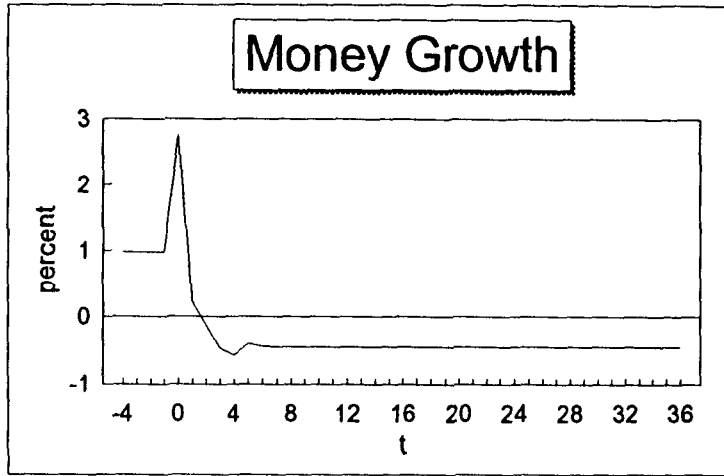


Fig. 1. Optimal Monetary Policy

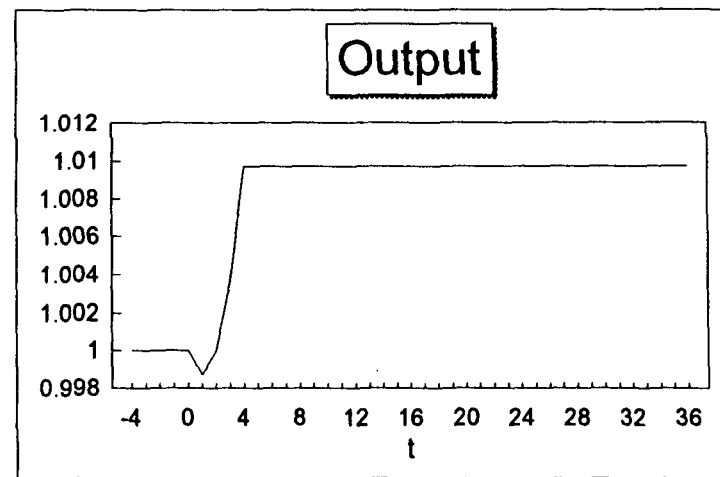
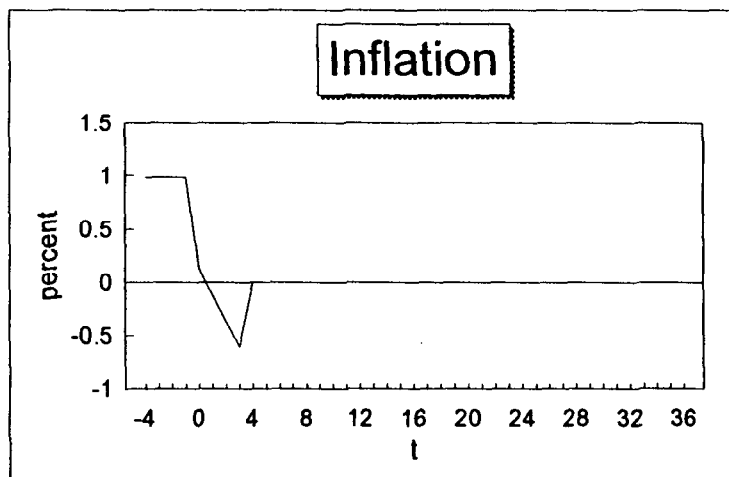
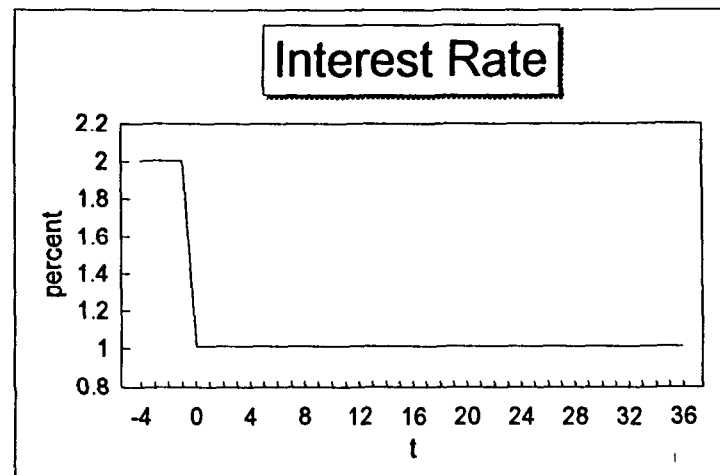
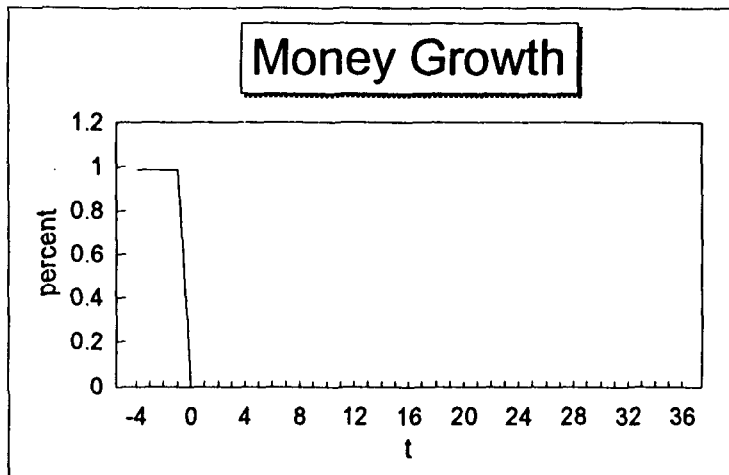


Fig. 2. Zero Money Growth Policy

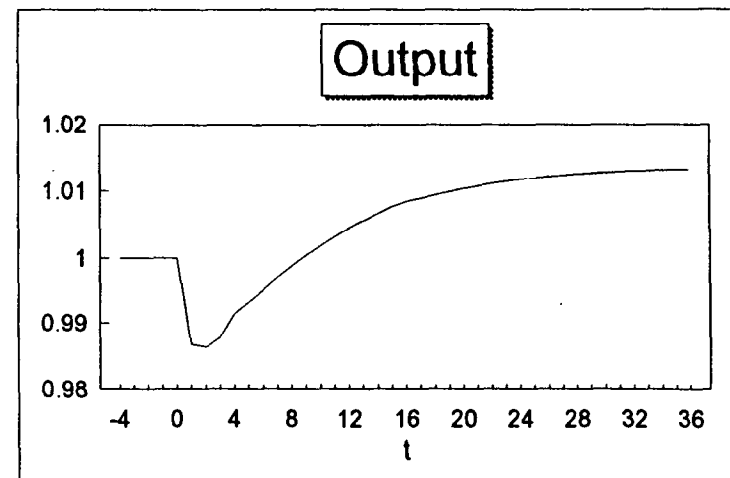
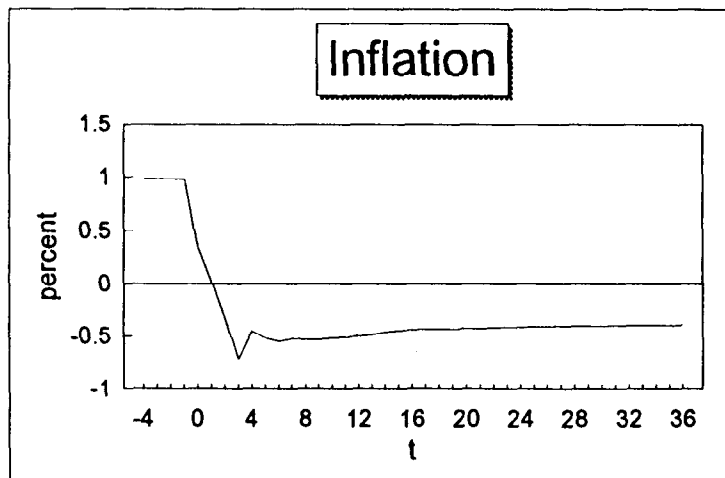
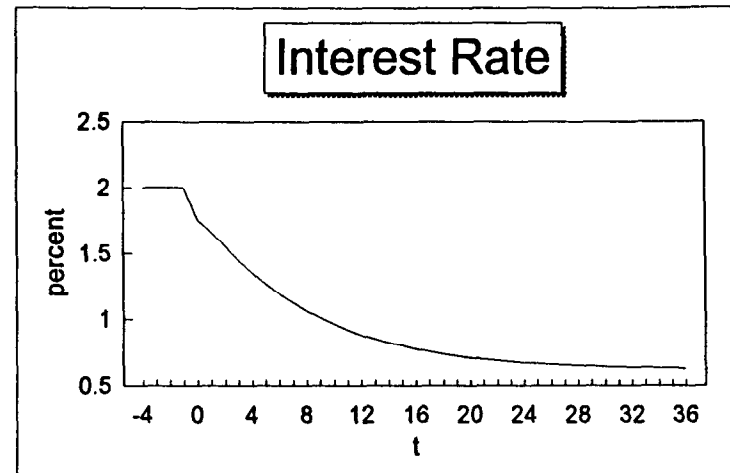
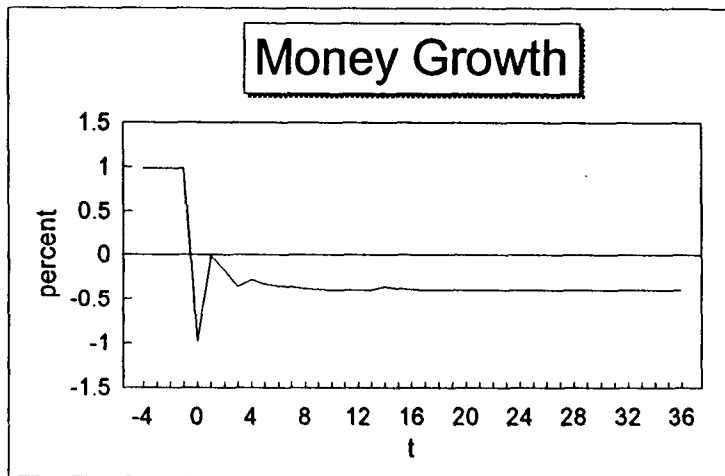


Fig. 3. Optimal Monetary Policy With Partial Credibility,  $\phi=0.125$