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Labor Supply Shifts and Economic Fluctuations

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Abstract:

We propose a new VAR identification scheme that distinguishes *shifts of* and *movements along* the labor demand schedule to identify labor-supply shocks. According to our VAR analysis of post-war U.S. data, labor-supply shifts account for about 30 percent of the variation in hours and about 15 percent of the output fluctuations at business cycle frequencies. To assess the role of labor-supply shifts in a more structural framework, estimates from a dynamic general equilibrium model with stochastic variation in home production technology are compared to those from the VAR.

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Key Words: Labor-Supply Shifts, VAR, Home Production, Bayesian Econometrics

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1 Introduction

A leading question in macroeconomics is the identification of forces that cause the cyclical allocation of time. Modern dynamic general equilibrium analysis emphasizes shifts in labor demand due to technological change. Empirical studies on the decomposition of sources of business cycles by Shapiro and Watson (1988) and Hall (1997) have called for an attention to labor-supply shifts. This paper examines the importance of labor-supply shifts as a source of economic fluctuations.

First, we develop and apply a new identification procedure for vector autoregressions (VAR). It decomposes the fluctuations of aggregate hours and output into *movements along* the labor demand schedule and *shifts of* the schedule itself. The former is interpreted broadly as response to a labor-supply shock. Our VAR identification is based on the notion that an increase in hours due to a labor-supply shock leads to a fall in labor productivity, as the production capacity is fixed in the short-run and the economy operates along the decreasing marginal-product-of-labor schedule. We place a prior distribution on the slope of the short-run labor-demand curve and on the reduced-form VAR parameters and conduct Bayesian inference.

Second, we impose additional restrictions by estimating a fully-specified dynamic stochastic general equilibrium model (DSGE). The DSGE model potentially yields a more precise estimate of the relative importance of labor supply shifts. We consider an aggregate home production model (Benhabib, Rogerson, and Wright, 1991, and Greenwood and Hercowitz, 1991)) in which labor-supply shifts are caused by the stochastic variation in home production technology.

The main empirical findings can be summarized as follows. Based on the VAR variance decomposition, temporary shifts in labor supply are an important source of hours fluctuations. They account for about 30 percent of the cyclical variation of hours worked. The DSGE model attributes more than 50 percent of the variation of hours to temporary labor-supply shifts. This larger estimate, however, may partly be due to misspecified over-identifying restrictions as the time series fit of the DSGE model is significantly worse than the VAR fit. According to both VAR and DSGE models, labor-supply shocks are less important for aggregate output as they explain only about 15 percent of its variation at business cycle frequencies.

Our estimates of the contribution of labor-supply shifts to economic fluctuaΩtions are somewhat smaller than those reported by Shapiro and Watson (1988) and Hall (1997). Shapiro and Watson (1988) identify labor-supply shocks through the stochastic trend in hours worked. While the empirical evidence on the stationarity of hours worked is not conclusive, we assume hours are stationary, which is conΩsistent with a large class of dynamic equilibrium models. In Hall (1997), Parkin (1988), and Baxter and King (1991), labor-supply shocks (or preference shocks) are identified as deviations from the optimality condition associated with the labor supply of competitive households. However, these residuals also reflect the extent to which a representative-agent model is inconsistent with aggregate hours and wages, potentially leading to a bigger estimate of labor-supply shifts. While we exploit the labor-market equilibrium as Hall, our identification scheme does not rely on a speΩcific form of households' preferences. Instead, it is based on the firms' production technology through the slope of the marginal-product-of-labor schedule.

The paper is organized as follows. Section 2 develops the VAR identification scheme. The home production model is presented in Section 3. Section 4 discusses the econometric estimation and inference. The empirical findings are summarized in Section 5 and Section 6 provides a conclusion.

2 A VAR Model of Labor Market Fluctuations

In this section we describe our identification scheme for the VAR using a labor demand and supply framework. The labor-market fluctuations are viewed as a series of equilibria generated by competitive households and firms whose tastes and technologies are perturbed by stochastic disturbances. To identify the sources of fluctuations we will fit a VAR and a DSGE model (specifically, an aggregate home production model) to three macroeconomic time series: hours worked, labor productivity, and expenditure on consumer durable goods. Expenditure on consumer durables serves as a proxy for the household's permanent income. In the context of the home production model, it represents the investment in home capital.

2.1 Identifying Assumptions

During the past four decades, labor productivity, spending on consumer durables, and aggregate output exhibited a pronounced trend, whereas aggregate hours and the consumption share did not show an apparent trend. Based on this observation, many dynamic macroeconomic models have been designed to evolve along a balanced growth path. A common stochastic trend in output, consumption, investment, capital, and labor productivity is induced by a labor augmenting technology and hours worked are stationary around this path. The VAR innovations are decomposed into three orthogonal shocks, denoted by $\epsilon_{a,t}$, $\epsilon_{b,t}$, and $\epsilon_{z,t}$.

Assumption 1 *The shock $\epsilon_{z,t}$ has a permanent effect on labor productivity and consumer durables whereas it has no effect on hours in the long run. The shocks $\epsilon_{a,t}$ and $\epsilon_{b,t}$ have transitory effects on hours, labor productivity, and consumer durables.*□

The shock $\epsilon_{z,t}$ induces a common stochastic trend, and it is subsequently interpreted as permanent technology shock in a DSGE model.

At time t, the competitive firms' inverse labor demand can be written as:

$$W_t = MPL_t = \varphi^D(L_{m,t}|K_{m,t}, S_t), \tag{1}$$

where W_t represents the real wage rate, MPL_t the marginal product of labor, $L_{m,t}$ hours employed, $K_{m,t}$ capital stock, and S_t state of technology at time t.[1] Similarly, the inverse labor supply of a competitive household is:

$$W_t = \varphi^S(L_{m,t}|\Omega(S_t, T_t)), \tag{2}$$

where Ω_t represents endogenous variables that influence the labor supply (e.g., real interest rate, consumption, and wealth) and T_t the exogenous stochastic component of tastes.

[1]Subscripts m and h, respectively, denote the market and home sector, consistent with the DSGE model introduced in Section 3.

Assumption 2 *The shock $\epsilon_{b,t}$ has a contemporaneous impact on T_t, but not on S_t and $K_{m,t}$. Thus, upon impact this shock shifts the labor-supply curve, but not the labor-demand curve (marginal-product-of-labor schedule).* □

We will interpret the shock $\epsilon_{b,t}$ broadly as a labor-supply shock, such as an unanticipated shift of the preference for leisure or the productivity of non-market activities. The capital stock $K_{m,t}$ is inherited from the previous period and therefore not affected by the current period's labor-supply shock. Given that production capacity is fixed in the short-run, firms operate along the downward sloping marginal-product-of-labor schedule.

