Working Paper Series

Macroeconomic Fluctuations and Bargaining

WP 01-04

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Federal Reserve Bank of Richmond Working Paper No. 01-04
July 2001
JEL Nos. C78, E30, E24
Keywords: Dynamic Bargaining, Decentralized Exchange, Sunspots

Abstract

I study the limit rule for bilateral bargaining when agents recognize that the aggregate economy (influencing the match surplus) follows a dynamic process that randomly switches back and forth between a finite number of possible states. The rule derived in this paper is of special importance for decentralized exchange economies with bargaining. Two simple applications are presented to illustrate this fact. The first example is a model of wage bargaining and trade externalities. I show that in those situations sophisticated bargaining tends to increase the volatility (due to extrinsic uncertainty) of the wage bill. The second example is based on the Kiyotaki-Wright model of money. I explain how equilibrium prices depend in a fundamental way on the dynamic bargaining solution.

*I would like to thank Karl Shell for helpful conversations and encouragement. Guido Cozzi, Todd Keister, Salvador Ortigueira, Luis Rivas, Neil Wallace, Randall Wright and, the participants at the Spring 1999 Cornell University-Penn State University Joint Macroeconomics Workshop also provided useful comments on an earlier draft. All errors are my own. A previous version of this paper circulated with the name "Bargaining when Sunspots Matter." Research support from the Thorne Fund and the Center for Analytic Economics at Cornell University is gratefully acknowledged. The views expressed here do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1 Introduction

In a stationary environment, there is a close relationship between the solutions to the sequential bargaining game (Rubinstein [15]) and the Nash Bargaining Rule (Nash [14]). See, for example, Binmore [1]. In fact, when either the period of time between each round of negotiations in the sequential bargaining game becomes small or when the discount factor goes to unity, the two theories deliver the same predictions.

In general, however, this equivalence breaks down when the agents interact in a non-stationary environment (Coles and Wright [4]). For the case in which the parameters of the bargaining game change *smoothly* over time, Coles and Wright develop a simple formula that can be applied to obtain the limiting solution (as the time interval between moves approaches zero) to the alternating-offers bargaining procedure (see also Coles and Muthoo [3] for a generalization of the arguments).

In this paper, using a similar method to the one used by Coles and Wright, I obtain a formula that can be used to derive the outcome of bargaining negotiations when the parameters of the game switch randomly among a *finite* number of possible states. The formula is simple and reduces to the Nash solution when there is just one state of nature. However, if there is more than one state, some important differences arise. Some of my results are similar to the findings in Coles and Wright, but they are much easier to interpret.

The dynamics of the parameters on the bargaining game may be induced by the evolution of the aggregate economy. For example, market prices may indirectly determine the feasible payoffs that each agent can obtain from negotiation. This fact makes the bargaining solution proposed in this paper useful for applications in macroeconomics. To illustrate this, I use the bargaining formula in two well-known models of decentralized exchange for which the equilibrium may be subject to extrinsic uncertainty (sunspots): (i) an economy with bilateral production matching and trade externalities (*à la* Diamond [6]), and (ii) a monetary random-matching economy (*à la* Kiyotaki and Wright [10]).

The two examples provide interesting insights into the fundamental factors that determine the influence of aggregate fluctuations upon equilibrium patterns of exchange. Nothing limits the applicability of the bargaining solution to environments where volatility is only driven by extrinsic uncertainty (as opposed to intrinsic "real" uncertainty). However, these sunspot examples are especially interesting due to their sheer simplicity. The underlying driving force for most of the findings in this part of the paper is that the relative position of agents on the two sides of the negotiation determines the final consequences of the macroeconomic fluctuations. In particular, if the agents in the bilateral bargaining are identical in all respects, the sunspot effects do not alter the surplus-splitting rules, but otherwise, the presence of sunspots does result in important changes in the relative bargaining power of the negotiators.

Bargaining power in the Rubinstein game depends on two factors: (i) the time discounting by agents, and (ii) the threat of delay if a proposal is not "fair" enough. In fact, agents who discount future payoffs relatively more will tend to have less bargaining power and will get worse deals. This same effect is what makes volatility (in the to-be-split surplus) affect the negotiation power when the agents are forward-looking in their bargaining. If an agent discounts the future less, then future capital gains (driven by the fluctuations in the environment) become relatively more important for her and she would have a natural desire to delay the deal, expecting those gains to be realized. This increases the bargaining power of patient agents when there is a high probability of capital gains in the future. In the same way, a high probability of future capital losses increases the bargaining power of impatient agents (patient agents discount the future losses more and will be eager to close a deal as soon as possible). This is the intuition for the main results of the paper.

The remainder of the paper is organized as follows. In Section 2, I develop the formula for the limiting case of a sequential bargaining game when the surplus of the match switches randomly between a finite number of states. Also in that section, I analyze a benchmark case: the simple splitting-the-cake problem. In Section 3, two macroeconomic examples are introduced: a labor market model with bargaining (that can be seen as a special case of the benchmark case), and a random matching model of money. Section 4 is reserved for concluding remarks.

2 The Bargaining Theory

In this section I derive the main result of the paper: a simple formula to obtain the surplus-splitting rule in the specific dynamic bargaining environment. I also present a benchmark example with a long tradition in the bargaining literature, the splitting-the-cake problem. This example is a partial equilibrium, microeconomic application of the theory. For its simplicity, though, it is especially useful to illustrate some of the implications of the result. In Section 3, I discuss two more interesting macroeconomic applications.

2.1 General Rule

Consider the following economic problem. Two agents, 1 and 2, are bilaterally matched. Out of this encounter there is some mutually beneficial surplus that can be produced; agents have to decide how to split that surplus between them. This is the typical bargaining situation studied by Nash and Rubinstein. Let $x \in \mathbb{R}$ represent a decision variable for the agents determining how the surplus is divided. Agent 1 has an instantaneous payoff function $u_1(x;\theta)$, where θ represents a vector of parameters that influence the size of the total surplus to be distributed.[1] Agent 2's payoff function is given by $u_2(x;\theta)$. Assume that $u_i : \mathbb{R}\times S \to \mathbb{R}$, where S is a finite set, and that u_i is twice continuously differentiable in the first argument, with $i = 1, 2$. Let u_1 be increasing and concave in x for every θ. Also, u_2 is decreasing and concave in x. Agent i discounts the future at rate r_i, $i = 1, 2$. Agents derive zero utility from no trade as well as from obtaining no surplus from the match. Hence, using the participation constraints we obtain that $x \in [\underline{x}, \overline{x}]$, where $x = \overline{x}$ corresponds to the situation where all the surplus goes to agent 1 and $x = \underline{x}$ when all the surplus goes to agent 2. Note that the values of θ, $\underline{x}$, and $\overline{x}$ may be changing through time.

The idea is to consider the solution to the *alternating offers* bargaining game (see Rubinstein [15]) and then study the limit of this solution when the time period between offers goes to zero. If θ_t is constant through time, then it is well known that the limit of the unique subgame-perfect equilibrium outcome of the alternating offers game is the Nash solution with bargaining power and threat points that depend on the details of the specific game (Binmore [1]). If θ_t follows a smooth non-constant path, then the Coles-Wright [4] solution applies. Alternatively, we will consider the case where θ_t takes a finite number of different possible values and randomly *jumps* among them. In fact, the main interest will be in the market equilibrium of an economy where agents get paired through a matching process and bargain to split the mutual benefits of that match.[2] In this sense, θ_t may represent the state at time t of the aggregate economy that for some reason determines the value of the match

[1] Note that these are not parameters of the utility function. They represent the state of the economy where these agents interact and they are taken as given at the individuals' level.

[2] This is in the spirit of Rubinstein and Wolinsky [16], although they allow for the possibility of exogenous breakdowns and deal only with constant surpluses. Merlo and Wilson [11] consider the case where the surplus follows a general stochastic process (in a discrete time framework). However, the focus in Coles and Wright [4] and the present paper is very different than that in Merlo and Wilson [11]. Here we are mainly interested in obtaining a simple *limiting* solution to the sequential bargaining game and then applying it to a macroeconomic model with the objective of investigating its possible implications.

(see the examples in the next section).

The alternating offers bargaining procedure operates as follows. First, agent 1 makes an offer at time t, $y(t)$, that may depend on time because θ_t does. Agent 2 either accepts or rejects the offer. If agent 2 accepts, then the game ends and the payoff vector is given by $[u_1(y(t);\theta_t), u_2(y(t);\theta_t)]$.[3] If agent 2 does not accept agent 1's offer, then a period Δ of time goes on and at time $t+\Delta$ agent 2 gets to make an offer $z(t+\Delta)$. Agent 1 accepts or rejects that offer, and the game goes on in that manner. However, θ_t, the state of the aggregate economy where these two agents interact, follows a stochastic dynamic process that switches back and forth between a finite set of possible values. Hence, between t and $t+\Delta$, the economy may transit from one state to another. This may then result in a more favorable situation for one of the two agents, who – aware of this possibility – will act accordingly when bargaining.

More specifically, assume the aggregate state of the economy depends on a random variable that can take values in the set $S \equiv \{s_a, s_b, ...\}$, and that follows a Poisson process with transition rates $\pi_{s,s'}$, with $s, s' \in S$.

We will restrict our attention to the case where there is no delay in the negotiations. Following Coles and Wright [4] we will call this situation an Immediate Trade Equilibrium (ITE). It is well known in the literature that delays are possible in this type of environment (see Merlo and Wilson [11]). However, since the focus of this paper is on comparing *dynamic* bargaining with the Nash solution, it is only reasonable to restrict attention to those equilibria that have immediate trade. When delays are part of the outcome, applying Nash bargaining is obviously inappropriate. We will provide conditions under which an ITE exists and we will briefly discuss how the

[3] Actually, the equilibrium payoff in state θ_s may depend on the payoff in the other possible states (see the examples in the next section). We will keep this simplified notation for clarity of exposition.

equilibrium with delays would work.

In an ITE there exists reservation values $[\widehat{x}_{1s},\ \widehat{x}_{2s}]$ such that if the economy is in state s, agent 1 accepts an offer x from agent 2 whenever $x \geq \widehat{x}_{1s}$, and agent 2 accepts an offer x from agent 1 whenever $x \leq \widehat{x}_{2s}$.[4] Also, from the properties of the payoff functions the best acceptable offer of an agent is always the reservation value of her partner, i.e., $y(t) = \widehat{x}_{2s}$ and $z(t) = \widehat{x}_{1s}$.

Define $H_s \equiv \sum_{s' \in S \backslash s} \pi_{ss'}$ to be the hazard rate at state s. The equilibrium reservation values in each state s satisfy the following two equations:

$$u_1(\widehat{x}_{1s};\theta_s) = \frac{1}{1+r_1\Delta}\left[(1-\Delta H_s)\,u_1(\widehat{x}_{2s};\theta_s) + \sum_{s' \in S\backslash s} \Delta \pi_{ss'} u_1(\widehat{x}_{2s'};\theta_{s'})\right], \qquad (1)$$

and

$$u_2(\widehat{x}_{2s};\theta_s) = \frac{1}{1+r_2\Delta}\left[(1-\Delta H_s)\,u_2(\widehat{x}_{1s};\theta_s) + \sum_{s' \in S\backslash s} \Delta \pi_{ss'} u_2(\widehat{x}_{1s'};\theta_{s'})\right], \qquad (2)$$

where $s \neq s'$ for $s,\ s' \in S$ and $S \backslash s$ denotes the set of all the elements of S excluding s. Clearly, $\widehat{x}_{1s}$ and $\widehat{x}_{2s}$ will be functions of Δ. To simplify notation, I choose not to write the dependence on Δ explicitly but it should be kept in mind for the upcoming arguments.

Let $h_s(\Delta) \equiv \widehat{x}_{2s} - \widehat{x}_{1s}$. It is shown in Appendix 1 that $h_s(\Delta) = O(\Delta)$ (i.e., $h_s(\Delta)/\Delta \to c \in \mathbb{R}$ as $\Delta \to 0$). Then, using the Taylor expansion of $u_i(\widehat{x}_{1s};\theta_s)$, $i = 1, 2$, around $\widehat{x}_{2s}$, from (1) and (2) one obtains,

$$\begin{aligned} r_1 u_1(\widehat{x}_{2s};\theta_s) \quad &= \quad (1+r_1\Delta)\left[u_1'(\widehat{x}_{2s};\theta_s)\frac{h_s(\Delta)}{\Delta} + \frac{o(\Delta)}{\Delta}\right] + \\ &\quad + \sum_{s' \in S\backslash s} \pi_{ss'} \left(u_1(\widehat{x}_{2s'};\theta_{s'}) - u_1(\widehat{x}_{2s};\theta_s)\right), \end{aligned}$$

[4]Only history independent strategies are considered in the equilibrium to be studied. Also note that I dropped the t argument from the offer functions because the only source of dynamics will be the switching states of the aggregate economy indicated with superscript.

and

$$r_2 u_2(\widehat{x}_{2s};\theta_s) = -u_2'(\widehat{x}_{2s};\theta_s)\frac{h_s(\Delta)}{\Delta} + \frac{o(\Delta)}{\Delta} + $$
$$+ \sum_{s'\in S\backslash s} \pi_{ss'}\left(u_2(\widehat{x}_{2s'};\theta_{s'}) - u_2(\widehat{x}_{2s};\theta_s) + O(\Delta)\right).$$

where $o(\Delta)/\Delta \to 0$ as $\Delta \to 0$.

Then, after some substitutions and taking limits as $\Delta \to 0$,[5] the following equation obtains:

$$\left[r_1 u_1(\widehat{x}_s;\theta_s) + \sum_{s'\in S\backslash s} \pi_{ss'}\left(u_1(\widehat{x}_s;\theta_s) - u_1(\widehat{x}_{s'};\theta_{s'})\right)\right]\frac{1}{u_1'(\widehat{x}_s;\theta_s)} =$$
$$\frac{1}{-u_2'(\widehat{x}_s;\theta_s)}\left[r_2 u_2(\widehat{x}_s;\theta_s) + \sum_{s'\in S\backslash s} \pi_{ss'}\left(u_2(\widehat{x}_s;\theta_s) - u_2(\widehat{x}_{s'};\theta_{s'})\right)\right], \qquad (3)$$

for $s \neq s'$ and $s, s' \in S$. Using expressions (1) and (2) it is not hard to show that the following set of inequalities guaranties the existence of an Immediate Trade Equilibrium:

$$\sum_{s'\in S\backslash s} \frac{\pi_{ss'}}{r_i + H_s}\frac{u_i(\widehat{x}_{s'};\theta_{s'})}{u_i(\widehat{x}_s;\theta_s)} < 1, \qquad (4)$$

for $i = 1, 2$ and all $s \in S$. To better understand this inequality we restrict our attention to the case where S has only two elements. Assume without loss of generality that $u_i(\widehat{x}_{s'};\theta_{s'}) > u_i(\widehat{x}_s;\theta_s)$. In this case, (4) reduces to only one restriction,[6]

$$\frac{\pi_{ss'}}{r_i + \pi_{ss'}}\frac{u_i(\widehat{x}_{s'};\theta_{s'})}{u_i(\widehat{x}_s;\theta_s)} < 1.$$

This inequality allows for a straightforward interpretation: large discount rates, small probabilities of switching to the states with higher surplus, and small disparities between the size of the surplus in the different states will all increase the chances of an ITE. This condition is related to Assumption 2 (Shrinking Pareto Frontiers) in Coles and Muthoo [3]. I summarize the previous discussion in the following proposition.

[5] Note that when $\Delta \to 0$ it becomes irrelevant who makes the first offer ($h(\Delta) \to 0$ as $\Delta \to 0$).

[6] The other inequality implied by expression (4) is always satisfied.

Proposition 1 *If the state of the aggregate economy, θ, follows a Poisson process with finite state space S and if inequality (4) holds for all $s \in S$ and $i = 1, 2$, then there exists an ITE and the limiting (as $\Delta \to 0$) splitting rule from bargaining, $\{\widehat{x}_s(t),$ $s \in S\}$, satisfies the set of equations given by (3).*

Remark 1 When $\theta_s = \theta_{s'}$ for all s, $s' \in S$, by using concavity of the payoff functions, it can be shown that $\widehat{x}_s = \widehat{x}_{s'}$. Equation (3) then reduces to the standard Nash Bargaining solution.

