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Incentives, Communication, and Payment Instruments

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Abstract

Alternative payment instruments are studied in an economy with private information, delayed communication, and limited commitment. Attention is restricted to checks and bank drafts, which differ in resource cost and communication characteristics. Checks are less costly but settlement delays create a limited commitment constraint. We find that drafts dominate at low wealths and checks at higher wealths. Applications to 19th century and modern payment systems are discussed.

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1. Introduction

A central function of non-cash payment instruments is to communicate information about an economic exchange between the time and place at which the exchange takes place to the institutions or people responsible for settling any claims created by the transaction.[1] This communication can run in either direction. A guaranteed payment instrument like a bank draft or a money order conveys information that funds have been allotted for the desired purchase. In contrast, a personal check contains no such guarantee. It only enables the seller to communicate the buyer's obligation to the buyer's bank. The delay inherent in this communication presents the seller (or the seller's bank) with credit risk that would not exist with a guaranteed payment.

The communication characteristics of payment instruments have been important in the past and despite continuing improvement in communication technology, they are still important today.[2] Consider the following two examples:

1. In the U.S. during much of the 19th century, interregional payments were made with bank drafts, often drawn on New York. A merchant would go to his bank, purchase a draft, often at a discount, from the bank for payment to be made in New York money. Upon receipt of this draft, the seller, or jobber as wholesalers were called then, would deposit the draft in his bank which would collect the funds from the bank's New York correspondent. Over the latter half of the 19th century, bank drafts were replaced with personal checks for interregional payments.[3] Checks were more flexible, saving the buyer a trip to the bank, as well as providing a receipt of payment. Checks, however, required the jobber's bank to collect funds from the merchant's bank.

2. In 1998, according to the Federal Reserve's Survey of Consumer Finances, 13.2% of U.S. households did not own checking accounts (Kennickell, et al (2000)). Account ownership is highly correlated with income. For example, Hogarth and O'Brien (1997) estimate that in 1995, 43.6% of households with less than $10,000 in annual income did not own checking accounts. These households make payments with cash and money

[1] This is not to say that cash does not have a communication role. Indeed, Townsend (1989) and, more recently, Kocherlakota (1998) have emphasized the communication properties of currency.

[2] Indeed, the resources used by the banking sector to supply payment services are not trivial. Radecki (1997) estimates that for the 25 largest bank holding companies, payment services account for 40% of operating revenues.

[3] Preston (1920) and testimony to the House Committee on Banking and Currency (1920) suggest that this transition occurred in the 1890's. A New York Clearing House Report (1873) contains evidence, however, that this transition had already started by the early 1870's.

orders, which can be purchased at such locations as post offices, check-cashing outlets, grocery stores, and even convenience stores.[4]

In both examples, there are multiple means of payment. The transition from bank drafts to checks for interregional payments occurs over time, during a period of technological progress. In present times, means of payment differs cross-sectionally with household's income. The examples suggest that the characteristics of payment instruments vary in importance with features in the economic environment. In this paper, we examine how these characteristics may depend on the environment. In particular, we focus on the importance of wealth and the information structure for determining means of payment.

While we consider other types of payment instruments, our main focus is on the cost and benefits of checks as a widely used means of payment. In both historical discussions and in contemporary observations about the resilience of checks in the face of rapidly changing technology for electronic payments, one often sees references to the convenience and flexibility of using checks. The checking account allows one to issue widely accepted claims on demand, without having to stop to verify balances or receive authorization. Our model focuses directly on this benefit of checks. The main cost of checks comes from the same characteristics that drive their benefits. A checking account gives an individual an opportunity to consume in excess of available balances. For sellers of goods to be willing to accept checks, the checking account contract must address this potential incentive problem. While other forms of payment allow the payor to pre-commit to a limited amount of spending, the delayed settlement of checks makes such pre-commitment difficult.

Clearly, the length of the clearing lag can be crucial to an account-holder's incentive to abuse checking privileges. Reductions in clearing times can make it possible to more easily control depositors' incentives. Hence, our model could provide insight into why it has been seen as important to spend resources in order to reduce check float.[5]

Our model shares some crucial elements with existing work in banking and payments theory. The basic preference structure follows Diamond and Dybvig (1983). As in that paper and the literature that follows, an important role of a depository intermediary in our model is to provide a form of insurance against preference shocks. However, we also consider a spatial dimension, as in Townsend (1987, 1989) and, more recently, Lacker (1997a). For our purposes, spatial separation provides a natural justification for delays in communication. During these delays, check writers may visit multiple sellers and make excessive purchases if they choose. These multiple visits are similar to the idea of side trading which has been

[4] For more information on the "unbanked" see Prescott and Tatar (1999).

[5] Check float has recently been studied by Lacker (1997b) and McAndrews and Roberds (1999a).

studied in moral hazard models without exclusive contracting.[6] Unlike this literature, in our model side trades are observable ex post and can then be punished.

The incentive problem arising from delayed settlement of checks is also related to some of the issues that arise in the comparison of net and real-time gross settlement of interbank payments, as examined by Kahn and Roberds (1998) and Kahn, McAndrews and Roberds (1999). Pre-paid instruments, like bank drafts, are analogous to gross settlement, as they eliminate credit risk but impose resource costs on participants. Checks, with delayed settlement, expose both sellers and the depositor's bank to the possibility that the depositor will choose to default. One key difference, however, is that in interbank settlement arrangements, information on obligations can flow continuously, independent of the settlement interval. In our environment, delayed settlement of checks is dictated by the flow of information.

2. Model

The model builds on the Diamond-Dybvig environment. There are three periods and two types of agents, buyers and sellers. There is a continuum of each type of measure one. At time $t = 0$, each buyer is endowed with w units of a storable good, and buyers reside at location one. At time $t = 1$, buyers travel to location two to meet sellers. At time $t = 2$, buyers return home. Sellers are always located at location two.

Preferences and endowments

Buyers' preferences are defined over state-contingent consumption in period one at location two, and consumption at home in period two. In period one, each agent experiences a shock θ, an element of the finite set Θ. The probability of shock θ is $p(\theta)$, and realizations are independent across agents. State-contingent consumption in period one is identified by a composite good $x(\theta)$, while second period consumption is identified by $y(\theta)$. Buyers preferences are $\theta u(x) + v(y)$, with $u(x)$ and $v(y)$ concave and increasing, and with $v(0) = 0$. The shock θ measures the intensity of preference for the period one good and is like the liquidity shock in the Diamond-Dybvig model. This shock is experienced after the buyers arrive at location two.

The composite good $x(\theta)$ can be either the amount purchased from a single seller or a function of the amounts purchased from up to N sellers. We assume that there are a variety of buyer types with regard to their ability and desire to visit multiple sellers, and that an individual's characteristics along this dimension are privately observed at the same time that they learn their θ values. There are some buyers who are able to visit up to N sellers. Let J denote the set of N sellers visited by such a buyer, and let x_j denote the buyer's purchases

[6] For example, see Bizer and DeMarzo (1992) and Kahn and Mookherjee (1998).

from seller j. For some of these buyers, $x = \sum_{j \in J} x_j$, while for others $x = N \min_{j \in J} (x_j)$. Hence, the latter part of the buyer population has a taste for variety, while the former cares only about their total consumption. Finally, some of the buyers who care only about total consumption are able to visit only a single seller. This heterogeneity means that, if buyers are limited in the number of purchases they can make (say, for instance, by being given a fixed number of tokens), there will be some buyers (those with the Leontief-like preferences) who will be badly disadvantaged. At the same time, if a payment arrangement could be devised to accommodate the buyers with a taste for variety (say by limiting the purchases from any individual seller to $1/N$ of an intended amount of x), then those with limited ability to make multiple visits would suffer. We limit our attention to allocations that are symmetric along this dimension of buyers' preferences.[7]

Sellers can create one unit of the good x with one unit of effort. They receive linear utility from consumption of y and linear disutility from effort. We can leave out effort and write their preferences as $y - x$. Sellers price competitively and will not accept any risk of non-payment. That is, they will produce a unit of x in period 1 only if payment of a unit of y in period 2 is certain.

Technology

There is a constant returns to scale storage technology at location one, which converts each unit of the period zero endowment into r units of the period two consumption good, y. The period two consumption good can be costlessly transported from location one to location two in order to compensate sellers.

The bank

The buyers may form into a coalition which we call a bank. This bank holds buyers' deposits and it honors any payment obligations entered into by its buyers, up to a buyer's specified entitlement. In particular, it will ship y to sellers who present evidence that a buyer made a payment promise. Furthermore, the bank can pre-commit to a fixed payment by issuing a non-falsifiable document, which we call a bank draft. Finally, after making payments to sellers the bank distributes the remaining y to buyers according to the multilateral contract agreed to by the coalition of buyers.

Information

Buyers' preference shocks θ are private information. Sellers do not know buyers' shopping history. While wealth w is public information among buyers, sellers may or may not know buyers' wealths. We examine both possibilities in our analysis.

[7] In principle, it would be possible to devise allocations that separate buyers along this dimension. While this would complicate the characterization of the optimal allocations, the key effect of limited commitment in the use of checks would be qualitatively the same.

Communication

Buyers can create two types of written documents. The first type is a check, which is written at the point of sale. It is a promise to pay a specified amount upon presentment to the bank. In our environment, this means that a seller who receives a check with a specified amount in return for producing some x may present it to the bank and receive the corresponding amount of y, up to the buyer's available balances.

The second type of written document is a bank draft. It is also a promise to pay a specified amount upon presentment to the bank. Unlike a check, a bank draft is not written at the point of sale but instead it is written before the buyer leaves location one. The bank guarantees payment for the specified amount. We assume that it costs d units of the endowment good to issue a draft. We have in mind the time costs of the buyer going to the bank and the bank writing out the bank draft. Since a draft must be drawn for a pre-set amount, a buyer will need to carry multiple drafts in order to achieve consumption that is contingent on the preference shock. This increases the costs of using bank drafts.

The use of drafts in the model, does not conform exactly to the way in which bank drafts were used in the 19^{th} century. Typically, a merchant (corresponding to a buyer in our model) would determine the merchandise desired and contract with a jobber (seller). He would then go to his bank and purchase a draft. Depending on the relationship between the buyer and the seller, goods might be transferred before or after the presentation of the draft to the seller. The main cost of using drafts, then, was the inconvenience (and perhaps delayed exchange of goods) of making a special trip to the bank before the finalization of the transaction between the buyer and the seller. Such an extra trip (in the middle of period 1) is not feasible in the context of our model, but the main idea, that it is costly to achieve fully contingent allocations with guaranteed payment instruments, carries over to our specification.

There is a one-period lag in presentment of checks and drafts by sellers to the bank. In the case of a check, this means there is a one-period lag before the seller discovers whether the buyer has sufficient funds to pay the check. In contrast, the draft guarantees the seller that payment will be made. Accordingly, a check potentially exposes the seller to credit risk, while a bank draft does not.[8]

Commitment

Buyers cannot commit to creating personal payment obligations that are limited to their

[8] In our model, the bank never fails in equilibrium so a draft will always be paid. If there was a chance of bank failure, a draft would still be a better guarantee of payment than a check, because with a check, payment depends on an individual's and the bank's balances, while with a draft it depends only on the bank's balances.

own entitlement. A buyer may want to "overdraw" his account by writing checks for large amounts. The bank's ability to punish a buyer for "binging" on the first period good x is limited to giving the buyer zero consumption of the second period good y. If checks were not accepted and all payments were made with bank drafts then there would be no commitment problem, since bank drafts are pre-funded for specified amounts.

Allocations

An allocation for buyers is a function $(x(\theta, w), y(\theta, w))$. We ignore the allocation for sellers because we assume competitive pricing so sellers always receive zero utility or profits. We assume that sellers accept checks only for amounts that will be repaid with certainty. A seller's decision on whether to accept checks for any given value will depend on common knowledge about the buyers' environment.

The set of feasible allocations depends on the set of available instruments. Buyers enter into a multilateral contract that specifies an allocation $(x(\theta, w), y(\theta, w))$, which maximizes expected utility on the feasible set. We first write out the problem for an economy in which drafts are used. As suggested above, this problem corresponds to one with full commitment with regard to the honoring of obligations presented by buyers for purchase of goods. Since wealth is common knowledge within the set of buyers, one can approach this allocation problem as a set of separate problems, one for each wealth class. We then write out the problem where checks are the only available means of payment. In this case, we can treat different wealth levels independently only if w is fully public information, known by sellers as well as buyers. We make comparisons between alternative payment instruments (checks and drafts) for both the public wealth and the private-wealth case, and we consider economies in which both instruments are available.

2.1. Pure bank draft economy

As discussed above, if bank drafts are the only available payment instrument, then buyers can commit to restraints on their consumption of x. They will be able to purchase no more than the amount they bring with them in drafts. An optimal allocation among buyers of a given wealth level, however, will typically involve some sharing of preference shock risks. Hence, allocations must respect the need for inducing truth-telling with respect to the preference shock θ. Under full information, an optimal allocation would set $y(\theta, w)$ equal to a constant $y(w)$ for all θ. Under private information about θ, constant y for different θ's cannot be incentive compatible. Therefore, risk sharing is limited by private information in this environment, even under full commitment. Treating the wealth level in $(x(\theta, w), y(\theta, w))$ as a parameter rather than an argument, the programming problem for wealth class w is

Program 1

$$\max_{x(\theta,w)\geq 0,y(\theta,w)\geq 0} \sum_{\theta} p(\theta)[\theta u(x(\theta,w)) + v(y(\theta,w))]$$

subject to the resource constraint (given competitive pricing by sellers)

$$\sum_{\theta} p(\theta)[x(\theta,w) + y(\theta,w)] - rw + D \leq 0, \tag{2.1}$$

and the truth-telling constraints

$$\theta u(x(\theta,w)) + v(y(\theta,w)) \geq \theta u(x(\theta',w)) + v(y(\theta',w)), \forall \theta, \theta'. \tag{2.2}$$

Optimal allocations in this economy for a wealth level w are identical to those of the full-commitment, private-information problem for wealth level $w - D/r$. The quantity D in represents the total resource cost of using drafts. That is, D is equal to the cost per draft, d, times the number of drafts a buyer must carry in order to achieve his desired consumption. Since a buyer may have to visit N stores, he will need enough drafts to make purchases at each store. Also, since the amount of a draft is pre-set, a buyer will need to bring many drafts for each potential visit, corresponding to the different purchases desired by buyers with different θ's. For instance, if $\Theta = \{\theta^l, \theta^h\}$, the buyer will need two drafts per visit, one with value $x(\theta^l, w)/N$ and the other with value $[x(\theta^h, w) - x(\theta^l, w)]/N$. The buyer with preference shock θ^l will use the first draft, and a buyer with θ^h will use both. If there are M possible values for θ, then to achieve a separating (according to θ) allocation, a buyer needs to take out MN drafts, with a total resource cost of $D = MNd$. The key feature of this specification of the cost of drafts is that drafts become more costly the more variable are buyers' preferences and desired purchases.[9]

2.2. Pure check economy

If buyers are allowed to write checks, then the allocations they receive must respect their ability to create excess obligations by overdrawing their accounts. Guarding against this incentive requires adding a limited commitment constraint to the optimization program. We call this constraint the "binge" constraint. Treating wealth as a parameter, as in Program 1, the program for a given wealth level when wealth is public information is

Program 2

$$\max_{x(\theta,w)\geq 0,y(\theta,w)\geq 0} \sum_{\theta} p(\theta)[\theta u(x(\theta,w)) + v(y(\theta,w))]$$

[9] We do not study the possibility of a buyer taking out fewer than MN drafts and exposing himself to more preference uncertainty, although for large enough M, a buyer would do this.

subject to the resource constraint

$$\sum_{\theta} p(\theta)[x(\theta, w) + y(\theta, w)] - rw \leq 0, \tag{2.3}$$

the truth-telling constraints

$$\theta u(x(\theta, w)) + v(y(\theta, w)) \geq \theta u(x(\theta', w)) + v(y(\theta', w)), \forall \theta, \theta', \tag{2.4}$$

and the binge constraints

$$\theta u(x(\theta, w)) + v(y(\theta, w)) \geq \theta u(\bar{b}(w)), \forall \theta, \tag{2.5}$$

where

$$\bar{b}(w) = N \max_{\theta} x(\theta, w).$$

The binge constraint stops a buyer from overdrawing his account. If a buyer is going to over-consume, he will seek to consume the maximum possible amount. This is achieved by purchasing, at each of N shops, the maximum amount that any buyer would ever purchase from a single shop. Hence, the intended consumption bundle (x, y) for a buyer of a particular type (θ, w) must give utility at least as great as the utility from the bundle $(\bar{b}, 0)$. The fact that $\bar{b}$ represents the maximum consumption for a binger implies that sellers' decisions about the maximum acceptable amount of a check must be consistent with the contracted allocation set by the buyers. That is, sellers independently choose which checks to accept, based on their knowledge of the arrangement agreed to by buyers. In effect, we have, as an equilibrium requirement, that the largest check that a seller will accept is equal to $\bar{b}/N$. This follows from our assumption that a seller will not accept a check for an amount that would give a buyer an incentive to binge.