The responses of the marginal product of labor and hours worked (both in logs) to a labor-supply shock $\epsilon_{b,t}$ have to satisfy the following relationship

$$\frac{\partial \ln MPL_t}{\partial \epsilon_{b,t}} = \left(\frac{1}{\varphi^D} \cdot \frac{\partial \varphi^D}{\partial \ln L_{m,t}} \right) \cdot \frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}}. \tag{3}$$

The factor in parentheses is the slope of the inverse labor demand function. For example, under a Cobb-Douglas production technology with labor share α one obtains:

$$\frac{\partial \ln P_t}{\partial \epsilon_{b,t}} = (\alpha - 1) \frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}}, \tag{4}$$

where $P_t = MPL_t$ is the labor productivity. Roughly speaking, conditional on the slope of the labor demand function, it is possible to identify $\epsilon_{b,t}$ through its joint effect on hours and productivity. The slope of the labor demand schedule itself, however, is imposed and not identifiable. Finally, $\epsilon_{a,t}$, namely a temporary labor demand shock, is identified by assuming that the three structural shocks are orthogonal.[2].

2.2 VAR Specification

Define the vector of stationary variables $\Delta y_t = [\Delta \ln P_t, \Delta \ln I_{h,t}, \ln L_{m,t}]'$. Moreover, let $\epsilon_t = [\epsilon_{z,t}, \epsilon_{a,t}, \epsilon_{b,t}]'$. The VAR can be expressed in vector error correction form as

$$\Delta y_t = \Phi_0 + \Phi_{vec} y_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta y_{t-i} + u_t, \quad u_t \sim iid\, \mathcal{N}(0, \Sigma_u). \tag{5}$$

The reduced form disturbances u_t are related to the structural disturbances ϵ_t by $u_t = \Phi_* \tilde{\epsilon}_t$, where $\tilde{\epsilon}_t$ is a standardized version of ϵ_t with unit variance.

According to Assumption 1, the shock $\epsilon_{z,t}$ generates a stochastic trend in productivity and expenditures on consumer durables. The two series are cointegrated with cointegration vector $\lambda = [1, -\lambda_{21}, 0]'$. Instead of restricting λ_{21} to one we estimate the parameter in the VAR analysis to allow for a possibly steeper Engle curve for expenditures on durable goods. We do not impose a cointegration relationship between the cumulative hours of work ($\sum_{\tau=0}^{t} L_{m,t}$) and labor productivity or consumer durables. Hence, the rank of $\Phi_{vec} = \mu\lambda'$ is one, which is in fact confirmed by a formal selection based on Bayesian posterior odds. The stochastic trend of y_t has

[2] Our analysis does not consider other disturbances such as monetary and fiscal policy shocks. For post-war U.S. data, government policy shocks are often considered to be of secondary importance in business-cycle analysis. For example, according to King, Plosser, Stock and Watson (1991), permanent nominal shocks account for only a small variation of real variables. The cyclical components of government spending is not highly correlated with output measures – it is less than 0.2 for Hodrick-Prescott filtered data. Also, expanding the list of shocks often invites arbitrary identifying restrictions in the VAR analysis.

the form $C_{LR}\Phi_* \sum_{\tau=0}^{t} \tilde{\epsilon}_t$. Since productivity and consumption expenditures have a common trend, the first two rows of the 3×3 long-run multiplier matrix C_{LR} are proportional.

The structural shocks $\tilde{\epsilon}_t$ are identifiable if the elements of the 3×3 matrix Φ_* can be uniquely determined based on $\Phi_0, \ldots, \Phi_p$, Φ_{vec}, and Σ_u. Let Ψ_* denote the unique lower triangular Cholesky factor of Σ_u. Any matrix Φ_* such that $\Phi_*\Phi_*' = \Sigma_u$ is an orthonormal transformation of Ψ_*, that is, $\Phi_* = \Psi_* B$ for some orthonormal matrix B. Let $[A]_{ij}$ denote the i'th row and j'th column of a matrix A. According to Assumption 1, the shocks $\epsilon_{a,t}$ and $\epsilon_{b,t}$ only have transitory effects on productivity and consumer expenditures. Thus, the elements $[(C_{LR}\Psi_*)B]_{12}$ and $[(C_{LR}\Psi_*)B]_{13}$ have to be zero. The contemporary effects of the labor-supply shock $\epsilon_{b,t}$ on productivity and hours worked are given by $\partial P_t/\partial\epsilon_{b,t} = [\Phi_*]_{13}$ and $\partial L_{m,t}/\partial\epsilon_{b,t} = [\Phi_*]_{33}$. Define $C_* = [1, 0, -(\alpha - 1)]$. According to Assumption 2 and Equation (4) the value of $[(C_*\Psi_*)B]_{13}$ has to be zero. These three orthogonality conditions uniquely determine the orthonormal transformation B.

3 A Fully Specified Model Economy

The DSGE model provides a more specific interpretation of the three structural shocks and their propagation. It also assists the understanding of the economic intuition behind our identification scheme. The model economy consists of identical, infinitely lived households who maximize the expected discounted lifetime utility defined over consumption C_t and pure leisure

$$\mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}(\log C_s+\kappa\log(1-L_{m,s}-L_{h,s}))\right],$$

where $L_{m,t}$ is the fraction of time supplied to the labor market and $L_{h,t}$ is hours spent on home production. $\mathbb{E}_t$ is the time t conditional expectation and β is the discount factor. Consumption is an aggregate of market consumption $C_{m,t}$ and the consumption of home produced goods $C_{h,t}$:

$$C(C_{m,t},C_{h,t})=[\chi C_{m,t}^{\frac{\upsilon-1}{\upsilon}}+(1-\chi)C_{h,t}^{\frac{\upsilon-1}{\upsilon}}]^{\frac{\upsilon}{\upsilon-1}}, \tag{6}$$

The substitution elasticity υ captures the households' willingness to substitute market and home-produced goods. The home production technology exhibits constant-returns-to-scale in home capital $K_{h,t}$ and labor $L_{h,t}$:

$$C_{h,t}=[\psi(X_{h,t}L_{h,t})^{\frac{\tau-1}{\tau}}+(1-\psi)K_{h,t}^{\frac{\tau-1}{\tau}}]^{\frac{\tau}{\tau-1}}, \tag{7}$$

where $X_{h,t}$ is a labor-augmenting home productivity process. We do not restrict the home technology to a Cobb-Douglas function (unlike the market technology below) because some consumer durable goods (e.g., dishwashers and microwaves) are substitutes for time, whereas others (e.g., DVD players) are complements. Our specification reduces to a separable-in-logs utility if $\nu=\tau=1$. The households own $K_{h,t}$ and the market capital stock $K_{m,t}$; and their budget constraint is:

$$C_{m,t}+I_{m,t}+I_{h,t}=W_tL_{m,t}+R_tK_{m,t}, \tag{8}$$

where R_t is the rental rate of market capital and $I_{m,t}$ and $I_{h,t}$ are capital investments. To avoid unreasonably volatile investments in a multi-sector model, capital accumulation is subject to a convex adjustment cost as in Baxter (1996):

$$K_{j,t+1} = \phi(I_{j,t}/K_{j,t})K_{j,t} + (1-\delta)K_{j,t}, \quad j = h, m \tag{9}$$

where δ is the depreciation rate of capital and $\phi' > 0, \phi'' \leq 0$.

The representative firm produces the market output Y_t according to a Cobb-Douglas technology in market capital and labor and maximizes the profit each period:

$$\max_{L_{m,t}, K_{m,t}} \; K_{m,t}^{1-\alpha}(X_{m,t}L_{m,t})^{\alpha} - W_t L_{m,t} - R_t K_{m,t}, \tag{10}$$

where $X_{m,t}$ represents a labor-augmenting market productivity process. The goods market equilibrium condition is:

$$Y_t = C_{m,t} + I_{m,t} + I_{h,t}. \tag{11}$$

Market and home productivity are, respectively, $X_{m,t} = \exp[z_t + a_t]$ and $X_{h,t} = \exp[z_t + b_t]$, where z_t represents a common trend that follows a random walk with drift:

$$z_t = \gamma + z_{t-1} + \epsilon_{z,t}. \tag{12}$$

The temporary components, a_t and b_t, follow stationary first-order autoregressions:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t}, \quad b_t = \rho_b b_{t-1} + \epsilon_{b,t}. \tag{13}$$

We assume that $\epsilon_t = [\epsilon_{z,t}, \epsilon_{a,t}, \epsilon_{b,t}]'$ is serially uncorrelated with diagonal covariance matrix Σ_ϵ. The diagonal elements are denoted by σ_z^2, σ_a^2, and σ_b^2, respectively.

As the VAR in Section 2, the log-linearized DSGE model provides a probabilistic representation for $\Delta y_t = [\Delta \ln P_t, \Delta \ln I_{h,t}, L_{m,t}]'$. The model economy also satisfies all the identifying assumption of the VAR described above. Moreover, the DSGE model imposes additional restrictions which potentially yield more precise estimates of variance decompositions and impulse responses.

4 Econometric Approach

The goal of the econometric analysis is to assess the relative importance of labor-supply shocks for the cyclical variation of output and hours.[3] The VAR is denoted by $\mathcal{M}_0$ and the over-identified log-linearized DSGE model by $\mathcal{M}_1$. To be consistent with the Cobb-Douglas production technology used in the DSGE model, we will assume that under the VAR specification the slope of the inverse labor demand function is also $\alpha - 1$. Hence, the parameter α appears in both $\mathcal{M}_0$ and $\mathcal{M}_1$. The parameters of model $\mathcal{M}_i$, except for α, are stacked in the vector $\theta_{(i)}$, $i = 0, 1$. $\theta_{(0)}$ contains the cointegration parameter λ_{12} and the non-redundant elements of the reduced-form matrices $\Phi_0, \dots, \Phi_p, \Sigma_u$ in Equation (5).

Variance decompositions and truncated impulse response functions, denoted by the $m \times 1$ vector φ, are transformations of the parameters $\theta_{(i)}$ and α, that is, $\varphi = \tilde{\varphi}_i(\theta_{(i)}, \alpha)$. Under both $\mathcal{M}_0$ and $\mathcal{M}_1$ the vector process Δy_t has a moving-average (MA) representation in terms of the standardized structural shocks $\tilde{\epsilon}_t$:

$$\Delta y_t = \mu(\theta_{(i)}, \alpha) + \sum_{j=0}^{\infty} C_j(\theta_{(i)}, \alpha)\tilde{\epsilon}_{t-j}. \tag{14}$$

[3] A Technical Appendix that summarizes the computational details is available from authors upon request.

The population mean μ and the moving average coefficients C_j are model-specific functions of $\theta_{(i)}$ and α. Define the vectors $M_z = [1,0,0]'$, $M_a = [0,1,0]'$, and $M_b = [0,0,1]'$. The impulse responses to the shock $\tilde{\epsilon}_{s,t}$ are given by $C_j M_s$.

The h-th order autocovariance matrix of Δy_t can be decomposed into the contributions of the three structural shocks:

$$\Gamma_{\Delta y}(h) = \sum_{s\in\{z,a,b\}} \Gamma^{(s)}_{\Delta y}(h) = \sum_{s\in\{z,a,b\}} \sum_{j=\max\{0,-h\}}^{\infty} C_j M_k M_k' C_{j+h}'. \tag{15}$$

The relative contribution of shock s to the unconditional variance of the j'th element of Δy_t is given by the ratio $[\Gamma^{(s)}_{\Delta y}(0)]_{jj}/[\Gamma_{\Delta y}(0)]_{jj}$. The spectrum of the stationary process Δy_t is

$$S_{\Delta y}(\omega) = \sum_{s\in\{z,a,b\}} S^{(s)}_{\Delta y}(\omega) = \sum_{s\in\{z,a,b\}} \sum_{h=-\infty}^{\infty} \Gamma^{(s)}_{\Delta y}(h) e^{-ih\omega} \tag{16}$$

and represents the contribution of frequency ω to the variance of Δy_t. To assess the relative importance of the three shocks at business cycle frequencies we consider the decomposition of $\int_{\underline{\omega}}^{\overline{\omega}} S_{\Delta y}(\omega)d\omega$, where $\underline{\omega}$ and $\overline{\omega}$ correspond to cycles of 32 and 6 quarters, respectively.[4]

The likelihood functions for the two models are denoted by $p(Y_T|\theta_{(i)}, \alpha, \mathcal{M}_i)$.

[4]According to $\mathcal{M}_0$ and $\mathcal{M}_1$ the level of output is integrated of order one and its autocovariances do not exist. Let $S^{(s)}_{\Delta y}(\omega)$ denote the three components of the spectrum of output growth. We define the spectrum of the level of output at frequencies $\omega > 0$ as

$$S^{(s)}_{y}(\omega) = \lim_{\phi\to 1} \frac{S^{(s)}_{\Delta y}(\omega)}{1+\phi^2 - 2\phi cos(\omega)}. \tag{17}$$

The term $1/[1+\phi^2 - 2\phi cos(\omega)]$ is the power transfer function of the AR(1) filter $[1-\phi L]^{-1}$, where L denotes the temporal lag operator.

We adopt a Bayesian approach and place a prior distribution with density

$$p(\theta_{(i)}, \alpha|\mathcal{M}_i) = p(\theta_{(i)}|\mathcal{M}_i)p(\alpha), \quad i = 0, 1 \tag{18}$$

on the parameters. Equation (18) reflects the assumption that α is *a priori* independent of $\theta_{(0)}$ and $\theta_{(1)}$. Moreover, the prior distribution of α is the same for both models. According to Bayes Theorem the posterior density of the parameters is proportional ($\propto$) to

$$p(\theta_{(i)}, \alpha|Y_T, \mathcal{M}_i) \propto p(Y_T|\theta_{(i)}, \alpha, \mathcal{M}_i)p(\theta_{(i)}|\mathcal{M}_i)p(\alpha). \tag{19}$$

The likelihood function of the VAR is uninformative about the slope of the inverse demand schedule $\alpha - 1$ and depends only on the reduced form parameters $\theta_{(0)}$:

$$p(Y_T|\theta_{(0)}, \alpha, \mathcal{M}_0) = \tilde{p}(Y_T|\theta_{(0)}, \mathcal{M}_0). \tag{20}$$

Straightforward manipulations using Bayes Theorem can be used to verify that the VAR posterior is the product of the posterior density of the identifiable reduced-form parameters obtained from $\tilde{p}(Y_T|\theta_{(0)}, \mathcal{M}_0)$ and the prior density of α:[5]

$$p(\theta_{(0)}, \alpha|Y_T, \mathcal{M}_0) = \tilde{p}(\theta_{(0)}|Y_T, \mathcal{M}_0)p(\alpha). \tag{21}$$

Since $\varphi = \tilde{\varphi}_i(\theta_{(i)}, \alpha)$, Equations (18) and (19) implicitly determine the prior and posterior of variance decompositions and impulse response functions. Rather than attempting to specify a prior on φ directly, as in Gordon and Boccanfuso (2001), we

[5]Our VAR based inference is a specific example of Bayesian analysis of a nonidentified econometric model. Poirier (1998) provides a comprehensive survey and many additional examples.

use economic intuition derived from assumptions on aggregate preferences, production technologies, and equilibrium relationships to specify the prior for φ indirectly. Since the distribution of reduced-form parameters $\theta_{(0)}$ is updated based on the sample information Y_T, the implied distribution of φ is updated with every observation and we learn about the relative importance of structural shocks and the response of the economy.

Our method is explicit about the direction of the parameter space in which learning does not occur. If the dimension of the nonidentifiable component of the parameter vector is low, as in our application, we can assess the robustness of our conclusion by tracing out, for instance, the relative importance of the labor-supply shock as a function of α. A similar approach was used by King and Watson (1992) who plotted their statistics of interest against a one-dimensional variable indexing VAR identification schemes.

The VAR identification proposed in this paper is based on the notion that productivity and hours worked move in opposite directions in response to a labor-supply shock. Equation (4) can be qualitatively interpreted as an inequality restriction on the impulse responses:

$$\frac{\partial \ln P_t}{\partial \epsilon_{b,t}} > 0 \quad \text{and} \quad \frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}} < 0 \tag{22}$$

Canova and DeNicolo (2002), Faust (1998), and Uhlig (2003) develop identification and inference procedures based on such inequality constraints. Our approach places a prior distribution on identification schemes that are consistent with (22) and averages the posterior distribution of population characteristics φ over *a priori* likely values of the unidentifiable parameter α that indexes the identification schemes.

5 Empirical Analysis

Both VAR and DSGE models are fitted to post-war quarterly U.S. data on labor productivity, expenditure on consumer durables, and hours worked.[6] The sample period ranges from 1964:I to 1997:IV. The overall sample size is $T = 136$ and the first $T_* = 20$ observations are used as training sample to initialize lags and parameterize the prior distributions.

5.1 Priors

The prior distribution used in the estimation of the DSGE model is summarized in columns 3 to 5 of Table 1. The shapes of the densities are chosen to match the domain of the structural parameters. The prior means for labor share, discount rate, productivity growth, capital depreciation, and the steady state ratio of home to market investment are respectively $\bar{\alpha} = 0.666$, $\bar{\beta} = 0.993$, $\bar{\gamma} = 0.004$, $\bar{\delta} = 0.025$, and $\bar{I_h}/\bar{I_m} = 0.7$. These values can be justified based on the training sample and

[6]Real gross domestic product (GDPQ), consumption of consumer durables (GCDQ), employed civilian labor force (LHEM), civilian noninstitutional population 20 years and older (PM20 and PF20) are extracted from the DRI·WEFA database. Define $POPQ = 1E6 * (PF20 + PM20)$, $Y_t = GDPQ/POPQ$ and $I_{h,t} = GCDQ/POPQ$. Average weekly hours, private non-agricultural establishments (EEU00500005) are obtained from the Bureau of Labor Statistics. Annual hours worked at monthly frequency are $L_{m,t} = 52 * EEU00500005 * LHEM \ / \ POPQ$ and converted to quarterly frequency by simple averaging. Labor productivity is $P_t = Y_t/L_{m,t}$.

are commonly used in the literature. Hence, we use fairly small standard deviations for the distributions of these parameters. Prior means for the steady state hours, $\bar{L}_m = 0.33$ and $\bar{L}_h = 0.25$, are obtained from the Michigan Time Use Survey. A larger standard deviation is allowed for L_h, as the hours spent on home work may be measured with a greater uncertainty.

We allow for large standard deviations in the prior distributions of home technology parameters because they are not easy to infer. The prior means for the substitution elasticities, $\bar{\nu} = 1$ and $\bar{\tau} = 1$, correspond to a one-sector model with separable-in-logs utility. The prior mean of the labor share ψ in the home production function is also set to 0.666. The weight on leisure χ in the utility function is implicitly determined by the other parameters. The steady state adjustment costs are assumed to be zero and the elasticity of the investment/capital ratio with respect to Tobin's q, $\eta = (|(I^*/K^*)\phi''/\phi'|^{-1})$ is estimated. The prior mean for η is 100, implying a small adjustment cost, with a large standard deviation of 100. We use diffuse priors for the exogenous technology processes a_t, b_t, and z_t. Finally, we introduce two additional parameters ξ_1 and ξ_2 to adjust the normalization of total hours to one and to capture the average growth rate differential between labor productivity and home investment in the data. The structural parameters are assumed to be *a priori* independent of each other.

The training-sample is used to construct a conjugate prior for the VAR parameters conditional on the cointegration parameter λ_{21}. While the DSGE model implies that $\lambda_{21} = 1$, we relax that restriction and choose the prior $\lambda_{21} \sim \mathcal{N}(1, 0.025^2)$. The prior for α is the same as in the DSGE model analysis (Table 1). Posterior odds were used to select the lag-length $p = 2$.

5.2 Parameter Estimation and Time Series Fit

The posterior means and standard errors of the parameters of the DSGE model are reported in columns 6 and 7 of Table 1.[7] The estimated substitution elasticity between market goods and home goods, υ, is 2.302, slightly higher than those of Rupert, Rogerson, and Wright (1995) and McGrattan, Rogerson, and Wright (1997). The substitution elasticity between capital and labor in home production, τ, is 2.446 suggesting that goods and time are substitutes in home production activity. The estimated labor share ψ in the home technology is 0.753 and the fraction of hours spent on home production activity L_h is 0.170. The temporary home production shock is somewhat more persistent than the market shock: $\hat{\rho}_a = 0.745$ and $\hat{\rho}_b = 0.865$. The 90 percent posterior confidence interval for the correlation (conditional on time $t-1$ information) between market productivity $\ln X_{m,t}$ and home productivity $\ln X_{h,t}$ ranges from 0.18 to 0.37, somewhat lower than the values used in the literature (e.g., 0.67 in Benhabib, Rogerson and Wright, 1991 and 1 in Greenwood and Hercowitz, 1991). Finally, the adjustment cost parameter estimate $\hat{\eta}$ is 30.70, implying a small adjustment cost in capital accumulation.

To assess the relative time series fit of the VAR and the DSGE model we compute

[7] While McGrattan, Rogerson, and Wright (1997) also estimate home production models based on aggregate time series, our analysis distinguishes itself from theirs in several dimensions. First, we focus on the comparison to a structural VAR, particularly, the variance decomposition. Second, microeconomic evidence is incorporated through prior distributions in our Bayesian estimation. Third, we are able to uncover the comovement of innovations to market and home productivity.

marginal data densities

$$p(Y_T|\mathcal{M}_i) = \int p(Y_T|\theta_{(i)}, \alpha, \mathcal{M}_i) p(\theta_{(i)}, \alpha|\mathcal{M}_i) d(\theta_{(i)}, \alpha) \tag{23}$$

conditional on the training sample 1964:I to 1968:IV. The log-marginal data density can be interpreted as a measure of one-step-ahead predictive performance $\ln p(Y_T|\mathcal{M}_i) = \sum_{t=T_*}^{T} p(y_t|Y_t, \mathcal{M}_i)$. The values are $\ln p(Y_T|\mathcal{M}_0) = 1087.2$ for the VAR and $\ln p(Y_T|\mathcal{M}_1) = 999.6$ for the DSGE model, implying that for a wide range of prior probabilities the posterior probability of the DSGE model is essentially zero.[8] To shed more light on the poor fit of the DSGE model, we computed in-sample, root-mean-squared-errors (RMSE) at the posterior mode estimates. The RMSE's for the growth rates of output and consumer durable expenditures are very similar for the two models, whereas the RMSE of hours is substantially higher for the DSGE model: 0.0080 versus 0.0059 for the VAR.

5.3 Variance Decompositions and Impulse Responses

Our main interest is to unveil the sources of cyclical variation in hours and output. Table 2 presents the variance decomposition (posterior means and 90 percent confidence intervals) for output and hours at business cycle frequencies, namely cycles of 6 to 32 quarters. Labor-supply shifts play an important role for the fluctuations of hours. The shock ϵ_b accounts for almost 30 percent (posterior mean) of the fluctuations in hours according to the VAR, and more than half according to the DSGE model. The relative contribution of the labor-supply shocks to output fluctuations

[8] The posterior probability of model $\mathcal{M}_i$ is $\pi_{i,T} = \frac{\pi_{i,0} p(Y_T|\mathcal{M}_i)}{\sum_{j=0,1} \pi_{j,0} p(Y_T|\mathcal{M}_j)}$, where $\pi_{i,0}$ is its prior probability.

is somewhat small albeit non-negligible; they account for about 15 percent of the output variation.

The posterior confidence intervals indicate that the VAR model based variance decompositions are associated with more posterior uncertainty than the DSGE decompositions. For instance, the 90 percent confidence intervals for the contribution of $\epsilon_{b,t}$ to hours fluctuations are $[0, 0.277]$ (VAR) and $[0.083, 0.186]$ (DSGE), respectively. As pointed out by Faust and Leeper (1997), VAR variance decompositions based on long-run restrictions are associated with a high degree of uncertainty. Conditional on the over-identifying restrictions embodied in the DSGE model, however, one obtains fairly sharp estimates.

While VAR and DSGE model analysis broadly agree upon the decomposition of output fluctuations, there are discrepancies in terms of the variance decomposition of hours. According to the marginal data densities and the RMSEs, the time series fit of the VAR is much better than the fit of the DSGE model, in particular in terms of hours worked. Hence, the DSGE-based decomposition probably overestimates the contribution of labor-supply shocks to the fluctuations of hours. The finding that the DSGE model analysis attributes only about 7 percent of the hours fluctuation to the permanent technology shock $\epsilon_{z,t}$ is partly due to the balanced-growth-path property of this class of models; common technology shocks tend to shift both labor demand and supply in a similar magnitude, leaving hours almost unaffected.

The VAR-based point estimates and confidence intervals reported in rows 1-3 and 10-12 of Table 2 were computed based on the reduced form parameters $\theta_{(0)}$ and the slope of the labor demand schedule, $\alpha - 1$. Conditional on the VAR, because the data provide no information about α, the inference is potentially sensitive to the choice of the prior $p(\alpha)$. Moreover, our identifying assumption for labor-supply shocks exploits the notion that the production capacity is fixed in the short run and that there exists a stable relationship between labor productivity and hours employed. This premise may be violated if firms heavily rely on the factor utilization. For example, if there is a significant variation in the level workers' effort (e.g., labor hoarding in Burnside, Eichenbaum, and Rebelo, 1993) in the face of exogenous shifts in labor supply, our identifying restriction is no longer valid. However, if the capital utilization is time-varying and the cost of intense utilization results in a faster depreciation of capital, our identifying restriction is still appropriate. In this case, the restriction has to be modified to accommodate the systematic capital utilization; the slope of the labor demand schedule is smaller in absolute value than $\alpha - 1$ (see Appendix). To assess the robustness of our VAR-based analysis we also report the contribution of the labor-supply shock to output and hours variation conditional on $\alpha = 0.3$, 0.6, and 0.9 in Table 2. The share of ϵ_b lies between 23 and 38 percent for hours and 12 to 20 percent for output. As we move to a higher value of α, i.e., utilization of capital is less costly, the importance of the labor-supply shock is reinforced.

Overall, both the VAR and DSGE model analysis suggests that the labor-supply shocks play an important role for economic fluctuations, especially for hours worked. Our estimates are, however, smaller than the previous estimates. Shaprio and Watson (1988) find that 60 percent of the cyclical variation in hours is due to permanent shifts in labor supply. In Shapiro and Watson, labor-supply shocks are identified by the stochastic trends in hours worked. While the empirical evidence on the stationarity of hours is not conclusive, we assume hours to be stationary. This is consistent with a large class of DSGE models. Hall (1997) attributes almost the entire variation of hours to preference shocks, which he identifies as deviations from the optimality condition associated with the labor supply of a competitive household. Hall's finding is broadly in line with our DSGE model analysis as both approaches impose more restrictions on the short-run labor market equilibrium and, as a result, require bigger shifts in labor supply. However, as mentioned above, the weak time series fit of the DSGE model indicates that some of these over-identifying restrictions are potentially misspecified and that the corresponding findings have to be interpreted with caution. Our VAR identification scheme does not rely on a specific form of household's preference. Instead, it is based on the firm's production technology through the slope of marginal-product-of-labor schedule.

To assess whether the structural shocks identified from the VAR conform with our economic intuition, Figure 1 depicts one-standard-deviation impulse responses of labor productivity, expenditure on consumer durable goods (investment in home capital), and market hours to the three structural shocks.[9] The graphs show the responses from the DSGE model (solid lines) and those from the VAR along with the 90 percent confidence interval (dotted lines). Looking at the first row, in response to a permanent shock, labor productivity both in the DSGE model and VAR approach the new steady state at a similar pace. Spending on consumer durables also

[9]The signs of the responses are normalized as follows: the initial responses of productivity to $\epsilon_{z,t}$, productivity to $\epsilon_{a,t}$, and hours to $\epsilon_{b,t}$ are positive, positive, and negative, respectively.

increases permanently. Market hours rises instantaneously and slowly returns to its steady state in the DSGE model, whereas its response is hump-shaped in the VAR. The VAR responses to a temporary market productivity shock (second row of the Figure) closely traces those from the DSGE model except for the delayed response of hours in the VAR. Finally, in response to a temporary home productivity shock, labor productivity rises in both the VAR and DSGE model. Home investment rises immediately in the DSGE model whereas it exhibits a slow hump-shape response. In sum, the VAR responses, by and large, conform with the economic intuition provided by the DSGE model. However, hours responses are delayed in the VAR for about 2 quarters, and the short-run dynamics of expenditure on consumer durables are somewhat different from the DSGE model prediction.

Based on the competitive labor market equilibrium, we identify exogenous shifts in labor supply. Yet the proposed identification scheme allows a more general – also an alternative – interpretation than labor-supply shocks. As an illustrative example, consider a model economy with sticky prices where firms have to produce goods to meet their demand. In this economy, the labor demand is no longer a simple reflection of the marginal product of labor. It is instead jointly determined by the demand for goods and the output-labor elasticity from the production technology. Suppose now there is an increase in the demand for goods that is not caused by a productivity shift. This will lead to an increase in the demand for labor at a given level of production capacity. The joint behavior of labor productivity and hours is still dictated by the marginal-product-of-labor schedule.[10]

[10]In this event, the real wage will increase given the upward sloping labor supply curve. However,

5.4 Evolution of Latent Technology Processes

According to the home production model, recessions may occur because agents find it optimal to allocate more time in non-market activities. In our DSGE model the attractiveness of non-market activity is measured by the latent home technology process.[11] We plot three technology indices in Figure 2 together with the NBER business cycle peaks and troughs. All five recessions during the sample period are associated with low levels of market productivity. Two business cycle troughs, March 1975 and November 1982, coincide with unusually high productivity of non-market activities. The strong interpretation of this finding is that unusually high non-market productivity or preference shift has contributed to a low employment and output. A weaker interpretation is that the economic downturns in March 1975 and November 1982 cannot be solely explained by an adverse technology shock in the market.

6 Conclusion

We investigate the sources of economic fluctuations in the context of a dynamic general equilibrium. A new VAR identification scheme which distinguishes between shifts of and movements along the marginal product of labor is proposed to identify

labor productivity falls as employed hours increases, justifying our use of labor productivity instead of a wage series under this more general interpretation.

[11] Ingram, Kocherlakota and Savin (1997) recover non-market variables (non-market consumption, non-market hours, and leisure) based on the households' optimality conditions and times series of market variables, whereas we obtain the time series of the latent home technology.

three structural disturbances: temporary labor-supply shifts, temporary labor demand shifts, and permanent productivity shocks that eventually move both demand and supply. According to the variance decomposition from the VAR, the labor-supply shifts are an important driving force of the cyclical fluctuation of hours, as they account for about 30 percent of the variation. For output fluctuations at business cycle frequencies, the role of labor-supply shifts is modest as their relative contribution is about 15 percent. To assess the importance of labor-supply shifts in the context of an equilibrium model, a home-production model with stochastic variation in non-market technology is estimated. While the DSGE model based decomposition of output resembles the VAR results, the structural model attributes a higher fraction of hours fluctuations to the labor-supply shifts than the VAR. This result, however, is partly due to misspecified over-identifying restrictions of the DSGE model.

Appendix: Labor Demand with Variable Capital Utilization

Consider a Cobb-Douglas production function with inputs in capital services and hours:

$$Y_t = (u_t K_{m,t})^{1-\alpha}(X_{m,t}L_{m,t})^{\alpha},$$

where u_t represents the utilization of the capital stock. Suppose the intensive use of capital results in a fast depreciation. At the cost of a more complicated notation, we could work with an alternative decentralization scheme in which firms make decisions on accumulation. However, since both decentralizations are essentially identical, as in the main text, suppose the firm rents the capital from the households. Yet the firm has to compensate households for faster depreciation when the capital is utilized more intensively:

$$\max_{L_{m,t},K_{m,t},u_t} (u_t K_{m,t})^{1-\alpha}(X_{m,t}L_{m,t})^{\alpha} - W_t L_{m,t} - (R_t + \delta(u_t))K_{m,t}.$$

For illustrative purposes, assume that the elasticity of depreciation is constant: $\delta(u_t) = \delta_0 \frac{u_t^{\lambda+1}}{\lambda+1}$, where $\lambda > 0$. As $\lambda \to \infty$, the utilization is held constant and the depreciation rate is fixed. The first order conditions of the profit maximization problem with respect to $L_{m,t}$ and u_t imply that the inverse labor demand schedule still depends on the predetermined capital stock and the market productivity shocks only. However, its slope changes:

$$\frac{\partial \ln W_t}{\partial \epsilon_{b,t}} = \mu(\alpha - 1)\frac{\partial \ln L_{m,t}}{\partial \epsilon_{b,t}}, \quad \mu = \frac{\lambda}{\lambda + \alpha} \leq 1.$$

Therefore, the proposed identification scheme is still valid but the slope of the labor demand schedule is smaller than in the constant utilization case, reflecting an extra margin for the firm to exploit.

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Table 1: Prior and Posterior Distribution for DSGE Model Parameters

Parameters		Prior			Posterior	
Name	Range	Density	Mean	S.D.	Mean	S.D.
α	[0,1]	Beta	0.666	0.020	0.741	0.018
β	[0,1]	Beta	0.993	0.002	0.979	0.003
γ	$\mathbb{R}$	Normal	0.004	0.0005	0.003	0.0004
δ	[0,1]	Beta	0.025	0.002	0.017	0.002
L_m	[0,1]	Beta	0.330	0.020	0.335	0.021
ρ_a	[0,1]	Beta	0.800	0.100	0.745	0.034
ρ_b	[0,1]	Beta	0.800	0.100	0.865	0.034
L_h	[0,1]	Beta	0.250	0.050	0.183	0.045
η	$\mathbb{R}^+$	Gamma	100.0	100.0	30.70	5.546
ψ	[0,1]	Beta	0.666	0.100	0.753	0.080
ν	$\mathbb{R}^+$	Gamma	1.000	2.000	2.302	0.388
I_h/I_m	$\mathbb{R}^+$	Gamma	0.700	0.020	0.686	0.020
τ	$\mathbb{R}^+$	Gamma	1.000	2.000	2.446	0.536
ξ_1	$\mathbb{R}$	Normal	2.960	1.000	3.145	0.006
ξ_2	$\mathbb{R}$	Normal	0.000	0.020	0.005	0.0003
σ_z	$\mathbb{R}^+$	InvGamma	0.01*	2.000*	0.007	0.001
σ_a	$\mathbb{R}^+$	InvGamma	0.01*	2.000*	0.009	0.001
σ_b	$\mathbb{R}^+$	InvGamma	0.015*	2.000*	0.021	0.007

Notes: For the Inverse Gamma (u, s) priors we report the parameters u and s. For $u = 2$ the standard deviation (S.D.) is infinite. The posterior moments are calculated from the output of the Metropolis algorithm.

Table 2: Variance Decomposition at Business Cycle Frequencies

Variable	Model	Shock	Mean	Conf. Interval	
$\ln L_{m,t}$	VAR	ϵ_z	0.205	[0.000	0.529]
	VAR	ϵ_a	0.510	[0.069	0.905]
	VAR	ϵ_b	0.285	[0.005	0.588]
	VAR ($\alpha = 0.3$)	ϵ_b	0.230	[0.006	0.454]
	VAR ($\alpha = 0.6$)	ϵ_b	0.268	[0.007	0.551]
	VAR ($\alpha = 0.9$)	ϵ_b	0.376	[0.021	0.738]
$\ln L_{m,t}$	DSGE	ϵ_z	0.066	[0.006	0.124]
	DSGE	ϵ_a	0.268	[0.132	0.403]
	DSGE	ϵ_b	0.666	[0.526	0.805]
$\ln Y_{m,t}$	VAR	ϵ_z	0.491	[0.089	0.911]
	VAR	ϵ_a	0.367	[0.000	0.703]
	VAR	ϵ_b	0.142	[0.000	0.277]
	VAR ($\alpha = 0.3$)	ϵ_b	0.118	[0.000	0.250]
	VAR ($\alpha = 0.6$)	ϵ_b	0.130	[0.000	0.256]
	VAR ($\alpha = 0.9$)	ϵ_b	0.204	[0.000	0.388]
$\ln Y_{m,t}$	DSGE	ϵ_z	0.448	[0.339	0.560]
	DSGE	ϵ_a	0.417	[0.311	0.519]
	DSGE	ϵ_b	0.135	[0.083	0.186]

Notes: Decomposition of aggregate output $\ln Y_t$ and market hours $\ln L_{m,t}$ at business cycle frequencies (6 to 32 quarters per cycle). The table reports posterior means and 90 percent confidence intervals. VAR ($\alpha = x$) signifies that α was fixed at the value x rather than integrated out with respect to its prior distribution.

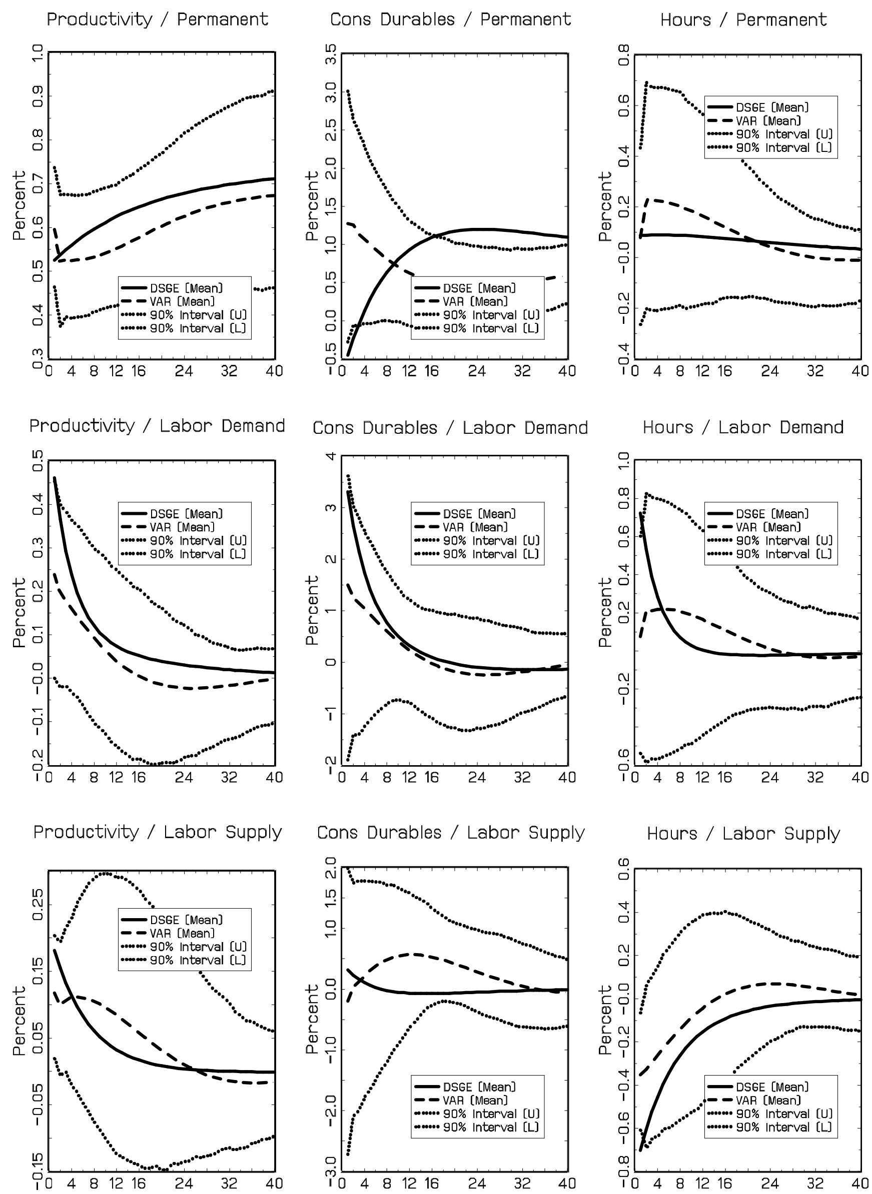

Figure 1: Impulse-response Functions

Notes: Figure depicts posterior mean responses for VAR (dashed) and DSGE model (solid). The dotted lines represent pointwise 90 percent Bayesian confidence intervals based on the VAR posterior.

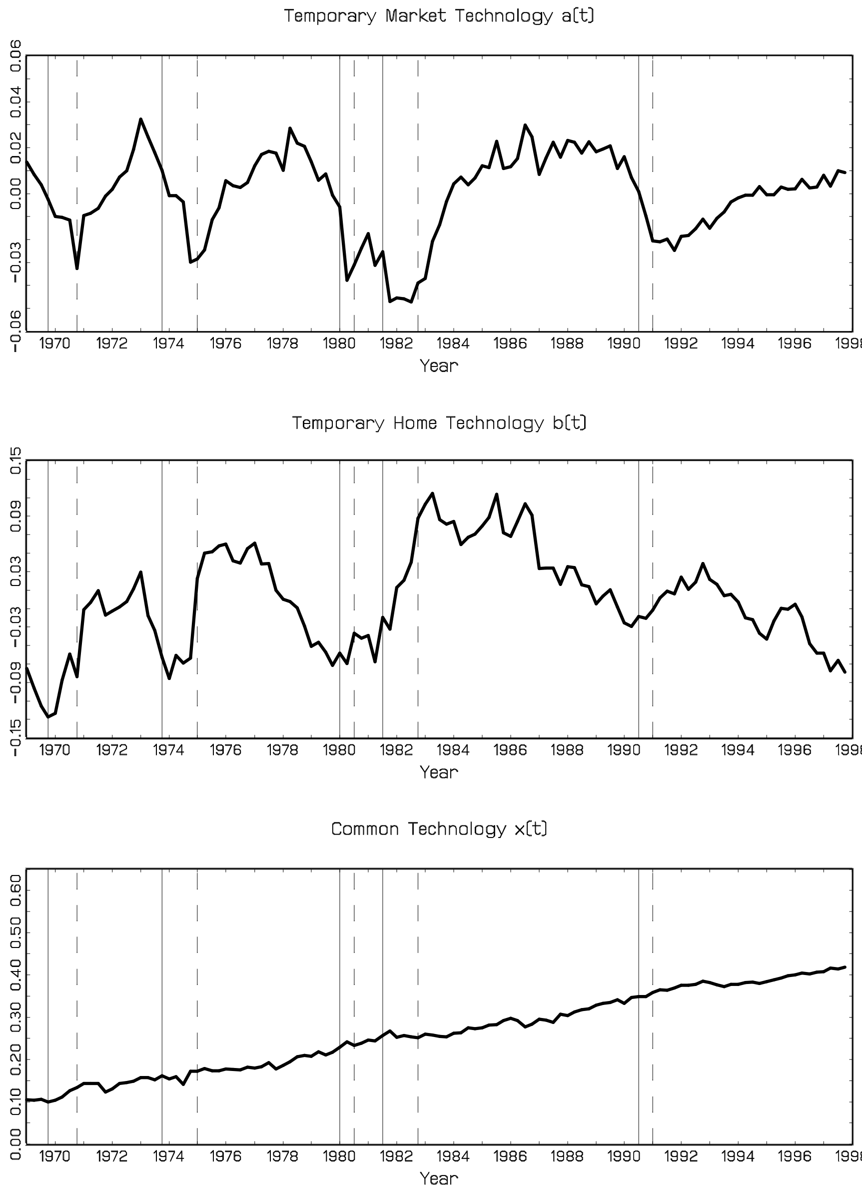

Figure 2: Filtered technology processes a_t, b_t, and x_t

Notes: The posterior mean estimates of the latent technology processes are based on the DSGE model. Solid vertical lines correspond to business cycle peaks, dashed lines denote business cycle troughs (NBER Business Cycle Dating).