Even when there is delay in equilibrium, there always exists a state $s \in S$ for which immediate trade will occur. The idea is that there is a state of nature for which waiting can only reduce the expected future surplus available. With this in mind, given an interest rate and the set of transition probabilities, it is always possible to divide the states of nature into those for which the agent would prefer to trade and those for which she would prefer to wait. Calculating the threshold state is then straightforward. In this paper I only study the implications of Proposition 1, which only applies when delays do not take place in equilibrium.

2.2 Benchmark Case: Splitting the Cake

One of the most studied examples of bargaining situations is the case where two agents with linear utility functions meet, produce a surplus (a cake) of size P, and have to decide how to split it between the two (see for example Rubinstein [15]). Let x be the part of the cake that goes to agent 1, and $P - x$ the corresponding share for agent 2. Consider a situation where the size of P depends on a random state variable taking two possible values (s_a or s_b) and following a Poisson process with transition rates $[\pi_{ab}, \pi_{ba}]$. Note that as it stands, the uncertainty in the present case is essentially *intrinsic*. Indeed the result in Proposition 1 is applicable to general environments (including, but not exclusively, the sunspots case).

It is easy to see that the problem just described is a particular case of the one in the previous subsection. The payoff functions are given by $u_1(x;\theta_s) = x$ and $u_2(x;\theta_s) = P_s - x$, $s = s_a, s_b$; and equations (3) are in this case,

$$r_1 x_s + \pi_{ss'}(x_s - x_{s'}) = r_2(P_s - x_s) + \pi_{ss'}\left[P_s - x_s - (P_{s'} - x_{s'})\right],$$

for $s, s' = s_a, s_b$, and $s \neq s'$. Define $\overline{r} \equiv (r_1 + r_2)/2$.

- **Result 1:** Suppose that $P_{s_a} = P_{s_b} = P$. Then, $x_{s_a} = x_{s_b} = (r_2/\overline{r})(P/2)$ and if $r_1 = r_2$, then they will split the cake in halves. These are the usual results of traditional Nash Bargaining Theory.

- **Result 2:** Suppose that $P_{s_a} \neq P_{s_b}$ and $r_1 = r_2$. Then $x_{s_a} = P_{s_a}/2$ and $x_{s_b} = P_{s_b}/2$, i.e. in spite of the dynamics in the size of the cake, they still split it in halves. (See Theorem 3 in Coles and Wright [4] for an analogous result.)

- **Result 3:** Suppose that $P_{s_a} \neq P_{s_b}$ and $r_1 \neq r_2$, then

$$x_{s_a} = \frac{r_2}{\overline{r}}\frac{P_{s_a}}{2} + \frac{\pi_{ab}}{\overline{r}}\left[\frac{r_2 - \overline{r}}{\overline{r} + \pi_{ab} + \pi_{ba}}\right]\frac{P_{s_b} - P_{s_a}}{2},$$

and

$$x_{s_b} = \frac{r_2}{\overline{r}}\frac{P_{s_b}}{2} - \frac{\pi_{ba}}{\overline{r}}\left[\frac{r_2 - \overline{r}}{\overline{r} + \pi_{ab} + \pi_{ba}}\right]\frac{P_{s_b} - P_{s_a}}{2}.$$

If, for example, $P_{s_b} > P_{s_a}$ and $r_2 > r_1$, then $x_{s_a} > (r_2/\overline{r})(P_{s_a}/2)$ and $x_{s_b} < (r_2/\overline{r})(P_{s_b}/2)$.[7] The equilibrium payoff of agent 1, who is relatively more patient, is higher in the "small cake" state because she would be willing to wait for the change in state relatively longer than agent 2 (this raises her reservation value, which is what she will end up getting in equilibrium). Similarly, when

[7] Using these expressions for x_{s_a} and x_{s_b} and the set of inequalities given by (4) it is possible to find restrictions over r_1, r_2, P_{s_a}, P_{s_b} and the transition rates π such that an ITE exists.

the economy is in state s_b, agent 1 will be eager to close a deal in the current situation: the risk of loss from the switch in states the following Δ-period is relatively more important for this agent (she is more patient, thus she cares relatively more about future losses). For this reason, her payoff in the "big cake" state tends to get smaller.

From this analysis, one can see that when the discount rates differ among agents, the disparity in payoff is either accentuated by the *variability* in the size of the cake, as in the "small cake" state s_a (as $x_{s_a} > (r_2/\overline{r})(P_{s_a}/2) > P_{s_a}/2$), or dissipated, as in the "big cake" state s_b (as usually $(1/2)P_{s_b} < x_{s_b} < (r_2/\overline{r})(P_{s_b}/2)$ when π_{ba} is relatively small). One prediction from this theory would be that one should expect more disparate 'surplus-splitting' conditions during a slowdown. This would be true whenever bargaining constitutes a substantial component of the transaction mechanisms operating in the economy and agents differ in their discounting of the future. Additionally, note that the payoff of the agent with the higher discount factor (agent 2) tends to show a higher volatility when dynamic bargaining is explicitly considered:

$$P_{s_a} - x_{s_a} < (r_1/\overline{r})(P_{s_a}/2) < (r_1/\overline{r})(P_{s_b}/2) < P_{s_b} - x_{s_b}.$$

The present discussion should serve as a preamble for the first example of the next section, which constitutes a special case of this problem but is embedded in a general equilibrium setup. It should be mentioned though that in the decentralized exchange economies that we shall study next, the sunspot effects are essentially exogenous to the match (once in the match, agents take as given that the economy where they interact is subject to sunspot effects). This makes the analysis very much comparable to the one in this subsection.

3 Application: The Sunspots Case

Several well-known examples of economies with decentralized trading show that equilibria are often sensitive to the influence of extraneous variables that coordinate expectations. After the seminal work of Cass and Shell (see Shell [17] and Cass and Shell [2]), it has been shown in the literature that rational expectations economies in which the usual Arrow-Debreu assumptions do not obtain generally allow for the existence of sunspot equilibria.[8] Trade frictions are one of the important factors that generate this sunspot-type of phenomena (see for example, Diamond [6] and Howitt and McAfee [8] for a labor market model, and Wright [22] and Shi [19] for random matching models of money). Not surprisingly, in these economies bilateral bargaining usually constitutes an essential part of the arrangements. The combination of these two phenomena – bargaining and sunspots – makes decentralized exchange economies a natural environment for illustrating the concepts developed in this paper.

In what follows, I will present two examples of economies where the formula obtained in Proposition 1 can be applied to determine the outcome of a bargaining procedure. The first application is based on extensive literature on trade externalities developed after Peter Diamond's seminal work on the possible macroeconomic consequences of Search Equilibrium Analysis (see Diamond [6], Howitt and McAfee [8], [9], and the references therein). Those models typically generate multiple equilibria and sunspot equilibria; however, the bargaining process is usually not explicitly modeled (although it is clearly implicit in the analysis; see Drazen [5]).[9] The first example in this section will then partly fill that gap in the literature.

[8] This is the so-called Philadelphia Pholk 'Theorem'. See Shell [18].

[9] Mortensen [12], in a model with similar characteristics as the ones considered in this paper, uses explicit Nash bargaining with the bargaining power of agents changing through time following an exogenous rule associated with certain indicators of the aggregate state of the economy (e.g., labor market tightness).

The second application is a random matching monetary economy similar to that presented in Kiyotaki and Wright [10]. The latest generation of these types of models has considered price determination through a bargaining procedure within each monetary match. One interesting result out of those papers is the possibility of multiple and sunspot monetary equilibria (see Shi [19], Trejos and Wright [20], and Ennis [7]). Following the analysis of the previous section, it becomes apparent that to study sunspot equilibria in these models one needs to take into account the potential effects of extrinsic uncertainty on the bargaining outcomes. A description of the consequences from following such strategy is provided in the second part of this section.

3.1 Wage Bargaining and Sunspots

The first example in this section constitutes a complete description of an economy where a large number of workers interact with entrepreneurs to produce some output. One of the distinguishing characteristics of this economy will be the presence of trade externalities as introduced by Diamond [6]. In fact, the present economy is a simplified version of the one in Howitt and McAfee [8], but one in which I *explicitly* analyze the bargaining process that determines wages. In this example a trade externality acts as the driving force for the existence of multiple equilibria originated in the possibility of coordination failures. The multiplicity of equilibria is what allows sunspots to affect the equilibrium allocations in the economy. Although the model is stylized, it will illustrate how the bargaining rule obtained in Section 2 can be introduced in a simple model to uncover some of its main implications.

Consider an economy with a large number of two types of agents: entrepreneurs and workers. For notational convenience we will identify entrepreneurs as type 1 agents and workers as type 2 agents. There are three tradable objects: output, homogeneous labor services, and money (used only for payments). The output produced by a firm (a match between an entrepreneur and a worker) has to be sold in the market. The proceeds are divided between the entrepreneur and the worker and they then use that money to buy other consumption goods in the market. The market for output is perfectly competitive, but firms incur a transaction cost during the sales process. The transaction cost is of the *iceberg type*; i.e., it takes the form of a proportion of total sales. Hence, a firm employing n units of labor will have a net revenue of

$$R(n, \overline{n}) = [1 - \xi(\overline{n})] f(n),$$

where f is the firm's production function and $\xi(\overline{n}) f(n)$ is the transaction cost. Assume that ξ is a continuously differentiable, strictly decreasing function and that f is continuously differentiable and strictly concave. The trade externality comes through $\xi(\overline{n})$, which depends upon the aggregate employment (per firm) $\overline{n}$: the higher the general level of employment in the economy, the easier it is to sell goods and therefore the lower the transaction cost $\xi(\overline{n}) f(n)$.

At every moment in time, an entrepreneur can potentially get matched with a worker according to a Poisson process with arrival rate β. Assume workers meet entrepreneurs also at rate β. After the match is formed, output is produced and the worker experiences disutility $v(n)$ from labor. The firm then sells its output in the market and pays wages. Finally, entrepreneurs and workers use the proceeds to buy goods for consumption.

Under this setup, entrepreneurs and workers develop a bilateral relationship when they get matched. The standard approach for this situation is to assume that they will bargain over the distribution of the match surplus. Let x be the payment to the workers after negotiations. Define V_i to be the value for agent type $i = 1, 2$

of being unmatched and waiting for a potential partner. Also, let r_j, $j = 1, 2$ be the time discount rates for firms and workers respectively, and $x(\overline{n})$ the equilibrium wage-bill when aggregate employment (per firm) is $\overline{n}$. Then, it is not hard to show that $V_1(\overline{n}) = (\beta/r_1)[R(\overline{n}, \overline{n}) - x(\overline{n})]$ and $V_2(\overline{n}) = (\beta/r_2)[x(\overline{n}) - v(\overline{n})]$ in equilibrium.

Assume $v(n)$ is continuously differentiable and strictly convex. One well accepted bargaining procedure is the Nash Bargaining Solution. The predictions from this solution concept are equivalent to the limiting outcomes in the alternating offers Rubinstein game (see Binmore [1] and Muthoo [13] for a review). Then, when the entrepreneur and the worker Nash-bargain over the amount of labor n and the payroll x, the resulting agreement (n, x) must solve the following problem:

$$\max_{x,n}[R(n, \overline{n}) - x + V_1(\overline{n})]^{\alpha}[x - v(n) + V_2(\overline{n})]^{1-\alpha}$$
$$\text{subject to } \; x + V_2(\overline{n}) \geq v(n) \quad \text{and} \quad x \leq R(n, \overline{n}) + V_1(\overline{n}).$$

From the first-order conditions (assuming an interior solution), we have that

$$x^* = (1 - \alpha)[R(n^*, \overline{n}) + V_1(\overline{n})] + \alpha[v(n^*) - V_2(\overline{n})], \tag{5}$$

and

$$R_1(n^*, \overline{n}) \equiv (1 - \xi(\overline{n}))f'(n^*) = v'(n^*). \tag{6}$$

Note that, as expected, the Nash solution is "efficient" (given $\overline{n}$, n^* maximizes net surplus $R(n, \overline{n}) + V_1(\overline{n}) - v(n) + V_2(\overline{n})$). The weight α represents the relative bargaining power of entrepreneurs. If one thinks of Nash bargaining as a simplified approximation to the solution of the Rubinstein game, then $\alpha/(1 - \alpha)$ is given by the ratio of workers' and entrepreneurs' discount rates (r_2/r_1). Considering the net payoff, x_N and z_N, for workers and firms respectively, we see that

$$x_N^* \equiv x^* - v(n^*) + V_2 = (1 - \alpha)[R(n^*, \overline{n}) + V_1 - v(n^*) + V_2],$$

and

$$z_N^* \equiv R(n^*, \overline{n}) + V_1 - x^* = \alpha[R(n^*, \overline{n}) + V_1 - v(n^*) + V_2].$$

That is, a proportion α of the match surplus goes to the entrepreneur and a proportion $(1-\alpha)$ goes to the worker. If both have the same discount factors (i.e., $\alpha = 1/2$), it can be shown that $x^* = [R(n^*, \overline{n}) + v(n^*)]/2$ when $\overline{n} = n^*$, and they split the surplus in half (see Result 1 in the Benchmark Case for a direct analogy). This result will become important later when I analyze a sunspot equilibrium with equal discount factors. Finally, note that n^* is independent of the splitting rule x^*.

Definition 1 *A certainty equilibrium for this economy is given by the set* $\{(\overline{n}, n^*, x^*), (V_1, V_2)\}$ *such that:* ***1)*** (n^*, x^*) *satisfies (5) and (6);* ***2)*** $\overline{n} = n^*$*;* ***3)*** $V_1^* = (\beta/r_1)[R(n^*, n^*) - x^*]$ *and* $V_2^* = (\beta/r_2)[x^* - v(n^*)]$.

It should be apparent that the trade externality can generate multiple equilibria in this economy.[10] In fact, the higher the aggregate employment in the economy, the easier it becomes to sell the produced goods and therefore the higher the marginal productivity of labor. This implies higher optimum levels of employment at the firm level and possibly a high employment equilibrium (n_H^*, n_H^*, x_H^*). Inversely, for low levels of $\overline{n}$, the marginal productivity of labor is low and this can result in low actual employment equilibrium levels n_L^* (see Figure 1). Clearly, where the economy is in any period depends exclusively on how the agents get coordinated.

(*insert Figure 1 here*)

Sunspot Equilibrium. Assume that there are multiple certainty equilibria and that expectations over the aggregate employment level in the economy follow a two-state Poisson process with transition rates $\{\pi_{LH}, \pi_{HL}\}$. In other words, the agents in

[10] Sufficient conditions for the existence of multiple equilibria are: $v'(0) > 0$, $(1-\sigma(0))f'(0) = 0$, $\lim_{n\to\infty} f'(n) = 0$, and there exist an $\widehat{n} > 0$ such $(1-\sigma(\widehat{n}))f'(\widehat{n}) > v'(\widehat{n})$.

the economy coordinate themselves to be in either of the two certainty equilibria (low or high employment) according to a two-state *sunspot* random variable. Assume also that regardless of the state of the economy, when an entrepreneur and a worker get matched they always decide to produce. If one considers the limiting solution of the alternating offers bargaining game, the outcome of the negotiations differs (in general) from that in a certainty equilibrium.[11] At the moment of the negotiations, agents take into account that the economy might switch to the other state at any time. This potentially affects their reservation values. In this paper we are especially interested in this type of effect. However, there is another important effect of sunspots on the equilibrium quantities in the economy. The value V_i of being unmatched waiting for an arrival depends on the dynamic properties of the aggregate economy. When sunspots matter, the value functions for entrepreneurs and workers are given by the following system of equations:

$$V_{1s} = \frac{\beta}{r_1}\left[R_s(\widehat{n}_s, \widehat{n}_s) - \widehat{x}_s\right] + \frac{\pi_{ss'}}{r_1}(V_{1s'} - V_{1s}),$$

$$V_{2s} = \frac{\beta}{r_2}\left[\widehat{x}_s - v_s(\widehat{n}_s)\right] + \frac{\pi_{ss'}}{r_2}(V_{2s'} - V_{2s}),$$

where $s, s' = L, H$, $s \neq s'$, and the hats indicate that these are values in a sunspot equilibrium (as opposed to the certainty, multiple equilibria of the previous definition). The fact that V_{1s} and V_{2s} depend on the sunspot variable will also influence the equilibrium splitting rule.[12]

[11] Note that if one assumes *myopic* behavior by the agent in a match, then a fixed exogenous rule for splitting the surplus obtains. This is what has been done in much of the previous literature (see Drazen [5]). Under that assumption, sunspot equilibria of this model constitute nothing more than a *trivial randomization* over certainty equilibria. However, endogenizing the splitting rule through explicit *sophisticated* bargaining (as in Section 2) will be shown to produce new possible observable equilibrium outcomes. See Shell [18] for a general discussion on the importance of this issue for the study of sunspot equilibrium.

[12] See Ennis [7] for a study on the influence of sunspots over the value of being unmatched in a decentralized exchange economy.

It can easily be shown that the equilibrium payoff for both the firm and the workers is increasing in net surplus $R(n,n) - v(n)$. Therefore, since the partners in the match will always agree to maximize that surplus, the condition for production efficiency is still satisfied. In particular, $\widehat{n}_L$ and $\widehat{n}_H$ solve versions of equation (6). Using the general proposition from Section 2, one can determine the payroll $\widehat{x}_s$ in each state. The version of equation (3) in the current setup is,

$$r_1[R(\widehat{n}_s,\overline{n}_s) - \widehat{x}_s + V_{1s}] + \pi_{ss'}[R(\widehat{n}_s,\overline{n}_s) - x_s + V_{1s} - (R(\widehat{n}_{s'},\overline{n}_{s'}) - \widehat{x}_{s'} + V_{1s'})] =$$

$$= r_2(\widehat{x}_s - v(\widehat{n}_s) + V_{2s}) + \pi_{ss'}[\widehat{x}_s - v(\widehat{n}_s) + V_{2s} - (\widehat{x}_{s'} - v(\widehat{n}_{s'}) + V_{2s'})],$$

where $s, s' = L, H,\ s \neq s'$, and $\widehat{n}_s$ solves $R_1(\widehat{n}_s,\overline{n}_s) = v'(\widehat{n}_s)$. Let $\alpha \equiv r_2/(r_1+r_2)$. Then, the equations determining the equilibrium values of $\widehat{x}_s$ are given by

$$\widehat{x}_s = (1-\alpha)\left[R(\widehat{n}_s,\overline{n}_s) + V_{1s}\right] + \alpha\left[v(\widehat{n}_s) - V_{2s}\right] +$$

$$\Phi\{R(\widehat{n}_{s'},\overline{n}_{s'}) + V_{1s'} - (v(\widehat{n}_{s'}) - V_{2s'}) - [R(\widehat{n}_s,\overline{n}_s) + V_{1s} - (v(\widehat{n}_s) - V_{2s})]\}, \quad (7)$$

for $s, s' = L, H\ ;\ s \neq s'$, $\Phi = [\pi_{ss'}(1-2\alpha)]/[r_1 + r_2 + 2(\pi_{ss'} + \pi_{s's})]$.

These equations should be compared with those obtained using expression (5). Note that when the discount rates of firms and workers are the same, the bargaining power index $\alpha = 1/2$, the coefficient $\Phi = 0$, and the third term in the sum disappears. Further calculations show that $\widehat{x}_s = [R(\widehat{n}_s,\widehat{n}_s) + v(\widehat{n}_s)]/2 = x_s^*$ with $s = L, H$, and the solution is immune to the existence of sunspots. (Both effects due to sunspots, the change on the value functions and the change in the bargaining rule, wash out when $r_1 = r_2$). In any other case, the existence of sunspot fluctuations affects the splitting rule from bargaining. The net effect is the result of the interaction of two channels through which sunspots influence the state of the match. On one hand, the value functions are different than in the non-sunspots case because they depend on the expected dynamics of the aggregate economy. On the other hand, and most important to this paper, the sunspots fluctuations affect the process of negotiations. This is represented by the last term in expression (7). In particular, if $r_2 < r_1$; i.e., if workers are more impatient than managers and if the total surplus from the match is bigger in the high employment equilibrium situation H, then $\widehat{x}_L$ tends to be lower in equilibrium than x_L^*, the certainty equilibrium value. The more patient side in the negotiation is more willing to delay a deal during bad times and this increases its bargaining power. One could say that the theory presented here predicts a tendency to lower payrolls as a proportion of total revenue during a slump (especially when there exists a perception among agents that the economy would, with considerable probability, recover from the current depression). Similarly, $\widehat{x}_H$ tends to be higher than x_H^*. During good times, firms discount less future losses and they are eager to close a deal before the economy switches to the bad state, sacrificing in this way some of their bargaining power. Note finally that these two implications (lower $\widehat{x}_L$ and higher $\widehat{x}_H$) tend to increase the variance of the labor share under the effect of sunspots. The wage bill is a lower proportion of a low surplus during bad times and a higher proportion of a high surplus during good times. However, our simplified structure has workers that are risk neutral with respect to income. Hence, this extra variability in income does not create any additional welfare losses for the agents.

3.2 Monetary Equilibrium, Bargaining and Sunspots

The example presented in this section consists of a random matching economy with money (see Kiyotaki and Wright [10]). The present discussion should be regarded as a complement to the paper by Trejos and Wright [20]. We extend their analysis to the case of sunspot equilibrium. Although Shi [19] proved the existence of sunspot equilibria for this model (using arguments of continuity), the actual characteristics of those equilibria have not been fully explored. (See Ennis [7] for a characterization of some of the steady-state properties of these sunspot equilibria.) We will show here that Proposition 1 can help us further understand the effects that ("excess") volatility have in the functioning of this type of monetary economy.

Consider the model in Trejos and Wright [20].[13] Time is continuous. A unit measure of infinitely-lived agents gets matched every period according to a Poisson process. There is specialization in production and consumption (no agent is able to consume what she herself produces). With probability y, a double coincidence of wants occurs between two randomly matched agents. Otherwise, matches are single coincidence meetings or no coincidence at all. Agents derive utility $u(q)$ from consuming and disutility $c(q)$ from producing, and they discount the future at rate r. Money is indivisible. At each date an agent can have either 0 or 1 units of money, but no more. Goods are non-storable and divisible. After production, agents have to consume to be able to produce again. Let $M \in (0, 1)$ be the total amount of units of money in the economy. The previous assumptions imply that in equilibrium there is an invariant distribution of money holdings: at every moment in time there is a fraction M of agents holding a unit of money and a fraction $(1 - M)$ of agents with a production opportunity and no money.

There are two classes of matches that originate trade in this economy: monetary matches and barter matches. In a *monetary match*, an agent with one unit of money meets an agent with an opportunity to produce the good that the former wants. They then decide how much of the good will be exchanged for the unit of money. For this, agents engage in a bargaining procedure. In a *barter match* none of the agents have money but there is a double coincidence of wants (one agent wants what the other

[13] For a good discussion of the general assumptions underlying the structure of that economy, see Wallace [21].

can produce and vice versa). In this case, traded quantities are also determined through bargaining.

Assume that $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$ for all $q \geq 0$ and that $c(0) = 0$, $c'(q) > 0$ and $c''(q) \geq 0$ for $q \geq 0$. Also assume that $u'(0) > c'(0) = 0$ and that there exists a $\overline{q} > 0$ such that $u(\overline{q}) = c(\overline{q})$.

Let V_0 be the value of being a producer (prior to a match) and V_1 the value of holding money (also prior to a match). In a steady-state (non-sunspots) monetary equilibrium with $q \leq \overline{q}$, (V_0, V_1) will satisfy

$$rV_0 = \Omega + M \ (V_1 - V_0 - c(Q))$$

and

$$rV_1 = (1 - M) \ [u(Q) + V_0 - V_1],$$

where $\Omega = (1 - M) \ y \ [u(q^*) - c(q^*)]$ is the barter payoff $(u'(q^*) = c'(q^*))$. Note that from these two equations one can solve for (V_1, V_2) as functions of Q, the quantity of goods for which a unit of money is exchanged in equilibrium.

The Nash bargaining problem in a monetary match is given by

$$\max[V_0(Q) + u(q)][V_1(Q) - c(q)]$$

subject to

$$V_1(Q) - c(q) \ \geq \ V_0(Q) \tag{8}$$

$$V_0(Q) + u(q) \ \geq \ V_1(Q) \tag{9}$$

where q is the quantity to be exchanged in this particular match. Restrictions (8) and (9) are individual rationality constraints for sellers and buyers, respectively.

When one takes the Nash solution as the bargaining rule for both types of matches, Proposition 3 in Trejos and Wright [20] (see also Shi [19]) establishes that, if there exists a monetary equilibrium, then there are indeed two of them, a high price constrained equilibrium and a low price unconstrained equilibrium. In the high price constrained equilibrium restriction (8) is binding.[14]

In this paper, however, we are most interested in studying sunspot equilibria and the implementation of the bargaining rule introduced in Section 2. Shi [19] established the existence of sunspot equilibria in this type of economy following ideas first developed in Wright [22]. The basic idea is to consider the case in which the economy randomly switches back and forth between the high and the low price situation. We now turn to the analysis of this case.

Assume that the quantity of goods (Q) exchanged in the typical monetary match follows a Poisson process with transition rates $\pi_{s,s'}$, where $s \neq s'$ and $s, s' \in \{H, L\}$. Here the index H indicates high quantity (low price) states and the index L, low quantity (high price) states.

For a sunspot equilibrium, the value of being a seller and the value of being a buyer will depend on the current state of the economy. Hence, one can show that the value functions are now given by

$$rV_{0s} = \Omega_s + M[V_{1s} - V_{0s} - c(Q_s)] + \pi_{ss'}(V_{0s'} - V_{0s}),$$

and

$$rV_{1s} = (1 - M)\ [u(Q_s) + V_{0s} - V_{1s}] + \pi_{ss'}(V_{1s'} - V_{1s}),$$

where $s, s' \in \{H, L\}$, $s \neq s'$, and Ω_s is the expected payoff from a barter match in state s. These value functions are in fact functions of (Q_H, Q_L), the inverse of the state-contingent equilibrium price level.

When agents meet in a mutually beneficial match (either barter or monetary), they will bargain over production. Naturally, these negotiations will be influenced

[14] Trejos and Wright [20] provide a threshold for the discount rate such that for given values of M and y, if the discount rate is smaller than the threshold, a monetary equilibrium exists.

by the fact that agents now know that the economy is switching states over time and that the current state is only temporary.

Consider first a *barter match*. We will now show that the barter quantities traded in equilibrium are independent of sunspots fluctuations. This is a direct consequence of the fact that the two agents meeting in a match with a double-coincidence-of-wants are symmetric across the negotiation table: Each agent is able to produce the good that the partner would like to consume. Symmetry then implies that both agents assess the benefits (and losses) of future price changes similarly. For this reason, sunspots have no consequences over the relative bargaining power in a barter match.

Let agents 1 and 2 be partners in a barter match. Define J_{is} to be the difference between agent i's payoff in state s and s',

$$J_{is} = u(q_{is}) - c(q_{js}) + V_{0s} - \left[u(q_{is'}) - c(q_{js'}) + V_{0s'}\right],$$

where $s, s' \in \{H, L\}$, $s \neq s'$, and $i, j = 1, 2$, $i \neq j$. Here, q_{is} is the quantity that agent i obtains from the match in state s. Using Proposition 1 we have that in this case equation (3) takes the form

$$[u(q_{is}) - c(q_{js}) + V_{0s} + \frac{\pi_{ss'}}{r} J_{is}] \frac{1}{u'(q_{is})} = \frac{1}{c'(q_{is})} [u(q_{js}) - c(q_{is}) + V_{0s} + \frac{\pi_{ss'}}{r} J_{js}], \quad (10)$$

with $s, s' \in \{H, L\}$, $s \neq s'$, and $i, j = 1, 2$, $i \neq j$.

Note that if $q_{1s} = q_{2s}$, for $s = H, L$, then $J_{1s} = J_{2s}$ for $s = H, L$. In this case, equation (10) becomes the traditional Nash bargaining rule with the unique solution q^* (satisfying $u'(q^*) = c'(q^*)$). In Appendix 2 we show that $q_{1s} = q_{2s}$ with $s = H, L$. Therefore, we conclude that barter trades are independent of sunspots and that $\Omega_s = \Omega_{s'} = \Omega$, the *non*sunspots expected payoff from barter.

For the *monetary match*, agents bargain only over the quantity of the good that will be changed for the indivisible unit of money. Agents' positions in the match are fundamentally disparate in this case: One has money and the other has a production opportunity. In consequence, sunspots are bound to have important effects in the bargaining outcome of the monetary match.

In this case, equation (3) takes the form

$$[V_{0s} + u(q_s) + \frac{\pi_{ss'}}{r}(V_{0s} + u(q_s) - V_{0s'} - u(q_{s'}))]\frac{1}{u'(q_s)} =$$

$$= \frac{1}{c'(q_s)}[V_{1s} - c(q_s) + \frac{\pi_{ss'}}{r}(V_{1s} - c(q_s) - V_{1s'} + c(q_{s'}))]. \tag{11}$$

The results of the bargaining procedure will be given by the solution to (11) as long as it satisfies the following participation constraints: $V_{1s} - c(q_s) \geq V_{0s}$ for the seller and $V_{0s} + u(q_s) \geq V_{1s}$ for the buyer. It can be shown that the latter constraint is binding only when the former is binding.[15] Hence, to find a solution to the bargaining problem only the sellers' constraint (the agent with the production opportunity) is relevant.

Define $\Delta_{iss'} \equiv V_{is} - V_{is'} = -\Delta_{is's}$, with $i = 0, 1$. These Δ's are again functions of (Q_H, Q_L), the inverse of the equilibrium price level. Define $T(Q)$ as the function

$$T(Q) = [(r + M)(1 - M)u(Q) - \Phi c(Q)]u'(Q) - [\Phi u(Q) - M(r + 1 - M)c(Q)]c'(Q)$$

with $\Phi = r(1+r)+M(1-M)$. Then, equation (11) can be rewritten as $T_s(Q_s, Q_{s'}) = 0$ where $s, s' \in \{H, L\}$, $s \neq s'$, and $T_s(Q_s, Q_{s'})$ is given by

$$T_s(Q_s, Q_{s'}) = T(Q_s) + [(1 - M)u'(Q_s) - (1 + r - M)c'(Q_s)](\Omega - \pi_{ss'}\Delta_{0ss'}) -$$

$$[(r + M)u'(Q_s) - Mc'(Q_s)]\pi_{ss'}\Delta_{1ss'} - \tag{12}$$

$$[u'(Q_s)(c(Q_s) - c(Q'_s) - \Delta_{1ss'}) + c'(Q_s)(u(Q_s) - u(Q_{s'}) + \Delta_{0ss'})](1 + r)\pi_{ss'}.$$

[15] This statement is associated to the fact that for a sunspot equilibrium to exist the transition rates need to be relatively small.

When there are no sunspots and no barter possibilities ($\Omega = 0$) the equilibrium values of Q are given by the solutions to $T(Q) = 0$. There is a unique monetary equilibrium in this case. In the case with barter ($\Omega > 0$) and no sunspots, there are two monetary equilibria, one low-price unconstrained equilibrium that solves $T_s(Q, Q) = 0$, and a high-price constrained equilibrium that satisfies the seller's participation constraint with equality (see Trejos and Wright [20] and Shi [19]). Generically, for small enough values of $\{\pi_{ss'}\}$, a sunspot equilibrium will exist switching from the constrained to the unconstrained equilibrium (this is essentially the existence result in Shi [19]).[16] Equation (12) then holds for $s = H$. The quantity traded at state L, Q_L, satisfies $V_{1L} - c(Q_L) = V_{0L}$. When agents act myopically during the bargaining process (i.e., when the traditional Nash solution is used), the last term in the RHS of equation (12) disappears and the rest of the analysis proceeds in the same manner. In general, we should be able to identify in this last term the same type of effects that resulted from using the dynamic bargaining formula (3) in the labor market example. In this model, one can expect that in the low-price (high quantity) state H, the agent holding a unit of money will be eager to close a deal. This is because the producer in the match would lower the quantities that she is willing to produce (in exchange for the unit of money) if she discovers that the economy has switched to the high-price (low quantity) state. In a sunspot equilibrium there is always a chance that the economy will presently switch states during the negotiations. For this reason, the money holder will try to speed up the deal in the low-price state and hence, lose bargaining power. As a consequence, the buyer (the money holder) will get relatively smaller quantities in equilibrium in this state (relatively smaller than when using the myopic Nash bargaining rule). For this intuition to hold the

[16] It is easy to see that when $\{\pi\} \to 0$, the function $T_s(Q_s, Q_{s'})$ converges to the function $T(q)$ used by Trejos and Wright [20] in the proof of their Proposition 3. The existence of a sunspot equilibrium is a direct implication of this.

last term in equation (12) would have to be negative. This requires that

$$u^{'}(Q_H)\left[c(Q_H)-c(Q_L)-\Delta_{1HL}\right]+c^{'}(Q_H)\left[u(Q_H)-u(Q_L)+\Delta_{0HL}\right]>0 \qquad (13)$$

It can be shown that $\Delta_{0HL} > 0$. This is because in state H the seller obtains a positive surplus from the match even though she has to produce a larger quantity. In state L, the seller gets zero surplus as the quantity for the trade is determined according to the participation constraint. It is also the case that $\Delta_{1HL} > 0$. This introduces an ambiguity in the sign of expression (13). For most cases though, the sign can be easily shown to be positive.

The quantity traded in state L does not directly depend on the fact that we use the dynamic bargaining formula. However, as the value functions depend on the equilibrium quantities in both states, there is actually an indirect effect. Generally we can expect that this effect will not be very significant (see Ennis [7]).

In summary, introducing dynamic bargaining in the model has important implications for the quantities traded, and hence the equilibrium price level. In fact, because the quantities in the low-price state will tend to be lower, we can conclude that forward-looking bargaining tends to reduce price volatility within this (extrinsic uncertainty) equilibrium.

4 Conclusions

In this paper I have developed a formula for the limiting immediate trade solution to the alternating-offers bargaining game (as the time interval between offers goes to zero) in an economy subject to stochastic dynamics. The aggregate state variable is assumed to follow a Poisson process defined over a finite set of possible values. The formula is relatively simple and intuitive. Although this dynamic bargaining solution has strong similarities with traditional Nash bargaining, there are important differences. I showed that as agents anticipate the switch in the state, they modify their reservation values for closing a deal during negotiations. This results in a different final outcome of the game.

To suggest the broad applicability of these results, I present examples of economies for which this bargaining rule applies. In particular, I analyze in depth the implications of dynamic bargaining in decentralized exchange economies. In these examples, aggregate variables are not "sticky" in that they can *jump* as *sunspots* realizations coordinate agents among possible equilibrium outcomes. It is very clear that the case of sunspot equilibrium is only one of the many possible applications of this sophisticated bargaining solution concept. In fact, shocks to fundamentals that follow our specific stochastic dynamics can easily be studied – with only a slight modification in our formula.

To start characterizing the consequences of the new bargaining solution, I introduce a benchmark case, the traditional splitting-the-cake problem. In this case, the impatient agent gets a smaller share from negotiations when the size of the cake is relatively small and there is a given probability that it will get larger in the near future. This is primarily a partial-equilibrium example and the uncertainty over the size of the cake is assumed to be exogenous (and in a way, purely *intrinsic*).

In the third section of the paper, I present a pair of fully specified economies where agents get matched and bargain over the splitting of a surplus that they jointly generate. These examples are interesting on their own. For the first application, I introduced a complete economy with bilateral production matching and wage bargaining. In this economy, a trade externality generates the possibility of sunspot equilibria as a result of a coordination failure. One of the main ideas illustrated by this application is that the effect of macroeconomic fluctuations over bargaining strongly depends on the agents' relative positions in the match. If the agents in a match are symmetric in all other respects, then the differences in discount factors play a very important role. In particular, when the discount rates are the same (so that agents have equivalent bargaining power), the bargaining outcomes are immune to sunspots and the sunspot equilibria are trivial randomizations over certainty equilibria. However, when the discount rates differ, similar conclusions to those in the splitting-the-cake problem are reached: if workers are more impatient than entrepreneurs, one would predict higher volatility in the wage bill (as a proportion of income) due to the forward-looking bargaining rule and the sunspot dynamics.

When agents have asymmetric positions in the bargaining game, the implications of the theory are less apparent, but still consequential. The second application in the paper is an attempt to understand one of these cases. It consists of a monetary random matching economy with bargaining and sunspots. The bargaining solution method proposed in this paper is applied to the example. I study the possible effects of dynamic bargaining and macroeconomic fluctuations upon state-contingent prices. Two types of matches need consideration in this case, barter matches (double coincidence of wants and no money) and monetary matches (single coincidence of wants and money). Again, the players' relative positions for negotiation are critical. In the barter match where agents are completely symmetric, sunspots fluctuations do not alter the bargaining solution. However, in monetary matches, agents are fundamentally different (one is a seller and the other is a buyer). In such a situation, price volatility will tend to be reduced in equilibrium when the presence of forward-looking bargainers is taken into consideration.

References

[1] Binmore, K., Nash Bargaining Theory II, in "The Economics of Bargaining," ed. K. Binmore and P. Dasgupta, Basil Blackwell, Oxford (1987), 61-76.

[2] Cass, D. and K. Shell, Do Sunspots Matter?, *Journal of Political Economy* **91** (1983), 193-227.

[3] Coles, M. and A. Muthoo, Bargaining Equilibrium in a Non-Stationary Environment, *mimeo*, University of Essex (2001).

[4] Coles, M. and R. Wright, A Dynamic Equilibrium Model of Search, Bargaining, and Money, *Journal of Economic Theory* **78** (1998), 32-54.

[5] Drazen, A., Self-Fulfilling Optimism in a Trade-Friction Model of the Business Cycle, *American Economic Review Papers and Proceedings* **87** (1988), 369-372.

[6] Diamond, P., Aggregate Demand Management in Search Equilibrium, *Journal of Political Economy* **90** (1982), 881-94.

[7] Ennis, H., On Random Matching, Monetary Equilibria and Sunspots, *Macroeconomic Dynamics* **5** (2001), 132-42.

[8] Howitt, P. and P. McAfee, Stability of Equilibria with Externalities, *Quarterly Journal of Economics* **103** (1988), 261-77.

[9] Howitt, P. and P. McAfee, Animal Spirits, *American Economic Review* **82** (1992), 493-507.

[10] Kiyotaki, N. and R. Wright, On Money as a Medium of Exchange, *Journal of Political Economy* **97** (1989), 927-54.

[11] Merlo, A. and C. Wilson, A Stochastic Model of Sequential Bargaining with Complete Information, *Econometrica* **63** (1995), 371-399.

[12] Mortensen, D., Equilibrium Unemployment Dynamics, *International Economic Review* **40** (1999), 889-914.

[13] Muthoo, A., "Bargaining Theroy with Applications," Cambridge University Press, Cambridge, U.K., 1999.

[14] Nash, J., The Bargaining Problem, *Econometrica* **18** (1950), 155-162.

[15] Rubinstein, A., Perfect Equilibrium in a Bargaining Model, *Econometrica* **50** (1982), 97-109.

[16] Rubinstein, A. and A. Wolinsky, Equilibrium in a Market with Sequential Bargaining, *Econometrica*, **53** (1985), 1133-50.

[17] Shell, K., Monnaie et allocation intertemporelle, Séminaire d'Econométrie Roy-Malinvaud, Paris, November, 1977 [Title and abstract in French. Text in English] Translation forthcoming in *Macroeconomic Dynamics*.

[18] Shell, K., Sunspot Equilibrium, in "The New Palgrave," Vol. 4, pp. 549-551, Macmillan & Co., New York, 1987.

[19] Shi, S., Money and Prices: A Model of Search and Bargaining, *Journal of Economic Theory* **67** (1995), 467-496.

[20] Trejos, A. and R. Wright, Search, Bargaining, Money and Prices, *Journal of Political Economy* **103** (1995), 118-141.

[21] Wallace, N., Absence-of-Double-Coincidence Models of Money: A Progress Report, Fed. Res. Bank of Mpls. *Quarterly Review*, Winter (1997).

[22] Wright, R., A Note on Sunspot Equilibria in Search Models of Fiat Money, *Journal of Economic Theory* **64** (1994), 234-241.

Appendix 1

In what follows, it is shown that $h_s(\Delta) \equiv \widehat{x}_{2s}(\Delta) - \widehat{x}_{1s}(\Delta)$ converges to zero at rate Δ as Δ approaches zero. First note that from (1),

$$u_1(\widehat{x}_{1s};\theta_s) - u_1(\widehat{x}_{2s};\theta_s) = -\Delta r_1 u_1(\widehat{x}_{1s};\theta_s) + \Delta \sum_{s'\in S\setminus s} \pi_{ss'}[u_1(\widehat{x}_{2s'};\theta_{s'}) - u_1(\widehat{x}_{2s};\theta_s)]. \tag{14}$$

Clearly, from this expression, we have

$$u_1(\widehat{x}_{1s};\theta_s) - u_1(\widehat{x}_{2s};\theta_s) \to 0 \tag{15}$$

as $\Delta \to 0$. Since $u_1(\bullet;\theta_s)$ is continuous and strictly increasing, this implies that we have $\widehat{x}_{1s}(\Delta) \to \widehat{x}_{2s}(\Delta)$. As a consequence, $h_s(\Delta) \to 0$ as $\Delta \to 0$. It is not difficult to see that $\widehat{x}_{is}(\Delta)$ converges ($i = 1, 2$ and $s \in S$): $\widehat{x}_{is}(\Delta)$ is a differentiable function of Δ with bounded derivative and hence uniformly continuous; this guarantees that $\lim_{\Delta\to 0} \widehat{x}_{is}(\Delta)$ exists. Then, by the continuity of u_1 again,

$$\begin{aligned}
\lim_{\Delta\to 0} \frac{u_1(\widehat{x}_{1s};\theta_s) - u_1(\widehat{x}_{2s};\theta_s)}{\Delta} &= -r_1 u_1(\widehat{x}_s;\theta_s) + \sum_{s'\in S\setminus s} \pi_{ss'}[u_1(\widehat{x}_{s'};\theta_{s'}) - u_1(\widehat{x}_s;\theta_s)] \\
&= \gamma_{1s} \in \mathbb{R}.
\end{aligned} \tag{16}$$

We also know that

$$\lim_{h_s(\Delta)\to 0} \frac{u_1(\widehat{x}_{2s} + h_s(\Delta);\theta_s) - u_1(\widehat{x}_{2s};\theta_s)}{h_s(\Delta)} = \gamma_{2s} \in \mathbb{R}$$

because $u_1(\bullet;\theta_s)$ is differentiable. Finally, since $h_s(\Delta)$ is a continuous function of Δ, we can write

$$\lim_{\Delta\to 0} \frac{u_1(\widehat{x}_{2s} + h_s(\Delta);\theta_s) - u_1(\widehat{x}_{2s};\theta_s)}{\Delta} \frac{\Delta}{h_s(\Delta)} = \gamma_{2s}. \tag{17}$$

Substituting (16) in (17), we get

$$\gamma_{1s} \lim_{\Delta\to 0} \frac{\Delta}{h_s(\Delta)} = \gamma_{2s},$$

which says that $h_s(\Delta) = O(\Delta)$.

Appendix 2

In this appendix we show that $q_{1s} = q_{2s}$ with $s = H, L$ in a *barter match* for the monetary economy of Section 2.2.

Suppose not. Suppose, without loss of generality, that we have $q_{1s} < q_{2s}$ for some s and every possible $\pi_{ss'}$. Define $A_{ijs} \equiv u(q_{is}) - c(q_{js}) + V_{0s}$ for $i \neq j$ and $i, j = 1, 2$. Also define

$$\xi_s \equiv \frac{(r + \pi_{ss'})A_{21s} - \pi_{ss'}A_{21s'}}{(r + \pi_{ss'})A_{12s} - \pi_{ss'}A_{12s'}}.$$

From expression (10) we have,

$$\frac{u'(q_{1s})}{c'(q_{1s})} = \frac{c'(q_{2s})}{u'(q_{2s})} = \xi_s < 1.$$

Since $A_{21s} > A_{12s}$ and $\xi_s < 1$ we have that $A_{21s'} > A_{12s'}$. This implies that $q_{1s'} < q_{2s'}$ and hence that $\xi_{s'} < 1$. Summarizing, we have that *(i)* $\xi_s < 1$,*(ii)* $\xi_{s'} < 1$,*(iii)* $A_{21s} > A_{12s}$, and *(iv)* $A_{21s'} > A_{12s'}$. We show now that these four inequalities can not all hold at the same time. From $\xi_{s'} < 1$ we obtain that

$$A_{21s'} < A_{12s'} + \frac{\pi_{s's}}{r + \pi_{s's}}(A_{21s} - A_{12s}).$$

Replacing this in the inequality given by $\xi_s < 1$ we have that

$$\left[\frac{r + \pi_{ss'}}{\pi_{ss'}} - \frac{\pi_{s's}}{r + \pi_{s's}}\right](A_{21s} - A_{12s}) < 0,$$

which implies that $A_{21s} < A_{12s}$. This contradicts inequality *(iii)* above. In other words, by assuming that $q_{1s} \neq q_{2s}$ for some s, we have reached a contradiction. Hence, we conclude that $q_{1s} = q_{2s}$ for all s.

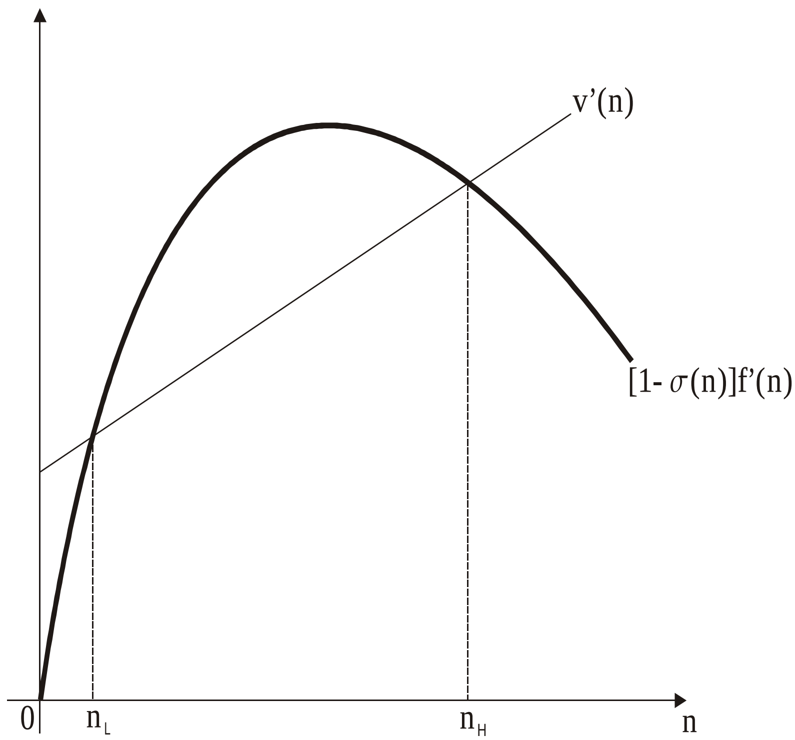

Figure 1: Multiplicity of Equilibria