In the case when wealth is private information (that is, not known to sellers), Program 2 would be written somewhat differently. First, w in $(x(\theta, w), y(\theta, w))$ would no longer be treated as a parameter but instead as an argument. Second, the objective function would in principle be a weighted sum of the expected utilities of the different wealth types. Third, the resource constraints would be combined across wealth types. However, for most of our discussion, we will be able to treat the problem as separate problems solved by each wealth class with its own resource constraint. Fourth, in the private-wealth case the programs for different wealth levels are interdependent through the binge constraint. Specifically the binge constraint for any wealth level and any θ is now $\theta u(x(\theta, w)) + v(y(\theta, w)) \geq \theta u(\bar{b})$,where $\bar{b} = N \max_{\theta, w} x(\theta, w)$.

Finally, the presence of the binge constraint means that we do not need truth-telling on buyers with respect to their wealth, so those constraints can be left out of the private-wealth version of Program 2. If someone lies about their wealth they consume $x(\theta', w')$ immediately but the bank finds out about this deception in the next period and can punish the buyer by giving them zero units of y. But since $x(\theta, w) < \hat{x}$ for all θ, w, if the binge constraint holds then constraints that prevent lying about wealth hold as well.

In most of what follows we simply treat each wealth level as solving a separate problem. With private wealths (that is, unknown by sellers) the problems may be related through the binge constraint. However, if we assume that there can be no resource transfers between wealth types then the binge constraint often (although not always) affects all buyers in the same direction. When this is true, we will be able to show that the allocation for the lowest wealth type is identical to their allocation when wealth is public. The interaction between wealths can then be summarized by a constraint that this particular wealth level's allocation places on the problem solved by other (higher) wealth classes. There are cases however, in which the existence of multiple wealth levels, levels unobserved by sellers, results in a Pareto frontier of allocations. The selection of a single allocation would then depend on welfare weights placed on different wealth-types. If the no-transfer assumption is dropped then, again, it is less reasonable to treat the problems for different wealth types as largely separable.

3. Analysis

We start our analysis with the public-wealth case. If wealth is public, that is known by the sellers as well as the buyers, then a seller's willingness to accept checks can be contingent on the buyer's wealth. This allows the allocations of buyers with different wealths to be unrelated and analyzed separately, just as Programs 1 and 2 are written. This means we can analyze the problem as a collection of separate public-wealth economies each with a single wealth level.

3.1. Public-wealth case

For simplicity, assume that there are two preference shocks, $\Theta = \{\theta^l, \theta^h\}$. It is helpful to define a full-commitment, private-information allocation, that is, a solution to Program 1 without the cost D in the resource constraint. We call such an allocation $(x^*(\theta, w), y^*(\theta, w))$. This allocation is characterized by $\theta^l u'(x^*(\theta^l, w)) = v'(y^*(\theta^l, w))$, $\theta^h u'(x^*(\theta^h, w)) > v'(y^*(\theta^h, w))$, the θ^l truth-telling constraint holds with equality, the resource constraint holds, and with some transfers from the θ^l state to the θ^h state, that is, $x^*(\theta^h, w) + y^*(\theta^h, w) > rw$, and $x^*(\theta^l, w) + y^*(\theta^l, w) < rw$. Figure 1 illustrates.

The binge constraint may become a problem when sellers are paid with a lag and do not know a buyer's previous purchasing history, that is, other sellers visited and amounts purchased. For example, if sellers freely gave credit to buyers for purchases up to $x^*(\theta^h, w)$, then buyers might have an incentive to "overdraw" on their balances at their home bank, in effect, binging on x.

The binge constraint requires the buyer to have enough y so that he has no incentive to overspend on x. Overspending on x entails buying N times the maximum $x(\theta, w)$ that a buyer of wealth w could purchase. This amount is $x(\theta^h, w)$. Hence, the allocation for the θ^h buyer must satisfy $v(y(\theta^h, w)) \geq \theta^h[u(Nx(\theta^h, w)) - u(x(\theta^h, w))]$. This turns out to be the only binge constraint that will ever bind as a solution to Program 2 for the public-wealth case. More specifically, any allocation $(x(\theta, w), y(\theta, w))$ which satisfies the truth-telling constraints and the binge constraint for θ^h will satisfy the binge constraint for θ^l. Furthermore, this constraint will hold with strict inequality.

Lemma 1. Any allocation $(x(\theta, w), y(\theta, w))$ with $x(\theta^h, w) \geq x(\theta^l, w)$, which satisfies the truth-telling constraints (2.4) and the binge constraint (2.5) for θ^h satisfies with strict inequality the binge constraint (2.5) for θ^l.

Proof: By the θ^h binge constraint $\theta^h u(x(\theta^h, w)) + v(y(\theta^h, w)) \geq \theta^h u(Nx(\theta^h, w))$. Since the right-hand side falls more rapidly with a decline in θ than the left-hand side, this implies $\theta^l u(x(\theta^h, w)) + v(y(\theta^h, w)) > \theta^l u(Nx(\theta^h, w))$. But by truth-telling, $\theta^l u(x(\theta^l, w)) + v(y(\theta^l, w)) \geq \theta^l u(x(\theta^h, w)) + v(y(\theta^h, w))$ so $\theta^l u(x(\theta^l, w)) + v(y(\theta^l, w)) > \theta^l u(Nx(\theta^h, w))$.

What this lemma allows us to do is to restrict attention to the θ^h binge constraint, along with the θ^l truth-telling constraint.

In general, the shape of the θ^h binge constraint will depend on the utility specification. However, for several reasonable utility functions this constraint can be represented as an increasing concave function in (x, y) space. Consumption bundles that satisfy this constraint must lie on or above the graph of this function. Figure 2 illustrates; here, the binge constraint is simply represented by a curve which gives the set of (x, y) points satisfying $v(y) = \theta^h[u(Nx) - u(x)]$. Examples of utility functions that yield a concave constraint include $u(x) = x^\alpha, v(y) = y^\beta, 0 < \alpha < \beta \leq 1$, and $u(x) = \ln(x + 1), v(y)$ concave.[10]

In general, we can characterize allocations according to the wealth range. There are three ranges.

[10] In all of the figures, the income-expansion paths – the sets of bundles for which marginal rates of substitution between x and y are equal to one – are drawn as rays through the origin. This is simply done for illustrative purposes. For results below regarding optimal allocation across wealth levels, a sufficient condition is that the binge constraint cross the θ^h expansion path from above. This condition is satisfied by the examples cited.

1. **Highest wealth levels** - Binge constraint does not bind. Here, the allocation is the same as the full-commitment allocation $(x^*(\theta, w), y^*(\theta, w))$. Note that this is also the same as the allocation achieved with drafts for buyers who have wealth $w + D/r$. See Figure 3.

2. **Intermediate wealth levels** - Binge constraint binds for θ^h, incentive constraint binds for θ^l. The low type's allocation satisfies $\theta^l u'(x(\theta^l, w)) = v'(y(\theta^l, w))$. The high-type's expenditures will exceed rw while the low-type's expenditures will be less than rw. That is, $x(\theta^h, w) + y(\theta^h, w) > rw$ and $x(\theta^l, w) + y(\theta^l, w) < rw$. Also, the high-type's allocation will not be tangent to his "new" budget line, because he is on the boundary of his binge constraint, that is, $\theta^h u'(x(\theta^h, w)) > v'(y(\theta^h, w))$. Compared to the full commitment allocations, $x(\theta^h, w) < x^*(\theta^h, w)$, and $x(\theta^l, w) > x^*(\theta^l, w)$. See Figure 4.

3. **Lowest wealth levels** - Binge constraint binds for θ^h, both types receive the same allocation. The allocation satisfies $x + y = rw$, and $v(y) = \theta^h[u(Nx) - u(x)]$. See Figure 5.

For buyers with sufficiently high wealth, limited commitment has no effect on the allocation. Final period consumption of the y good serves as a type of collateral. If buyers can "set aside" enough y, then they will be deterred from binging, and high-wealth buyers have plenty of y to set aside. As w falls, so does the ability to set aside y and deter binging. When the binge constraint binds, $x(\theta^h, w)$ and $x(\theta^l, w)$ are pushed closer together than would be desired under full commitment. Eventually, they collapse to a single allocation. This happens for a wealth w_0 such that the binge constraint crosses the budget line at the point where the binge constraint meets the θ^l expansion path. As wealth continues to fall, θ^h and θ^l continue to have identical consumption, determined by the intersection of the binge constraint and the budget line.

3.2. Checks versus Drafts

Now suppose that buyers, as a group, choose whether to allow checking or to require individuals to make payments with bank drafts. As discussed above, draft allocations for buyers with wealth w are full-commitment allocations for wealth level $w - D/r$. For high-wealth cases, check allocations are also full-commitment allocations, but without a resource cost. The desirability of checks versus drafts depends crucially on wealth, as well as on the magnitude of D. At the highest wealths, checks are clearly preferred.

Once wealth is low enough so that the binge constraint binds, the limited commitment aspects of checks begin to introduce an expected utility cost. At any such wealth level, drafts could be preferred, for low enough D. Certainly, at or below w_0, as defined above, the expected utility cost of using checks is quite large. Let us assume that the resource costs of drafts are low enough so that the expected utility of using checks is less than the expected utility of using drafts at w_0. We then have the following, which can be stated without proof.

Proposition 2. Let $V^c(w)$ and $V^d(w)$ denote the value (expected utility) functions for using checks and drafts, respectively. Suppose $V^c(w_0) < V^d(w_0)$, and define $w_1 > w_0$ by $v(y^*(\theta^h, w_1)) = \theta^h[u(Nx^*(\theta^h, w_1)) - u(x^*(\theta^h, w_1))]$. Then, there exist $\hat{w}_0, \hat{w}_1$, with $w_0 < \hat{w}_0 \leq \hat{w}_1 < w_1$, such that $V^c(w) < V^d(w)$ for all $w \in [w_0, \hat{w}_0)$, and $V^c(w) > V^d(w)$ for all $w > \hat{w}_1$.

The wealth level w_1 is that at which the binge constraint is just satisfied for the (x^*, y^*) allocation. For all wealths above w_1, the binge constraint does not bind, and checks implement the corresponding (x^*, y^*) allocations. We expect, although we have not proved it, that there is a single crossing of $V^c(w)$ and $V^d(w)$ (that is, $\hat{w}_0 = \hat{w}_1$). Still, the interpretation of this analysis is that with public wealth levels, low-wealth buyers use drafts, while high-wealth buyers use checks.

3.3. Private Wealth Levels

The existence of buyers with varying initial endowments that are unknown to sellers complicates the use of checks. If sellers are unable to distinguish buyers according to their levels of wealth, then their willingness to accept checks may be limited. In the public-wealth case, the maximum amount of a check payment that a seller is willing to accept is increasing in the buyers' wealth. With multiple wealth levels, a high-wealth individual's ability to write a check for the purchase of x can be constrained by the low-wealth individual's binge constraint. Suppose, for instance, that a high-wealth buyer's allocation is not constrained by the binge constraint in the public-wealth case. If a low-wealth buyer's public-wealth allocation is constrained, and if sellers are willing to accept checks up to $x^*(\theta^h, w)$ for some higher wealth level, then the low-wealth, high-θ buyer will certainly overdraw. Hence, the existence of poorer buyers may keep wealthy buyers from attaining the unconstrained allocation through the use of checks. To avoid this limitation, wealthy buyers may have an incentive to subsidize poor buyers, so as to loosen the binge constraint. We first consider the allocations when no such wealth transfers are feasible.

3.4. No transfers between wealth classes

Suppose that there are multiple buyers' locations, each with a single wealth level. There is no possibility of communication or transfers among buyer locations. At some of these locations, buyers have an endowment of w^h, and at others, $w^l < w^h$. Sellers do not know a buyer's place of origin and, therefore, do not know a buyer's wealth. Let $(x^s(\theta, w), y^s(\theta, w))$ denote the optimal allocation in the public-wealth case with wealth w. Analogously, we let $(x^m(\theta, w), y^m(\theta, w))$ represent an optimal allocation for the private-wealth case.

When the low-wealth, θ^h binge constraint binds in the public-wealth case, the optimal allocation for the low-wealth type in the private-wealth case will be their public-wealth allocation, since they cannot do any better, as a group, without using more aggregate resources than they have available from their aggregate endowment.

Lemma 3. If $v(y^s(\theta^h, w^l)) = \theta^h[u(Nx^s(\theta^h, w^l)) - u(x^s(\theta^h, w^l))]$ then $(x^m(\theta, w^l), y^m(\theta, w^l)) = (x^s(\theta, w^l), y^s(\theta, w^l))$.

Proof: Since $(x^s(\theta, w^l), y^s(\theta, w^l))$ solves the low-wealth buyers' public-wealth expected utility maximization problem, allocations with $x(\theta^h, w^l) < x^s(\theta^h, w^l)$ are feasible, but cannot provide greater expected utility. Since the binge constraint binds, an allocation with $x(\theta^h, w^l) > x^s(\theta^h, w^l)$ would also need to have $y(\theta^h, w^l) > y^s(\theta^h, w^l)$, so as not to violate the binge constraint. This, however, would violate the truth-telling constraint, which could not be restored without giving the θ^l type greater lifetime consumption, violating the resource constraint.

In the private-wealth case, the existence of low-wealth buyers puts potentially severe constraints on the allocation of high-wealth buyers. In particular, sellers will not accept checks for more than $x^s(\theta^h, w^l)$; a low-wealth, high-θ buyer that is allowed to buy more than that amount will overdraw his balances in purchases of x, leaving sellers less than fully compensated. This gives rise to the following.

Proposition 4. If $v(y^s(\theta^h, w^l)) = \theta^h[u(Nx^s(\theta^h, w^l)) - u(x^s(\theta^h, w^l))]$ then $x^m(\theta, w^h) \leq x^m(\theta^h, w^l), \forall\theta$, and $x^m(\theta^h, w^l) = x^m(\theta^h, w^h)$.

Proof: For the first part of the Proposition, if $x^m(\theta, w^h) > x^m(\theta^h, w^l)$ for some θ then $v(y^s(\theta^h, w^l)) < \theta^h[u(Nx^m(\theta^h, w^h)) - u(x^s(\theta^h, w^l))]$ which is a contradiction. From the first part, we know that $x^m(\theta^h, w^h) \leq x^m(\theta^h, w^l)$. If $x^m(\theta^h, w^h) < x^m(\theta^h, w^l)$, then $\theta^h u'(x(\theta^h, w^h)) > v'(y(\theta^h, w^h))$ which means $x(\theta^h, w^h)$ can be increased keeping $x(\theta^h, w^h) + y(\theta^h, w^h)$ equal to a constant, while increasing expected utility for w^h without violating truth-telling for (θ^l, w^h) buyers.

The effect of the binge constraint here is striking. The high-wealth buyer's consumption of x is much smaller than in the public-wealth case. One should note that the binge constraint continues to bind only for the (θ^h, w^l) buyers. As long as $x(\theta, w) \leq x(\theta^h, w^l)$ for all (θ, w), then $\hat{x} = x(\theta^h, w^l)$ where $\hat{x}$ is as defined in the specification of the binge constraint in Program 2. If an allocation seeks to set $x(\theta, w) > x(\theta^h, w^l)$ for any (θ, w), then the (θ^h, w^l) buyer cannot be deterred from binging without violating either the truth-telling constraint for the (θ^l, w^l) buyer or the aggregate resource constraint for the w^l buyers. Hence, at an optimal allocation in the private-wealth case $\hat{x} = x(\theta^h, w^l)$, and it will clearly be the case that the binge constraint will not bind for any type other than (θ^h, w^l). Figure 6 illustrates the private-wealth solution when the binge constraint binds for the (θ^h, w^l) buyers in the public-wealth case.

If w^h is much greater than w^l, then the result in the preceding proposition implies a severe limitation on the high-wealth consumption. In the extreme, this limitation could drive high-wealth agents to use costly drafts while the poor continue to use checks. To illustrate, consider the case in which w^l is so low that both the θ^h and the θ^l buyers consume the same (x, y). Then the check allocation for high-wealth buyers is so constrained that $(x^m(w^h), y^m(w^h))$ is independent of θ. Consequently, subject to the resource cost of drafts, drafts will dominate checks.

Remark 1. Two important parameters for this result are the distribution of wealth and the magnitude of the lowest wealth level. In particular, the lowest wealth has to be low enough that the cost of drafts is large relative to wealth. If not, then both types may prefer drafts.

If the binge constraint on the low-wealth, high-θ type does not bind, then there is a range of Pareto optimal allocations. The best of these for low-wealth buyers has $x^m(\theta^h, w^l) = x^s(\theta^h, w^l)$ and is described by the following.

Proposition 5. Suppose $x^m(\theta^h, w^l) = x^s(\theta^h, w^l)$ and $y^s(\theta^h, w^l) > \theta^h[u(Nx^s(\theta^h, w^l)) - u(x^s(\theta^h, w^l))]$. Let $(\tilde{x}, \tilde{y})$ satisfy $\theta^h u(\tilde{x}) + v(\tilde{y}) = \theta^h u(x^s(\theta^h, w^l)) + v(y^s(\theta^h, w^l))$ and $v(\tilde{y}) = \theta^h[u(N\tilde{x}) - u(\tilde{x})]$. Then $x^m(\theta, w) \leq \tilde{x}$, $\forall \theta, w$.

Proof: If $x^m(\theta, w) > \tilde{x}$ then the binge constraint for the low-wealth agent would not be satisfied.

The best one can do for low-wealth types is to give them their public-wealth allocation. However, other allocations are feasible that increase the utility of the high-wealth individuals. The best of these for the high-wealth buyers sets $x^m(\theta^h, w^h) = x^m(\theta^h, w^l) > \tilde{x}$, where $\tilde{x}$ is defined above. This allocation entails increasing the consumption of x by the low-wealth θ^h

buyer in order to loosen the constraint on the high-wealth's consumption of x. The precise allocation that is best for the high-wealth individuals is defined by: θ^l, w^l types being on their expansion path, that is, $\theta^l u'(x(\theta^l, w^l)) = v'(y(\theta^l, w^l))$; truth-telling binding for θ^l, w^l; and the binge constraint binding for θ^h, w^l. Figure 7 illustrates the end points of this set of allocations. In this graph, the allocation favored by w^l has $x^m(\theta^h, w^l) = x^s(\theta^h, w^l)$ and $x^m(\theta^h, w^h) = \bar{x}$. The allocation favored by w^h has $x^m(\theta^h, w^l) = x^m(\theta^h, w^h) = \bar{x}'$. In each case, the (θ^l, w^l) allocation lies at the intersection of the θ^l expansion path and the θ^l indifference curve through the (θ^h, w^l) allocation. Note that even when the binge constraint does not bind in the public-wealth case for the lowest wealth, it can still bind and have a dramatic effect on allocations in the private-wealth case.

3.4.1. Case with transfers between wealths

In the previous subsection, we restricted our analysis to the case where there were no resource transfers across wealth types. Because the binge constraint for low-wealth buyers strongly affects feasible allocations available to high-wealth buyers, high-wealth buyers may want to transfer resources to the low-wealth buyers in order to increase the amount of x that high-wealth buyers can consume. Such a transfer loosens the binge constraint for low-wealth buyers, allowing all wealth types to consume more x.

In addition to a direct ex ante wealth transfer, one can imagine other methods of transferring resources between wealth types. For example, some binging could be tolerated as long as sellers are adequately compensated. Compensation, here, would take the form of the bank covering bingers' overdrafts. It seems likely that an ex ante transfer is the preferred way to transfer resources between wealth classes. Consider two transfers that give the low-wealth buyers the same lifetime utility. One does this with an allocation that satisfies the binge constraint, while the other allows buyers to binge. The latter will typically involve a greater transfer of resources to the low-wealth buyers. We can establish such a result more formally for the special case of preferences that satisfy $\theta u(x) + v(y) = \theta \ln x + y$.

3.5. Hybrid payment systems

In a pure check system, low-wealth buyers can present themselves to sellers as high-wealth buyers. This potential for misrepresentation severely restricts the allocations of high-wealth individuals. Consequently, we would expect to see combinations of bank drafts and checks. For example, in the private-wealth case high-wealth buyers will choose bank drafts to avoid the consumption limits imposed by the binge constraint. Similarly, if the banking sector was able to coordinate who it offered accounts to, it might want to issue checking accounts to the wealthy and exclude the poor from the banking sector. This would limit some of the negative effects of the binge constraint, but would require that the bank have the ability to certify account holders in a way that can be communicated to sellers.

4. Discussion and Interpretation

Our results are characterized by a strong connection between wealth and the acceptability of checks. In the public-wealth case, we found that wealth level alone was a deciding factor while in the private-wealth case, we found that the distribution of wealth mattered as well. In particular, the existence of low-wealth individuals with the ability to write checks could severely limit the value of acceptable checks. In thinking about the observations mentioned in the introduction the above models are helpful.

4.1. 19th century payment instruments

Throughout much of the 19th century, checks were accepted locally but not for interregional trade. It was only in the late 19th century that checks replaced bank drafts as the primary means of interregional payments. We interpret the local acceptance of checks in terms of our public-wealth model. Local information was high enough in quality for sellers to have an idea about whether or not a buyer could settle a claim he issued.

We interpret the transition from bank drafts to checks in the late 19th century United States as a transition, for interregional exchange, from the private-wealth model to the public-wealth model. The latter half of the 19th century was a period of great technological advance. With the expansion of the railroads and the development of the telegraph, communication costs greatly declined. Furthermore, the quality of information improved through, for instance, the development of credit reporting services like those offered by Dun and Bradstreet. We interpret these advances as providing distant wholesalers better information about potential buyers' wealths, making them more willing to accept checks, even from merchants that they might not have previously had business relationships with. In this sense, the model presented above should be viewed as one of exchange outside of relationships. During a period of growing interregional trade, one would expect a growing amount of such trade.

4.2. Paper solutions

Despite the technological advances of the late 19th century there were still substantial communication and informational obstacles to making payments and payment instruments were also devised that attempted to deal with these obstacles. One such instrument was a traveller's letter of credit. Letters of credit were designed to help travelers obtain funds at distant locations, particularly from correspondents of a traveler's home bank. The letter of credit was a letter from officers of the traveler's home bank stating the traveler had been extended credit up to some amount and the correspondent should advance funds if requested by the traveler. The letters contained flexibility in case a traveler did not want all of his funds at once. Attached to the letter would be a list that each bank which forwarded money to the traveller would make an entry in, indicating the advanced amount. Subsequent banks would see this list and know how much money was left available to the traveller. As banks advanced the funds they would send drafts to the home bank for the advanced funds. Once the line of credit was exhausted, the final advancing bank would then keep the letter of credit and its attached list and then send both to the home bank for payment. Hence, a letter of credit solves the commitment problem by both setting a maximum amount for purchases and allowing sellers to communicate with each other regarding a buyer's history of transactions. Dewey and Shugrue (1922) contains a description of this payment instrument.

4.2.1. The Cheque Bank

In late 19th century England, the checking system was highly developed but its use was mainly restricted to the upper classes and merchants. The Cheque Bank, extensively described in Jevons (1897), was a highly interesting experiment in extending the use of checks to the lower and middle classes. This bank developed a version of the check designed to communicate to sellers the amount of funds the buyer had available to him. Upon receipt of a deposit, the bank would issue checks with limits on the amount each could be written for. The limits on the issued checks summed to the amount of the deposit and the limit on each check was conveyed by perforating this number on the check. Unlike a bank draft, however, the Cheque Bank checks could be written for amounts less than their limit. This allowed payments to be made for irregular amounts, particularly through the mail, where currency or bank notes were unsafe. The Cheque Bank kept track of two balances, the amount of the deposit and the amount of credit extended by the issued but uncleared checks.

Cheque Bank checks combined the flexibility of a check with the certification of a bank draft. Despite these advantages, however, the Cheque Bank ultimately failed in the late 1890's. Not much information is known about this bank's operations and failure. But, according to liquidation proceedings reported in the Banker's Magazine (1901), the bank failed because of forgery problems and increased competition for less wealthy depositors from the rest of the banking sector.[11]

4.3. The unbanked

Presently in the U.S., ownership of a checking account is highly correlated with income, which in our model is equivalent to wealth. We interpret this observation in terms of our public-wealth model. In practice, wealths are not literally observed by sellers but there are close substitutes. For example, databases exist that allow a retailer to find out if a check writer has a history of writing bad checks. Some retail establishments are catered to by different income groups.

As in the data, there is a positive correlation between income and account ownership in our model. For payments in which cash is not practical, low-wealth individuals use money orders (bank drafts in the model) because they cannot commit to not binge on a credit extension.

In the environment presented above, the punishment for writing bad checks is limited to the loss of final period consumption. Additional punishments would relax the binge constraint, allowing for even better allocations. In many U.S. states it is illegal to willfully overdraw an account.[12] In practice, small overdrafts are not forwarded to legal authorities for prosecution, but larger overdrafts, binging in our model, may be, and if large enough, can be considered fraud. This suggests modifying the binge constraint so that buyers could only consume a fixed amount over their allowed allocation. In this version of the model, the wealth effects would be even stronger.

This discussion suggests that there are other ways to model the binge constraint. In addition to changing the penalties, the amount of binging could be modeled as a declining function of the length of time it takes to send a check to the buyer's bank. This modeling strategy would allow us to compare the costs and benefits of efforts to reduce check float time.

5. Conclusion

In this paper, we have emphasized the communication and commitment characteristics of alternative payment instruments. We think these are important characteristics for understanding the nature of payment services provided by the banking system. Our focus has

[11] There is some indication that other banks began offering similar perforated checks around the time that the Cheque Bank closed. We do not know how long this practice persisted.

[12] In some states like Virginia, insufficient funds in one's account is considered evidence of intent.

been on how the characteristics of payment instruments affect the deposit account relationship and the consumption patterns that can be supported. The analysis suggests some conditions under which checks can be expected to perform well compared to other instruments, in spite of their limited commitment problems. These conditions essentially require that a buyer have something to lose if he takes advantage of the limited commitment.

In the model presented above, what a misbehaving buyer has to lose is balances that remain at the bank. One might imagine a dynamic version of this model in which punishment comes not from the denial of final period consumption (as above) but from the denial of rights to future interaction with the bank. In this regard, our model is related to some recent work that has examined the effects of periodically updated public information in a random matching environment.[13] Like our model, this body of work considers the effect of a (perhaps random) delay in the updating of public information about an agent's history. Most of this literature has focused on either the general characterization of optimal allocations or on the comparison of privately issued and fiat monies. The random-matching nature of such environments tends to preclude the type of payment instrument considered here, namely, the issue of an obligation from one particular agent to another particular agent. Furthermore, the focus on private money emphasizes the incentives of the bank but not the incentives of banks' customers. By placing the delayed communication feature into a standard banking model, we hope to have developed a framework well suited to study the characteristics of payment instruments used by bank depositors.

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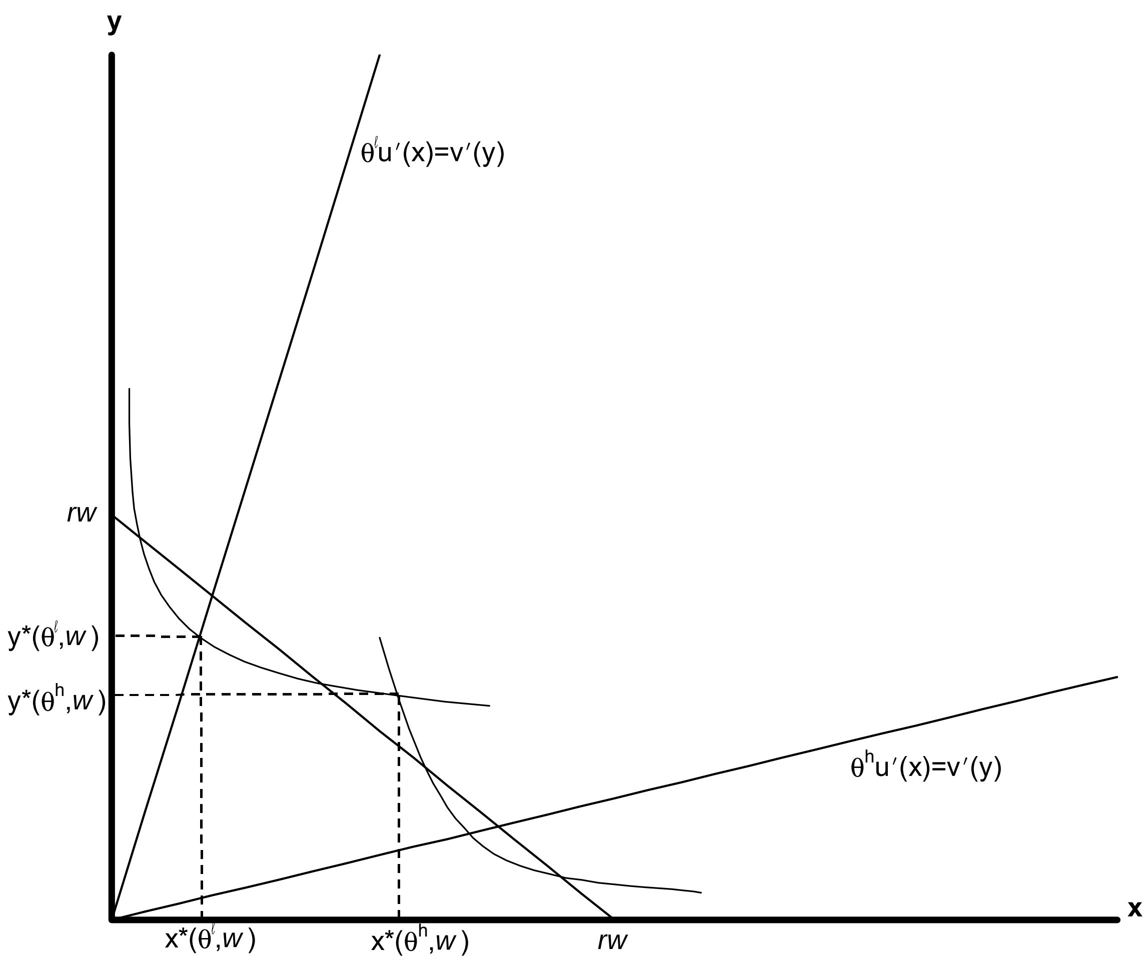

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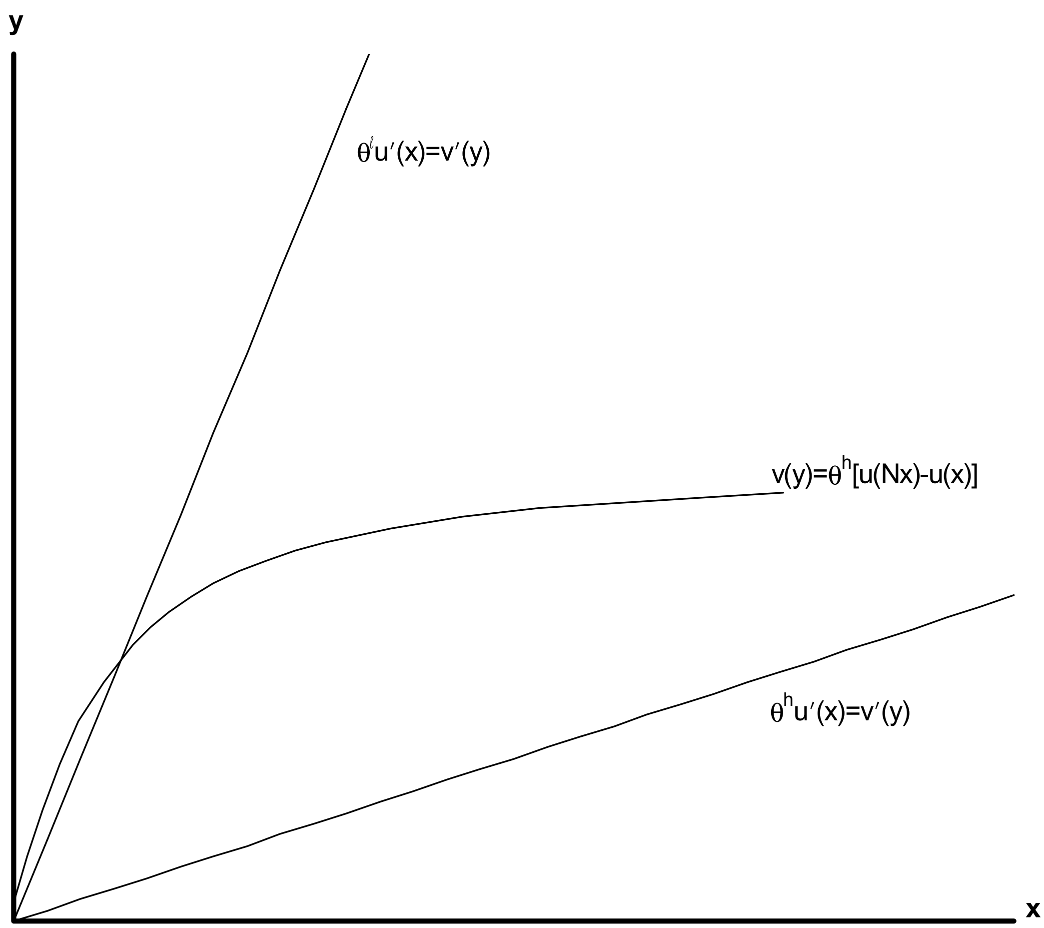

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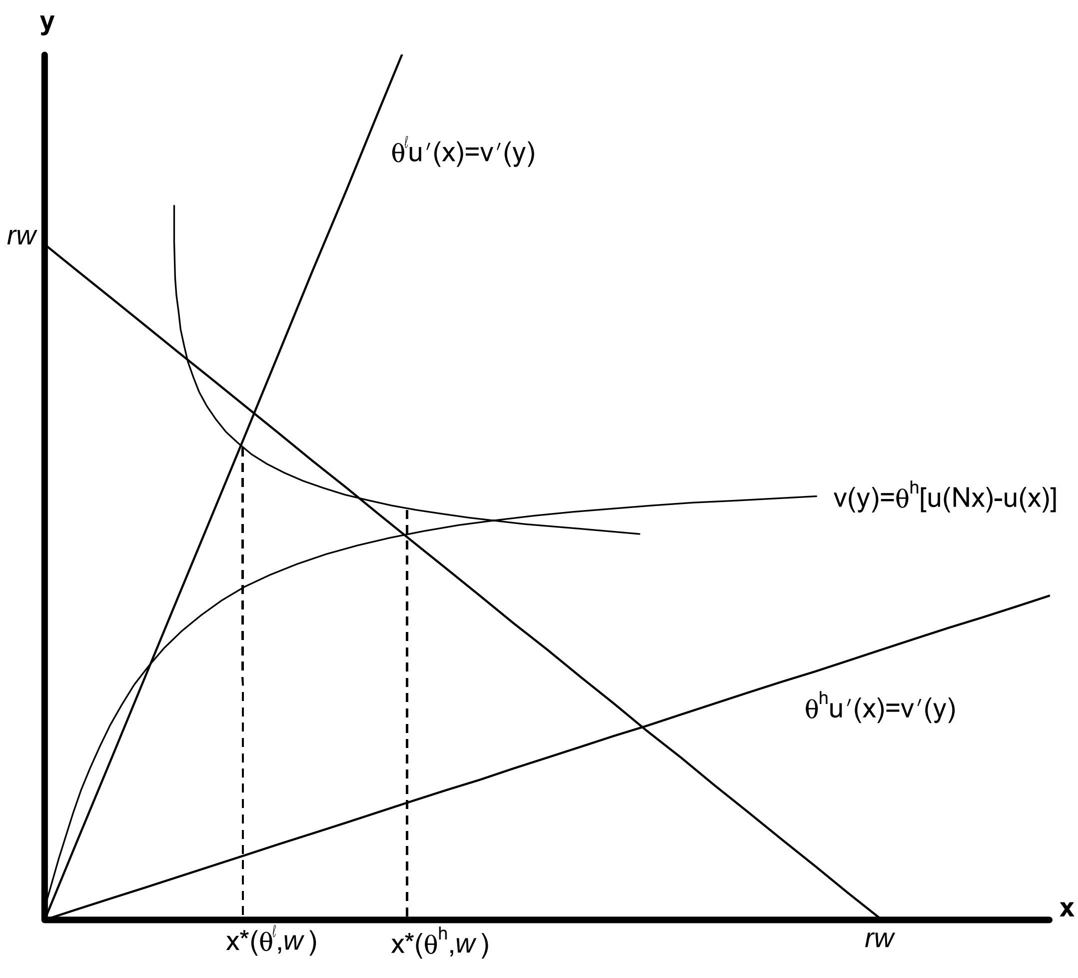

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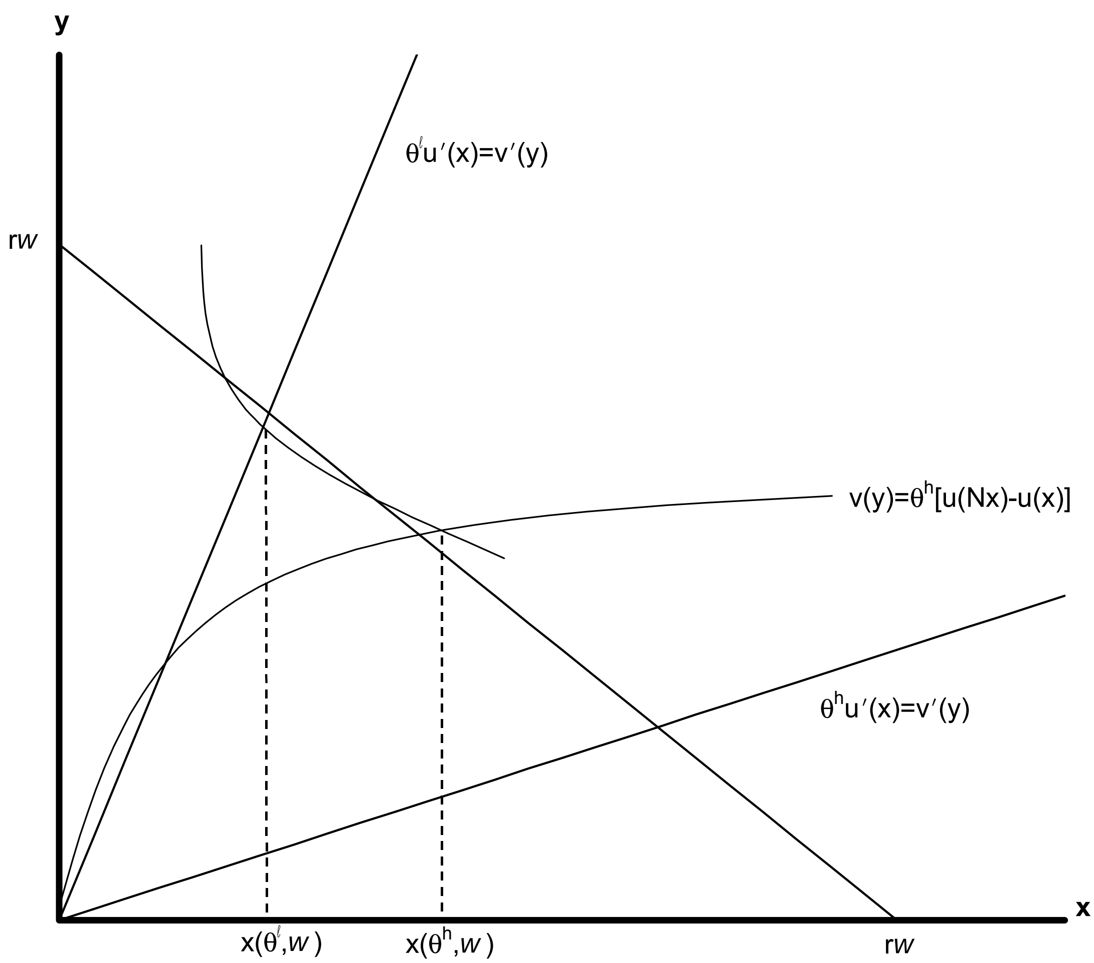
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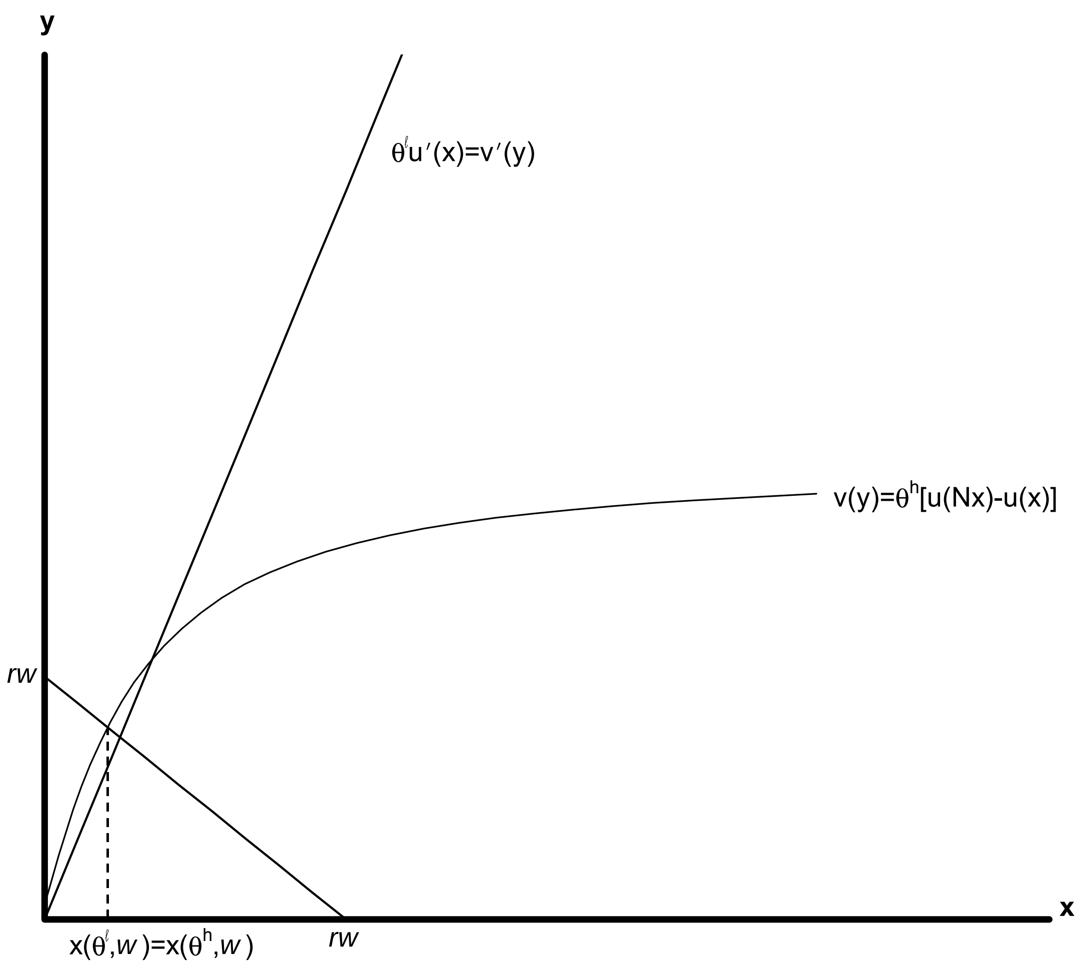

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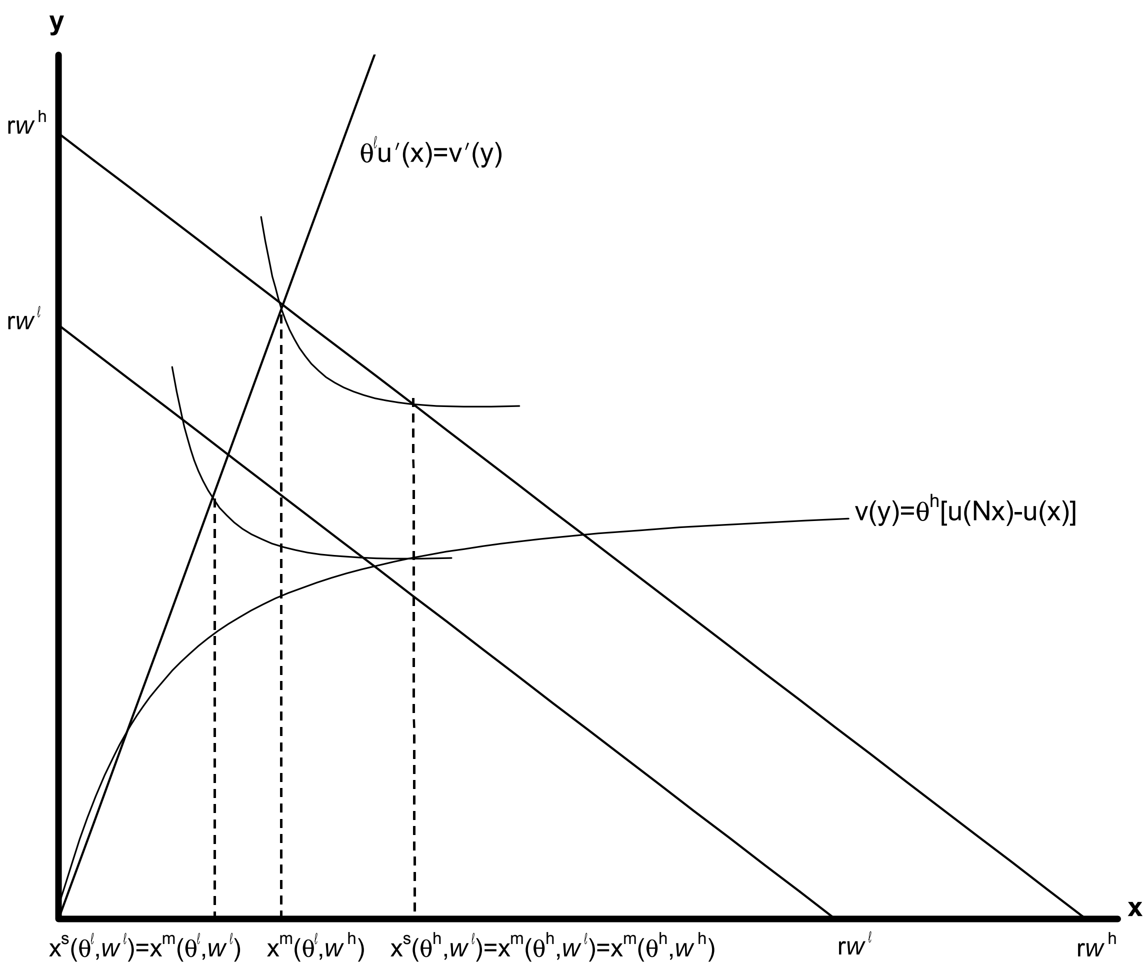
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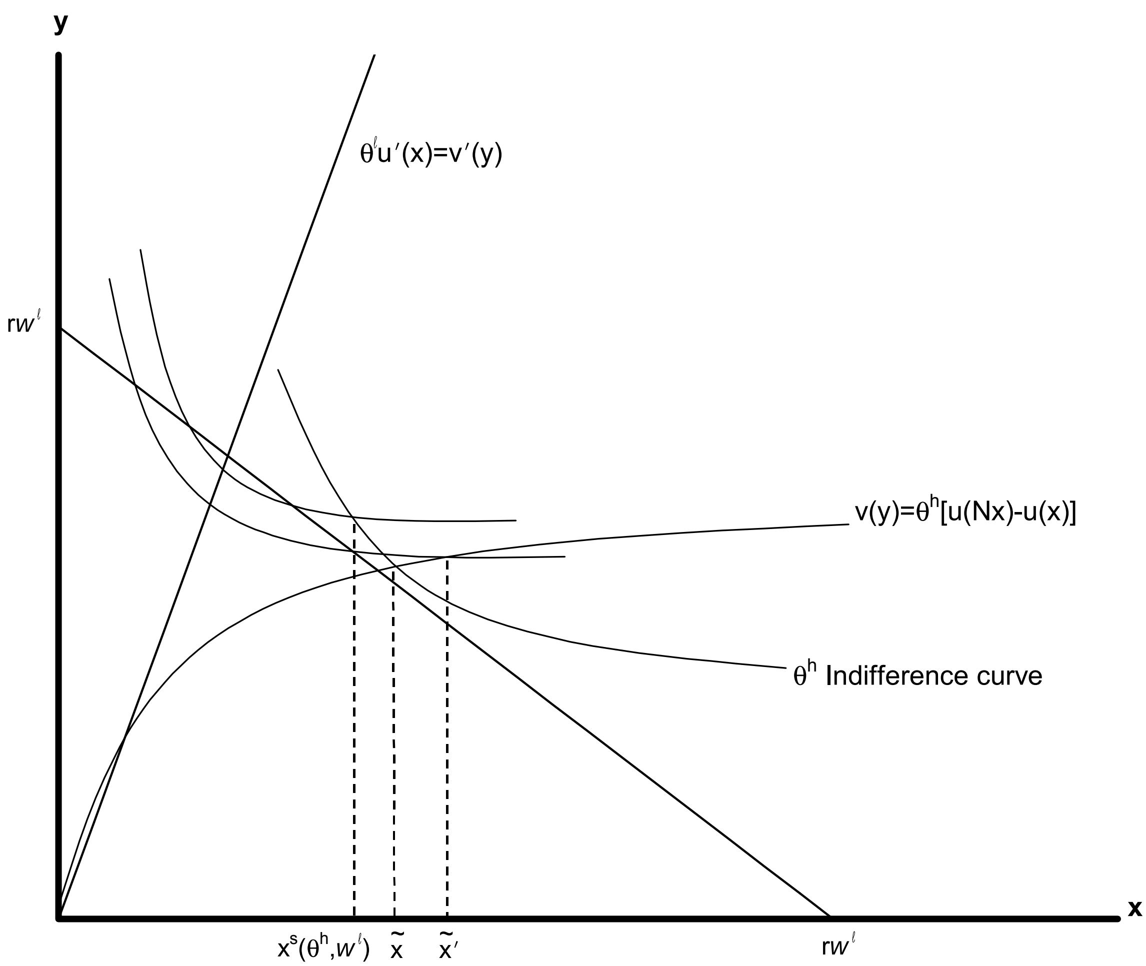

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Figure 7: