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Abstract

This paper analyzes the effects of inflation variability on economic growth in a model where money is introduced via a cash-in-advance constraint. In this setting, we find that inflation adversely affects long-run growth, even when the cash-in-advance constraint applies only to consumption. At the same time, we find that inflation and growth are positively related in the short-run. In addition, variability tends to increase average growth through a precautionary savings motive. Since inflation and inflation variability tend to be highly correlated, this latter effect attenuates the negative long-run relationship between inflation and real growth. It also provides a partial rationale for the notorious lack of robustness in cross-country regressions of growth and inflation.

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1. Introduction

The advent of endogenous growth theory has given economists a new framework for studying the relationship between public policy and economic growth. Early papers in the area have demonstrated that the differences in growth rates across countries could be partially explained by differences in national policies[1]. Such policies might include fiscal, financial, trade and other forms of government programs. Most of this early literature concentrates on policies that are fixed over time, but one notices that public policies not only differ across economies but also over time. With the notable exceptions of Eaton (1981), Gomme (1993), Aizenman and Marion (1993), and Hopenhayn and Muniagurria (1996), the analysis of such variations and their effects on economic growth has received much less attention. Other than Gomme, most of this literature has focused on the effects of fiscal policies. In this paper, we analyze the implications of variable monetary policy for economic growth.

In studying the effects of monetary policy on real growth, we desire a model that is consistent with a number of stylized facts. First, the model should display a negative relationship between average inflation and average growth. However, typical estimates of this relationship are relatively weak and we want a model that is consistent with this particular feature of the data. Second, in developed economies, inflation and output growth are positively correlated at cyclical frequencies and we should be able to simultaneously account for this fact. Finally, the model we study should respect the notion that countries with less developed financial systems appear to suffer more severely from the effects of inflation than do more developed economies [see Boyd, Levine and Smith (1996)].

Perhaps surprisingly, these features of the data naturally emerge within the confines of a conventional linear neoclassical growth model where money is introduced via a cash-in-advance constraint. For the case of logarithmic utility and white noise money growth shocks, the model admits closed form solutions that allow us to provide intuition concerning the general channels through which inflation and inflation variability affect real growth.

[1] Early studies include, in particular, Jones and Manuelli (1990), and Rebelo (1991).

Moreover, these closed form solutions also allow us to clearly contrast the behavior of the stochastic economy relative to its deterministic counterpart.

Accounting for the variability of monetary policy introduces some important consequences for real growth. Specifically, we find that when the cash-in-advance constraint applies only to consumption, inflation is no longer neutral. In equilibrium, the effects of inflation are similar to those of a stochastic tax on consumption. Therefore, in contrast to the equivalent deterministic case, the mean growth rate of output is decreasing in average money growth. In addition, the growth rate of output is state dependent, with higher growth emerging when money growth is high. This result obtains because households substitute away from money balances, and hence consumption, in periods of relatively high expected inflation. Analogous to the results in Eaton (1981), but in contrast to Aizenman and Marion (1993), we further show that, even without persistence, the mean growth rate is increasing in the variance of the shocks to money growth.

When the liquidity constraint also applies to investment and money growth is white noise, we find that the negative effect of average money growth on real growth decreases with the variability of inflation. However, growth rates are no longer contingent upon the particular realization of money growth. This is due to the fact that when the inflation tax applies to both consumption and investment, and given that current shocks to money growth do not contain any information about future shocks, households are no longer able to substitute between consumption and investment in order to avoid the inflation tax across various states. Therefore, the state dependence of real growth varies with the fraction of investment subject to the inflation tax. As in the previous case where consumption faces a cash-in-advance constraint, the mean growth rate of output increases with the volatility of inflation.

Using numerical simulations, we also examine the growth implications of variable monetary policy when shocks to money growth are persistent and a more general utility function is specified. In performing these simulations, we take into account the fact that the mean and volatility of money growth are strongly and positively correlated across countries. The results show that when a significant fraction of investment is subject to the cash-in-advance constraint, real growth can also be lower when the high money growth shock occurs. This feature of the model arises because the persistence of shocks implies that being in the high money growth regime today indicates that the inflation tax is likely to be high in the future as well. In addition, since real growth increases with the variance of money growth, this model provides a partial rationale for the notorious lack of robustness in cross-country regressions of growth and inflation. In essence, the effects of increased variability partially offset the effects of higher inflation.

This paper is organized as follows. Section 2 presents our basic model and briefly reviews the effects of deterministic inflation rates on economic growth. In section 3, we analyze the growth implications of monetary variability when shocks to money growth have no persistence and compare these results with those obtained in section 2. Section 4 extends the analysis to persistent shocks through numerical simulations. Finally, some concluding remarks are offered in section 5.

2. Basic Framework

In this section we present the basic model and analyze its behavior in a deterministic setting. Our model economy is populated by a continuum of infinitely-lived households. These households are identical in terms of their preferences and their technology. Time is discrete and indexed by $t = 1, 2, \ldots$. The timing of production and trade follows that of the cash-in-advance economy described in Svensson (1985). Specifically, households must decide on their cash holdings before knowing their current state. They then purchase goods and after the goods market closes, they receive a lump sum monetary transfer and rearrange their portfolios. This timing of events is particularly important since money demand generally embodies transactions, precautionary, and store-of-value motives[2]. The representative household solves

$$\max\ E_t \sum_{j=t}^{\infty} \beta^{j-t} U(C_j),\ \beta \in (0,1)$$

subject to the constraints:

$$C_t + \psi[K_{t+1} - (1-\delta)K_t] \leq \frac{M_t^d}{P_t}, \tag{2.1}$$

[2] Our model, therefore, resembles that of Chari, Jones, and Manuelli (1995), but differs from Gomme (1993).

$$C_t + K_{t+1} - (1-\delta)K_t + \frac{M^d_{t+1}}{P_t} = AK_t + \frac{M_t + \nu_t}{P_t}, \tag{2.2}$$
$$\delta,\ \psi \in [0,1],$$

where C_t and K_t denote consumption and the capital stock respectively, and E_t is the conditional expectations operator. The time t information set includes all variables dated t and earlier as well as the end of period capital stock. Money is introduced via lump sum transfers, ν_t at the end of period t. Thus $M_{t+1} = M_t + \nu_t$ represents the post-transfer nominal money holdings at the end of period t, and M^d_t is the amount of nominal money held at the beginning of the period.

Output is produced using a linear technology and capital depreciates at the rate δ. In this setting, the capital stock is viewed as incorporating an expanded concept of capital as first described in Jones and Manuelli (1990), and Rebelo (1991). ψ controls the fraction of investment, in both physical and human capital, which is subject to the cash-in-advance constraint. This fraction may be substantial in developing countries that do not have well functioning credit markets. For simplicity, we abstract from the fact that ψ may itself be endogenous[3].

The first order conditions for the household's problem are given by:

$$U'(C_t) = \lambda_{1,t} + \lambda_{2,t}, \tag{2.3}$$

$$\psi\lambda_{1,t} + \lambda_{2,t} = \beta(1-\delta)\psi E_t\lambda_{1,t+1} + \beta[(1-\delta) + A]E_t\lambda_{2,t+1}, \tag{2.4}$$

$$\frac{\lambda_{2,t}}{P_t} = \beta E_t \frac{(\lambda_{1,t+1} + \lambda_{2,t+1})}{P_{t+1}}, \tag{2.5}$$

and the resource constraint (2.2). $\lambda_{1,t}$ and $\lambda_{2,t}$ denote the multipliers associated with the first and second constraints respectively. Equation (2.5) indicates that it is optimal to equate the marginal value of an additional dollar today with the discounted expected value of an additional unit of consumption tomorrow. Equation (2.4) is the condition determining

[3] For examples where the degree of financial sophistication may be contingent on inflation, see Bencivenga and Smith (1992), Ireland (1994), and Boyd and Smith (1996).

optimal capital accumulation and equates the cost of investing one more unit today with the value of that additional investment tomorrow. That value not only includes the extra product available, but the value of relaxing next period's cash in advance constraint.

We can use equations (2.3) and (2.5) to rewrite equation (2.4) as

$$\begin{aligned} \psi U'(C_t) &= -(1-\psi)\beta U' E_t(C_{t+1})\frac{P_t}{P_{t+1}} + \beta(1-\delta)\psi E_t U'(C_{t+1}) \\ &\quad +\beta[\beta(1-\delta)(1-\psi)+\beta A]E_t U'(C_{t+2})\frac{P_{t+1}}{P_{t+2}}. \end{aligned} \tag{2.6}$$

Equation (2.6) may be interpreted as the fundamental Euler equation governing the law of motion for consumption.

Let us assume that transfers of money are issued by the government at rate $z_t \geq \beta$ so that[4]

$$M_{t+1} = z_t M_t. \tag{2.7}$$

The solution to the competitive equilibrium involves finding the household's policy function, $K_{t+1} = \phi(K_t, z_t)$, which satisfies (2.6) while respecting market clearing in both goods and money markets. This function should also satisfy the cash-in-advance constraint in (2.1). The market clearing conditions are

$$M_t^d = M_t \tag{2.8}$$

and

$$C_t + K_{t+1} - (1-\delta)K_t = AK_t. \tag{2.9}$$

[4] A necessary condition for the cash in advance constraint to always bind is that $[U'(C_t)/P_t]/[\beta E_t U'(C_{t+1})/P_{t+1}] \geq 1$. With CRRA utility a necessary condition would be $z_t \geq \beta\gamma_{\max}^{1-\alpha}$, where $\gamma_{\max}$ is the maximum growth rate of consumption and α is the coefficient of relative risk aversion. With log utility, this condition simplifies to $z \geq \beta$. For the purposes of this paper, nothing is lost by confining attention to the case of a binding cash in advance constraint.

2.1. A deterministic solution with logarithmic utility

As a point of reference, we display the solutions for the growth rate of this economy when money growth is deterministic and utility is logarithmic. We choose logarithmic utility because it allows us to find closed-form solutions in the equivalent stochastic economy. This will be especially useful in providing intuition into how various realizations of money growth rates affect real growth. The solution is obtained by conjecturing that $K_{t+1} = \phi A K_t + (1-\delta)K_t$ and $C_t = (1-\phi)AK_t$, and finding the constant ϕ which is consistent with our description of equilibrium. We then use this solution for ϕ to calculate the deterministic growth rate of output. This yields

$$\begin{aligned}\gamma_d &= \frac{Y_{t+1}-Y_t}{Y_t} \\ &= \frac{\beta(1-\delta)\psi z + \beta[\beta(1-\delta)(1-\psi)+\beta A]}{\psi z + (1-\psi)\beta} - 1. \qquad (2.10)\end{aligned}$$

Equation (2.10) indicates that except in the case where $\psi = 0$, the economy's growth rate is negatively related to the growth rate of money. At the Friedman rule, where $z = \beta$, the optimal growth rate of the economy is $\gamma_d^* = \beta(A + (1-\delta)) - 1$.[5]

2.2. Two familiar cases:

2.2.1. $\psi = 0$: A cash-in-advance constraint on consumption

In this case, investment is not subject to the cash-in-advance constraint and the growth rate of output is independent of the money growth rate. The fraction of output devoted to investment, ϕ_d, is $\beta - ((1-\beta)(1-\delta)/A)$ and the economy wide growth rate is γ_d^*. The solution corresponds to one that would obtain in an analogous non-monetary economy with a constant consumption tax with the proceeds of that tax being rebated lump-sum to agents.

[5] The Friedman rule obtains when $[U'(c_t)/P_t]/[\beta E_t U'(c_{t+1})/P_{t+1}] = 1$ which implies that the nominal interest on a bond that sells in an asset market prior to the opening of the goods market is zero [see Svenson (1985)]. This is also the condition that implies that the cash-in-advance constraint is just binding. In the deterministic case and for CRRA utility $z = \beta\gamma^{1-\alpha}$, where γ is the growth rate, yields the Friedman rule. For log utility the Friedman rule obtains when $z = \beta$.

The consumption tax does not distort the allocation between saving and investment and hence the economy attains the first best outcome.

2.2.2. $\psi = 1$: A cash–in-advance constraint on consumption and investment

In this case, money growth implicitly acts as a tax on investment as well as consumption. A higher rate of money growth, therefore, reduces the rate of return to investment [see Stockman (1981)]. The solutions for ϕ_d and the growth rate of the economy, γ_d are

$$\phi_d = \frac{1}{A}\{\frac{\beta(1-\delta)z + \beta^2 A}{z} - (1-\delta)\} \tag{2.11}$$

and

$$\gamma_d = \frac{Y_{t+1} - Y_t}{Y_t} = \frac{\beta^2 A}{z} + \beta(1-\delta) - 1. \tag{2.12}$$

Thus, when investment is subject to the cash in advance constraint, growth is less than optimal. More generally, the degree to which monetary policy adversely affects real growth ultimately depends on the fraction of investment subject to the inflation tax.

Therefore, in comparing inflation and growth effects across countries, one must be cognizant of the degree of financial sophistication present. For the US and most OECD countries, ψ is probably close to zero so that, in a deterministic world, sustained inflation should not be expected to yield significant growth effects. This relationship underpins the results in Chari, Jones and Manuelli (1996). However, ψ significantly greater than zero may better characterize the payments technology in many developing economies and it is important to consider these cases as well. It is doubtful that a single value of ψ is applicable across widely differing countries, industrialized and developing alike, especially when the relevant concept of capital contains a human component. That is, access to higher education in developing economies is often subject to stringent liquidity constraints. Although Chari, Jones and Manuelli (1996) do make a distinction between cash and credit goods which helps attenuate the real effects of money growth, this distinction is again likely to be much less relevant for developing countries. In fact, and as suggested in Hansen (1996), note that when all investment is subject to the cash-in-advance constraint, the real effects of higher average money growth can be quite significant. This may explain why the effects of inflation on growth are sensitive to the subsample of countries examined [see Barro (1996)].

The theoretical results presented above though do make it surprising that empirical studies involving a wide range of countries have not been able to incontrovertibly document the harmful effects of inflation on real growth. Most studies only find a small effect of inflation on growth. In particular, through the use of Extreme-Bound Analysis, Levine and Zervos (1993) suggest that the relationship between inflation and growth may be particularly fragile. As we shall see, this seemingly weak relationship may be partially explained by considering the variability of monetary policy.

3. Endogenous growth with stochastic monetary policy: the case of iid shocks

We now address the issue related to the effects of inflation on real growth when money growth is stochastic. Similar to the fiscal policy analysis contained in Eaton (1981), Aizenman and Marion (1993), and Hopenhayn and Muniagurria (1996), we show that variable monetary policy introduces some important implications for growth relative to the deterministic case.

Let us suppose that government transfers of money are issued stochastically according to the rule

$$M_{t+1} = (z + \eta_t)M_t, \tag{3.1}$$

where η_t is an identically and independently distributed random variable. In addition, we assume that η_t lies in the two point set, $\eta_t \in \{-\sigma, \sigma\}$, such that $\text{prob}\{\eta_t = \sigma\} = \text{prob}\{\eta_t = -\sigma\} = 1/2$. We can think of this stochastic representation as a two-sate Markov chain with no persistence. The more general case in which persistence is allowed is examined below. Consistent with the notation in the previous section, we choose a process for money growth that has mean z and variance σ^2. Moreover, we impose the restriction that $z - \sigma \geq \beta$ and focus our attention on cases where the cash-in-advance constraint is binding. This specification on monetary policy allows us to consider the effects of money growth and volatility separately.

The representative household solves the problem depicted in the previous section. The functional equation that governs the accumulation of capital is given by (2.6) and is reproduced below.

$$\begin{aligned}\psi U'(C_t) &= -(1-\psi)\beta E_t U'(C_{t+1})\frac{P_t}{P_{t+1}} + \beta(1-\delta)\psi E_t U'(C_{t+1}) \\ &\quad +\beta[\beta(1-\delta)(1-\psi)+\beta A]E_t U'(C_{t+2})\frac{P_{t+1}}{P_{t+2}}. \qquad (3.2)\end{aligned}$$

The representative household's policy function will now generally be contingent upon the particular realization of money growth shocks. Consequently, let us conjecture feasible shock contingent solution paths such that $K_{t+1} = \phi(\eta_t)AK_t + (1-\delta)K_t$ and $C_t = [1-\phi(\eta_t)]AK_t$. Since monetary shocks can only take on one of two values, finding the competitive equilibrium amounts to solving for $\phi(\sigma)$ and $\phi(-\sigma)$ in a manner consistent with money market clearing and the cash-in-advance constraint.

To provide some intuition regarding the effects of stochastic monetary policy on real growth, we now re-examine the two corner cases, $\psi = 0$ and $\psi = 1$.

3.1. Two familiar cases revisited

3.1.1. $\psi = 0:$ A cash-in-advance constraint on consumption

In this case, the functional equation (3.2) reduces to

$$E_t U'(C_{t+1})\frac{P_t}{P_{t+1}} = [\beta(1-\delta)(1-\psi)+\beta A]E_t U'(C_{t+2})\frac{P_{t+1}}{P_{t+2}}. \qquad (3.3)$$

This equation implies that when agents receive a high money transfer today, next period's nominal balances will grow at a relatively fast rate. Inflation is therefore expected to be higher than average and agents consequently demand less real balances which increases investment. As a result, consumption is lower and growth is higher in the high transfer state. Specifically, for the case of log utility, equation (3.3) can be written as

$$\frac{1}{C_t} = \beta[A+(1-\delta)]E_t\{\frac{z+\eta_t}{C_{t+1}[z+\eta_{t+1}]}\}. \qquad (3.4)$$

From (3.4), it is clear that the feature of particular interest is the *relative* variability of monetary policy as captured by $\frac{z+\eta_t}{z+\eta_{t+1}}$. As z becomes arbitrarily large, $\frac{z+\eta_t}{z+\eta_{t+1}}$ approaches one and equation (3.4) tends to that which obtains in a deterministic setting. Using the definition of ϕ, solving equation (3.4) yields[6]

$$\phi(-\sigma) = \frac{\beta Az - (1-\beta)(1-\delta)(z+\sigma)}{A[z+(1-\beta)\sigma]} \tag{3.5}$$

and

$$\phi(\sigma) = \frac{\beta Az - (1-\beta)(1-\delta)(z-\sigma)}{A[z-(1-\beta)\sigma]}. \tag{3.6}$$

Observe that when $\sigma = 0$, $\phi = \beta - ((1-\beta)(1-\delta)/A)$ as in the previous section. As noted, $\phi(\sigma) > \phi(-\sigma)$ so that a higher growth rate obtains when money growth is high. That is, real growth and money growth are positively related in the short run. Nevertheless, this result is consistent with the fact that on average, money growth adversely affects economic growth. Specifically, let γ_m denote the long-run mean balanced growth rate for this economy. Then[7]

$$\begin{aligned} \gamma_m &= \frac{E(Y_{t+1} - Y_t)}{E(Y_t)} \\ &= \{\frac{\beta[A+(1-\delta)]z^2 - (1-\delta)z^2 + (1-\beta)^2(1-\delta)\sigma^2}{z^2 - (1-\beta)^2\sigma^2}\} - \delta, \end{aligned} \tag{3.7}$$

where the last equality makes use of the assumption that money growth shocks are independently distributed while K_t only depends on shocks up to time $t-1$.

In contrast to the analogous deterministic case, money is no longer neutral with respect to the mean real growth rate. When $\sigma = -\sigma = 0$, γ_m reduces to the deterministic growth rate γ_d^*. It is straightforward to show that γ_m is now a decreasing function of average money growth. To see why this is true, consider Figure 5.1 which plots $\phi(\sigma)$ and $\phi(-\sigma)$ as functions of z. Since monetary policy is now variable, agents undertake some precautionary saving and hence save relatively more than in the deterministic case. Observe that $|\phi(\sigma)| > |\phi(-\sigma)|$ so that the expected fraction of output that is invested in any one period is greater than its deterministic counterpart. However, it is also true that both functions converge monotonically to the value of ϕ under certainty, ϕ_d, as z becomes arbitrarily large. This is because the

[6] For details of the solution, see Appendix A.

[7] It can be easily established that the shares of consumption and investment in output are stationary in expected value terms.

relative variability of monetary policy tends to vanish when this occurs. Following Jensen's Inequality, the unconditional expectation of real growth is, therefore, declining in z.

Further, (3.5) and (3.6) implies that all else equal, a rise in the variability of money growth increases the average level of savings and investment. As shown in (3.7), average real growth therefore increases with σ^2. This result naturally follows since less real money balances are held on average as the return to money balances become more uncertain. As a consequence, more resources are invested. This finding is analogous to that of Merton (1969), who shows that precautionary savings tend to increase with the volatility of the return to investment in a non-monetary economy. Eaton (1981), and Hopenhayn and Muniagurria (1996), find similar results in the analysis of variable fiscal policy. More recently, Rebelo and Xie (1996) also show that these results extend to the case of a monetary economy with a cash-in-advance constraint on consumption where variability is induced through real shocks. It is interesting to note that a number of empirical studies, notably by Grier and Tullock (1989), Levine and Renelt (1992), and Barro (1996) have been unable to verify the more conventional view that greater volatility in the inflation rate should be associated with lower growth, while McTaggart (1992) finds that variability has a positive effect on growth.

It should be pointed out that, in contrast to deterministic endogenous growth models, there no longer exist a straightforward link between the real return to savings and growth in this environment. To see this, let $q(\eta_t)$ denote the price of a bond that returns one unit of output and sells in the asset market after the goods market closes. Defining $1+r(\eta_t) = \frac{1}{q(\eta_t)}$, it follows that $1 + r(\eta_t)$ is constant and equal to $A + 1 - \delta$ as expected. This is because investment in physical capital is not subject to the inflation tax when $\psi = 0$. Nevertheless, given that in any time period the household undertakes more investment when the return to real balances is low and vice versa, real growth is state dependent.

Throughout our discussion, we have used the notion that through the liquidity constraint, money growth plays a role similar to that of a consumption tax in affecting real growth. To see this, suppose that our representative household were to maximize its lifetime utility over an infinite horizon subject to the constraint,

$$C_t(1 + \tau_t) + K_{t+1} - (1 - \delta)K_t = AK_t + TR_t \tag{3.8}$$

where $\tau_t \in (0,1)$ follows a process similar to that which describes money growth above. TR_t represents current period tax proceeds remitted to the household as a lump sum transfer. When utility is logarithmic, the relevant Euler equation is identical to (3.4) and is given by

$$\frac{1}{C_t} = \beta[A + (1-\delta)]AE_t\{\frac{1+\tau_t}{C_{t+1}[1+\tau_{t+1}]}\}, \tag{3.9}$$

where the tax on consumption now replaces the rate of money growth. In a deterministic setting, where $\tau_t = \tau$ for $t = 1, 2...$, the consumption tax does not affect real growth. This is the expected result. However, note from (3.4) and (3.9) that a variable tax rate would affect mean real growth in precisely the same way as variable money growth. In particular, from equation (3.4), we can think of the proceeds generated by the inflation tax in period t as $[z + 1 + \eta_t]C_t$. These proceeds exactly equal the amount of seignorage rebated to the household as a lump sum transfer since, using the cash-in-advance constraint, we have

$$[z - 1 + \eta_t]C_t = [z - 1 + \eta_t]\frac{M_t}{P_t} = \frac{\nu_t}{P_t} \tag{3.10}$$

In a sense, this relationship therefore creates an intuitive link between the growth effects of variable monetary policy and those of variable fiscal policy as investigated in Eaton (1981), Dotsey (1990), and Hopenhayn and Muniagurria (1996).

3.1.2. $\psi = 1$: A cash–in-advance constraint on consumption and investment

As mentioned earlier, when the relevant notion of capital involves a human component, the fraction of investment subject to the liquidity constraint may be substantial, especially in less developed economies. This subsection offers some insights into the growth effects of stochastic monetary policy when this is true. The functional equation (3.2) can be written as

$$U'(C_t) = \beta(1-\delta)E_tU'(C_{t+1}) + \beta^2 AE_tU'(C_{t+2})\frac{P_{t+1}}{P_{t+2}} \tag{3.11}$$

Observe that with white noise shocks, equation (3.11) implies that economic decisions are invariant to the realization of money growth. This result is in fact quite intuitive. Since current money growth shocks do not contain any information about future shocks, and since investment and consumption are both subject to the liquidity constraint, the household cannot substitute between consumption and investment to avoid the inflation tax across states. More importantly, this means that in developing economies, the fact that real growth and money growth may be positively related at cyclical frequencies tends to vanish.

This aspect of the model is reminiscent of the analysis of stochastic taxes in Dotsey (1990). To be specific, and setting $\delta = 1$ for notational convenience, equation (3.11) can be expressed as

$$\frac{1}{C_t} = \beta^2 A E_t \{ \frac{1}{C_{t+1}[z + \eta_{t+1}]} \Lambda_{t+2} \}, \tag{3.12}$$

where $\Lambda_{t+2} = \frac{K_{t+2}}{C_{t+2}} \frac{C_{t+1}}{K_{t+1}}$. Now, consider the problem of maximizing utility in a real economy when both consumption and investment are taxed. For the case of logarithmic utility, the relevant stochastic Euler equation is

$$\frac{1}{C_t} = \beta A E_t \{ \frac{1}{C_{t+1}[1 + \tau_{t+1}]} \}. \tag{3.13}$$

Hence, when a tax applies to investment as well as consumption, the optimal intertemporal decision does not depend on the current state of the world. That is, neither the current money growth rate nor the current tax rate appear in equations (3.12) or (3.13). Therefore, in both cases, the policy function is identical across states. As in Dotsey (1990), we shall see that the invariance of the policy function with respect to money growth states disappears when shocks are correlated.

A closed-form solution for $\phi(.)$ can be found by solving (3.12) for either state of the world. The solution is

$$\phi(\sigma) = \phi(-\sigma) = \frac{\beta^2 z}{z^2 - \sigma^2} + \frac{(1-\delta)(\beta - 1)}{A}. \tag{3.14}$$

Although $\phi(.)$ is not state contingent, this solution nevertheless does depend on the variance of money growth. In particular, it is clear that less consumption, and thus more savings and investment, takes place the more volatile the process describing monetary policy. Hence, as in the previous subsection, the mean balanced growth rate is increasing in the degree of monetary variability. Formally, the expected long-run growth rate is given by

$$\gamma_m = \frac{E(Y_{t+1} - Y_t)}{E(Y_t)} = \frac{\beta^2 A z}{z^2 - \sigma^2} + \beta(1-\delta) - 1. \tag{3.15}$$

When $\sigma = -\sigma = 0$, $\phi(.)$ in (3.14) reduces to (2.11) while γ_m in (3.15) reduces to the deterministic growth rate γ_d in (2.12). On a more significant note, observe from equation (2.12) that

$$\left|\frac{\partial \gamma_d}{\partial z}\right| = \frac{\beta^2 A}{z^2}, \tag{3.16}$$

while from (3.15):

$$\left|\frac{\partial \gamma_m}{\partial z}\right| = \frac{\beta^2 A[z^2 + \sigma^2]}{(z^2 - \sigma^2)^2}. \tag{3.17}$$

It can be established that since we have confined ourselves to $\sigma < z - \beta < z$, $\left|\frac{\partial \gamma_m}{\partial z}\right| > \left|\frac{\partial \gamma_d}{\partial z}\right|$ and the impact of average money growth on real growth is always stronger in the presence of monetary variability. This result can be best explained by referring to Figure 5.2. By introducing some variability in the monetary process, more saving and greater investment takes place relative to the deterministic case. Therefore, for $\sigma > 0$, γ_m tends to exceed γ_d. However, both measures converge monotonically to $\beta(1-\delta) - 1$ as z becomes arbitrarily large. This behavior implies that real growth in the stochastic world must decline faster as the money growth rate increases. This result is in fact analogous to that which obtains with a cash-in-advance constraint strictly on consumption since, when $\psi = 0$, we showed that $\frac{\partial \gamma_d}{\partial z} = 0$ while $\frac{\partial \gamma_m}{\partial z} < 0$.

As pointed out earlier, it is still true that the link between the return to savings and real growth differs from that which emerges in a deterministic world. In this case, investment in physical capital is subject to the inflation tax. Therefore in states where money growth is high, bonds are relatively more attractive which results in higher prices and lower returns. Formally, we have $1 + r(\sigma) = \beta A[\frac{1}{z+\sigma} + \frac{(z-\sigma)(1-\delta)}{\beta A z}] < \beta A[\frac{1}{z-\sigma} + \frac{(z+\sigma)(1-\delta)}{\beta A z}] = 1 + r(-\sigma)$, where $1 + r(\eta_t)$ is defined as in the previous subsection. Nonetheless, since the household cannot substitute between consumption and investment to avoid the inflation tax across states, real growth is constant.

Before discussing the empirically relevant cases in the next section, we summarize our results in the following table.

Effects of Inflation on Real Growth

	Deterministic Case	*Variable Regime*
$\psi = 0$	No effect	Negative effect of inflation on average growth
		Positive effect of variability
		Higher real growth in high money growth state
$\psi = 1$	Negative effect of inflation	Stronger negative effect of inflation
		Positive effect of variability
		Growth does not depend on money growth state

Thus far, we have been able to show that for the cases where $\psi = 0$ and $\psi = 1$, the adverse effects of inflation on long-run growth are stronger when monetary policy is variable. In addition, and except for the corner case where $\psi = 1$, we have also found that real growth is state dependent. Of significant importance is that in all cases, the variability of money growth tends to increase average real growth through a precautionary savings motive.

At this stage, one may wonder more specifically about the features of the environment which have helped in generating these results. We briefly discuss two of these features. First, the shock dependence of investment in the case where $\psi = 0$ implicitly relies on the fact that today's monetary transfer cannot be used for consumption. If, as in Gomme (1993), transfers could be used in the goods market, then variability would have no effect on real growth when money growth shocks are white noise. In this case, $\phi(\sigma) = \phi(-\sigma) = \phi_d = \beta - ((1-\beta)(1-\delta))/A$. Modeling money transfers as in Svensson (1985) implies that our specification is consistent with what Easterly and Bruno (1995) refer to as "conventional wisdom." That is, "at business-cycle frequencies, inflation and growth may be positively related, while the relationship should be negative for the medium and long-run." It is interesting to note that the results for the case where $\psi = 1$ are identical irrespective of whether monetary transfers can be used in today's goods market. Second, an alternative specification for the money growth rule might have been $M_{t+1} = z(1 + \eta_t)M_t$. This would imply that money growth was irrelevant in the case where $\psi = 0$. Because the empirical relationship between growth rates and volatility is non-linear, we chose to work with the specification in (3.1).

Panels a) and b) of Figure 5.3 depict a strong positive relationship between the log of mean money growth and the log of its standard deviation for both low and high inflation countries. As in Barro (1996), we define high inflation countries as those having average inflation rates exceeding 40 percent and conversely for low inflation countries. This distinction is important since Boyd, Levine, and Smith (1996) also observe that "countries with inflation rates in excess of this threshold on average observe a discrete and substantial reduction in financial depth." Since average money growth and the variability of money growth are positively correlated, our model suggests that the growth effects of higher average money growth may be ambiguous when a large number of countries is considered. This would provide a partial explanation for the findings reported in Levine and Zervos (1993). In the next section, we attempt to quantify the real effects of average money growth through a numerical simulation. In doing so, we show that accounting for the variability in the monetary process reduces the strength of the relationship between average inflation and real growth.

4. Endogenous growth with stochastic monetary policy: the case of correlated shocks

In this section, we make use of a numerical simulation to solve a more general version of the basic model presented above. We allow for the less restrictive utility function exhibiting constant relative risk aversion, $U(C) = \frac{C^{1-\alpha}-1}{1-\alpha}$, $\alpha > 0$, and take the relationships in Figure 5.3 as given. Appendix B shows that in spite of having CRRA utility, and as a result of the linear technology, the functional equation analogous to (3.2) does not depend on the capital stock. Therefore, a policy function contingent upon the various money growth states can be found using a simple non-linear equation solver.

In order to investigate the implications that our model has for the growth effects of inflation in an empirically relevant setting, we must calibrate our model. We first describe our specification for the money supply rule. For simplicity, we assume that the shocks to money growth follow a two-state Markov process. Defining η_l and η_h as the low and the high shocks respectively, we let the state transition probabilities be given by prob$\{\eta_{t+1} = \eta_l | \eta_t = \eta_l\} = 1 - p$ and prob$\{\eta_{t+1} = \eta_h | \eta_t = \eta_h\} = 1 - q$. This stochastic process has the advantage of simplicity and at the same time is able to match cross country information on the mean and standard deviation of money growth. Formally, let η_m and σ^2 denote the long-run mean and variance of the shocks generated by the Markov chain. Then, it follows that η_l and η_h can also be expressed as

$$\eta_l = \eta_m - \sigma\sqrt{\frac{p}{q}} \text{ and } \eta_h = \eta_m + \sigma\sqrt{\frac{q}{p}}. \tag{4.1}$$

Given η_l and q, equation (4.1) can be used to find the values of η_h and p which are consistent with any pair (η_m, σ). In the following examples we calibrate $1 - q$ to be .85 implying that when money growth is in the high state, it tends to stay in that state. Bruno and Easterly (1995) provide evidence that this is indeed the case in the sense that once in a high inflation range, countries typically find it very difficult to revert back to lower inflation rates. This is due in part, no doubt, to the fact that drastic measures are usually needed in such circumstances.

We then let the cross-country relationship between the standard deviation and the mean of money growth be given by

$$\log \sigma_i = \zeta + \rho \log(1 + \eta_m)_i, \tag{4.2}$$

where σ_i and $(1 + \eta_m)_i$ respectively denote the long-run standard deviation and mean of money growth of country i. Observe that in terms of the notation used in sections 2 and 3, we now have $z = 1 + \eta_m$. Least Square values of ζ and ρ can then be obtained for both the low and high inflation countries. Thus, in varying the mean of money growth in the simulations below, we will be careful to set η_h and p using (4.1) so as to preserve the relationship depicted in (4.2) as applying to low and high inflation samples. As far as the other parameters in the model are concerned, we set the coefficient of relative risk aversion α to 1.25, the rate of impatience β to .96, and the depreciation rate δ to .08. Choosing A equal to .175 generates reasonable growth rates across countries with varying levels of inflation.

We now examine two numerical examples that portray the interaction between money growth and its variability on economic growth. In doing so we are able to highlight some key behavioral features that should be present across various monetary models. The first example analyzes the growth implications of variable monetary policy for countries where the liquidity constraint only applies to a small fraction of investment with $\psi = .1$. With this parameterization, approximately 31% of output is subject to the cash-in-advance constraint, which is about the same percentage as in Chari, Jones, and Manuelli (1995). This case is meant to encompass most developed nations as well as some newly industrialized countries where credit markets function relatively well. Next, we carry out the same numerical exercise for countries with either limited or ineffectual credit markets in which case we set $\psi = .65$. This latter scenario is likely to be encountered mainly in developing countries where sophisticated financial institutions have not yet emerged. This point is especially relevant with respect to investments in human capital.

Solving equation (3.2) yields the solutions $\phi(\eta_l)$ and $\phi(\eta_h)$ with the associated growth rates

$$\gamma(\eta_t) = \begin{cases} \gamma(\eta_l) = \phi(\eta_l)A - \delta & \text{when } \eta_t = \eta_l \\ \gamma(\eta_h) = \phi(\eta_h)A - \delta & \text{when } \eta_t = \eta_h \end{cases}. \tag{4.3}$$

Given the distribution of money growth specified above, the long-run mean growth rate is computed as

$$\gamma_m = \frac{q}{p+q}\gamma(\eta_l) + \frac{p}{p+q}\gamma(\eta_h). \tag{4.4}$$

With this in mind, we now turn our attention to the results.

4.1. $\psi = .1$: A weak liquidity constraint on investment

In this simulation, we set the growth rate of money in the low state, η_l, to .03, and consider average inflation rates ranging from 1 to 40 percent. The estimated values of ζ and ρ in equation (4.2) are .10 and 2.71 with $t-$values of 19.3 and 3.2 respectively. Panel d) of Figure 5.4 shows that, as in the simple cash-in-advance case, real growth is positively related to the particular realization of money growth. As we have already noted, when a small fraction of investment is subject to the inflation tax, the representative household can substitute between consumption and investment to avoid the inflation tax across different states. More importantly, note that as the average inflation rate increases, and as a result the variance of inflation also rises, the degree of substitution becomes more intensive creating a wider dispersion between the two levels of state contingent growth rates. Therefore, countries such as Columbia, which have managed to live with relatively high levels of inflation[8], should exhibit greater volatility in their real growth rate than most industrialized countries. This is especially true since Panel c) in Figure 5.4 shows that nations which possess low average money growth spend virtually all their time in the low state. In contrast, countries which display average levels of inflation around 20 to 30 percent per year spend roughly equal amounts of time in each state. Thus, this model suggests that as average money growth rises, which by equation (4.2) also leads to an increase in nominal variability, real growth rates become more volatile. The fact that variable monetary policy has implications for the volatility of real growth rates has thus far been overlooked in empirical studies.

We also note that in Figure 5.4, panel d), the long-run mean growth rate displays less sensitivity with respect to average inflation than does the deterministic growth rate. We know from the previous section that, all else equal, the adverse impact of average money growth on real growth is stronger when monetary policy is variable. However, as equations (3.7) and (3.15) suggest, the rise in variability which accompanies higher average money growth tends to raise real growth. When the link between the mean and volatility of money growth is calibrated as in (4.2), the net effect is to create a weaker relationship between average inflation and real growth relative to the deterministic case. This result becomes even more apparent in the next example. As far as quantitative estimates of the effects of inflation on real growth are concerned, we find that a 10 percent rise in inflation is associated with a .13 percent decrease in real growth. Barro (1996) reports that for low inflation countries, a similar increase in inflation results in a .23 percent decrease in growth but that latter value is not statistically different from zero.

[8] Around 22 percent on average for the case of Columbia.

4.2. $\psi = .65$: Limited or ineffectual credit markets

Here, we explore a relationship between average inflation and real growth which may be more relevant for economies with severely limited financial markets. As mentioned earlier, Boyd, Levine, and Smith (1996) observe that these countries typically possess average inflation rates in excess of a 40 percent threshold. As a result, we run the simulations for inflation rates above 40 percent and set the growth rate of money which prevails in the low state, η_l, to .2. The estimated values of ζ and ρ in equation (4.2) are .084 and 4.32 in this case. The associated $t-$values are 6.9 and 5.4 respectively. First, note from panel d) in Figure 5.5 that in contrast to the case where $\psi = .1$, a higher growth rate now emerges when the low money growth shock occurs [i.e. $\gamma(\eta_l) > \gamma(\eta_h)$]. To understand this result, it is useful to refer to the analysis of stochastic taxes carried out in Dotsey (1990). In that paper, a greater fraction of output is invested in the low tax state if the low tax state implies that taxes are likely to be low in the future. In this example, both consumption and a large fraction of investment are subject to the inflation tax. We have already seen that when shocks to money growth are identically and independently distributed, current shocks do not contain any information about future shocks so that the household cannot effectively avoid the inflation tax across states. In this case however, the relationship depicted in (4.2) implies that money growth shocks are persistent in the sense that $q + p < 1$. The values of p consistent with equation (4.1) for various levels of average money growth are shown in panel b) of Figure 5.5. Therefore, the current realization of a low shock to money growth implies that the inflation tax is likely to be low in the future as well. Consequently, more investment takes place in that state of the world. Thus, our analysis not only suggests that the behavior of real growth depends upon specific realizations of money growth, but also that the nature of this dependence varies with the fraction of investment subject to the inflation tax.

It should be remarked that, in this framework, the quantitative amplitude of real growth, $|\gamma(\eta_l) - \gamma(\eta_h)|$, is too small to explain the sharp decline in growth rates which typically follow inflation crises as documented in Easterly and Bruno (1995). However, in a world where countries' monetary policies display some variability, this aspect of the model helps explain why real growth may respond negatively to a temporary boost in inflation in developing economies.

Panel d) in Figure 5.5 also shows that the negative effect of higher average inflation on the deterministic growth rate are significant. In contrast, note that the mean long-run growth rate which emerges when money growth is variable exhibits much less sensitivity. This is due to the mitigating effect of higher nominal volatility. Panel d) shows that as the average inflation rate first rises, the growth rate begins to fall. However, as the average inflation rate continues to rise, the effects of higher volatility begin to dominate and real growth even starts to increase. Hence, in contrast to the deterministic framework, this stochastic model suggests that even when a substantial fraction of investment is subject to the cash-in-advance constraint, the relationship between inflation and real growth remains relatively weak and is only about half as large as in the deterministic case. For example, an 10 percent increase in inflation is associated with a .13 percent decrease in growth in the stochastic environment, but with a .45 percent in the deterministic environment. The mitigating effect of monetary variability may help partially explain why Levine and Zervos (1993) find that even when countries with average inflation in excess of 40 percent are considered, inflation seems unrelated to growth.

It remains that most empirical work on the effects of inflation on growth is conducted using the entire sample of countries. Calculations of the effects of inflation on growth in this case may be approximated by the slope of the line, say, between growth rates at 10 percent inflation in developed economies and growth rates at 60 percent inflation in less developed countries. The slope of this line implies a .42 percent decrease in growth for evert 10 percent increase in inflation, a number which is within the range of most empirical studies.

5. Conclusion

In this paper we investigate the theoretical linkages between inflation, inflation volatility, and economic growth in a basic monetary growth model. As in the literature on growth and fiscal policy, we find that volatility introduces some new implications for economic growth. For instance, the model shows that inflation and growth may be negatively related in the long run while positively related at cyclical frequencies. In addition, the positive short run relationship between inflation and growth dissipates, and may even be negative, in economies with limited financial markets. The negative long run relationship between growth and inflation that is generated in this framework is of the order of magnitude found in much of the empirical literature. Since the model abstracts from many important avenues through which money may affect growth, we do not wish to make too much of this result. We do, however, regard the theoretical channels isolated in this paper as potentially important for other models of money. Moreover, it should be clear that the analysis presented in this paper easily extends to the variety of specifications included within the class of AK growth models. An obvious example of such a variation is one where the relative price of capital is endogenous.

Our analysis also indicates that the volatility of money and the volatility of growth are directly linked and this observation deserves empirical attention. Furthermore, volatility can offset the effects of average inflation if these two features of monetary policy are sufficiently correlated. That is, the precautionary savings effects can be large and potentially influence the size of the coefficient in a naive regression of growth on inflation that does not adequately account for the relationship between these two moments. The interlinkage between inflation rates and inflation volatility may, therefore, be an important element masking the growth effects of inflation.

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Appendix A:

Using the cash-in-advance constraint, and the fact that $C_t = [1-\phi(\eta_t)]AK_t$ while $K_{t+1} = \phi(\eta_t)AK_t + (1-\delta)K_t$, the functional equation in (3.4) reduces to

$$\frac{\phi(\eta_t)A + (1-\delta)}{[1-\phi(\eta_t)][z+\eta_t]} = \beta[A+(1-\delta)]E_t\{\frac{1}{[1-\phi(\eta_{t+1})][z+\eta_{t+1}]}\}$$

Define the transformation $\phi(\eta_t) = \frac{\beta f(\eta_t)}{1+\beta f(\eta_t)}$. This yields

$$f(\eta_t) = E_t\{[1+\beta f(\eta_{t+1})]\frac{z+\eta_t}{z+\eta_{t+1}}\} - \frac{1-\delta}{\beta[A+(1-\delta)]},$$

which, in this context, represents a set of two linear equations in tow unknowns. That is

$$f(-\sigma) = \frac{1}{2}(1+\beta f(-\sigma)) + \frac{1}{2}(1+\beta f(\sigma))\frac{z-\sigma}{z+\sigma} - \frac{1-\delta}{\beta[A+(1-\delta)]}$$

and

$$f(\sigma) = \frac{1}{2}(1+\beta f(-\sigma))\frac{z+\sigma}{z-\sigma} + \frac{1}{2}(1+\beta f(\sigma)) - \frac{1-\delta}{\beta[A+(1-\delta)]}.$$

Solving for f then yields the solutions for ϕ in the text.

Appendix B:

This appendix shows that even with CRRA utility, the functional equation analogous to (3.2) does not depend on the current capital stock. Therefore computations of equilibria across different monetary regimes can be carried out relatively simply.

The Euler equation which governs the optimal evolution of consumption in (3.2) is

$$\begin{aligned}\psi U'(C_t) &= -(1-\psi)\beta E_t U'(C_{t+1})\frac{P_t}{P_{t+1}} + \beta(1-\delta)\psi E_t U'(C_{t+1}) \\ &\quad +\beta[\beta(1-\delta)(1-\psi)+\beta A]E_t U'(C_{t+2})\frac{P_{t+1}}{P_{t+2}}\end{aligned}$$

Letting $U(C) = \frac{C^{1-\alpha}-1}{1-\alpha}$, $C_t = [1-\phi(\eta_t)]AK_t$, $K_{t+1} = \phi(\eta_t)AK_t + (1-\delta)K_t$, and using the cash-in-advance constraint, we get

$$
\begin{aligned}
\psi C_t^{-\alpha} &= -(1-\psi)\beta \frac{[\phi(\eta_t)A + 1 - \delta]}{[z+\eta_t][1-\phi(\eta_t)+\psi\phi(\eta_t)]} \times \\
& E_t\{([1-\phi(\eta_{t+1})]AK_{t+1})^{-\alpha}[1-\phi(\eta_{t+1})+\psi\phi(\eta_{t+1})]\} \\
& +\beta(1-\delta)\psi E_t\{[1-\phi(\eta_{t+1})]^{-\alpha}A^{-\alpha}K_{t+1}^{-\alpha}\} \\
& +\beta[\beta(1-\delta)(1-\psi)+\beta A] \times \\
& E_t\{\frac{[\phi(\eta_{t+1})A+1-\delta][1-\phi(\eta_{t+2})+\psi\phi(\eta_{t+2})][1-\phi(\eta_{t+2})]^{-\alpha}A^{-\alpha}K_{t+2}^{-\alpha}}{[z+\eta_{t+1}][1-\phi(\eta_{t+1})+\psi\phi(\eta_{t+1})]}\}
\end{aligned}
$$

Substituting for C_t, K_{t+1}, and K_{t+2}, the above expression simplifies to

$$
\begin{aligned}
\frac{\psi}{[1-\phi(\eta_t)]^{\alpha}} &= -(1-\psi)\beta \frac{[\phi(\eta_t)A+1-\delta]^{1-\alpha}}{[z+\eta_t][1-\phi(\eta_t)+\psi\phi(\eta_t)]} E_t\{\frac{[1-\phi(\eta_{t+1})+\psi\phi(\eta_{t+1})]}{[1-\phi(\eta_{t+1})]^{\alpha}}\} \\
& + \frac{\beta(1-\delta)\psi}{[\phi(\eta_t)A+1-\delta]^{\alpha}} E_t\{\frac{1}{[1-\phi(\eta_{t+1})]^{\alpha}}\} \\
& + \frac{\beta[\beta(1-\delta)(1-\psi)+\beta A]}{[\phi(\eta_t)A+1-\delta]^{\alpha}} \times \\
& E_t\{\frac{[\phi(\eta_{t+1})A+1-\delta]^{1-\alpha}[1-\phi(\eta_{t+2})+\psi\phi(\eta_{t+2})]}{[z+\eta_{t+1}][1-\phi(\eta_{t+1})+\psi\phi(\eta_{t+1})][1-\phi(\eta_{t+2})]^{\alpha}}\}
\end{aligned}
$$

which is independent of K_t.

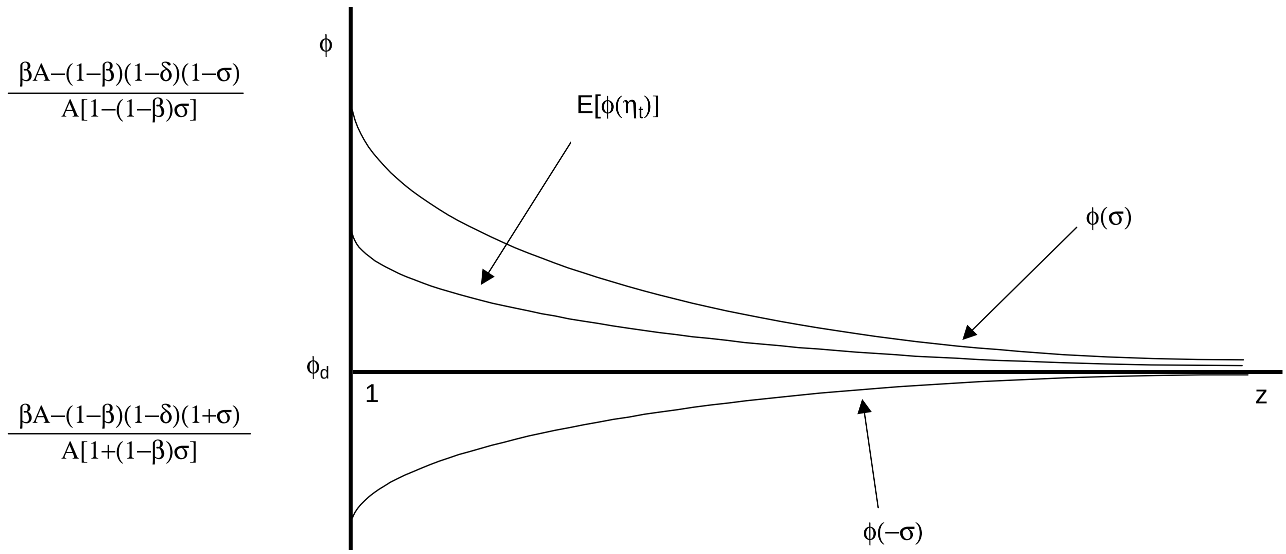


Figure 5.1:

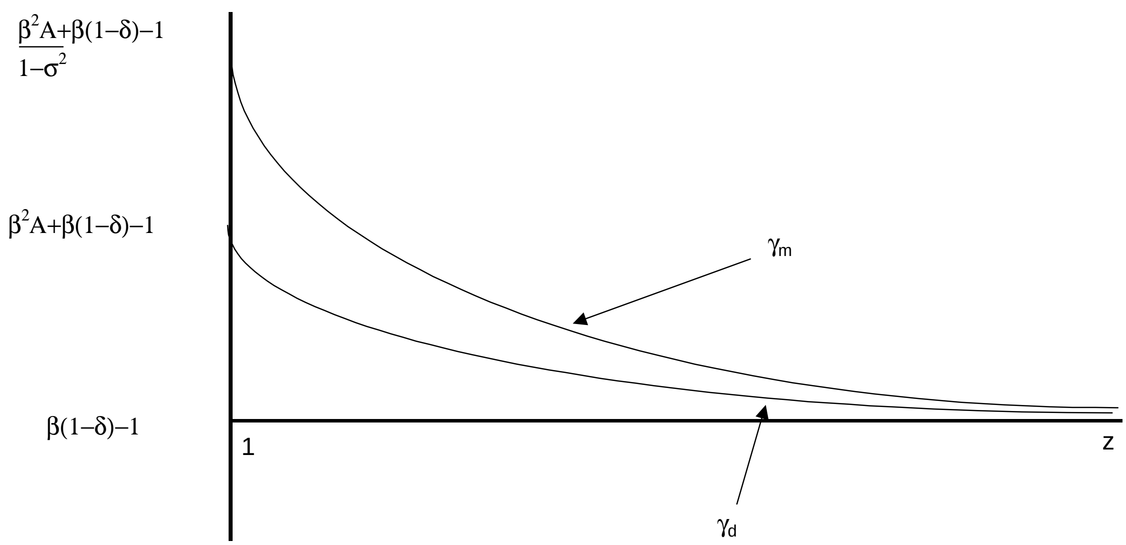


Figure 5.2:

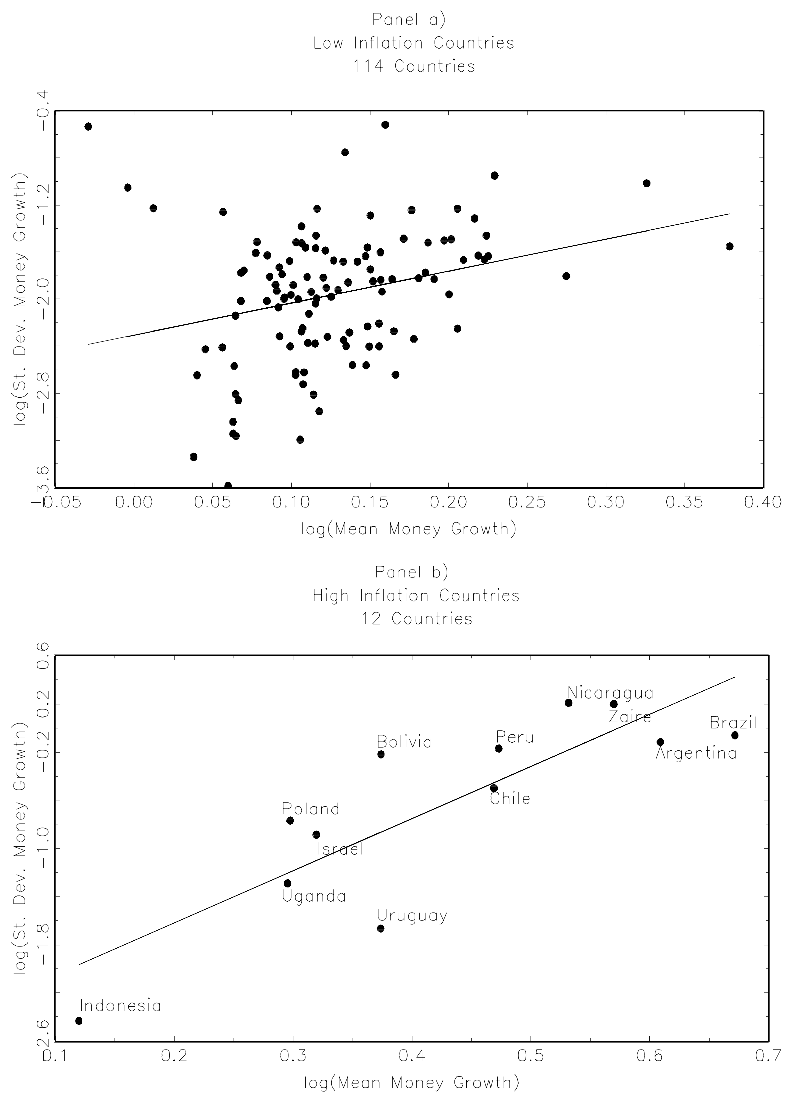

Panel a)
Low Inflation Countries
114 Countries
log(St. Dev. Money Growth)
log(Mean Money Growth)
Panel b)
High Inflation Countries
12 Countries
Nicaragua
Zaire
Brazil
Argentina
Peru
Bolivia
Chile
Poland
Israel
Uganda
Uruguay
Indonesia
log(St. Dev. Money Growth)
log(Mean Money Growth)

Figure 5.3:

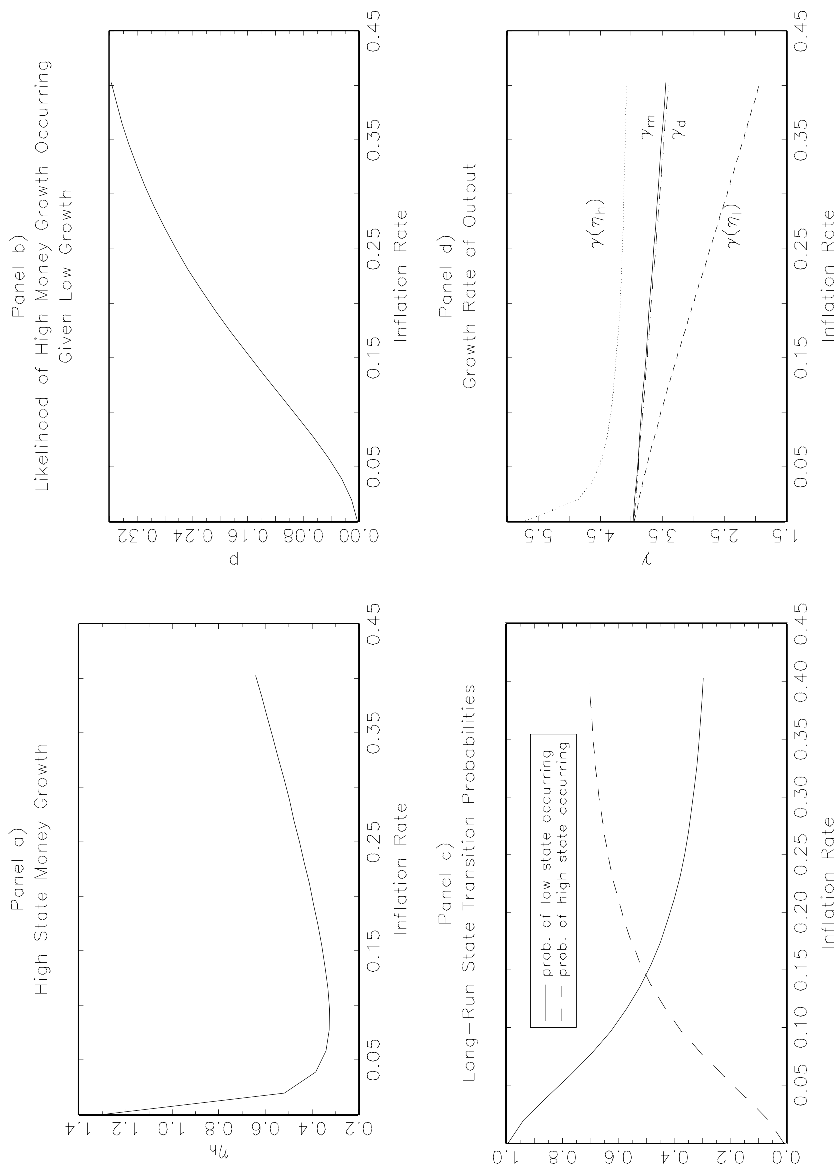
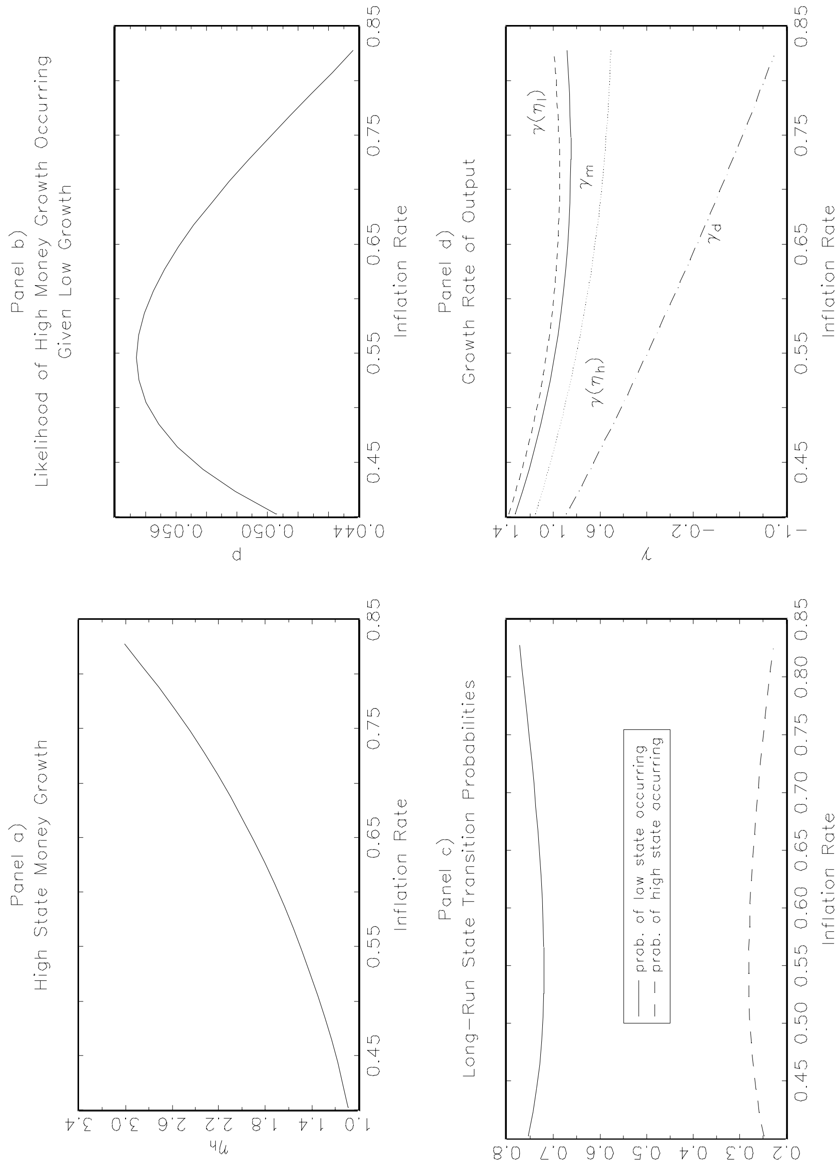

Panel a)
High State Money Growth
Inflation Rate
Panel b)
Likelihood of High Money Growth Occurring
Given Low Growth
Inflation Rate
Panel c)
Long-Run State Transition Probabilities
prob. of low state occurring
prob. of high state occurring
Inflation Rate
Panel d)
Growth Rate of Output
Inflation Rate

Figure 5.4:



Panel a)
High State Money Growth
Inflation Rate
Panel b)
Likelihood of High Money Growth Occurring
Given Low Growth
Inflation Rate
Panel c)
Long-Run State Transition Probabilities
prob. of low state occurring
prob. of high state occurring
Inflation Rate
Panel d)
Growth Rate of Output
Inflation Rate

Figure 5.5: