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Heterogeneity and Aggregation in the Labor Market: Implications for Aggregate Preference Shifts*

Federal Reserve Bank of Richmond Working Paper 03-17

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Abstract:

The cyclical behavior of hours of work, wages, and consumption does not conform with the prediction of the representative agent with standard preferences. The residual in the intra-temporal first-order condition for commodity consumption and leisure is often viewed as a failure of labor-market clearing. We show that a simple heterogeneous agent economy with incomplete markets and indivisible labor generates an aggregation error that looks much like the preference residual in aggregate data. Our results caution against viewing the preference residual as a failure of labor-market clearing or a fundamental driving force of business cycles.

JEL Classification: E24, E32, J21, J22
Key Words: Aggregation, Heterogeneity, Incomplete Market, Preference Shifts

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1 Introduction

The equilibrium outcome of the representative-agent model with standard preferences is hard to reconcile with the joint behavior of aggregate hours of work, consumption, and wages over the business cycle (Barro and King, 1984; Mankiw, Rotemberg and Summers, 1989). The accounting method that uses the optimality condition for the choice between hours of work and leisure measures a significant residual in preference (e.g., Hall, 1997). This is often viewed as a failure of the labor-market clearing.[1]

In this paper, we show that a heterogeneous-agent economy with incomplete capital market and indivisible labor can generate an aggregation error that looks like a preference shock. Although there are no inherent preference shifts, in the face of aggregate technology shocks, the model generates an aggregation error that is comparable to the preference residual in aggregate data: it is highly correlated with hours of work and as volatile as the cyclical component of hours. Our results caution against viewing the preference residual as a failure of labor-market clearing or a fundamental driving force of business cycles.

Our analysis quantitatively complements the theoretical work by Sheinkman and Weiss (1986) who show that capital-market incompleteness can lead to a stochastic term in aggregate preferences.[2] According to our model, however, an incomplete capital market alone cannot generate an aggregation error quantitatively comparable to the preference residual in the data. With divisible labor, hours of work, in response to aggregate shocks, are highly correlated across households, allowing for a fairly good aggregation. Indivisible labor alone cannot account for the preference residual in the data either. Entry and exit of workers with heterogeneous skill creates a compositional bias in aggregate wages and hours (Bils, 1985; Solon, Barsky and Parker, 1989). The composition bias, however, has an impact mostly on the magnitudes, not the correlations. In the data, the preference

[1] For example, Mankiw et al. proposed four hypotheses: (1) measurement errors (2) aggregation (3) time-varying preference (4) failure of market clearing. We discuss each of these within the context of heterogeneous dynamic general equilibrium model in Section 4 and 5.

[2] It also complements Nakajima (2003) who derives aggregate preference shocks and the variation of TFP in a two-type households model with capacity utilization and government spending shocks.

residual arises not only because hours of work moves little relative to wages, but also because it is not strongly correlated with wages. The aggregation error is significantly magnified when the incomplete market is combined with indivisible labor.

The paper is organized as follows. Section 2 briefly discusses the preference residual in the data. Section 3 lays out the model economy. In Section 4, we calibrate the model economy, compute the aggregation error, and compare them to the preference residual in the data. In Section 5, we distinguish the roles of market incompleteness and indivisible labor by investigating the economies with and without the complete market and indivisible labor. Section 6 is the conclusion.

2 Preference Residual in Aggregate Data

One of the leading topics in macroeconomics is the identification of the fundamental driving forces behind economic fluctuations. Economists adopt accounting procedures that use aggregate time series data together with equilibrium conditions of a prototype model. A prominent example is Solow residual. Under the assumptions about market competition and the production function, shifts in technology are identified (e.g., Kydland and Prescott, 1982).[3] Likewise, the standard optimality condition on the choice between commodity consumption and leisure (together with the assumption that household efforts are paid for by their marginal product) identifies the preference residual (Hall, 1997). To illustrate, consider a representative agent who maximizes the utility under the competitive labor market:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - B \frac{H_t^{1+1/\gamma}}{1+1/\gamma} \right\},$$

where β is the discount factor, C_t is consumption, and H_t is hours worked.[4] The parameter γ represents the compensated labor-supply elasticity and B is the utility parameter. Given the real wage rate W_t, hours of work and consumption satisfy the static intra-temporal first-order condition

[3] The Solow residual is subject to specification errors due to, for example, imperfect competition, cyclical utilization, and aggregation across firms (e.g., Hall, 1987.

[4] While this form of utility is popular in both business cycle analysis and empirical labor supply literature, time non-separability does not account for the joint behavior of hours, wages and consumption in the data (Mankiw, Rotemberg and Summers, 1989).

(which is also referred to as Frisch labor-supply function):

$$BH_t^{1/\gamma} = \frac{W_t}{C_t}. \tag{1}$$

Figure 1 exhibits the cyclical components of total hours worked H and real wage-consumption W/C—marginal disutility from work according to (1)—for U.S. for 1948:I-2003:II.[5] Both series are detrended by the Hodrick-Prescott filter. The quantity and price of hours tend to move in opposite directions (the correlation coefficient is -0.53), suggesting a serious departure from a labor-supply schedule. This is often viewed as a failure of labor-market clearing or existence of preference shocks. Under the assumption that the aggregate labor-supply elasticity γ is 1, we compute the implied time series of B that accommodates (1).[6] Figure 2 shows the cyclical components preference residual $(1/B)$. As the residual is highly correlated with labor supply and its volatility is in the same order of magnitude as hours worked, it is often listed as a candidate driving force of economic fluctuations (Hall, 1997; Chari, Kehoe and McGrattan, 2002). A strong interpretation of the residual would be shifts in marginal rate of substitution between commodity consumption and leisure at the aggregate level (e.g., fads, changes in home production technology, demographic changes). A weak interpretation would be labor-market frictions, the extent to which the labor market deviates from a competitive equilibrium.[7] In the next section, we present a model economy in which the preference residual arises as an aggregate error.

[5]Hours represent the total hours employed in the non-agricultural sector. Labor productivity is used for W as in Hall (1997); Expenditure on nondurable goods and services for C. The use of real wage instead of labor productivity does not change the conclusion of the paper.

[6]This elasticity is higher than a typical estimate in the micro data (e.g., MaCurdy, 1981), which is less than 0.5. Assuming an inelastic labor supply (a smaller value of γ) leads to a bigger residual. Hall (1997) used $\gamma = 1/1.7$. Conversely, elastic labor supply produces a smaller residual. However, a particular choice of γ does not resolve the issue. The preference residual arises not only because hours of work moves little relative to its price but also because it moves in the opposite direction to its marginal disutility.

[7]We note two important premises in the calculation of the preference residual: the competitive labor market and the aggregation. In this paper, we focus on the latter only. This does not necessarily reflect our view on the importance of noncompetitive natures in the labor market. We maintain the competitive market to isolate the effect of aggregation error.

3 The Model

There is a continuum (measure one) of workers who have identical preference but different productivity. Individual productivity varies exogenously according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. A worker maximizes his utility over consumption c_t and hours of work h_t:

$$\max_{\{c_t,h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - B \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right\} \tag{2}$$

subject to

$$a_{t+1} = w_t x_t h_t + (1+r_t)a_t - c_t. \tag{3}$$

Workers trade claims for physical capital, a_t, which yields the rate of return r_t and depreciates at δ. The capital market is incomplete. Physical capital is the only asset available to workers who face a borrowing constraint: $a_t \geq \bar{a}$ for all t. Following Hansen (1985) and Rogerson (1988), we abstract from the intensive margin and assume that labor supply is indivisible; i.e., h_t takes either zero or $\bar{h}(<1)$. If employed, a worker supplies $\bar{h}$ unit of labor and earns $w_t x_t \bar{h}$, where w_t is wage rate per effective unit of labor.[8]

The representative firm produces output according to a constant returns Cobb-Douglas technology in capital, K_t, and efficiency units of labor, L_t:[9]

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha},$$

where λ_t is the aggregate productivity shock with a transition probability distribution function $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$.[10]

[8] In general, the labor supply decision operates on both the extensive and the intensive margins. However, it is rare for workers to be allowed to choose completely flexible work schedules or to supply a tiny amount of hours. Furthermore, it is well known that the variation in the number of employees is the dominant source of fluctuations in total hours worked (e.g., Heckman, 1984).

[9] This implicitly assumes that workers are perfect substitutes for each other. While this assumption abstracts from reality, it greatly simplifies the labor-market equilibrium.

[10] In this model economy, aggregate productivity shock is the only source of economic fluctuations. However, this

The value function for an employed worker, denoted by V^E, is:

$$V^E(a,x;\lambda,\mu) = \max_{a'\in\mathcal{A}} \Big\{ \ln c - B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma} + \beta E\Big[\max\big\{V^E(a',x';\lambda',\mu'), V^N(a',x';\lambda',\mu')\big\}\big|x,\lambda\Big]\Big\} \tag{4}$$

subject to

$$c = wx\bar{h} + (1+r)a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbf{T}(\lambda,\mu),$$

where $\mathbf{T}$ denotes a transition operator that defines the law of motion for the distribution of workers $\mu(a,x)$.[11] The value function for a non-employed worker, $V^N(a,x;\lambda,\mu)$, is defined similarly with $h=0$. Then, the labor-supply decision is $V(a,x;\lambda,\mu) = \max_{h\in\{0,\bar{h}\}}\big\{V^E(a,x;\lambda,\mu), V^N(a,x;\lambda,\mu)\big\}$

Equilibrium consists of a set of value functions, $\{V^E(a,x;\lambda,\mu),\ V^N(a,x;\lambda,\mu),\ V(a,x;\lambda,\mu)\}$, a set of decision rules for consumption, asset holdings, and labor supply, $\{c(a,x;\lambda,\mu),\ a'(a,x;\lambda,\mu),\ h(a,x;\lambda,\mu)\}$, aggregate inputs, $\{K(\lambda,\mu),\ L(\lambda,\mu)\}$, factor prices, $\{w(\lambda,\mu),\ r(\lambda,\mu)\}$, and a law of motion for the distribution $\mu' = \mathbf{T}(\lambda,\mu)$ such that:

1. Individuals optimize:

   Given $w(\lambda,\mu)$ and $r(\lambda,\mu)$, the individual decision rules $c(a,x;\lambda,\mu)$, $a'(a,x;\lambda,\mu)$, and $h(a,x;\lambda,\mu)$ solve $V^E(a,x;\lambda,\mu)$, $V^N(a,x;\lambda,\mu)$, and $V(a,x;\lambda,\mu)$

2. The representative firm maximizes profits:

$$w(\lambda,\mu) = F_1\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big) \tag{5}$$

$$r(\lambda,\mu) = F_2\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big) - \delta \tag{6}$$

   for all (λ,μ).

does not reflect our view on the cause of business cycles. As we would like to show that the preference residual contains a significant specification error rather than true shifts in aggregate labor supply, we intentionally exclude the shocks that may shift the labor supply itself (e.g., shifts in government spending or changes in income tax rate) here.

[11]Let $\mathcal{A}$ and $\mathcal{X}$ denote sets of all possible realizations of a and x, respectively. The measure $\mu(a,x)$ is defined over a σ-algebra of $\mathcal{A}\times\mathcal{X}$.

3. The goods market clears:

$$\int \{a'(a,x;\lambda,\mu) + c(a,x;\lambda,\mu)\} d\mu = F\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big) + (1-\delta)K \tag{7}$$

for all (λ, μ).

4. Factor markets clear:

$$L(\lambda,\mu) = \int x h(a,x;\lambda,\mu) d\mu \tag{8}$$

$$K(\lambda,\mu) = \int a d\mu \tag{9}$$

for all (λ, μ).

5. Individual and aggregate behaviors are consistent:

$$\mu'(A^0, X^0) = \int_{A^0, X^0} \left\{ \int_{\mathcal{A},\mathcal{X}} \mathbf{1}_{a'=a'(a,x;\lambda,\mu)} \; d\pi_x(x'|x) d\mu \right\} da' dx' \tag{10}$$

for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

4 Quantitative Analysis

4.1 Calibration

We briefly explain the choice of the model parameters. The details can be found in Chang and Kim (2003) in which a similar model economy is used to study the mapping from individual to aggregate labor supply elasticities. The unit of time is a quarter. We assume that x follows an AR(1) process: $\ln x' = \rho_x \ln x + \varepsilon_x$, where $\varepsilon_x \sim N(0, \sigma_x^2)$. As we view x reflecting a broad measure of earnings ability in the market, individual wages from the PSID for 1971-1992 are used to estimate the stochastic process of x. We set $\rho_x = 0.95$ and $\sigma_x = 0.225$.[12]

Other parameters are in accordance with the business cycle analysis and empirical labor supply literature. A working individual spends one-third of discretionary time: $\bar{h} = 1/3$. The individual compensated labor-supply elasticity of hours, γ, is 0.4. The labor share of output, α, is 0.64, and

[12] In Chang and Kim (2003) we report various estimates for ρ_x and σ_x, controlling for selection bias and individual effects. The aggregate time series behavior of consumption, hours of work, and wages—which is our primary interest in this paper—is not sensitive to the values of ρ_x and σ_x we considered.

the depreciation rate, δ, is 2.5 percent. We search for the weight parameter on leisure, B, such that the steady state employment rate is 60 percent, the average from the CPS for 1967:II-2000:IV. The discount factor β is chosen so that the quarterly rate of return to capital is 1 percent. Aggregate productivity shock, λ_t, follows an AR(1) process: $\ln \lambda' = \rho_x \ln \lambda + \varepsilon_\lambda$, where $\varepsilon_\lambda \sim N(0, \sigma_\lambda^2)$. We set $\rho_\lambda = 0.95$ and $\sigma_\lambda = 0.007$ following Kydland and Prescott (1982). Table 1 summarizes the parameter values of the benchmark economy.

4.2 Cross-sectional Earnings and Wealth Distribution

As we investigate the aggregation issue, we desire for the model economy to possess a reasonable heterogeneity. We compare the cross-sectional earnings and wealth—two important (observable) dimensions of heterogeneity in the labor market—between the model and the data.

Table 2 summarizes the detailed information on wealth and earnings from the PSID and the model. Family wealth in the PSID (1984 survey) reflects the net worth of house, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. For each quintile group of wealth distribution, we calculate the wealth share, the ratio of group average to economy-wide average, and the earnings share.

Both in the data and model, the poorest 20 percent of the wealth distribution owns virtually nothing. In fact, households in the first quintile of the wealth distribution are in debt both in the model and data. On the contrary, households in the 4th and 5th quintile of the PSID own 18.74 and 76.22 percent of total wealth, respectively. According to the model, they own 24.48 and 66.31 percent, respectively. The average wealth of the 4th and 5th quintile are, respectively, 0.93 and 3.81 times larger than that of a typical household, while these ratios are 1.22 and 3.33 according to our model. The 4th and 5th quintile groups of the wealth distribution earn, respectively, 24.21 and 38.23 percent of total earnings in the PSID. The corresponding groups earn 23.44 and 30.39 percent, respectively, in the model.

Overall, the wealth distribution is more skewed in the data. In particular, the model fails to match the highly concentrated wealth in the right tail of the distribution. About half of the total wealth is held by the top 5 percent of the population (not shown in Table 2). In our model, only 20 percent of the total wealth is held by them. However, our primary objective is not to explain the behavior of the top 1 or 5 percent of the population. We argue that the model economy possesses reasonable heterogeneity to study aggregation effects in the labor market.

4.3 Cyclical Properties of the Model

We numerically solve the equilibrium of the model using the "bounded rationality" method of Krusell and Smith (1998), and generate the aggregate time series through simulations.[13]

Table 3 shows the volatility of key aggregate variables from our model economy. In the face of aggregate productivity shocks, the model exhibits output volatility of 1.31, accounting for about two-thirds of cyclical output variations in the data (2.04), similar to the findings based on the standard representative-agent models (e.g., Kydland and Prescott, 1982). Other statistics are also similar to those from the standard models: consumption is about half as volatile as output and investment is about three to four times as volatile as output.

A distinguishing feature of our model is the labor market. The (relative) volatility of hours and productivity are very close to those in the data, which is hard to achieve in a standard model unless unreasonably high labor-supply elasticity is assumed. The volatility of hours relative to output is 0.84, almost identical to that in the data (0.85), and the volatility of labor productivity relative to output is 0.57, again close to that in the data (0.50). This is due to the composition effect. On average, less-productive workers tend to participate in the labor market during expansions and exit during recessions, creating a pro-cyclical composition bias in measured hours (relative to the effective unit of hours) and a counter-cyclical bias in aggregate wages (Bils, 1985; Solon, Barsky and Parker, 1995). When we measure the hours in efficiency unit, hour of work is about half as volatile as output—which is similar to that in the representative-agent model. While the composition effect allows for the model to match the relative volatility better, it cannot account for the preference

[13] The model generated aggregated time series is based on the individual choices of 5,000 agents over 3,500 periods.

residual in the data. We discuss this in detail in Section 5 when we compare the economies with and without the complete capital market.

Table 4 shows the correlations. The correlations among output, consumption, and investment are higher than those in the data, a feature common in the standard RBC models. However, the correlations of hours and labor productivity with output are 0.85 and 0.55, respectively, close to those in the data. In particular, hours and labor productivity exhibit virtually no correlation (0.04). This is surprising because a near-zero correlation between hours and labor productivity has been noted as one of the most salient failures of the RBC models.

4.4 Preference Residual from the Model

If we were to account for the model-generated aggregate time series of hours, wages, and consumption by a competitive equilibrium of a representative agent, we confront the same problem: hours of work and marginal disutility from working, measured by W/C, do not strongly co-move. If fact, they are negatively correlated as in the data. We compute the preference residual from the model-generated aggregate data under $\gamma = 1$ based on (1) as we did from the actual data.[14]

Figure 3 shows hours and preference residual ($1/B$) from the model. The residual (due to aggregation error) in the model exhibits similar stochastic properties with that in the data. The standard deviation of the residual relative to output in the model is 1.18, close to 1.23 in the data. In absolute terms, the residual in the model is about two-thirds as volatile as those in the data. The correlation between the residual and total hours worked in the model is -0.94, close to -0.92 in the data. The correlation between total hours and aggregate wage to consumption ratio is -0.54, very close to -0.53 in the data.

Despite there being no inherent preference shifts, (in the face of aggregate technology shocks) the model generates aggregation error that looks much like the preference residual in the data. Our result calls for some caution against viewing the preference residual as the fundamental driving

[14] In Chang and Kim (2003) we find that this type of economy exhibits an aggregate labor-supply elasticity around 1.

force of business cycles, as well as to the view that the residual reflects a failure of labor-market clearing.

5 Role of Incomplete Market and Indivisible Labor

The residual in our model stems from an incomplete capital market and indivisible labor. To investigate the marginal contributions of each, we consider three additional model economies. For comparison, we refer the benchmark economy to HII that stands for "Heterogeneity-Incomplete Market-Indivisible Labor."

5.1 Alternative Model Specifications

Heterogeneity + Complete Market + Indivisible Labor (HCI) The second model we consider allows for a complete capital market but maintains indivisible labor: "Heterogeneity-Complete Market-Indivisible Labor" (HCI). Thanks to perfect risk sharing, agents enjoy the same level of consumption regardless of their employment status, productivity, and asset holdings.[15]

Equilibrium of this economy is identical to the allocation of the social planner who chooses the sequence of aggregate consumption $\{C_t\}_{t=0}^{\infty}$ and cut-off productivity levels $\{x_t^*\}_{t=0}^{\infty}$ for labor-market participation. For an efficient allocation, the planner assigns workers who have comparative advantage in the market (more productive workers) to working. If a worker's productivity is above x_t^*, he supplies $\bar{h}$ hours of labor.

The planner's value function in the complete market, $V^C(K,\lambda)$, and the decision rules for consumption, $C(K,\lambda)$, and cut-off productivity, $x^*(K,\lambda)$, satisfy the following Bellman equation:

$$V^C(K,\lambda) = \max_{C,x^*}\left\{\log C - B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}\int_{x_t^*}^{\infty}\phi(x)dx + \beta E\Big[V^C(K',\lambda')\big|\lambda\Big]\right\} \tag{11}$$

subject to

$$K' = F(K,L,\lambda) + (1-\delta)K - C, \tag{12}$$

[15] The distribution of workers is no longer a state variable in the individual optimization problem. Moreover, due to the ergodicity of the stochastic process for idiosyncratic productivity, the cross-sectional distribution of workers is always stationary.

where $L = \bar{h}\int_{x^*}^{\infty} x\phi(x)dx$ is the aggregate efficiency unit of labor, and $\phi(x)$ is the productivity distribution of workers. The planner chooses the cut-off productivity x^* so that:

$$\frac{1}{C}F_L(K, L, \lambda)\bar{h}x^*\phi(x^*) = B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}\phi(x^*). \tag{13}$$

The left-hand side is the (society's) utility gain from assigning the marginal worker to production. There are $\phi(x^*)$ number of workers with productivity x^* in the economy. Each of them supplies $\bar{h}x^*$ units of effective labor and the marginal product of labor is F_L. The right-hand side represents the disutility incurred by those workers.

Heterogeneity + Incomplete Market + Divisible Labor (HID) The third model economy we consider allows for divisible labor supply, but the capital market is incomplete: "Heterogeneity-Incomplete Market-Divisible Labor (HID)". The equilibrium of this economy can be defined similarly to that of HII in Section 3—equations (5) through (10)—with the worker's value function with divisible labor, $V^D(a, x; \lambda, \mu)$:

$$V^D(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}, h \in (0,1)} \left\{ \ln c - B\frac{h^{1+1/\gamma}}{1+1/\gamma} + \beta E\Big[V^D(a', x'; \lambda', \mu')\big|x, \lambda\Big] \right\} \tag{14}$$

subject to

$$c = wxh + (1+r)a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbf{T}(\lambda, \mu).$$

Representative-Agent Model (RA) The last model we consider is "Representative-Agent (RA)" model. The value function of the representative agent, $V^R(K, \lambda)$, is:

$$V^R(K; \lambda) = \max_{C,H} \left\{ \ln C - B\frac{H^{1+1/\gamma}}{1+1/\gamma} + \beta E\Big[V^R(a', x'; \lambda', \mu')\big|x, \lambda\Big] \right\} \tag{15}$$

subject to

$$K' = F(K, H, z) + (1-\delta)K - C. \tag{16}$$

5.2 Comparison of Four Model Economies

Except for β and B, the same parameter values are used across the models. In the RA model β is 0.99 and B is chosen so that steady-state hours of work is the same as the aggregate hours in the benchmark economy, which is 0.2 ($= \bar{h} \times 60\%$). For the HCI model, β is 0.99 and B is chosen to be consistent with 60% employment along with $\bar{h} = 1/3$.[16] For HID, β and B are jointly searched to be consistent with average hours of 0.2 and a real rate of return of one percent in the steady state. The equilibrium of the HCI economy is solved by the Parameterized Expectation Algorithm of Marcet and Lorenzoni (1999). The equilibrium of HID economy is solved by the "bounded rationality" method. The equilibrium of RA is solved by a value function iteration.

Figure 4 exhibits the preference residuals from the four models. The residuals are computed using the labor-supply elasticity $\gamma = 0.4$ for divisible-labor economies (RA and HID), and $\gamma = 1$ for indivisible-labor economies (HCI and HII).[17] The residuals from the representative-agent (RA) are zero, as expected. The residual from the HCI reflects the aggregation error due to composition effect only. The residual from the HID indicates the aggregation error due to an incomplete market only. These residual are not large enough to account for the preference residual in the data. The volatilities (relative to output) are 0.2 for the HCI and 0.1 for the HID, which are only one sixth and one tenth, respectively, of that in the data.

An incomplete capital market alone cannot generate a large aggregation error. With divisible labor, hours of work are highly correlated across households, allowing for a fairly precise aggregation. Indivisible labor alone cannot account for the preference residual in the data either. Even under the complete market, aggregate hours and wages are subject to the composition bias due to entry and exit of workers with heterogeneous skill. On average, low-wage and less-skilled workers participate during expansions, which makes the aggregate wages less cyclical (e.g., Bils 1985; Solon, Barsky,

[16] Specifically, we find the steady state cut-off productivity x^* from the 60 percentile of $\phi(x)$: $\int_{x^*}^{\infty} x\phi(x)dx = .6$. Then, find B that satisfies (13).

[17] When labor supply is indivisible, the aggregate labor-supply elasticity can be very different from the individual elasticity: e.g., the aggregate elasticity depends on the reservation wage distribution. We use $\gamma = 1$ to make the statistics of the models to be compatible with those from the data. However, a particular choice of γ does not alter the main conclusion.

and Parker, 1995) and aggregate hours more cyclical than the hours measured in effective unit.[18] The composition bias, however, has an impact mostly on the volatility, not the correlation. In the data, the preference residual arises not only because hours move too little relative to wages, but also because it is not strongly correlated with wages.

Figure 5 shows the sample path (the percentage deviations from their sample averages) of aggregate hours across four models. The sample paths are comparable to each other because economies are under the identical sequence of aggregate technology shocks. Total hours of work fluctuates the most in the HII economy. Hours in HCI also moves a lot, whereas hours in HID moves closely with that in RA, suggesting that indivisible labor is more important than capital market incompleteness for the volatility of hours. Figure 6 plots the aggregate hours in efficiency units. Despite indivisible labor, the cyclical behavior of the effective unit of hours in HCI is identical to the hours worked in RA. The effective unit of hours is more volatile in HII than in HCI because under the incomplete market high-skilled workers (as well as low-skilled) perform inter-temporal substitution in labor-market participation; whereas under the complete market only those at the margin (whose productivity is around x^*, which is much lower than the average productivity of the existing workers) enter and exit the labor market in response to shifts in aggregate productivity.

Confronted with the inability of the standard equilibrium model to account for the joint behavior of aggregate hours of work, wages, and consumption, Mankiw, Summers and Rotemberg (1985) proposed four hypotheses: (1) measurement errors (2) aggregation (3) time-varying preference (4) failure of market clearing. Our analysis sheds some lights on these hypotheses: measurement errors (1) due to composition effect is not enough; (2) can be misinterpreted as (3); the residual does not necessarily reflect (4).

[18] Hansen (1993) computes effective unit of hours by controlling for worker characteristics based on the CPS. He finds that such adjustment does not resolve the discrepancy between the data and the standard RBC model. The quality adjustment based on micro data is limited because the observed characteristics accounts for only a fraction of worker's ability. For example, the R^2 of cross-sectional wage regression is usually below 0.4.

6 Summary

The cyclical behavior of aggregate hours of work, wages, and consumption is hard to reconcile with the equilibrium outcome of the representative-agent model with standard preference. The accounting method that uses aggregate time series, together with the optimality condition, measures a significant residual in preference. We show that a heterogeneous-agent economy with incomplete capital market and indivisible labor can generate an aggregation error which is quantitatively comparable to the preference residual in the data. Our result calls for some caution against viewing the residual as a fundamental driving force or a failure of labor-market clearing.

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Table 1: Parameters of the Benchmark Model Economy

Parameter	Description
$\alpha = 0.64$	Labor share in production function
$\beta = 0.979852$	Discount factor
$\gamma = 0.4$	Labor-supply elasticity
$B = 170.39$	Utility parameter
$\overline{h} = 1/3$	Labor supply if working
$\bar{a} = -2.0$	Borrowing constraint
$\rho_x = 0.95$	Persistence of idiosyncratic productivity shock
$\sigma_x = 0.225$	Standard deviation of innovation to idiosyncratic productivity
$\rho_\lambda = 0.95$	Persistence of aggregate productivity shock
$\sigma_\lambda = 0.007$	Standard deviation of innovation to aggregate productivity

Table 2: Characteristics of Wealth Distribution

	Quintile					
	1st	2nd	3rd	4th	5th	Total
PSID						
Share of wealth	-0.52	0.50	5.06	18.74	76.22	100
Group average/population average	-0.02	0.03	0.25	0.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
Model						
Share of wealth	-2.68	1.78	10.11	24.48	66.31	100
Group average/population average	-0.13	0.09	0.51	1.22	3.33	1
Share of earnings	10.79	15.94	19.43	23.44	30.39	100

Note: The PSID statistics reflect the family wealth and earnings from the 1984 survey.

Table 3: Volatilities of Aggregate Variables

Variable	Data	Model
σ_Y	2.04	1.31
σ_C/σ_Y	0.42	0.39
σ_I/σ_Y	3.59	3.02
σ_H/σ_Y	0.85	0.83
σ_L/σ_Y	–	0.54
σ_W/σ_Y	0.50	0.57
$\sigma_{W/C}/\sigma_Y$	0.55	0.48
σ_B/σ_Y	1.23	1.16

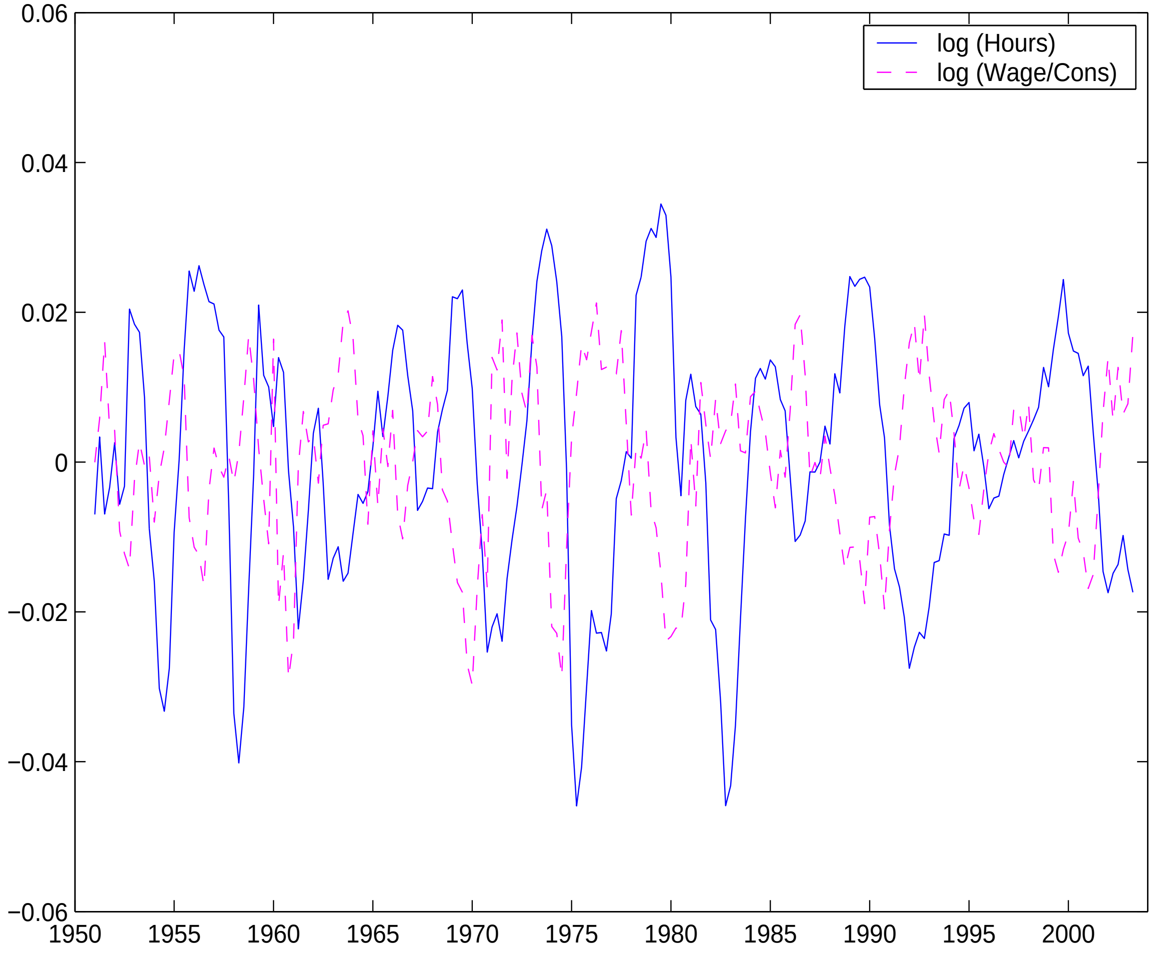
Note: All variables are logged and de-trended by the H-P filter. The volatility of output is measured by its standard deviation, and those of all other variables are by the relative standard deviations to output. The variable L denotes the effective unit of labor. The variable W denotes the average labor productivity.

Table 4: Correlations of Aggregate Variables

Variable	Data	Model
$cor(Y, C)$	0.77	0.84
$cor(Y, I)$	0.88	0.98
$cor(Y, H)$	0.87	0.85
$cor(Y, L)$	–	0.90
$cor(Y, W)$	0.52	0.55
$cor(H, W)$	0.03	0.04
$cor(H, W/C)$	-0.53	-0.54
$cor(B, H)$	-0.92	-0.94
$cor(B, Y)$	-0.64	-0.65

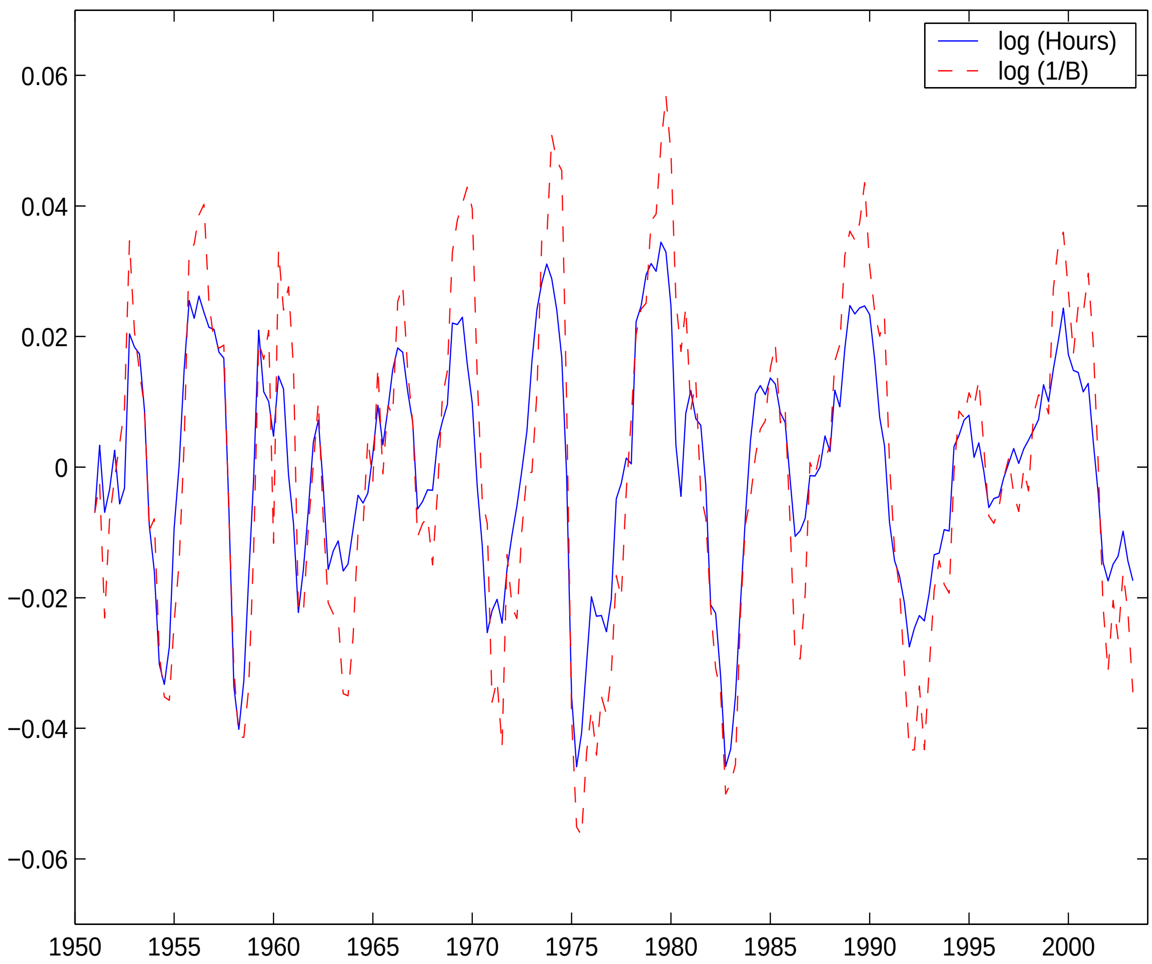
Note: See the note for Table 3 for variable descriptions.

Figure 1: Cyclical Components of Total Hours and Marginal Disutility from Work



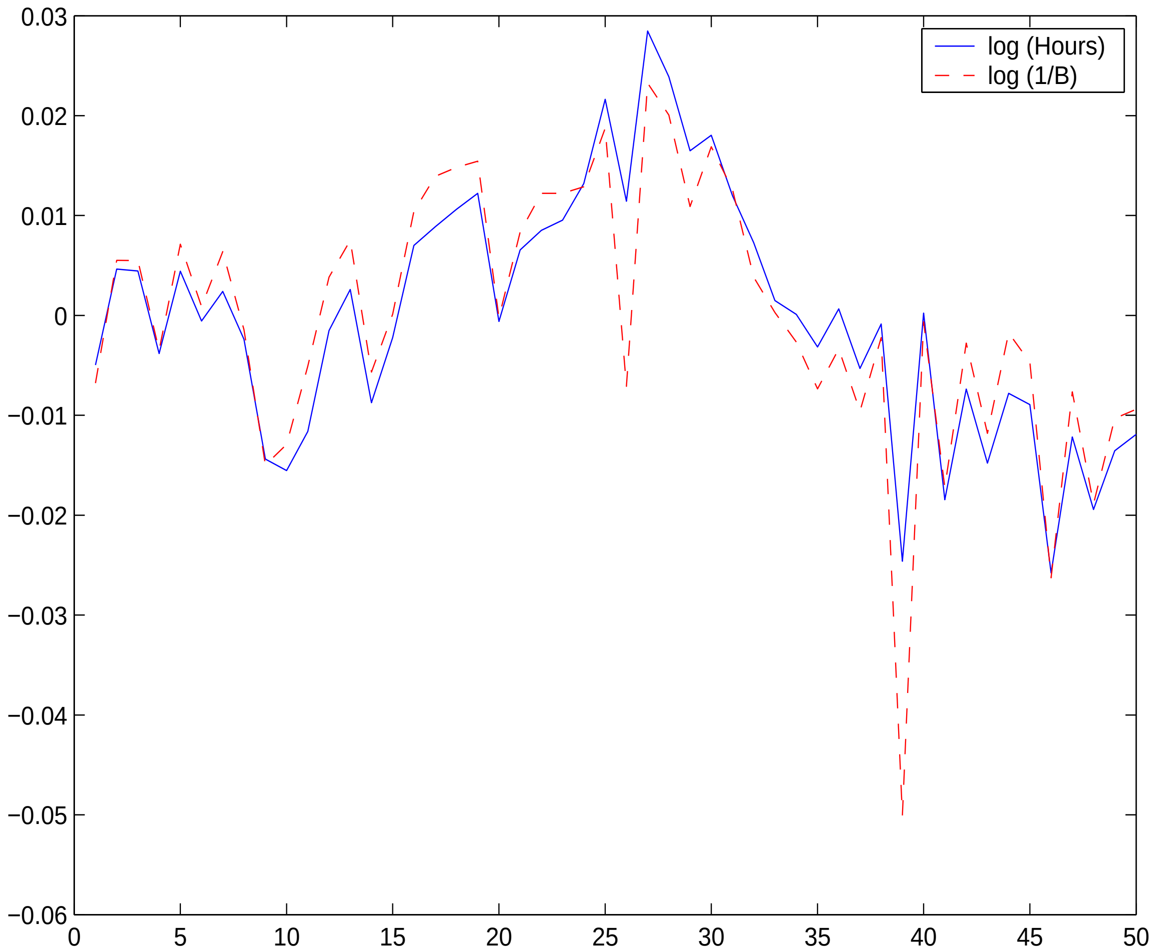
Note: Hours represent the total employed hours in non-agricultural private sector. Wage rate is measured by the labor productivity. Consumption refers to the expenditure on non-durable goods and services.

Figure 2: Cyclical Components of Preference Residual for U.S.



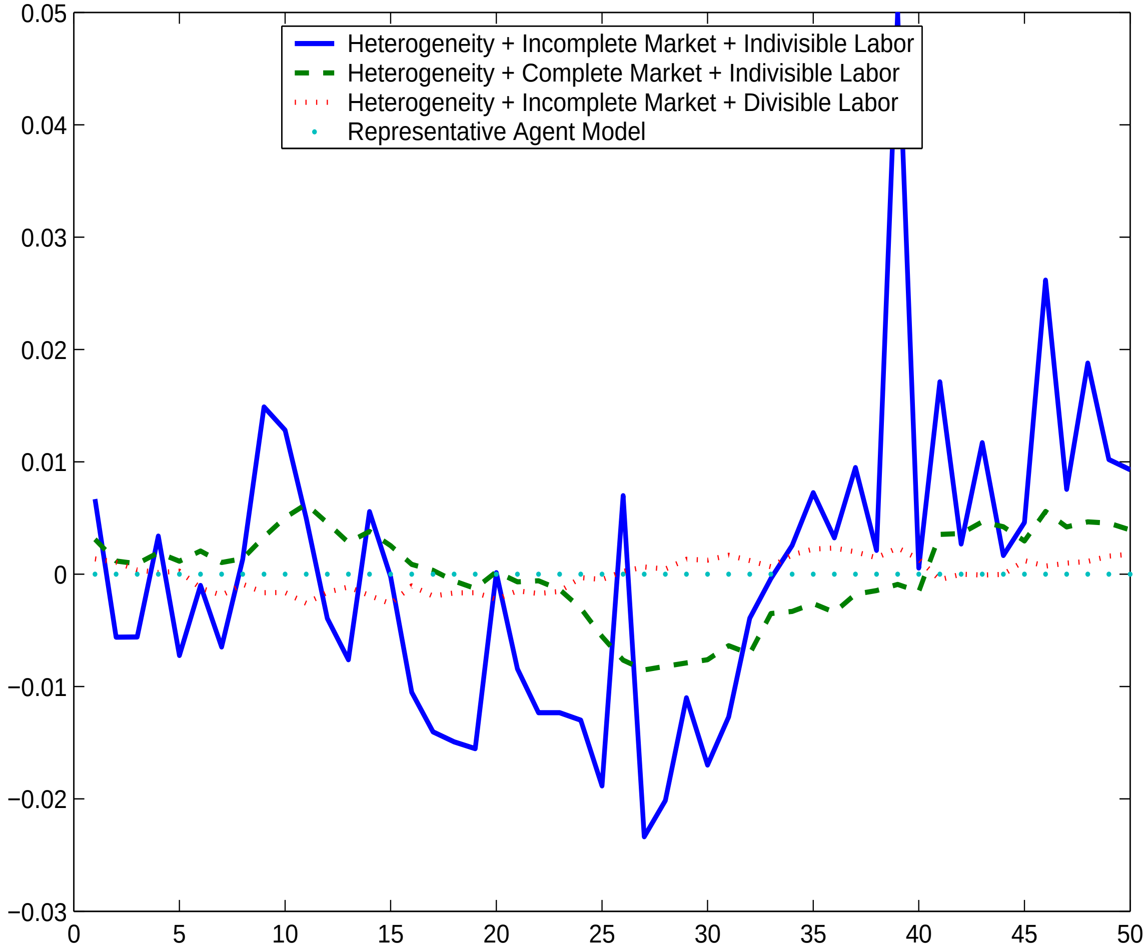
Note: Preference residual (B) is computed from (1) with the compensated labor-supply elasticity (γ) of 1.

Figure 3: Total Hours and Preference Residual from the Benchmark Model



Note: The preference residual (B) is computed from Equation (1) with the labor-supply elasticity (γ) of 1.

Figure 4: Preference Residuals (B) from the Models



Note: The preference residual (B) is computed from Equation (1). For divisible labor models, we use $\gamma = 0.4$; for indivisible labor models, we use $\gamma = 1$.

Figure 5: Total Hours of Work from the Models

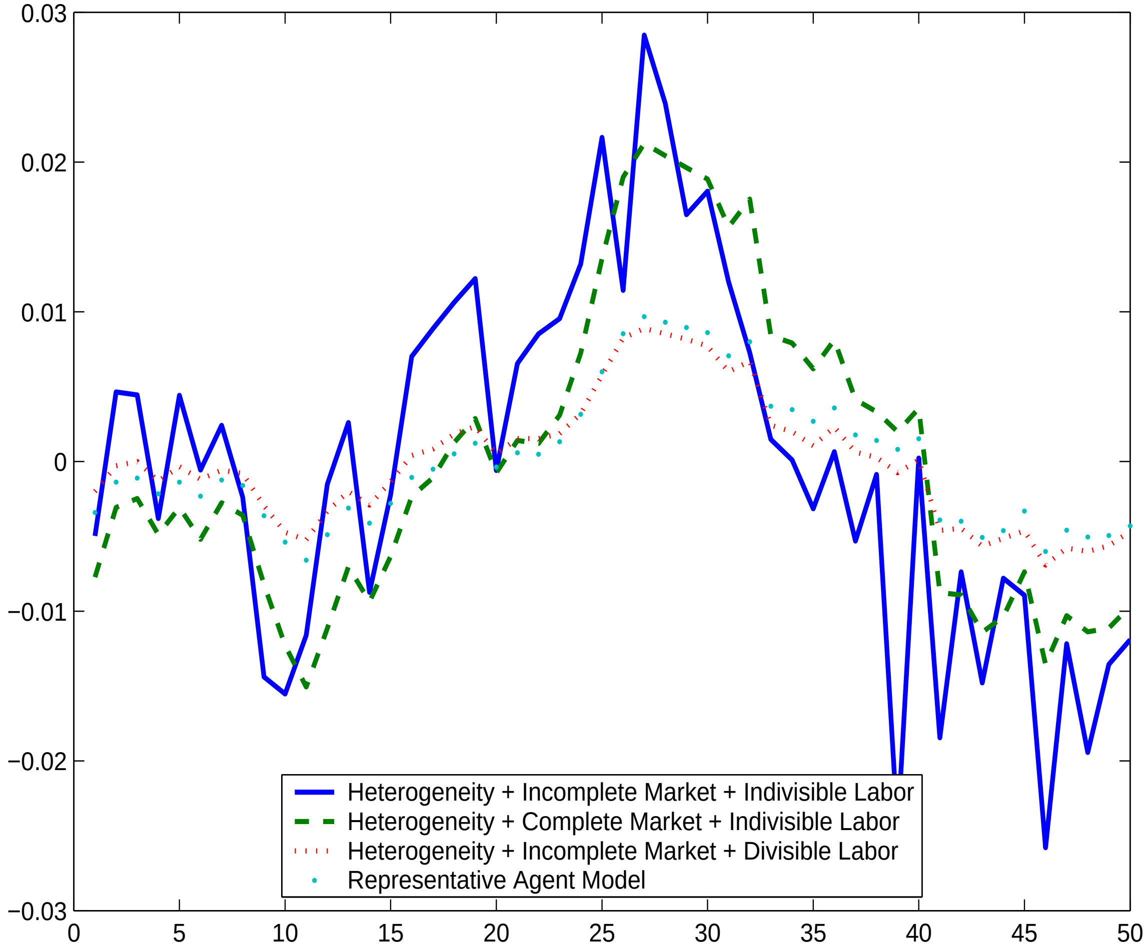


Figure 6: Effective Unit of Hours from the Models