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Informality and Rent-Seeking Bureaucracies in a Model of Long-Run Growth*†

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Abstract

This paper explores the links among growth, the informal economy, and rent-seeking bureaucracies. The presence of congestion associated with the enforcement of property rights implies that informality can be useful. Whether bureaucratic rent-seeking is detrimental to growth then depends on how good a substitute informality is to production in the formal sector. In order to create profits which can be appropriated, rent-seeking bureaucrats limit entry into the formal economy. As a result, firms operate in the informal sector even when the cost of informality is high, in which case lower growth emerges. However, when the cost of informality is low, a large number of firms choose to operate informally irrespective of entry conditions. In the latter case, growth is unaffected by a rent-seeking bureaucracy as entry restrictions in the formal economy do not bind.

JEL Classification: E13, O10

Keywords: Informal sector, Rent-seeking, Economic growth

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† The views expressed in this paper are solely those of the author and do not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1 Introduction

This paper introduces a simple economic model that can be used to explore the links among growth, bureaucracy, and the interplay between the formal and informal sectors. These links have long been thought important factors in the process of economic development. In this model, growth occurs through the costly adoption of technology already available in more advanced economies. As is generally the case in the endogenous growth literature, the resulting equilibrium growth rate need not be efficient. Hence, there is a potentially beneficial role for a bureaucracy that controls entry and exit by firms in the formal economy. However, while an efficient bureaucracy may be able to improve welfare, a rent-seeking bureaucracy can lower both welfare and economic growth.

In the economy presented below, firms must choose between operating formally or informally. As in Braun and Loayza (1993), the informal sector is defined as "a set of economic units which do not comply with one or more government imposed taxes and regulations, but whose product is considered legal." As a result of their illegal status, informals cannot benefit from enforceable property rights over their product. This often results in profit losses due to vandalism, theft, as well as the fact that contracts cannot be enforced. By having access to law enforcement services, formal firms do not generally suffer such losses. However, the latter firms must contend with various government taxes which are used to finance the provision of a police force and courts of law. The model determines an equilibrium fraction of firms allocated to each sector and traces its dependence on the cost of informality,[1] as well as other factors. In particular, when entry in the formal sector is regulated, the model indicates how the scale of the informal sector changes.

The analysis carried out in this paper leads to some interesting positive conclusions. First, empirical work on the links between bureaucratic corruption and growth suggest that there is a negative relationship in the international cross-section.[2] Our framework predicts this relationship when locating production in the informal sector involves similar and relatively high costs in all countries. Second, these empirical studies also indicate that there are significant outlier countries which experience relatively high corruption, high growth, and large informal sectors. Our framework implies precisely this scenario when the costs of operating informally are relatively low. In the latter case, informality is a viable substitute for production in the formal sector and the model further implies that rent-seeking has negligible effects on economic growth. This third result illustrates that there may exist a more subtle nonlinear relationship between bureaucracy and growth than previously investigated

[1] This terminology is adopted from De Soto's (1989) well known study, "The Other Path."

[2] See Mauro (1995), or Keefer and Knack (1995).

in empirical work.

The analysis below also yields some interesting normative conclusions. First, policymakers are sometimes concerned by the large scale of informal sectors in developing economies. Contrary to this concern, the model indicates that the size of the informal sector alone has little to say about growth. On the one hand, a large informal sector can arise as a result of firms seeking to escape a predatory bureaucracy in the formal economy. In this case, the country's growth rate may be low due to the rent-seeking behavior of the bureaucracy, but would be even lower with fewer informals contributing to economic activity. On the other hand, if the informal sector is large because the costs of operating informally are relatively small, then it is efficient to locate economic activity in this sector. Second, our framework does contain some cases where a decline in economic growth, produced by an efficient bureaucracy, leads to higher welfare. Therefore, one must be cautious of equating growth and welfare.

This paper is organized as follows. Section 2 specifies the technology, the market structure underlying the interplay between formal and informal firms, and the equilibrium growth rate that emerges with free entry in both sectors. This section is meant to serve as a benchmark case and stands as a reasonable proxy for most European and North American countries. By contrast, in section 3, the size of the formal sector is determined by rent-seeking bureaucrats and the resulting effects on long-run growth are analyzed. The latter scenario more closely resembles the nature of bureaucratic procedures in many parts of Asia and Latin America. The emphasis in this paper is specifically on the nature of bureaucrats' activities and how they effect the relative sizes of the formal and informal sectors. We find it useful, therefore, to take the polar cases defined by free entry in the formal sector on the one hand, and regulated entry by rent-seeking officials on the other, as given.[3] Section 4 addresses the question of the optimal size of the formal sector from the standpoint of a benevolent government. Finally, some concluding remarks and directions for future research are offered in section 5.

2 Long-run growth with formal and informal firms

Consider a closed economy in which a large number of agents, who are identical and uniformly distributed on the unit interval, directly operate the final goods technology. This production process uses a range of intermediate inputs which agents learn in a sequential fashion. The accumulated stock of knowledge, therefore, may be interpreted as a measure of human capital. Endogenous growth will stem from domestic investments devoted to the

[3] Acemoglu and Verdier (1998) provide an analysis which focuses instead on the factors that determine the endogenous emergence of a bureaucracy.

adoption of new technologies. In this framework, new technologies are not being discovered but rather copied from what is already available elsewhere. We assume that each new technology creates a corresponding industry. Following Easterly et al. (1993), we further assume that the economy is relatively removed from the technology frontier. This eliminates the need to deal with a transition phase in which some resources may be channeled towards innovation.[4]

To ensure that both formal and informal firms interact in the same market, we assume that at each date in time, and within each industry, all firms compete in a Cournot fashion. The implications of this assumption are twofold. First, each firm can potentially earn a profit depending on how many firms ultimately operate in its industry. This feature of the environment is crucial since, as Mauro (1998) points out, "it is the existence of rents that motivates rent-seeking behavior." Second, this assumption allows the return to technology adoption to be contingent upon the equilibrium number of operating firms. Therefore, economic growth will be partially driven by the number of firms that ultimately make use of newly acquired technology. Because the game is infinitely repeated, there may exist many equilibria. We focus on the scenario where firms play the unique single-period Nash equilibrium in every period.[5]

2.1 Final goods technology

Time is continuous and the horizon in infinite. As in Romer (1990), a single final good Y is produced by combining labor with a range of intermediate goods $i \in [0,\mathrm{A}]$ according to[6]

$$Y = \left[\int_0^{\mathrm{A}} x(i)^{\alpha} di\right] L^{1-\alpha} \text{ with } 0 < \alpha < 1, \tag{1}$$

where $x(i)$ is viewed as a productive input, which may or may not be some form of capital, and which includes services. L denotes labor input. Since learning occurs sequentially, being able to work with the input $x(\mathrm{A})$ also implies being able to work with any input in the interval $[0,\mathrm{A}]$. For simplicity, we set $L = 1$ in the remainder of the analysis.

The optimal choice of intermediate goods utilization by agents yields an inverse demand function for the i^{th} good,

$$p(i) = \alpha x(i)^{\alpha-1}, \tag{2}$$

[4] Since in this analysis we have in mind a developing economy, this modeling assumption seems appropriate.

[5] This unique single-period Nash equilibrium is also the single equilibrium of a repeated finite-horizon game.

[6] To economize on notation, we omit the time subscripts.

where $p(i)$ is the price of $x(i)$ in terms of the final good. It follows that output, net of the cost of intermediate goods, is simply given by

$$\text{Net Output} = (1-\alpha)\int_0^{\text{A}} x(i)^{\alpha} di. \tag{3}$$

Once a new technology is successfully adopted from elsewhere, the corresponding intermediate input is produced domestically. The price of intermediate inputs, $p(i)$, in (2) is hence endogenous in equilibrium. In particular, it will depend on the manner in which intermediate goods producing firms interact with one another. We now address the nature of this interaction.

2.2 Imperfect competition at the firm level

We assume that each industry $i \in$ [0,A] is composed of both a formal and an informal sector. We denote by n^F and n^I the number of formal and informal firms respectively, with $N = n^F + n^I$. In order that some intermediate goods be produced, it is assumed that there exists at least one firm so that $N \geq 1$.

For analytical tractability, we model the difference in cost structure between formals and informals as occurring through the fixed portion of total costs. Allowing for marginal costs to differ instead would not alter the substance of our conclusions. The symmetry in marginal costs, both within and across sectors, implies that production levels will be identical for all firms. Moreover, the Cournot game implies that each firm's output in a given industry will depend upon the total number of firms in that industry. Letting X denote this common output level, and assuming symmetry across industries, it then follows that $x(i) = NX$ for $i \in$ [0,A].[7] However, it is important to note that from the standpoint of each firm, the output of all other firms is taken as given so that, from equation (2), it views the price it faces as $p = \alpha\left(X + \sum_{j \in n^F, n^I} X_j\right)^{\alpha-1}$. In this last expression, X_j denotes the production level of the j^{th} firm in either sector. We now turn to the description of formals and informals in more detail, as well as the outcome of the Cournot game.

In any industry i, per period profits generated by a typical firm in the informal sector are given by

$$\pi^I = pX - \upsilon X - \lambda, \text{ with } \lambda > 0. \tag{4}$$

Thus, once copied from an already existing blueprint, an intermediate good of any type requires $\upsilon > 0$ units of final output to produce domestically. In addition, informal firms do

[7] Goods in the formal and informal sectors are, therefore, perfect substitutes in this framework.

not pay any taxes but must incur a fixed cost per period, λ, which Braun and Loayza (1993) refer to as "the disadvantages inherent to informality." By virtue of their illegal status, informals do not benefit from access to a police force or courts of law. As a result, they do not possess complete and enforceable property rights over their product. We can therefore think of λ as a fixed profit loss due to criminal offences such acts of terrorism or theft [see De Soto (1989)]. Under this interpretation of the cost of informality, the size of λ describes how well the informal sector manages to cope outside the legal system. As noted by De Soto, an informal judicial system which relies on extralegal norms can sometimes emerge to resolve land disputes and criminal acts. The cost of informality may, therefore, vary from country to country. In this paper, we do not investigate the factors that might ultimately determine the extralegal organization of informals. Instead, losses due to the lack of property rights enforcement are taken as exogenous from the point of view of an informal firm. Equilibrium growth rates are then derived, under both free entry in the formal sector and regulated entry by rent-seeking officials, for different values of λ.

Profits for the typical formal firm in any industry i are given by

$$\pi^F = pX - \upsilon X - T. \tag{5}$$

These firms, therefore, face marginal costs similar to those of the informals as well as a fixed tax cost per period, T. The tax cost T amounts to an outflow of funds inherent to operating in the formal sector. As such, taxes are collected by the government which then uses this revenue to provide courts of law and a police force. Hence, in this paper, the defining characteristic of formal firms lies in the fact that they do not incur any of the costs associated with λ. As in Barro (1990), access to a police force and judicial services is subject to congestion. In particular, we assume that the resource costs attached to the provision of public services are quadratic in the size of the formal economy. Total government spending on public services, across all industries, can accordingly be expressed as $G = \int_0^{\mathrm{A}} \eta \left(n^F\right)^2 di$, with $\eta > 0$. We further assume that the government maintains a balanced budget policy so that, in every period, government spending exhausts total tax revenue, $\mathrm{A}\eta \left(n^F\right)^2 = \mathrm{A}n^F T$. It follows that

$$T = \eta n^F. \tag{6}$$

In other words, firms in the formal sector pay taxes equal to the average cost of providing them with public services.

Profit maximization by firms in either sector yields a set of N identical first-order condi-

tions for the exact Cournot form,

$$\alpha(\alpha-1)\left(X+\sum_{j\in n^F,n^I}X_j\right)^{\alpha-2}X+\alpha\left(X+\sum_{j\in n^F,n^I}X_j\right)^{\alpha-1}-\upsilon=0, \tag{7}$$

from which it follows that each firm's production level is given by

$$X=\left(\frac{\alpha}{\upsilon}\right)^{\frac{1}{1-\alpha}}\left[N^{\alpha-1}-(1-\alpha)N^{\alpha-2}\right]^{\frac{1}{1-\alpha}}. \tag{8}$$

As alluded to earlier, X is a function of the total number of firms in the industry, including both the formal and informal sectors, and will thus be denoted $X(N)$ throughout the remainder of the paper. In a similar fashion, the price of intermediate goods in (2) can now be denoted $p(N)$. Appendix A shows that the net sales margin for intermediate goods producers, defined as

$$\chi(N)=\left[p(N)-\upsilon\right]X(N), \tag{9}$$

decreases with N.[8] Given the different nature of fixed costs in the formal and informal economies, this feature of the model plays a key role below in determining the equilibrium sizes of the two sectors. Specifically, since the use of a police force and courts of law is subject to congestion, some firms will generally prefer to locate production in the informal economy.

2.3 Determination of the growth rate

To close the model, we now need to address the problem faced by a representative agent. At each point in time, the representative agent receives both formal and informal firms' profits, across all industries, as dividends. Using the notation in (4) and (5), the sum of profits can be expressed as $\Pi^F+\Pi^I=\mathrm{A}n^F\pi^F+\mathrm{A}n^I\pi^I$, where Π^F and Π^I are taken as given. Moreover, from equation (3), the net output obtained from the use of intermediate goods is given by $(1-\alpha)\left[NX(N)\right]^{\alpha}\mathrm{A}$. We assume that to adopt a new technology from abroad, it is necessary to invest one unit of final output. The problem of a representative agent with discounted, constant elasticity preferences, can therefore be expressed as

$$\max\ \mathcal{U}^c=\int_0^{\infty}e^{-\rho t}\frac{C^{1-\sigma}-1}{1-\sigma}dt \text{ with } \sigma\geq 0 \text{ and } \rho>0, \tag{P1}$$

[8] The intuition behind this result is relatively straightforward. As the number of firms increases, each firm operates on a lower residual demand curve and, consequently, attempts to reduce output in an effort to maximize profits and curb falling prices. In equilibrium, prices fall faster than the rate at which firms are able to adjust their output. The net sales margin for each firm, therefore, falls with the total number of firms in both sectors.

$$\text{subject to } C + \frac{d\mathrm{A}}{dt} = (1-\alpha)\left[NX(N)\right]^{\alpha}\mathrm{A} + \Pi^{F} + \Pi^{I}, \text{ and } \mathrm{A}_0 > 0 \text{ given}, \tag{10}$$

where C denotes the agent's consumption level.[9] The solution to the above dynamic optimization problem yields the following growth rate formula for consumption,

$$\gamma_c = \frac{(1-\alpha)\left[NX(N)\right]^{\alpha} - \rho}{\sigma}, \tag{11}$$

with no transitional dynamics. Using the resource constraint for the economy as a whole, it is clear that (11) represents a balanced growth rate for this economy which, in the remainder of the analysis, we denote by γ. At this stage, one should note that the return to acquiring new technology, $(1-\alpha)\left[NX(N)\right]^{\alpha}$, is not yet an equilibrium solution. Indeed, the size of the formal sector and that of the underground economy have yet to be determined. Nevertheless, a first interesting result arises:

Result 1: *The return to acquiring technology,* $(1-\alpha)\left[NX(N)\right]^{\alpha}$, *increases monotonically with the total number of firms,* N, *and eventually asymptotes to a constant.*

The proof of this result is in appendix A. In essence, there are two important factors that determine the degree to which this economy takes advantage of newly acquired technology: the number of firms that actually make use of this technology, and the degree to which each firm uses it. Two opposing forces are in effect. On the one hand, a greater number of firms directly helps in raising the return to technology adoption. On the other hand, and because of imperfect competition, a greater number of firms implies that each firm will produce a smaller quantity of the associated intermediate input. The growth rate then rises at a decreasing rate with the total number of firms -*including informal firms*- involved in the production of intermediate inputs. The important point here is that the informal sector may play a legitimate role in promoting economic growth. Therefore, equation (11) may lend some credibility to support programs that explicitly recognize the role of the informal economy in the process of economic development. Of course, and as we now show, whether or not an informal sector exists in equilibrium ultimately depends on how well informals can cope outside of official laws and regulations.

Equilibrium with free entry in the formal economy requires that $\pi^F = 0$ or, equivalently,

$$\chi(N) = \eta n^F. \tag{12}$$

If it were the case that $\pi^F > 0$, additional firms would attempt to join the formal sector while if $\pi^F < 0$, some firms would exit. In addition, it must also be the case that $\pi^I = 0$

[9] Equation (10) implies that the goods market clears since the right-hand side simply equals $Y - G - \mathrm{A}N\upsilon X - \mathrm{A}n^I\lambda$ or net GDP.

in equilibrium. Again, there should be no incentive for informals to either enter or exit the underground economy. Furthermore, with $\pi^I = 0$, no firm has an incentive to switch between sectors. Thus, equilibrium in the informal sector implies that the following condition must simultaneously hold,

$$\chi(N) = \lambda. \tag{13}$$

Letting $N(\lambda)$ denote the solution to (13), it can be shown that the equilibrium number of firms in both sectors varies with respect to λ as follows,[10]

$$n^F = \begin{cases} \lambda/\eta & \text{when } \lambda < \lambda^* \\ \lambda^*/\eta & \text{when } \lambda \geq \lambda^* \end{cases}, \text{ while } n^I = \begin{cases} N(\lambda) - \lambda/\eta & \text{when } \lambda < \lambda^* \\ 0 & \text{when } \lambda \geq \lambda^* \end{cases}, \tag{14}$$

where λ^* represents a threshold cost above which, under free entry, no informal finds it profitable to operate. Figure 1 helps follow the intuition underlying the sizes of the two sectors in (14).

When the cost of informality is relatively low so that $\lambda < \lambda^*$, the size of both sectors is simply found by solving (12) and (13) simultaneously while recalling that $n^F + n^I = N$. This immediately yields $n^F = \lambda/\eta$ and $n^I = N(\lambda) - \lambda/\eta$. However, as the cost of operating without enforceable property rights rises, two factors come into play. First, an increasing number of informals prefer to switch to the formal economy. The size of the formal sector, therefore, naturally rises with λ. Second, since both formal and informal firms interact in the same market, an increase in the cost of informality means that the combined size of the two sectors must fall. That is, since $\chi(N)$ falls monotonically with N, equation (13) implies that $N(\lambda)$ falls monotonically with λ. These two factors directly imply that the number of informals declines as doing business in the underground economy becomes increasingly difficult. Eventually, a threshold λ^* is attained at which point the informal sector vanishes and the size of the formal sector stabilizes at λ^*/η.

Having characterized how the number of firms in either sector endogenously responds to variations in the cost of informality, equation (11) implies that the equilibrium growth rate is given by

$$\gamma = \begin{cases} \left((1-\alpha)[N(\lambda)X(N(\lambda))]^\alpha - \rho\right)/\sigma & \text{when } \lambda < \lambda^* \\ \left((1-\alpha)\left[(\lambda^*/\eta)X(\lambda^*/\eta)\right]^\alpha - \rho\right)/\sigma & \text{when } \lambda \geq \lambda^* \end{cases}, \tag{15}$$

where $X(\lambda^*/\eta)$ solves (8) when $N = \lambda^*/\eta$ and similarly for $X(N(\lambda))$. Hence, the rate of growth declines as the cost associated with the lack of legal protection rises. This follows from the fact that $N(\lambda)$ is decreasing in λ while, as indicated earlier, the return to technology

[10] The construction of this equilibrium is shown appendix A.

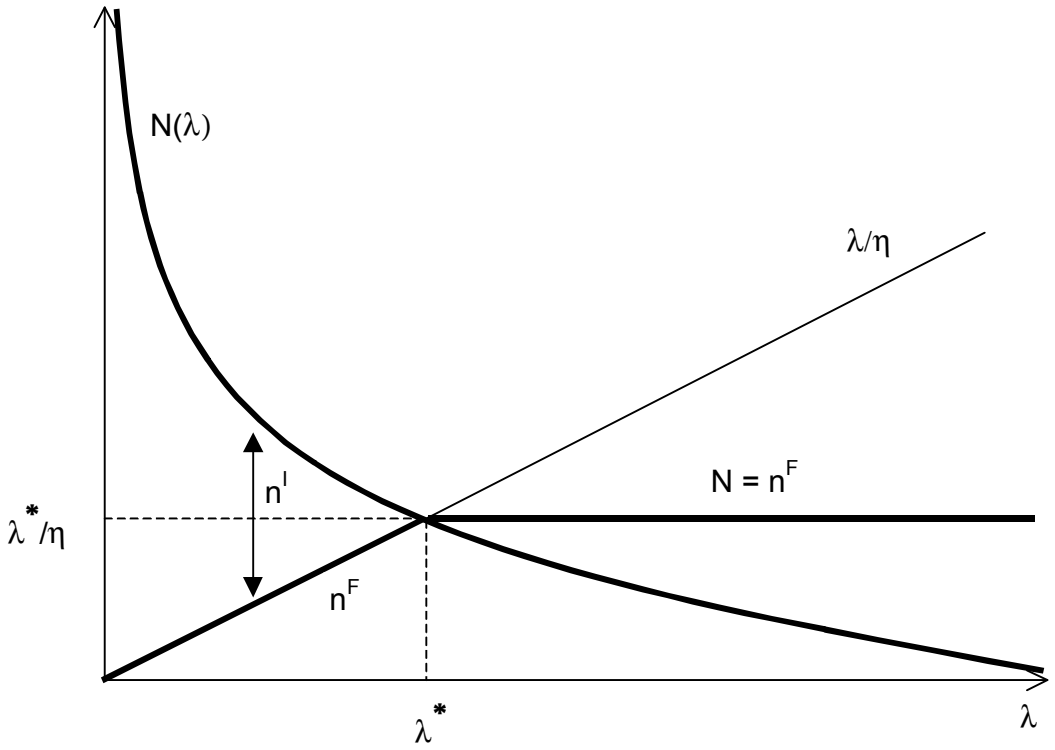

Figure 1

adoption, $(1-\alpha)\left[NX(N)\right]^{\alpha}$, increases with N. These results directly lead to the following observation:

Result 2: *When $\lambda \geq \lambda^*$, free entry in the formal economy endogenously rules out the existence of an informal sector. In that case, the rate of growth simply reflects formal economic activity. However, when the cost of operating informally falls relatively low and $\lambda < \lambda^*$, an informal economy emerges and, by increasing the number of available intermediate inputs, actually raises the return to acquiring new technology.*

Therefore, with no particular restrictions placed on being formal, the presence of an underground economy is not necessarily inconsistent with a high rate of growth. If free entry truly exists in the formal sector, there is a sense in which informality can never be detrimental to economic growth. Informals either contribute to growth when the cost of informality is low enough, or simply disappear endogenously if this cost is sufficiently high. It is worth noting that since profits are zero for all firms under free entry, the representative agent's lifetime utility can simply be integrated out in (P1) to yield $\mathcal{U}^c = \frac{A_0^{1-\sigma}(\rho-\gamma(1-\sigma))^{-\sigma}}{(1-\sigma)}$. Therefore, since $\rho > \gamma(1-\sigma)$ must hold,[11] $\mathcal{U}^c$ increases with γ in this case.

Of course, the equilibrium described in this section is useful only as a benchmark scenario. In practise, large informal sectors tend to be associated with bureaucratic corruption, questionable payments, as well as substantial amounts of red tape. According to Bayley (1966), the situation in Delhi was such that "giving presents and gratuities to government officers was an indispensable courtesy and respectable, civilized way of carrying on business." In such circumstances, some firms may prefer to produce informally even when the absence of legal protection proves relatively costly; and it is the latter distortion that may well lead to lower growth. Therefore, we now turn our attention to the rate of growth that emerges when rent-seeking public officials control entry in the formal sector.

3 Long-run growth with rent-seeking officials and regulated entry

In the case of free entry in the formal sector, it was implicitly assumed that access to the formal economy could be obtained at zero cost and without any further complications. This situation, however, is far from that which is described by De Soto (1989) in regard to the formal sector in Peru. Thus, a typical industrial company can use "only \$23.00 of every \$100.00 of the surpluses generated and potentially convertible to profits, and they spend the

[11] This restriction ensures that lifetime utility is bounded. See Rebelo (1991).

remaining $76.00 on remaining legal. *Contrary to what one might assume, only $17.60 of this amount goes to pay taxes; $59.60 is spent on other legal costs.*"[12] As we shall now argue, De Soto's observations are consistent with the outcomes of a rent-seeking bureaucracy.

To investigate the effects of rent-seeking bureaucratic behavior on growth, suppose an alternative scenario in which, at the beginning of time, a finite number $S > 1$ of public officials is randomly drawn from the population of agents. Here, one is born either a consumer or a government official and cannot later switch types. For convenience, we suppose that each bureaucrat has jurisdiction over A/S industries.

To understand at least some of the motivations behind the remarkably high levels of red tape which De Soto (1989) associates with the formal sector, we now make four important assumptions concerning the environment.

First, suppose that a bureaucrat associated with a particular industry can regulate the size of the formal sector in that industry. To do so, he would issue licenses and create additional procedural requirements while charging firms accordingly. De Soto (1989) writes that to remain in the formal sector in Peru, "business people have to satisfy a number of regulations, ranging from processing an endless succession of documents in government offices to administering their staff inflexibly." On the one hand, this latter assumption may reflect a fair amount of decentralization in the regulation of different industries. This may be especially true of nations whose bureaucratic system is largely disorganized. On the other hand, it may simply reflect the illegal undertakings of corrupt officials. In either case, this assumption allows a rent-seeking official to charge each formal firm with an artificial cost per period, ψ, for the provision of unnecessary administrative services.

Second, we assume that officials can act without any real risk of detection or punishment. Shleifer and Vishny (1993) point out that where public pressure to stop corruption is relatively weak -and we believe this to be the case in most developing countries- this is in fact a fitting assumption.

Third, suppose, for simplicity, that bureaucrats consume only out of the revenues they acquire from charging firms with non-tax legal costs in their industry. This assumption follows an early observation by Myrdal (1968) who writes that in "nearly all Asian countries, there has always been a tradition of corruption. Public office meant perquisites. Officials were not well paid and had to make ends meet. The well-timed bribe -which was often almost a conventional fee- was the emollient which made the wheels of administration turn more efficiently."

Finally, as in Braun and Loayza (1993), we assume that revenues collected by officials in the form of procedural or administrative costs are not reinvested in this economy. This

[12] Emphasis added.

assumption captures the idea that bureaucratic graft typically finds its way to foreign bank accounts instead of being invested domestically.

Having assumed that public officials can freely create artificial administrative procedures, one might naturally expect such red tape to emerge as a way to capture the rents generated in the formal economy. Moreover, corrupt officials can also decide on how to maximize the size of these rents by controlling entry in the formal sector. Recall that, in each time period, a formal firm earns profits in the amount of π^F. Therefore, taking n^F as given, we can think of a first optimization stage faced by a typical rent-seeking bureaucrat as

$$\max_{\psi} \mathcal{U}^b = \int_0^{\infty} e^{-\rho t} \frac{[C^b]^{1-\sigma} - 1}{1-\sigma} dt \text{ with } \sigma \geq 0 \text{ and } \rho > 0, \tag{P2}$$

$$\text{subject to } C^b = \int_0^{\mathrm{A}/S} n^F \psi di, \; \psi \leq \pi^F, \text{ and } \mathrm{A}_0 > 0 \text{ given.}$$

Under the assumption that no danger of detection or punishment exists, a corrupt official will naturally set the amount of artificial service costs applied to each firm, ψ, to its upper bound, π^F.

A question which immediately follows is: what is the optimal size of the formal sector from the perspective of rent-seeking government officials? As a result of imperfect competition among intermediate goods producers, formal firms can potentially make positive profits as long as entry in the formal economy is limited. Thus, we may imagine a second optimization stage in which a corrupt official solves a new version of the problem in P2,

$$\max_{n^F} \mathcal{U}^b = \frac{\left[(\mathrm{A}_0 n^F \pi^F)/S\right]^{1-\sigma}}{(1-\sigma)\left(\rho - \gamma(1-\sigma)\right)}. \tag{P2'}$$

The equation in (P2') has been obtained by setting $\psi = \pi^F$ in problem (P2) and integrating out the resulting expression for the lifetime utility of a rent-seeking official. It is straightforward to establish that $\mathcal{U}^b$ is increasing in $n^F \pi^F$ for all values of the parameters. It follows that the maximization of $\mathcal{U}^b$ with respect to n^F is equivalent to the maximization of $n^F \pi^F$. In other words, a public official regulating a given industry will choose the number of licenses he issues so as to maximize the sum of formal firms' profits in that industry. Note that rent-seeking officials do not internalize the effects of their actions on γ. Rather, they take the rate of economic growth as given in solving their optimization problem.

In order to determine the size of the informal economy as well as that of the formal sector, we assume that there still is free entry in the informal economy. In practice, this is a

distinguishing feature of the underground economy relative to that of the formal sector. As in section 2, the no entry/exit condition for informals is

$$\chi(N) = \lambda. \tag{16}$$

Furthermore, as we have just indicated, bureaucrats choose the size of the formal sector so as to maximize the total value of formal firms' profits,

$$\max_{n^F} = n^F \left[\chi(N) - \eta n^F\right]. \tag{17}$$

Observe that officials still provide law enforcement services and, therefore, need to take into account their associated resource costs. Without these services, all firms would choose to operate informally and there would be nothing for public officials to gain. Given these equilibrium conditions, (16) determines the total number of active firms. As before, we let this solution be denoted by $N(\lambda)$. Appendix A then shows that the equilibrium sizes of both the formal sector and the underground economy can be characterized as functions of λ as follows,

$$n^F = \begin{cases} \lambda/2\eta & \text{when } \lambda < \overline{\lambda} \\ N(\lambda) & \text{when } \lambda \geq \overline{\lambda} \end{cases}, \text{ while } n^I = \begin{cases} N(\lambda) - \lambda/2\eta & \text{when } \lambda < \overline{\lambda} \\ 0 & \text{when } \lambda \geq \overline{\lambda} \end{cases}, \tag{18}$$

where $\overline{\lambda}$ represents a threshold cost above which, under bureaucratic rent-seeking, the informal sector ceases to exist. A direct comparison with our initial results in section 2 shows the following:

Result 3: *All else equal, the size of the informal economy is larger when the formal sector must contend with a rent-seeking bureaucracy than when it is determined by free entry.*

Figure 2 provides some intuition behind this result. First, suppose that $\lambda < \lambda^*$. Consistent with their rent-seeking behavior, officials find it optimal to restrict the size of the formal economy to $\lambda/2\eta$ in order to maximize the sum of formal firms' profits. These profits amount to $\lambda^2/4\eta$ per industry and can then be extracted with the use of red tape as described above. In Figure 2, the size of the informal sector is therefore given by the difference between the curve $N(\lambda)$ and the ray $\lambda/2\eta$, and exceeds that which emerges under free entry, $N(\lambda) - \lambda/\eta$. Moreover, recall from Figure 1 that when the cost of informality reached λ^*, informals no longer found it profitable to operate under free entry. In this case, however, an informal sector still exists when $\lambda \geq \lambda^*$. Indeed, by artificially restricting entry in the formal economy, a predatory bureaucracy makes it worthwhile for some firms to operate informally even when the cost of informality is relatively high. Eventually, the costs of doing business

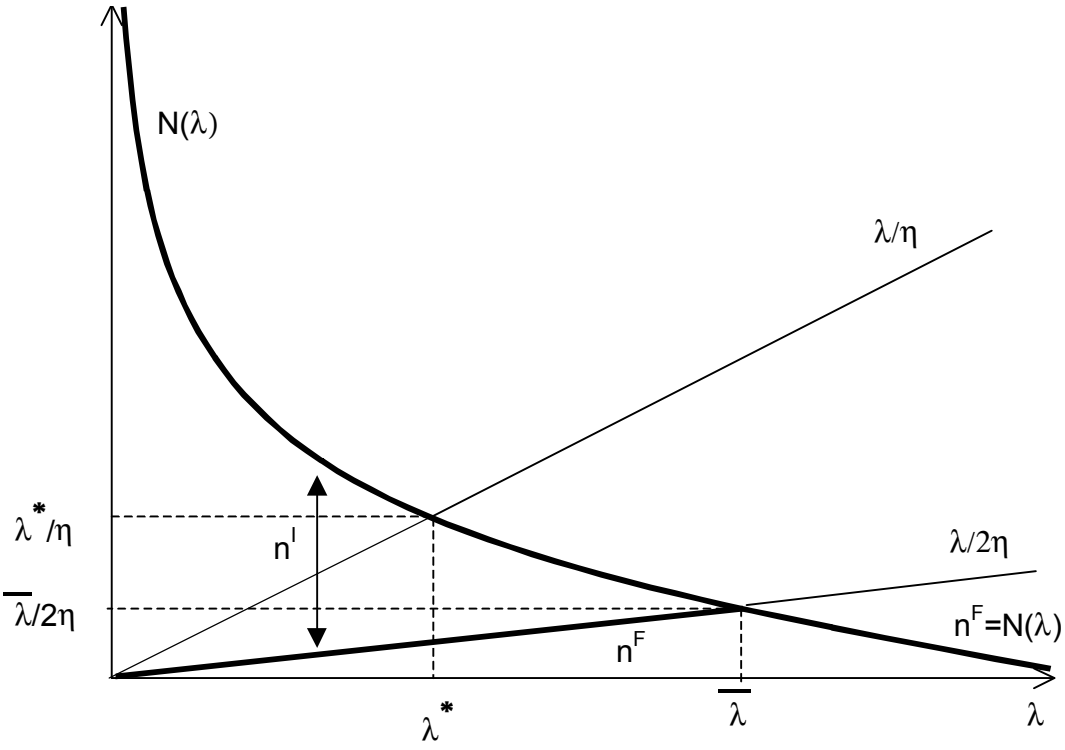

Figure 2

in the informal sector can reach a high enough threshold, denoted by $\overline{\lambda}$, that, rent-seeking officials notwithstanding, the underground economy endogenously vanishes. It should be clear that $\overline{\lambda} > \lambda^*$. Appendix A shows that when $\lambda \geq \overline{\lambda}$, corrupt officials find it optimal to set the size of the formal sector to $n^F = N(\lambda)$. These equilibrium results are in some ways reminiscent of the intuition presented in Shleifer and Vishny (1993). In particular, they are a good example of the allocative distortions that may result in an environment characterized by pervasive corruption. In contrast to Shleifer and Vishny (1993), however, we shall see that whether or not such bureaucratic rent-seeking is necessarily detrimental for growth still depends importantly on how well the informals manage in an extralegal environment.

To recover the rate of economic growth in this case, note that the problem facing the representative agent is essentially the same as that depicted in (P1), except for the fact that formal firms' profits now accrue to public officials. Thus, the return to acquiring new technology is still given by $(1-\alpha)\left[NX(N)\right]^{\alpha}$. Given the equilibrium sizes of the formal and underground economies in (18), the equilibrium growth rate in equation (15) is now,

$$\gamma = \frac{(1-\alpha)\left[N(\lambda)X(N(\lambda))\right]^{\alpha} - \rho}{\sigma}, \tag{19}$$

where $X(N(\lambda))$ solves (8) when $N = N(\lambda)$.

Figure 3 shows a comparison of the growth rates that emerge under the scenarios described in this section as well as section 2.That is, equations (15) and (19). In particular, the following result is worth stressing:

Result 4: *The rate of growth that arises when rent-seeking officials control entry to the formal economy is either the same or lower than when its size is determined by free entry.*

First, observe that when the costs of operating outside the formal economy are relatively high (i.e. $\lambda \geq \lambda^*$), bureaucratic corruption and red tape necessarily lead to lower growth relative to the free entry case in section 2. This result directly stems from the notion that, by artificially restricting entry in the formal economy, rent-seeking officials induce informals to operate even when informality proves relatively costly. This forces an overall reduction in the level of economic activity and leads to the production of fewer intermediate inputs. These mechanisms in turn lower the degree to which the economy can take advantage of newly adopted technology. Since, in this model, rent-seeking also corresponds to the creation of unnecessary procedures, this result provides a theoretical foundation for the various findings documented by Mauro (1995) on growth and bureaucratic quality. Specifically, by using a panel of 70 countries over the period 1971-1983, Mauro (1995) presents detailed empirical work which suggests that bureaucratic corruption and red tape are negatively correlated

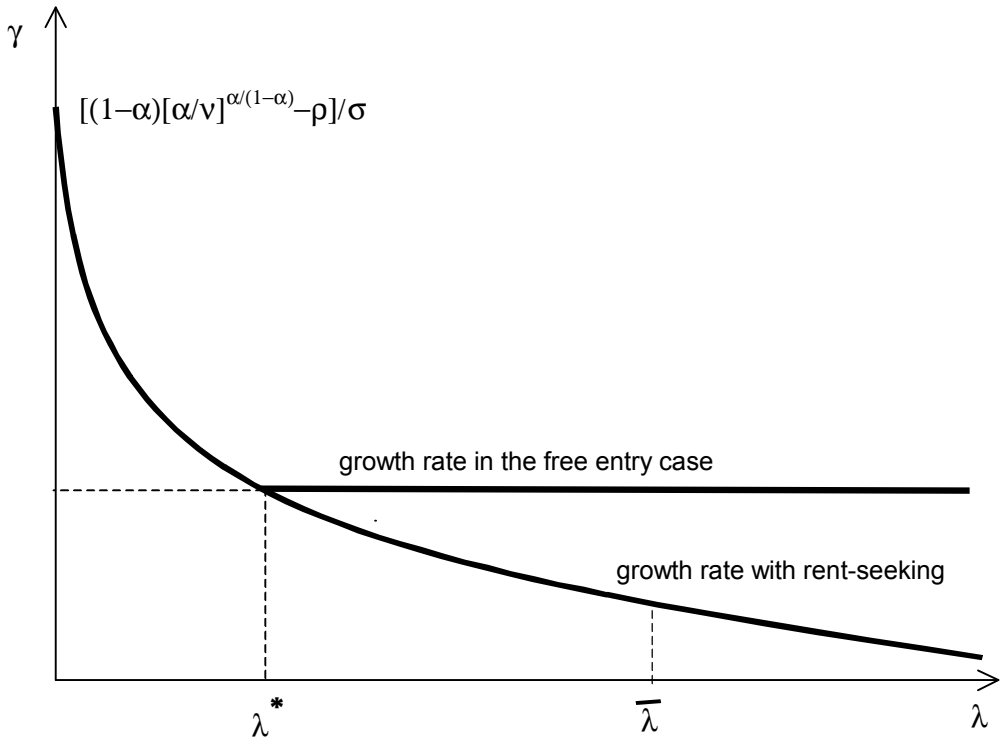
γ
[(1−α)[α/ν]^{α/(1−α)}−ρ]/σ
growth rate in the free entry case
growth rate with rent-seeking
λ*
λ̄
λ

Figure 3

with country growth performance. Although difficult to assess directly, one would therefore expect the cost of locating production in the informal sector to be relatively high for most countries.

Second, when the cost of informality falls below λ^*, a relatively high growth rate prevails irrespective of whether the formal economy is in the hands of rent-seeking officials. To understand the nature of this result, observe that when $\lambda < \lambda^*$, a large number of firms prefer to locate production in the informal sector even in the free-entry equilibrium. Hence, the act of limiting entry in the formal economy does not bind in this case. Recall from section 2 that, in this model, it is the *total* number of intermediate inputs produced that helps determine the return to acquiring new technology. It may be that the costs of carrying on business informally are relatively small for countries such as China, or even Italy. This would partially explain why these countries do not seem to suffer despite pervasive corruption.

Third, as we stressed earlier, bureaucratic rent-seeking always leads to larger informal economies relative to the free entry case. This finding supports the empirical evidence offered by Loayza (1996) on Latin American countries. However, a low equilibrium growth rate does not result from problems inherent to the size of the informal sector. Rather it is the outcome of a high cost of informality combined with the burden of a rent-seeking bureaucracy.

Finally, and since informal firms make zero profits in equilibrium, the representative agent's welfare is still given by $\mathcal{U}^c = \frac{A_0^{1-\sigma}(\rho-\gamma(1-\sigma))^{-\sigma}}{(1-\sigma)}$. As pointed out earlier, this expression is increasing in the growth rate. Therefore, when the informal sector operates in relatively difficult conditions, that is when the cost of informality is high, bureaucratic rent-seeking not only leads to lower growth but also lower welfare. In contrast, when the cost of informality is relatively low, corruption is irrelevant for both economic growth and agents' lifetime utility.

3.1 Rent-seeking, informality, and growth in related literature

The notion that corrupt bureaucracies play an important role in the process of economic development is not necessarily new. Leff (1964) for instance, argued that if bribes could be collected, corruption might actually enhance growth by raising the productivity of government officials. A few years later, Bayley (1966), and Nye (1967), attempted to systematically categorize channels through which political corruption could affect economic growth. This literature has recently seen a resurgence in interest, especially with regard to the question of occupational choice. Acemoglu and Verdier (1998), for example, show that corruption usually induces a persistent misallocation of talent in general equilibrium. This misallocation is then associated with a reduction in productive investments. In a similar vein, Murphy et al. (1993) stress that because of increasing returns to rent-seeking activities, corrupt practices

render productive activity less attractive and, consequently, lead to lower growth. In contrast, our model remains silent on the matter of occupational choice as agents cannot switch between activities. Nevertheless, rent-seeking still has a significant impact on growth since it affects the degree to which firms take advantage of technological adoptions. Our view of a corrupt bureaucracy, therefore, is complementary to that of Parente and Prescott (1994) who stress the importance of barriers to such adoptions. Of course, in this paper, results depend crucially on the relative cost conditions that characterize informal firms. In tying the actions of self-interested government officials to the environment in which they operate, this model has addressed two of the more pervasive characteristics of developing nations: corruption and the informal economy.

The fact that the informal economy constitutes a significant portion of economic activity in developing countries is well established. While many consider the informal sector as potentially harmful to development, others, including Montiel et al. (1993), have instead come to view the informal economy as a crucial determinant of growth. The latter authors suggest that the mere "size of these markets argues against the hypothesis that they are unproductive or undesirable." As a good example of just how important a role the underground economy can play, De Soto (1989) notes that in the case of the Peruvian transport industry, informal minibuses have come to be recognized by the state as "the main vehicles for mass transportation." We have shown in this paper that informal firms can indeed play a legitimate role in the process of economic growth.

4 Long-run growth and economic welfare in a second best world

Thus far, we have established that bureaucratic rent-seeking neither leads to higher growth nor higher utility relative to the case of free entry in the formal sector. This is true irrespective of the cost of informality. However, because the enforcement of property rights is subject to congestion externalities, it would be erroneous to conclude that restricting entry in the formal economy is always detrimental for economic welfare. A question which then arises is: can economic welfare ever be improved by restricting entry in the formal sector and allowing firms to keep the resulting profits? In answering this question, we may imagine a benevolent government whose aim is to maximize consumers' lifetime utility. This benevolent government would take the nature of firms' interactions as given and thus solve a second-best policy problem. This is in contrast to a much less realistic first-best planning problem in which the government could freely choose the size of the informal economy. Under these conditions,

the no entry/exit condition would hold for informal firms as in sections 2 and 3,

$$\chi(N) = \lambda. \tag{20}$$

In addition, a benevolent government would solve

$$\max_{n^F} \mathcal{U}^c = \frac{\left([\rho - \gamma(N)(1-\sigma) + n^F\pi^F]\mathrm{A}_0\right)^{1-\sigma}}{(1-\sigma)\left(\rho - \gamma(N)(1-\sigma)\right)}, \tag{P3}$$

The expression in (P3) is obtained by integrating out lifetime utility in (P1) while taking the (imperfectly) competitive equilibrium as given.[13] Recall using (11) that the growth rate, γ, is ultimately a function of the total number of intermediate inputs producers, N. Therefore, given an equilibrium in which both formal and informal firms exist, the solution to (P3) is simply found by maximizing $n^F\pi^F$.[14] Put another way, in solving its second-best policy problem, a benevolent government ends up having the same objective as that of corrupt bureaucrats. The difference, of course, is that the resulting rents that are generated in the formal sector now contribute to increasing consumers' initial consumption level. We already know what the equilibrium sizes of both sectors are in this case. In particular, they are given by (18). Note that the size of the formal sector implies that $2\eta n^F = \lambda$ for $\lambda < \overline{\lambda}$. A welfare-maximizing government thus chooses the size of the formal economy so as to equate the *marginal* cost of entry across sectors. In contrast, the size of the formal sector under free entry was determined by equating the *average* cost of entry across sectors, $\eta n^F = \lambda$ for $\lambda < \lambda^*$. As in section 3, the resulting rate of growth is

$$\gamma = \frac{(1-\alpha)[N(\lambda)X(N(\lambda))]^\alpha - \rho}{\sigma}. \tag{21}$$

However, since agents now receive formal firms' profits as dividends, lifetime utility is greater than that which obtains with bureaucratic rent-seeking. Specifically,

$$\mathcal{U}^c = \frac{\left([\rho - \gamma(1-\sigma) + \frac{\lambda^2}{4\eta}]\mathrm{A}_0\right)^{1-\sigma}}{(1-\sigma)\left(\rho - \gamma(1-\sigma)\right)} > \frac{\mathrm{A}_0^{1-\sigma}\left(\rho - \gamma(1-\sigma)\right)^{-\sigma}}{(1-\sigma)} \tag{22}$$

[13] Since a finite number of public officials is randomly drawn from a continuum of agents, the welfare of rent-seeking bureaucrats does not enter in (P3). In other words, in looking at the welfare of the economy as a whole, we discount the corrupt few who, in this model, are randomly put in a position of control.

[14] The expression in (P3) is strictly increasing in $n^F\pi^F$.

The rents $\frac{\lambda^2}{4\eta} > 0$ in equation (22) represent the maximized value of formal firms' profits that previously accrued to rent-seeking officials. In this case, however, these profits capture the efficiency gain that results from internalizing the congestion inherent to the provision of a public force and courts of law. Although profits never enter the determination of the growth rate, it certainly matters for welfare who ultimately turns out to be the beneficiary of those rents.

In section 3, we saw that when the cost of informality was sufficiently high, that is when $\lambda \geq \overline{\lambda}$, no informal firm found it worthwhile to operate. In that case, a complication arises in (P3) since all firms are formal and $n^F = N$. To solve for the second-best equilibrium with no informal firms, we can think of the problem faced by a benevolent government as

$$\max_N \mathcal{U}^c = \frac{\left([\rho - \gamma(N)(1-\sigma) + N(\chi(N) - \eta N)]\, \mathrm{A}_0\right)^{1-\sigma}}{(1-\sigma)\left(\rho - \gamma(N)(1-\sigma)\right)}, \tag{P3'}$$

$$\text{subject to } \chi(N) \leq \lambda,$$

where the constraint ensures that informals never find it profitable to re-enter. Considering the functional forms for $\gamma(N)$ and $\chi(N)$, an analytical solution to (P3') is difficult to obtain. However, a numerical formulation of (P3') can easily be carried out. The results from this simulation are shown in Figures 4a) through 4d).[15]

The second-best welfare solution is always greater than the welfare solutions that emerge in sections 2 and 3. Moreover, long-run growth is the same as that which arises under bureaucratic rent-seeking. The reason is that, in deciding on the size of the formal sector, a well-meaning government faces the same incentives as rent-seeking officials. Both adopt the same decision rule and generate rents in the formal economy in the same fashion. In one case, the efficiency gain accrues to all consumers whereas, in the other case, it accrues to a small number of rent-seeking officials (i.e. with measure zero in our framework). The analysis of this second-best solution, therefore, is especially helpful in pointing out the dangers of equating growth and welfare.

[15] The parameter values used in this particular example are as follows: $\alpha = .30$, $\rho = .02$, $\sigma = 4$, $\eta = 7.5 \times 10^{-5}$, $\nu = 20$. The range of values for λ was chosen so as to generate a total number of firms between 1 and 100 in each industry. For these parameter values, the constraint in (P3') always binds. Therefore, and for all values of λ, the second-best solutions for the sizes of both the formal sector and the underground economy mimic those which emerge under a rent-seeking bureaucracy.

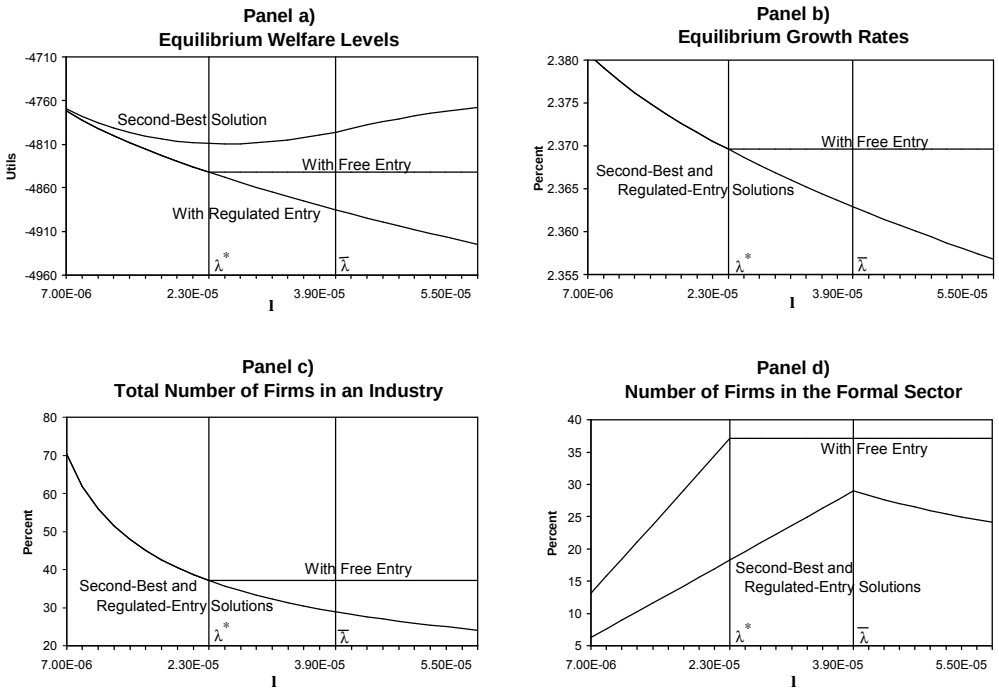

Figure 4

5 Conclusions and directions for future research

The objective in this paper has been to draw a link between bureaucratic rent-seeking, the informal economy, and economic growth. In addressing the issues first set out in the introduction, our framework provides the following observations.

First, large informal sectors, in and of themselves, are not necessarily detrimental to economic growth. With imperfect competition among firms, the return to technology adoption increases with the number of firms that are able to take advantage of newly acquired technology. These firms may well include informal firms. Therefore, in principle, higher growth is not inconsistent with having large informal economies. When the costs associated with operating outside of official laws and regulations are too high, the informal sector simply vanishes endogenously.

Second, in a world with little monitoring over bureaucracies, rent-seeking officials may find it advantageous to create artificial bureaucratic procedures as a way to appropriate firms' profits. In such an environment, it is optimal for these officials to restrict entry in the formal economy in order to maximize the rents that are extracted as red tape. This finding helps explain why large informal sectors are typically associated with both high levels of red

tape and inefficient bureaucracies.

Finally, we find that whether or not bureaucratic rent-seeking is associated with lower growth depends crucially on the costs associated with the absence of property rights protection. On the one hand, when these costs are relatively low, a large informal sector exists even in the free-entry equilibrium. Therefore, the effects of restricting entry in the formal sector do not bind in this case. On the other hand, when the costs resulting from the lack of legal protection are relatively high, bureaucratic rent-seeking necessarily leads to lower growth. By restricting entry in the formal sector, rent-seeking behavior allows the informal economy to operate when, under free entry, it would otherwise cease to exist.

Given the results in this paper, a natural question is: what factors ultimately determine the cost of not having access to complete and enforceable property rights? In this paper, lack of such access directly resulted in profit losses due to acts of vandalism and theft that were exogenous from the standpoint of an informal firm. However, the costs related to the absence of property rights may vary substantially across countries. De Soto (1989) writes that "in some adverse situations, a spirit of cooperation emerges that enables community leaders to demand of their members an altruism which permits surprisingly high levels of cooperation." Nevertheless, since there always remains "conflicts of interest which cannot be resolved by informal coordination," such collective organizations can never fully compensate for their extralegality. Hence, understanding the true endogenous nature of informal organization will be an important next step in exploring the role of the informal economy in the process of economic development.

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Appendix A.

A.1. Proof that the net sales margin, $\chi(N) = [p(N) - v]\, X(N)$, is monotonically decreasing in N.

To see this, we use the first order condition (7) and the fact that all firms produce the same output level to re-write the net sales margin as

$$\chi(N) = \alpha(1-\alpha)N^{\alpha-2}(X(N))^{\alpha}.$$

Using the solution for $X(N)$ in (8) then yields

$$\chi(N) = \alpha(1-\alpha)\left(\frac{\alpha}{v}\right)^{\frac{\alpha}{1-\alpha}}\left[N^{\frac{2(\alpha-1)}{\alpha}} - (1-\alpha)N^{\frac{\alpha-2}{\alpha}}\right]^{\frac{\alpha}{1-\alpha}}.$$

It follows, after some manipulations, that

$$\frac{\partial \chi}{\partial N} = -\alpha(1-\alpha)\left(\frac{\alpha}{v}\right)^{\frac{\alpha}{1-\alpha}}\left[N^{\frac{2(\alpha-1)}{\alpha}} - (1-\alpha)N^{\frac{\alpha-2}{\alpha}}\right]^{\frac{2\alpha-1}{1-\alpha}} N^{\frac{-2}{\alpha}}\left[2N + \alpha - 2\right].$$

Observe that $[2N + \alpha - 2] > 0$ since $N \geq 1$ and $\alpha < 1$. In addition, $N \geq 1$ also implies that

$$\left[N^{\frac{2(\alpha-1)}{\alpha}} - (1-\alpha)N^{\frac{\alpha-2}{\alpha}}\right]^{\frac{2\alpha-1}{1-\alpha}} = N^{\frac{\alpha-2}{\alpha}}\left[N - (1-\alpha)\right] > 0.$$

Therefore, $\frac{\partial \chi}{\partial N} < 0$.

Since $\chi(N)$ can also be expressed as

$$\chi(N) = \alpha(1-\alpha)\left(\frac{\alpha}{v}\right)^{\frac{\alpha}{1-\alpha}}\frac{1}{N^2}\left[1 - \frac{1-\alpha}{N}\right]^{\frac{\alpha}{1-\alpha}},$$

we also have that $\chi(1) = \alpha(1-\alpha)\left(\frac{\alpha^2}{v}\right)^{\frac{\alpha}{1-\alpha}} > 0$ while $\lim_{N\to\infty}\chi(N) = 0$.

A.2. Proof that the return to technology adoption, $(1-\alpha)\,[NX(N)]^{\alpha}$*, increases monotonically with* N *and asymptotes to a constant.*

Note that $X(N)$ in (8) can also be written as

$$X(N) = \left(\frac{\alpha}{v}\right)^{\frac{1}{1-\alpha}}\frac{1}{N}\left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{1-\alpha}}.$$

Therefore, $NX(N) = \left(\frac{\alpha}{v}\right)^{\frac{1}{1-\alpha}}\left[1 - \frac{1-\alpha}{N}\right]^{\frac{1}{1-\alpha}}$ and increases with N since $\alpha < 1$. In addition, it immediately follows that $\lim_{N\to\infty} NX(N) = \left(\frac{\alpha}{v}\right)^{\frac{1}{1-\alpha}}$.

A.3. Construction of the equilibrium sizes of the formal and informal sectors with free entry.

Since $N(\lambda)$ solves (13), it is straightforward to establish that $N(\lambda)$ is decreasing in λ while $\lim_{\lambda \to 0} N(\lambda) = \infty$ by the properties of $\chi(N)$ above. Now, consider an equilibrium in which both sectors exist. Then, by solving (12) and (13) simultaneously, it immediately follows that

$$n^F = \frac{\lambda}{\eta}$$

while

$$n^I = N(\lambda) - \frac{\lambda}{\eta},\ n^I \geq 0.$$

Given that $N(\lambda)$ falls with λ while $\frac{\lambda}{\eta}$ increases with λ, the size of the informal sector declines as the cost of informality rises, and eventually vanishes. Thus, define λ^* such that when $\lambda = \lambda^*$,

$$N(\lambda) = N(\lambda^*) = \frac{\lambda^*}{\eta}.$$

Then, when $\lambda = \lambda^*$, $n^I = \frac{\lambda^*}{\eta} - \frac{\lambda^*}{\eta} = 0$. Next, consider the situation where $\lambda \geq \lambda^*$. In this case, the cost of informality is too high for informals to carry on business. The number of formal firms must therefore solve the no entry/exit condition (12) with $n^F = N$, or

$$\chi(N) = \eta N. \tag{A.1}$$

We now show that the solution to (A.1) is given by $n^F = N = \frac{\lambda^*}{\eta}$. To see this, observe that, by definition of $N(\lambda)$ and λ^*, when $N = \frac{\lambda^*}{\eta} = N(\lambda^*)$, $\chi(N) = \lambda^*$. That is, the left-hand side of (A.1) is equal to λ^*. Moreover, when $N = \frac{\lambda^*}{\eta}$, ηN, or alternatively the right-hand side of (A.1), is also equal to λ^*. Hence, $N = \frac{\lambda^*}{\eta}$ solves (A.1) and the equilibrium sizes of both sectors can be described as in (14).

A.4. Construction of the equilibrium sizes of the formal and informal sectors with rent-seeking officials in the formal economy.

As indicated in the text, an equilibrium where both formals and informals operate must simultaneously satisfy

$$\chi(N) = \lambda \tag{A.2}$$

and

$$\max_{n^F} \; n^F \left[\chi(N) - \eta n^F\right]. \tag{A.3}$$

As before, $N(\lambda)$ solves (A.2). Moreover, we can use (A.2) to substitute for $\chi(N)$ in (A.3) and, consequently, obtain the first order condition,

$$\lambda - 2\eta n^F = 0.$$

It follows that $n^F = \frac{\lambda}{2\eta}$ while $n^I = N(\lambda) - \frac{\lambda}{2\eta}$, $n^I \geq 0$. As in the construction of the previous equilibrium, the size of the informal economy falls as λ rises and eventually ceases to exist. Hence, we can define $\overline{\lambda}$ such that when $\lambda = \overline{\lambda}$,

$$N(\lambda) = N(\overline{\lambda}) = \frac{\overline{\lambda}}{2\eta}.$$

It follows that $n^I = 0$ when $\lambda = \overline{\lambda}$. In another words, when the cost of having no access to legal protection is high enough, no informal finds it profitable to operate, even when the size of the formal economy is artificially reduced by a rent-seeking bureaucracy. It is obvious that $\overline{\lambda} > \lambda^*$. Moreover, when $\lambda \geq \overline{\lambda}$, all firms operate in the formal economy and $n^F = N$. Therefore, in determining an equilibrium with no informals, we can think of the problem faced by rent-seeking officials as

$$\max_{N} N\left[\chi(N) - \eta N\right] \tag{A.4}$$

$$\text{subject to } \chi(N) \leq \lambda.$$

The additional constraint in the above problem ensures that in determining the size of the formal economy, corrupt officials never choose a value of N which might induce informals to re-enter. That is, since $\chi(N)$ is decreasing in N, the cost of informality places a lower bound on the extent to which officials can reduce the size of the formal sector in an effort to drive up formal firms' profits.

It is straightforward to show that

$$N\chi(N) = \alpha(1-\alpha)\left(\frac{\alpha}{\upsilon}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{N}\left[1 - \frac{1-\alpha}{N}\right]^{\frac{\alpha}{1-\alpha}},$$

which decreases monotonically with N for $N \geq 1$. Hence, the difference $N\chi(N) - \eta N^2$ also falls with N. This implies that in solving (A.4), rent-seeking officials will try to reduce the size of the formal sector as far as they can. However, in so doing, they run up against the possibility of inducing informal firms to enter. That is, given λ and for N low enough, $\chi(N) > \lambda$. Hence, when $\lambda \geq \overline{\lambda}$, the constraint binds and corrupt bureaucrats set the size of the formal sector so as to satisfy $\chi(N) = \lambda$.