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Clearing, Settlement, and Monetary Policy

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Abstract: This paper develops a general equilibrium model of the clearing and settlement of private payment instruments. Spatial separation, heterogeneous preference shocks and limited communication provide a role for private credit as a means of payment. Although this method could be applied to various settlement arrangements, the use of central bank deposit liabilities in settlement is studied here. Various tools of payment system policy, such as intraday overdraft limits and fees, collateral requirements, reserve requirements, and interest on reserves, are examined.

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1. Introduction

Central bank liabilities generally consist of currency and deposits. One striking feature of central bank deposits, as opposed to central bank note issue, is that their predominant use is to *settle* private credit instruments used in payment. This paper outlines an approach to understanding the use of central bank deposit liabilities in clearing and settlement.

One reason to be interested in such models is the recent attention central banks have devoted to the operations of their funds transfer systems. The volume and value of payments settled on central bank books has increased dramatically in recent years--much faster than the underlying national economies (Borio and Van den Bergh [1993]). In response, various payment system policy measures have been proposed or adopted. For example, in the United States the Federal Reserve has introduced intraday overdrafts limits (Richards [1995]). In Europe there has been a movement toward requiring that overdrafts be collateralized by high-grade securities (Folkerts-Landau, et al [1996]). A notable exception is Switzerland, which has for many years disallowed daylight overdrafts (Vital [1994]). Some of the basic tools of payment system policy, such as overdraft limits and fees, minimum balance requirements, interest rates, and the like, are contractual features intrinsic to deposit banking but inapplicable to currency issue. A model of central bank liabilities as book-entry deposits would allow us to evaluate payment system policy. Along the way, we are forced to broaden the notion of "monetary policy regime" to encompass the basic contractual terms on which the central bank offers deposit accounts.

The current trend in some countries toward lower or nonbinding reserve requirements also makes this topic timely.[1] In the absence of binding reserve requirements, holdings of central bank deposit liabilities are governed by the clearing function they serve. The demand for central bank clearing balances can have very different qualitative features, and will depend critically on the contractual terms on which deposits are offered. One purpose of this paper then, is to develop the microfoundations of a model of central bank liabilities used in clearing and settlement. This obviously requires a model in which there is meaningful clearing and settlement of credit payment instruments. The model presented here builds on recent work on the role of credit as a means of payment in environments with spatial separation and limited communication (Townsend [1987], Prescott [1987], Schreft [1992]), and incorporates the feature, absent in other models of multiple means of payment, that private payment instruments are usually settled in outside money.

The idea of central banks as clearing facilities seems both realistic and faithful to the historical roots of central banking. The early precursors of European central banks included the sixteenth century public clearing banks of Germany and the Mediterranean, municipal institutions that transferred book-entry credit among merchants and bankers. The *Amsterdamsche Wisselbank* (Amsterdam Exchange Bank, founded 1609), was chartered explicitly to clear and settle bills of exchange, the ubiquitous payment instrument of late medieval and early modern commerce, and benefited from a regulation that all bills of exchange of 600 florins or more were to be settled at the Bank.[2] The Bank of England began discounting bills soon after its founding in 1695, a function that combines lending with clearing (Richards [1934]). The goldsmith notes the Bank accepted were immediately presented for payment, much as in modern check clearing operations. Private

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[1]There are some notable exceptions such as Switzerland.

[2] On the early public banks see van Dillen [1934] and van der Wee [1977]. Adam Smith [1976] contains a perceptive description of the workings of the Amsterdam Exchange Bank.

clearinghouses, in many ways prototypes of the modern central bank, often issued their own liabilities with which members could settle (Cannon [1901]). In the U.S., the Federal Reserve Banks, founded to issue currency and accept banks' deposits, quickly entered the business of clearing checks (Spahr [1926]). Goodhart [1988] has argued that central banks evolved in part out of the need for a independent provider of payments services.

Private settlement arrangements have often coexisted with central bank clearing facilities. Commercial banks in many U. S. cities operate clearinghouses which exchange checks and then settle by paying their net obligations. On a larger scale, the New York Clearinghouse operates CHIPS, a multilateral net settlement system often used for the dollar leg of foreign exchange trades (Bank for International Settlements [1993]). Private settlement arrangements have raised concerns among bank regulators because they inherently involve multilateral credit extensions. Prudential regulatory concerns about the magnitude of the risks routinely incurred by large-value arrangements has motivated many payment system policy initiatives.[3] Changes in the terms on which central bank deposits are offered--intraday overdraft fees, for example--are often evaluated in light of the effect on the incentive for banks to settle through private intermediary arrangements.

One lesson that emerges from the model is that the policy regime has an important effect on the incentive to form private multilateral arrangements to clear and settle payments. Banks hold positive central bank clearing balances in part because of the probability of binding central bank constraints on daylight credit. Private net settlement arrangements allow their participants to economize on central bank balances by making greater use of private intraday credit. Thus, under regimes in which central bank balances earn no interest, part of the incentive to settle privately is the ability to avoid holding "sterile" reserves. The model thus identifies yet another effect of the inflation tax; an excess incentive to settle payments privately. As one would expect, the magnitude of the incentive to clear privately is critically affected by the details of central bank policies, such as intraday overdraft terms and the reserve requirement regime. The theory is not yet ready, however, for a complete welfare analysis of policies aimed at containing payment system risk or influencing the distribution of clearing between private and public facilities. That would require an adequate account of settlement risk and the phenomenon policymakers refer to as "systemic risk." We must content ourselves with a first step towards such an analysis. The path outlined here aims at capturing both the settlement arrangements and the payments that give rise to them in a coherent general equilibrium economy.[4]

In closely related work, Freeman [1996] presents a model in which consumers issue their own notes to pay for purchases. Notes are then settled by final payment in fiat money. Freeman finds parameterizations under which it is welfare improving for the central bank to temporarily increase the money supply by buying notes, which are then presented directly to the issuer for payment. Green [1996] reexamines Freeman's model and shows that a private settlement arrangement is also capable of achieving the desired outcome. Freeman's model seems to relate most closely to central bank note issue, in the sense that the policies considered could just as well be carried out with paper bearer instruments. In this paper buyers issue bills which the seller's bank presents to the buyer's bank; payment is by the transfer of value at a centralized book-entry deposit

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[3]See, for example, the Lamfalussy Report (Bank for International Settlements [1990]) on regulatory standards for netting schemes.

[4]On risks in interbank lending see Rochet and Tirole [1996] and Summers [1996]. On risk in interbank settlement systems, see Kahn and Roberds [1996], Schoenmaker [1995], Angelini, et al [1996] and the citations they contain.

facility. The central bank is modeled as a public monopoly deposit facility. This approach, while it is consistent with the notion that legal restrictions are an important determinant of the valuation of government monetary liabilities, is not meant to prejudge the inherent desirability of central bank provision of payment services. The latter must be regarded as an open question at this time; the answer is likely to turn on the questions of governance structure and incentives, as suggested by Green [1996].

Other recent work has also focused on the economics of payment arrangements. Williamson [1992] presents a model of private notes used in payment, and compares a free banking regime to government currency monopoly under private information. In an earlier paper of his [1989] banks function as record-keeping institutions that transfer asset ownership to pay for purchases by their depositors, very much like the banks in this paper. McAndrews and Roberds [1995] present a model of a clearinghouse aimed at capturing the "panics" of the U.S. National Banking Era; their model also relies on spatial separation. Kahn and Roberds [1995] explore the role of limited information in explaining observed interbank settlement arrangements. More recently, Kahn and Roberds [1996] have uncovered useful insights concerning settlement risk in a partial equilibrium framework; they emphasize the trade-off to participants between risk and the interest cost of holding central bank reserve balances. This latter cost, and how it is affected by the particulars of central bank payment system policy, is the focus of attention here.

2. Central banks as clearing facilities: motivating the model

The objective, then, is a model of the use of central bank deposit liabilities to settle payments. As noted above, this requires a model in which the underlying transaction is paid for by the exchange of a credit instrument, which is then separately *cleared* (presented for payment) and *settled* (repaid).

The work of Townsend [1980, 1987, 1989] and Williamson [1992] suggest that spatial separation and limited communication should be important in understanding the use of credit as a means of payment. In their models agents travel to transact, and want to acquire a good at one time and place in exchange for a promise to supply something at a different time and place. There is a natural role for credit instruments as a means of carrying out such trades. In particular, Townsend [1987] has emphasized that written instruments can ensure reliable (incentive-compatible) communication across time and locations. I follow Townsend here by focusing on written payment instruments in a model with spatial separation and limited communication. Alternatively, payment instruments can be interpreted as electronic instructions that are processed some time after the underlying transaction.[5]

Heterogeneity would also seem to be important for understanding settlement. If every agent in the model was a symmetric replica of every other agent, then claims would routinely offset and no net transfers would be required. I adopt the device of having consumers experience preference shocks of the type introduced by Lucas [1980] and Diamond and Dybvig [1983]. The shocks are private information, for otherwise insurance arrangements would remove the heterogeneity in payment claims. Private preference shocks also play a role in motivating the need for payment instruments to ensure that announcements are consistent across locations, as in

[5] For models of credit as a means of payment see Schreft [1992], Ireland [1994], and Lacker and Schreft [1996]. For treatments of electronic payments in particular see Townsend [1986], Wallace [1986], and Lacker [1996].

Townsend [1987]. Atkeson and Lucas [1992] and Taub [1994] study the same type of shocks in infinite horizon settings. In order to facilitate comparison with standard monetary models, I also adopt an infinite horizon. For simplicity, the model has no role for currency or any other central bank liability.

The next section formally describes the economic environment. In this paper I will limit attention to a particular class of institutional arrangements within this environment: agents buy goods with bills, which banks then settle. In section 4 I justify this assumption by describing limitations on communication and commitment capabilities under which the stipulated arrangements would emerge in this environment. The analysis of the remainder of the paper is self-contained, however, so that the reader can skip the section 4 if so desired.

3. The economic environment

The agents in this economy live on N distinct islands, indexed by i=1,2,..., and on each island there are N agents. When necessary, agents will be identified by their island of origin and by their name on that island, so that agent j from island i is referred to as ij. Time is discrete and indexed by t=1,2,.... Each agent on each island is endowed each period with a perishable consumption good that is specific to that island. Thus there are N types of goods at each date. I wish to abstract from aggregate uncertainty, at least initially. One way to do so is to suppose that there are a countably infinite number of agents and islands; I will assume $N=\infty$. The $N=\infty$ economy is envisioned as the limit of finite N-sized economies as $N\to\infty$, but this limit is never taken explicitly.

At each date t the agents want only to consume a good from a particular island. Specifically, each agent ij receives a shock to their preferences $h_{ijt}\in\{1,2,...,N\}$, indicating the island whose good they desire. These shocks are such that no two agents from a given island want the same island's good. Thus one agent from each island travels to every other island. One agent from each island desires their own island's good, and thus stays at home. Agents simultaneously travel to acquire the good they desire, which must be consumed before returning home. Let c_{ijt} denote the consumption by ij at date t of good h_{ijt}. The endowment of ij at date t (of good i) is equal to the same positive constant y for all i, j, and t.

In addition, each period each agent experiences a private preference shock, a positive number θ_{ijt}, drawn from a finite set Θ, which determines how patient they are. All agents on the same island receive the same preference shock, so $\theta_{ijt}\equiv\theta_{it}$ for all ij. The shock is known directly only to the agent that receives the shock, but since the structure of the economy is common knowledge an agent's shock is known to all other agents on the same island, since they receive the same shock as well. Given the shocks h_{ijt} and θ_{it}, agents care about consumption within period t according to the function $u(c_{ijt})\theta_{it}$, where u is strictly increasing, strictly concave, continuously differentiable, nonnegative and bounded. The shocks θ_{it} are independent and identically distributed over time and across islands, and are drawn according to the strictly positive distribution μ on Θ. In addition, preference shocks are assumed to have a mean of one:

$$1 \quad = \quad \int_{\Theta} \theta\mu(d\theta)$$

The preferences of agent ij are given by

$$\mathrm{E} \;\; (1-\beta)\sum_{t=1}^{\infty} \;\; \beta^{t-1} u(c_{ijt})\,\theta_{it}$$

where the discount factor β lies between 0 and 1. When θ_{it} is high the agent is impatient and consumption is especially urgent, and when θ_{it} is low the agent is patient and willing to delay consumption.

Events take place within a given time period according to the following schedule. First, agents are together on their home island and can transfer endowment. Agents learn which island's good they desire that period. The agent that remains behind distributes that island's good to visiting agents from other islands; these sedentary agents can be thought of as "merchant-bankers." Then the $N-1$ traveling agents from each island depart; these can be described as "shoppers." On their way, they learn their preference shock, θ_{it}. Agents arrive at their destination separately and in random order, and must depart before other agents arrive. Thus the interaction of shoppers and merchants is governed by the sequential service constraint, which here, as in Wallace [1988], is viewed as a physical characteristic of the environment. After acquiring their desired consumption good, the shoppers leave the island they are visiting and consume the good before returning home, where they are together with all other agents on their island. The merchant also must consume before agents return. After travelers return to their home islands, time remains for one or more agents from each island to travel again, although no consumption goods are available at this point. During this last "settlement" portion of the period it will be sufficient for just one agent to represent each island; without loss of generality I will assume that it is the merchant-banker. Communication between islands while the shoppers are obtaining goods from merchants and consuming them is impossible, as in Townsend [1987, 1989]. The "government" is a selfless agent that is active only during the settlement period.

The agents on a given island know each other's preference shocks, since all of them receive the same shock. This raises the possibility of mechanisms that force them to inform on each other by playing agents off one another. To avoid these complications, I will assume that the agents on an island always collude among themselves in their relations with the rest of the economy. Allocations must respect the ability of agents to conspire in choosing reporting strategies. E. S. Prescott and Townsend [1996] give a rigorous treatment of such an assumption in a mechanism design context. This approach allows us to treat each island as a single "bank," and to focus on the interbank/interisland settlement process.

Note a convenient feature of this environment. By the law of large numbers the fraction of islands receiving a given preference shock is the same as the ex ante probability of receiving that shock. Shoppers arrive at a given island from every other island, and thus the distribution of shopper characteristics on a given island is the same as the economy-wide distribution of shopper characteristics. Each island is a replica of each other island, except for consumption preferences. As a result, resource feasibility will require that consumption, averaged across all island characteristics, is equal to y; this is enough to ensure resource feasibility on each island. Another result of this assumption is that the evolution of the aggregate state of the economy can be treated as determinant, even though individual islands face uncertainty.

Exchange is organized by means of bills and deposit accounts. Shoppers pay for consumption goods by providing bills drawn on the deposits they hold with the merchant-banker on their home island. During the final stage of each period clearing and settlement occur; merchants travel to a central location and present bills. At a facility there each island's merchant keeps an account with which the island's bills are paid. Account balances must be nonnegative when settlement is completed each period, but islands can borrow or lend account balances. Because an island's consumption fluctuates while its sales of consumption good are constant (in equilibrium), an island's loan balance will rise or fall depending on the sequence of preference shocks it receives.

4. Payment arrangements

The analysis of the paper takes as a starting point that exchange is organized in the manner just described. This section attempts to justify these arrangements. The beneficial role of this exercise is to confirm that there is at least one internally consistent set of assumptions that generate these payments arrangements. A reader willing to accept the institutional arrangements as given, however, can skip this section without great harm.

The task in this section, then, is to sketch the mechanism design problem for this economy. The focus, at first, is on the outcomes or *allocations* that can be delivered by any arbitrary institutional arrangement. An allocation for this economy is a set of contingent consumption plans specifying each agent's consumption under all possible sequences of shocks. Restrictions on feasible and incentive compatible allocations can be derived directly from the primitives of the economy--the preferences, endowments, and technologies. A natural assumption to make is that agents will agree to an arrangement that provides an allocation that is Pareto optimal, subject to the constraints derived from the primitives of the economy. After finding the optimal allocations, the next step is to consider just what institutional arrangements are capable of delivering such outcomes. The exposition proceeds, for the sake of brevity, without formal proofs. Three distinct economies are considered.

4.1 A full communication economy

To begin, consider a *full communication* version of this economy, in which costless communication across islands is possible, but the preference shock, θ, remains private information. Communication is useful in this economy because it allows intertemporal tie-ins between consumption at different dates. The usefulness of intertemporal tie-ins to help cope with the incentive constraints in multi-period economies is now well-known: see Townsend [1982, 1987], Atkeson and Lucas [1992], Taub [1994].

Individuals and their home islands can be characterized at any date by their shock history. Let $\theta^t = (\theta_1, \theta_2, ..., \theta_t)$ be a history of preference shocks t periods long, an element of the space Θ^t. Suppressing the indexes i and j for a moment, a consumption plan for a typical agent is a collection of functions $c_t : \Theta^t \rightarrow \mathbb{R}_+$ for t=1,2,..., where $c_t(\theta^t)$ is the consumption after the shock sequence θ^t. Resource feasibility requires that consumption not exceed endowment on each island, but since the population of shoppers attending any given island is an exact replica of the economy-wide population of shoppers, each island's resource feasibility condition is the same; average consumption must not exceed average endowment.

$$\int c_t(\theta^t)\, \mu^t(d\theta^t) \quad \leq \quad y \qquad \forall t$$

Incentive compatibility is the same here as well, and requires that at every date t, announcing θ_t truthfully is weakly preferred to making some other announcement $\hat{\theta}_t$:[6]

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[6]Misreporting means that every agent from that islands identically misreports θ, consistent with the assumption that agents on an island collude together. Otherwise merchants could compare the reports of agents from a given island. Merchants also must reveal their own preference shock to visiting shoppers.

$$(1-\beta)u(c_t(\theta^{t-1},\theta_t))\theta_t \quad + \quad \mathrm{E}_t \sum_{s=1}^{\infty} (1-\beta)\beta^s u(c_{t+s}(\theta^{t-1},\theta_t,\theta^s))\theta_{t+s}$$

$$\geq \quad (1-\beta)u(c_t(\theta^{t-1},\hat{\theta}_t))\theta_t \quad + \quad \mathrm{E}_t \sum_{s=1}^{\infty} (1-\beta)\beta^s u(c_{t+s}(\theta^{t-1},\hat{\theta}_t,\theta^s))\theta_{t+s} \qquad \textbf{(1)}$$

$$\forall\theta^{t-1}\in\Theta^{t-1}, \quad \forall\theta_t,\hat{\theta}_t\in\Theta, \quad \forall t$$

The value of intertemporal tie-ins can be seen plainly here. If the θ_t announcement has no effect on consumption at future dates t+s, s=1,2,..., then current consumption can not vary with the θ_t announcement in an incentive compatible allocation.[7]

Written as in (1), the incentive compatibility constraints are quite complicated, but Atkeson and Lucas prove that the problem can be given a much simpler and for our purposes instructive recursive formulation. They imagine a social planner responding to shock announcements with an allocation of current consumption and a promised expected lifetime utility at the beginning of the next period. Each agent begins a period with a previously promised lifetime utility, w say, in some feasible set D. Shock-contingent schedules for consumption and future utility are functions of w; an agent that announces preference shock θ receives current consumption $c(w,\theta)$ and future expected utility $w'(w,\theta)$. This provides the agent with utility $(1-\beta)u(c(w,\theta))\theta$ + $\beta w'(w,\theta)$. Incentive compatibility is just

$$(1-\beta)u(c(w,\theta))\theta \;+\; \beta w'(w,\theta) \quad \geq \quad (1-\beta)u(c(w,\hat{\theta}))\theta \;+\; \beta w'(w,\hat{\theta}) \qquad \textbf{(2)}$$

$$\forall\theta,\hat{\theta}\in\Theta, \quad \forall w\in D, \quad \forall t$$

The planner now also faces the consistency constraint that the previously promised lifetime utility must equal the expected utility of current consumption plus discounted future utility.

$$w \quad = \quad \int_{\Theta} [(1-\beta)u(c(w,\theta))\theta \;+\; \beta w'(w,\theta)]\mu(d\theta)$$

The planner would like to provide full insurance each period against θ shocks, but this is not possible without intertemporal tie-ins. One way to provide more to impatient agents, while ensuring truthful revelation, is to reduce future utility for agents announcing a high θ; that is, make $c(w,\theta)$ increasing and $w'(w,\theta)$ decreasing in θ. Impatient agents receive high current consumption but lower future utility. While this allocation falls short of providing full insurance, it does provide limited insurance against becoming impatient.

What is noteworthy, for the purpose at hand, is that the optimal allocation involves keeping a running score on each agent, with deductions in periods when consumption is high and accruals when consumption is low. This suggests the usefulness of an accounting system to keep track of agents' entitlement to utility. In the world of Atkeson and Lucas, agents are all together, so that (costless) centralized record-keeping is feasible. Townsend [1987] argues that bills of exchange, bank notes, and deposit banking also serve as record-keeping mechanisms in exactly this sense, by allowing mutually beneficial intertemporal tie-ins. Moreover, in Townsend's models these

[7] The alert reader may have noticed that the model is mathematically identical to Atkeson and Lucas [1992].

mechanisms are essential due to spatial separation, in the sense that without verifiable communication across locations the optimal full-communication allocation cannot be achieved; welfare is lower in the absence of such institutions.[8]

Although the Atkeson and Lucas allocation does have agents facing a trade-off between current and future consumption, it is in one respect glaringly inconsistent with typical banking arrangements. Agents are not choosing consumption bundles along an intertemporal budget constraint, as they would if they were paying for consumption at given prices with bills drawn on interest-earning account balances. The trade-off between current and future consumption is nonlinear--indeed, there is no uniform intertemporal rate of substitution with which to discount future consumption. Atkeson and Lucas [1992, pp. 442-45] show that in general agents with different preference shocks will have different intertemporal rates of substitution under the optimal allocation. In short, the Aktinson and Lucas allocation does not resemble market trade.

4.2 A full communication economy with unobserved side-trading

This observation, along with the results of Gale [1980], Allen [1985], Hammond [1987], and Townsend [1989], suggests that unobserved side-trading will lead to linear consumption menus. Specifically, if agents are able to form coalitions, unobserved by other agents, that make enforceable side agreements to transfer received consumption goods, agreements that are themselves subject to incentive and feasibility constraints, then available consumption bundles must be linearly related.[9] In other words, there exist prices (discount rates) such that all shock announcements result in consumption plans with the same present discounted value. More precisely, the result is that an allocation will be vulnerable to side trades unless there exists a set of numbers $\{\rho_{t,s}, t=1,2,...; s=1,2,...\}$ that satisfy[10]

$$c_t(\theta^{t-1},\theta_t) + \sum_{s=1}^{\infty} \rho_{t,s} \mathrm{E}_t c_{t+s}(\theta^{t-1},\theta_t,\theta^s) = c_t(\theta^{t-1},\hat{\theta}_t) + \sum_{s=1}^{\infty} \rho_{t,s} \mathrm{E}_t c_{t+s}(\theta^{t-1},\hat{\theta}_t,\theta^s) \qquad \forall\theta^{t-1}\in\Theta^{t-1}, \quad \forall\theta_t,\hat{\theta}_t\in\Theta, \quad \forall t \tag{3}$$

$$\rho_{t,s}\theta_t u'(c_t(\theta^{t-1},\theta_t)) = \mathrm{E}_t[\beta^s\theta_{t+s}u'(c_{t+s}(\theta^{t-1},\theta_t,\theta^s))] \qquad \forall\theta^{t-1}\in\Theta^{t-1}, \quad \forall\theta_t\in\Theta, \quad \forall t \tag{4}$$

The reasoning is straightforward. If intertemporal marginal rates of substitution are not equal for all agents, the allocation will be vulnerable to unobserved borrowing and lending; thus immunity to

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[8]See also Gale [1980].

[9]Gale [1980], working with sequence economies, assumed that the "planner" could not observe announcements at earlier dates and showed that paper assets, such as money, could be useful. Allen [1985] argues that in multiperiod principal-agent models the possibility of unobserved borrowing and lending will further restrict the set of incentive compatible contracts, and that agents will evaluate allocations according to their present discounted value. Hammond [1987] proves that if an allocation in a private-information continuum economy is to be incentive compatible against side trades by finite coalitions, then there must be a decentralizing budget set with linear prices. Townsend [1989], in a model of tokens (or coins) as incentive-compatible communication devices, shows that if agents can make unobserved side trades then the schedule relating tokens to consumption goods must be linear, as in a price system.

[10]This result was verified in unpublished research with Peter Ireland.

side trading requires that intertemporal marginal rates of substitution be equal across agents. The entire set of intertemporal discount rates $\{\rho_{t,s}\}$ can be calculated in the standard fashion from the marginal rate of substitution between consumption at adjacent dates. Now observe that the left side of (3) is the expected present value of consumption given history θ^{t-1} and announcement θ_t, while the right side is the same quantity for the alternative announcement $\hat{\theta}_t$. If these are not equal for all announcements, then coalitions can collectively improve on their allocation. For example, suppose the quantity in (3) were larger for $\hat{\theta}_t$ under some resource feasible and incentive-compatible allocation. Agents could falsely announce $\hat{\theta}_t$, although the resulting allocation would not provide them with greater expected utility, since by assumption the original allocation satisfies the incentive constraints. But they could then trade to obtain a consumption bundle that they strictly prefer to their original allocation. Thus unless (3) holds, allocations are vulnerable to side trades.

Conditions (3) and (4) imply that in choosing a preference shock announcement, agents are choosing a consumption bundle subject to an intertemporal budget constraint that allows for risk-free borrowing and lending. The present discounted value of consumption from date *t* onward is the same, no matter what the date *t* preference shock. This allows rewriting the problem with wealth levels rather than utility levels as the state variable. A typical agent enters period *t* with a given present discounted value of promised consumption given by (3). After consuming c_t, the remaining account balance earns interest at the rate $r_t = (1-\rho_{t,1})/\rho_{t,1}$. Agents therefore face a sequence of intertemporal budget constraints:

$$a_{t+1} \quad = \quad (1+r_t)(a_t - c_t), \qquad \forall t \tag{5}$$

where a_t is equal to the quantity in (3), and a_1 is given.[11] Agents face an additional constraint at each date from the fact that they have to be able to afford nonnegative consumption from date *t* forward. This requires that $a_t \geq 0$, for all *t*. Note that this "borrowing constraint" follows naturally from the feasibility constraints of the mechanism design problem.

Incentive compatibility can now be replaced with the requirement that allocations represent utility-maximizing choice subject to the sequence of budget constraints in (5). Feasibility of those choices requires that average consumption not exceed per capita output *y*, which will pin down the sequence of interest rates. The optimal allocation, then, is also an equilibrium outcome in a decentralized market for risk-free borrowing and lending. Much of the intertemporal incentives in (1) have been undone. Moreover, because of the private preference shocks there is still a nontrivial demand for intertemporal exchange--agent and island balances will fluctuate over time in response to preference shocks.

4.3 A limited communication economy

The next step is to return to the model in which interisland communication is impossible while shoppers are visiting other islands, and observe that there is a role for portable forgery-proof messages as communications devices. The need is to communicate the announcements made to merchants, or, equivalently, the amount of consumption good merchants transferred to shoppers. One way to use such messages to implement the optimal full-communication allocation is for shoppers to record their preference shock announcement in a document that is exchanged directly for consumption. The merchant, in turn, presents the document to the merchant from the home

[11] The initial balance a_1 determines initial lifetime expected utility. Since all are initially identical, symmetric treatment requires that a_1 be the same for all agents. Market clearing then requires that a_1 equal the present discounted value of the endowment stream.

island of the traveling agent during the settlement period, resulting in a deduction from the account of the agent's island. The document could specify the shock announcement, but it is equivalent, and more realistic, for it to specify the amount of consumption transferred to the shopper, that is, the amount deducted from the island's account.

By itself, however, this mechanism does not quite constitute clearing and settlement. An amount is deducted from the island's account but does not result in a credit to the account of the merchant that presents the document. In other words, the merchant does not receive payment upon presenting the document. The mechanism resembles more closely a cost-allocation accounting scheme, where invoices communicate the amount of common resources used by various agents.

Note, however, that an invoice mechanism relies heavily on the commitment of merchants to disburse the amount of consumption good that corresponds to the message contained in the document. Since the merchant and the shopper are together and unobserved, it is quite natural to consider the possibility that the two of them might conspire against the mechanism. Suppose then that unobserved side trades between the merchant and the visiting shopper are possible. They would choose the contents of the document in a way that is jointly advantageous to the two of them. Since the shoppers future consumption is strictly declining in the recorded announcement θ while the merchant's future consumption is invariant with respect to θ, they strictly prefer to have the merchant accept a writing that states that the lowest possible θ was announced. The mechanism fails to communicate preference shock announcements and the scheme unravels.

To avoid this possibility, the mechanism can be modified to keep track of transfers of endowment, not just consumption. Suppose the account of a traveling agent is credited with the amount of the endowment they transfer to the merchant on their island before leaving to visit other islands. The account balance of a typical traveling agent (again suppressing the index *ij*) now evolves according to

$$a_{t+1} \quad = \quad (1+r_t)(a_t+y-c_t), \qquad \forall t$$

The merchant's account is in turn debited for the amount of endowment the merchant acquires. Then the merchant's account is credited with the amount of good transferred to visiting shoppers from other islands. Let s_{ijt} be the amount transferred by the merchant on island i to the visiting shopper from island j at date t. Then the account of island i's merchant (agent $i1$) evolves according to[12]

$$a_{i1,t+1} \quad = \quad (1+r_t)\Big[a_{i1t} + \sum_{j=1, j\neq i}^{N} s_{ijt} - y(N-1)\Big], \qquad \forall t \tag{6}$$

The arrangement now requires that the amount deducted from a shopper's account be added to the account of the merchant presenting the document. A merchant that accepts a document that understates the amount transferred to the shopper suffers real consequences; the credit to their account balance is lower than it would have been, and their future consumption is commensurately reduced. Merchants and shoppers now have opposing interests in the amount of consumption reported in the document. Their joint choice of consumption transfer and announcement report will necessarily satisfy the first order conditions equating their intertemporal marginal rate of substitutions.

Note, incidentally, that resource feasibility at date t requires

[12]For clarity, I revert temporarily to a finite economy.

$$c_{i1t} + \sum_{j=1, j\neq i}^{N} s_{ijt} = yN \qquad \forall t \tag{7}$$

Substituting (7) in (6) yields the merchant's budget constraint

$$a_{i1,t+1} = (1+r_t)(a_{i1t}+y-c_{i1t}), \qquad \forall t$$

exactly parallel to the budget constraints of a traveling agent. Note also that the borrowing constraint in this mechanism is now

$$a_{ijt} \geq -y - \sum_{s=1}^{\infty}\left(\prod_{j=1}^{s}\frac{1}{(1+r_{t+j-1})}\right) y \equiv -\phi_t, \qquad \forall i,j,t \tag{8}$$

In other words, net borrowed balances must not exceed the present discounted value of the endowment stream. Again, this constraint follows directly from the feasibility requirement that an agent's balance must allow for nonnegative future consumption.

To summarize, in the full-communication version of the economy, taking into account the possibility of unobserved side trading, the optimal allocation is identical to that obtained allowing only risk-free borrowing and lending; each agent has an account balance that records their consumption, sale of endowment, and interest earnings. In the limited communication economy, there is a role for written documents to facilitate exchange. A shopper creates a writing that the merchant later presents for payment, resulting in a transfer of value from the shopper's account to the merchant's account. This mechanism is able to implement the optimal full-communication immune-to-side-trading allocation. The essential frictions are: the private nature of the preference shock θ, the inability to communicate across islands while shoppers are obtaining consumption, and the inability to observe or prevent side agreements between agents.

The payments mechanism has up to this point been described as paper-based, corresponding most closely to bills of exchange or checks. The arrangement could easily be thought of as an electronic payment scheme, however, such as debit or credit cards. A debit card transaction results in a transfer of deposit funds to the seller from the buyer, while a credit card transaction generates a loan balance for the buyer. On-line systems immediately commence the transaction, and thus resemble the full-communication version of the model. Off-line systems transmit information later in the day, like the model with bills.

5. A simple model of clearing and settlement

Consider the decision problem of a typical traveling agent. This shopper begins period t with an account balance a_t (where I have suppressed the shopper's index ij), is credited for endowment deposit y and is debited c_t for the bills issued to purchase consumption. The agent's balance evolves according to $a_{t+1}=(1+r_t)(a_t+y-c_t)$, where r_t is the interest rate paid on account balances. Note that as an innocuous normalization account balances are denominated in units of consumption good each period. A negative balance, or overdraft, is limited by the amount the agent can repay with future transfers of endowment; the agent faces the borrowing constraint, $a_t \geq -\phi_t$, t=1,2,..., where ϕ_t is the present discounted value of current and future endowments, and is given in (8) above. Consumption at date t depends on $\theta^t=(\theta_1,\theta_2,...,\theta_t)$, the history of preference shocks up through time t, an element of the space Θ^t. A consumption plan, therefore, is a function c_t:$\Theta^t \rightarrow \mathbb{R}_+$, specifying consumption $c_t(\theta^t)$ after shock history θ^t. A typical traveling agent therefore chooses consumption plans to solve

$$\text{MAX} \quad \mathrm{E}\ (1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}\theta_t u(c_t)$$

$$\text{s.t.} \qquad a_{t+1} = (1+r_t)(a_t+y-c_t), \qquad \textbf{(P1)}$$

$$c_t \geq 0, \qquad a_t \geq -\phi_t$$

where $a_1=0$ is given. Since all agents from a given island are identical, an island's (per capita) balance is identical to that of a representative agent from that island. Since all islands experiencing the same shock history behave in the same way, (P1) can also be viewed as the decision problem of a representative island.

The state of the economy at any given date t can be described by the distribution of initial balances, ψ_t, a probability measure on $[-\phi_t,+\infty)$. Because individual island balances are buffeted by preference shocks over time, the distribution of balances will, in general, vary over time. Because of the size of the economy, however, it will evolve in a deterministic way; there is no aggregate uncertainty. An equilibrium therefore consists of determinate sequences of interest rates $\{r_t\}$, balance distributions $\{\psi_t\}$, and contingent consumption plans $\{c_t\}$. Consumption plans and the evolution of balances must be consistent with utility maximization in (P1), and in addition consumption must not exceed endowment:

$$\int c_t(\theta^t)\,\mu^t(d\theta^t) \quad \leq \quad y, \qquad \forall t$$

where the probability measure μ^t is defined on Θ^t.

For some intuition into behavior of the model, consider the case of logarithmic utility: $u=\ln(c)$, for which the interest rate is constant and given by[13]

$$1+r^* \quad = \quad \frac{1}{\mathrm{E}[h(\theta)]}, \qquad \text{where } h(\theta) \equiv \frac{\beta}{\beta+(1-\beta)\theta}$$

Given r^*, the evolution of an agent's consumption and account balance can be written as time-invariant functions of beginning balances a_t the current shock θ_t, and the borrowing constraint $-\phi=-y/r$:

$$c_t \quad = \quad c^*(a_t,\theta_t) \quad \equiv \quad (1-h(\theta_t))\big(a_t+\phi\big)$$

$$a_{t+1} \quad = \quad A^*(a_t,\theta_t) \quad \equiv \quad \frac{h(\theta_t)}{\mathrm{E}[h(\theta)]}\big(a_t+\phi\big) \quad - \quad \phi$$

Consumption and account balances vary in opposite directions with the preference shock, as expected; agents consume more in response to a high θ_t, drawing down their accounts. Consumption and end-of-period balances are both increasing in beginning of period balances.

Perhaps the most striking feature of the model is the nonstationarity of the distribution of wealth. Account balances "fan out" over time in response to preference shocks. To see this, note that

$$\ln(a_{t+1}+\phi) \quad = \quad \ln[h(\theta_t)] \;-\; \ln\{\mathrm{E}[h(\theta)]\} \;+\; \ln(a_t+\phi)$$

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[13]See Lucas [1992], pp 240-41. See Taub [1994] for a version with linear u.

The logarithm of an agent's net worth, $a_t+\phi$, is a random walk with drift. Moreover, the drift term $E\{\ln[h(\theta)]\} - \ln\{E[h(\theta)]\}$ is negative, implying that net worth and consumption go to zero with probability one even though the population averages of net worth and consumption are both constant. This is a well-known property of this and other dynamic private information models (see Green [1987] for example), and although it may seem unappealing, nothing that follows depends on it. What is essential is that there be some heterogeneity of net trades each period so that net clearing obligations are not trivial.[14]

Returning to the model with general preferences, payment obligations behave as follows. Shoppers from an island i with shock history θ_i^t issues bill in the amount of $c_t(\theta_i^t)$ each. Merchants from every other island come to the settlement with bills in this amount drawn on island i. Merchant i has bills drawn on every other island as well: $c_t(\theta_j^t)$, for $j \neq i$, for a per capita total of y. The bilateral net obligation of merchant i to merchant j is $c_t(\theta_i^t) - c_t(\theta_j^t)$. After netting out across all other merchants, i's net obligation is $c_t(\theta_i^t) - y$. Note that without the uninsured idiosyncratic risk, θ, settlement is trivial: c_{it} always equals y.

6. The central bank as a clearing facility

At this point little has been said about the particulars of settling up. One institutional arrangement that can coordinate the necessary clearing and settlement is a clearing bank, a merchant that both facilitates the presentment of claims and intermediates between islands. Islands with a run of patience (low θ shocks) run up positive balances, while islands with a run of impatience run up negative balances. Positive balances represent clearing bank liabilities, while negative balances represent overdraft lending by the clearing bank. An alternative arrangement is to separate lending from clearing and settlement. The clearing bank could offer accounts in which negative balances are not permitted, but with merchants allowed to freely borrow and lend account balances, either bilaterally or through a separate intermediary. Note that there would be no reason for positive clearing balances if they paid a lower rate of return than loans, since each island would know at the beginning of the settlement period what claims are coming due. If one were to introduce explicit frictions affecting clearing and settlement activities, it would be possible to pin down arrangements more precisely. For example, if there were costs associated with each bilateral communication as in Townsend [1978], distinct intermediaries structures might emerge.

The presentment process could involve bilateral transfers, with the two counterparties to each transaction jointly appearing at the clearing facility and ordering the transfer. Negative balances in this case would correspond to overdraft loans from the facility, or bilateral loans between merchants. Alternatively, presentment could take place as in a clearinghouse, with simultaneous exchange of bills followed by settlement of net obligations, either an accepted token currency or transfer of deposits, perhaps held at the clearinghouse itself: see Cannon [1901], or, for a particularly vivid description, Gibbons [1859]. Again, if one were to introduce explicit frictions affecting the presentment process, it would be possible to pin down arrangements more precisely.

To explore the applicability of this framework to late twentieth century central banking, I will assume that the clearing facility acts as a deposit bank only, accepting deposit accounts but making no direct loans or advances itself, except as part of its clearing operations as described below. Thus account balances must be nonnegative; banks arrange separately for loans of account

[14] There are other models of uninsurable idiosyncratic risk with stationary, nondegenerate, equilibrium wealth distributions: see, for example, Aiyigari [1994].

balances between them. Furthermore, I will suppose that the facility is operated as a legal monopoly by the central bank, and that this monopoly is exploited by paying a below-market rate of return on account balances. The monopoly could be enforced by regulatory fiat or by the underpriced provision of clearing services, although I do not model these explicitly.[15] Although this approach reflects the view that legal restrictions are essential to explaining the fact that monetary assets, such as reserve account balances, are dominated in rate of return (Wallace [1983]), I do not mean to prejudge the question of whether there is a beneficial role to be played by a governmental clearing bank.

By shifting approach here, a certain unavoidable tension is introduced. Up until this point, the analysis has been positive, deriving institutional arrangements as Pareto optimal outcomes. I depart at this point in order to analyze the effects of apparently suboptimal monetary policies like reserve requirements and noninterest-bearing reserves. The outcomes can clearly be improved upon, but the implicit assumption from here on is that the government is capable of preventing such improvements. While this is the usual assumption underlying normative policy analysis, it is a bit more problematic here than usual, because of the ability of coalitions to engage in unobserved side trading. Such side trading, if feasible, could circumvent a central bank's authority and completely undo the intended effects of monetary policy. Accepting this tension can be viewed as a compromise that allows us to explore the components of a deeper theory of monetary institutions.[16]

For each bank, then, the central bank maintains a *reserve account* that is used to settle all bills due to or from each bank. To allow for a varying rate of exchange between reserves and consumption goods, let p_t be the period t price of consumption goods in units of reserves. Reserve accounts do not earn interest. I allow the government to borrow and lend by issuing risk-free one-period bonds and levying lump-sum taxes. Government bonds will compete directly with risk-free private loans. The model can display rate of return dominance because of the legal restriction that bills must be cleared in reserve accounts. I will assume without loss of generality that individual agents' bank account balances are denominated in the same monetary units as reserves. During the settlement period the bank presents a total of $p_t y$ worth of bills drawn on other banks, and bills worth a total of $p_t c_t$ are presented to the bank for payment.

The clearing and settlement of bills takes place according to the following process. There is a fixed subinterval of the settlement period, call it the *clearing period*, during which the central bank accepts bills. For the entire clearing period each bank sends bills at the same continuous rate to the central bank. When the central bank receives a bill it immediately transfers the funds.[17] A

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[15]The Federal Reserve appears to have relied on both during its history. Many services were underpriced or entirely unpriced until the Monetary Control Act of 1980, and some regulatory constraints, such as par checking rules, appear to have bolstered the Fed's market share early on (Spahr [1926]).

[16]It is worth noting that many of the early public banks cited in the introduction share features with the central bank in this model. They were government-chartered banks of deposit, and they transferred deposits and cleared bills of exchange among merchant-bankers. Many at some time held a legal monopoly on clearing payment instruments; bills of exchange over a certain amount had to be paid by deposit transfer at the public bank. They were universally involved in funding the government, and many were not allowed to make private loans. On the *Taula de Canvi de Barcelona* see Usher [1943], Rui [1979]; on the *Rialto* bank in Venice see Luzzatto [1934]; on the Amsterdam Exchange Bank see van Dillen [1934], van der Wee [1977], and Smith [1976].

[17]This is therefore a delivery-versus-payment system.

bank's balance can fluctuate as bills are cleared during this interval. At each moment each bank selects an arbitrary note to send from those it has on hand. Clearing results in a net credit of $p_t(y-c_t)$ over the course of the clearing subinterval. Moreover, a bank's reserve account balance changes linearly over the clearing period at the constant rate $p_t(y-c_t)$: a fraction δ of the clearing period results almost surely in a net credit of $\delta p_t(y-c_t)$.

When does the clearing period occur within the settlement period? One possibility is for the clearing period to begin after the settlement period begins, thus allowing borrowing and lending to take place before clearing. When bankers come together at the beginning of the settlement period, however, they already know the amount of bills drawn on them, and thus they know what net accrual of funds to expect during the clearing period. Any constraint by the central bank on intraday account balances, such as a limit on daylight overdrafts, could be costlessly circumvented by bank borrowing at the beginning of the settlement period. In order for intraday account constraints to have any force (as distinct from end-of-day constraints), there must be some intraday debits that banks are unable to prevent by borrowing.

I therefore adopt the assumption that the clearing period occurs at the beginning of the settlement period, before any borrowing or lending take place. This arrangement allows central bank restrictions on intraday balances to have some force, and can be viewed as a technological feature of central bank operations. It is intended to capture the common characteristic of many central bank funds transfer systems that a bank can not completely control the debits to its account, or at least can not instantly offset debits via a securities market transactions.[18] Note that I am not disallowing credit market "transactions" before or during the clearing period; bankers are free to contact each other and make any promises they wish. The assumption is that the central bank does not open its ledger for transfers at time *t* before the clearing is complete. This assumption is designed to enable us to study issues having to do with daylight overdraft policy; for other purposes one could imagine other assumptions about the structure and timing of clearing operations.

A typical bank begins period *t* with a central bank balance of m_{t-1}. During the clearing period at *t* a typical bank's reserve balance begins at m_t and changes at the constant rate $p_t(y-c_t)$ to finish at

$$\hat{m}_t \quad = \quad m_{t-1} + p_t(y-c_t), \qquad \forall t \tag{9}$$

The bank made investments in loans and government bonds of b_{t-1} the previous period. These earned interest at the nominal rate i_{t-1}. The resulting reserve account balance at the end of the clearing period is divided between investments, b_t, new reserve balances, m_t, and lump-sum tax payments, $p_t\tau_t$. A typical bank therefore faces the following sequence of budget constraints

$$m_t + b_t \quad = \quad \hat{m}_t + (1+i_{t-1})b_{t-1} - p_t\tau_t, \qquad \forall t \tag{10}$$

The central bank imposes a constraint on overnight balances:

$$m_t \quad \geq \quad \Phi_{mt} \qquad \forall t \tag{11}$$

where $\{\Phi_{mt}\}$ is a nonnegative sequence. In addition, there is an daylight overdraft limit:

[18]Many clearing and settlement systems are automated in such a way that payments, once initiated, proceed autonomously. The assumption is in no way meant to detract attention from the fact that such technological arrangements are adopted voluntarily by payment system participants, who face trade-offs between various costs and risks in designing the architecture of system components.

$$\hat{m}_t \quad \geq \quad \Phi_{\hat{m}t} \qquad \forall t \tag{12}$$

where $\{\Phi_{\hat{m}t}\}$ is a nonpositive sequence. For convenience, I assume that both constraints are constant in real terms,

$$\Phi_{mt}=\phi_m\, p_{t+1}, \qquad \Phi_{\hat{m}t}=\phi_{\hat{m}}\, p_t, \qquad \forall t$$

where $\phi_m \geq 0$ and $\phi_{\hat{m}} \leq 0$ are constants, but variations could easily be considered.[19] Since the balance changes linearly over the clearing period, the constraint $\hat{m}_t \geq \phi_{\hat{m}}$ is sufficient to assure that the clearing balance is greater than $\phi_{\hat{m}}$ for the entire clearing period. After the clearing period is over, the bank can send and receive reserve account balances as it wishes.

The borrowing constraint must be modified in this setting to account for the effect of the monetary arrangement on a borrower's capacity to repay a debt. What is the smallest total end of period balances, m_t+b_t, that the borrower can repay with certainty, while meeting the constraints (9) and (10), and still being able to afford nonnegative consumption? The answer can be obtained by solving the budget constraints forward, substituting the smallest feasible consumption at each date: $c_{t+s}=0$, $s=1,2,....$ The result is

$$\frac{(m_t+b_t)}{p_t} \quad \geq \quad -\phi_t \quad \equiv \quad -\sum_{s=0}^{\infty}\left[\prod_{j=0}^{s}\left(\frac{1}{(1+r_{t+j})}\right)\right]\left[y-\tau_{t+s+1}-(1+r_{t+s})\left(\frac{i_{t+s}}{1+i_{t+s}}\right)\phi_m\right] \quad \forall t \tag{13}$$

where $1+r_t=(1+i_t)p_t/p_{t+1}$. The borrowing limit is the present discounted value of the stream of future endowments net of lump sum taxes and net of the inflation tax on the minimum reserve account balance of ϕ_m.

The government budget constraint for this economy is standard:

$$M_t \;+\; B_t \quad = \quad M_{t-1} \;+\; (1+i_{t-1})B_{t-1} \;-\; p_t\tau_t, \qquad \forall t \tag{14}$$

where M_t and B_t are aggregate reserve balances and government bond issues at t respectively. In addition, note that the assumptions about clearing operations imply that $\hat{M}_t=M_{t-1}$ for all t, where $\hat{M}_t$ is the net total intraday account balance, that is, including overdrafts as negative balances. This effectively disallows monetary operations within the clearing period. An alternative interpretation, however, is that the money supply is the sum of the nonnegative account balances, and that this balloons during the day with intraday overdrafts. While there is implicit intraday lending going on, there are no open market operations during the clearing period.

The problem facing the typical agent, or, equivalently, the typical island, is to solve

$$\begin{aligned} &\text{MAX} \quad \text{E}\ (1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}\theta_t u(c_t) \\ &\text{s. t.} \quad c_t \geq 0, \quad \text{and } (9)-(13) \end{aligned} \tag{P2}$$

where the island subscript i has again been suppressed. An equilibrium consists of: (i) deterministic sequences for the price level, the nominal interest rate, government liabilities, and taxes, $\{p_t,i_t,M_t,B_t,\tau_t\}$; and (ii) plans for consumption c_t, overnight balances m_t, intraday balances $\hat{m}_t$, and net lending b_t, all functions on Θ^t, that solve (P2) given the sequences $\{p_t,i_t,\tau_t\}$, such that average holdings of reserves equals M_t

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[19]The price level in (11) is dated t+1 for convenience, in view of (9).

$$M_t \quad = \quad \int m_t(\theta^t)\,\mu^t(d\theta^t), \qquad \forall t$$

the loan market clears

$$B_t \quad = \quad \int b_t(\theta^t)\,\mu^t(d\theta^t), \qquad \forall t$$

and the government budget constraint (14) holds.

The conventional specification of policy here would have the government and the central bank jointly choose sequences for money, bonds and taxes. Equilibrium conditions then determine the sequence of prices and interest rates, although not all policy sequences allow the existence of an equilibrium that satisfies the government budget constraint. A natural alternative is to suppose that the government and the central bank jointly choose sequences for taxes and the nominal interest rate. Equilibrium conditions determine the sequence of real holdings of government bonds and money. The choice of (arbitrary) monetary units in which the original government liabilities are issued then determines the price level sequence.

Let $\lambda_{0t}(\theta^t)$, $\lambda_{1t}(\theta^t)$, and $\lambda_{2t}(\theta^t)$ be the Lagrangian multipliers on constraints (13), (10), and (9) respectively, and let $\eta_t(\theta^t)$ and $\hat{\eta}_t(\theta^t)$ be the multipliers on (11) and (12) respectively. The first order conditions for (P2) are

$$\beta^t\theta_t u'(c_t(\theta^t)) \quad = \quad \lambda_{1t}(\theta^t)p_t \tag{15}$$

$$\lambda_{2t}(\theta^t) \quad = \quad \mathrm{E}_t[\lambda_{1,t+1}(\theta^{t+1})] \;+\; \lambda_{0t}(\theta^t) \;+\; \eta_t(\theta^t) \tag{16}$$

$$\lambda_{1t}(\theta^t) \quad = \quad \lambda_{2t}(\theta^t) \;+\; \hat{\eta}_t(\theta^t) \tag{17}$$

$$\lambda_{2t}(\theta^t) \quad = \quad \mathrm{E}_t[(1+i_t)\lambda_{2,t+1}(\theta^{t+1})] \;+\; \lambda_{0t}(\theta^t) \tag{18}$$

where E_t is the expectation taken after the time *t* shocks are revealed.

By combining (16), (17), and (18), we obtain an expression relating the nominal interest rate to the multipliers on the reserve balance constraints:

$$i_t\mathrm{E}_t[\lambda_{2,t+1}(\theta^{t+1})] \quad = \quad \eta_t(\theta^t) \;+\; \mathrm{E}_t[\hat{\eta}_{t+1}(\theta^{t+1})], \qquad \forall t,\ \forall\theta^t\in\Theta^t \tag{19}$$

Here we see that if the nominal interest rate is positive, then at every date and for every bank at least some reserve constraints must bind. Overnight balances face the opportunity cost i_t. Either the overnight balance constraint is binding [$\eta_t(\theta^t)>0$] and $m_t=\phi_m$, or the bank holds excess reserves because the intraday balance is expected to bind next period [$\hat{\eta}_{t+1}(\theta^t,\theta_{t+1})>0$ for some θ_{t+1}]. Which constraints bind will vary across banks, but will depend crucially on $\phi_m-\phi_{\hat{m}}$, the gap between the overnight limit and the intraday limit. This difference is the extent to which the intraday constraint is more lenient than the overnight constraint. Note the similarity between (19) and a similar condition that arises in cash-in-advance models. There the nominal rate is proportional to the Lagrangian multiplier on the cash-in-advance constraint: see, for example, equation (5.17) in Townsend [1987]. If the nominal rate is positive, then the cash-in-advance constraint must be binding, at least in some states. The economics are in some respects similar here. In the presence of a positive nominal rate banks want to minimize reserve balances. Here, however, whether the constraint is binding or not depends on the behavior of *net* trade at *t*, c_t-y, not gross purchases c_t as in the cash-in-advance model. Note also that the model here shares features with models of the "precautionary" demand for money (for example, Svensson [1985]), in that the money holding decision is made, ex ante, under uncertainty about the magnitude of the ex post money holding constraint. Again, however, the constraint here is on *net* ex post trades.

If a bank has little need for intraday credit, the minimum end-of-day balance may be sufficient to meet its clearing needs. This is the case if

$$\mathrm{MAX}_{\theta\in\Theta}[c_{t+1}(\theta^t,\theta)] - y \quad < \quad \phi_m - \phi_{\hat{m}} \tag{20}$$

which implies that $\hat{\eta}_{t+1}(\theta^t,\theta_{t+1})=0$, for all θ_{t+1}. Then (19) implies that $\eta_t(\theta^t)>0$, and thus $m_t/p_{t+1}=\phi_m$. The inequality (20) states that the bank's maximum net clearing obligation at t+1 is less than the difference between the minimum overnight balance requirement, ϕ_m, and the minimum daylight balance, $\phi_{\hat{m}}$. Such a bank will not be constrained by the intraday limit; instead the overnight limit is binding. What sort of bank holds the minimum overnight balance? In this model consumption is positively related to the wealth an island has built up. Since sales are fixed at y, the maximum net clearing need measured by the left side of (20) also varies positively with wealth. Thus it is the poorest islands for whom (20) holds and who maintain only the minimum overnight balances.

A bank holding more than the minimum overnight balance must be expecting the intraday credit limit to bind sometimes next period, since $m_t/p_{t+1}>\phi_m$ implies that $\eta_t(\theta^t)=0$, and thus by (19), $\hat{\eta}_{t+1}(\theta^t,\theta_{t+1})>0$ for some θ_{t+1}. For such a bank the overnight balance is related to clearing needs according to

$$\frac{m_t(\theta^t)}{p_{t+1}} \quad = \quad \mathrm{MAX}_{\theta\in\Theta}[c_{t+1}(\theta^t,\theta)] - y + \phi_{\hat{m}} \tag{21}$$

which states that the overnight balance is equal to next period's maximum net clearing obligation minus the daylight overdraft limit $-\phi_{\hat{m}}$. The right side of (21) will be largest for the wealthiest islands, who will therefore be the most likely to hold more than the minimum overnight balance. This is consistent with empirical evidence that a few large banks were responsible for a large portion of intraday overdrafts in the U.S.[20] Of course the relationship in the model between bank size and intraday overdrafts is heavily dependent on the fact that overnight and daylight balance constraints are constants that do not vary with the size of the bank; other assumptions will give rise to different predictions for the cross-sectional distribution of overdrafts.

The fact that reserve balances do not bear interest creates a "wedge" that can distort resource allocations. The effects of the wedge in this model can be seen by collecting first order conditions:

$$\beta^t\theta_t u'(c_t(\theta^t)) - \hat{\eta}_t(\theta^t)p_t \;=\; (1+i_t)\frac{p_t}{p_{t+1}}\mathrm{E}_t\Big[\beta^{t+1}\theta_{t+1}u'(c_{t+1}(\theta^{t+1})) - \hat{\eta}_{t+1}(\theta^{t+1})p_{t+1}\Big] + \lambda_{0t}(\theta^t)p_t \tag{22}$$

In the absence of the multipliers in (22), this is a standard intertemporal asset pricing equation relating the real interest rate to the expected intertemporal marginal rate of substitution. The multipliers drive a wedge between the intertemporal marginal rate of substitution and the real rate of return. All else held constant, a binding intraday constraint ($\hat{\eta}_t(\theta^t)>0$) tends to depress current consumption. Similarly, if the intraday constraint is expected to bind next period ($\hat{\eta}_{t+1}(\theta^{t+1})>0$ for some θ_{t+1}), future consumption tends to be depressed. Note that for some banks all multipliers in (22) are zero. These are the less wealthy islands for whom overnight balances are sufficient and

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[20]Richards [1995], p. 1072. Note that without the heterogeneity induced by the idiosyncratic preference shocks, we have $c_t=y$, and $\hat{m}_t=m_t$ for all banks, and money demand is trivially $m_t=\phi_m$.

intraday balances never bind.[21]

Note what happens as the daylight overdraft limit varies. At one extreme, if there is no limit at all then $\phi_{\hat{m}}=-\infty$ and $\hat{\eta}_t(\theta^t)=0$ for all t. From (19), it is clear that $\eta_t(\theta^t)>0$, which implies that overnight balances are zero for all banks, as one would expect. With no constraint on intraday balances, and plenty of time to borrow before tomorrow's overnight balance requirement must be met, there is no reason to hold overnight balances tonight.[22] At the other extreme, a policy of no intraday overdrafts sets $\phi_{\hat{m}}=0$. If the overnight constraint is $\phi_m=0$ as well, then positive overnight balances are required by any island whose consumption the following period will ever exceed output. In this case it is as if banks face a simple cash-in-advance constraint on their net settlement obligation.

7. Policy tools

Many aspects of payments system policy can be seen as components of the contractual terms on which central bank deposits are offered. The framework developed here allows a glimpse of what the model is capable of and how various policy tools might fare under general equilibrium welfare analysis. The multipliers on the balance constraints play a crucial role here. They measure what a bank would be willing to pay to avoid, say, the inflation tax on reserve balances. As it stands now the model provides banks only one way to avoid holding reserves; reducing the consumption corresponding to the largest preference shock. In more elaborate models in which banks can make costly alternative arrangements to avoid using central bank reserve, such as private settlement arrangements, the balance constraint multipliers would measure the pecuniary incentive to do so.

7.1. Interest on Reserves

Not surprisingly, paying interest on reserves can ameliorate the effects of reserve account balance constraints. Suppose banks are credited at the end of the settlement period with interest at rate i_t^r on the previous night's account balance. The budget constraint is now

$$m_t + b_t = \hat{m}_t + i_{t-1}^r m_{t-1} + (1+i_{t-1})b_{t-1} - p_t\tau_t, \qquad \forall t$$

The critical condition (19) becomes

$$(i_t - i_t^r)\mathrm{E}_t[\lambda_{2,t+1}(\theta^{t+1})] = \eta_t(\theta^t) + \mathrm{E}_t[\hat{\eta}_{t+1}(\theta^{t+1})], \qquad \forall t,\ \forall\theta^t\in\Theta^t$$

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[21]There is a similarity between the distortion to asset prices in (22) and the distortion in Lacker and Schreft [1996]. There the costliness of credit used as a means of payment drives a wedge between the value of claims in securities markets and the marginal utility of consumption. Here, the costliness of clearing payment instruments again drives a wedge between the value of settlement period claims and the marginal utility of consumption.

[22]Note that in a model in which clearing takes place at the end of the settlement period, with no subsequent borrowing allowed, there would be a positive demand for precautionary intraday balances, and in some states banks would carry overnight balances as well. Another approach to studying issues related to intraday credit is to assume that a day corresponds to $T>1$ periods in the model.

As the overnight interest rate on reserves approaches the market rate, the balance constraint multipliers vanish. In the limit, the overnight rate equals to the market rate, the balance constraints have no effect at all, and the division of assets between reserves and loans is indeterminate.

7.2. Reserve requirements

A reserve requirement in this model links the minimum overnight balance to some bank balance sheet quantity. With noninterest-bearing reserves, a reserve requirement will have the well-known effect of taxing the quantity that is reservable. For example, suppose the reserve requirement applies to total assets;

$$m_t \quad \geq \quad \Phi_{mt} \quad = \quad \delta\left[\mathrm{MAX}\left(0,b_t\right) + p_t\phi_t\right], \qquad \forall t$$

where $0<\delta<1$.[23] (Note that real nonfinancial wealth is ϕ_t.) This leads to the addition of the term $-\delta\eta_t(\theta^t)$ on the right side of (22) for banks that are net lenders. For them, $b_t>0$ and lending is taxed. The same result holds for a reserve requirement that applies just to financial wealth, $\mathrm{MAX}[0,b_t]$.

Another effect emerges here as well, however. By increasing the gap between the overnight and the intraday minimum balances, a reserve requirement tends to loosen the intraday constraint. Recalling the discussion of (20) and (21), this will tend to increase the number of banks for which the overnight constraint binds, and reduce the number of banks for which the intraday constraint binds, all else held constant. This implies lower average overdrafts, a prediction roughly consistent with the findings of Hancock and Walkaways [1995] that the level of required reserves is negatively correlated with daylight overdrafts.

Assuming the comparison is between two regimes with identical nominal rate sequences, the demand for excess reserves ($m_t-\Phi_{mt}$) will be higher in the regime with the smaller reserve requirement. This is consistent with recent trends in the U.S., where the introduction of sweep accounts has allowed many depository institutions to reduce their level of required reserves (Seiberg [1996]). Many such banks are setting up "clearing accounts," special subaccounts that do not count towards reserve requirements. Since these constitute excess reserve holdings, they accord well with the model prediction. Another inducement to setting up clearing accounts is provided by the interest they earn in the form of credits towards other Federal Reserve fees.

7.3. Daylight overdraft fees

Federal Reserve Banks began charging fees for daylight overdrafts in April, 1994. A simple overdraft fee can be implemented here by charging interest at rate f_t on negative intraday balances.[24] The budget constraint (10) is modified accordingly:

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[23]This specification binds on a daily basis rather than on an average basis over a number of days, as in the U.S., for example. The modifications necessary to account for averaging reserves over a maintenance period are straightforward.

[24]This ignores the deductible in the Fed's overdraft scheme; implementing it would be straightforward.

$$m_t + b_t = \hat{m}_t - f_t\text{MAX}[0,-\hat{m}_t] + (1+i_{t-1})b_{t-1} - p_t\tau_t, \qquad \forall t \tag{23}$$

With this substitution the first order conditions are as before, but with (17) replaced by

$$\lambda_{1t}(\theta^t) = (1+f_t\iota_t)\lambda_{2t}(\theta^t) + \hat{\eta}_t(\theta^t) \qquad \forall t \ \forall\theta^t\in\Theta^t$$

where ι_t=1 if $\hat{m}_t(\theta^t)\leq 0$ and ι_t=0 otherwise. Condition (19) now becomes

$$\text{E}_t[(i_t - f_{t+1}\iota_{t+1})\lambda_{2,t+1}(\theta^{t+1})] = \eta_t(\theta^t) + \text{E}_t[\hat{\eta}_{t+1}(\theta^{t+1})], \qquad \forall t,\ \forall\theta^t\in\Theta^t \tag{24}$$

The two balance constraint multipliers are here related to the gap between the current nominal rate and tomorrow's expected marginal overdraft charge. One might have expected the gap between the current nominal rate and the current marginal overdraft charge to be the relevant comparison, but this ignores the fact that preventing daylight overdrafts requires accumulating balances the night before; reserve balances at the end of period t help reduce overdrafts during period t+1. Thus the expected marginal overdraft cost this period is compared to the marginal opportunity cost of accumulating balances at the end of the previous period.

More generally, since the nominal interest rate is a tax on reserve balances, then an overdraft fee can be seen as a tax credit. A one dollar overdraft receives an implicit subsidy of i_t, the flip side of the inflation tax. A reduction in daylight overdraft is therefore taxed at the rate i_t. An overdraft charge is a marginal rebate on this subsidy. For the portion of the day during which the account is overdrawn the net tax on overdraft reduction is $i_t - f_{t+1}$. This suggests that overdraft fees might ameliorate the effects of the inflation tax. Can the inflation tax be completely offset by setting the overdraft fee equal to the nominal rate? For some banks the answer is yes. Imagine setting the intraday overdraft fee equal to the overnight market rate, $f_{t+1}=i_t$, and suppose that ϕ_m=0. If the left side of (24) is zero, then the effect of the inflation tax has been eliminated for this bank, since then the intertemporal substitution condition (22) is undistorted by nonzero balance-constraint multipliers (assuming the borrowing constraint does not bind so that $\lambda_{0t}(\theta^t)$=0). For the left side of (24) to be zero would require $\iota_t(\theta^{t-1},\theta_t)$=1 for all θ_t. For this to occur the bank would have to always overdraft at t. This requires

$$\text{MIN}_{\theta\in\Theta}[c_{t+1}(\theta^t,\theta)] > y + \frac{m_t}{p_{t+1}}, \tag{25}$$

Thus it is the wealthy banks (net payers) for whom the overdraft fee eliminates the marginal effect of inflation. Not all banks can be net payers, however, and many banks will still face a positive shadow value on the left side of (24).

Discussion of the appropriate level of daylight overdraft fees has traditionally focused on credit risk. Although the model contains no credit risk, it is able to shed some light on the appropriate "risk-free" reference rate to which credit risk adjustments should be applied. The model suggests that the risk-free daylight overdraft fee should at least equal the nominal rate. In comparison, the U.S. daylight overdraft fee is 15 basis points, which is less than three one-hundredths of the current Fed Funds rate.

Note that one cannot gauge the optimal overdraft fee by inspecting market rates for intraday loans under a no-fee regime. Market rates for intraday loans will be heavily influenced by f_t. To see this, imagine that banks can borrow and lend *during* the clearing period (after θ_t is known), with repayment after the clearing period is over. Budget constraints are then

$$\hat{m}_t = m_{t-1} + p_t(y - c_t) + \hat{b}_t, \qquad \forall t$$

$$m_t + b_t = \hat{m}_t - f_t \mathrm{MAX}[0, -\hat{m}_t] + (1+\hat{i}_t)\hat{b}_t$$
$$+ (1+i_{t-1})b_{t-1} - p_t\tau_t, \qquad \forall t$$

where $\hat{b}_t$ is the amount lent and $\hat{i}_t$ is the interest rate. From the first order conditions we find

$$\hat{i}_t\lambda_{2t}(\theta^t) = f_t\iota_t\lambda_{2t}(\theta^t) + \hat{\eta}_t(\theta^t) \qquad \forall t \ \forall\theta^t\in\Theta^t$$

Banks in need will borrow their way away from the intraday balance constraint, so $\hat{\eta}_t(\theta^t)=0$. Then banks expecting an overdraft (ι_t=1) will be willing to pay an interest rate up to f_t, but no more. The fact that the Fed is willing to supply intraday credit at price f_t puts a ceiling on the market rate for intraday loans. Thus under a no-fee regime one would observe an intraday risk-free rate of zero, consistent with the findings of Mengle et al [1987]. This is *not*, however, evidence that the proper intraday risk-free rate *should* be zero. Finding the right daylight overdraft rate given the nominal overnight rate is a second-best problem; i_t constitutes a pre-existing tax on overnight balances. Since intraday overdrafts are a close substitute for overnight balances, standard public finance theory suggests that the optimal intraday rate--that is, the optimal tax rate on intraday overdrafts--should be about the same.

7.4. Collateralized daylight overdrafts

An alternative method of discouraging daylight overdrafts is to require collateral. Overdraft limits would be based on the amount of eligible assets pre-assigned as cover. To see how this would affect the demand for reserve account balances, suppose that government bonds are the only eligible securities. Let b_t and d_t be a representative bank's holdings of government and private debt, respectively. The daylight overdraft limit (12) is replaced by

$$\hat{m}_t \geq \Phi_{\hat{m}t} \equiv \mathrm{MIN}[0, -b_{t-1}] \qquad \forall t$$

The budget constraint (10) is now replaced by

$$m_t + b_t + d_t = \hat{m}_t + (1+i^b_{t-1})b_{t-1} + (1+i^d_{t-1})d_t - p_t\tau_t, \qquad \forall t$$

where bonds and debt can now earn different rates of return. To keep banks from creating eligible government securities, we need to impose a short sale constraint.

$$b_t \geq 0 \qquad \forall t$$

This constraint implements the restriction on eligible collateral; otherwise banks could create synthetic eligible government bonds.[25]

Under this regime the two novel first order conditions are

$$\lambda_{2t}(\theta^t) = \mathrm{E}_t[(1+i^d_t)\lambda_{2,t+1}(\theta^{t+1})] + \lambda_{0t}(\theta^t) \qquad \forall t \ \forall\theta^t\in\Theta^t$$

[25]Banks can create perfect substitutes for government bonds in the sense that they can issue risk-free intertemporal claims. The policy regime imposes a legal distinction between risk-free private debt and risk-free government debt.

$$\lambda_{2t}(\theta^t) \quad = \quad E_t[(1+i_t^{\,b})\lambda_{2,t+1}(\theta^{t+1})] \; + \; \lambda_{0t}(\theta^t) \; + \; E_t[\hat{\eta}_{t+1}(\theta^{t+1})] \qquad \forall t \;\; \forall\theta^t \in \Theta^t$$

These yield two conditions.

$$(i_t^{\,d} - i_t^{\,b})E_t[\lambda_{2,t+1}(\theta^{t+1})] \quad = \quad E_t[\hat{\eta}_{t+1}(\theta^{t+1})] \qquad \forall t \;\; \forall\theta^t \in \Theta^t$$

$$i_t^{\,b}E_t[\lambda_{2,t+1}(\theta^{t+1})] \quad = \quad \eta_t(\theta^t) \qquad \forall t \;\; \forall\theta^t \in \Theta^t$$

By restricting the supply of eligible debt, the central bank can open up a wedge between the rate of return on eligible and ineligible securities. Alternatively, if there is more eligible debt than banks need for overdrafts then the rates of return must be identical: $i^d_t = i^b_t$. In this case the economy has more than enough eligible securities to provide for intraday overdrafts with no opportunity cost, and $\hat{\eta}_t = 0$, $\forall t$. In either case there is no reason to hold overnight balances since the cost of increasing the overdraft limit, $i^d_t - i^b_t$, is less than the cost of overnight balances, i^d_t. Thus $\eta_t > 0$ and $m_t = \Phi_{\hat{m}t}$. Enhanced models would recover a positive demand for overnight balances, however. A separate limit on collateralized daylight overdrafts would constrain banks and provide a reason for holding sterile reserves.

8. Private net settlement arrangements

Central banks often do not completely monopolize clearing and settlement. Private settlement arrangements have often existed alongside a central bank clearing facility. It is not just a matter of public and private competition, however, because private settlement arrangements generally settle their net obligations on the books of the central bank. Check clearinghouses, for example, exchange checks and then settle their net obligation with a Fedwire transfer. CHIPS records transactions during the day and then settles at the end of the day on the books of the Federal Reserve Bank of New York. Note that such arrangements amount to settling one credit instrument--the original check or payment order--with yet another credit instrument--the promise to pay the clearinghouse at the end of the day. They amount to substituting multilateral private credit for the use of central bank balances and overdrafts.

Private settlement arrangements have sparked concern among central bankers. With large transactions volumes and significant heterogeneity of net obligations, multilateral credit exposures can be substantial. Safety and soundness issues naturally arise, even apart from the perception that settling on the central bank's books makes central bank lending in a crisis more likely (see Roberds [1993] and Kahn and Roberds [1996]). These concerns have at times inhibited efforts to constrain intraday central bank credit, for fear of driving more clearing onto private systems (Folkerts-Landau, et al [1996]).

Although settlement risk is beyond the scope of this paper, the model can be used to explore the determinants of the incentive to clear privately rather than through the central bank. There are a few different ways of formalizing private settlement; I will start with a simple one. A private clearinghouse allows banks to settle among themselves. Banks decide whether to exchange bills with a given bank through the clearinghouse or through the central bank. Exchanging bills through the clearinghouse requires the consent of both banks; otherwise exchange is through the central bank. Net obligations to the clearinghouse are paid by transfer at the central bank later in the settlement period, after central bank clearing. As before, central bank clearing runs down intraday balances, risking a binding daylight overdraft constraint. The benefit of exchanging through the clearinghouse is the avoidance of central bank constraints.

Suppose that banks must choose whether or not to join well before knowing their clearing position. In the simplest case, banks must decide at the beginning of time, before the first shock arrives. Since all islands are identical before the first shock has arrived, a bank merely has to consider how many other identical banks with which it wishes to exchange privately. Let α_t be the fraction of banks with which a given bank is willing to exchange privately at t. If all other banks are willing to exchange privately with this bank, then the bank's budget constraints are:

$$\hat{m}_t \;=\; m_{t-1} + (1-\alpha_t)p_t(y-c_t) \qquad \forall t$$

$$m_t + b_t \;=\; \hat{m}_t + (1+i_{t-1})b_{t-1} + \alpha_t p_t(y-c_t) - p_t\tau_t \qquad \forall t$$

With these constraints and with λ_{3t} and λ_{4t} as the Lagrangian multipliers on the constraints $\alpha_t \geq 0$ and $\alpha_t \leq 1$, respectively, the first order conditions yield

$$\begin{aligned} \mathrm{E}[p_t(c_t(\theta^t)-y)(\lambda_{1t}(\theta^t)-\lambda_{2t}(\theta^t))] &= \mathrm{E}[p_t(c_t(\theta^t)-y)\hat{\eta}_t(\theta^t)] \\ &= \lambda_{4t} - \lambda_{3t} \qquad \forall t \;\; \forall\theta^t \in \Theta^t \end{aligned} \tag{26}$$

The left side of (26) is strictly positive, because the intraday constraint only binds ($\hat{\eta}_t(\theta^t)>0$) when consumption exceeds output. This bank therefore wants to set $\alpha_t=1$. Since all other banks are similarly situated, this verifies that the other banks are willing to exchange privately. Thus all banks exchange privately at all dates.

Banks would be willing to incur costs in order to clear privately. The left side of (26) is the amount a bank would be willing to "pay" in time t reserve balances, at the margin, for the ability to exchange bills privately. For example, banks in this model would be willing to acquire their own hardware and communications systems to facilitate private clearing. Clearing privately might also involve additional costs associated with credit risk, as Kahn and Roberds [1996] argue -- for example, information gathering costs or deadweight losses associated with contract enforcement.[26]

Central bank policy measures aimed at containing risk in private multilateral settlement arrangements, such as those advocated in the Lamfalussy Report (Bank for International Settlements [1990]), seem motivated by concern that the credit risk incurred in such arrangements is socially excessive. The model illustrates one possible reason for such concern. Here, the fact that reserves do not earn interest leads banks to economize on overnight balances and intraday overdrafts by organizing private substitutes for clearing at the central bank. The resultant overuse of private clearing arrangements could involve greater credit risk than is socially desirable. It is not immediately obvious, however, why the excessive costs incurred by private settlement arrangements should come exclusively in the form of credit risk; this model predicts that excessive costs could just as well come in the form of excess resources devoted to the alternative clearing systems.

As one would expect from the analysis of Section 7, the policy regime has important effects on the incentive to clear privately. To consider just one example, recall the daylight overdraft fee studied earlier. In this regime the equation (26) becomes

$$\mathrm{E}[p_t(c_t(\theta^t)-y)(\lambda_{1t}(\theta^t)-\lambda_{2t}(\theta^t))] \;=\; \mathrm{E}[p_t(c_t(\theta^t)-y)(f_t\iota_t\lambda_{2t}+\hat{\eta}_t(\theta^t))] \;=\; \lambda_{4t} - \lambda_{3t} \tag{27}$$

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[26]Rochet and Tirole [1996] argue that private settlement brings with it the benefits of mutual monitoring by participants.

Now any bank with a positive probability of a daylight overdraft is willing to pay to join a private clearing arrangement. Imposing a daylight overdraft fee encourages private clearing.

9. Concluding remarks

This paper was motivated by questions regarding central bank policy toward clearing and settlement arrangements, and by the idea that answering such questions will require models that capture both settlement arrangements and the origin of the underlying payment arrangements. The approach taken here rests on the presumption that payment system policy is a branch of monetary policy, where I take monetary policy to refer broadly to (1) the terms on which governments issue their own payment instruments, such as open market operations, overdraft policy, or the interest (if any) paid on reserves, and (2) the way governments regulate private payment instruments, by, for example, prohibiting private bearer notes or imposing reserve requirements. This presumption has the implication that some of the lessons taught by monetary economics will be useful in studying payments systems. One such lesson highlighted here is the importance of the "wedge" of inefficiency associated with inflation; its affect on payments arrangements has received scant attention to date. Sections 7 and 8 of this paper sketch out how the particulars of central bank policy interacts with this wedge in various ways.

The examples presented above certainly are not exhaustive treatments of the policy questions at hand. Nevertheless, they suggest that operational answers can be obtained from this model, or from versions of it.

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